The Evolution of the World Trade Web. A Weighted-Network Analysis.

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Abstract

This paper employs a weighted network approach to study the empirical properties of the web of trade relationships among world countries, and its evolution over time. We show that most countries are characterized by weak trade links; yet, there exists a group of countries featuring a large number of strong relationships, thus hinting to a core-periphery structure. Also, better-connected countries tend to trade with poorly-connected ones, but are also involved in highly-interconnected trade clusters. Furthermore, rich countries display more intense trade links and are more clustered. Finally, all network properties are remarkably stable across the years and do not depend on the weighting procedure.

Sommario

Il paper utilizza l'analisi delle reti complesse per studiare le proprietà empiriche della rete formata dagli scambi commerciali tra i diversi paesi, e la sua evoluzione nel tempo. Si trova che la maggior parte dei paesi caratterizzata da legami commerciali deboli. Tuttavia, vi un gruppo di paesi che mostra un gran numero di legami intensi. Questa caratteristica suggerisce l'esistenza di un centro e di una periferia nel sistema. Inoltre, le economie connesse in modo pi intenso al sistema tendono a scambiare beni con quelle periferiche, ma, allo stesso tempo, fanno parte di agglomerazioni in cui il commercio particolarmente intenso. Le economie pi ricche mostrano un grado maggiore sia di integrazione che di agglomerazione. Da ultimo, il lavoro mostra come le propriet della rete degli scambi commerciali siano molto stabili nel tempo e non dipendano dalla particolare procedura di ponderazione.

Keywords: Networks; World trade web; international trade; weighted network analysis; integration; trade openness; globalization.

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1 Introduction

This paper employs network analysis to study the statistical properties of the web of trade relationships among a large set of world countries in the period 1981-2000. We employ data on import and export flows to build, in each year, a network of links between pairs of countries, where each link is weighted by the amount of trade that it carries. This enables us to apply novel statistical techniques developed in the framework of weighted network analysis and thus to characterize some robust stylized facts of international trade patterns.

In the last decades, a large body of empirical contributions have increasingly studied socio-economic systems in the framework of network analysis.[1] A network is a mathematical description of the state of a system at a given point in time in terms of nodes and links. The idea that real-world socio-economic systems can be described as networks is not new in the academic literature (Wasserman and Faust, 1994). Indeed, sociologists and psychologists have been employing social network analysis since the beginning of the last century to explore the patterns of interactions established among people or groups (Freeman, 1996; Scott, 2000).[2]

More recently, however, the empirical study of networks has flourished thanks to the considerable contribution stemming from physics and computer science. Scholars from such academic disciplines have begun to extensively explore the statistical properties of technological, biological and information networks with new and more powerful statistical tools (Albert and Barabási, 2002; Dorogovtsev and Mendes, 2003; Newman, 2003; Pastos-Satorras and Vespignani, 2004). Fields of application here include – among others – the Internet and the WWW, peer-to-peer networks, power grids, train routes and airline connections, electronic circuits, neural networks, metabolism and protein interactions, and so on.

These new methods have been subsequently applied to social and economic systems (Watts, 1999). As a result, the idea that systems like markets, industries, or the world economy, might be considered as networked structures has become increasingly accepted also among empirical economists. In this context, a network-based approach has been recently employed in empirical studies of international trade (Serrano and Boguñá, 2003; Li et al., 2003; Garlaschelli and Loffredo, 2004a, 2005; Kastelle et al., 2005; Serrano et al., 2007; Bhattacharya et al., 2007a,b). Here the idea is to depict the web of trade relations as a network where countries play the role of nodes and a link describes the presence of an import/export relation between any two countries (and possibly the intensity of that flow). We call this network the World Trade Web (WTW).

What can a network approach add to our economic understanding of international trade dynamics? We claim that this methodology allows for a better description of the existing heterogeneity in the degrees of

[1] A survey of this enormous literature is beyond the scope of this paper. The interested reader is referred to Scott (2000), Barabási (2003), Watts (2003), Carrington et al. (2005), among others.

[2] Well-known examples of such studies include networks of friendship and social acquaintances (Rapoport and Horvath, 1961; Milgram, 1967), marriages (Padgett and Ansell, 1993), and job-market interactions (Granovetter, 1974).

connectivity and, hence, of international economic integration. For example, total trade to GDP ratios only provide a measure of openness of an economy, but fail to capture the ways in which each country is connected within the WTW. This subtler evidence can be instead fully evaluated by means of higher (than one) order measures of connectivity, i.e. indicators that take into account trade relationships that are one, two or three steps away from the node/country under analysis. In other words, what matters for integration into the WTW is not only how much a country trades but also the specific distribution of trade volumes across trading partners. Additionally, for a full picture of integration, the specific characteristics of the trading patterns of every partner must also be considered. From this perspective, it is important to characterize the number of countries with which a given country trades, whether or not the partners of a country trade – more or less intensively – with each other, and the degree of dependency of the whole network on a specific country. These various dimensions of connectivity can be characterized using complex network indicators that measure the number and the intensity of the trading relationships, the level of clustering (bilateral or multilateral trade), their dispersion or concentration, the centrality for a given node (country).

The relevance of these specific characteristics from an economic perspective emerges from the fact that trade relationships define a certain degree of dependency of other countries on a given country, or a certain degree of influence of one country on others. For example, node (country) centrality, discussed in detail later in the paper, denotes the likelihood of a given country to appear along a randomly selected trade chain within the trade network. The higher this likelihood, the more influential the country is within the network (or the more the network depends on this country). Hence, shocks hitting more central countries are more likely to be transferred to many other countries. This idea of network dependency or network fragility can also be related to the presence and size of trade blocks in the network. If trade blocks are well defined, then shocks originated in one block would not affect other blocks. Therefore it is important to look at the degree of clustering within the WTW.

Knowledge of such topological properties is not only important per se (e.g., because it enhances our descriptive knowledge of the stylized facts pertaining to the WTW, and allows us to give a more detailed account of the processes of integration and globalization), but it may also be relevant to better explain macroeconomic dynamics. As suggested by Kali and Reyes (2007) and Serrano et al. (2007), a full description of the WTW can be used to highlight global interdependencies that explain the propagation of financial and economic crises, since an economic slowdown of a given country can easily disrupt production and consumption chains within the WTW. Indeed, as shown by Forbes (2002) and Abeysinghe and Forbes (2005), economic shocks to any single country can be easily transmitted —via trade linkages of any order and non-linear multiplier effects— to countries that are relatively minor bilateral trading parters. Alternatively, international trade flows can also be seen as cash flows exchanges among the countries involved. Therefore a reduction of exports reduces the capabilities of a given node to import from other countries. Finally, the arguments that have emerged from growth theory (see for instance Rivera-Batiz and Romer, 1991) linking the benefits from trade to the exposure to new technologies and the expansion of market access can be verified and fully characterized by using network analysis. Empirically, recent studies have in fact suggested that gains from trade do not depend only on the degree of trade openness, but also on the number of trading partners and their characteristics (Arora and Vamvakidis, 2005). Also, Kali et al. (2007) find that a higher number of trading partners (which corresponds to node degree in complex network analysis) is associated with higher growth rates and suggests that this is the result of exposure to better technologies, expanded market access, and higher levels of competition.

In this paper, we present a detailed study of the WTW for the period 1981-2000 using a weighted network approach. From a purely descriptive perspective, we attempt to single out some robust stylized facts pertaining to trade relationships and their evolution over time. The empirical regularities displayed by the data could then be used as a starting point to model the structure and the evolution of the WTW and therefore provide a theoretical explanation of why the network of trade flows is organized in this way. In other words, empirical regularities can provide some guidance for theoretical models that seek to explain the evolution of world trade linkages and the benefits (and costs) arising from them. For example, the evolution of free trade agreements has been the objective of recent theoretical studies that follow an endogenous network-formation approach (Goyal and Joshi, 2006; Furusawa and Konishi, 2007)[3]. Our findings indicate that countries with similar industrialization levels tend to form trade blocks (clubs) in the WTW.

More specifically, this paper attempts to answer the following questions: Are rich countries more connected than poor ones – both in terms of the number and intensity of their trade relationships? Do well-connected countries entertain trade relationships with partners that are themselves well-connected (i.e. hold many and intense trade relationships)? How large is the likelihood that rich countries tend to trade with countries that preferentially trade only among them? Or, in other words, how large is the probability to find groups of rich countries that form trade clusters? What are the most central countries in the WTW? Have the structural properties of the WTW been changing across time? Is the WTW more connected today than in the past (both in terms of number of connections and trade flows)? What has all that to do with the would-be process of globalization?

From a methodological point of view, we employ novel techniques that allow us to study the WTW as a weighted network.[4] The bulk of the relevant literature on international trade networks has indeed studied a binary version of the WTW, where each directed link from country i to country j is either in place or not

[3]See Jackson (2004) for an introduction.

[4]The analysis of weighted networks was introduced in Barrat et al. (2004, 2005); Barthélemy et al. (2005), and further developed in Dall'Asta et al. (2006); Saramaki et al. (2007); Onnela et al. (2005); DeMontis et al. (2005).

according to whether the trade flow from i to j is larger than a given threshold. On the contrary, we weight the importance of each directed link by using actual trade flows and GDPs. A similar approach has been adopted by Li et al. (2003), Bhattacharya et al. (2007b), Bhattacharya et al. (2007a) and Serrano et al. (2007). In this paper, conversely, we build on Fagiolo et al. (2008) and we present a more thorough analysis of the WTW which expand upon previous results along several dimensions. First, we present a description of WTW properties spanning across several measures/indicators (e.g., node connectivity, assortativity, clustering and centrality). Second, we look at how WTW properties correlate with country/node characteristics such as country income (as measured by per capita GDP). Third, we study the extent to which empirical regularities are robust to alternative ways of weighting existing links. Finally, we statistically check whether the inherent directionality of WTW flows is so strong to justify an analysis that takes explicitly into account the direction of trade flows or, by contrast, one can safely study the WTW as if it were an undirected graph.

Our results show that weighted network analysis allows one to obtain very different conclusions as compared to a binary-network framework. Furthermore, our main (qualitative) findings seem to be quite robust to a number of alternative, economically meaningful, weighting schemes. In addition, we also find that the WTW is a strongly symmetric network, so that disregarding the direction of links does not alter the main results. Our weighted analysis indicates that weak trade relationships dominate for the vast majority of countries; yet, there exists a group of countries (identifying the core of the network) featuring a number of strong relationships, thus hinting to a core-periphery structure. Moreover, poorly-connected countries tend to trade with better-connected ones, whereas the latter are involved in relatively highly-interconnected trade triples. In addition, high-income countries tend to form more intense trade links and to be more clustered. Cliques are thus built along the lines of both connectivity and income level and can be seen as a sign of the persistent relevance of local relationships. However, the growing importance of global links is testified by the disassortative feature of WTW: poorly connected nodes tend to connect to central ones and use them as hubs to access the rest of the network. Finally, all structural properties of the WTW display a remarkable stationarity across the years. The stability of the WTW structure suggests that international goods market integration has not increased dramatically over the last 20 years or, viewed from a different vantage point, that despite increased economic integration the core of the WTW has remained mostly unaffected.

Some of the concepts used throughout the paper are not new to the international trade literature, though they assume here a different connotation. So, for instance, the network of international trade to which we refer, is not the same developed in the works of Rauch and co-authors (see Rauch, 2001; Rauch and Casella, 2003), where the authors focus on the role played by business and social networks in alleviating informational asymmetries. Here, on the contrary, we exploit network analysis to describe the patterns of aggregate goods trade among countries. Similarly, core countries are not necessarily those where most of productive activities are located as in the New Economic Geography framework, bur rather those involved in a large number of trade linkages (Ottaviano et al., 2002).

The rest of the paper is organized as follows. In Section 2 we briefly introduce, in a rather informal way, the main concepts related to the empirical analysis of networks (more details are contained in Appendix A). Section 3 briefly surveys the relevant literature on international trade networks. Data are described in Section 4. We report our main results in Section 5. Finally, Section 6 concludes and discusses future work.

2 An Introduction to the Statistical Analysis of Weighted Networks

A socio-economic network is usually described by means of a graph, that is a collection of N nodes, possibly connected by a set of links.[5]

The simplest type of graph is binary and undirected. This means that any two nodes can be either connected by a link or not, and link directions do not count. If two nodes are connected, we say that they are "partners" or "nearest neighbors". To formally characterize such type of networks, it is sufficient to provide the so-called adjacency matrix, i.e. a symmetric $N \times N$ binary matrix A whose generic entry $a_{ij} = a_{ji} = 1$ if and only if a link between node i and j exists (and zero otherwise).[6]

If the researcher has good reasons to justify her/his empirical analysis by using binary undirected networks (BUNs), the most immediate statistics is the *node-degree* (ND) distribution (and its moments). ND is simply defined as the number of links that a given node has established (i.e., how many connections it holds). The shape of the ND distribution can convey a lot of information on the structural properties of a network. For example, BUNs that are created totally at random have unimodal, bell-shaped ND distributions.[7] On the contrary, the so-called scale-free networks (Barabási, 2003) are characterized by right-skewed (power-law) distributions, with a majority of small-ND nodes and a minority of large-ND nodes (i.e., the hubs).

If one is instead interested in a graph-wide measure of the degree of connectivity of the network, a simple way to proceed is to compute the *density* of the graph. The latter is defined as the total number of links that are actually in place divided by the maximum number of links that there can exist in an undirected graph with N nodes.

The ND statistic only counts nodes that are directly linked with the one under analysis. In this respect, it can be considered as a first-order indicator, as it only takes into account information about nodes that

[5] We refer the reader to Appendix A for more formal definitions and notation.

[6] Self-loops, i.e. links connecting i with itself are not typically considered. This means that $a_{ii} = 0$, for all i.

[7] For example in random networks where each link is in place with a certain given probability, independently on all the others (i.e., according the simplest Erdös-Renyi random-graph model: see e.g. Bollobás, 1985). In what follows, we employ the term "random network" as a synonym for the Erdös-Renyi random-graph model.

are one step-away from the original one. However, any two nodes with the same ND can acquire a different importance in the network to the extent their partners are themselves connected in the network, i.e. if they also have a high ND. To measure how much the partners of node i are themselves very connected in the network, one may compute the *average nearest-neighbor degree* (ANND), that is the average of ND of all partners of i. Nodes with the largest degree and ANND are typically the ones holding the most intense interaction relationships.

A third important feature of network structure concerns the extent to which a given node is clustered, that is how much the partners of a node are themselves partners.[8] This property can be measured by the *clustering coefficient* (Watts and Strogatz, 1998; Szabó et al., 2005), that is the percentage of pairs of i's nearest neighbors that are themselves partners. Node clustering is very important, as geographically-structured networks are typically highly-clustered, with short-distance links counting more than long-distance ones. Unlike ND, ANND and node clustering are both second-order indicators, because they look at statistics concerning nodes lying two steps away from the one under consideration.

So far, we have only considered binary networks, i.e. graphs where what counts is the mere presence or absence of an interaction between any two nodes. Many researchers have argued, however, that the majority of socio-economic relationships also involve an assessment of how intense is an interaction between two nodes (if any). If one studies such relationships using a BUN approach, it is likely that a lot of important information will be disregarded (see Barrat et al., 2004, for an introduction). Conversely, in many other networks like the internet, airline traffic, scientific citations, or the WTW, links are characterized by a non-reducible heterogeneity. If we use a BUN analysis, we run the risk of considering the same way links that instead carry very weak or very strong flows. In those cases, we need to move from a BUN perspective to a weighted (undirected) network (WUN) approach. A WUN is simply defined by means of a symmetric $N \times N$ "weight" matrix W, whose generic entry $w_{ij} = w_{ji} > 0$ measures the intensity of the interaction between the two nodes (and it is zero if no link exists between i and j).

The three statistics above (degree, ANND, and clustering) can be easily extended to a WUN approach. First, ND can be replaced by node *strength* (NS), defined as the sum of weights associated to the links held by any given node. The larger the NS of a node, the higher the intensity of interactions mediated by that node. It is easy to see that, given the same ND, any two nodes can be associated to very different NS levels. Incidentally, we note that strength is only an aggregate measure of the interaction intensity mediated by a node. Thus one can also measure the extent to which a node holds links associated with a very dispersed (resp. concentrated) weight profile. To do that, each node i can be associated with the Herfindahl strength-

[8] Network clustering is a well-known concept in sociology, where notions such as "cliques" and "transitive triads" have been widely employed (Wasserman and Faust, 1994; Scott, 2000). For example, friendship networks are typically highly clustered (i.e. they display high cliquishness) because any two friends of a person are very likely to be friends.

concentration index.[9]. The index will be equal to one if a country concentrates all its trade relationships with one partner, and it will decreases towards zero the more differentiated is its trade portfolio (for any given number of trade partners).

Second, one might assess how much the partners of a node are themselves characterized by a high strength by computing either the *weighted average of nearest-neighbor node degrees* (WANND, see Barthélemy et al., 2005) or the arithmetic *average of nearest-neighbor strengths* (ANNS). Once again, any two nodes with the same ANNS (and ANND) can end up having very different levels of ANNS or WANND.

Third, one can straightforwardly compute a *weighted clustering coefficient* by suitably weighting each triangle using the weights w_{ij} associated to its three edges (see Appendix A, and Fagiolo, 2007; Saramaki et al., 2007).

Another important notion in network analysis concerns the extent to which a given node is "central" in the graph. However, the meaning of "centrality of a node" is rather vague and has consequently generated many competing concepts and indicators (Scott, 2000). The two most commonly employed definitions of centrality refer to a local notion (a node is central if it has a large number of connections) or to a global notion (a node is central if it has a position of strategic significance in the overall structure of the network). Local centrality can be easily measured by node degree (in BUNs) or node strength (in WUNs). As far as global centrality in BUNs is concerned, the most used indicator is *node betweenness centrality* (BC), defined as the proportion of all shortest paths between any two nodes that pass through a given node. BC thus measures how much a given node acts as intermediary or gatekeeper in the network. It is easy to see that low-ND nodes, which are not locally central, can have a large BC, and therefore be globally central.

Despite its importance, BC is not straightforwardly extendable to WUNs. Therefore, in this paper, we build on recent works by Newman (2005) and Fisher and Vega-Redondo (2006), who have put forward a notion of centrality that nicely fits both BUN and WUN analyses. In a nutshell, they develop an index called *random walk betweenness centrality* (RWBC), which easily captures the effects of the magnitude of the relationships that each node has with its partners as well as the degree of the node in question. Newman (2005) offers an intuitive explanation of this centrality measure. Assume that a source node sends a message to a target node. The message is transmitted initially to a neighboring node and then the message follows a link from that vertex, chosen randomly, and continues in a similar fashion until it reaches the target node. The probabilities assigned to outgoing links can be either all equal (as in BUNs) or can depend on the intensity of the relationship (i.e., link weights in WUNs), so that links representing stronger ties will be chosen with higher probability. In this respect, RWBC – being a global centrality indicator – is a

[9] See Herfindahl (1959) and Hirschman (1964). For early applications of the disparity index to networks, see Almaas et al. (2004) and Barthélemy et al. (2005).

highest-order indicator, as it embodies information about across-node paths of any length.

Finally, notice that the "undirected" nature of both BUNs and WUNs approaches requires the matrices A and W to be symmetric. This implies that it is reasonable to assume that binary or weighted relationships are bilateral or reciprocal. However, the majority of interaction relationships that can be captured in network analyses are in principle directed (i.e., not necessary symmetric or reciprocal). For example, exports from country i to country j in a given year (e.g., as a share of i's GDP) can be substantially higher or lower than exports from country j to country i (e.g., as a share of j's GDP). As discussed in Fagiolo (2006), deciding whether one should treat the observed network as directed or not is an empirical issue. The point is that if the network is "sufficiently" directed, one has to apply statistics that take into account not only the binary/weighted dimension, but also the direction of flows. As this analysis can often become more convoluted, one ought to decide whether the "amount of directedness" of the observed network justifies the use of a more complicated machinery. There can be several ways to empirically assess if the observed network is sufficiently symmetric or not (cf. Appendix A, and Garlaschelli and Loffredo, 2004b; Fagiolo, 2006). In many cases, networks that can be thought to be asymmetric turn out to appear sufficiently symmetric to justify a BUN/WUN approach (see also below for the WTW). If this happens, the common practice is to symmetrize the original observed network.[10]

3 Related Literature

The idea that international trade flows among countries can be conceptualized by means of a network has been originally put forth in sociology and political sciences. Most of this literature, however, did not address the study of trade networks by using a strategy rooted in the statistical analysis discussed in Section 2. Nevertheless, since the seminal paper by Snyder and Kick (1979), an increasing number of scholars have argued that relational variables are more relevant than (or at least as relevant as) individual country characteristics in explaining the macroeconomic dynamics ensuing from import-export patterns.

This strand of trade-network studies has been deeply influenced by the so-called "world system" or "dependency" theories, i.e. the notion that one can distinguish between core and peripheral countries. In this view, core countries can appropriate most of the surplus value added produced by peripheral ones, which are thus prevented from developing.

For example, Snyder and Kick (1979) study the BUN stemming from aggregate data on trade relationships among 118 countries in 1965 and employ a core-periphery setup to classify countries in three groups (core, semi-core, and periphery). They obtain a clear-cut three-tiered structure for the network, with core countries

[10]That is (a_{ij}, w_{ij}) are replaced by $(max\{a_{ij}, a_{ji}\}, 0.5(w_{ij} + w_{ji}))$, see De Nooy et al. (2005).

nearly identified with OECD members. Regression analyses show that the position of a country in the network is able to explain GNP growth, a result interpreted as a sort of confirmation of dependency theories. The importance of network position in explaining growth and development from a core-periphery approach is also stressed by Nemeth and Smith (1985), who apply their analysis to 1970 data of trade flows disaggregated over 5 distinct product classes.[11] A similar approach is followed by Breiger (1981), who studies the composition of trade flows in 1972. Notably, he shows that country classification into blocks is not robust to the introduction of weighted links: if one employs a WUN, where link weights are defined as total trade flows (depurated by average imports and exports for that product class in order to account for size effects), two competing blocks emerge. The first one is dominated by the US (and comprises Canada and Japan), while the second accounts for the (then young and small) European Community. More recently, Smith and White (1992) explore in a dynamic framework the core-periphery approach to network analysis by comparing results in three different years (1965, 1970 and 1980). They document an enlargement of the core over time, a reduction of within-core variability, and a progressive marginalization of very peripheral countries. A binary, directed approach is instead followed by Kim and Shin (2002), who study three snapshots of trade flows (1959, 1975 and 1996) disaggregated over a large set of commodities for 105 countries. Interestingly, they find that both the density of the network and the variance of ND distributions have increased through time, a result which is associated by the two authors to the globalization process. Moreover, the creation of new links does not appear to be evenly distributed. Core countries are more likely to create outward links (i.e., to initiate an export link), while peripheral countries are more likely to create an inward link (i.e., to receive it), with Asian countries accounting for the majority of newly created links.[12] The effects of globalization are further explored by Kastelle et al. (2005) who perform a binary network analysis on IMF data to test baseline hypotheses on the dynamics of the topological properties of the WTW. They study the period 1938-2003 and find that the evolution of the international trade network has not reached any steady state implying a fully-globalized pattern. Rather, the WTW has been slowly changing and seems to have the potential to continue to do so in the future.

The study of international trade as a relational network has been recently revived in the field of econophysics, where a number of contributions have explored the (notionally) complex nature of the WTW. The common goal of these studies – well in line with the strategy described in Section 2 – is to empirically analyze the mechanics of the international trade network and its topological properties, by abstracting from any social and economic causal relationships that might underlie them (i.e., a sort of quest for theory-free

[11] Sacks et al. (2001) build a measure of country position in the network based on the concept of "structural autonomy" and show that it has a positive effect on country's per capita GDP.

[12] Very similar results are obtained by Mahutga (2006), who shows that the globalization process has induced structural heterogeneity and thus inequality.

stylized facts). From a methodological perspective, almost all these contributions carry out their analysis using a binary approach. In other words, a link is either present or not according to whether the trade flow that it carries is larger than a given lower threshold.[13] For instance, Serrano and Boguñá (2003) and Garlaschelli and Loffredo (2004a) study the WTW using binary undirected and directed graphs. They show that the WTW is characterized by a disassortative pattern: countries with many trade partners (i.e., high NDs) are on average connected with countries with few partners (i.e., low ANNDs). Furthermore, partners of well connected countries are less interconnected than those of poorly connected ones, implying some hierarchical arrangements. In other words, a negative correlation emerges between CC and ND distributions. Remarkably, Garlaschelli and Loffredo (2005) show that this evidence is quite stable over time. This casts some doubts on whether economic integration (globalization) has really increased in the last 20 years. Furthermore, the ND distribution appears to be very skewed. This implies the coexistence of few countries with many partners and many countries with only a few partners.

More recently, a few contributions have adopted a weighted-network approach to the study of the WTW. The motivation is that a binary approach cannot fully extract the wealth of information about the intensity of the trade relationship carried by each edge and therefore might dramatically underestimate the role of heterogeneity in trade linkages. This seems indeed to be the case: Fagiolo et al. (2008) show that the statistical properties of the WTW viewed as a WUN crucially differ from those exhibited by its weighted counterpart. For example, the strength distribution is highly left-skewed, indicating that a few intense trade connections co-exist with a majority of low-intensity ones. This is confirmed by Bhattacharya et al. (2007a) and Bhattacharya et al. (2007b), who find that the size of the group of countries controlling half of the world's trade has decreased in the last decade. Serrano et al. (2007) study the network of bilateral trade imbalances.[14] They note that also the international trade-imbalance network is characterized by a high level of heterogeneity: for each country, the profile of trade fluxes is unevenly distributed across partners (i.e., the disparity index is typically low). At the network level, this prompts to the presence of high-flux backbones, i.e. sparse subnetworks of connected trade fluxes carrying most of the total flux in the network.

A common problem of all these studies is that the directed or undirected nature of the observed international-trade networks is not thoroughly addressed. In other words, a directed (or undirected) analysis is pursued without statistically assessing the underlying observed nature of the WTW. An exception is the

[13]There is no agreement whatsoever on the way this threshold should be chosen. For example, Kim and Shin (2002) use cutoff values of US$ 1 million and 10 million. Kastelle et al. (2005) endogenously set a cutoff so as to have, in each year, a connected graph. Kali and Reyes (2005) experiment with different lower thresholds defined as shares of country's total exports. On the contrary, other papers (Serrano and Boguñá, 2003; Garlaschelli and Loffredo, 2004a, 2005; Kali and Reyes, 2007) straightforwardly define a link whenever a non-zero trade flow occurs.

[14]That is, they weight each link by the difference between exports and imports. Notice that, as happens also in Bhattacharya et al. (2007b,a), their across-year comparison may be biased by the fact that trade flows are expressed in current U.S. dollars and do not appear to be properly deflated.

paper by Garlaschelli and Loffredo (2005), who explore the conditions under which one can recover the directed character of a network from its undirected description. However, they fall short from providing a directed analysis using ad-hoc indicators (see for example Fagiolo, 2007).

This work builds on the findings and methodologies discussed so far and presents a more thorough analysis of the WTW that expand the previous ones along (at least) four dimensions. First, we present a more complete characterization of WTW statistical properties. This is done by discussing several measures/indicators pertaining to weighted-network analysis (e.g., node connectivity, assortativity, clustering and centrality). Second, we explicitly study how these properties correlate with country/node characteristics. For example, we ask the question whether high-income countries (e.g. in terms of per-capita GDP) are also highly-connected or highly-clustered. Third, we study the extent to which the statistical properties that we find are robust to alternative ways of weighting existing links. Finally, we explicitly assess via a statistical indicator whether the observed WTW network is sufficiently undirected to justify a WUN analysis instead of a WDN one.

4 Methodology and Data

We employ international trade data provided by Gleditsch (2002) to build a sequence of weighted directed networks from 1981 to 2000. Original data report imports and exports from/to a large set of world countries for the period 1950-2000. The choice of the subperiod to be used in the study is driven by three related considerations. First, data for small countries suffer from many missing values, both on trade flow and GDP/population. Second, the number of countries for which trade data are available increases over the years. This might be a problem if one wants to analyze the dynamics of the topological properties of the WTW. Third, the country sample size must be as large as possible to achieve statistical significance. As a result, we decided to build a balanced panel by focussing on $T = 20$ years (1981-2000) and $N = 159$ countries (see Appendix B for more details).

For each country and year, data report trade flows in current US dollars. Therefore, we have deflated all nominal figures so as to allow for meaningful across-year comparisons. Whenever exports from country i to j do not match imports of j from i, we averaged the two figures. To build adjacency and weight matrices, we followed the flow of goods. This means that rows represent exporting countries, whereas columns stand for importing countries.

As to link weighting, in order to avoid any ambiguity stemming from the choice of a particular lower threshold, we define a "trade relationship" by setting the generic entry of the adjacency (binary) matrix $\tilde{a}_{ij}^t = 1$ if and only if exports from country i to country j (labeled by e_{ij}^t) are strictly positive in year t.

Moreover, we experiment with a few economically-meaningful weighting systems and explore the robustness of our results to these alternatives. Our baseline results will refer to weights defined as $\tilde{w}_{ij}^t = e_{ij}^t / GDP_i^t$, where e_{ij}^t are time-t (deflated) exports from country i to country j, and GDP_i^t is i's real gross domestic product in year t. This weighting procedure allows us to control for exporter country's size and tells us how economy i depends on economy j as a buyer (as j is buying from i). Furthermore, we check if our results change when we divide e_{ij}^t by the importing country's output (GDP_i^t). This can provide information on how economy j depends on economy i as a seller. Finally, we study what happens when we do not scale exports by GDPs and we just weight a link from i to j with exports e_{ij}^t.

For any particular choice of the weighting setup, we end up with a sequence of $N \times N$ adjacency and weight matrices $\{\tilde{A}^t, \tilde{W}^t\}$, $t = 1981, ..., 2000$, which fully describe the evolution of the WTW from a binary and weighted directed perspective.

5 Results

5.1 Global Properties of the WTW

We begin with a quick overview of the main global properties of the WTW. From a binary perspective, the WTW appears to be a highly connected network, with an extremely high density, which has been slightly increasing over the years (cf. Figure 1). This witnesses for the increasing participation of countries to world trade over the last 20 years of the last century. It is easy to see that the majority of links are reciprocated: for instance, in the second half of the 1990, almost all countries export to partners that in turn export to them.

The almost-symmetric pattern of the WTW is statistically detected also by the S index studied in Fagiolo (2006), which for all years stays very close to zero for both the binary and the weighted version of the WTW, thus strongly testifying in favor of symmetry.[15] If anything, the WTW seems to have become more symmetric during the years. This evidence indicates that a directed analysis is not necessary. Therefore, in what follows, we will explore the statistical properties of *symmetrized* versions of the WTW. This means that, in the binary case, any entry a_{ij} of the new adjacency matrix A is set to 1 if and only if either $\tilde{a}_{ij} = 1$ or $\tilde{a}_{ji} = 1$ (and zero otherwise). Accordingly, the generic entry of the new weight matrix W, originally defined as $\tilde{w}_{ij}^t = e_{ij}^t / GDP_i^t$, is replaced by:[16]

[15] See Appendix A for the technical details. Note that the corresponding standardized index takes values at least 10 standard deviations below zero.

[16] Due to the extreme symmetry of the network, results do not change if one symmetrizes the export matrix first and then divides by the GDP of the exporting country.

$$w_{ij}^t = \frac{1}{2}(\tilde{w}_{ij}^t + \tilde{w}_{ji}^t) = \frac{1}{2}(\frac{e_{ij}^t}{GDP_i^t} + \frac{e_{ji}^t}{GDP_j^t}). \tag{1}$$

In order to have well-behaved weights, we also employ the convention of dividing all entries in W by their maximum value. This does not introduce any bias in our analysis and ensures that $w_{ij}^t \in [0, 1]$ for all (i, j) and t (Onnela et al., 2005).

5.2 Connectivity

The first issue we address concerns the study of the shape of the degree and strength distributions. More specifically, we explore the extent to which countries are more or less connected (i.e., if they are more or less locally-central in the WTW) both in terms of number of partners (ND) and interaction intensity (NS), and whether these patterns have changed through time.

To begin with, we note that ND distributions do not appear to be as skewed as expected (see Figure 2). In fact, they can hardly be proxied by lognormal or Pareto densities. A power-law behavior is detected only in the middle of the distribution, as the sharp cutoff reported by Garlaschelli and Loffredo (2005) is present. Remarkably, ND distributions display some bimodality: beside a modal value of 50-100 partners, there is a large group of countries that trade with almost everyone else (hence a second peak around 150). This evidence is more pronounced in the middle of the period. Note also that the shape of the ND distribution remains quite stable over time. Figure 3 displays the time evolution of the first four moments of the ND distribution: average ND has slightly increased over the years, meaning that the number of trade relationships have been weakly but steadily growing during the observed time frame. Moreover, the standard deviation has remained stable, which suggests that integration has increased rather evenly, without resulting in any rise in the heterogeneity of the number of trade links established by each country. This conclusion is reinforced by the reduction in both skewness and kurtosis that characterizes the last few years in the sample: the ND distribution has thus become more symmetric and the tails have thinned out to signify that fewer countries now display extreme ND values.

The picture substantially changes when we measure connectivity in the weighted version of the network. The distribution of NS among countries is now much more lognormal, see Figure 4, even though in the right tail (high strengths) there seems to be many more countries than a lognormal model would predict. Furthermore, bimodality disappears: strength distributions are more right-skewed, with the majority of countries characterized by weak trade relationships.

The structural difference between degree and strength distributions can be better appreciated by looking at how the degree-strength correlation varies through time. As Figure 5 (left) shows, this correlation is significantly larger than zero and quite stable around 0.5. This means that *on average* countries with many trade partners tend to hold also more intense trade relationships. However, as shown by the degree-strength scatter plot for 2000, the strength variability for any given degree level is quite high (Figure 5, right). This implies that only a subset of those countries holding many trade relationships (high ND) actually have a very high strength.[17]

Notice also that the weak increase in ND is not matched by a similar behavior for average NS, which remains quite stable in the period 1981-2000 (Figure 6, left panel). Interestingly, average strength is relatively low (at least in a [0,1] scale) as compared to the relatively high average degree. Furthermore, the observed drop of skewness and kurtosis of ND distributions does not have a counterpart as far as NS is concerned (compare Figure 3 and 6, right panels). Since this phenomenon is mainly concentrated in the 1990s, it seems to suggest that the recent wave of globalization resulted in an increased number of connections, but did not have any sizable effect on their magnitude. In terms of NS there are many more countries in the tails of the distribution, which is also much more skewed than in the case of ND.

Right-skewness of NS distributions maps into a relatively high average node disparity (i.e., a relatively low Herfindahl index). Indeed, a majority of countries holding a portfolio of very dispersed trade relationships typically coexists with a few countries that concentrate almost all their trade flows within a small numbers of partners. Node disparity distributions can in fact be very well approximated by log-normal densities. It is interesting to note that in general node disparity is negatively correlated with both node degree and node strength. Therefore, the more partners a country holds, and the more intense its trade relationships, the more dispersed is its trade profile. This is partially expected, because in the presence of equally-distributed weights node disparity should scale as the inverse of node degree.

This first set of results allows us to make an important methodological point. If the study of the WTW is carried out from a BUN perspective, thus loosing a lot of information, one runs the risk of getting a misleading picture of the underlying relational patterns. A weighted network perspective, instead, allows one to better appreciate how the intensity of the interaction structure is distributed across the population (cf. also Fagiolo et al., 2008).

5.3 Assortativity

The foregoing results suggest that the WTW, if viewed as a BUN, is a relatively strongly connected and dense network. On the contrary, if we give weights to these trade links, the picture changes completely: the WTW, now viewed as a WUN, is characterized by a majority of relatively weak trade flows coexisting with a few

[17]As the right panel of Figure 5 shows, there seems to be a subset of countries featuring low ND and relatively high strength.

strong trade connections.[18] This reflects results reported in Serrano et al. (2007) and, interestingly, reminds of the distinction between intensive and extensive margins found in the microeconomic trade literature, with the former (export intensity) being much more important than the latter (number of exporting firms) in explaining aggregate export performances.

Degree and strength statistics, however, are only first-order indicators. In other words, they just take into account links to one-step-away partners and do not convey any information on the finer structure of the WTW. Indeed, it may well happen that countries holding many links only trade with poorly-connected countries (we call such a network "disassortative"). Conversely, it may be the case that better connected countries also tend to trade with other well-connected countries (i.e., an "assortative" network).

In order to explore assortativity in the WTW, let us begin with a BUN perspective and study the behavior of average nearest-neighbor degree (ANND), and how it correlates with other network statistics. As expected, ANND is very high and quite stable in the period considered (Figure 7, left). Average ANND weakly increases from 110 to 120 and stays always above the average degree. However, the degree-ANND correlation pattern clearly indicates a strongly disassortative network: correlation figures are very close to -1 and their magnitude increases over time (Figure 7, right). In the WTW viewed as a BUN, countries that hold many trade relationships definitely trade with poorly-connected countries. This results confirms previous findings by Serrano and Boguñá (2003) and Garlaschelli and Loffredo (2004a).

If the WTW is now studied as a WUN, its disassortative nature remains evident but results are much weaker. As Figure 8 shows, population-averages of both weighted average nearest-neighbor degree (WANND) and average nearest-neighbor strength (ANNS) are quite stable over time and mimic the behavior of degree and strength. However, their correlation with degree and strength is still negative but weaker in magnitude in all years (Figure 9). This means that countries holding a lot of trade relationships do not tend to establish very intense trade links with all their partners. Again, the study of the WTW from a WUN perspective is able to offer a different (and more insightful) picture.

The disassortative nature of the WTW implies that countries that are less (and more weakly) connected tend to form trade relationships with well and more intensively connected countries (i.e., the hubs). This feature is relevant, since it suggests that the WTW has a core-periphery structure (at least in terms of link intensity). This feature, which is common in many social and economic networks, finds theoretical backing in a recent model by Hojman and Szeidl (2008), where the authors prove that under fairly general assumptions, the unique equilibrium network has a star-like structure.[19] From an economic and policy point of view, this

[18] As discussed in Section 5.7, this holds true even if one replaces the baseline weighting procedure with a few, economically meaningful, alternative schemes.

[19] The key assumptions are that the benefits from connections exhibit decreasing returns, and that they depend negatively on distance. Contrary to the predictions of the model, the WTW does not display a single country as its center. This is due to the fact that in the (real) world of international trade, the benefit from connecting to a country is not monotonically increasing

results is likely to imply that peripheral countries suffer from a sort of marginalization. Consistently with some recent results in the field of economic geography (see Ottaviano et al., 2002, p.411), we interpret our finding as suggesting that such a polarized structure is not necessarily the most efficient outcome, and that a more balanced structure of trade relations would allow both developing and industrialized countries to exploit more completely the gains from trade.

To further investigate this feature, we plot correlation patterns of ANND, WANND and ANNS vs. node degree and strength. As Figure 10 shows for year 2000, the ANND-ND correlation presents a very limited variability. Conversely, both the WANND-ND and the ANNS-NS scatter plots are characterized by a much more dispersed cloud of points. In particular, there seems to exist a not negligible number of medium/high-degree or medium/high-strength countries that, despite the overall disassortativity, tend to trade with countries that are themselves more and better connected. This seems to support the hypothesis that, within the core-periphery structure of the WTW, there exists an intermediate periphery that is well connected to high degrees nodes (and trades heavily with them).

5.4 Clustering

We now turn to exploring clustering patterns, and their relations with connectivity. This entails asking whether more and better connected countries tend to build trade relationships with pairs of countries that themselves trade with each other.

Figure 11 (top-left) shows the behavior of the average CC for the BUN description of the WTW. Average CC is very high in all years. Furthermore, it is always larger than network density (cf. Figure 1). Since in a random network the expected CC is equal to its density, this result implies that the WTW (viewed as a BUN) is statistically more clustered than if it were random. Therefore, countries tend to form – on average – trade relationships with partners that also trade with each other. This sort of "cliquishness" suggests that regional or local ties still play a very relevant role, where localism does not not necessarily have a geographic meaning, but can very well be read as a tendency to interact with traditional partners.[20] These can be members of a regional group, countries with similar degree of development, or simply partners that are historically close.

Does this result hold also when we take into account that trade relationships are very heterogeneous in their intensity? The answer is no.[21] What is more, the supports of node-clustering distributions lies, in every

in the number of its trading partners. This suffices for a network to display more that one hub.

[20]This interpretation is further corroborated by the fact that geographically-structured networks are typically highly clustered, with short-distance links counting more than long-distance ones.

[21]Indeed, the weighted version of the CC, albeit quite stable over time, is significantly smaller (from a statistical point of view) than its expected value in a random network. Indeed, average clustering ranges from 3.8776×10^{-4} (in 1994) to 5.5106×10^{-4} (in 1982) whereas the expected value of weighted clustering in random networks goes in the same years from 0.2272 to 0.2717 – that is, $\frac{27}{64}$ times network density (see Appendix A for the details).

year, to the left of random-network expected values, indicating that no country ever exhibits a clustering coefficient that is above the expected value.

Yet, the comparison with random networks may not be very meaningful in the weighted case. In fact, we already know that link weights and strengths are far from being uniformly-distributed across existing links. Therefore, a fairer comparison requires the computation of the expected clustering coefficient across all weighted networks that are characterized by the same (observed) network density and the same (observed) links, but by a random allocation of the (observed) weights across the existing links[22]. As Figure 11 (top-right) shows, this comparison indicates that, on average, the weighted version of the WTW is slightly less clustered than expected. Nevertheless, the across-node distribution of clustering coefficients is (in every year) quite skewed to the left, as the bottom-left panel of figure 11 shows for the year 2000. Thus, even if on average the WTW is weakly clustered, a small portion of countries are characterized by "excess" clustering, i.e. a clustering larger than its expected value in networks with the same binary structure and weight distribution. As Figure 11 (bottom-right) shows, this percentage oscillates around 25%, suggesting that only a small fraction of countries are actually clustered. Out of the network jargon, there seems to exist a strong heterogeneity in the way countries form and maintain trade cliques, consistently with the idea of the existence of a minority of prominent nodes acting as strongly clustered hubs.

If one looks at the correlation between clustering and degree/strength, the striking mismatch between BUNs and WUNs noticed above still emerges. Indeed, as found also by Serrano and Boguñá (2003) and Garlaschelli and Loffredo (2005), countries that hold more trade partners (high degree) are less clustered than those holding few partners. The correlation is very strong and negative, as it is close to -0.96 throughout the whole period (Figure 12, left panels). From a BUN perspective, thus, a core-periphery, star-shaped trade network seems to be in place. Countries that hold a small number of trade relationships do not trade with each other but are connected to the hubs. Again, if one takes into account the actual trade intensities associated to these connections, the conclusion is reversed (Figure 12, right panels). The correlation between the weighted CC and strength is now positive, statistically-significant, and sharply increasing across time. Therefore, countries with high-intensity trade relationships are typically involved in highly-interconnected triples, that is clustering levels that are statistically larger than their expected value in comparable random structures. This is a pattern that somewhat reminds the "rich club phenomenon" (where "richness" is now interpreted in terms of intensity of trade relationships). The fact the the magnitude of the CC-strength correlation is increasing through time suggests that the "rich club phenomenon" continues to be an issue for international trade. This finding is consistent with a recent model by Furusawa and Konishi (2007) where

[22]To do so, for each year we generated a sample of 10000 random networks whose adjacency matrices have been kept fixed and equal to the *observed* one, whereas *observed* link weights have been randomly reshuffled across the links.

the authors find that – if countries are asymmetric and industrial commodities are independent from each other – pairs of countries sign a trade agreement only if their industrialization levels are similar.

5.5 WTW Properties and Country-specific Characteristics

An interesting issue to explore concerns the extent to which country specific characteristics relate to network properties. In fact, we expect not only the former to determine the latter, as usually claimed in the international trade literature (Baier and Bergstrand, 2004), but also the position of each country within the WTW to shape economic dynamics (Kali et al., 2007). We focus here on the correlation patterns between network indicators and country per capita GDP (pcGDP) in order to see whether countries with a higher income are more integrated into world trade or more clustered.

As far as degree and strength are concerned, the outcomes are very clear: there seems to be a relatively high and persistent positive correlation between connectivity levels and pcGDP (Figure 13), both in terms of the number of trade partners a country holds and in terms of the intensity of its trade interactions. However, the strength-pcGDP correlation appears to be stronger than the degree-pcGDP one.[23] Therefore, high-income countries tend to hold more, and more intense, trade relationships.

Results for clustering-pcGDP correlations mimic instead those obtained for the correlation between clustering and degree/strength. High-income countries tend to be less clustered from a BUN point of view, while they are more clustered (and increasingly so over the years) from a weighted perspective (Figure 14). This result supports the "rich club phenomenon" interpretation discussed above. The fact that this correlation is increasing over the years suggests that cliquishness among richer countries has been augmenting so that, as long as the strength of trade relations is concerned, further integration for the overall network can be attributed to stronger ties among advanced countries.

5.6 Centrality

So far we have treated nodes as if they were anonymous, not considering which countries display higher or lower network properties. Now we address the role each country plays in the WTW by means of a measure of global *centrality*. By doing so, we will be able to explicitly characterize the core and the periphery of the network, whose existence is hinted at by our results, and to compare them.

We compute *random walk betweenness centrality* (RWBC) for each of the countries in the sample and use the results to classify them as part of the core and or of the periphery. It turns out that – due to the high density that characterizes the WTW – the binary version of RWBC is almost perfectly correlated with

[23] Also the shape of the underlying relation is different. While degree seems to be linearly related to pcGDP, a log-log relation holds between strength and pcGDP. This means that pcGDP influences more heavily node strength than node degree.

ND[24]: as a result, in what follows we will focus only on the weighted version of RWBC. A second reason to look at weighted RWBC only is that so far weighted indicators seemed to give a better representation of the network structure, and in particular to hint more directly to a core-periphery structure.

RWBC is the highest order measure of integration considered in this study, since it takes into consideration paths of any given length that go through the country under analysis. In other words it measures the likelihood of a given country appearing in a randomly selected trade chain within the network. This likelihood is determined by the number and intensity of trading relationships, those of country i and those of all other countries. Therefore more central countries are more influential because they have a higher number of direct connections, which are also characterized by high intensity.

Figure 15 presents the distribution of weighted RWBC for 1981, 1990, and 2000. The observed patterns have not changed over time and the distributions are heavily skewed to the right, confirming the hypothesis of a clear-cut core-periphery structure. To identify the countries actually belonging to the core we (arbitrarily) impose a threshold at the 95th percentile of RWBC: hence, only countries with a value of centrality within the top 5% are considered core.[25] Table 2 displays the 8 countries forming the core between 1981 and 2000. Interestingly, this simple information turns out to be very powerful in describing the evolution of international trade integration in the last two decades of the XXth century, and can actually trace a number of relevant economic episodes. For instance, unification allows Germany to overcome Japan in this special ranking and gain the second place, whereas the dissolution of Soviet Union marked the exit from the core, as Russia (which took its place in the sample) never comes close to reaching the first 5% of the sample. Moreover, the 1981 peak in oil prices that followed the second shock and the beginning of the Iran –Iraq war results in Saudi Arabia being briefly included into the core, though it drops quickly out of it and further away in the following years as the price of crude oil drops down.

More recently, the increasing importance acquired by Asian countries –most notably China, but also South Korea– in international trade is captured by our centrality index. Both countries have become part of the core in 2000, after having been close to achieve this already in 1995. Other Asian countries such as Malaysia, India and (above all) Thailand have experienced a remarkable increase in their RWBC over the last twenty years. On the contrary, Latin American countries (i.e., another classical group of emerging markets) did not manage to climb the ranking as fast as their Asian counterparts. For instance, Brazil displays a very stable measure of centrality, whereas Mexico and Argentina are characterized by wide fluctuations both in the absolute value of the RWBC index and in the relative position in the ranking. Among Latin American countries only Chile and, to a lesser extent, Colombia do appear to improve their status within the WTW

[24]The correlation between the two indicators is not statistically different from 1.

[25]A very similar result is obtained if one attributes the core status to those countries displaying values of RWBC above the mean plus one standard deviation.

network, although they have remained quite peripheral. Qualitatively, the picture does not change if we let the data "choose" core countries. This can be done by ranking them in terms of RWBC and re-define the core as the set of countries commanding at least 50% of world trade. The outcome is very similar to the previous one and suggests the presence of a small (and stable) number of countries playing a pivotal role in the WTW.

Finally, the analysis of the correlation between per capita GDP and RWBC reveals a similar pattern to that observed for the relationship between node strength and pcGDP.[26]

5.7 Robustness

All results obtained so far refer to a particular weighting procedure. To recall, each directed link from node i to j is weighted by total exports of country i to country j and then divided by the country i's GDP (i.e., the exporter country). Such a weighting setup allows one to measure how much economy i depends on economy j as a buyer.

Are our findings robust to alternative weighting schemes? To address this issue, we consider the two alternative setups discussed in Section 4. In the first setup, we still remove size effects from trade flows, but we now divide by the GDP of the *importer* country (j's GDP, in the above example). In the second setup, we retain the size effect and we simply define the weight of link (i, j) as total exports from i to j.

All our main results turn out to be quite robust to all these alternatives.[27] This is an important point, as a weighted network analysis might in principle be sensible to the particular choice of the weighting procedure.

To begin with, we compare the symmetry index for the three weighting schemes across the years. If one scales exports by the importer's GDP the symmetry index stays very close to the one found in the baseline weighting schemes, whereas if one does not scales by the GDP the index is surprisingly lower, indicating that raw trade matrices are even more symmetric that rescaled ones. This indicates that under all three schemes a WUN analysis is appropriate.

As a further illustration, Figure 16 reports the correlation structure between ANNS, clustering, node strength and pcGDP across years.[28] Left panels refer to the first alternative weighting scheme (exports scaled by importer GDP) whereas right panels shows what happens under the second alternative setup (no GDP scaling at all). All previous results are confirmed. Notice that if we do not scale exports, an even stronger correlation emerges in all years between weighted clustering and strength. Of course, we

[26]This is expected since one of the interpretations of node strength is related to the degree of influence that a given node has on the network or to what extent other nodes depend on a given node; also, the correlation between RWBC and NS is very high.

[27]As mentioned, we have also experimented with another weighting scheme where we have symmetrized the graph before dividing by exporter (or importer) GDP. All these alternatives did not result in any significant change of our main findings.

[28]More detailed results are available from the authors upon request.

do not expect our results to hold irrespective of *any* weighting scheme to be adopted. In fact, the BUN characterization of the WTW is itself a particular weighting scheme, one that assigns to each existing link the same weight.[29]

6 Concluding Remarks

In this paper, we have explored the empirical properties of the world trade web (WTW) using network analysis. This allows us to better characterize the degree of international trade integration and to track its evolution over time. Following a stream of recent literature, we have conceptualized the web of trade relationships across countries as a weighted network where countries play the role of nodes, and trade flows represents links between nodes. To that end, we have studied imports/exports flows between all pairs formed out of 159 countries, from 1981 to 2000.

From a methodological point of view, our paper is among the first to address a thorough empirical investigation of the statistical properties of the WTW as a *weighted* network. This means that instead of accounting for the mere presence of a trade relationship between any two countries, we estimate the intensity of any trade relationship by some function of the value of trade flows carried by that link. Our results show that a weighted analysis can deliver a different insight as far as the topological structure and statistical properties of the WTW are concerned. Indeed, many findings obtained by only looking at the number of trade relationships that any country maintains are completely reversed if one takes into account the relative intensity of trade links. Furthermore, our main (qualitative) findings seem to be quite robust to a number of alternative, economically meaningful, weighting schemes.

From a descriptive point of view, this paper can be considered as an attempt to single out some robust stylized facts pertaining to the evolution of the WTW structure. As compared to standard international-trade statistical investigations, network analysis allows the researcher to explore not only first-order phenomena associated to import-export patterns of any given country (e.g., the degree of openness to trade) but also second- and higher-order empirical facts concerning, for example, the extent to which highly connected countries tend to trade with poorly connected ones, the likelihood that trade partners of highly connected countries are themselves parters, and so on. In this respect, this study can be considered as a preliminary step towards a modeling exercise that attempts to replicate and explain the statistical regularities that we empirically observe (e.g. in the framework of endogenous-network formation models à la Jackson, 2004).

Our exercises show that the WTW is an extremely symmetric network, where almost all trade relationships tend to be reciprocated with similar intensities. This allows one to study the WTW as if it were a

[29] In this respect, an interesting exercise would imply to find (if any) a proper rescaling or manipulation of original trade flows that makes WUN and BUN results looking the same.

weighted *undirected* network. Notwithstanding a very high density, the average strength of nodes is rather poor. Indeed the majority of countries holds mainly weak relationships, whereas only a selected core on nodes combine high degree and high strength. This hints to a core-periphery structure for the weighted WTW. This insight is confirmed by the finding regarding the "disassortative" nature of the WTW: our data show that countries holding many (and more intense) trade relationships preferably trade with poorly-connected countries.

From a policy perspective the polarized structure of international trade is sub-optimal. From a standard Ricardian point of view it prevents all countries to fully exploit the benefits of comparative advantages and factor endowments. Similarly, if one takes an approach closer to endogenous growth theory and assumes that the main benefit from trade comes from the flows of ideas and from market size, it is clear that occupying a peripheral place in the WTW can hinder growth and development.

Furthermore, while the average number of trade relationships has increased through time, their average intensity has remained quite stable. More generally, all structural properties of the WTW display a remarkable stationarity across the years. This stability implies that trade integration has not increased dramatically over the last 20 years or, in other words, that its change has not had a significant impact on the structure of the WTW. A possible explanation is that trade integration has been steadily growing since the 1950s and the bulk of it had been achieved before the period under consideration here. This means that the recent wave of globalization has not altered significantly the structure of the WTW, though one observes countries such as China and India rapidly gaining ground in terms of centrality in the network.

We also find that the WTW, viewed as a binary undirected network, is highly clustered. Moreover, countries that hold more trade partners (high degree) are less clustered than those holding few partners. These conclusions are completely different when we account for the importance of each link. Indeed, the weighted version of the WTW displays a relatively weaker clustering level and (the few) countries with high-intensity trade relationships are typically involved in highly-interconnected trade triples. Hence, there exists a small group of tightly connected countries that play a pivotal role in the network of world trade.

Finally, we have studied the relationships between network properties and country income. We have shown that high-income countries tend to form more (and more intense) trade links and to be more clustered (and increasingly so over the years).

As mentioned, this work represents a preliminary step towards a better understanding of the topological properties of the WTW and its dynamics. The agenda of interesting issues to address in the future is therefore quite rich. Firstly, one would like to explore in more details the topological properties of the WTW, both cross-sectionally and time-series. Meaningful questions here concern the role of geographical proximity in shaping the structure of international trade, the degree of fragility of the network, and so on. Furthermore, trade flows could be disaggregated across product classes to explore how trade composition affects network properties.

Secondly, one could abstract from aggregate statistical properties and analyze at a finer level the role of single countries in the network structure. For instance, how does the dynamics of degree, strength, clustering, etc. behave for single relevant countries in different World regions? Do country-specific network indicators display the same time-stationarity of their aggregate counterparts?

Finally, to dig deeper in the policy implications outlined here, and in line with work like Kali and Reyes (2007), one can ask whether the topological properties of the WTW, viewed as a weighted network, are able to explain the macroeconomic dynamics of growth and development.

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Appendix A: Statistical Analysis of Binary and Weighted Networks

Preliminaries

In this appendix, we present some more formal definitions of the statistics introduced in Section 2 for both binary and weighted networks, and we provide a compact matrix-notation useful to compute them (see also Albert and Barabási, 2002; Newman, 2003; Fagiolo, 2007, for an introduction).

Consider a notionally-directed and possibly weighted network composed of N nodes. Let $\tilde{W} = \{\tilde{w}_{ij}\}$ be a $N \times N$ weight matrix (not necessarily symmetric), where $\tilde{w}_{ij} \in [0, 1]$ and $\tilde{w}_{ii} = 0$ for all i. The binary case will imply that $\tilde{w}_{ij} \in \{0, 1\}$. We assume that a directed link from i to j exists if and only if $\tilde{w}_{ij} > 0$. The adjacency $N \times N$ matrix $\tilde{A} = \{\tilde{a}_{ij}\}$, where $\tilde{a}_{ij} \in \{0, 1\}$, is thus defined from $\tilde{W}$ by letting $\tilde{a}_{ij} = 1$ iff $\tilde{w}_{ij} > 0$ (and zero otherwise).

In what follows, we will also define $X_{(i)}$ as the i-th row of matrix X; $X^{[k]}$ as the matrix obtained from X by raising to k each entry; and $\frac{\{\mathbf{u}\}}{\{\mathbf{v}\}}$ as the vector obtained by dividing the two vectors entry by entry.

Checking for Symmetry

To check if an empirically-observed weighted network W is sufficiently symmetric to justify an undirected analysis, we employ the index developed in Fagiolo (2006). The index is based on the following idea. If a network is symmetric then any norm of the (suitably normalized) difference between $\tilde{W}$ and $\tilde{W}^T$ (i.e., its transpose) should vanish.

To build the index, define, without loss of generality:

$$Q = \{q_{ij}\} = \tilde{W} - (1 - \tilde{W})I_N, \tag{2}$$

where I_N is the $N \times N$ identity matrix. Notice that $q_{ij} = \tilde{w}_{ij}$ for all $i \neq j$, while now $q_{ii} = 1$ for all i[30].

Consider then the square of the Frobenius (or Hilbert-Schmidt) norm:

$$\|Q\|_F^2 = \sum_i \sum_j q_{ij}^2 = N + \sum_i \sum_{j \neq i} q_{ij}^2, \tag{3}$$

where all sums (also in what follows) span from 1 to N. The index used to check for symmetry is defined as:

[30] The need for recovering self-loops is only required to have an index which is strictly increasing in the degree of asymmetry of the underlying graph, see Fagiolo (2006) for details.

$$\widetilde{S}(Q) = \frac{\|Q - Q^T\|_F^2}{\|Q\|_F^2 + \|Q^T\|_F^2} = \frac{\|Q - Q^T\|_F^2}{2\|Q\|_F^2} = \frac{1}{2}\left[\frac{\|Q - Q^T\|_F}{\|Q\|_F}\right]^2. \tag{4}$$

It is easy to see that:

$$\widetilde{S}(Q) = 1 - \frac{\sum_i \sum_j q_{ij} q_{ji}}{\sum_i \sum_j q_{ij}^2}. \tag{5}$$

Furthermore, the scaled version of $\widetilde{S}(Q)$

$$S(Q) = \frac{N+1}{N-1}\widetilde{S}(Q), \tag{6}$$

ranges from 0 (full symmetry) to 1 (full asymmetry). In order to use the index as a statistically-sound check for symmetry, let us suppose that entries in $\tilde{W}$ are independently and identically distributed as a uniform random variable defined in the unit interval. In that case, one can find coefficients $(m_B(N), s_B(N))$, which depend both on N and on the binary (B) vs. weighted (W) nature of the underlying graph (i.e. of $\tilde{W}$), such that

$$S_B(Q) = \frac{S(Q) - m_B(N)}{s_B(N)} \tag{7}$$

$$S_W(Q) = \frac{S(Q) - m_W(N)}{s_W(N)} \tag{8}$$

are distributed as a standardized Normal random variable. This can help one in assessing the extent to which an empirically-observed binary/weighted graph is directed or not. Positive (respectively, negative) values of the standardized index (e.g., $k = 1, 2, ...$ standard deviations away from zero) would suggest that the graph is directed (respectively, undirected).

Notice that, in the case the notionally-directed graph $\tilde{W}$ turns out to "look" as an undirected graph, common practice calls for a symmetrization of binary/weighted links. In the case of binary graph, we will let:

$$A = \{a_{ij}\} = max\{\tilde{a}_{ij}, \tilde{a}_{ji}\}, \tag{9}$$

whereas if the graph is weighted we define:

$$W = \{w_{ij}\} = \frac{1}{2}(\tilde{w}_{ij} + \tilde{w}_{ji}). \tag{10}$$

Binary Undirected Networks, BUNs

Let us suppose that the underlying graph is binary and undirected and let A be its adjacency matrix.

The degree of node i (or node degee, ND) is defined as

$$d_i = \sum_j a_{ij} = A_{(i)}\mathbf{1}, \tag{11}$$

where $\mathbf{1}$ is the N-vector made of all ones.

Similarly, the average nearest-neighbor degree (ANND) of node i reads:

$$annd_i = d_i^{-1} \sum_j a_{ij} d_j = d_i^{-1} \sum_j \sum_h a_{ij} a_{jh} = \frac{A_{(i)} A \mathbf{1}}{A_{(i)} \mathbf{1}}. \tag{12}$$

Finally, node i's clustering coefficient (CC), defined as the ratio of the number of triangles with i as one vertex, to the maximum number of triangles that node i could have formed given its degree (Fagiolo, 2007), is equal to:

$$C_i(A) = \frac{\frac{1}{2}\sum_{j\neq i}\sum_{h\neq(i,j)} a_{ij}a_{ih}a_{jh}}{\frac{1}{2}d_i(d_i-1)} = \frac{(A^3)_{ii}}{d_i(d_i-1)}. \tag{13}$$

Notice that in a random graph where links are in place, independently of each other, with a probability $p > 0$, the expected value for the CC is equal to p.

Weighted Undirected Networks, WUNs

Let us now assume that the underlying graph is weighted and undirected and let W be its weight matrix.

Firstly, node strength of i is defined as :

$$s_i = \sum_j w_{ij} = W_{(i)}\mathbf{1}. \tag{14}$$

Furthermore, the average nearest-neighbor strength (ANNS) of i is computed as the arithmetic mean of strengths of i's neighbors as follows:

$$anns_i = d_i^{-1}\sum_j a_{ij}s_j = d_i^{-1}\sum_j\sum_h a_{ij}w_{jh} = \frac{A_{(i)}W\mathbf{1}}{A_{(i)}\mathbf{1}}. \tag{15}$$

Similarly, the weighted average of nearest-neighbor degrees (WANND) of i reads:

$$wannd_i = s_i^{-1}\sum_j w_{ij}d_j = s_i^{-1}\sum_j\sum_h w_{ij}a_{jh} = \frac{W_{(i)}A\mathbf{1}}{W_{(i)}\mathbf{1}}. \tag{16}$$

Sometimes, it is also useful to define "node disparity" among (concentration of) i's weights as follows:

$$h_i = \frac{(N-1)\sum_j\left(\frac{w_{ij}}{s_i}\right)^2 - 1}{N-2} = \frac{(N-1)\frac{1}{s_i^2}\sum_j w_{ij}^2 - 1}{N-2} = \frac{(N-1)\frac{W_{(i)}^{[2]}\mathbf{1}}{(W_{(i)}\mathbf{1})^2} - 1}{N-2} \tag{17}$$

As far as the weighted version of the CC for WUNs is concerned, we focus here on the extension of the CC to WUNs originally introduced in Onnela et al. (2005):

$$\tilde{C}_i(W) = \frac{\frac{1}{2}\sum_{j\neq i}\sum_{h\neq(i,j)} w_{ij}^{\frac{1}{3}}w_{ih}^{\frac{1}{3}}w_{jh}^{\frac{1}{3}}}{\frac{1}{2}d_i(d_i-1)} = \frac{(W^{[\frac{1}{3}]})^3_{ii}}{d_i(d_i-1)}, \tag{18}$$

where we define $W^{[\frac{1}{k}]} = \{w_{ij}^{\frac{1}{k}}\}$, i.e. the matrix obtained from W by taking the k-th root of each entry. As discussed in Saramaki et al. (2007), the index $\tilde{C}_i$ ranges in $[0,1]$ and reduces to C_i when weights become binary. Furthermore, it takes into account weights of all edges in a triangle (but does not consider weights not participating in any triangle) and is invariant to weight permutation for one triangle. The expected value of the weighted CC in a random graph where links are in place, independently of each other, with a probability $p > 0$, is equal to $(\frac{3}{4})^3 p$.

Random-Walk Betweenness Centrality (RWBC)

Suppose the underlying graph, interpreted as a current circuit, is a WUN and let W be its weight matrix and s the $N\times 1$ strength vector. Following Newman (2005) and Fisher and Vega-Redondo (2006), consider a generic node i for which we want to compute the RWBC and an impulse generated from node h (the source) and working its way to node k (the target). Let $f(h,k)$ be the "source" $N\times 1$-vector such that $f_i(h,k) = 1$ if $i = h$, $f_i(h,k) = -1$ if $i = k$, and 0 otherwise. Define by $v(h,k)$ the $N\times 1$-vector of node voltages. Newman (2005) shows that Kirchoff's law of current conservation implies that:

$$v(h,k) = [D-W]^{-1}f(h,k), \tag{19}$$

where $D = diag(s)$ and $[D-W]^{-1}$ is computed using the Moore-Penrose pseudo-inverse.

This in turn implies that the current (i.e. intensity of interaction) flowing through node i, originated from h and getting to k, is given by:

$$I_i(h,k) = \frac{1}{2}\sum_j |v_i(h,k) - v_j(h,k)|, \tag{20}$$

where $I_h(h,k) = I_k(h,k) = 1$.

It is then straightforward to define node-i RWBC as:

$$RWBC_i = \frac{\sum_h \sum_{k \neq h} I_i(h,k)}{N(N-1)}. \tag{21}$$

Appendix B: Countries in the Balanced Panel (1981-2000)

The dataset provided by Gleditsch (2002) includes 196 countries for which there are data on trade flows from 1948 to 2000. However, trade data contain many missing (or badly reported) values before 1970. In addition, there are some countries with zero total exports in some years.

Notice also that our analysis requires to match trade data with real GDP (both in levels and per capita). This is because: (i) weights are defined as exports divided by GDP; (ii) one wants to cross-sectionally correlate network measures with country-specific variables like per-capita GDP.

We have therefore selected countries in such a way to have: (i) a time horizon and a country sample size as long as possible; (ii) no missing values in trade data and GDP (both in levels and per capita); (iii) non-zero total exports.

By applying conditions (i) and (ii) we get only 83 countries from 1960-2000. This number becomes 138 for the period 1970-2000; 152 for the period 1970-2000; 163 for the period 1981-2000; and 168 for the period 1990-2000. We thus decided to select the time interval 1981-2000 using 163 countries. However, 4 of them (San Marino, Andorra, Liechtenstein, Monaco) have total exports equal to zero in some years. This leaves us with N=159 countries, whose list is in Table 1.

Table 1: List of Countries in the Balanced Panel.

Id	Acro	Name	Id	Acro	Name	Id	Acro	Name	Id	Acro	Name	Id	Acro	Name	Id	Acro	Name
2	USA	United States	150	PAR	Paraguay	395	ICE	Iceland	520	SOM	Somalia	678	YEM	Yemen	910	PNG	Papua
20	CAN	Canada	155	CHL	Chile	402	CAP	Cape Verde	522	DJI	Djibouti	690	KUW	Kuwait	920	NEW	New Zealand
31	BHM	Bahamas	160	ARG	Argentina	403	STP	Sao Tome	530	ETH	Ethiopia	692	BAH	Bahrain	935	VAN	Vanuatu
40	CUB	Cuba	165	URU	Uruguay	404	GNB	Guinea-Bissau	540	ANG	Angola	694	QAT	Qatar	940	SOL	Solomon's
41	HAI	Haiti	200	UKG	United Kingdom	411	EQG	Eq. Guinea	541	MZM	Mozambique	696	UAE	Arab Emirates	950	FJI	Fiji
42	DOM	Dominican Rep.	205	IRE	Ireland	420	GAM	Gambia	551	ZAM	Zambia	698	OMA	Oman	970	KBI	Kiribati
51	JAM	Jamaica	210	NTH	Netherlands	432	MLI	Mali	552	ZIM	Zimbabwe	700	AFG	Afghanistan	971	NAU	Nauru
52	TRI	Trinidad/Tobago	211	BEL	Belgium	433	SEN	Senegal	553	MAW	Malawi	710	CHN	China	972	TON	Tonga
53	BAR	Barbados	212	LUX	Luxembourg	434	BEN	Benin	560	SAF	South Africa	712	MON	Mongolia	973	TUV	Tuvalu
54	DMA	Dominica	220	FRN	France	435	MAA	Mauritania	570	LES	Lesotho	713	TAW	Taiwan			
55	GRN	Grenada	225	SWZ	Switzerland	436	NIR	Niger	571	BOT	Botswana	731	PRK	North Korea			
56	SLU	Saint Lucia	230	SPN	Spain	437	CDI	Cote DIvoire	572	SWA	Swaziland	732	ROK	South Korea			
57	SVG	St. Vincent	235	POR	Portugal	438	GUI	Guinea	580	MAG	Madagascar	740	JPN	Japan			
58	AAB	Antigua	260	GFR	Germany	439	BFO	Burkina Faso	581	COM	Comoros	750	IND	India			
70	MEX	Mexico	290	POL	Poland	450	LBR	Liberia	590	MAS	Mauritius	760	BHU	Bhutan			
80	BLZ	Belize	305	AUS	Austria	451	SIE	Sierra Leone	591	SEY	Seychelles	770	PAK	Pakistan			
90	GUA	Guatemala	310	HUN	Hungary	452	GHA	Ghana	600	MOR	Morocco	771	BNG	Bangladesh			
91	HON	Honduras	325	ITA	Italy	461	TOG	Togo	615	ALG	Algeria	775	MYA	Myanmar			
92	SAL	El Salvador	338	MLT	Malta	471	CAO	Cameroon	616	TUN	Tunisia	780	SRI	Sri Lanka			
93	NIC	Nicaragua	339	ALB	Albania	475	NIG	Nigeria	620	LIB	Libya	781	MAD	Maldives			
94	COS	Costa Rica	345	YUG	Yugoslavia	481	GAB	Gabon	625	SUD	Sudan	790	NEP	Nepal			
95	PAN	Panama	350	GRC	Greece	482	CEN	Centr African Rep.	630	IRN	Iran	800	THI	Thailand			
100	COL	Colombia	352	CYP	Cyprus	483	CHA	Chad	640	TUR	Turkey	811	CAM	Cambodia			
101	VEN	Venezuela	355	BUL	Bulgaria	484	CON	Congo	645	IRQ	Iraq	812	LAO	Laos			
110	GUY	Guyana	360	RUM	Rumania	490	DRC	Congo (Zaire)	651	EGY	Egypt	816	DRV	Vietnam			
115	SUR	Surinam	365	RUS	Russia	500	UGA	Uganda	652	SYR	Syria	820	MAL	Malaysia			
130	ECU	Ecuador	375	FIN	Finland	501	KEN	Kenya	660	LEB	Lebanon	830	SIN	Singapore			
135	PER	Peru	380	SWD	Sweden	510	TAZ	Tanzania	663	JOR	Jordan	840	PHI	Philippines			
140	BRA	Brazil	385	NOR	Norway	516	BUI	Burundi	666	ISR	Israel	850	INS	Indonesia			
145	BOL	Bolivia	390	DEN	Denmark	517	RWA	Rwanda	670	SAU	Saudi Arabia	900	AUL	Australia			

Table 2: Countries in the core

1981	1985	1990	1995	2000
USA	USA	USA	USA	USA
Japan	Japan	Germany	Germany	Germany
Germany†	Germany†	Japan	Japan	Japan
UK	UK	France	France	France
France	France	UK	UK	UK
USSR	USSR	Italy	Italy	China
Italy	Italy	USSR	Belgium	Italy
Saudi Arabia	Netherlands	Netherlands	Netherlands	Korea

† Up to 1989 data refers to West Germany only.

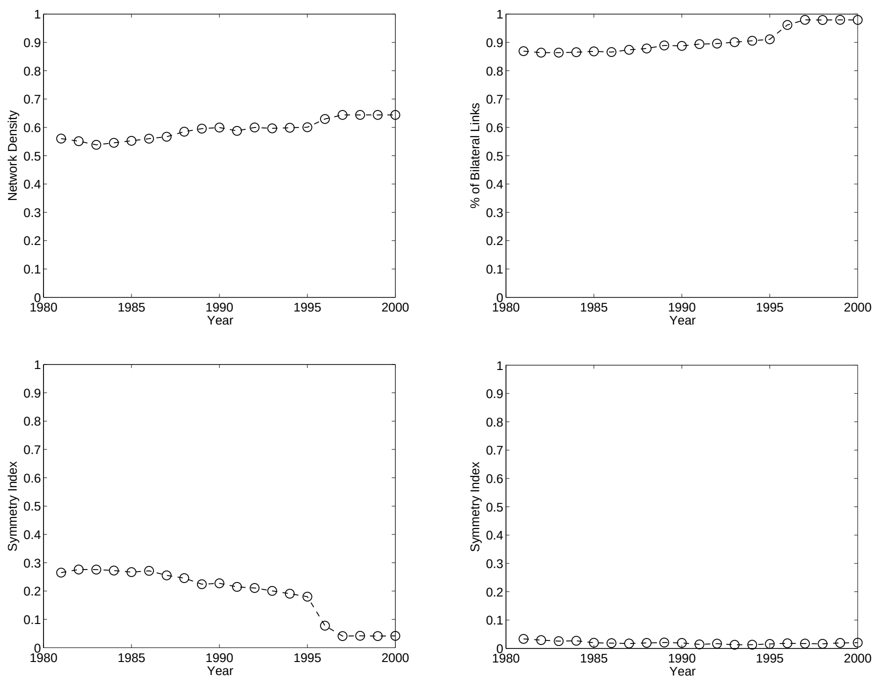
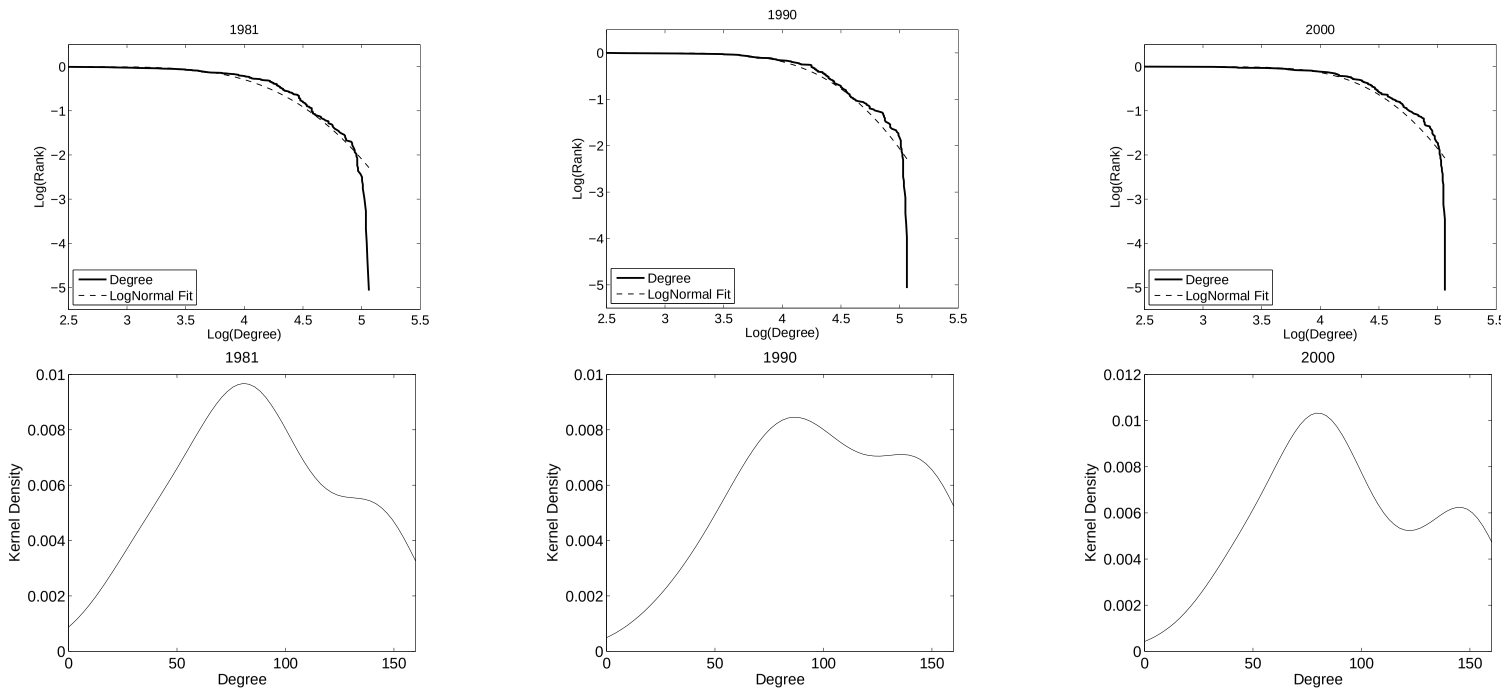


Figure 1: Global network indicators vs. years. Top-left: Network density. Top-right: Percentage of bilateral links. Bottom-left: S index (not standardized) for BUNs. Bottom-right: S index (not standardized) for WUNs.

Figure 2: The WTW as a BUN. Degree distributions in 1981, 1990, 2000. Top panels: Size-rank plots (dashed line: lognormal fit). Bottom panels: Kernel density estimates.



Note. Size-rank plots. X-axis: log of degree. Y-axis: Log of rank of x-axis observation.

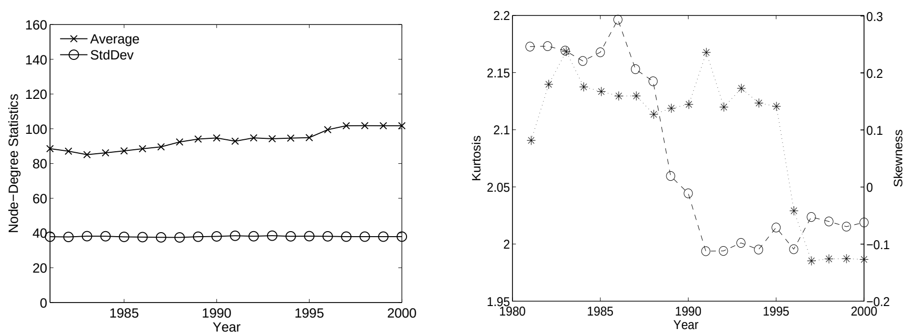
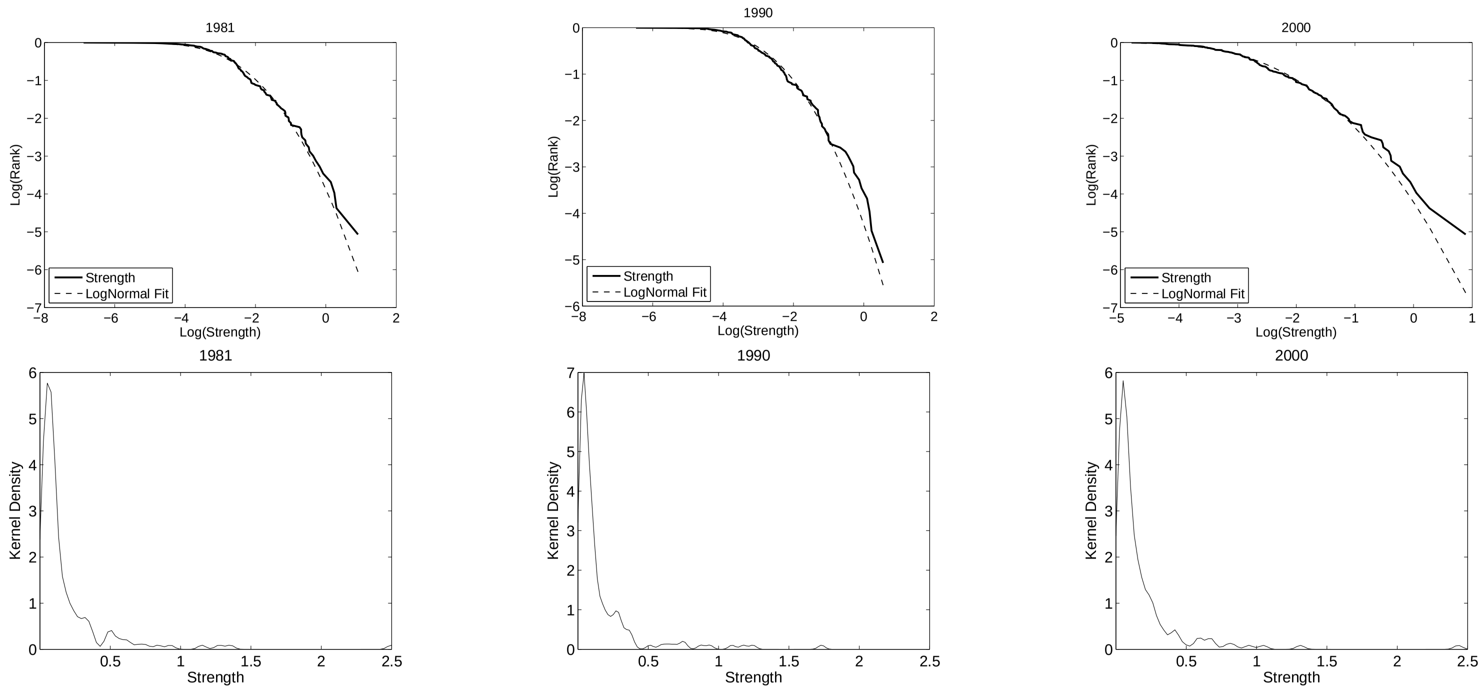


Figure 3: The WTW as a BUN. Left panel: Average and standard deviation of degree distributions. Right panel: kurtosis (circles) and skewness (asterisks) of degree distributions.

Figure 4: The WTW as a WUN. Strength distributions in 1981, 1990, 2000. Top panels: Size-rank plots (dashed line: lognormal fit). Bottom panels: Kernel density estimates.



Note. Size-rank plots. X-axis: log of strength. Y-axis: Log of rank of x-axis observation.

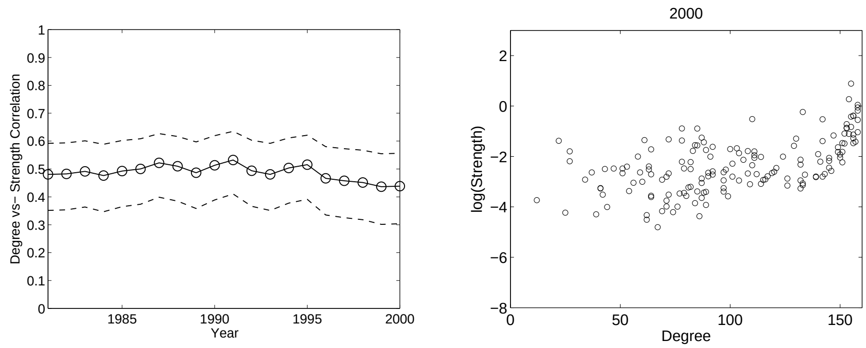


Figure 5: BUN vs. WUN. Left panel: Degree-strength correlation vs. years (dashed lines: 5% and 95% confidence intervals). Right panel: Degree-strength scatterplot in 2000.

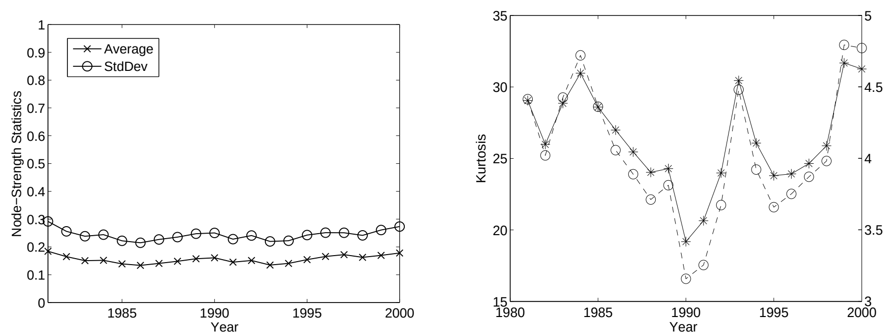


Figure 6: The WTW as a WUN. Left panel: Average and standard deviation of strength distributions. Right panel: kurtosis (circles) and skewness (asterisks) of strength distributions.

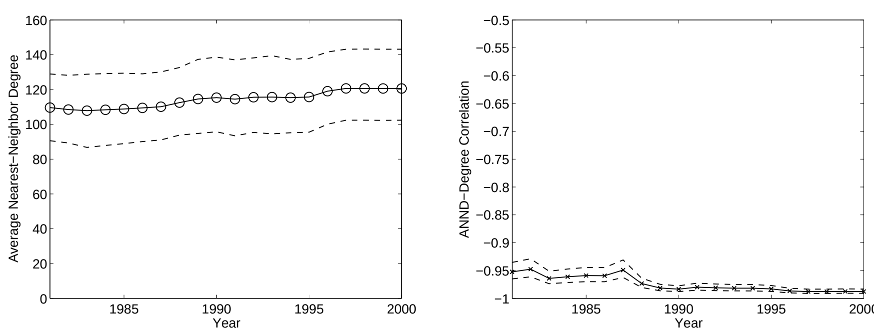


Figure 7: Average Nearest-neighbor degree (ANND). Left: Population average vs. years. Right: Correlation between (ANND) and degree vs. years. Dashed lines: 5% and 95% confidence intervals.

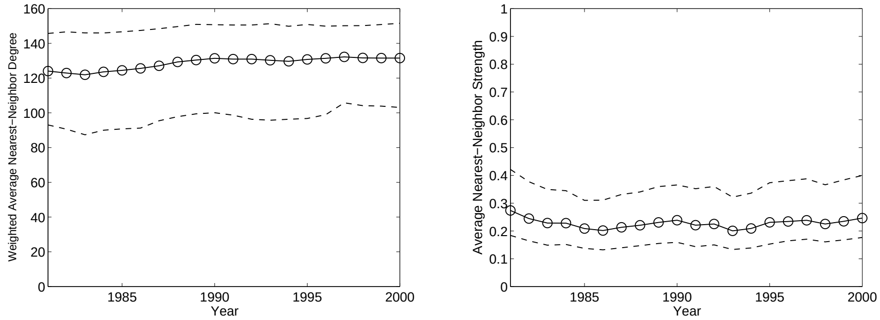


Figure 8: Left: Population-average of weighted average nearest-neighbor degree (WANND) vs. years. Right: Population-average of average nearest-neighbor strength (ANNS) vs. years. Dashed lines: 5% and 95% confidence intervals.

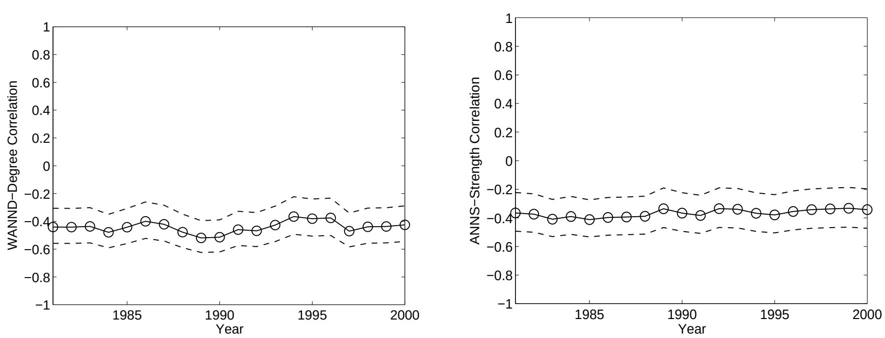


Figure 9: Left: WANND-degree correlation vs. years. Right: ANNS-strength correlation vs. years. Dashed lines: confidence intervals.

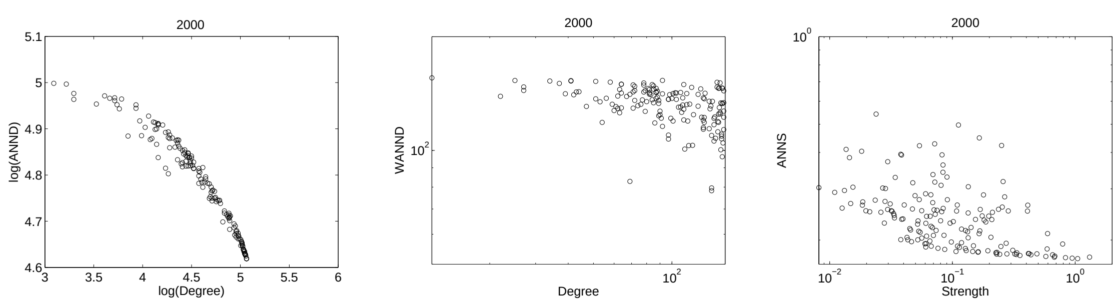


Figure 10: Left: ANND-degree scatter plot. Mid: WANND-degree scatter plot. Right: ANNS-strength scatter plot. Year: 2000.

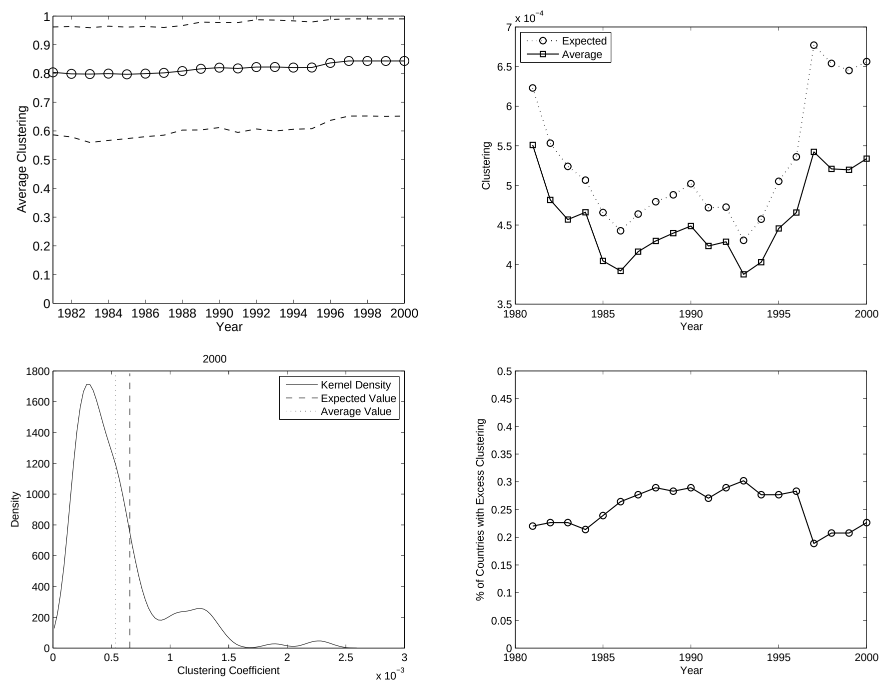


Figure 11: Top-Left: Average of BUN (binary) clustering coefficient vs. years. Dashed lines: 5% and 95% confidence intervals. Top-Right: Average and expected value of WUN (weighted) clustering coefficient vs. years. Bottom-Left: Kernel density of the WUN (weighted) clustering coefficient in 2000 vs. average and expected values. Bottom-right: Percentage of countries characterized by node clustering above its expected value. *Note*: Expected values computed by randomly reshuffling for 10000 times (in each year) the observed weights across the existing links.

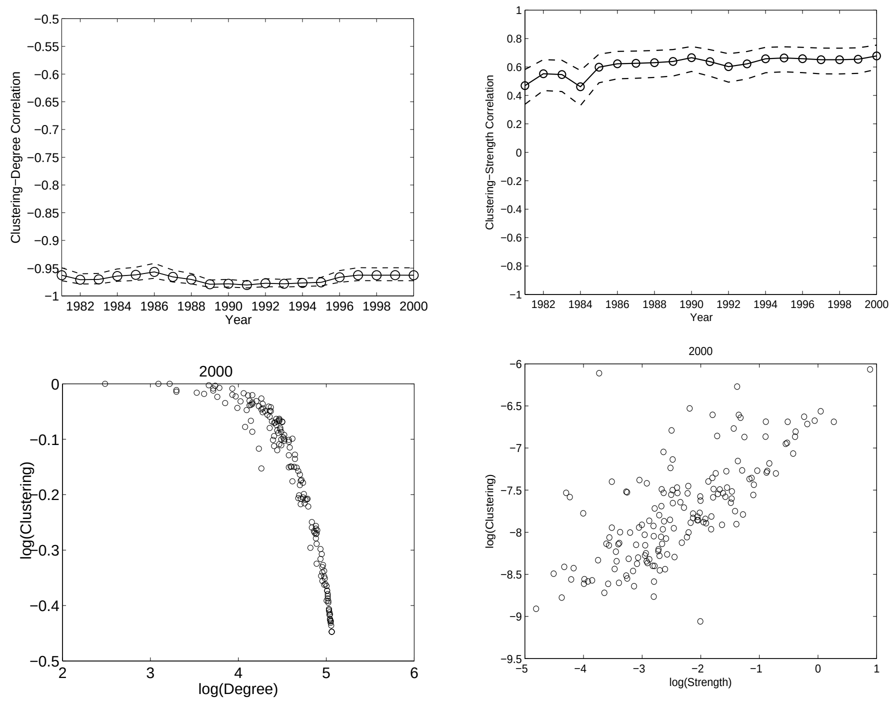


Figure 12: Top-left: Correlation between (binary) clustering coefficient and degree vs. years. Top-right: Correlation between (weighted) clustering coefficient and strength vs. years. Dashed lines: 5% and 95% confidence intervals. Bottom-left: Scatter plot of (binary) clustering coefficient and degree in year 2000. Bottom-right: Scatter plot of (weighted) clustering coefficient and strength in year 2000.

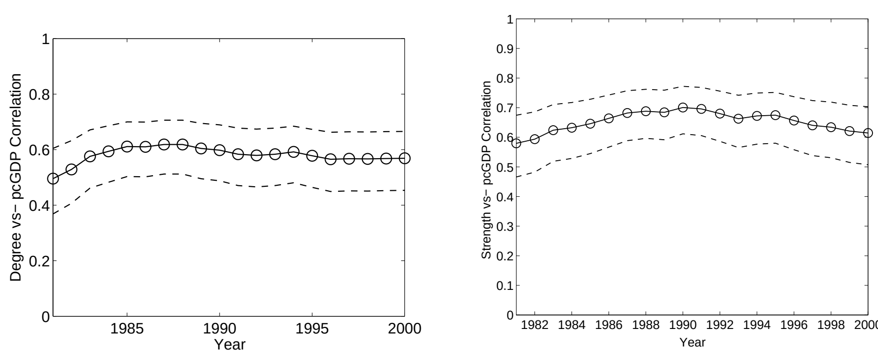


Figure 13: Correlation between degree-pcGDP and strength-pcGDP vs. years. Dashed lines: 5% and 95% confidence intervals.

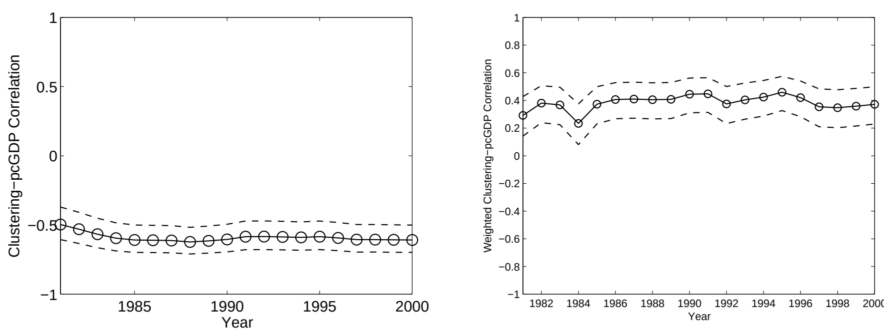
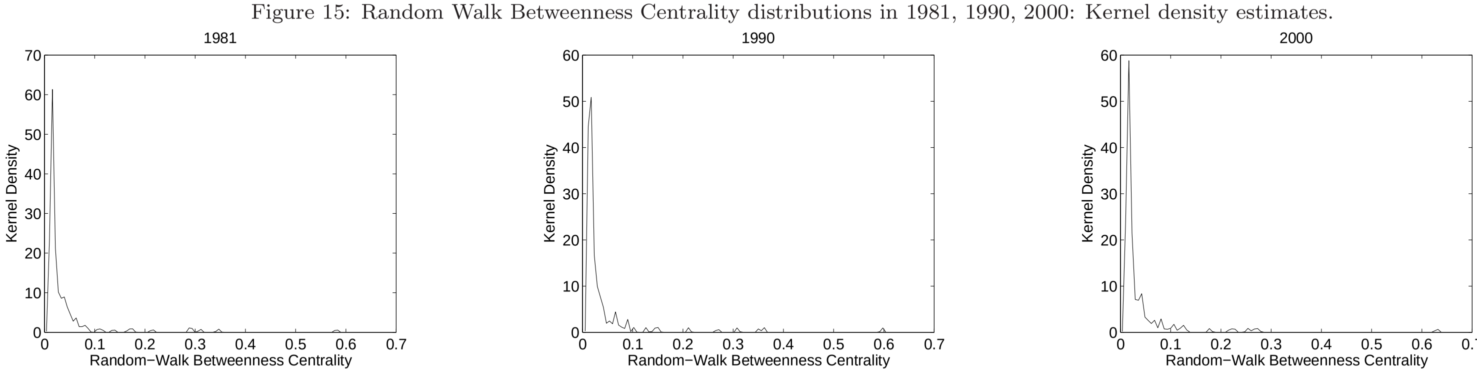


Figure 14: Left: Correlation between clustering and pcGDP in a BUN vs. years. Right: Correlation between clustering and pcGDP in a WUN vs. years. Dashed lines: 5% and 95% confidence intervals.

Figure 15: Random Walk Betweenness Centrality distributions in 1981, 1990, 2000: Kernel density estimates.



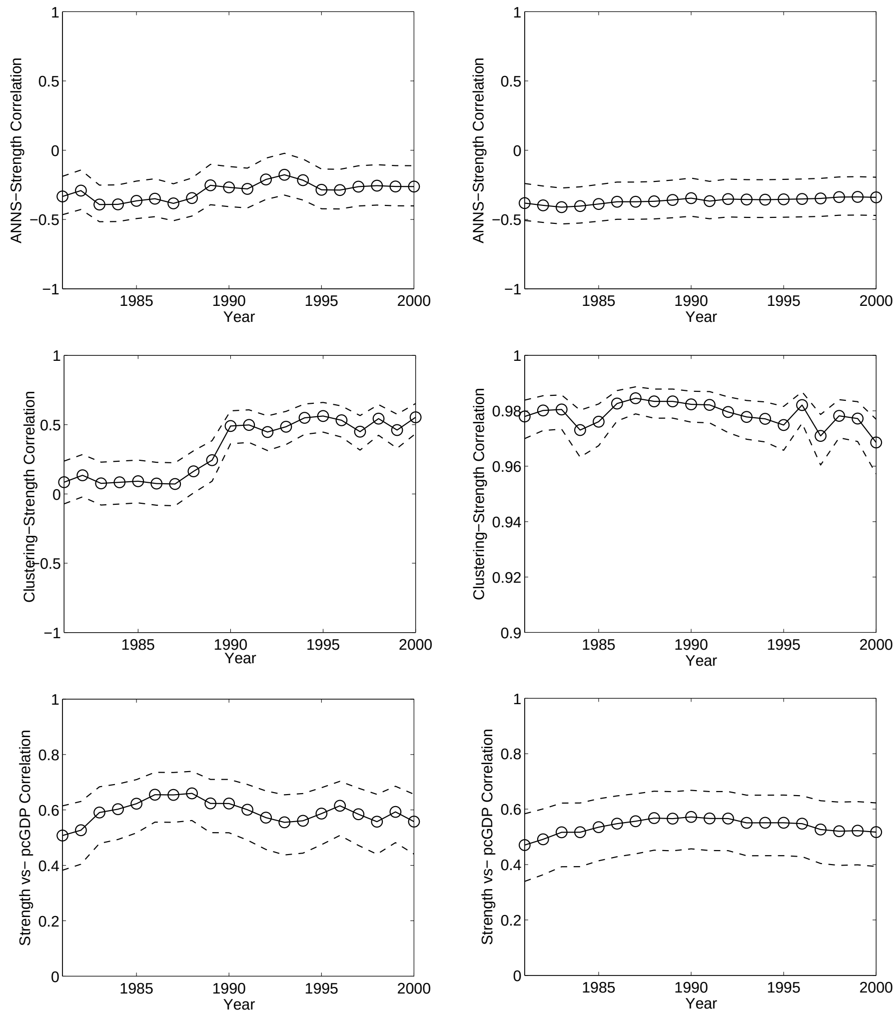


Figure 16: Alternative weighting schemes. Left panels: Exports divided by importer country GDP. Right panels: Exports not scaled by any country size measure. Top: Correlation between strength and ANNS vs. years. Mid: Correlation between clustering and strength vs. years. Bottom: Correlation between strength and pcGDP. Dashed lines: 5% and 95% confidence intervals.