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Intra-daily dynamics of dollar-sterling exchange market [☆]

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Abstract

This report describes the results of an initial exercise to explore the stylized facts of the dynamic properties of exchange rates employing a behavioral finance framework and high-frequency, second-scale, data. It uses a heterogeneous agent model, where chartist and fundamentalist traders coexist, in order to offer insights on their intra-daily movements and responses, to different market states. We find significant variations between the variables used to describe the agent's behavior in daily scales and that of intra-daily scales. The evolutionary stability of the otherwise *irrational* chartist behavior is demonstrated.

Keywords: Heterogeneous agents, Exchange rate decomposition, Kalman Filter

1. Introduction

Contrary to stock markets, where the trading times are determined by the locality of the market itself, foreign exchange markets trade continuously. Furthermore, according to the Bank of International Settlements, the USD-GBP pair accounts for 12% of the world total foreign exchange market turnover[1]. Therefore it is of great interest to analyze the underlying dynamics of the price formation mechanism for such a market.

Employing the standard chartist-fundamentalists' approach developed in the work of Brock and Holmes [2], the arithmetic mean between best Bid and best Ask is used to derive the market clearing price using the available tick-by-tick data. The data are then aggregated to 1-, 5-, 12- and 30- second intervals in order to be analyzed. Data of this nature are machine recorded, and while highly noisy and prone to extreme fluctuations, are not prone to errors, except in cases where traders themselves make mistakes which they then enter the system. Therefore these data exhibit a fertile field to analyze the effect of news and how these news influence intra-daily changes among traders' strategies. News is defined as interest rate announcements, made by the Bank of England. The work presented will attempt to address in a stylized manner the question of the short range effects of news, on players' behavior. To achieve this, we focus specifically on the data available for days, when it is known that news were released by the Bank of England[3].

There is a surprising absence of literature examining the second-scale data of intra-daily foreign exchange rates and how beliefs are formed on such a fine scale. Works

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by Sato [4, 5] and Mizuno [6] take an econophysics approach in the utilization of such high-frequency data. Nevertheless they focus mostly on results concerning statistical distributions and correlations of exchange rate fluctuations and do not address issues regarding the behavior of the traders.

The works of Frankel and Froot in 1986 [7] and in 1990 [8], introduced the modeling notion of a foreign exchange market that is structured around agents/traders employing a chartists-fundamentalists' belief framework. Based on this model, insight into traders behavior can be provided. De Grauwe and Dewachter [9] extended this framework, providing evidence about its chaotic nature and put a significant emphasis on the role of each individual strategy. If the reader wishes to probe further into a bibliographic survey of agent-based computational finance, LeBaron offers an invaluable reference [10].

The paper will be organized as follows: The next Section provides a basic description of the model employed. Section 3 will offer an outline of the Kalman Filter used here as the technique to derive the fundamental exchange rate. Section 4 will present the numerical estimation results and the effects that news arrivals appears to have on the system. The discussion and interpretation of the estimation results follows in Section 5. Finally section 6 will offer some concluding remarks.

2. Heterogeneous Agent Model

Following the theoretical framework introduced by De Grauwe and Grimaldi [11] and which was extended by Kozhan and Salmon [12], we consider the interaction between domestic and foreign currency through the exchange rate, which is denoted by s_t and is set to be the price of one unit of foreign currency in units of domestic currency. In the current case, for convenience the domestic currency is assumed to be the US dollar and foreign currency the GB Pound (£/\$) . The market is populated by N traders who can assume two distinctive strategies or "beliefs": chartists or fundamentalists.

Quoting Sarno and Taylor[13]: *Chartist, or "technical," analysis uses charts of financial-asset price movements to infer the likely course of future prices and thus to derive forecasts and trading strategies.* In essence, the chartist trader acts as a noise trader. They ignore the fundamentals of the exchange rate and extrapolate past movements of the market exchange rate s_t into the future. It is safe to say that the chartist approach is effectively a positive feedback loop. In the current setting, the simplest possible chartist rule is used mainly for analytic tractability. The chartist assumes that the change in the exchange rate will be equal to the arithmetic mean of the two most recent changes in the exchange rate Δs_{t-1} and Δs_{t-2} . Therefore their forecast is explicitly given by

$$E_t(s_{t+1}|C) = s_{t-1} + \beta \frac{1}{2}((s_{t-1} - s_{t-2}) + (s_{t-2} - s_{t-3})), \quad (1)$$

where $0 \leq \beta \leq 1$ represents the degree of chartist extrapolation. Clearly, modern day chartists use more highly sophisticated rules. On the one hand Kozhan and Salmon in their model use a simple long-short moving average rule, De Grauwe and Grimaldi use of an exponential decay memory mechanism. Brock and Hommes on the other hand in their seminal work used a *single* lag, in a simple linear forecasting rule [14]. Furthermore it seems rather improbable that all (or even enough) agents will coordinate their forecasts under the same rule, regardless of its sophistication. Nevertheless such a rule does not

prohibit certain stylized facts to be derived and quite possibly *amplifies* the tendency of chartists to trade on *noise*, emphasizing the differences in the two approaches.

A fundamentalist assumes the existence of an underlying equilibrium/latent price (fundamental value) s_t^* . Based on his estimation about the fundamental rate's true value, they then compare the present market clearing exchange rate with the fundamental exchange rate and forecasts the movement of the future market exchange rate as a drift towards the fundamental rate. Their forecasting rule can be explicitly given by

$$E_t(s_{t+1}|F) = s_{t-1} - \psi(s_{t-1} - s_{t-1}^*), \quad (2)$$

where $0 \leq \psi \leq 1$, represents the speed with which fundamentalists expect the market exchange rate to return to the fundamental exchange rate. In this case it can be easily argued that a fundamentalist approach is practically a negative feedback loop trying to anchor the market clearing exchange rate back to its fundamental value. The fundamentalists *know that there is a band of inactivity in the goods market* [11]. The meaning of this is, that if the market clearing exchange rate is not deviating from the fundamental exchange rate *enough*, there are no market mechanisms to drive the realized market clearing rate back to the exact latent fundamental rate. Thus, the fundamentalists realize that there are no profit opportunities for them and predict no actual change in the market clearing rate. More formally, if :

$$|s_{t-1} - s_{t-1}^*| < C \Rightarrow E_t(s_{t+1}|F) = s_{t-1} \quad (3)$$

where clearly if $|s_{t-1} - s_{t-1}^*| \geq C$, the original fundamentalists forecasting rule is employed as fundamentalists recognize profit opportunities.

One can easily observe that in both formulas the value of s_t is not present despite t being the actual moment of prediction. That is because under a Walrasian market equilibrium frame, as here, the agents have access only to publicly available information up to time $t - 1$ and also because the actual realized foreign exchange rate at time t depends on the forecasts for s_{t+1} [11]. This idea regarding the informational inefficiency of the markets, was put forward by Hellwig [15] and has proved central in recent research in the field [12, 16].

As argued by recent papers [16–19] there is increasing empirical evidence supporting the validity of this model. It appears, that determinants of dispersion involving the traders' expectations, show consistency with the assumptions of a exchange market environment that is populated by chartists and fundamentalists traders. Work by Taylor and Sarno [13] and earlier Taylor and Allen [20] have demonstrated the presence of technical traders in the market presenting a number of examples from foreign exchange markets around the world.

Going back to the technical details of the model it is clear that the proportion of these two types of agents are changing over time and are subject to their relative performance in profit-making. In each case the agents strive to maximize their utility function :

$$U(W_{t+1}^i) = E_t(W_{t+1}^i) - \frac{1}{2}\mu V^i(W_{t+1}^i), \quad (4)$$

where W_{t+1}^i is the wealth of agent i at time $t + 1$, E_t is the expectation operator, μ is the risk aversion coefficient, and $V^i(W_{t+1}^i)$ represents the conditional variance of the

wealth of agent i . This utility function is in direct relation, regarding its mean variance framework, with the Capital Asset Pricing model proposed by Treynor.

The exact wealth of an agent i is itself given by the following formula:

$$W_t^i = (1 + r^*)s_t d_{i,t-1} + (1 + r)(W_{t-1}^i - s_{t-1} d_{i,t-1}) \quad (5)$$

where r and r^* denote the domestic and foreign interest rate respectively and $d_{i,t}$ are the holdings of the foreign assets by agent i at time t . Using this setting, the first term in the above equation describes the value of the foreign portfolio in terms of the domestic currency and the second term, the value of the domestic portfolio itself.

One of the advantages of this setting is that it allows the analytical tractability of the model and the derivation of the optimal holdings of foreign assets in time t . That is done by substituting Eq. 5 in Eq.4 and then maximizing with respect to $d_{i,t}$. Therefore, more formally the optimal holdings of foreign assets are given by the equation:

$$d_{i,t} = \frac{(1 + r^*)E_t(s_{t+1}|i) - (1 + r)s_t}{\mu\sigma_{i,t}^2} \quad (6)$$

where $0 \leq \mu \leq 1$ is the coefficient of risk aversion and $\sigma_{i,t}^2$ the risk variable associated with this demand (more details on the calculation of the risk will follow). We also note that the optimal holdings depend on the expected market demand for the foreign asset D_t ; that is the sum of the weighted demands of all agents:

$$D_t = \sum_{i=1}^N n_{i,t} d_{i,t} \quad (7)$$

where $n_{i,t}$ is the number of agents employing the specific strategy i at time t . For equilibrium the market demand is always met by the market supply Z (So $Z_t = D_t$).

One of the cornerstones of the model is the behavior-specifying fitness criteria. The rules used here are a straightforward adaptation of the work of Brock and Hommes [2, 14] but a number of different criteria have been proposed [21, 22].

The current fitness criterion reflects the fraction of the agents in the population utilizing a specific strategy. It is a function depending on the risk adjusted profits and falls into the greater category of *logit* rules. One of the first to apply the logit rules in finance was Theil [23] and has ever since served as a common ground for econometric research. In the current case, the fraction of the population using chartist (w_c) or fundamentalist (w_f) forecasting rules are described as :

$$w_{i,t} = \frac{e^{\gamma\pi'_{i,t}}}{e^{\gamma\pi'_{C,t}} + e^{\gamma\pi'_{F,t}}}, \quad 0 \leq \gamma, i = \{F, C\} \quad (8)$$

where γ is the coefficient of update intensity and $\pi'_{i,t}$ the risk adjusted profit for agents using strategy i at time t . As γ increases agents choose the more profitable rule more quickly. On the other hand, as γ decreases agents are more hesitant to change their behavior preference based on the prior relative profitability of their strategy. While the reader might believe that it is reasonable for traders to have a $\gamma \rightarrow \infty$, as this would mean that the agents instantly switch their behavior to the most profitable strategy, empirical data show that traders do not behave according to this. Furthermore, and quite counter-intuitive, from initial simulations and in line with the current literature

[24], it is clear that the larger the γ , the higher the probability for the market clearing exchange rate to be in a bubble-regime and away from the fundamental price; this results in a more volatile and uncertain environment.

The risk-adjusted profit $\pi'_{i,t}$ is formally given by the following formula:

$$\pi'_{i,t} = \pi_{i,t} - \mu\sigma_{i,t}^2 \quad (9)$$

where $\pi_{i,t}$ is the profit prior to risk adjustment, μ as before is the risk aversion coefficient and $\sigma_{i,t}^2$ the variable representing the risk associated with each strategy. The profit function is formally given as:

$$\pi_{i,t} = [s_{t-1}(1+r^*) - s_{t-2}(1+r)]\text{sgn}[(1+r^*)E_{t-1}(s_t|i) - (1+r)s_{t-1}], \quad (10)$$

where:

$$\text{sgn}[x] = \begin{cases} -1 & \text{if } x < 0 \\ 0 & \text{if } x = 0 \\ 1 & \text{if } x > 0 \end{cases} \quad (11)$$

Examining first the profit function itself it is made clear that profit opportunities arise when the agents are able to exploit differences in the changes of the foreign exchange rate s . Also, on a more technical note, under the current model, profits are given in a *per unit* investment framework and not under total profit yield that can actually be based on the initial wealth accumulated by an agent. Despite that, during the initialization of the model we do assume all agents have an equal initial wealth. Another feature presented is that this model penalizes wrong but seemingly not directly wealth-losing decisions. If the market exchange rate rises, while the agent predicts that it would drop, then the agent makes a *per unit* loss, because it appears as if he has sold foreign assets at a lower price rather than actually holding (or buying), while the foreign assets have actually increased their value.

Equally important is the term $\sigma_{i,t}^2$ representing the risk values. Following the logic of DeGrauwe and Grimaldi [24] it is assumed that agents represent risk through the last period's forecast error:

$$\sigma_{i,t}^2 = (E_{t-1}(s_t|i) - s_t)^2 \quad (12)$$

This is the most simplified risk assumed form in the literature but it is clearly one of the most intuitive and less computationally demanding. It has to be mentioned here that this procedure of fitness measurement serves ultimately as a form of learning through social interaction. The agents employing a given strategy learn the merit/goodness of the other strategy and based on this knowledge, revise their own strategy preference.

It is debatable whether or not such rules as the one presented here based on *logit* principles or others, such as rules based on the Brown-von-Neumann-Nash principle [22] can be applied as a real life approximation, because they rely on assumptions about the agents having a full understanding about the other agents' success (or failures). Also, even if people shared all their information regarding their strategies' outcome, the fine time-scale of the model would raise certain issues about the efficiency of information propagation. Possibly more accurate would be a *regret* rule [25, 26] ; where agents decide their strategy based on whether or not certain criteria were met. This would though break away from the existing norm in the literature and make the results incompatible with the existing macrostructure literature.

While the idea behind the technical means used by the chartists in order to make a prediction is relatively obvious even in some of the most sophisticated cases (in the end, it will be some ARIMA or GARCH variant), fundamentalists rely on some basic *laws of motion* in order to model the future movement of the latent fundamental price. UIP, the Uncovered Interest Parity, condition serves as such. The UIP is a basic algebraic identity relating interest rates and exchange rates in form of expectations. The UIP, utilizing the the current exchange rate and the fraction between the interest rates per unit of time of the two currencies traded, allows the estimation of the exchange rate's future movement. In the current setting UIP is expressed as: $s_t^* = \frac{1+r}{1+r^*} s_{t-1}^*$. (where as before s_t^* is the latent fundamental exchange rate at time t and r and r^* denote the domestic and foreign currency interest rates respectively). Therefore, assuming a Gaussian expectation error term $N(0, Q)$ for the noise process ϵ_t , the UIP can be re-written as:

$$s_t^* = \frac{1+r}{1+r^*} s_{t-1}^* + \epsilon_t \quad (13)$$

Through the UIP, and always assuming that the risk premium and the transactions costs are negligible, the market moves to nullify the effect of carry trade. Clearly the difference in interest rates is one of the fundamental profit generating phenomena in the foreign exchange market that traders exploit. The model presented here provides an indirect explanation as to how the carry traders actually act. More details about the exact calculation of the fundamental exchange rate follow in the next section.

The final element of this model is clearly the Market Clearing rate determination. The market mechanism employed, clears the market numerically assuming that the realized change in the market equals the combined market forecast at time t by chartist and fundamentalist traders with the addition of white noise errors $N(0, R)$. Because of that the market clearing exchange rate is given as:

$$s_t = w_{t,C} E_t(s_{t+1}|C) + w_{t,F} E_t(s_{t+1}|F) + N(0, R) \quad (14)$$

As LeBaron [10] suggests, this kind of price determination process overcomes *issues of rationing, or market-maker inventories that need to dealt with*. Furthermore this framework is rather easy to implement (a cumbersome issue in some settings) and it is not numerically costly. The only main draw-back, as the same author notes, is that *it may impose too much market clearing*, but this is an issue outside the scope of the current work.

Summarizing the model, it is acceptable that a series of simplifications have been made. Nevertheless the model is based on solid behavioral theoretical frameworks. It overcomes the standard problem of the REEM (Rational Expectations - Efficient Market) model that has dominated financial analysis since the 60's and has caused it to fail to provide predictive and explanatory power in recent years. The presented model is a member of the greater family of models known as Adaptive Belief Systems in Finance as they were presented by Rieck [27]. Making a final comment about the two different strategies/behaviors it is important to mention that empirical evidence does imply that in many cases, traders perceive chartist and fundamentalist analysis techniques as complementary approaches.

3. Kalman Filter

The Kalman filter plays a crucial role to the estimation of the latent fundamental price; the price that is actually used by the fundamentalists in order to make their forecast. The Kalman filter itself is a discrete, recursive linear filter proposed by Rudolf Kalman in 1960 [28] and has played a crucial role in modern implementations of navigation systems and in Control Theory in general.

In the filter's original application, the position/state of the system had to be established using noisy measurements from sensors and basic estimates relying on the knowledge of the processes describing the system itself (i.e. laws of motions). The original implementation also assumed that the measurement and process error are Gaussian distributed. The filter utilizes the last known measurement of the state of the system and forecasts a prior estimate of the next period's value of the system's latent variable, then it combines this estimate with the actual realization of the next period to calibrate its forecast. It has to be noted that the Kalman filter itself is optimal in the sense that it utilizes all available information in a mean square sense.

Cheung [29] used a Kalman filter algorithm in order to estimate exchange rate risk premia. In a similar manner, Faust et al. [30] have recently used a Kalman filter approach to analyze the underlying fundamentals regarding the dollar risk premia also. To draw an analogy with the previous example coming from navigation, here the system's noisy measurements are the market clearing exchange rate values. The UIP estimate has the role of the system's underlying process. Chartists methods are typically applied in short horizon decision making, therefore using the Kalman filter to derive fundamental exchange rate information is done because we are focusing on days where news announcements are done. Because of that, the latent fundamental price has a significant variance despite the short term horizon. Also traders that failed to make correct market predictions about interest rates changes, are put into a price discovery process that further amplifies the changes in the market exchange rate. Here it is worth mentioning the work done by Stratonovich in the 60's and by Julier and Uhlman [31] in recent years to account for non-linear cases. Because of the analytically simple model, that we are using it is not necessary to follow this route; however, any real time financial application of a Kalman filter should definitely focus on the implementation of a Stratonovich-Kalman-Bucy or a Unscented Kalman filter.

The implementation of a Kalman filter is relatively straightforward conceptually. The main theoretical notions behind it are widely known: Bayes Rule, Gaussian distributions and linear equations of motion (or in the present case the UIP). First we show the prediction step of the Kalman filter, modified to our specific system²:

$$s_t^{*-} = s_{t-1}^* \frac{1+r}{1+r^*} \quad (15)$$

$$P_t^- = P_{t-1}^- \left(\frac{1+r}{1+r^*} \right)^2 + Q \quad (16)$$

$$n_t = s_t - A_1 s_t^{*-} - A_2 s_{t-1} - A_3 s_{t-3} \quad (17)$$

$$f_t = (A_1)^2 P_t^- + R, \quad (18)$$

²The setting employed is the one based on the suggested setting by Kim and Nelson on the book on State-Space Models [32]

where s^{*-} is the expected (estimated) value of the latent fundamental exchange rate s^* utilizing information up to time $t-1$, P^- is the a priori estimated error variance, n is the prediction error, f is the conditional variance of the prediction error and R and Q are the measurement and process noise variances.

In this first step the filter estimates the latent variable's value at time t utilizing all available information up to time $t - 1$. The crucial update step is as follows:

$$K_t = \frac{A_1 P_t^-}{f_t} \quad (19)$$

$$s_t^* = s_t^{*-} + K_t n_t \quad (20)$$

$$P_t = (1 - K_t A_1) P_t^-, \quad (21)$$

where K_t is the Kalman gain, and P_t is the a posteriori estimation of the error variance. The Kalman gain is therefore chosen in a way that minimizes the a posteriori error variance. A high conditional error variance f will *numb* the effect that the measure s_t has on the final estimate of the latent variable and the final estimation will rely mostly on the initial prediction. Vice versa, if P^- , the error estimate in the process is very high, the final estimation will rely on the measurement (the market clearing exchange rate), more heavily.

The simplicity of the model renders the incorporation of news effects quite easy. Changes in r or r^* , the interest rates concerning the domestic and foreign currency respectively, can be propagated instantly in the prediction stage of the algorithm, affecting the derivation of s^{*-} and P^- .

The Kalman filter itself is then optimized in order to reproduce results by stochastic simulations that exhibit the features apparent in real data. This is done by trying to minimize concurrently the difference between two indicators of the statistical properties of the time-series examined. The Standard Deviation, and it's Kurtosis, both of them formally being calculated by the following formulas:

$$\sigma = \sqrt{\frac{1}{N} \sum_{i=1}^N (x_i - \bar{x})^2}, \quad K = \frac{\frac{1}{N} \sum_{i=1}^N (x_i - \bar{x})^4}{(\frac{1}{N} \sum_{i=1}^N (x_i - \bar{x})^2)^2} \quad (22)$$

The exact dates used in the analysis are the following : 1. January 2, 2003 (no news announcements present) 2. January 9, 2003 (news announcement for stable interest rates) 3. February 6, 2003 (news announcement for decreased interest rates (-.25%)) and 4. November 6, 2003 (news announcement for increased interest rates (+.25%))³. While the four different aggregation scales that were mentioned in the Introduction were examined, the data and results shown in the current study were based on the 5-second aggregated data.

The Kalman filter is therefore used to derive the fundamental rate of the examined and the simulated exchange rates. Real data are used in order to provide a reference to ensure that the simulated series are indeed in line with real data (Table 4).

³More information available at [3]

4. Numerical Results

The complexity of the model implies it is impossible to approach analytically. Therefore, we rely on numerical simulations to investigate the model's behavior. In order to achieve this, the model was implemented in C++. The questions formulated address different subjects regarding the behavioral characteristics of the model. First we focus on the differences between the known coefficients from cases where the model was used to interpret data coming from daily averages, and the newly computed coefficients derived during the fine-tuning of the model. Second, we focus on the effect that news arrivals have in the market and how they affect agents' choice of strategy. The third and final question focuses on the per unit profitability of each agents' strategy.

Addressing the first question, it has to be noted that the basic behavior of the model is in line with DeGrauwe's findings [11]. This is a standalone result on its own and is quite interesting given the fact that the known behavior of the model regards cases of macro-scale (daily scale) rather than micro-scale (second scale) data (Table 1).

The stylized facts about a Chartist-Fundamentalist market framework as the one proposed here, are as follows:

Sensitivity to β . The lower β , the coefficient of chartists extrapolation, is known to anchor the market clearing foreign exchange rate closer to the otherwise latent fundamental rate and results in a less volatile market clearing rate. It also suggests that the kurtosis of the distribution of s_t moves further away from the expected gaussian kurtosis of Kurt=0, exhibiting a platykurtotic behavior. Despite the apparent deviation from the known literature, this is truly in line with the experimental data that suggest kurtosis values between -1.19 and 0.36, showing platykurtotic behavior also. (See Appendix A - Table 4)

Sensitivity to γ . The higher γ , the coefficient of choice update intensity, results in higher volatility in the market clearing foreign exchange rate. It also causes the market rate to have a greater disconnection from the fundamental rate.

Sensitivity to ψ . The higher ψ , the coefficient of fundamentalists belief's intensity, does lead to higher volatility in the market clearing rate s , but it forces it to move closer to the fundamental rate s^* also. The kurtosis of the sample is forced further away from the norm of 0. It appears as if fundamentalists traders, force the latent fundamental rate to behave accordingly to their beliefs, resulting a fundamental rate that is itself more volatile.

Insensitivity to μ . In contrast with the above mentioned parameter, the risk aversion coefficient μ , as also noted in the case of daily data, does not appear to have a significant effect on the final market clearing rate. De Grauwe himself *downplays* its importance in his model. Clearly if $\mu_c \neq \mu_f$ then the model would have a whole different dynamic behavior but this scenario is not investigated in the current study.

Sensitivity to Q . Clearly Q , the underlying process noise, has an important impact on the whole behavior of the model. Lower values of Q , result in less volatile behavior in the market price formation mechanism, more *gaussian*-like distribution, and less disconnections between the market clearing rate and the latent fundamental rate.

Sensitivity to R . As in the case of Q , R also affects the market clearing rate in a significant way. Less measurement noise, allows the agents to *trust* the realized rate more. As a result the market clearing rate becomes less volatile but in addition it exhibits larger deviations from the fundamental rate. The kurtosis of the realized rate appears significantly changed pushing the (log) values of it to an even more *bernoulli*-like distribution.

Table 1 shows our numerical findings ⁴. The reference values associated with the model are as follows: 1. $\beta = 0.94$, 2. $\gamma = 500$, 3. $\mu = 0.915$, 4. $\psi = 0.002$, 5. $Q = 3 * 10^{-10}$ and 6. $R = 1.5 * 10^{-9}$. The values derived are based on the log of the actual values of s_t . In each case all the variables of the model are set to be equal to the reference values and only the one denoted for the specific column is changed.

Reference Setting		$\beta = 0.84$	$\gamma = 1000$	$\mu = 0.715$
Standard Dev.	0.001268 (.0004416)	0.001148 (.000394)	0.001275 (.0004445)	0.001268 (.0004416)
Kurtosis	-0.670048 (.5235370)	-0.671943 (.512261)	-0.669449 (.523935)	-0.670048 (.5235370)
$\Sigma(s - s^*)^2$	0.002799 (.0032512)	0.002496 (.001650)	0.002828 (.00185347)	0.002799 (.0032512)
Reference Setting		$\psi = 0.02$	$Q = 9.01 * 10^{-10}$	$R = 8 * 10^{-9}$
Standard Dev.	0.001268 (.0004416)	0.001502 (.0005728)	0.001493 (.00056307)	0.001754 (.0004401)
Kurtosis	-0.670048 (.5235370)	-0.666427 (.517974)	-0.701636 (.528471)	-0.494171 (.468305)
$\Sigma(s - s^*)^2$	0.002799 (.0032512)	0.000253 (.000900)	0.001136 (.0009075)	0.017939 (.0105638)

Table 1: Basic numerical results displaying the different responses of the simulated exchange rate s , to individual parameter changes

It is quite important to stress here the differences between the known coefficient for daily and for intra-daily data; examining first the coefficient β and μ the proposed model are in line with values proposed by De Grauwe [24] in his seminal work on exchange rates.

On the contrary γ and ψ appear significantly different, differing by at least a scale of magnitude from the values suggested by DeGrauwe's models. The coefficient of update intensity has to be raised by almost two scales of magnitude; from a value $\gamma = 5$ to 10 suggested in most models based on daily closing price, to the current model use of γ 's that moves up to 1000. Also the coefficient of fundamentalists belief intensity is clearly decreased, from approximately 0.2 in De Grauwe's models it is reduced to 0.002. Both these facts are easily explained based on the idea that on the one hand larger γ 's are necessary for the model to actually cause the agents to *move*. Otherwise the model remains firmly close to its steady state of $w_c = w_f = \frac{1}{2}$. The intra-daily interest rate (if assumed that there are indeed such rates) is so small that the agents do not have an incentive to actually update their strategy. Also, ψ is definitely smaller. It will be almost infeasible for a fundamentalist trader to believe that the market clearing rate will indeed fluctuate so much in the course of one day, and that the market itself will be able to regulate itself back to the fundamental in such a small time-scale. As shown in the Figure 1, the market clearing price is not *firmly* attached on the fundamental rate in the same way that it is in the case of daily exchange rates. Little to no new fundamental information may be available during a *small* 8 hour interval, and therefore according to standard economic theory, no significant changes should be apparent in the fundamental price itself (except obviously by the drift imposed by the UIP conditions).

⁴As derived by the aggregation of 100 runs of each set-up

Q and R finally are significantly smaller, than most models encountered in the literature that deal with daily data [12, 17], this is obviously due to fact that much finer scale data are examined. It should be stressed however that the fine tuning of the model relies finally on the correct choice of Q and R . The noise in this kind of model is the key factor changing the whole behavior of the model itself.

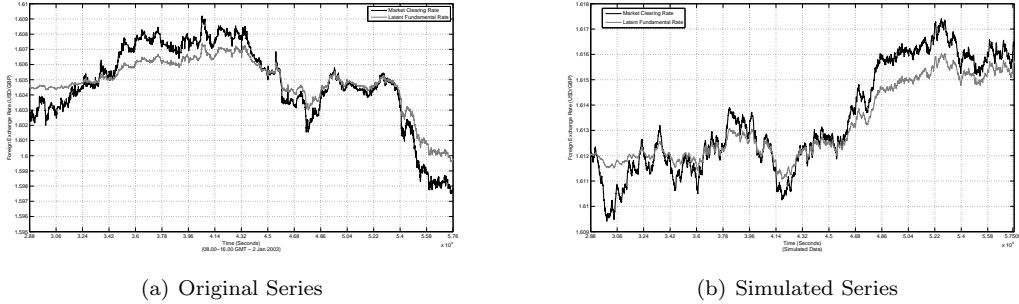


Figure 1: Calculating fundamental exchange rate s^* in real and simulated s

The next question focuses on the effect of news. News are phenomena that substantially change the financial environment in which agents act. In this case news are defined as the news announcements regarding the changes in sterling's interest rate by the Bank of England. This type of news provide the luxury of knowing the exact timing when they are released in the public (midday) and are well documented and easy to quantify. They contain a problem though; traders do have the ability to anticipate them. Agents are able to make prior predictions about the magnitude of the change in the interest rate. That way, based on their predictions they are able to plan ahead and subsequently not lose their connection to the fundamental rate. In essence, the volatility of the market clearing price in most real cases is a product of the agents that wrongly predicted the subsequent changes in s^* and their consequences on s and therefore enter a price discovery procedure. Simulating this situation we focus on two different scenarios: One where there is a drop in the interest rate and another where there is a rise. We also focus on a *smaller* time interval. Clearly the market agents have the ability to calibrate their future strategies quite efficiently and adjust quickly to the new fundamentals; therefore an arbitrary 1 hour interval before and after the news announcement is examined.

Figure 2 exhibits market conditions after a drop (-0.25%) and after a rise (+0.25%). These changes are coherent with the actual changes that are announced by the Bank of England. It is quite clear that we fail to observe news arrival effects. This is further emphasized by the almost identical basic statistical measurements used for characterization of the systems⁵. In Figure 2 the top diagram shows the absolute difference between market exchange rate s and latent fundamental value s^* . In addition, the lower half shows the proportion of agents using chartists behavior. It is easily seen that the proportion of chartists increase during times when there are significant drops or jumps between s and s^* . It also suggests that fundamentalist traders are able to remain in an otherwise

⁵For these cases and the following, the simulation depth was increased 20-fold, reaching now 2000 runs

overpriced market by exploiting small fluctuations due to random noise. Nevertheless, in the case of large jumps between s and s^* , only the chartists apparently have the ability to exploit them.

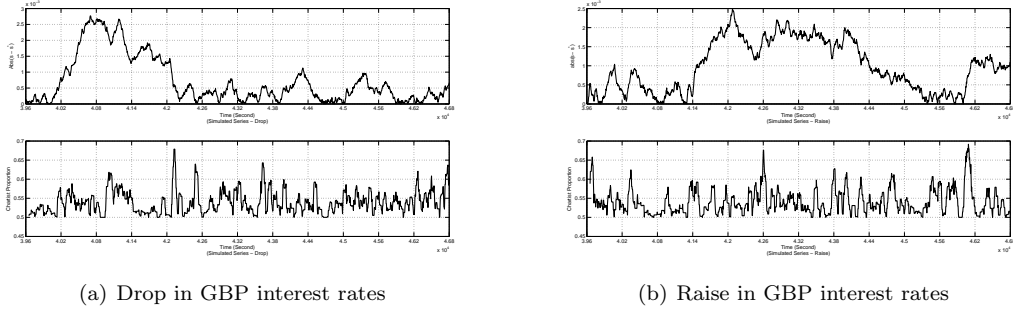


Figure 2: Top diagram : Absolute difference between s_t and s_t^* - Bottom diagram : Chartist market proportion (w_c) (using simulated series)

Simulated Data	No Changes	Raise	Drop
Standard Dev.	0.0007012 (.00021688)	0.0006976 (.00021553)	0.0007021 (.0002159)
Kurtosis	-0.546445 (.6021)	-0.537724 (.602013)	-0.540222 (.606914)
$\Sigma(s - s^*)^2$	0.0008851 (.00082631)	0.0008754 (.00082766)	0.0008842 (.000833181)
w_c	0.54339 (0.0923494)	0.54651 (0.0923494)	0.54028 (0.0913788)

Table 2: Numerical results of news effects on s

The final question is to consider the profitability which each behavior enjoys. The same three scenarios as before are used. In all three cases both the chartists and the fundamentalists appear to be losing money (Table 3). In the cases examined, the average of the sum of the total risk adjust profits per unit invested under both chartist and fundamentalist approaches, suggest negative values. While this is clearly disturbing for the validity of the model in real application, it is important to stress the fact that the fundamentalists always appear to *lose* more.

Simulated Data (2000x)	No Changes	Raise	Drop
Chartists Risk Adjust Profits	-0.158067 (.00233209)	-0.158066 (.0023297)	-0.158068 (.00233171)
Fundamentalists Risk Adjust Profits	-0.222021 (.00259213)	-0.222021 (.00259183)	-0.222021 (.00259218)

Table 3: Average of total risk-adjusted profitability per unit

5. Discussion

The most solid results of this model were the apparent importance of the successful estimation of the process (Q) and of the measurement noise (R) in order to get meaningful results that correspond to empirical data. Noise (and thus the risk) in the system is by far the most prevalent factor of any decision making procedure. It is clear that there are no well documented techniques in order to fine tune it. All techniques proposed, ultimately rely on heuristics rather than a solid analytical framework. This raises the

argument about the usage of Uncertainty principles, as proposed by Knight [33, 34] and has been the focus of the paper by Kozhan and Salmon [12]. The noise in the process itself changes the whole model's behavior because it dictates the drift of the otherwise random walk that the fundamental values follow within the course of a day. Surprisingly, this feature has been downplayed in the literature. Furthermore the measurement error, is equally important. While it is widely accepted that the market players do not know the full mechanics of the market formation mechanism and thus act under *uncertainty* rather than *risk*, only a few research papers deal with Knightian uncertainty.

One rather welcome byproduct of the model is that as in the case of daily data, it succeeds in reproducing bubble-crash asymmetries. What the model suggests takes place, is that in a bubble-run the chartists exploit the trend moving away from the fundamental rate *amplifying* its movement, while the fundamentalists *push* the rate back to the fundamental value. In that way the exchange rate s is standing between two different opposing trends. On the contrary, when in a crash, both fundamentalists and chartists, *push* the market exchange rate back to its fundamental rate. Under this scenario, the exchange rate s is under two co-directional trends and therefore the movement back to the fundamental rate is more steep and violent.

The main inability of the model came when trying to effectively model what occurs in the case of news arrivals in real life markets. The model, in its proposed format, failed to reproduce news arrivals effects. While it is clear that news arrivals do influence the market, the current model does not exhibit such behavior. This is quite possibly because the model does not contain order flow parameters and also because the chartist rule used, is too simplistic.

An interesting finding under the examination of news arrivals effects was the volatile 4th moment behavior of the exchange rate. This further signifies the truly chaotic behavior of the price formation mechanism. It further provides evidence for the apparent inability of standard statistical measurements to explain the market mechanism fully.

An important finding though was the actual prevalence of chartists traders during times of turmoil. As the market fluctuates, fundamentalists cannot *rationalize* it and therefore the only group exploiting this trend is the chartist one. This is further enforced by the fact that it appears that the market clearing rate and the fundamental rate are not tightly bound with each other. Fluctuations in the market clearing rate, can be introduced due to noise effects that are subsequently amplified and are incomprehensible to fundamentalist traders.

As mentioned above, possibly the chartist rule is too simplistic; nevertheless the chartist group exhibit lower losses than the fundamentalists. Their *irrational* behavior, contrary to the prediction of the REEM⁶ framework, allows them to survive in such markets (if not prevail in times of big fluctuations). Chartists are therefore evolutionary stable. Even in the cases that the market clearing price is not completely disconnected by the fundamental rate, a chartist trader, still exploits the *stable* trend and thus remains in the market. The bounded rational fundamentalists agents, on the other hand appear to be almost unrealistic at times. It has to be noted at this point that these measurements are measures of the risk adjusted profitability and not of the actual profits or losses sustained by the agents. In addition, the chartists' forecast does incorporate less risk

⁶Rational Expectations Efficient Market

on average in comparison with that of the fundamentalists' and therefore the chartist behavior remains favorable in cases where a chartist strategy would not substantially favor an agent.

As a concluding remark it has to be reported that the proportion of chartists is increased by noise in the market. Interestingly, this remark is also reversible. The noise in the market is itself amplified by the chartists.

6. Conclusions and further remarks

In this report we have estimated a model for the US dollar British pound exchange rate, and examined the dynamics of the exchange rate. Using the well-established model of Brock and Hommes as a basis, we present a simplified version in order to apply it to intra-daily exchange rate movements.

Our main finding is that chartist and fundamentalist strategies ultimately coexist in the market despite the markets' chaotic nature. In addition, the fundamental price does not appear to be the sole source of volatility in the market, emphasizing the importance of empirical finance in understanding the underlying mechanism of the exchange rates determination.

There are a number of fruitful routes for future research. Introducing co-integration between the market clearing rate and the fundamental rate and employing a more advanced agent behavior framework are the most obvious. Furthermore, the integration of uncertainty and the incorporation of real unevenly spaced tick-by-tick data, will also allow us to determine the role of uncertainty and order flows in explaining real intra-daily exchange rate fluctuations as occur in reality.

Appendix A.

Real Data	2 Jan 03	7 Jan 03	6 Feb 03	6 Nov 03
Standard Dev.	0.001678	0.001249	0.001887	0.001180
Kurtosis	0.368105	-0.431407	-1.191557	-0.615029
$\Sigma(s - s^*)^2$	0.01610	0.011683	0.013422	0.010590

Table 4: Basic measurements of actual intra daily data

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