# HERITAGE OF THE BLOCH EQUATIONS IN QUANTUM OPTICS

*by E. L. Hahn*

ABSTRACT

The Bloch equations as originally applied to nuclear magnetic resonance have wide application to a variety of physical effects that do not necessarily involve gyromagnetic spin systems. In particular, they are useful in predicting effects involving electric dipole laser resonance phenomena, which in many cases are analogs of effects observed in nuclear magnetic resonance.

## I. INTRODUCTION

Among the versatile contributions of Felix Bloch to physics, the Bloch equations of nuclear induction[1] (phrased more commonly today as "nuclear magnetic resonance" [NMR]) have played a major role in guiding researchers and students in the interpretation of resonance experiments. The direction of my research in physics would have been quite different without recourse to the Bloch equations. As a graduate and one-year postdoctoral student at the University of Illinois, my start in NMR research was based on these equations. Since I did not know how to use quantum mechanics, the classical pictures allowed by Bloch's equations saved my neck because they afforded a reasonable macroscopic interpretation of spin echo experiments. It was my good fortune to continue these experiments at Stanford, and there my association with Felix had a profound effect on the way I have done physics ever since. At the beginning of my two years (1950–1952) at Stanford, I remember distinctly that my propensity for arm-waving vector models somewhat overwhelmed Felix. Nevertheless, with dis-

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cussions seasoned every now and then with dubious jokes, we soon developed a rapport. Eventually, I learned how to use quantum mechanics from Felix after numerous priceless sessions at the blackboard. His effectiveness as a teacher was evident from his broad ability to shift back and forth between the thinking of phenomenological models and fancy formalism.

The Bloch equations have wide application to a number of physical effects that do not necessarily involve gyromagnetic spins. They apply directly to any two quantum level or equally spaced quantum level system, and are particularly useful in predicting effects involving electric dipole laser resonance phenomena. It follows also that the Bloch-type equations are generally descriptive of systems that contain bistable states. For example, the phase transition in ferroelectric $KH_2PO_4$, associated with the two potential well model of the hydrogen bond, can be deduced[2] from a set of Bloch equations. Free electron laser action from a relativistic electron beam that passes through a spatially periodic magnetic field can be analyzed[3] in terms of coupled Bloch-Maxwell equations. In special situations concerning resonance radiation interacting with unequally spaced multilevel systems, particularly three level ones (see figures 6 and 7, pp. 36-37), such systems can be formulated in terms of the density matrix to give an equivalent two level Bloch equation description.[4] Of course, it is common knowledge that the Bloch equations have a historical root in the mechanical torque equation for the motion of a physical top. The torque equation for the description of gyromagnetic resonance phenomena as presented by Bloch provides a powerful phenomenological means for connecting the linear and nonlinear regimes of resonance response in one formulation. His equations introduced the idea that the otherwise coherent time dependence of quantum mechanical macroscopic moment expectation values could be coupled to phenomenological damping terms. They describe resonance and dispersion in classical terms that connect with the quantum mechanical interpretation through the Liouville equation for the density matrix from which they are derived. Bloch's famous damping time constants $T_1$ (longitudinal spin lattice relaxation time) and $T_2$ (transverse phase memory total relaxation time) are common physics language of today among resonance researchers. Although the original phenomenological Bloch equations work very well for fluids, in many cases they are not rigorous for all systems, particularly solids. Nevertheless, the Bloch formulation has stimulated new statistical investigations with the density matrix that are more rigorous for the particular system under investigation. Exceedingly useful is the property that the Bloch equations enable predictions of nonlinear quantum macroscopic phenomena[5] that no amount of fastidious quantum mechanical perturbation theory could predict as handily.

## II. THE HERITAGE OF SPIN RESONANCE IN QUANTUM OPTICS

Numerous effects in quantum optics known today are analogs of spin resonance phenomena discovered in former days and are often predicted by Bloch equations. Although considerations of propagation and fluorescence are not present in NMR cavity resonance experiments, there remains a host of similar effects in the time and frequency domain that also appear in quantum optics. One can list as examples adiabatic inversion, Rabi (nutation) oscillations, Bloch-Siegert shift, free precession echo effects, and superposition states. Researchers in quantum optics are sometimes unaware that they rediscover old NMR phenomena. Or they may have a tendency to ignore similar known NMR effects as unimportant because quantum optical effects are governed by matrix elements anywhere from $10^8$ to $10^{15}$ times larger than those in NMR. I am reminded of the story of a visit by the Sultan of Morocco to Berkeley. After he was introduced to a number of high energy physicists, he chided another man, who said he was a low energy physicist, for being too modest! It is my intent in this essay to discuss with immodesty a few points about the impact of low energy magnetic resonance on the development of quantum optics.

Coherent quantum spectroscopy was initiated by the Rabi[6] atomic molecular beam radio frequency (rf) resonance method. For the first time coherent electromagnetic radiation was applied to excite atomic spin resonance transitions. A coherent spectroscopy was achieved with unprecedented resolution, and yet it was apparatus-limited because of the finite beam excitation time in its path from source to detector. (Later a modification by Ramsey[7] reduced this effect considerably.) The Bloch[1] and Purcell[8] groups introduced the science of NMR in condensed matter by extending the Rabi resonance principle to the continuous excitation of spin ensembles in a resonant cavity. The higher resolution capability of this approach very soon revealed sharp NMR spectra; and numerous effects on the resonance by the chemical and solid state environment were discovered. Microwave electric dipole spectroscopy of rotational and vibrational states in molecules showed a parallel development. During these early investigations, the discovery of new spectra, spins, moments, line shifts, and line shape characteristics dominated the attention of resonance research workers. Interest was focused on atomic properties and not on changes in the resonance radiation itself, which exchanges energy with the excited atoms. With the applied radiation fields assumed as fixed driving terms in the analysis, a number of physical ideas that might have led to an earlier realization of the maser-laser principle were suppressed. The impasse was broken somewhat when the idea of coherent radiation reaction, though quite old, was given prominence by Dicke[9] (1954) with regard to coherent emission of electric dipole radiation. He championed the point that an ensemble of atomic electric dipole moments radiates coherently in the same manner as a precessing spin ensemble, and that coherently excited electric dipoles can be identically described by the same superposition coherence states that apply to NMR. It was nothing new, and yet it was instructive to reiterate that any resonance signal in the presence of a driving field (stimulated absorption or emission), or a coherently emitted field from an array of phased dipoles in the absence of a driving field, is the result of a reaction field produced by a macroscopic radiating source polarization term in Maxwell's equations. In addition, Dicke posed a very interesting new question: How does an ensemble of atomic particles in the completely inverted excited state evolve from a condition of incoherent spontaneous emission to a condition of coherent spontaneous emission, where only the latter emission is predicted by Maxwell's equations and the former emission is treated by quantum electrodynamics? Unfortunately, while Dicke meant the term "superradiance" to be applied to this particular evolution process from incoherence to coherence, "superradiance" is now applied to any process of coherent emission involving light. Of course by use of the Bloch equations coupled to a tuned circuit, superradiance would then apply as well to NMR free precession effects.[10] In NMR an oscillating reaction field is produced by a current, which is induced in an inductance by a precessing magnetization according to Faraday's law. For ordinary NMR experiments the effect of the reaction field (see figure 5, p. 35) in directly altering the Zeeman energy of the coherent precessing magnetization is imperceptible (except under very special circumstances[11]) because of the dominating effects of shorter damping times $T_1$, $T_2$ and of inhomogeneous broadening. Yet it is this very change in energy that makes possible the observation of NMR signals, whether they result from driven resonance or from free precession. In quantum optics, rapidly varying electric reaction fields from dense ensembles of electric dipole moments can be sufficiently intense to "tip" the polarization completely to the ground state (or to the excited state) in times short compared to the optical $T_1$ and $T_2$ time constants. In an inverted two level system, the electric fields that derive from stored energy in the atoms of sufficient density are intense enough to act like "180° pulses," which convert populations completely from one state to the other. This is the mechanism inherent in the self-induced transparency phenomenon.[12]

Although Dicke's paper contained all the ingredients that showed the equivalence of an ensemble of spin $I = ½$ particles to that of an optical two level system, his analysis in using the rules of angular momentum did not explicitly spell out the torque equation of motion for a giant macroscopic electric dipole moment. In 1957, Feynman, Vernon, and Hellwarth[13] showed explicitly that the Bloch equation form was applicable to a two level system with any type of dipole matrix element, involving circularly polarized radiation resonance (angular momentum change $\Delta m = \pm 1$) or linearly polarized radiation resonance ($\Delta m = 0$). Although the discussion in their paper was restricted to a particular MASER problem, they promoted the viewpoint of the semi-classical method, in which the electric field is a self-consistent solution of both the optical type Bloch equations, originating from the density matrix, and Maxwell equations. The work of Jaynes[14] and Lamb[15] extended this semi-classical approach later to quantum maser and laser analysis. And to complete this brief history, the maser-laser principle was finally realized in 1954 (Townes,[16] Basov and Prokhorov,[17]), when proposals for pumping to maintain inverted quantum states combined with efficient coherent radiation feedback were implemented in the microwave ammonia maser and later in an optical solid state system.

## III. THE OPTICAL BLOCH-MAXWELL EQUATIONS

The Bloch-Maxwell semi-classical equations of quantum optics consist of a set of equations in the form of Bloch's original nuclear induction equations, which describe the dynamics of optical electric polarization of a two level system, coupled to Maxwell's equations. In addition to the usual time dependence, the spatial dependence of the electric polarization and the propagating electric field must be considered over two level medium dimensions greater than the optical wavelength $\lambda = 2\pi/k$, where $k = \omega/c$ is the propagation constant in vacuum, and $\omega$ is the optical frequency. The original Bloch equations

$$\frac{d\mathbf{M}}{dt} = \gamma(\mathbf{M}\times\mathbf{H}) - \frac{\hat{x}_o M_x + \hat{y}_o M_y}{T_2} - \frac{\hat{z}_o(M_z - M_o)}{T_1} \qquad (1)$$

for the macroscopic magnetization $\mathbf{M}$ apply to an ensemble of $N$ spins/$cc$ subjected to a magnetic field

$$\mathbf{H} = \hat{z}_o H_o + \hat{x}_o H_1(t).$$

Define the applied linear polarized rf field along the $\hat{x}_o$ direction as

$$H_1(t) = 2H_1 \cos \omega t$$

perpendicular to the polarizing field $\hat{z}_o H_o$. As usual we specify $\gamma$ as the gyro-magnetic ratio, $\mu_o = \gamma \hbar I$ the magnetic moment, $I$ the spin quantum number, and $\omega_o = \gamma H_o$ as the transition Larmor frequency. Angular momentum changes $\Delta m = \pm 1$ correspond to the choice of either sign of the gyromagnetic ratio $\pm|\gamma|$. A particular sign signifies the secular interaction of right (+) or left (−) circularly polarized fields. A sum of opposite senses of precessing polarizations describes the dynamics of a system undergoing $\Delta m = 0$ transitions.

The Hamiltonian translation of terms in Eq. (1) to electric dipole notation is presented as follows: For the total Hamiltonian let

$$\mathcal{H} = -\gamma\hbar I_z H_o - \gamma\hbar I_x H(t) \rightarrow \mathcal{H}_o - 2\sigma_x p_o \mathcal{E} \cos[\omega t - kz + \phi(z,t)],$$

where $\phi(z,t)$ is an arbitrary propagation phase assigned to a plane wave propagating in the $z$ direction, and $\boldsymbol{\sigma}$ is the Pauli operator. Important equivalences are

$$\mu_o \rightarrow p_o,$$

$$\gamma \rightarrow \varkappa = 2p_o/\hbar,$$

$$\gamma\hbar I_x \rightarrow p_o\sigma_x = (p_o/2)(\sigma_+ + \sigma_-),$$

$$\mathbf{M} \rightarrow \mathbf{P},$$

$$H_1(t) \rightarrow 2\mathcal{E}\cos[\omega t - kz + \phi(z,t)],$$

$$\text{Energy: } W = -M_z H_o \rightarrow N\,\mathrm{Tr}\{\mathcal{H}_o\varrho\} = (-N\hbar\omega_o/2)\mathrm{Tr}\{\sigma_z\varrho\}$$

where $\varrho$ is the density matrix; and

$$M_z \rightarrow P_z = -W\varkappa/\omega_o.$$

The diagonal energy Hamiltonian operator $\mathcal{H}_o = -\hbar\omega_o\sigma_z/2$ transforms like $I_z = \sigma_z/2$ for $I = 1/2$. The quantity $P_z$, analogous to $M_z$, appears as a pseudo-electric polarization component. Figure 1 shows the similarity between magnetic and electric dipole two level superposition. Here the pure electronic eigenstates $\psi_1$ and $\psi_2$ do not display permanent electric dipole moments. The induced electric polarization

$$P_+ = P_x + iP_y = Np_o\mathrm{Tr}\{\sigma_+\varrho\} = Np_o(u + iv)\exp(-i\omega_o t)$$

is obtained from use of the density matrix $\varrho$, where $i\hbar\dot{\varrho} = [\mathcal{H},\varrho]$, by applying the transformation

$$\mathcal{H}_r = T^{-1}\mathcal{H}T$$

where

$$T = \exp\{-i(\mathcal{H}_o/\hbar\omega_o)[\omega t + \phi(z,t)]\}.$$

This results in the interaction Hamiltonian $\mathcal{H}_r$ matrix (in the "rotating frame")

$$\mathcal{H}_r = -\hbar \begin{pmatrix} \Delta\omega/2 & \varkappa\mathcal{E} \\ \varkappa\mathcal{E} & -\Delta\omega/2 \end{pmatrix},$$

where $\Delta\omega = \Delta\omega_o - \phi(z,t)$ and $\Delta\omega_o = \omega_o - \omega$.

A set of polarization components $Np_o u$, $Np_o v$, $Np_o w$ appear as slowly varying functions of $t$ and $z$ in a frame of reference rotating at frequency $\omega + \dot{\phi}(z,t)$ where $Np_o u$ is directed along the field modulus $\mathcal{E}$. Therefore

$$\mathbf{P}_r = (\hat{u}_o u + \hat{v}_o v - \hat{z}_o w)Np_o.$$

A schematic of the components of $u$, $v$, and $W$ is shown in figure 2. In terms of the components of $\tilde{\varrho} = T^{-1}\varrho T$,

$$u = \tilde{\varrho}_{12} + \tilde{\varrho}_{21},$$

$$v = -i(\tilde{\varrho}_{12} - \tilde{\varrho}_{21}),$$

$$w = -(\tilde{\varrho}_{11} - \tilde{\varrho}_{22}).$$

For small thermal energy $k_B T << \hbar\omega_o$, complete optical ground state occupation gives $w_o \mp \tilde{\varrho}_{11}(0) = -1$, and total ground state energy $W_o = -N\hbar\omega_o/2$. The radiative polarization $P_r$ of the undamped system is coupled to the energy density

$$W = Nw\hbar\omega_o/2$$

according to the conservation condition

$$(Np_o)^2(u^2 + v^2) + [(\varkappa/\omega_o)W]^2 = [(\varkappa/\omega_o)W_o]^2.$$

With damping terms included, the conservation condition no longer applies, and the coupling in general appears in the optical Bloch equations (written for $+|\gamma|$):

$$\dot{u} = \Delta\omega v - u/T_2,$$

$$\dot{v} = -\Delta\omega u - \varkappa \mathcal{E} w - v/T_2,$$

$$\dot{w} = \omega_o \mathcal{E} v - (w - w_o)/T_1.$$

These scalar equations may be given in compact form as

$$\frac{d\mathbf{P}_r}{dt} = \mathbf{P}_r \times [\hat{u}_o \varkappa \mathcal{E}(z,t) + \hat{z}_o(\Delta\omega_o + \dot{\phi})]$$

$$- \frac{(\hat{u}_o u + \hat{v}_o v) N p_o}{T_2} - \frac{(W - W_o)}{T_1} \hat{z}_o.$$

When these equations are coupled to the second order Maxwell's equation in the case of a traveling plane wave propagating in the $+z$ direction (ignoring negligible diffraction effects from sample boundaries), the following first order differential equations result:

$$\frac{\partial \mathcal{E}}{\partial z}(z,t) = -\frac{2\pi\omega N p_o}{\eta c} \int_{-\infty}^{\infty} v g(\Delta\omega_o)\, d\Delta\omega_o, \tag{2a}$$

$$\mathcal{E} \frac{\partial \phi}{\partial z}(z,t) = \frac{2\pi\omega N p_o}{\eta c} \int_{-\infty}^{\infty} u g(\Delta\omega_o)\, d\Delta\omega_o. \tag{2b}$$

where $\eta$ is the host medium background refractive index. The individual polarizations $Np_o u$ and $Np_o v$ are summed over a normalized inhomogeneous line distribution function $g(\Delta\omega_o)$ where

$$\int_{-\infty}^{\infty} g(\Delta\omega_o)\, d\Delta\omega_o = 1.$$

These equations apply in the slowly varying envelope approximation

$$\frac{\partial \mathcal{E}}{\partial z} << \frac{\mathcal{E}}{\lambda}; \quad \frac{\partial \mathcal{E}}{\partial t} << \varkappa \mathcal{E},$$

where $t$ above corresponds to the retarded time, namely $t_{lab} \rightarrow t + \eta z/c$. Generalization of these equations can be made to cases where the states $\psi_1$ and $\psi_2$ are possibly degenerate, and not necessarily characterized by a single dipole matrix element that connects them. If the slowly varying envelope approximation is not made, then terms of the type $\omega\dot{v}$, $\omega\dot{u}$, $\ddot{u}$, and $\ddot{v}$ appear in Eqs. (2a) and (2b) along with dominating terms $\omega^2 v$ and $\omega^2 u$. The size of these higher order terms is usually of the same order of magnitude as the off-resonance response of other quantum levels in the same atom, and these terms are therefore neglected. If the contribution of these higher order terms is to be included, then it is necessary to include the response of other off-resonance quantum levels as well. One may also include the effects of transverse mode character of a non-plane wave, not treated here.

A unique feature of Eq. (2a) shows that any time dependence of $\phi$ will cause frequency shifts and modulation in any situation where $u$ is not proportional to $\mathcal{E}(z,t)$, and is therefore nonlinear. The effective propagation vector is given by

$$k_e = k + \frac{\partial\phi}{\partial z}(z,t);$$

and the modified frequency is $\omega_e = \omega + \dot{\phi}(z,t)$. Nonlinear deviations from the classical linear field attenuation given by Beer's law

$$\mathcal{E}^2(z) = \mathcal{E}^2(0)e^{-\alpha z}$$

also result from the nonlinear behavior of $Np_o v$ in Eq. (2b), where $\alpha = 8\pi p_o^2 N g(0)\omega/\eta\hbar c$ is the linear absorption coefficient at exact resonance.

## IV. ANALOGOUS NMR EFFECTS IN QUANTUM OPTICS

A full review of all the quantum optics experiments that are similar to those known in NMR would take too much space. Instead, mention of the physical importance of a few highlights is the best that this essay can offer. One way of picking out the optical analogs is simply to thumb through the "Old Testament" of *The Principles of Nuclear Magnetism* by Abragam.[18] And even from the "New Testament" on electron spin resonance written by Abragam and Bleaney,[19] for example, the dynamics of phonons interacting with paramagnetic centers in solids can be directly applied to optical photons. (Once at a seminar in Saclay, after I mentioned that Abragam was the only person I knew who was an author of both of the "Old" and "New Testaments," he pointed out that I had overlooked a Higher Being.)

A list is given below of various effects and phenomenology that occur in quantum optics. A good number of the listed items are coincident with those in NMR. Those which do not relate at all to NMR concern fluorescence (spontaneous emission), certain propagation effects, and multilevel Raman processes in unequally spaced multilevel systems. The list does not pretend to be complete, nor is there any assignment of relative importance to each item. Wherever an asterisk (*) is noted, the Bloch formulation has little or no applicability. Although the Bloch-Maxwell equations may be applied directly in describing numerous effects in quantum optics of a semi-classical nature, in some instances where a more rigorous application of the density matrix is required, a resemblance to the Bloch formulation nevertheless may be extracted from the general analysis under certain approximations.

When an asterisk appears with a dagger (*†), it means for certain exceptions the item can be connected usefully with the Bloch formulations, while in most cases it cannot. These assignments are perhaps arbitrary in some cases.

**Effects and phenomonology in quantum optics**

Linear absorption, dispersion
Nonlinear saturation, hole burning, spectral and spatial diffusion
Adiabatic following and inversion
Rabi oscillation "nutations"
Bloch-Siegert shift*
AC Stark shift*†
Autler-Townes effect*†
Free precession photon (spin-like echo); coherent superposition of quantum states
Double resonance:
- (a) Double excitation with two fields in the laboratory frame of a two level system
- (b) Rotary saturation double excitation with one field in the laboratory frame and one field in the rotating frame of a two level system

Self-induced transparency (manifestation of an electromagnetic soliton)
Coherent radiation "reaction field" damping
Two photon transition dynamics
Stimulated Raman effect*†
Quantum fluorescence beats*
Coherent beats, Raman beats, Raman echoes*†
Raman and Brillouin scattering*
Doppler free Lamb dip and two photon spectroscopy*†
Two level laser Lamb theory
Free electron laser theory
Laser threshold "phase transition"
Fourier transform spectroscopy

Holography*† (in $\omega$-$t$ space the spin-photon echo is an analog of information storage assessed by Fourier analysis in $k$-$r$ space of optical holography)
Optical self-focusing*
Nonlinear optical pulse chirping, frequency modulation, propagation dependence of phase*†
Nonlinear-optical mixing, harmonic generation, optical rectification*
Homogeneous, inhomogeneous broadening
Relaxation dynamics
Rotating frame polarization locking (production of electric polarization along $\mathfrak{E}$ in the rotating frame)

For some of the items listed above it is difficult to make a clear distinction between NMR- and non-NMR-like behavior. For example, in an unequally spaced three level system the Autler-Townes effect concerns the observation of a splitting of a single resonance transition line (connecting level 2 and level 3) into two lines because of the Rabi flop modulation imposed on level 2 by coherent resonance saturation of the $1 \leftrightarrow 2$ transition. This mechanism explains the optical rotary saturation effect[20,21] in a two level system (transition $1 \leftrightarrow 2$) that can be analyzed by the Bloch equations. Figure 3 (p. 33) gives a vector diagram schematic of the rotary saturation mechanism.

The enormous amount of physics implied in the items above cannot be properly outlined here. I hope the reader will have some familiarity with most of them. Wherever the Bloch equations might apply may be seen from the analytical route by the block diagram of figure 4 (p. 34). Various routes of analysis may be taken in the semi-classical (SC) approach. A perturbation approach is often carried out without use of the Bloch equations, given the dotted arrow route. Those not familiar with the direct validity of Bloch equations sometimes take this route by insertion of perturbation source terms into Maxwell's equations directly. Later one may discover that by taking the dashed arrow route, one can include all orders of perturbation simultaneously. This procedure resulted in the discovery of self-induced transparency.[22] The Bloch-Maxwell equations were first solved by computer, and then shown later to be analytically soluble. And finally a limited number of experiments can be analyzed only by quantum electrodynamics (QED), which takes the solid arrow route. These include quantum fluorescence beats, Lamb shift, spontaneous emission, and Dicke superradiance.

Now turn the tables and ask the question: What NMR effects and concepts have not been observed or do not apply in quantum optics, and why? Here is a list of a few of them:

1. Analog of spin temperature in the rotating frame

2. Conserved dipolar energy reservoir
3. Adiabatic cooling
4. Double resonance cross-relaxation coupling between different atomic (spin) species
5. Overhauser effect
6. Solid effect (similar to no. 4)
7. Motional narrowing

In a limited sense one might be surprised to find that items 1, 4, and 6 could be realized in future experiments. In general the analog of NMR "spin temperature" equilibrium in solids does not carry over into the optical case. A single species optical dipolar energy reservoir probably cannot be realized. In a solid, electric dipole-dipole interaction energy is not readily conserved within an isolated two level ensemble of dipoles, a phenomenon that prevents the onset of a quasi-equilibrium temperature different from the lattice temperature. Naturally, such an equilibrium would be possible if the isolated system energy would remain intact for times long compared to the formally estimated dipole-dipole coupling time, designated as $T_2$. For optical dipoles this equilibrium is not favored because of rapid spontaneous emission, or strong coupling to broad collisional phonon degrees of freedom, which introduce competitive lifetimes comparable to or shorter than the optical system electric dipole-dipole coupling time. (Because of strong coupling, adiabatic cooling and motional narrowing cannot be achieved.) However, a very dilute system of optical dipoles may be assigned a temperature if it is prepared in such a way that it interacts exclusively, for example, by double resonance coupling[20] with an abundant species of spins, where the spins are in dipolar thermal equilibrium contact among themselves. The diagonal susceptibility of the optical system in the laboratory frame provides the coupling mechanism. In the frame of reference rotating with the optical frequency a two level transition occurs in the rf range. The optical polarization appears to be diagonal with respect to the direction of an effective field. The polarization dynamics can be handled by detailed balance or by a set of transformed Bloch equations with relaxation times appropriate to the new frame of reference.[21] A temperature may therefore be assigned to the dilute optical dipoles even though they do not couple directly among themselves. This would require a doped constituent in a solid with a forbidden transition that provides a long fluorescence lifetime.

## V. CONCLUSIONS

From this brief summary of quantum optics, we have but one example of the universality of ingenious theoretical formulations by Felix Bloch that

(text continues p. 37)

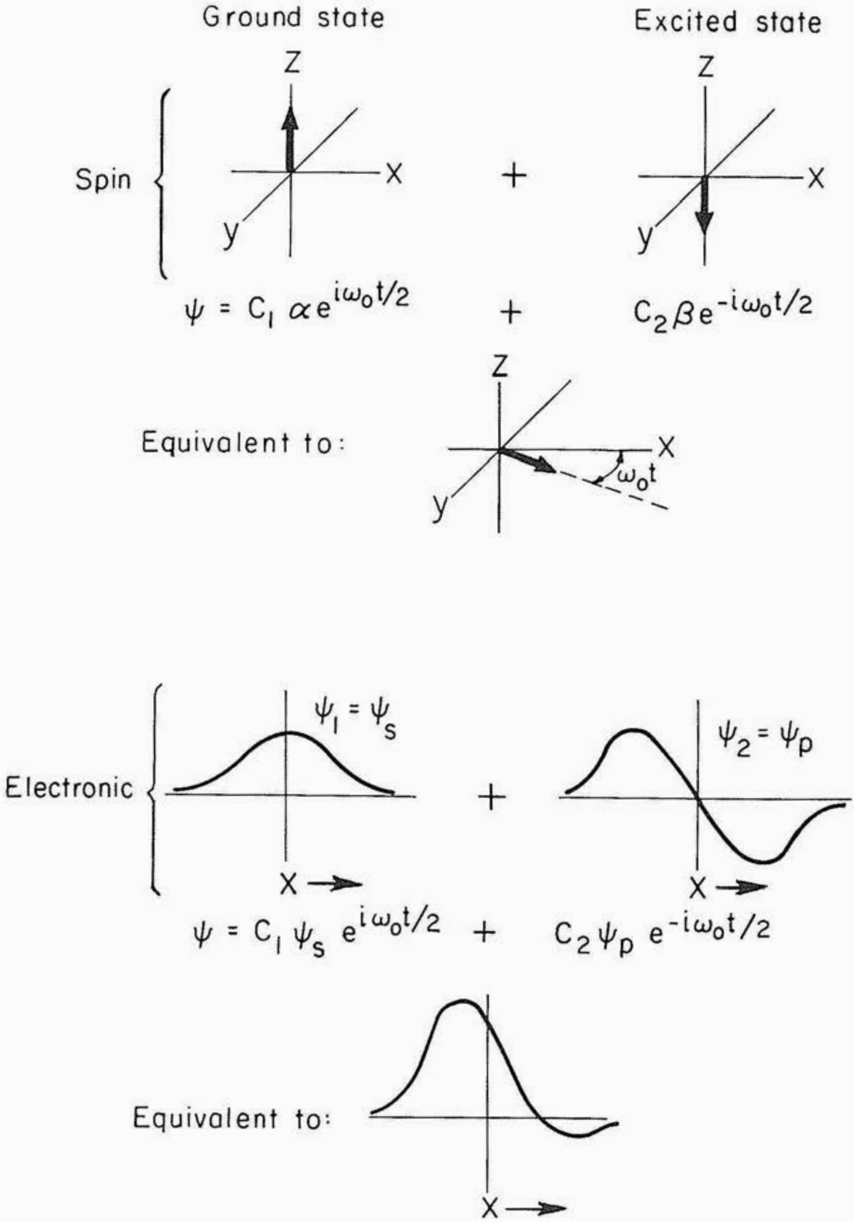
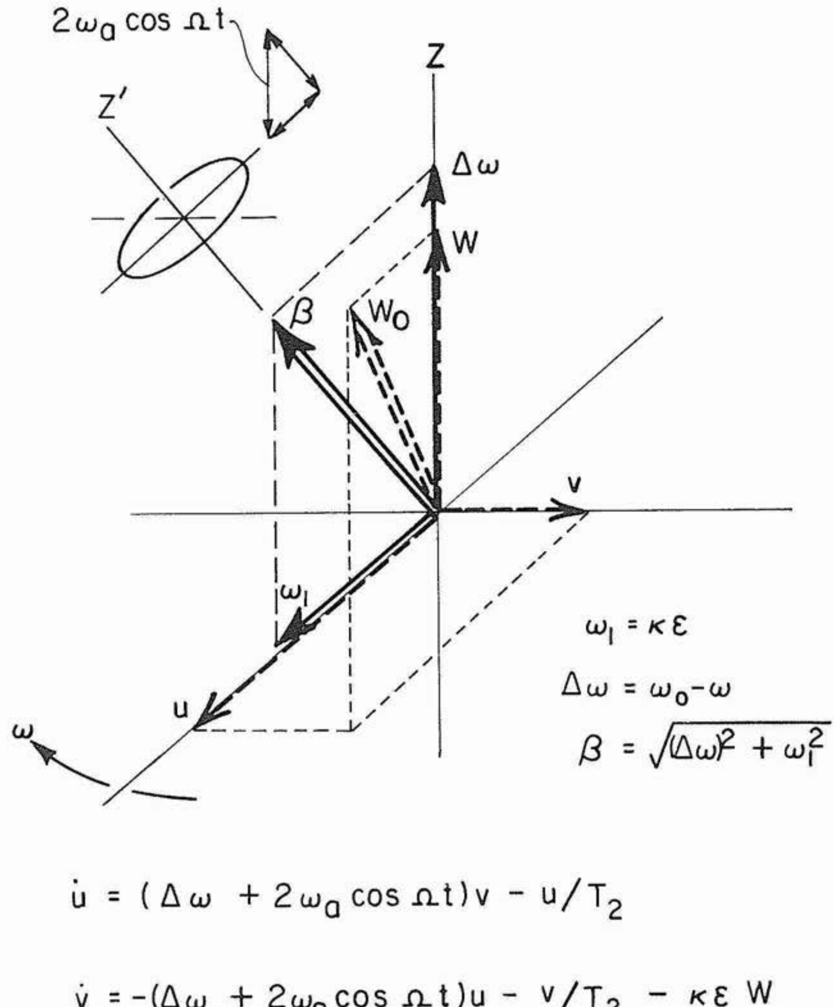



FIG. 1. COMPARISON BETWEEN MAGNETIC SPIN DIPOLE AND OPTICAL ELECTRIC DIPOLE TWO LEVEL SUPERPOSITION STATES. Superposition of spin states $\psi_1 = \alpha$ (spin up) and $\psi_2 = \beta$ (spin down) correspond to superposition of $\psi_1 = \psi_S$ and $\psi_2 = \psi_P$ electronic states, for example. For $C_1 = C_2$, after a 90° pulse, maximum oscillating dipole moments radiate (precess) at the Bohr transition frequency $\omega_o$.




$$\dot{u} = (\Delta\omega + 2\omega_a \cos \Omega t)v - u/T_2$$

$$\dot{v} = -(\Delta\omega + 2\omega_a \cos \Omega t)u - v/T_2 - \kappa\varepsilon W$$

$$\dot{W} = \kappa\varepsilon v - (W - W_0)/T_1$$

FIG. 2. COMPONENTS OF THE VECTOR $\mathbf{P}_r = (\varkappa/\omega_o)\mathbf{W}_o$ in terms of $W$, $u$, and $v$ in the reference frame rotating at frequency $\omega$. With damping these components are defined as steady state values from the Bloch equations, where $W_o$ is no longer conserved. If the two level separation is modulated at frequency $\Omega$ by an amount $\Delta E = 2\hbar\omega_a \cos \Omega t$ (see figure 3), rotary resonance transitions are induced at frequency $\Omega = \Omega_R = \beta$, and rotary saturation of $\mathbf{P}_r$ occurs. The Bloch equations are shown with the rotary perturbation term included. Here, $u$, $v$, and $W$ correspond to $u$, $v$, and $w$ (as density matrix elements defined in the text) multiplied by $Np_o$.

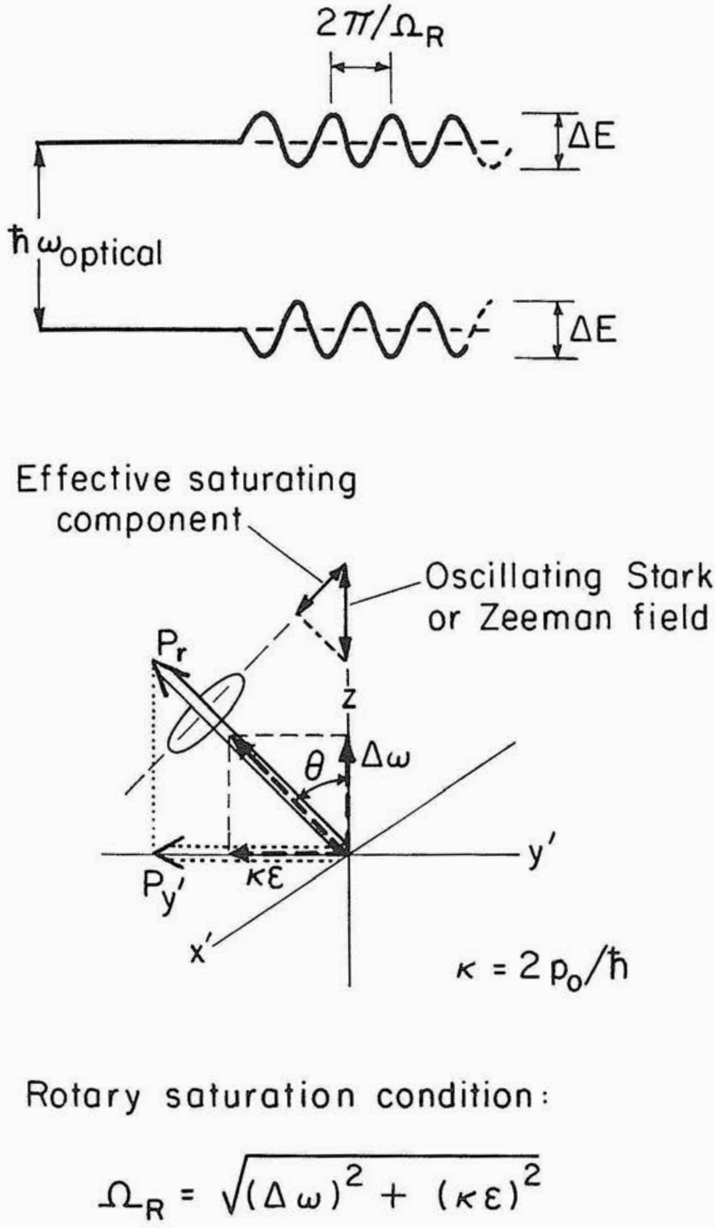



FIG. 3. OPTICAL ROTARY SATURATION OF THE EFFECTIVE POLARIZATION $P_r$, represented as polarized along the "effective frequency" direction $\boldsymbol{\beta}$ (see figure 2) in the rotating frame.

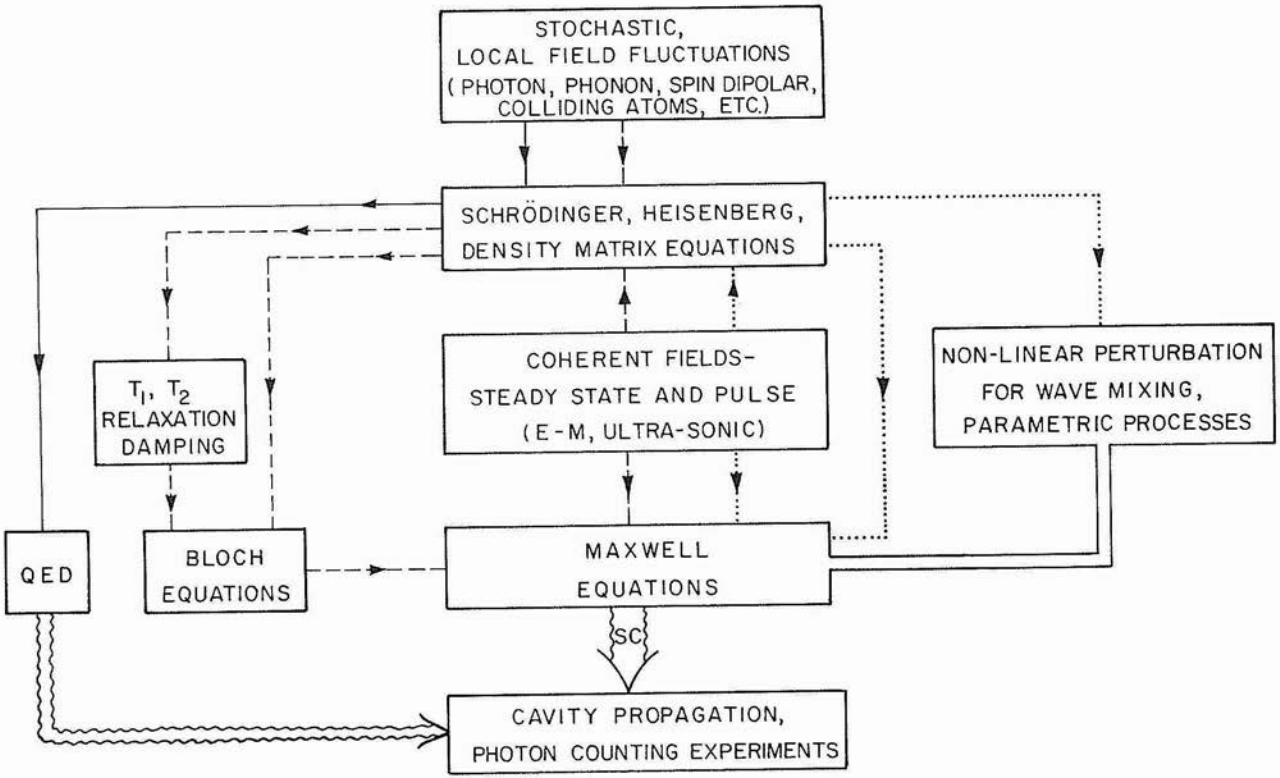



FIG. 4. VARIOUS ROUTES OF ANALYSIS IN QUANTUM OPTICS.

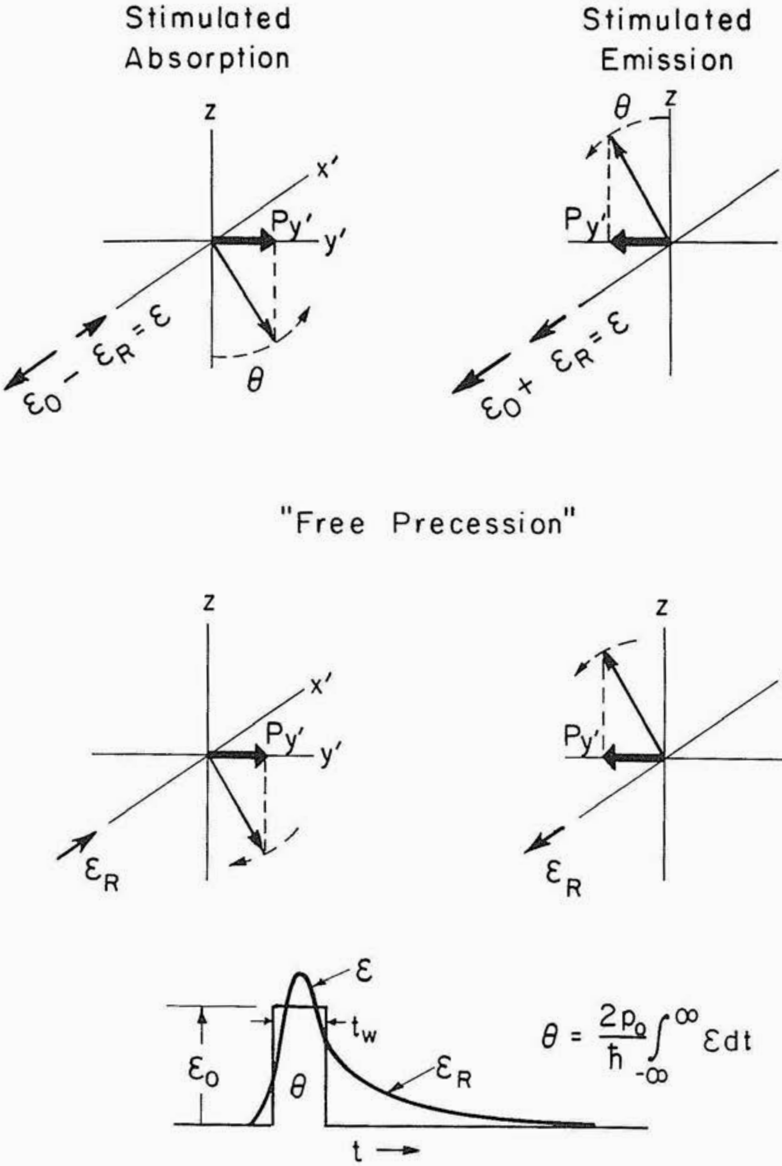
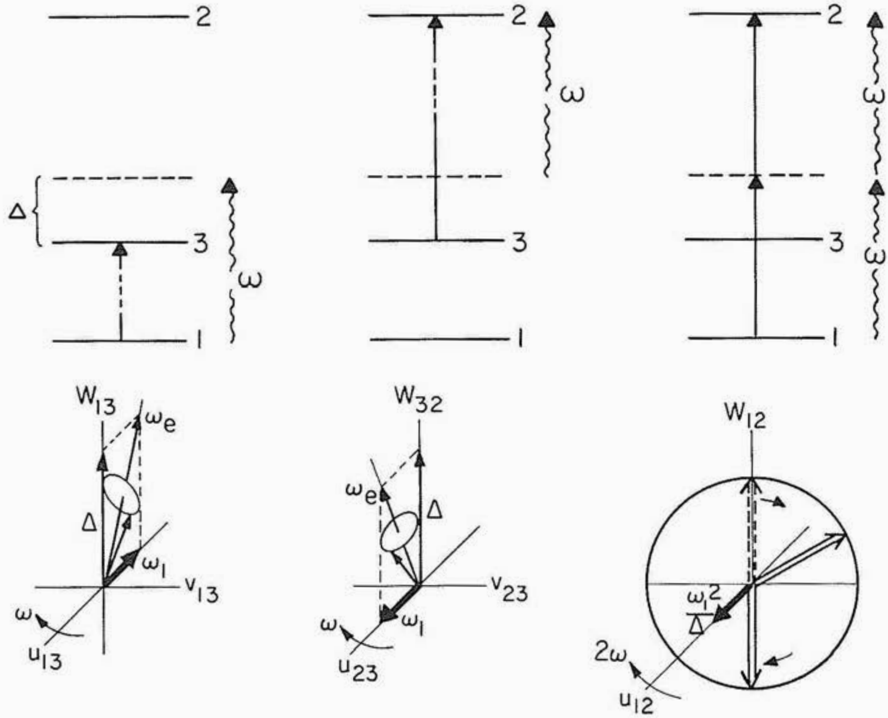



FIG. 5. (top) Stimulated absorption from the ground state and stimulated emission from the inverted state at exact resonance, corresponding to opposite signs of superposition state polarizations $P_{y'}$ in the rotating frame. $\mathcal{E}_o$ is the applied field; $\mathcal{E}_R$ is the reaction field. (bottom) Coherent emission from source polarization $P_{y'}$ after $\mathcal{E}_o$ is suddenly cut off in a time $<< T_2$. Net field $\mathcal{E}$ is a pulse formed from the sum of $\mathcal{E}_o$ (as an applied rectangular pulse) and reaction field $\mathcal{E}_R$. Polarization tipping angle $\theta$ is determined by the time integral of $\mathcal{E}$.




One photon effective field

$$\omega_e = \sqrt{\Delta^2 + \omega_1^2} \sim \Delta \text{ for } \omega_1 << \Delta$$

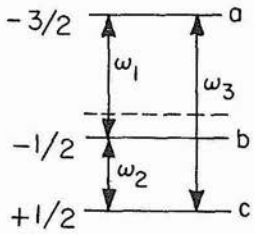
$$W_{13} + W_{32} = W_{12}$$

Two photon effective field

$$\Omega_e = \omega_1^2/\Delta \approx 1/\hbar^2 \cdot \frac{\langle 1|\mu_{13}H_1|3\rangle\langle 3|\mu_{32}H_1|2\rangle}{\Delta} \text{ for } \delta = 0$$

$$\Omega_e = \sqrt{\left(\omega_1^2/\Delta\right)^2 + \left(2\delta\right)^2} \text{ for } \delta \neq 0$$

FIG. 6. TWO PHOTON PROCESS compared to one photon processes in a three level system. In terms of NMR, states 1 and 2 are placed in coherent superposition, describable by Bloch equations. The $W_{12}$ polarization precesses at frequency $2\omega$. The effective frequency $\Omega_e$ appears in the rotating frame for two photons interacting slightly off resonance by amount $2\delta << \Delta$. Populations between level pairs are indicated by $W_{ij}$ with corresponding $u_{ij}$, $v_{ij}$ components. At exact two photon resonance, nutation of $W_{12}$ takes place at the effective Rabi frequency $\omega_1^2/\Delta$, where $\omega_1 << \Delta$.



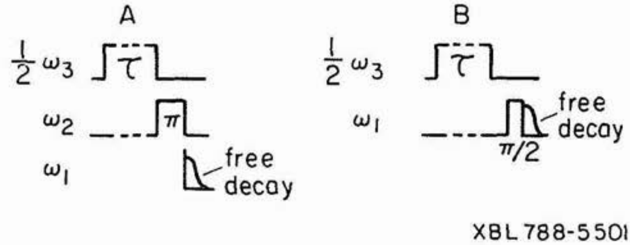




FIG. 7. TRANSFER OF SUPERPOSITION STATE COHERENCE in a three level system with unequally spaced states designated by quantum numbers $m = -3/2,\ -1/2,\ +1/2$ (see ref. 24). A two photon transition with each photon at frequency $\omega_3/2$ is induced by a pulsed field. States $a$ and $c$ are placed in coherent superposition. The far-off resonance conditions $\omega_1 - \omega_3/2 \neq 0$ and $\omega_3/2 - \omega_2 \neq 0$ create negligible direct coupling to state $b$. No dipole signal is obtained from the $a \leftrightarrow c$ superposition. If a $\pi$ pulse at frequency $\omega_2$ transfers coherence from level $c$ to $b$ (immediately after the $\omega_3/2$ pulse is shut off), superposition is created between $a$ and $b$ which gives a dipole radiation decay signal at frequency $\omega_1$. Or a $\pi/2$ pulse at $\omega_1$ will give a free precession dipole signal also at $\omega_1$. Conversely, other combinations for coherence transfer are possible. The principle of coherence transfer is useful in coherent Raman phenomena for measurements of relaxation times between levels not directly observable (i.e., $a \leftrightarrow c$).

penetrate many branches of physics. The Bloch equations as they were originally applied to nuclear magnetic resonance crop up in one form or another as a means for solving problems not necessarily related to spin resonance. The Bloch equations have some of the character of the Boltzmann equation, and lend themselves to modification and approximations suitable for the particular problem at hand. As one applies the Liouville equation in a general way, and as the number of elements of the density matrix increases with the number of quantum states, the bewilderment in finding a reasonable solution[23] can sometimes be reduced by seeking Bloch-type equations from suitable combinations of these elements.

It was a personal pleasure and honor to have been affiliated with Felix Bloch in those early days of excitement and enthusiasm when NMR was in its infancy.

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