NUCLEAR HYDRODYNAMICS IN A RELATIVISTIC MEAN FIELD THEORY

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ABSTRACT

We review a model relativistic quantum field theory of the nuclear many-body problem containing baryons and vector and scalar mesons. We also review the properties of nuclear and neutron matter at all temperatures and densities and of finite nuclei computed in the mean-field approximation. The justification for this approximation is discussed. The model is then used as a basis for discussing the *hydrodynamic flow* of nuclear matter. This is of interest in heavy-ion reactions, in nuclear spectroscopy, and in astrophysics. The mean-field equations obtained by minimizing the energy functional at fixed baryon and momentum densities are derived and used to study the hydrodynamic mass at all densities and flow velocities. The limiting cases of non-relativistic and extreme-relativistic motion are examined in detail. The model is explicitly covariant, and the results are shown to be equivalent to those obtained with appropriate Lorentz transformations.

I. INTRODUCTION

In 1933 Felix Bloch used the linearized equations for hydrodynamic flow to describe the electron cloud in the atom and to carry out his calculation of the energy loss of a charged particle in matter.[1] His elegant treatment of the collective dipole excitations about the mean-field ground state is a major contribution to the quantum theory of many-particle systems. Bloch's approach can be extended to describe collective charge oscillations of any multipolarity in atoms,[2] and also collective spin-waves.[3] If the

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electron cloud in an atom is divided into a set of core electrons treated collectively with Bloch's approach and a set of loosely bound valence electrons, then a reasonable picture is obtained of real neutral atoms including binding energies, charge densities, and photoionization cross sections.[4]

There are many reasons why it is desirable also to have a model for the hydrodynamic flow of *nuclear* matter. The study of collective excitations of nuclei about the mean-field ground state is central to nuclear physics. Furthermore, heavy-ion reactions where large aggregates of nuclear matter are fired into contact offer the possibility of directly observing nuclear hydrodynamics. In addition, the hydrodynamics of nuclear matter forms an essential ingredient in astrophysics in the discussion of supernovae and the formation of neutron stars.

Consider first the case of a *static* neutron star, which is nothing more than an enormous nucleus held together by gravitational attraction. The interior density in a neutron star can be a factor of 10-20 higher than in the interior of observed terrestrial nuclei. To discuss the properties of cold, condensed stellar objects such as neutron stars, it is necessary to know the stress tensor $T_{\mu\nu}$, the source in Einstein's field equations, from nuclear matter densities upwards. For a uniform fluid at rest, which locally describes the situation in a static neutron star, the stress tensor takes the form

$$T_{\mu\nu} = P\delta_{\mu\nu} + (\varrho + P/c^2)u_\mu u_\nu \tag{1.1}$$

where P is the pressure, ϱ the mass density, and $u_\mu = (\mathbf{o}, ic)$ is the four velocity of the fluid. What is thus required at the outset is $P(\varrho)$, or the equation of state of nuclear matter at high density.

Any description of nuclear matter under extreme conditions of density, flow, and temperature requires a more general starting point than the usual theory of nuclear structure based on non-relativistic nucleons interacting through static two-body potentials. There are several reasons for this. For example:

1) At nuclear densities, the velocity is already $v/c \sim 1/4$ for the fastest nucleons at the Fermi surface. As the density is raised the velocities increase, and the relativistic propagation of the baryons, and relativistic propagation and retardation of the mesons generating the interactions, must be taken into account.

2) It is an approximation to describe nuclei in terms of only nucleon coordinates, and a proper description, particularly at high densities where real mesons may be present, must explicitly take into account the meson degrees of freedom.

3) Correct, causal restrictions on the propagation of the modes of excitation of the interacting system must be contained in the theory.

The only consistent theoretical framework for dealing with this nuclear system that I am aware of is a relativistic quantum field theory. Given an appropriate Lagrangian density, we in principle have a complete calculational framework with which to answer any physical question that can be asked about the system. With a relativistic theory that explicitly includes the meson degrees of freedom, we can in principle make reliable extrapolations away from the narrow window on the equation of state of nuclear matter provided by observed nuclei. We must, of course, work with a renormalizable quantum field theory in order to obtain finite results for physical quantities.

The purpose of the present paper is to calculate the properties of hydrodynamic flow of cold nuclear matter within the framework of a model relativistic quantum field theory under study at Stanford for the past few years.[5-14] Many of the ideas in the model are similar in spirit to those of Johnson and Teller[15] as developed by Duerr,[16] and important recent applications of the present model and similar ones[17,18] to the nuclear surface and properties of finite nuclei have also been made by Kerman and Miller,[19] Boguta and Bodmer,[20] and Boguta and Rafelski.[21]

II. THE MODEL

As a model, consider the fields in table 1. We assume that the neutral scalar meson couples to the scalar baryon density through a coupling $g_s\bar{\psi}\psi\phi$ and that the vector meson couples to the conserved baryon current through $ig_v\bar{\psi}\gamma_\lambda\psi V_\lambda$.

TABLE 1

FIELDS IN THE MODEL

Field	Description	(Particles)	Mass × $(c/\hbar)$
ψ	baryon	$(p,n,\ldots)$	M
ϕ	neutral scalar meson	(σ)	m_s
V_λ	neutral vector meson	(ω)	m_v

As motivation for this model, we observe that in the limit of heavy, static baryons the one-meson exchange graphs give rise to an effective nucleon-nucleon potential of the form

$$V_{\text{eff}} = \frac{g_v^2}{4\pi}\frac{e^{-m_v r}}{r} - \frac{g_s^2}{4\pi c^2}\frac{e^{-m_s r}}{r} \tag{2.1}$$

and for appropriate choices of the coupling constants and masses, this potential can be made attractive at large separations and repulsive at short distances in accord with the observed nucleon-nucleon force. The reader will ask, "where are the pions?" In fact, the effects of the pion interaction with baryons largely average out in the description of the bulk properties of nuclear matter because of the strong spin dependence of the coupling. Eventually we must include additional fields for (π, ϱ, etc.) if we hope to achieve a truly quantitative description of the nuclear system.

The field equations for this model are

$$(\Box - m_s^2)\phi = -(g_s/c^2)\bar{\psi}\psi \tag{2.2}$$

$$\frac{\partial}{\partial x_\nu} F_{\mu\nu} + m_v^2 V_\mu = ig_v \bar{\psi}\gamma_\mu\psi \tag{2.3}$$

$$[\gamma_\mu(\frac{\partial}{\partial x_\mu} - \frac{ig_v}{\hbar c} V_\mu) + (M - \frac{g_s}{\hbar c}\phi)]\psi = 0 \quad . \tag{2.4}$$

Here $F_{\mu\nu} \equiv \partial V_\nu/\partial x_\mu - \partial V_\mu/\partial x_\nu$ is the antisymmetric field tensor. Equation (2.2) is simply the Klein-Gordon equation with a scalar source; Eq. (2.3) looks like massive QED with the conserved baryon current

$$B_\mu = i\bar{\psi}\gamma_\mu\psi \tag{2.5}$$

rather than the (conserved) electromagnetic current as source; and Eq. (2.4) is the Dirac equation with the scalar and vector fields introduced in a "minimal" fashion. The free field Lagrangians for this system are contained in any text, and the interaction Lagrangian density is just

$$\mathfrak{L}' = ig_v\bar{\psi}\gamma_\mu\psi V_\mu + g_s\bar{\psi}\psi\phi \quad . \tag{2.6}$$

Knowing the Lagrangian density, we can construct the stress tensor $T_{\mu\nu}$ in the canonical fashion, and from this the four-momentum operators for the system ($\mathbf{P}$, iH$/c$); from the expectation value of $T_{\mu\nu}$ for a uniform system, we have the equation of state through Eq. (1.1).

The theory is Lorentz covariant, and the procedure for quantizing to get the quantum field theory is well known. The theory is renormalizable, for it is similar to massive QED with a conserved current and an additional scalar interaction. Since the dimensionless coupling constants $C_v^2 = (g_v^2/\hbar c)(M^2/m_v^2)$ and $C_s^2 = (g_s^2/\hbar c^3)(M^2/m_s^2)$ are large, we have a strong-coupling theory and have not really made much progress unless a sensible starting solution to the theory can be found. The main point of the present work is that we can find a simple solution to the field equations that *becomes increasingly valid as the density of the system increases*, and which thus provides a meaningful starting point for computing modifications of the equation of state through the relativistic quantum field theory and standard many-body techniques[22] in a consistent fashion.

Consider a uniform system of baryons in a box of volume Ω. As the baryon density increases, so do the source terms on the right-hand sides of Eqs. (2.2, 2.3). When the sources are large, the meson fields can be replaced by their expectation values, which are then classical fields.

$$\phi \rightarrow \langle\phi\rangle \equiv \phi_0 \tag{2.7}$$

$$V_\lambda \rightarrow \langle V_\lambda\rangle \equiv i\delta_{\lambda 4} V_0 \tag{2.8}$$

and Eqs. (2.2, 2.3) can be immediately solved for a static, uniform system to give

$$\phi_0 = \frac{g_s}{m_s^2 c^2} \langle\bar{\psi}\psi\rangle \equiv \frac{g_s}{m_s^2 c^2} \varrho_s \tag{2.9}$$

$$V_0 = \frac{g_v}{m_v^2} \langle\psi^\dagger\psi\rangle \equiv \frac{g_v}{m_v^2} \varrho_B \quad . \tag{2.10}$$

When these classical fields are substituted into the Dirac equation (2.4), this equation is linearized and may be solved exactly. The stationary state solutions for a uniform system $\psi = \exp(-i\epsilon ct + i\mathbf{k}\cdot\mathbf{x})\, U(\mathbf{k},\lambda)$ satisfy a modified Dirac equation with effective mass

$$M^* = M - (g_s/\hbar c)\phi_0 \tag{2.11}$$

and energy spectrum

$$\epsilon_k^{(\pm)} = (g_v/\hbar c)V_0 \pm (\mathbf{k}^2 + M^{*2})^{1/2} \quad . \tag{2.12}$$

The solutions satisfy the relation

$$\bar{U} U = \frac{M^*}{(M^{*2} + \mathbf{k}^2)^{1/2}} U^\dagger U \quad . \tag{2.13}$$

The scalar density is reduced relative to the baryon density by Lorentz contraction. The Dirac fields may be expanded in terms of the solutions to the

Dirac equation with coefficients $A_{k\lambda}$, $B_{k\lambda}$ in standard fashion, and the Hamiltonian density becomes

$$\hat{\mathcal{H}}_{MFT} = g_v V_0 \varrho_B +$$

$$(\hbar c/\Omega)\Sigma_{k\lambda}(\mathbf{k}^2 + M^{*2})^{1/2}(A^{\dagger}_{k\lambda}A_{k\lambda} + B^{\dagger}_{k\lambda}B_{k\lambda})$$

$$+ (m_s^2c^2/2)\phi_0^2 - (m_v^2/2)V_0^2 \quad . \tag{2.14}$$

The new normal-mode amplitudes can be interpreted as creation and destruction operators and the system quantized in the canonical manner. Here we have chosen to define the *mean field theory* (MFT) by normal ordering in these creation and destruction operators. Since the Hamiltonian density in Eq. (2.14) is diagonal, the exact solution for the mean field theory is known and all properties of the system can be computed within this exactly solvable model.

Consider uniform nuclear matter. The ground state of Eq. (2.14) is obtained by filling the states with wave number l and spin-isospin degeneracy γ up to the Fermi level l_F. Here we choose to measure all lengths in units of $1/M \equiv \hbar/m_pc = .2103F$ and all energies in units of $m_bc^2 \equiv m_pc^2 = 938.3$ MeV. The equation of state computed from Eqs. (2.14, 1.1) in the MFT is

$$e = \frac{1}{2}C_v^2n_B^2 + \frac{1}{2C_s^2}(1-\chi)^2 +$$

$$\frac{\gamma}{(2\pi)^3}\int_0^{l_F} d\mathbf{t}(t^2+\chi^2)^{1/2} \tag{2.15}$$

$$p = \frac{1}{2}C_v^2n_B^2 - \frac{1}{2C_s^2}(1-\chi)^2 +$$

$$\frac{1}{3}\frac{\gamma}{(2\pi)^3}\int_0^{l_F} d\mathbf{t}\frac{t^2}{(t^2+\chi^2)^{1/2}} \tag{2.16}$$

$$n_B = \frac{\gamma}{(2\pi)^3}\int_0^{l_F} d\mathbf{t} \tag{2.17}$$

where e, p, and n_B are the dimensionless energy density, pressure, and baryon density. Here

$$\chi \equiv M^*/M \tag{2.18}$$

is the effective mass. The first two terms in Eqs. (2.15, 2.16) arise from the mass terms for the vector and scalar fields. The final two terms are those of a relativistic Fermi gas of baryons of mass χ. The equation of state is given in parametric form $e(n_B)$, $p(n_B)$. It remains to determine the effective mass. This can be done by minimizing $e(\chi)$ with respect to the parameter χ (equivalently, with respect to ϕ_0), which leads to the self-consistency relation

$$\chi = [1 + C_s^2 \frac{\gamma}{(2\pi)^3} \int_0^{l_F} dt \frac{1}{(t^2 + \chi^2)^{1/2}}]^{-1} \quad . \tag{2.19}$$

This is identical to the scalar meson field equation (2.9) where relation (2.13) has been used in the computation of the scalar density. It is evident that at high density $l_F \to \infty$, the effective mass goes to zero and the energy density is dominated by the vector repulsion, as is the pressure. In this limit $p \to e$, implying that the thermodynamic speed of sound approaches the velocity of light from below. This property of a pure vector interaction at high density was first pointed out by Zel'dovich.[23] At lower densities, the attractive scalar interaction can be made to dominate if the coupling constants are chosen properly. The nuclear medium then *saturates*. The coupling constants can be chosen to reproduce the equilibrium properties of nuclear matter[22]:

$$\{\gamma = 4,\ (E - Bm_b c^2)/B = -15.75\,\text{MeV},\ k_F = 1.42 F^{-1}\} \quad .$$

Accordingly,

$$C_v^2 \equiv (g_v^2/\hbar c)\,(M^2/m_v^2) = 195.7 \tag{2.20}$$

$$C_s^2 \equiv (g_s^2/\hbar c^3)\,(M^2/m_s^2) = 266.9 \quad . \tag{2.21}$$

Note that only the ratios of coupling constants to meson masses enter here. The resulting saturation curve is shown in figure 1.[5,6] The corresponding curve for neutron matter obtained by simply replacing $\gamma = 2$ is also shown in figure 1. Many other properties of nuclear matter are now predicted from the two determined constants in Eqs. (2.20, 2.21). These are the only two parameters in the MFT of nuclear matter, and all other properties may now be calculated. The calculated properties are compared with experiment in table 2. The quantity a_4 is the coefficient in the symmetry energy of the semi-empirical mass formula, and K_v is the bulk modulus.[22] If the scalar field is identified as the σ meson (a broad two-pion resonance at $m_s/M \cong .59$) and the vector field with the ω meson ($m_v/M = .835$), then we obtain from Eqs. (2.20, 2.21) the coupling constants in table 2. These are

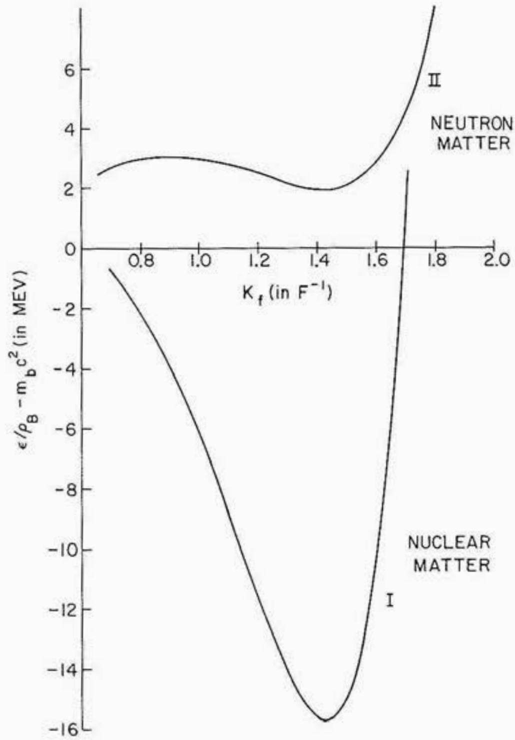

FIG. 1. SATURATION CURVE FOR NUCLEAR MATTER. Coupling constants chosen to fit value and position of minimum. Prediction for neutron matter ($\gamma = 2$) also shown.[6]

TABLE 2

CALCULATED PROPERTIES OF NUCLEAR MATTER AND BASIC COUPLINGS[6]

	a_4	M^*/M	K_v^{-1}	$g_s^2/4\pi\hbar c^3$	$g_v^2/4\pi\hbar c$
Theory	22.0 MeV	0.56	550 MeV	7.39	10.8
Experiment	23.5 MeV	(0.6)	(200-300 MeV)	8.2(σ)	17.3(ω)

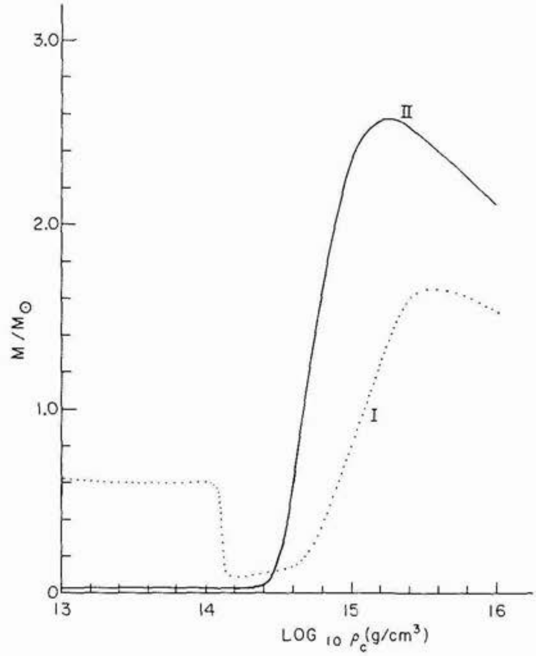

FIG. 2. CALCULATED MASS OF NEUTRON STAR as function of central density-curve II. Curve I comes from integrating the equation of state of Baym et al.[6]

compared with values obtained from a phase-shift analysis of N-N scattering.[6]

The equation of state for neutron matter gives the stress tensor through Eq. (1.1). The Tolman-Oppenheimer-Volkoff equations for the metric in general relativity can now be integrated to give the mass of a static neutron star as a function of central density. The result is shown in figure 2.[6] The MFT gives a maximum mass for a neutron star of $M = 2.57\ M_{\odot}$ There are arguments that the x ray binary pulsar in Vela X1 has a mass $M > 1.4 M_{\odot}$

To extend the previous analysis to finite systems with a fixed number of baryons, we allow spatial variations in the mean fields $\phi_0(|\mathbf{x}|)$, $V_0(|\mathbf{x}|)$. This adds the terms $(c^2 m_s^2/2)\ (\nabla\phi_0)^2\ -\ (m_v^2/2)\ (\nabla V_0)^2$ to the Hamiltonian density. If it is assumed that these fields vary slowly enough so that at each point the baryons can be treated as moving in locally constant fields, then the previous analysis from nuclear matter applies. The ground-state of the system is obtained by minimizing the total energy E as a function of the local Fermi wave number $k_F(|\mathbf{x}|)$. The constraint of a fixed number of baryons B can be incorporated through a Lagrange multiplier μ. Thus we set

$$\delta E - \mu \delta B = 0 \quad . \tag{2.22}$$

Since ϕ_0 and V_0 satisfy the field equations

$$(\nabla^2 - m_s^2)\phi_0(r) = -(g_s/c^2)\varrho_s(r) \tag{2.23}$$

$$(\nabla^2 - m_v^2)V_0(r) = -g_v \varrho_B(r) \tag{2.24}$$

the variations with respect to these quantities vanish and they may be held constant during the variation. The result of the variational principle is that $k_F(r)$ must satisfy the equation

$$g_v V_0 + \hbar c\,(k_F^2 + M^{*2})^{1/2} = \mu = \text{const.} \tag{2.25}$$

which can be used to find $k_F(\phi_0, V_0, \mu; r)$. The local baryon and scalar densities are expressed in terms of $k_F(r)$ and $M^*(r)$ as

$$\varrho_B = \frac{\gamma}{(2\pi)^3}\int_0^{k_F(r)} d\mathbf{k} \tag{2.26}$$

$$\varrho_s = \frac{\gamma}{(2\pi)^3}\int_0^{k_F(r)} d\mathbf{k}\,\frac{M^*}{(M^{*2} + k^2)^{1/2}} \quad . \tag{2.27}$$

Since in this Thomas-Fermi MFT the baryon and scalar densities vanish identically past a certain radius r_0, the asymptotic form of the solution is known, and hence $(V_0'/V_0)_{r_0}$ and $(\phi_0'/\phi_0)_{r_0}$ are determined. The coupled, non-linear, differential Eqs. (2.23, 2.24) can be integrated in from r_0, and μ and $V_0(r_0)$ (or $\phi_0(r_0)$) varied until a solution is found that satisfies the interior boundary conditions $V_0'(0) = \phi_0'(0) = 0$. The total number of baryons is determined at the end of the calculation by integrating Eq. (2.26). This has been carried out by Serr and Walecka.[13] There are now two additional parameters in the theory, since m_s and m_v must be specified independently. In Serr and Walecka[13] it is assumed that the properties of nuclear matter are described as in the previous discussion and that Eqs. (2.20, 2.21) hold; m_s/M is then chosen to fit the observed surface thickness of nuclei $t \cong 2.4F$. The dependence on m_v/m_s is not strong, and a value $m_v/m_s = 1.5$ is chosen in rough accord with the observed meson masses ($m_v/m_s \cong 1.42$). Calculated nuclear properties for the system with $B = 40$ are shown in table 3.[13] (Here the Coulomb interaction is neglected so that in equilibrium $N = Z = B/2$). a_2 is the coefficient in the surface energy in the semi-empirical mass formula[22] and is obtained by fitting the calculated energy to an expression $E - (m_b c^2)B = (-15.75\text{MeV})B + a_2 B^{2/3}$. R_0 is the parameter in the half-

TABLE 3

CALCULATED QUANTITIES FOR B = 40.[13] Here $N = Z$.

	a_2/t	R_0	α_{max}	m_s/M
Theory	13.1 MeV/F	1.03F	1.91 MeV	.518
Experiment	7.4 MeV/F	1.07F	1.80 MeV*	.59

*Obtained from $<\alpha(r)>$ in Sc^{41}.

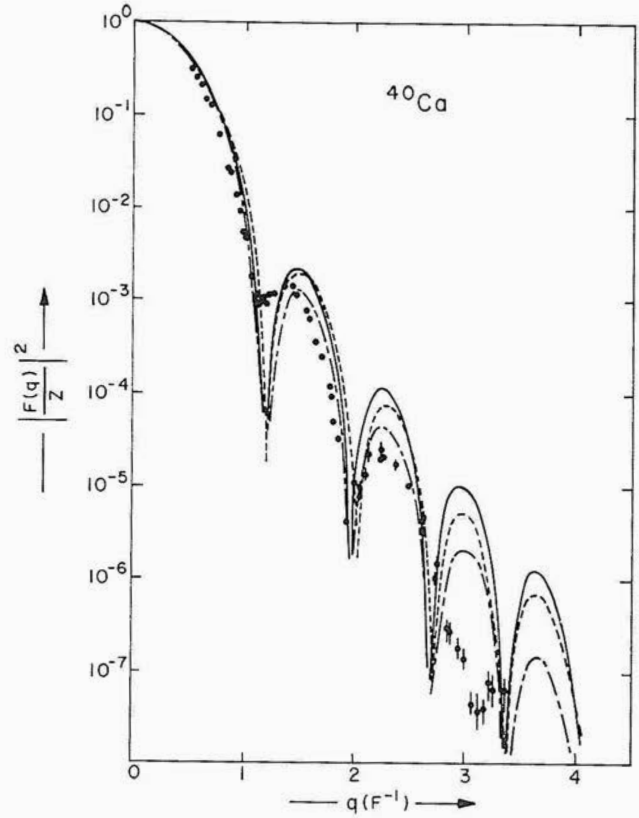

FIG. 3. CALCULATED AND EXPERIMENTAL FORM FACTOR FOR $_{20}Ca^{40}$. Dot-dashed curve includes single-nucleon form factor. Dashed curve includes vacuum fluctuations.[13]

density radius $R_{1/2} = R_0 B^{1/3}$. To test the quality of the calculated charge density, the Fourier transform of the calculated density is compared with the experimental values of the elastic electron-scattering form factor $[d\sigma/d\Omega]_{el}/[d\sigma/d\Omega]_{point}$ for ${}_{20}Ca^{40}$ in figure 3. Also shown is the result of folding in the internal electromagnetic structure of the nucleons through the experimental single-nucleon form factor. The model evidently provides a decent description of the nuclear form factor except for the highest momentum transfers, which probe short-distance spatial variations of the charge density.

The Foldy-Wouthuysen reduction of the single-particle Dirac equation for non-relativistic nucleons moving in the potential generated by the scalar and vector fields $\phi_0(r)$ and $V_0(r)$ allows us to identify the effective single-particle *spin-orbit interaction*

$$V_{so}(r) = \frac{1}{2M^2 r}\left(g_v \frac{dV_0}{dr} + g_s \frac{d\phi_0}{dr}\right)\mathbf{l}\cdot\mathbf{S} \equiv -\alpha_{so}(r)\mathbf{l}\cdot\mathbf{S} \quad . \tag{2.28}$$

The maximum value α_{max} is compared in table 3 with $<\alpha_{so}(r)>$ obtained from the experimental spin-orbit splitting of the $f_{7/2} - f_{5/2}$ levels in Sc^{41}. Note that whereas the effects of V_0 and ϕ_0 cancel in the binding energy, they add in the spin-orbit interaction which, of course, forms the basis of the nuclear shell model.

The extension for nuclei with $N \neq Z$ is discussed by Walecka.[14] The calculation of the equation of state of nuclear matter at all temperatures as well as densities is presented in references 7 and 11.

Corrections to this mean-field theory coming from the effects of the mean meson fields on virtual baryon-antibaryon pairs can be taken into account through the addition of a "vacuum fluctuation" term, which provides a proper treatment of the vacuum energy remaining after extracting the normal-ordered part of the Hamiltonian in Eq. (2.14). This is carried out in references 8, 9, 11, 13, and 18. If the coupling constants are again required to fit the saturation properties of nuclear matter, the effect on the charge density for ${}_{20}Ca^{40}$, although essential to the mathematical consistency of the theory, is not large and is illustrated in figure 3. The inclusion of exchange corrections to the MFT is investigated in references 8, 9, 10, 11, and 13, and the effects of two-body correlations have been examined by Brittan.[10] Collective modes of the system in this model have been investigated by Chin,[8,9] and the thermal conductivity has been calculated recently by Freedman.[12]

III. NUCLEAR HYDRODYNAMICS

With this introduction, we proceed to the main purpose of the present paper, which is to calculate the flow properties of cold nuclear matter at various densities. The canonical stress tensor for the field theory in section II can be written as[5]

$$T_{\mu\nu} = \left\{ -\frac{1}{2}c^2\left[(\frac{\partial\phi}{\partial x_\lambda})^2 + m_s^2\phi^2\right] - \frac{1}{4} F_{\lambda\varrho}F_{\lambda\varrho} - \frac{1}{2}m_v^2 V_\lambda V_\lambda\right\} \delta_{\mu\nu}$$

$$+ \hbar c\, \bar{\psi}\gamma_\mu \frac{\partial\psi}{\partial x_\nu} + c^2(\frac{\partial\phi}{\partial x_\mu})(\frac{\partial\phi}{\partial x_\nu}) - \frac{\partial V_\lambda}{\partial x_\nu}F_{\lambda\mu} \quad . \tag{3.1}$$

This form is unsymmetrized, but that makes no difference for the present discussion of uniform systems. The stress tensor is conserved by construction:

$$\frac{\partial}{\partial x_\mu} T_{\mu\nu} = 0 \quad . \tag{3.2}$$

The four-momentum operator for the system is defined by

$$P_\mu = (\mathbf{P}, \frac{i}{c}\mathrm{H}) \tag{3.3a}$$

$$\equiv \frac{1}{ic}\int d\mathbf{x}\, T_{4\mu} \quad . \tag{3.3b}$$

Differentiation with respect to time and the use of Eq. (3.2) shows that P_μ is a constant of the motion. Furthermore, the integral in Eq. (3.3) is independent of the particular space-like hypersurface because of Eq. (3.2) and Gauss's theorem in four dimensions. Hence P_μ transforms like a Lorentz four-vector.

Consider now *uniform nuclear matter in a state of uniform flow*. We again make the mean-field approximation discussed in section II. The new features of the problem are a finite baryon current

$$\mathbf{B} = \text{const.} \tag{3.4}$$

and since there is now a direction in the problem, a finite spatial part to the vector field [cf. Eq. (2.8)]

$$<\mathbf{V}> \rightarrow \mathbf{V} = \text{const.} \tag{3.5}$$

These two quantities can be related through the use of the field equation (2.3), which implies

$$\mathbf{V} = \frac{g_v}{m_v^2}\mathbf{B} \quad . \tag{3.6}$$

This relation now augments Eqs. (2.9, 2.10). The required modifications of Eqs. (2.12) and (2.13) are

$$\epsilon_k^{(\pm)} = (g_v/\hbar c)\, V_0 \pm [(\mathbf{k} - (g_v/\hbar c)\mathbf{V})^2 + M^{*2}]^{1/2} \tag{3.7}$$

$$\bar{U}U = \frac{M^*}{[M^{*2} + (\mathbf{k} - (g_v/\hbar c)\mathbf{V})^2]^{1/2}}\, U^\dagger U \quad . \tag{3.8}$$

The Hamiltonian density and momentum density in the MFT follow from Eqs. (3.1, 3.3) as in Eq. (2.14).

$$\hat{\mathcal{H}}_{\text{MFT}} = g_v V_0 \varrho_B +$$

$$(\hbar c/\Omega)\, \Sigma_{\mathbf{k}\lambda}\, [(\mathbf{k} - g_v/\hbar c)\mathbf{V})^2 + M^{*2}]^{1/2}\ (A^\dagger_{\mathbf{k}\lambda} A_{\mathbf{k}\lambda} + B^\dagger_{\mathbf{k}\lambda} B_{\mathbf{k}\lambda})$$

$$+\frac{1}{2}m_s^2 c^2 \phi_0^2 - \frac{1}{2}m_v^2 V_0^2 + \frac{1}{2}m_v^2 \mathbf{V}^2 \tag{3.9}$$

$$\hat{\mathcal{P}}_{\text{MFT}} = (1/\Omega)\Sigma_{\mathbf{k}\lambda}\hbar\mathbf{k}(A^\dagger_{\mathbf{k}\lambda} A_{\mathbf{k}\lambda} + B^\dagger_{\mathbf{k}\lambda} B_{\mathbf{k}\lambda}) \quad . \tag{3.10}$$

Note the modification of the baryon energy spectrum in Eq. (3.9) and the presence of the final mass term for the field $\mathbf{V}$.

Let us assume that the ground state of this uniform moving medium is obtained by filling the energy levels up to a Fermi surface $k_F = k_F(\theta_k,\phi_k)$ whose shape remains to be determined. In this case the energy, momentum, baryon, and scalar densities, and baryon current (calculated from Eq. (2.5) and the Dirac equation) become

$$\mathcal{E} = (\hbar c/\Omega)\Sigma_{\mathbf{k}_\lambda}^{k_F}((g_v/\hbar c)V_0 + [(\mathbf{k} - (g_v/\hbar c)\mathbf{V})^2 + M^{*2}]^{1/2})$$

$$+ \frac{1}{2}m_s^2c^2\phi_0^2 - \frac{1}{2}m_v^2V_0^2 + \frac{1}{2}m_v^2\mathbf{V}^2 \tag{3.11}$$

$$\mathcal{P} = (1/\Omega)\,\Sigma_{\mathbf{k}_\lambda}^{k_F}\hbar\mathbf{k} \tag{3.12}$$

$$\varrho_B = (1/\Omega)\,\Sigma_{\mathbf{k}_\lambda}^{k_F}1 \tag{3.13}$$

$$\varrho_s = (1/\Omega)\,\Sigma_{\mathbf{k}_\lambda}^{k_F}\frac{M^*}{[M^{*2} + (\mathbf{k} - (g_v/\hbar c)\mathbf{V})^2]^{1/2}} \tag{3.14}$$

$$\mathbf{B} = (1/\Omega)\,\Sigma_{\mathbf{k}_\lambda}^{k_F}\frac{(\mathbf{k} - (g_v/\hbar c)\mathbf{V})}{[M^{*2} + (\mathbf{k} - (g_v/\hbar c)\mathbf{V})^2]^{1/2}} \quad . \tag{3.15}$$

The Fermi surface enters as an unknown function in these equations

$$\Sigma_{\mathbf{k}_\lambda}^{k_F} \xrightarrow[\Omega\to\infty]{} \frac{\Omega}{(2\pi)^3}\Sigma_\lambda\int\int\sin\theta_k d\theta_k d\phi_k\int_0^{k_F(\theta_k,\phi_k)}k^2dk \quad . \tag{3.16}$$

The shape of this Fermi surface may be obtained by using the thermodynamic argument that at a *fixed* baryon density ϱ_B and momentum density $\mathcal{P}$ (recall that both these quantities are constants of the motion), the system in equilibrium will minimize the energy density. We may speak of densities, since for a uniform system the integral quantities involve only multiplication by the fixed volume Ω. This is now a straightforward variational problem. We minimize $\mathcal{E}$ and incorporate the constraints through Lagrange multipliers μ and $\mathbf{v}$ [cf. Eq. (2.22)]

$$\delta\mathcal{E}(k_F;\phi_0, V_0, \mathbf{V}) - \mu\delta\varrho_B(k_F) - \mathbf{v}\cdot\delta\mathcal{P}(k_F) = 0 \quad . \tag{3.17}$$

Here the additional explicit dependence of $\mathcal{E}$ on the variables $\{\phi_0, V_0, \mathbf{V}\}$ has been noted; $\delta\varrho_B$ and $\mathcal{P}$ contain no explicit dependence on these quantities. The calculation is greatly simplified when it is realized that the mean field equations (2.9, 2.10, and 3.6) ensure that the variations of the energy density with respect to these quantities vanish.

$$\left(\frac{\delta \mathcal{E}}{\delta \phi_0}\right)_{k_F;V_0,\mathbf{V}} = \left(\frac{\delta \mathcal{E}}{\delta V_0}\right)_{k_F;\phi_0,\mathbf{V}}$$

$$= \left(\frac{\delta \mathcal{E}}{\delta \mathbf{V}}\right)_{k_F;\phi_0,V_0} = 0 \quad . \tag{3.18}$$

Thus they may be held constant during the variation of the surface $\delta k_F(\theta_k,\phi_k)$. The remaining calculation using Eq. (3.16) is elementary and yields for the equation of the Fermi surface

$$g_v V_0 + \hbar c[M^{*2} + (\mathbf{k}_F - (g_v/\hbar c)\mathbf{V})^2]^{1/2} - \hbar \mathbf{v}\cdot\mathbf{k}_F = \mu = \text{const.} \tag{3.19}$$

It is convenient now to introduce a new variable

$$\mathbf{t} \equiv \mathbf{k} - (g_v/\hbar c)\mathbf{V} \tag{3.20}$$

and again introduce dimensionless variables as in section II. Equations (3.11, 3.12, 3.13, and 3.15) can then be put in the form

$$e = \frac{1}{2}C_v^2 n_B^2 + \frac{1}{2C_s^2}(1-\chi)^2 + \frac{1}{2}C_v^2\mathbf{b}^2 +$$

$$\frac{\gamma}{(2\pi)^3}\int_0^{t_F} d\mathbf{t}(t^2+\chi^2)^{1/2} \tag{3.21}$$

$$\mathbf{p} = C_v^2 n_B \mathbf{b} + \frac{\gamma}{(2\pi)^3}\int_0^{t_F}\mathbf{t}\, d\mathbf{t} \tag{3.22}$$

$$n_B = \frac{\gamma}{(2\pi)^3}\int_0^{t_F} d\mathbf{t} \tag{3.23}$$

$$\mathbf{b} = \frac{\gamma}{(2\pi)^3}\int_0^{t_F}\frac{\mathbf{t}}{(t^2+\chi^2)^{1/2}}d\mathbf{t} \tag{3.24}$$

where **b** is now the dimensionless baryon current. The equation for the Fermi surface (3.19) becomes

$$(\chi^2 + \mathbf{t}_F^2)^{1/2} - \mathbf{v}\cdot\mathbf{t}_F = \mu_{\text{eff}} \tag{3.25}$$

with

$$\mu_{\text{eff}} \equiv (\mu/m_b c^2) - C_v^2 n_B(1 - \mathbf{v}\cdot\mathbf{b}/n_B) \tag{3.26}$$

and $\mathbf{v} \equiv \mathbf{v}/c$ in dimensionless units. The self-consistency equation for the effective mass χ can be obtained from the minimization of the energy density with respect to that parameter.

$$\chi = [1 + C_s^2 \frac{\gamma}{(2\pi)^3} \int_0^{t_F} d\mathbf{t} \frac{1}{(t^2 + \chi^2)^{1/2}}]^{-1} \quad . \tag{3.27}$$

This is identical to the scalar meson field equation (2.9) where the right-hand side has been calculated with Eq. (3.14). The previous results for uniform nuclear matter at rest in section II can be recovered simply by setting $\mathbf{v} = 0$ in the expressions above, in which case the Fermi surface defined in Eq. (3.25) becomes spherically symmetric, implying $\mathbf{b} = 0$ and then $\mathcal{P} = 0$ from Eqs. (3.24, 3.22).

To examine the structure of these equations consider first the *non-relativistic limit* (NRL) where

$$(\chi^2 + t^2)^{1/2} \cong \chi + t^2/2\chi \quad . \tag{3.28}$$

A simple change of variables

$$\mathbf{t} \equiv \mathbf{s} + \chi \mathbf{v} \tag{3.29}$$

with $d\mathbf{t} = d\mathbf{s}$ then reduces Eq. (3.25) to

$$s_F^2/2\chi = \mu_{\text{eff}} - \chi + \frac{1}{2}\chi v^2 \tag{3.30}$$

and the Fermi surface is spherically symmetric in the variable s. Equation (3.23) then yields

$$n_B = \frac{\gamma}{6\pi^2} s_F^3 \tag{3.31}$$

which expresses s_F in terms of the baryon density. The baryon current is evaluated from Eq. (3.24) in this limit to give

$$\mathbf{b} \cong n_B \mathbf{v}(1 - \frac{1}{2\chi^2}\left(\frac{6\pi^2 n_B}{\gamma}\right)^{2/3}) \tag{3.32}$$

where we here work to lowest order in v assuming $v^2 << 1$. We next define the *hydrodynamic flow velocity* through the relation

$$\mathbf{b} \equiv n_B \mathbf{v}_{\text{hyd}} \quad . \tag{3.33}$$

Equation (3.32) therefore relates the Lagrange multiplier $\mathbf{v}$ to the flow velocity $\mathbf{v}_{\text{hyd}}$ according to

$$\mathbf{v}_{\text{hyd}} \cong \mathbf{v}\left(1 - \frac{1}{2\chi^2}\left(\frac{6\pi^2 n_B}{\gamma}\right)^{2/3}\right) \quad . \tag{3.34}$$

The solution to the self-consistency Eq. (3.27) in this non-relativistic limit is

$$\chi \cong 1 - C_s^2 n_B \quad . \tag{3.35}$$

The momentum density is obtained from Eq. (3.22) as

$$\mathscr{P} = n_B C_v^2 \mathbf{b} + \frac{\gamma}{(2\pi)^3} \int_0^{t_F} \mathbf{t}\, dt \quad . \tag{3.36}$$

We may use this relation to define the *mass of the hydrodynamic flow* according to

$$\mathscr{P} = \chi_{\text{hyd}}\, \mathbf{b} \tag{3.37a}$$

$$\mathscr{P} = \chi_{\text{hyd}}\, n_B\, \mathbf{v}_{\text{hyd}} \tag{3.37b}$$

and a combination of Eqs. (3.34, 3.35, 3.36) then yields

$$\chi_{\text{hyd}} \cong 1 + \frac{1}{2}\left(\frac{6\pi^2 n_B}{\gamma}\right)^{2/3} + n_B(C_v^2 - C_s^2) \qquad ;\text{NRL} \tag{3.38}$$

as the non-relativistic limit of the hydrodynamic mass. Note that the formal expansion parameter is the baryon density [Eq. (3.31)].

The energy density of the system given by Eq. (3.21) differs only by terms of order v^2 from that of nuclear matter at rest at the same baryon density, which can be written in this limit as

$$e \cong n_B + \frac{3}{10}\left(\frac{6\pi^2 n_B}{\gamma}\right)^{2/3} n_B + \frac{1}{2}\, n_B^2(C_v^2 - C_s^2) \qquad ;\text{NRL}. \tag{3.39}$$

We observe from Eqs. (3.38) and (3.39) that the hydrodynamic mass in this non-relativistic limit can be written as

$$\chi_{\text{hyd}} = \left(\frac{\partial e}{\partial n_B}\right)_0 \qquad ;\text{NRL} \qquad (3.40)$$

where the subscript zero denotes the rest frame of the fluid. The pressure of nuclear matter at rest at the baryon density n_B is obtained from Eq. (2.16) as

$$p \cong \frac{1}{5}\left(\frac{6\pi^2 n_B}{\gamma}\right)^{2/3} n_B + \frac{1}{2} n_B^2 (C_v^2 - C_s^2) \qquad ;\text{NRL}. \qquad (3.41)$$

Thus an alternative expression for the hydrodynamic mass is

$$\chi_{\text{hyd}} = \left(\frac{e+p}{n_B}\right)_0 \qquad ;\text{NRL}. \qquad (3.42)$$

Consider next the *extreme relativistic limit* (ERL) of this theory where $\chi \to 0$ and v is arbitrary (with $v < 1$). In this case the Fermi surface is given by Eq. (3.25) as

$$t_F - \mathbf{v}\cdot\mathbf{t}_F = \mu_{\text{eff}} \quad . \qquad (3.43)$$

If we choose $\mathbf{v}$ as the z-axis in momentum space, then the Fermi surface becomes

$$t_F = \mu_{\text{eff}}/(1 - v\cos\theta_t) \quad . \qquad (3.44)$$

This is the equation of an ellipse. In the present dimensionless units Eq. (3.20) implies $\mathbf{k} = \mathbf{t} + n_B C_v^2 \mathbf{v}$ and hence the focus of the ellipse is at the point $n_B C_v^2 \mathbf{v}$ in momentum space. The semi-minor axis is given by $\mu_{\text{eff}}/(1-v^2)^{1/2}$ and semi-major axis $\mu_{\text{eff}}/(1-v^2)$. The baryon density is obtained from Eq. (3.23) as [cf. Eq. (3.16)]

$$n_B = \frac{\gamma}{6\pi^2}\frac{\mu_{\text{eff}}^3}{(1-v^2)^2} \qquad (3.45)$$

which expresses μ_{eff} in terms of n_B at any v. The baryon current in Eq. (3.24) is

$$\mathbf{b} \cong n_B\,\mathbf{v} \qquad (3.46)$$

which identifies the Lagrange multiplier $\mathbf{v}$ as the hydrodynamic flow velocity defined through Eq. (3.33). The momentum density follows from Eq. (3.22):

$$\not{p} = n_B C_v^2 \mathbf{b} + n_B \mathbf{v} \frac{\mu_{\text{eff}}}{(1 - v^2)} \quad . \tag{3.47}$$

We again define the mass for hydrodynamic flow according to Eqs. (3.37) and hence can identify in this limit

$$\chi_{\text{hyd}} \cong n_B C_v^2 + \left(\frac{6\pi^2 n_B}{\gamma} \frac{1}{1 - v^2} \right)^{1/3} \quad ; \text{ERL.} \tag{3.48}$$

Note in this case we are discussing the high-density expansion of the hydrodynamic mass. It is just in this high-density regime that the present mean field theory is expected to become most valid. The solution to the self-consistency equation (3.27) in this limit is

$$\chi \cong [1 + C_s^2 \left(\frac{9\gamma}{16\pi^2} n_B^2 (1 - v^2) \right)^{1/3}]^{-1} \tag{3.49}$$

and indeed $\chi \to 0$ as $n_B \to \infty$ for any v, verifying our initial ERL assumption.

Let us now try to relate the quantities above to corresponding quantities in the rest frame of the fluid (denoted with a superscript zero). First, since the number of baryons dN_{baryons} in a given element $d\mathbf{x}^0$ of fluid is unchanged when the fluid is set into motion, we have (recall all lengths are in units of $1/M$)

$$dN_{\text{baryons}} = n_B^0 d\mathbf{x}^0 = n_B d\mathbf{x} \quad . \tag{3.50}$$

The Lorentz contraction of the longitudinal dimension of the volume tells us that $d\mathbf{x} = d\mathbf{x}^0 (1 - v^2)^{1/2}$ and hence the baryon density in the rest frame is related to the baryon density of the moving fluid by

$$n_B = n_B^0 / (1 - v^2)^{1/2} \quad . \tag{3.51}$$

The hydrodynamic mass in Eq. (3.48) can thus be rewritten as

$$\chi_{\text{hyd}} = \chi_{\text{hyd}}^0 / (1 - v^2)^{1/2} \tag{3.52a}$$

where the hydrodynamic rest mass is defined by

$$\chi^0_{\rm hyd} \equiv n^0_B C^2_v + \left(\frac{6\pi^2 n^0_B}{\gamma} \right)^{1/3} \qquad ; \text{ERL.} \qquad (3.52b)$$

It is evident from Eq. (3.52a) that this mass shows the correct relativistic increase with velocity. The energy and pressure of nuclear matter at rest with density n^0_B are given by Eqs. (2.15, 2.16)

$$e^0 \cong \frac{1}{2} C^2_v (n^0_B)^2 + \frac{3}{4} \left(\frac{6\pi^2 n^0_B}{\gamma} \right)^{1/3} n^0_B \qquad (3.53a)$$

$$p^0 \cong \frac{1}{2} C^2_v (n^0_B)^2 + \frac{1}{4} \left(\frac{6\pi^2 n^0_B}{\gamma} \right)^{1/3} n^0_B \qquad ; \text{ERL.} \qquad (3.53b)$$

Thus the hydrodynamic mass in Eq. (3.52b) can be expressed either as [cf. Eq. (3.40)]

$$\chi^0_{\rm hyd} = \left(\frac{\partial e}{\partial n_B} \right)_0 \qquad ; \text{ERL} \qquad (3.54)$$

or as [cf. Eq. (3.42)]

$$\chi^0_{\rm hyd} = \left(\frac{e+p}{n_B} \right)_0 \qquad ; \text{ERL.} \qquad (3.55)$$

Equations (3.48) and (3.52) are the main results of the present work. The hydrodynamic mass gets very large, and hence the medium gets very stiff, either for high density ($n_B \to \infty$, any v) or high velocity (fixed large n^0_B, $v \to 1$). This is the mass that enters into Newton's second law for the rate of change of momentum and the basic equations of hydrodynamics for the system.

We may understand Eq. (3.54) in the following manner: Suppose a few baryons are added to a volume $d\mathbf{x}^0$ of nuclear matter at rest so that the density in this region is changed by δn^0_B and the energy density by

$$\delta e^0 = \left(\frac{\partial e}{\partial n_B} \right)_0 \delta n^0_B \qquad . \qquad (3.56)$$

The change in energy of the system $\delta\epsilon^0$ is given as

$$\delta\epsilon^0 = \delta e^0 d\mathbf{x}^0 \quad . \tag{3.57}$$

Now the energy and momentum of the system defined by Eqs. (3.3) form a four-vector

$$\delta p_\mu^0 = (\mathbf{0}, i\delta\epsilon^0) \tag{3.58}$$

where we henceforth denote the dimensionless stress tensor and four-momentum by $t_{\mu\nu}$ and $p_\mu \equiv (\mathbf{p}, i\epsilon)$ respectively. Thus for a Lorentz transformation in the z-direction (along $\mathbf{v}$)

$$a_{\mu\nu}(v) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \dfrac{1}{(1-v^2)^{1/2}} & \dfrac{-iv}{(1-v^2)^{1/2}} \\ 0 & 0 & \dfrac{iv}{(1-v^2)^{1/2}} & \dfrac{1}{(1-v^2)^{1/2}} \end{bmatrix} \tag{3.59}$$

after which the fluid appears to be moving with velocity v in the z-direction, the Lorentz transformed four-momentum

$$\delta p_\mu = a_{\mu\nu}\delta p_\nu^0 \tag{3.60}$$

is given with the aid of Eq. (3.50) by

$$\begin{aligned} \delta\mathbf{p} &= \frac{\mathbf{v}}{(1-v^2)^{1/2}}[\delta e^0 d\mathbf{x}^0] \\ &= \frac{\mathbf{v}}{(1-v^2)^{1/2}}\left(\frac{\partial e}{\partial n_B}\right)_0 \delta n_B d\mathbf{x} \quad . \end{aligned} \tag{3.61}$$

Therefore the additional momentum density in the moving fluid is obtained as

$$\delta \mathcal{p} = \frac{\mathbf{v}}{(1 - v^2)^{1/2}} \left(\frac{\partial e}{\partial n_B} \right)_0 \delta n_B \tag{3.62}$$

and we recover our previous result for χ_{hyd} in Eq. (3.54).

Equation (3.55) may be understood as follows. The stress tensor in Eq. (3.1) forms a second-rank tensor. For a uniform fluid at rest, the stress tensor in Eq. (1.1) takes the form

$$t^0_{\mu\nu} = p\delta_{\mu\nu} + (e+p)\, u_\mu u_\nu \tag{3.63}$$

with $u_\mu = (\mathbf{0}, ic)$. After the Lorentz transformation (3.59), the stress tensor for the moving fluid is

$$t_{\mu\nu} = a_{\mu\mu'}\, a_{\nu\nu'}\, t^0_{\mu'\nu'} \quad . \tag{3.64}$$

Since the medium is uniform before and after the Lorentz transformation, we need not worry about the point at which the tensor is evaluated. Explicit evaluation of the sum in Eq. (3.64) using Eqs. (3.59) and (3.63) yields

$$t_{4j} = \frac{iv_j}{(1 - v^2)} (p+e)_0 \quad . \tag{3.65}$$

The momentum is given by Eq. (3.3) as

$$p_j = \frac{1}{i} \int d\mathbf{x} t_{4j} \tag{3.66}$$

or equivalently, with the aid of Eq. (3.51)

$$\mathbf{p} = \int \left[\frac{\mathbf{v}}{(1 - v^2)^{1/2}} \left(\frac{p+e}{n_B} \right)_0 \right] n_B d\mathbf{x} \quad . \tag{3.67}$$

Thus the momentum density is

$$\mathcal{p} = \frac{\mathbf{v}}{(1 - v^2)^{1/2}} \left(\frac{p+e}{n_B} \right)_0 n_B \tag{3.68}$$

and we recover Eq. (3.55).

The equivalence of Eqs. (3.54) and (3.55) is just thermodynamics, for if the number of baryons B is kept constant and the volume Ω is varied, we have

$$\frac{\partial e}{\partial n_B} = \frac{\partial(\epsilon/\Omega)}{\partial(B/\Omega)} = -\frac{\Omega^2}{B}\frac{\partial}{\partial\Omega}\left(\frac{\epsilon}{\Omega}\right)$$

$$= -\frac{1}{n_B}\left(\frac{\partial\epsilon}{\partial\Omega}\right)_B + \frac{e}{n_B} = \frac{p+e}{n_B} \tag{3.69}$$

by the definition of the pressure as $p = -(\partial\epsilon/\partial\Omega)_B$.

In summary, we have developed a general analysis for the uniform flow of nuclear matter in this mean field theory, and we have shown that the mass for hydrodynamic flow is given at high densities by Eqs. (3.48) and (3.52). One can apply the present results to non-uniform flow with the same approach as used in the discussion of finite nuclei in section II. Although the present field theory severely oversimplifies the actual meson degrees of freedom in the nucleus, it is interesting to have a model that (1) describes observed nuclear matter, (2) never introduces the concept of a static two-nucleon potential, (3) explicitly takes into account the meson degrees of freedom, and (4) exhibits Lorentz covariance at every stage. The model thus, at least in principle, allows an extrapolation away from ordinary nuclear matter to regimes of very high density and very large flow velocities.

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