II

THE SIZES AND MASSES OF THE STARS

WE described yesterday the close relations which exist between the spectral types, colors, and temperatures of the stars, but we said little or nothing about the actual temperatures of their surfaces. At the beginning of the twentieth century, there was really very little that could be said; but since then our knowledge has increased rapidly.

The laws of radiation from heated bodies are qualitatively familiar to everyone—especially in these days when electric heating is a commonplace. If we heat a body to a moderate temperature, it will radiate energy, which the hand held near by can appreciate as heat—though the eye cannot see it, because it is carried by waves too long, and too slowly vibrating, to affect this organ of special sense. Raise the temperature higher, and the heat radiation increases, and light begins to appear, at first dull and red, then brighter. As the temperature still increases, the light becomes much brighter and changes from red to yellow, and at last to white—till finally our experiment (as in the case of an "overvolted" lamp) is brought to an end by the melting or volatilization of the heated body.

The accepted formulation of these laws in mathematical language is due to Planck. His formula—tested both by observation and by thermodynamic reasoning—is rather complicated to discuss here, but its principal consequences, so far as they affect astronomical phenomena, may be stated in simple form. They apply primarily to the behavior of a perfect radiator (or "black body" as it is often called, because such a body would be also a perfect absorber of incident radiation, and hence look perfectly black), and, as might be expected in any physical formula, the temperature which appears in it is measured, not from the arbitrarily chosen zero of our thermometers, but from the absolute zero (—273° centigrade, or —460° Fahrenheit) at which a body would contain no heat energy.

Considering first the total radiation of energy, from a unit area, and in a unit time, it is found that this is proportional to the fourth power of the absolute temperature. At 1000° absolute (on the Centigrade scale) the emission is 1.37 calories per square centimeter per second, according to the excellent measures of Coblentz, which should be accurate within a fraction of one per cent. Since the Sun radiates 1490 calories per second from every square centimeter of its surface, a simple calculation shows that, if it were a perfect radiator, its temperature would be 5740° absolute. An imperfect radiator would emit less heat at this temperature, and have to be heated hotter in order to give out the same amount. Hence this figure may be regarded as a lower limit for the actual surface temperature of the Sun.

Taking next the amount of light radiated per unit of surface—that is, the surface-brightness of the body—it appears that the relation between this and the temperature is most simply defined by expressing the surface brightness in stellar magnitudes. We may then say that the changes in surface brightness are proportional to those in the reciprocal of the temperature, so that we have an equation of the form $J = A + B/T$, where J is the surface brightness, T the temperature, and A and B are constants. This equation is not absolutely exact, but the error only becomes important at temperatures exceeding 10,000°, when the formula gives values which are a little too low. Applying this method to the Sun, using Abbot's measures of the energy which it sends us in different wave-lengths, we find a temperature of 6100°. This also should be a little too low if the Sun is not a perfect radiator. The discordance between this and the previous value shows that such is indeed the case.

For light of different wave-lengths the coefficient B in the above equation varies inversely as the wave-length, indicating that the change of violet light (and hence of photographic magnitude) with the temperature is more rapid than that of visual magnitude. It follows that the variation of the color index with the temperature can also be represented by an equation of the same form, but with different values of the numerical coefficients A and B, the latter being about one-quarter as great as in the case of the visual surface brightness. Applying this method again to the Sun (still using Abbot's data), we get a decidedly lower temperature, about 5200°. The discrepancy can be explained by the fact that the violet part of the solar spectrum is full of dark lines, while there are not nearly so many in the green. If we could get rid of these, the violet rays would be considerably more strengthened than the green, the Sun's light would appear blue, and the "color-temperature" would come out higher. All things considered, we may adopt 6000° as the surface temperature of the Sun, with reasonable assurance that we are within a few per cent. of the truth. We must remember, however, that this is only a sort of average temperature of the various layers in the sun's atmosphere. The deeper layers are undoubtedly hotter than those nearer the surface, and it is therefore not surprising that at the centre of the Sun's disk, where we look straight down into its atmosphere, and probably see deeper into it, the light is brighter and bluer, corresponding to a temperature about 400° higher than calculated for the integrated light of the whole Sun.

When we attempt to estimate the temperatures of the stars, our first reliance must be upon the method depending upon the color of their light—for, without a knowledge of their diameters, we cannot directly determine the surface brightness, nor the emission of energy per square centimeter. Extensive studies of the distribution of energy in stellar spectra—which is a more precise specification of the color of their light—have been made by Wilsing and Scheiner at Potsdam, and by other observers. Their results are in good general agreement, and indicate that the surface temperature of the M-stars is about 3000° or a little higher; of the K-stars 4000°, of the G's 5400°—agreeing substantially with the color-temperature of the Sun—; for Class F about 7000°, and Class A, 11,000°. The typical stars of Class B must be exceedingly hot, but it is very difficult to determine their temperatures, as a small error in the observed colors makes in this case a large one in the temperature. From all the evidence, it may be estimated that in some cases their temperature reaches 20,000°, or perhaps more. The observed color indices of the stars agree excellently with those which would be exhibited by perfect radiators at these temperatures—which is not surprising, because they depend on observation, in another manner, of the very characteristics by means of which the temperatures were calculated.

Another, and more important, check upon the estimates of temperature can be found in the *luminous efficiency* of the stars—that is, the ratio of their emission of light to their whole out-put of radiant energy, that is, of heat. It is only in recent years that apparatus delicate enough to measure the heat of the stars has been devised. The most successful observations have been those of Coblentz at the Lick Observatory in 1914. With an exceedingly delicate thermopile, placed at the focus of the three-foot Crossley Reflector, and connected with a galvanometer of extreme sensibility, he obtained not merely indications, but measures, of the heat radiation from white stars as faint as the third magnitude and red stars at the limit of visibility to the unaided eye. More than eighty stars in all were measured, distributed among all the principal spectral classes. From these results it is found that stars of the same spectral class all give about the same amount of heat in proportion to their light, but from one class to another the proportion is very different. The whiter stars, of classes B, A, and F, give the most light in proportion to their heat, and are very similar to one another. But, for the same visual radiation, stars of Class G send out about 70 per cent. more heat, those of Class K more than twice as much, and the M-stars six times as much. The extreme case is Alpha Herculis, which has a spectrum of very pronounced M type, and gives out more than thirty times the heat, in proportion to its light, which an A-star does—or only three per cent. as much light, in proportion to the total heat.

Now it is well known to all illuminating engineers that the luminous efficiency of artificial sources of light increases very rapidly with the temperature—which is why tungsten lamps, which work at a higher temperature, give us several times as much light for our money as the old carbon filaments. The same principle, applied to the stars, confirms the low temperatures of Classes K and M. But the white stars present an apparent difficulty. As a black body is raised to very high temperatures—well above that of the Sun—most of the energy which it radiates passes into the ultra-violet, where it does not affect our eyes. Hence, theoretically, the luminous efficiency should be a maximum for a temperature not far from the Sun's, and fall off again for the hotter stars. The solution is found in the fact that the observations were made with light which had come through the earth's atmosphere, and been reflected from a silvered mirror. Our atmosphere is entirely opaque to all ultra-violet light of wave length less than 3000 Angstrom units (¾ that of the visible violet) and silver, though an admirable reflector for all visible rays, hardly reflects at all in the ultra-violet below 3000 and 3300. Hence practically no radiation of shorter wage length than this and very little beyond 3500, reaches the thermopile, and the powerful ultra-violet radiation of the white stars fails to record itself. When allowance is made for this, and also for the absorption of the long infra-red wave in the atmosphere, we get the ratio between the amount of the light of a distant hot body and that portion of its heat which could have reached the measuring instrument. Calculating this, with the temperatures already assigned to the various spectral classes, we obtain results which are in excellent agreement with Coblentz's observations throughout their whole range. For the coolest stars, this affords a better method of measuring temperature than the color index, which may be considerably affected by the presence of heavy dark bands in the green, yellow and red. In this way it may be calculated that the surface temperature of the one N-star which was observed by Coblentz is about 2500°, and that of Alpha Herculis but a little more than 2000°.

With our estimates of temperature thus confirmed, we may proceed to calculate the relative surface brightness of the stars. Taking the Sun as typical of Class G, we find that a star of the same size, but of Class B0, should be perhaps thirty times as bright, one of Class A0 ten or twelve times as bright, one of Class F0 about three times. The surface brightness of a K0 star should be about 1/7 of the Sun's and that of an M-star about 1/50. All these values, of course, are averages, and individual stars may sometimes run somewhat above or below them, especially at the extremes of the spectral scale. As an extreme example (to judge by its luminous efficiency), Alpha Herculis may have a surface brightness as low as 1/500 that of the Sun.

We are now in a position to form an idea of the real sizes of the stars. Take, for example, Sirius, which is 25 times as bright as the Sun, and of Class A. Taking its surface brightness as ten times the Sun's, its superficial area comes out 2½ times the Sun's and its diameter 60 per cent. greater than that of the Sun, or 1,200,000 miles. Procyon, of Class F5, six times as bright as the Sun, and probably twice as bright per unit of surface, must be nearly equal to Sirius in diameter. Alpha Centauri, which is almost exactly similar to the Sun in brightness as well as in spectrum, must be also very nearly of the Sun's diameter, 866,000 miles. Passing to redder stars, we may take 61 Cygni. The components of this well-known double star, one of our closest neighbors in space, give out respectively 1/20 and 1/40 of the Sun's light. Their spectra are K7 and K8, and their probable surface brightness something like 1/30 and 1/35 of the Sun's, which would make their diameters about 1,100,000 and 800,000 miles. A typical dwarf M-star may be taken as giving 1/150 the Sun's light, which, with a surface brightness of 1/50, would make its diameter 60 per cent. of the Sun's, or 500,000 miles. Finally, we may take "Barnard's star," the nearest after the system of Alpha Centauri, which is only 1/2700 as bright as the Sun. Its spectrum is of an advanced M type, and we may perhaps estimate its surface brightness as 1/300 of the Sun's, which would make its diameter some 300,000 miles. All these are dwarf stars, and have been chosen as fairly typical examples. The conclusion that such stars were usually from a couple of million to half million miles in diameter would be confirmed by further evidence.

For the giant stars, however, things must be very different. Take first the stars in Orion's belt, which average 4000 times as bright as the Sun. They are of Class B0, and their surface brightness may be estimated as thirty times the Sun's, which would correspond to a diameter of just about ten million miles. For Rigel, of Class B8, and 13,000 times the Sun's luminosity, the surface brightness may be estimated as 12 times the Sun's and the diameter about 30 million miles. Arcturus, a good example of the ordinary run of the redder giants, is about 90 times as bright as the Sun. Taking its surface brightness as 1/7 of the Sun's, since it is of Class K0, we find a diameter of 25 million miles. At the extreme of size come those stars which are at the same time extremely bright and very red. Betelgeuse, in Orion, is of Class M, and, according to the best measures of parallax, appears to be 900 times as bright as the Sun. With a surface brightness 1/50 that of the latter, the estimated diameter reaches the enormous value of 210 million miles. Antares, of similar spectral type, and 3000 times brighter than the Sun, by the same token comes out 390 million miles in diameter. These last values are, however, exceptional. We know of no other stars which bid fair to rival them, except perhaps Alpha Herculis, which according to Adams, is 250 times as bright as the Sun. With our previous estimate of surface brightness, its diameter would be 300 million miles —but this particular estimate is exceedingly uncertain.

These conclusions are so remarkable that they require to be tested in every available way. Until within a year or two, there was only one direct way of finding the diameter of a star, and this could be used in only a few instances. There are a good many stars which show regular, periodic, variations in their light, which give every evidence of arising from eclipse—the star being really double, with components so close together that they hide one another, partially or completely, during every revolution. If we have good measures of the brightness of such a star throughout its changes, we can find the relative diameters and brightness of the two stars—can indeed make a complete map or model of the system, except that we do not know the actual dimensions, that is, on what scale our model is drawn. But when, in addition, the star has been observed with the spectroscope, and both components are bright enough to be recorded, we may find the velocity of their orbital motion, and so put a scale of miles upon our model, and obtain the actual diameters of the stars. About a dozen such systems have so far been investigated. All these stars but one are bigger than the

Sun, and most of them are between twice and four times its diameter. As they are of Classes B and A, this confirms our estimates, when it is remembered that the stars in Orion's belt are unusually luminous, even for B-stars, and therefore probably larger than the average. The one star which is smaller than the Sun (about two-thirds its diameter) is a dwarf (W Ursae Majoris) of spectrum F8, and again supporting our conclusion.

But until last year, there was no direct confirmation of the great diameters computed for the redder giant stars. This has been furnished by one of the most notable advances in observational astronomy in the present generation—the direct measurement of the angular diameters of stars by Michelson and his associates at Mount Wilson. The great difficulty in the way of such measurements lies partly in the enormous distances of the stars, but mainly in the properties of the waves of light themselves. Though these travel in straight lines in empty space, yet, when they pass a sharp obstacle they have a slight tendency to bend inwards into the shadow: just as sea-waves passing the end of a long breakwater would spread sidewise into the calm space behind it. This spreading, called diffraction, is greater the narrower the aperture through which the light is admitted. With a very narrow slit, it becomes conspicuous, as may be seen by looking at a distant object through the gap below two lead-pencils held close in front of the eye, and pressed so close that light barely gets between them. With larger apertures, it is evident only under considerable magnifying power. The "diffraction pattern" produced by the circular aperture of a telescope, when sufficiently magnified, appears as a circular disk of light, brightest at the centre, and fading off gradually toward the edge, surrounded by a succession of concentric luminous rings, separated by dark spaces and each fainter than the last. Even if the object at which the telescope was directed were a mathematical point, its image would still show this spurious disk, and, for a larger object, the image of each point is similarly expanded, so that the whole effect is blurred. No perfection of workmanship can eliminate this difficulty, which is inherent in the very nature of light; but it may be reduced by using a telescope of large size, for, the larger the aperture of the telescope, the smaller is the spurious image. For a one-inch aperture, it is 4″.5 in diameter, but for a ten-inch it is only one-tenth as large. For the 40-inch Yerkes telescope, the diffraction disk measures only 0″.11, and for the 100-inch at Mount Wilson it should be slightly less than 0″.05. A large telescope, therefore, is inherently superior to a small one in its capacity for revealing fine detail, such as close double stars. When, with a small instrument, the spurious disks would overlap and conceal all trace of duplicity, a great one shows the components sharply separated—provided, at least, that the air is steady. On nights when the air is not optically homogeneous—which, alas, are in the large majority at most places—there is a continuous "boiling" or disturbance of the image, which seriously increases the blurring, and too often renders all refined observations impossible.

From the known distances of the stars, and the real diameters already estimated, it is easy to show that, even in the most favorable instances, the angular diameter of a star will not exceed 0″.05, and that cases in which it is greater than 0″.01 are rare. The real disks of the stars, therefore, must be smaller than the spurious disks produced by diffraction in even the greatest existing telescopes, and all that we can hope to see, even under ideal atmospheric conditions, is the latter. This makes the problem seem hopeless—barring the construction of huge telescopes, far larger than any now in existence, and fabulously costly. But Michelson—a life-long student of light waves and their uses—has flanked the obstacle by the apparently mad course of blocking up most of the opening of the telescope, and letting the light come in only through two apertures on opposite sides of the centre. The light from one of these apertures alone would produce a diffraction disk (or a figure of some other shape, if the opening were not circular) of much larger size than that given by the full aperture of the telescope. But, when the images from the two apertures are superposed, a new phenomenon appears. The large fuzzy spurious disk is crossed by fine, parallel, equidistant, dark bands, or "fringes" of exquisitely clear definition. These fringes are produced by the "interference" of the light waves of the pencils proceeding from the two apertures, very much as the spurious disk is produced by the interference of waves from all parts of the full aperture: but the distance between the fringes is inversely proportional to the separation of the two apertures, and when these are set at opposite sides of the full opening, the fringe width is eighty per cent. of the diameter of the spurious disk due to the unobstructed aperture.

Suppose now that we have a very close double star—so close that, even with the full aperture, the spurious images of the components are confused. If we pass to the two apertures, we shall have larger spurious disks than ever, and more completely superposed, but each will be crossed by its own system of fringes, and the central fringe of each system will pass through the geometrical image of the corresponding star. If these geometrical images are separated by only half the distance of the fringes, the bright fringes produced by one star's light will just occupy the dark spaces between the fringes given by the other, and the system of fringes will disappear. This happens when the components of the double star are of the same magnitude. If one of the stars is slightly brighter than the other, the "visibility" of the fringes diminishes, and they assume a washed-out appearance, without vanishing entirely. In either case, the existence of a double star can be detected, although the components are separated by less than half the distance at which their spurious images would begin to be confused, if observed with the full aperture. By making the distance between the apertures adjustable, and arranging them so that the line joining them can be turned to cross the main aperture in any direction, it is possible to measure the distance of a close double star (changing the distance of the slits until the fringes vanish). In this way measures were made upon the bright star Capella, which has long been known, from spectroscopic measures, to be a very close binary, with a period of 104 days. No telescope has ever resolved the components, but, with the interferometer, immediate evidence of the duplicity was secured. What is more, and better, it was found that very precise measures could be made, following an ingenious method devised by Anderson, although the distance was as small as 0".04. Indeed, the precision of the results appears to be fully twenty times greater (as regards the distance measures, at least) than can be secured by any method of observation previously known. Best of all, it was found that atmospheric disturbances—contrary to expectation—had much less influence upon the definition of the fringes than upon that of the star-image as a whole. This is conspicuous even to the novice who has the privilege of observing with the instrument. While the star images dance about, the fringes remain almost fixed, and present a singular appearance of objective reality.

So far we have spoken as if the single stars of the pair were mere luminous points. If the interferometer is turned upon an object of finite angular diameter, and the distance between the apertures increased, the fringes gradually fade away and finally disappear. How this happens may be seen by imagining the star-disk to be divided into two semi-circles by a line perpendicular to the position angle of the interferometer apertures. Each half-disk is an independent source of light, and will give its own system of fringes. As the apertures are separated and the fringes become narrower the two sets will finally become superposed, giving a uniform illumination. When this happens, the reading of our instrument (assuming that we interpret our observations as in the case of a double star) will give the distance between the centre of the luminous area of one semi-circle and that of the other. It is easy to see that this distance will be rather less than half the diameter of the whole circle—in fact it is about forty-one per cent. of it. Our readings may, therefore, be calibrated to give at once the diameter of the star-disk.

The resolving power of the interferometer is less in this case than for a double star, and a greater base line than even the diameter of the 100-inch telescope was desirable. An auxiliary apparatus was therefore adopted, consisting of a rigid steel beam twenty feet long, fixed across the outer end of the telescope, carrying two mirrors inclined at 45° to the axis of the beam, and capable to being set at any desired distance apart, up to the full light of the beam. These reflect the star's light inward along the beam to another pair of diagonal mirrors, which send it into the telescope. With this device the effective distance between the "apertures" is that between the centres of the outer mirrors, and the power of the instrument is more than doubled. Working with this, Michelson and Pease, in December 1920, found that the fringes disappeared in the light of Betelgeuse (Alpha Orionis) when the mirrors were ten feet apart, while in the case of other stars the fringes were clearly visible with a much wider separation—proving that the cause of their disappearance was not any defect of the instrument, or atmospheric disturbance, but a measurable diameter of Betelgeuse. Calculation showed the diameter to be 0″.047—a small quantity according to ordinary standards, but corresponding to enormous dimensions in miles, for the parallax of this star (according to the average of the results of several good observers) is only 0″.020. The diameter of the star is therefore 2.3 times the distance which separates the Earth and the Sun, or 215 millions of miles. In the following summer Pease extended this success by measuring the diameter of Arcturus, 0″.022, and of Antares, 0″.040. The parallaxes of these stars, ″.097 and ″.009, are known with greater percentage accuracy than that of Betelgeuse, and the computed diameters are correspondingly more trustworthy. That of Arcturus comes out 21 million miles and of Antares 400 million—considerably larger than the orbit of Mars.

A few other stars, notably Aldebaran, probably have angular diameters large enough to measure in the same fashion. To measure smaller stars, it might appear neccessary to build a still larger interferometer. But this has been avoided by Michelson's ingenuity. By a very simple device, he is able to produce, in the same field of view as the fringes already described, another set, formed by light from the same star and adjustable to any desired degree of visibility. In this way the visibility of the main fringes may be accurately measured, and this tells us how far towards disappearance they are when the mirrors are set at a given distance—say twenty feet—and hence at what distance the mirrors would have to be set to give complete disappearance, even though this may be far beyond the mechanical limits of the apparatus. This will multiply the power of the instrument three or four fold, if not more, and may make it possible to measure a few of the white stars, such as Sirius. This solution of a century-old problem, in its brilliancy, simplicity, and rapid advancement, deserves to rank with the most noteworthy triumphs of physical investigation, and would doubtless have brought a swarm of honours upon the head of its inventor if he had not already received almost all that the scientific world has to give. With its aid one may reasonably expect to have, within a few years more, a sufficient number of measures of stellar diameters to tell us whether the theoretical predictions will be as good in other cases as they are in these three. It may be mentioned in passing that the estimates of temperature for the various classes, upon which the calculations given above depend, are taken from some notes of mine prepared about three years before Michelson's apparatus was projected. Indeed, before Michelson's results appeared, Wilsing in Germany, Eddington in England, and Shapley and I in this country, had all published predictions concerning the diameters of Betelgeuse, none of which differed by more than about thirty per cent. from the value later observed. Unless further observations with the interferometer reveal larger discordances than have so far appeared, it should soon be possible to obtain thoroughly reliable values for the surface brightness of the stars of the various spectral classes, and then to calculate the diameters in miles of all stars of known parallax, and the diameters in seconds of arc of all the stars in the sky—except perhaps for a few abnormal objects.

We can be sure then, that, while the dwarf stars are all much alike in size, the giants are huge affairs—so huge, in some cases, that their size would be almost incredible if the evidence were not conclusive. The bulk of these bodies is amazing. Arcturus—not a large star as giants go—must have about fourteen thousand times the cubic content of the Sun, while Antares exceeds the Sun in volume a hundred million fold! One naturally asks, Is the quantity of matter in such a star—its mass—proportionally as great as its bulk? To answer this question in the case of any star, we must have recourse to gravitational theory. We can measure the mass of an astronomical body only by the effects of its attraction upon the motion of some neighboring body—and this limits our knowledge of stellar masses to double stars. Fortunately these are so numerous and so well distributed among the various spectral classes, both giant and dwarf, that we can be reasonably sure that they present a fair sample of the general run. When the stars of a binary pair have completed a large part of a revolution (or more) while under observation, their orbit can be computed, and then, if the parallax is known, the mass of the pair is easily found. Even if the motion is so slow that only a few degrees of the orbit have been described in a century, it is still possible to get valuable information by a statistical process, which gives values which may be considerably too large or too small for any individual star, but leads to an average which will be nearly correct for groups of twenty stars or more.

Combining these methods, several hundred systems become available. Recent work by Jackson at Greenwich shows that the average mass of a pair of dwarf stars is very close to twice that of the Sun, so that the individual components are strictly comparable with the Sun in mass. There is evidence, however, that the average mass differs for the various spectral classes. Ludendorff showed, a decade or more ago, from the data regarding spectroscopic binary systems, that the average mass of a pair of B-stars was about three times that of those of Class A and more than ten times that of the Sun. This conclusion was confirmed and extended by some work of mine a little later. Seares, who has recently gone over the same ground with more extensive data, confirmed these results and finds that the mean mass of a pair of stars of Class B0 is eighteen times that of the Sun, 10 for A0, 4.4 for F0, and 1.7 for G0, then diminishing slowly to 1.4 for M. For classes B and A these values apply to the giants and dwarfs together, for the later classes to the dwarf stars. For the redder giants the average mass of a pair is about 9 times the Sun's, according to a determination of mine, which I hope to revise in a year or two, but which is probably fairly reliable. These mean masses show that the stars are far more similar to one another in mass than in any other characteristic. This conclusion is confirmed by the individual values. If we confine ourselves to cases in which the errors of observation are not so large as to vitiate the results, we find no case of a mass exceeding twenty times that of the Sun, and none less than one-seventh of the Sun's. There appears to be a definite correlation between mass and brightness. The large masses all belong to B-stars of great luminosity—the smallest one to a tiny M-star, about 1/3000 as bright as the Sun. The average values for the giant and dwarf stars point in the same direction.

We are now in a position to evaluate another very important physical characteristic of the stars—their mean densities. Among the dwarfs, we have seen that the average mass of an individual star diminishes slowly from twice that of the Sun for Class F to half the Sun's for Class M, and that the radii ran from sixty per cent. greater than the Sun to forty per cent. less than the Sun's. This shows that the densities of all these stars are of the same order of magnitude. If we set the average density of a dwarf F-star as half of the Sun's, of a G-star as equal to the Sun's and of an M-star as twice as great, we shall be in good agreement with our data. For a very faint star like that mentioned above, we may estimate a diameter one-third and a mass one-seventh of the Sun's, giving a density four times that of the Sun, or about equal to the density of the Earth. Our knowledge of the surface brightness of these dwarf stars is not accurate enough to justify much reliance on the differences between these figures, but the general conclusion that the dwarf stars are comparable in density with the Sun, and most of them denser than water, appears certain. For the stars of Class A, we are on firmer ground, for most of the eclipsing variables belong here. In every such system, where we know the period and the relative sizes of the stars and orbit, a fortunate mathematical relation permits us to compute the density without having to know the dimensions or distance. For 69 stars, of spectrum between B5 and A5, the average density is 1/5 that of the Sun. They are decidedly similar to one another—7/8 of them having densities between four times the mean and 1/4 of it. The few eclipsing variables of early B type indicate a much smaller mean density—about 1/40 that of the Sun.

Passing to the giant stars, we may take as an example of

Class G, the brighter component of Capella. This star is 70 times as bright as the Sun and has 4.6 times its mass. Assuming that its surface brightness equals the Sun's we obtain a volume 600 as great, and a mean density 1/130 of the Sun's, or between eight and nine times that of air under standard conditions. For Arcturus, with a volume 14,000 times the Sun's 10 times the Sun's mass would be a liberal estimate. The corresponding density is 4/5 that of air. Finally for Antares, with its enormous bulk, even if we assign the exaggerated and improbable mass of 100 times that of the Sun, the density comes out one-millionth of the Sun's, or 1/900 that of ordinary air. The actual density is probably somewhat less, but more than 1/5 as great.

These values are so extraordinarily small that confirmation is desirable—and the eclipsing variables provide it. Of those of "later" spectral type, the majority are dense stars, one or two nearly equaling the Sun, and one having almost twice the Sun's density. But in a few cases the density is very much less. In one case it is about 1/100 that of the Sun, in two others 1/600 and 1/2000, and finally in one remarkable star, W Crucis, there is clear evidence of a density only 1/500,000 of the Sun's, or about 1/400 that of air. With this independent evidence, the reality of these low densities is placed beyond doubt. But when we realize that the mean density of Antares or W Crucis is less than that of what was once considered a fairly good vacuum, before air-pumps reached their present degree of perfection, we can but ask, Can a body of such low density possibly be opaque, and shine as if it were solid? We must not forget, however, that even a perfectly pure gas inevitably scatters some light away from a beam which passes through it, owing to the action of the very molecules themselves. The atmosphere above our heads—equivalent to five miles of air of standard density—scatters about one-eighth of the incident light, even when perfectly free from dust. After passing through fifty miles of air, only thirty per cent. of the original beam would remain; at a hundred miles, nine per cent.; after two hundred, less than one per cent.; and at the end of six hundred miles, not one part in a million. Such a thickness of perfectly clear air would be substantially opaque—the Sun itself would be barely visible through it. Now an easy calculation shows that a ray of light which passed centrally through Antares (taking the star as of uniform density, simply for the purpose of this calculation) would in our lowest estimate have passed through a quantity of gas equivalent to 800,000 miles of air—or more than a thousand times enough to secure substantial opacity. We need, therefore, trouble ourselves no more upon this score.

The very low densities of the giant stars permit an explanation of the spectroscopic differences upon which Adams has based his determination of the distances of the stars. The temperature of the atmosphere of a star of a given spectral class is probably much the same, whatever its absolute magnitude, but the density of this atmosphere, like that of the whole mass, must be very much greater in a dwarf star than in a giant. What spectroscopic differences might arise from this can be seen upon Saha's theory.

The relative strength of the ordinary lines and enhanced lines of any element depends upon the relative number of the atoms of this element, in the star's atmosphere, which are in the neutral and ionized states. Now an atom becomes ionized when its internal energy becomes so great as to lead to the ejection of an electron, and the rate at which this happens will be practically independent of the density of the gas, though it will increase rapidly with the temperature. The rate of recombination of the ions thus produced with the free electrons, to reconstitute neutral atoms, will, however, be greatly influenced by the density. It is obvious that the farther the atoms and electrons are apart, the longer, on the average, they will have to travel before meeting one another. Hence, though the atoms will not *enter* into the ionized state oftener in the gas of low density, they will *remain* in it longer, and the percentage of all the atoms which are ionized will increase as the density diminishes. This will favor the absorption of the enhanced lines at the expense of the lines of the neutral atom. Now the lines which Adams has picked out, on the basis of his wide experience, as characteristic of giant stars, are almost without exception enhanced lines of a pronounced type. The general character of the physical process which lies behind his beautiful discovery appears therefore to be understood. Many details remain obscure —for example, the prominence in the giant stars of the hydrogen lines, which belong to the neutral atom. But in so new a field of work, the presence of such unexplained phenomena is only a stimulus to further research, and a comforting reminder that there are still "new worlds to conquer."