III

ON THE STRUCTURE OF CONTINUA

LET us suppose that space is metric. The non-degenerate proper subcontinuum K of the continuum M is called a continuum of condensation of M if every point of K is a limit point of $M-K$.

The following definition is given on page 355 of P. S. T.

DEFINITION. A subcontinuum K of a compact continuum M is said to be an *essential continuum of condensation of type* 1 if there exists a non-degenerate subcontinuum T of K such that if G is an upper semi-continuous collection of mutually exclusive continua filling up M and G is, with respect to its elements, a continuum with no continuum of condensation then some continuum of G contains T.

In the present treatment an essential continuum of condensation of type 1 will be called merely an essential continuum of condensation.

This terminology is in some respects unfortunate. For an essential continuum of condensation of a continuum M is not necessarily a continuum of condensation of M. Let K denote an indecomposable continuum, let AB denote an arc having only the point B in common with K and let M denote $K+AB$. The continuum K is an essential continuum of condensation of M. For suppose G is an upper semi-continuous collection of mutually exclusive continua filling up M and such that G is, with respect to its elements, a continuum with no continuum of condensation. Since every continuum with no continuum of condensation is a regular curve therefore G is a regular curve with respect to its elements. Let H denote the collection of all the elements of G that have one or more points in common with K. Let h_B denote the element of H that contains the point B. Let h_B' denote the common part of K and h_B. Let H' denote the collection whose elements are the continuum h_B' and the continua of the collection H that lie wholly in K. Since every non-degenerate subcontinuum of a regular curve is itself a regular curve therefore H is a regular curve with respect to its elements. But clearly H' is isomorphic with H and no regular curve is an indecomposable continuum. Therefore if H is non-degenerate there exist two proper subcollections U and V of the collection H' such that every element of H' belongs either to U or to V and such that U and V are continua with respect to their elements. The point set U^* obtained by adding together all the point sets of the collection U is a continuum (of points) and so is the set V^* obtained in the same manner from the collection V. But $K=U^*+V^*$ and U^* and V^* are proper subsets of K. This is impossible since K is indecomposable.

Thus the supposition that H is non-degenerate has led to a contradiction. Therefore, for every G, K is a subset of some one element of G. Hence, by definition, K is an essential continuum of condensation of M. But no point of K except B is a limit point of $AB-B$. Therefore K is not a continuum of condensation of M.

However, the following theorem holds true.

THEOREM 1. *If K is an essential continuum of condensation of the compact continuum M then K contains a continuum of condensation of M.*

Proof. Suppose M is a compact continuum containing a continuum K which contains a non-degenerate continuum

T such that if G is an upper semi-continuous collection of mutually exclusive continua filling up M and, with respect to its elements, G is a continuum with no continuum of condensation then some continuum of the collection G contains T. Suppose some subcontinuum L of T contains no continuum of condensation of M. Then L contains no continuum of condensation of itself. Hence L is a continuous curve. Let AB denote an arc lying in L. Since AB is not a continuum of condensation of M there exists an interval XY of AB such that no point of XY except its endpoints is a limit point of $M-XY$. If Z is either X or Y let M_Z denote the component of $M-(XY-X-Y)$ that contains Z. Let G denote the upper semi-continuous collection whose elements are the points of the segment XY and either the continuum M_X or the continua M_X and M_Y according as M_X is or is not identical with M_Y. The collection G is, with respect to its elements, either an arc or a simple closed curve and, therefore, a continuum with no continuum of condensation. But T is not a subset of any element of G. Thus the supposition that not every subcontinuum of T contains a continuum of condensation of M leads to a contradiction.

It is not true, however, that if M is a compact continuum and T is a subcontinuum of M such that every non-degenerate subcontinuum of T contains a continuum of condensation of M then T is an essential continuum of condensation of M. Consider, for example, the continuum M consisting of a square together with its interior. Every non-degenerate subcontinuum of M contains a continuum of condensation of M but M has no essential continuum of condensation. As a second example consider a dendron M which is the sum of a straight line interval T and a countable number of mutually exclusive straight line intervals

A_1B_1, A_2B_2, A_3B_3, $\cdots$ such that (1) for each n, A_nB_n has only the point B_n in common with T, (2) the length of A_nB_n approaches 0 as n increases indefinitely and (3) every segment of T contains points of the sequence B_1, B_2, B_3, $\cdots$. Here not only is it true that every non-degenerate subcontinuum of T *contains* a continuum of condensation of M but every one *is* a continuum of condensation of M. Nevertheless neither T nor any other subcontinuum of M is an essential continuum of condensation of M.

To see that there exists a regular curve with an essential continuum of condensation, consider the following example.

EXAMPLE. Let AB denote a straight line interval of length 1. For each positive integer n, divide the interval AB into 2^n equal subintervals each of length $1/2^n$ and let Q_n denote this set of subintervals of AB. Let Q denote the infinite set whose elements are the intervals of all the sets Q_1, Q_2, Q_3, $\cdots$. For each interval of the set Q construct a semi-circle whose endpoints are the extremities of that interval. Let W denote the set of all such semi-circles. Let M denote the point set which is the sum of AB and all the semi-circles of the set W. Let T denote the interval AB. If G is an upper semi-continuous collection of mutually exclusive continua filling up M and such that, with respect to its elements, G is a continuum with no continuum of condensation then some continuum of the collection G contains T. For suppose G is such a collection for which this is not true. Let H denote the set of all continua of G that intersect AB. Let g_1 denote the continuum of the set H that contains P_1, the mid-point of AB. Let J_1 denote the semi-circle of the set W whose endpoints are P_1 and B, let J_2 denote the one whose endpoints are P_1 and P_2, the mid-point of P_1B, and let J_3 denote the one whose endpoints are P_2 and B. Suppose J_1 is not a subset of g_1. Then there are uncountably many continua of the set G each containing a point of J_1. Let K denote the set of all such continua. The continuum M does not contain uncountably many mutually exclusive non-degenerate continua. Hence only a countable number of continua of K belong to H. But K is a continuum of elements of G. Hence, in the space whose elements are the continua of the collection G, the element g_1 is a limit element of the set of elements $K-H$. Therefore if g_1 is not a limit element of the set of elements $G-H$ then g_1 contains J_1 and, similarly J_2 and therefore J_3 and so on and consequently AB. Similarly if there is a single point P of M, lying on AB and on some semi-circle of the set W, such that the continuum g_P of H that contains P is not a limit element of $G-H$, then g_P contains AB. If, on the other hand, for every such point P, g_P is a limit element of $G-H$ then H is a continuum of condensation of the set G of elements. It follows that some element of G contains AB. Hence AB is an essential continuum of condensation of M.

By the *diameter* of a closed and compact point set M is meant the smallest number d such that no two points of M are at a distance apart more than d.

A collection of continua is said to be a *contracting* collection provided it is true that if e is any positive number whatsoever then there are not more than a finite number of continua of G which are not of diameter less than e.

Consider (1) the collection of all point sets which are either arcs or points, (2) the collection of all point sets which are either simple closed curves or points, (3) the collection of all dendrons, (4) the collection of all regular curves and (5) the collection of all continuous curves. If Q is any one of these five collections and G is an upper semi-continuous collection of mutually exclusive continua filling up a continuum of the collection Q then, with respect to its elements, G is also a continuum of the collection Q.

THEOREM 2. *Every collection of mutually exclusive subcontinua of a regular curve is upper semi-continuous.*

This theorem follows from the fact that every collection of mutually exclusive subcontinua of a regular curve is contracting and every contracting collection of mutually exclusive compact continua is upper semi-continuous.

Two closed point sets are said to be *topologically equivalent* if there exists a one to one correspondence between them preserving limit points. A collection G of continua is said to be a *topological collection* provided it is true that every continuum which is topologically equivalent to an element of G is itself an element of G.

DEFINITION. If M is a continuum and Q is a topological collection of continua then the subcontinuum K of M is said to be a *Q-atomic subset* of M provided there exists an upper semi-continuous collection G of mutually exclusive continua filling up M such that, with respect to its elements, G belongs to Q and such that (1) K is an element of G and (2) if H is any other collection of mutually exclusive continua filling up M such that H is, with respect to its elements, a continuum of the collection Q then every continuum of the collection G is a subset of some continuum of the collection H or, in other words, each continuum of H is either an element of G or the sum of two or more elements of G.

DEFINITION. By a *dendratomic* subset of a continuum M is meant a Q-atomic subset of M where Q is the collection of all dendrons.

DEFINITION. The compact continuum M is said to be *webless* provided it is true that if G_1 and G_2 are two upper semi-continuous collections of mutually exclusive continua filling up a subcontinuum of M and such that each of them is an acyclic continuous curve with respect to its elements, then there does not exist an uncountable subcollection H of G_1 such that no continuum of H is a subset of any continuum of G_2.

It is shown in P. S. T. that every compact and webless continuum has dendratonic subsets.

THEOREM 3. *If G is a collection of mutually exclusive subcontinua of a compact and webless continuous curve and, for each continuum g of G, there exists an upper semi-continuous collection H of mutually exclusive continua filling up M such that H is an acyclic continuous curve with respect to its elements and such that g is an element of H then there exists an upper semi-continuous collection W of mutually exclusive continua filling up M such that W is an acyclic continuous curve with respect to its elements and every element of G is an element of W.*

Proof. Every element of G is the sum of the dendratomic subsets of M that intersect it and if the dendratomic subsets of M are regarded as points the set of "points" so obtained is an acyclic continuous curve T and every element of G is, with respect to these "points," a subcontinuum of this acyclic continuous curve and hence, by Theorem 2, if U denotes the collection of all these "subcontinua" of T and all the "points" (dendratomic subsets of M) of T which belong to no one of these subcontinua then U is an upper semi-continuous collection which is a dendron with respect to its elements. Let W denote the set of all continua x in the original sense such that, for some element y of U, x is the sum of all the dendratomic subsets of M which are subsets of y.

This theorem does not remain true if the stipulation that M be webless is omitted. Let L denote a square whose vertices are (0, 0), (0, 1), (1, 1) and (1, 0). Let M denote the square L together with its interior. Each point of M is an element of some upper semi-continuous collection H of mutually exclusive continua filling up M such that H is an arc with respect to its elements. But the collection of all points of M is not a dendron. Again, let G denote the set of all intervals q such that the endpoints of q are either $(x, 0)$ and $(x, 1)$ where x is an irrational number, or $(x, 0)$ and $(x, 1/2)$ where x is a rational number, between 0 and 1. Each interval of the set G is an element of some upper semi-continuous collection H filling up M such that H is an arc with respect to its elements. But in this case G is not upper semi-continuous.

The following theorem may be established.

THEOREM 4. *If Q is the collection of all arcs and all points then in order that the compact continuum M should have Q-atomic subsets it is necessary and sufficient that M should be a webless continuum which is an arc with respect to its dendratomic subsets.*

Thus if Q is the collection of all arcs and points then no dendron which is not an arc has Q-atomic subsets.

THEOREM 5. *Suppose Q is a topological collection of continua such that every compact and webless continuum has Q-atomic subsets. If all arcs and simple closed curves belong to Q then so does every compact continuum with no essential continuum of condensation.*

Proof. Suppose M is a non-degenerate compact continuum with no essential continuum of condensation. Suppose K is a non-degenerate subcontinuum of M. Since K is a regular curve, it contains an arc AB. Since AB is not an essential continuum of condensation of M there exists an upper semi-continuous collection G of mutually exclusive continua filling up M and such that (1) G is, with respect to its elements, a continuum with no continuum of condensation, (2) the arc AB is not a subset of any element of G. Let W denote the collection of all elements of G that contain one or more points of AB. The collection W is itself a regular curve of elements of G and it has no continuum of condensation. Let x and y denote two distinct elements of W. There exists an arc t of elements of W from x to y. Since the arc t of elements of G is not a continuum of condensation of G there exist two elements α and β of t such that if q denotes the segment of t with α and β as endelements then no element of q is a limit element of $G-q$. Let C_α denote the component of $G-q$ that contains α and let C_β denote the one that contains β. If C_α and C_β are distinct let H denote the collection whose elements are C_α, C_β and the elements of q. If C_α is identical with C_β let H denote the collection whose elements are C_α and the elements of q. In either case, the collection H is upper semi-continuous. In the first case it is an arc, and in the second case a simple closed curve, with respect to its elements. In either case the collection H is, with respect to its elements, a continuum of the collection Q. But there are uncountably many continua in the collection H and each of them contains at least one point of the arc AB. Hence AB is not a subset of any continuum of this collection. It follows that if K is non-degenerate it is not a Q-atomic subset of M.

In other words if a continuum M with no essential continuum of condensation has Q-atomic subsets then every Q-atomic subset of M is a point of M. But the set of all the Q-atomic subsets of a continuum belongs to the collection Q. Hence M itself belongs to Q. But M is a compact and webless continuum.

THEOREM 6. *Suppose Q is a topological collection of continua such that every compact and webless continuum has Q-atomic subsets. If all regular curves belong to Q so do all compact and webless continua.*

Proof. Suppose M is a compact and webless continuum. If P is any point of M there exists[1] an upper semi-continuous collection G of mutually exclusive continua filling up M such that G is a regular curve with respect to its elements and such that the point P is an element of G. Hence every point of M is a Q-atomic subset of M. Therefore M belongs to the collection Q.

The following result has now been established.

The collection of all dendrons is a topological collection Q such that every compact and webless continuum has Q-atomic subsets. But if Q is a topological collection including the class of all dendrons and also including simple closed curves then in order that every compact and webless continuum should have Q-atomic subsets it is necessary that Q should include the set of all regular curves with no essential continuum of condensation and that it should not include the collection of all regular curves except for the trivial case where it is the entire collection of all compact and webless continua.

Definition. The continuum M is said to have *property* N if for every positive number e there exists a finite set G of mutually exclusive non-degenerate subcontinua of M such that every subcontinuum of M of diameter greater than e contains some continuum of the set G.

Theorem 7. *Suppose the arc AB is a subset of the regular curve M and every segment of AB contains a point which is an endpoint of an arc in M which has only its endpoints in common with AB. Then every segment of AB contains two points which are the extremities of an arc lying, except for its endpoints, in $M-AB$ and if s is a segment of AB and K is a closed subset of $M-s$ there exists an arc having no point in common*

[1] Cf. J. H. Roberts, "Concerning non-dense plane continua," *Trans. Am. Math. Soc.*, XXXII (1929), p. 30, and P. S. T., p. 446.

with K and having in common with AB only its endpoints, which lie in s.

Proof. Suppose X and Y are two points lying on AB in the order $AXYB$. There exist two points C and D and a sequence of points $X_1, X_2, X_3, X_4, \cdots$ lying on AB in the order $AXCX_1X_2X_3X_4 \cdots DYB$. For each n, there exists, between X_n and X_{n+1}, a point P_n which is an endpoint of an arc Z_nP_n lying in M and having only Z_n and P_n in common with AB. If any two arcs of the sequence $P_1Z_1, P_2Z_2, P_3Z_3, \cdots$ have a point other than an endpoint in common, their sum contains an arc with endpoints between X and Y and lying, except for its endpoints, in $M-AB$. If no two of them have in common any point other than one of their endpoints and no one of them has both endpoints between X and Y then each of them has one endpoint between C and D and the other one in the closed point set which is the sum of the intervals AX and YB of the arc AB. Thus, in this case, there exist infinitely many arcs $P_1Z_1, P_2Z_2, P_3Z_3, \cdots$ each having one endpoint in one and the other endpoint in the other of the two mutually exclusive closed point sets CD and $AX+YB$, no two of these arcs having any point in common other than a common endpoint. But this is impossible since M is a regular curve.

Suppose now that X and Y are two points of the segment s. Suppose every arc which lies in M and has in common with AB only its endpoints, which lie between X and Y, intersects K. Then there exist in M infinitely many arcs $X_1C_1, X_2C_2, X_3C_3, \cdots$ such that $X_1, X_2, X_3, \cdots$ are distinct points lying between X and Y on AB and, for each n, C_n belongs to K and $(X_nC_n) \cdot (AB+K) = X_n + C_n$. Since M is a regular curve there exist two distinct numbers i and j such that X_iC_i and X_jC_j have in common a point Z between C_i and X_i. The point set $X_iC_i + X_jC_j$ contains an arc X_iZX_j which contains no point of K and lies, except for its endpoints, in $M-AB$.

THEOREM 8. *If the regular curve M contains an arc AB such that every segment of AB contains an endpoint of an arc in M which has only its endpoints in common with AB then M does not have property N.*

Proof. Suppose M has property N. Let d denote one half the distance from A to B. There exists a finite set W of mutually exclusive non-degenerate subcontinua of M such that every subcontinuum of M of diameter more than d contains some continuum of the set W. The arc AB contains at least one continuum of the set W. The set W_1 of all continua of W that are subsets of AB is a set of arcs A_1B_1, $A_2B_2, \cdots, A_mB_m$ with endpoints lying on AB in the order $A_1B_1A_2B_2A_3B_3\cdots$, the point A_1 being on the interval AB_1. For each n less than or equal to m there exists, in M, an arc $X_{1n}Z_{1n}Y_{1n}$ having in common with AB only the points X_{1n} and Y_{1n} which lie in the order $A_nX_{1n}Y_{1n}B_n$. Let AA_1 denote the point A or the interval AA_1 of AB according as A and A_1 are identical or distinct, let B_mB denote B or the interval B_mB of AB according as B_m and B are identical or distinct and, for each n $(1\leqq n\leqq m)$, let A_nX_n and Y_nB_n denote intervals of AB with endpoints as indicated. The continuum

$$AA_1+(A_1X_{11}+X_{11}Z_{11}Y_{11}+Y_{11}B_1)+B_1A_2+$$
$$(A_2X_{12}+X_{12}Z_{12}Y_{12}+Y_{12}B_2)+\cdots+$$
$$(A_mX_{1m}+X_{1m}Z_{1m}Y_{1m}+Y_{1m}B_m)+B_mB$$

contains an arc t_1 from A to B. The arc t_1 contains no continuum of the set W_1. But it is of diameter greater than d. Hence it contains at least one continuum of the set W. Let W_2 denote the set of all the continua of W that it contains. Each continuum of the set W_2 is an arc having no point in common with AB. For each n less than or equal to m there exists, in M, an arc $X_{2n}Z_{2n}Y_{2n}$ having in common with AB

only the points X_{2n} and Y_{2n}, which lie in the order $X_{1n}X_{2n}Y_{2n}Y_{1n}$, and containing no point of any continuum of the set W_2. The continuum

$$AA_1+(A_1X_{21}+X_{21}Z_{21}Y_{21}+Y_{21}B_1)+B_1A_2+ (A_2X_{22}+X_{22}Z_{22}Y_{22}+Y_{22}B_2)+\cdots+ (A_mX_{2m}+X_{2m}Z_{2m}Y_{2m}+Y_{2m}B_m)+B_mB$$

contains an arc t_2 from A to B. The arc t_2 contains no continuum either of the set W_1 or of the set W_2. But it contains at least one continuum of the set W. Let W_3 denote the set of all the continua of W that it contains. This process may be continued. Thus there exists an infinite sequence $W_1, W_2, W_3, \cdots$ such that, for each n, W_n is a non-vacuous set of continua of the set W and such that, if i is distinct from j, no continuum of W belongs both to W_i and to W_j. Hence W is an infinite set, contrary to hypothesis.

It may be shown that every dendron has property N. But this is not true of every regular curve. Indeed the following theorem will be proved.

THEOREM 9. *No regular curve with an essential continuum of condensation has property N.*

Proof. Suppose M is a regular curve with an essential continuum of condensation. Then M contains an arc AB such that if G is an upper semi-continuous collection of mutually exclusive continua filling up M and G is, with respect to its elements, a continuum with no continuum of condensation, then some element of G contains AB. Suppose now that there exist two points X and Y in the order $AXYB$ on AB such that no point of the interval XY of AB is an endpoint of an arc lying in M which has only its endpoints in common with AB. Then if P is a point of the interval XY either P is a limit point of no component of $M-AB$ or there exists a component L of $M-AB$ such that $L+P$ is a continuum. In the first case, if $X\neq P\neq Y$, let M_P denote the point P. In the second case, if $X \neq P \neq Y$, let M_P denote the continuum obtained by adding P to the sum of all components of $M-AB$ of which P is a limit point. Let L_X denote the component of $M-XY$ that contains A and let L_Y denote the one containing B. Let M_X and M_Y denote $\bar{L}_X$ and $\bar{L}_Y$ respectively. For each point Z of $M-AB$ there exists one and only one point P such that M_P contains Z unless Z belongs to L_X and $L_X = L_Y$ in which case there are two points P such that M_P contains Z. Let G denote the collection of all the continua M_P for all points P of XY (M_X and M_Y not being counted twice if $M_X = M_Y$). The collection G is an upper semi-continuous collection of mutually exclusive continua and it is either an arc or a simple closed curve with respect to its elements and no one of its elements contains AB. This involves a contradiction. Hence every segment of AB contains an endpoint of an arc lying in M and having only its endpoints in common with AB. Hence, by Theorem 8, M does not have property N.

In view of the above argument it is clear that the following theorem holds true.

THEOREM 10. *If the regular curve M has property N then not only does it have no essential continuum of condensation but if T is any subcontinuum whatsoever of M there exists an upper semi-continuous collection of mutually exclusive continua filling up M such that no element of G contains T and furthermore G is either an arc or a simple closed curve with respect to its elements.*

There exists a regular curve M without property N such that if T is any subcontinuum of M then there exists an upper semi-continuous collection of mutually exclusive continua filling up M such that, with respect to its elements, G is either an arc or a simple closed curve but no element of G contains T. But the following theorem holds true.

THEOREM 11. *If M is a regular curve such that if T is any subcontinuum of M there exists an upper semi-continuous collection of mutually exclusive continua filling up M such that no element of G contains T and such that G is, with respect to its elements, a dendron or some other continuum of which every subcontinuum has uncountably many cut points, then M is a dendron.*

Proof. Suppose M contains a simple closed curve J. By hypothesis there exists an upper semi-continuous collection G of mutually exclusive continua filling up M and such that G is an acyclic continuous curve with respect to its elements but no element of G contains J. Let H denote the set of all the elements of G that intersect J. Since M is a regular curve all but possibly a countable number of the elements of H are points. If P is any point belonging to H, $J-P$ is connected and therefore, by Theorems 1 and 12 of Chapter V of P. S. T., the collection $H-P$ is connected. Thus the compact continuum H of elements of G does not have more than a countable number of cut elements. Hence it is not acyclic. This involves a contradiction.

THEOREM 12. *If M is a regular curve and G is an upper semi-continuous collection of mutually exclusive continua filling up M and G is a dendron with respect to its elements then every simple closed curve that lies in M is a subset of some element of G.*

The set of all regular curves with property N includes the set of all dendrons and all regular curves without continua of condensation and it is included in the set of all regular curves with no essential continua of condensation.

THEOREM 13. *Suppose M is a regular curve with an essential continuum of condensation and AB is an arc lying in M and such that if G is an upper semi-continuous collection of mutually exclusive continua filling up M such that G is with* *respect to its elements a continuum with no continuum of condensation, then some element of G contains AB. Then if XY is a segment of the arc AB and K is a closed subset of $M-XY$, there exists in M an arc having no point in common with K and having in common with AB only its endpoints which belong to the segment XY.*

Theorem 13 may be established with the help of Theorem 7 together with the argument employed to prove Theorem 9.

In his paper "Concerning collections of cuttings of connected point sets,"[1] G. T. Whyburn has proved that if M is any compact continuum and G is any uncountable collection of mutually exclusive connected subsets of M each of which contains a cutting of M, then there exists an upper semi-continuous collection G_O of mutually exclusive compact sub-continua of M such that (1) all save possibly a countable number of elements of G are elements also of G_O, (2) the sum of all the elements of G_O is identical with M, and (3) M is an acyclic continuous curve with respect to the elements of G_O.

With the help of this theorem together with the fact that every compact and webless continuum has dendratomic subsets it is possible to prove the following theorem.

THEOREM 14. *If P is a point of the compact and webless continuum M the dendratomic subset of M containing P is the set of all points X of M such that there do not exist uncountably many mutually exclusive subcontinua of M each separating X from P in M.*

In his paper "Concerning the structure of a continuous curve,"[2] Whyburn has defined what he calls the cyclic elements of a continuous curve. If the term cyclic element of M is restricted so as to apply only to those which are not

[1] *Bull. Am. Math. Soc.*, XXXV (1929), pp. 87–104.
[2] *Am. Jour. of Math.*, L (1928), pp. 167–194.

points belonging to only one non-degenerate cyclic element of M then the following theorem holds true.

THEOREM 15. *The set of all cyclic elements of a compact continuous curve is an upper semi-continuous collection of the second kind as defined in* II.

With the help of this theorem, Theorem 13 of II and certain results established in the above mentioned paper of Whyburn's it follows that the following theorem holds.

THEOREM 16. *If the cyclic elements of a continuous curve M are regarded as "points" and two such "points" p and q are regarded as contiguous if and only if one of the continua p and q is a point of the other one then the set of all such "points" is an acyclic continuous curve.*

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