Modeling and Centralization of Strategic Inventory for Repairable and Long Lead-Time Spare Parts

by
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Bachelor of Science Industrial and Operations Engineering
University of Michigan, 2003

Submitted to the MIT Sloan School of Management and the Department of Civil and Environmental Engineering
in Partial Fulfillment of the Requirements for the Degrees of
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AND
Master of Science in Civil & Environmental Engineering

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Abstract

This thesis develops an optimal inventory model for repairable and long lead-time spare parts for an Engine overhaul business. In addition, it presents a business case for centralization of inventory.

Pratt & Whitney purchased the Norway Engine Center (NEC) in 2000. Two new engine centers, the Shanghai Engine Center (SEC) and the Turkey Engine Center (TEC) opened as joint ventures in 2009. While all three engine centers overhaul the same engine, they each make independent decisions regarding material strategy.

Operations are expected to grow substantially at the two newest centers. Current inventory practices are not sustainable as operations expand. In addition, the overhaul business is a competitive market and there is growing pressure to decrease engine turn-around-time (TAT). An optimal material strategy is needed to reduce the material sourcing time and therefore reduce overall TAT.

This project develops an inventory strategy that will significantly reduce TAT with minimal additional inventory investment. To accomplish this, an inventory model was developed to determine the optimal inventory level and then using this model, the business case for using centralization to reduce both holding cost and material sourcing time was investigated.

All inventory in the engine centers were considered in this project, however rotable material became the focus of this research as it has the largest impact on the engine center through its high value and long lead-times. Rotable material is inventory used to buffer against the lead-time of parts out for repair.

In the engine overhaul business material sourcing time is built into the process. This means that material is not needed immediately but rather after some specified amount of time. This feature is central to the rotable inventory model. The model determines the mean and variance of the excess lead-time – the portion of the lead-time that occurs after the specified time allotted. The excess lead-time is used to determine the optimal reorder point.

Using this model, we show that centralization of rotable material will reduce inventory value by more than 30% over the current decentralized system both using the current TAT as well as the proposed TAT.

Advisor: Don Rosenfield, Director of Leaders for Global Operations Program
Advisor: David Simchi-Levi, Professor of Civil & Environmental Engineering

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I would like to thank Pratt & Whitney for allowing me to spend 6 months on site. I learned a lot about inventory management and airplane engines. In particular, I would like to thank Stephen Hill, Gary Bosse, and Patricia Wu-Murad for their willingness to share their extensive knowledge.

I would also like to thank my academic advisors, Don Rosenfield and David Simchi-Levi. Don Rosenfield, the director of the LGO program was particularly helpful during this research. I was very fortunate to have an advisor that was as excited about this project as I was. I have learned so much during the past two years and I am very grateful to my LGO classmates.

Finally, I would like to thank my significant other, Joseph Tyler Ruthven for being willing to marry me in the middle of MBA core and for remaining his usual laid-back and kind self in the midst of the all the craziness of grad school.

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Biographical Note

The author, Tyeliah Duncan, grew up in Oklahoma City, OK. She graduated from the University of Michigan with a Bachelor of Science degree in Industrial and Operations Engineering. Tyeliah taught junior-high math and biology in a rural school in Burkina Faso as a Peace Corps volunteer. She then left her small village in Africa to teach English at a university in Chengdu, China, before finally returning to the US. She was most recently a project manager for Northrop Grumman supporting the Federal Aviation Administration before entering graduate school.

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Chapter 1 Introduction

This paper researches inventory optimization of repairable parts, focusing on three Pratt & Whitney engine centers located in Norway, Turkey, and China that all overhaul the same engine model.

1.1 Problem Motivation

The engine overhaul business is becoming an increasingly competitive market. Engine centers compete on both cost and engine turn-around-time (TAT). The time an engine spends in an engine center being serviced is very costly to airlines. Currently the average engine TAT for each of the three engine centers is longer than the competitive target.

The competitiveness of the engine centers is dependent on increasing the number of shop visits (the number of engines that are overhauled annually). To do this, the engine centers must generate additional demand by decreasing engine TAT while maintaining costs. A significant cost to the engine centers is the inventory, which consists of largely expensive low usage parts. To achieve competitiveness, shop visits need to be increased without a corresponding increase in inventory, requiring a comprehensive inventory strategy.

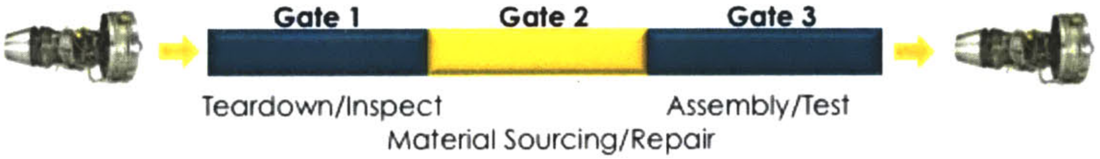
1.2 Hypothesis

This research posits first that there is a material solution to the TAT problem and second that centralizing repairable inventory will make this solution cost effective.

There are two ways to reduce TAT, improve the lead-time of repair and sourcing of parts or optimize inventory. The option to improve lead-time, while not a focus of this research, is discussed in Section 2.3. Our research focuses on the potential benefits to TAT of an optimal inventory strategy. In addition, because this is a cost sensitive business we look at the cost savings generated from centralizing inventory. We posit that by holding the optimal inventory centrally it is possible to improve TAT without an increase in cost. Centralizing inventory aggregates demand and reduces the total inventory required in the system. At the same time, reducing TAT reduces the engines in the system, which reduces the work-in-progress inventory (WIP).

When an engine comes into an engine center, it is processed through three gates before being shipped as a fully serviceable engine. Figure 1 is an illustration of the three gates.

Figure 1: Engine Overhaul Process



Gate 1 is engine teardown and inspection of parts. Gate 2 is the time allotted for parts being sent out for repair and new parts being ordered and delivered to the engine center. Gate 3 is reassembly of the engine and testing.

All parts must be on site in serviceable condition in order for gate 3 to be initiated. Each gate has an expected time to completion that the engine center works towards. The time allotted for this entire performance is known as the network. The network is not the actual TAT but rather the TAT goal. A network of 55 days for example might consist of 15 days for gate 1, 20 days for gate 2, and 20 days for gate 3 and will be shown as 55 days (15,20,20) in this paper. The hypothesis assumes that the delays in TAT happen during gate 1 and gate 2 where stocking inventory would be beneficial. By definition gate 3 begins when the parts arrive, therefore improving gate 3 can only be done through process improvements.

1.3 Research Methodology

The author spent six months on site at Pratt & Whitney facility in East Hartford, CT working in the aftermarket materials management group. Initially we defined the problem and collected data. We then divided the project into three sections:

1. Identifying current state
2. Establishing future state
3. Developing a business case for centralization

In identifying the current state, we looked at the types of inventory in an engine center as well as current stocking practices. We also analyzed the current lead-times for repairs.

To establish the future state, we developed two inventory models. The rotable inventory model focuses exclusively on repairable material as it has the most impact on TAT and these parts are generally high cost. The long lead-time model is a fairly simple model as there are significantly fewer parts for this type of inventory and less data was available.

The business case for centralization includes a full cost comparison of centralized versus decentralized inventory.

1.4 Thesis Outline

Chapter Two discusses the current state. This includes the type and quantity of material in the engine centers as well as the way this decision is made today. There is also a description of the organizational structure.

Chapter Three begins with a literature review of inventory optimization approaches. From there we describe each of the inventory models that were developed – the rotable inventory model and the long lead-time inventory model.

Chapter Four examines the potential for establishing a central warehouse for the three engine centers. We look at the benefits to planning and procurement gained through centralization. We also compare the inventory levels required in a centralized versus decentralized system and analyze the costs associated with each system.

In Chapter Five we discuss our findings and recommendations. We also describe some of the remaining questions and possible follow up investigations associated with this research.

Chapter 2 Current State

This chapter describes the landscape as it was when we began this research, beginning with an in depth discussion of the types of material in an engine center. We then look at the performance of the repair units for parts sent out for repair. Finally, in this chapter we look at the Pratt & Whitney organization and discuss the relevant factors to this research.

2.1 Inventory Analysis

To understand the benefits of centralizing inventory, we first need to understand what types of material are in an engine center.

Table 1: Types of Material in an Engine Center

Material	Description	Volume	Cost	Lead-Time
Point-of-Use	Nuts, bolts, and other common small parts	High	Low	Short
Peggable	Parts that can be ordered and arrive within gate 2	Low	High	Short
Rotable	Buffer against repairable parts that do not return within gate 2	Low	High	Long
Long LT	Consumable parts with a lead-time greater than gate 2	Low	High	Long
Slow moving	No demand in 12 Months	Low	High	

Point-of-use parts are managed using a min/max inventory model. This inventory system was put into place 3 years ago and has successfully reduced the amount of this type of inventory while maintaining a high service level. Because these are low-cost, high-volume parts, centralization for point-of-use-parts would not be beneficial. Therefore we do not consider this type of inventory further in this thesis.

Peggable material consists of parts that have a short enough lead-time that they can be pegged (assigned) directly to an engine. When an engine needs these parts, they can be ordered and arrive within gate 2 before reassembly is scheduled to begin. These parts are also high cost and low volume, which removes them as candidates for point-of-use classification. An engine center should not hold these parts in inventory, yet about 17% of the inventory in the Norway engine center by value is peggable material. Table 2 shows the percentage of each type of material in Norway.

Table 2: Percentage of Inventory Value by Type at Norway[1] Engine Center

Type	% of Total Value
Point-of-Use	14%
Peggable	17%
Rotable	22%
Long LT	13%
Slow moving	26%
Unidentified	8%

The percentages shown in Table 2 are estimations based on rules used to differentiate the material by types. We were unable to identify 8% of the material.

26% of the inventory value consists of slow moving inventory - parts that have been on the shelf without being used for at least 12 months. This material is not

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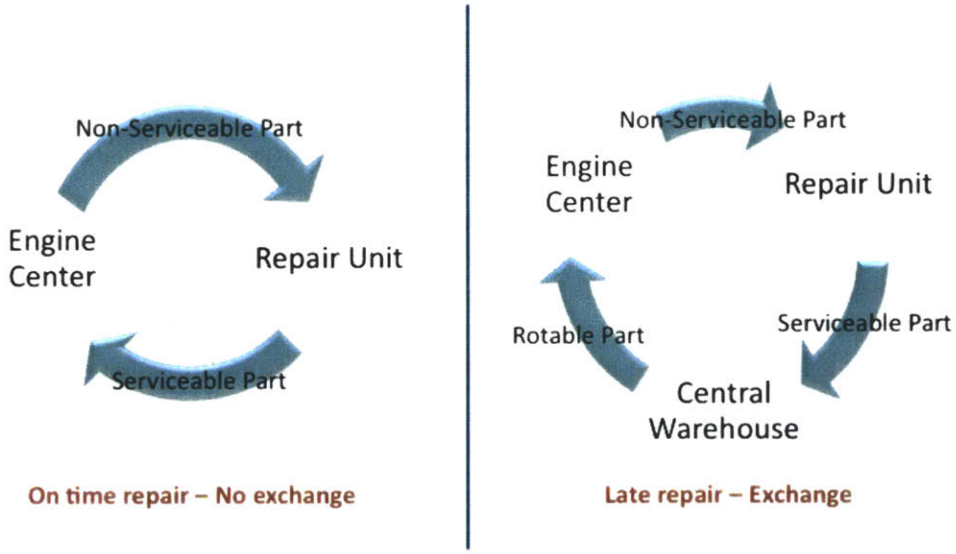
[1] The Norway Engine Center's inventory value is shown because it is the most established engine center, having been in operation by Pratt & Whitney for 10 years. Similar analysis on the other 2 engine centers is not shown here.

creating value and should be minimized to the extent possible. The high percentage of slow moving material reflects the many possible routes the material has for entering the facility and the very few routes it has for leaving the facility. For example, excess material may enter the facility because of over forecasting or buying material to expedite. There are weak processes for selling this material. Review of material is done rarely and the pipeline for transferring the material out for resell is not well established.

For both slow moving and peggable material the optimal inventory level is very low. In an ideal system, there would be little peggable or slow moving material. In the case of peggable material, the part can be ordered and arrive before it is needed so there is no need to hold inventory. In the case of slow moving material, the amount of material to hold is dependent on the holding cost, the current salvage cost and the ultimate sales cost. Rosenfield (1989) develops an excess inventory model to determine the number of parts to be held. However, we are focused on whether to centralize inventory. Clearly Pratt & Whitney would benefit more from establishing processes to reduce these types of inventory rather than centralizing this material. Therefore our research focuses on the remaining two types of inventory – rotable and long lead-time parts.

Rotable material is inventory that is held for parts out for repair. When a part is sent out for repair and does not return within gate 2, inventory is needed. Figure 2 shows the repair process with and without an exchange. An exchange of a rotable part happens when the lead-time for the repair is longer than the time allotted in gate 2.

Figure 2: Repair Process with and without Exchange



Current inventory value for rotable material is the most difficult to estimate. The philosophy for this type of inventory is that the repair unit should complete the repair within gate 2 making this inventory unnecessary. This assumption will be investigated in the next section. Due to this philosophy, the inventory has accumulated on an ad hoc basis, making it difficult to identify. An interesting note about this inventory is that it is not consumed. When inventory is used it is replaced by the repaired part once the repair is complete, shown in Figure 2. This makes a systematic approach to this type of material even more beneficial as once the material is in the inventory it remains in inventory until it is sold off.

Long lead-time parts are parts that cannot be pegged directly to an engine because the lead-time is longer than the time allotted for sourcing. These are replacement parts for parts on the engine that cannot be repaired. While the lead-time is longer than gate 2, the variance is much less than for repair parts. The supply of these parts is fairly consistent while repairs can vary greatly in duration.

Rotable and long lead-time parts have a lot of similarities. They are both low volume, high cost parts with long lead-times. In fact a few parts are both rotable and long lead-time parts since sometimes the part is repaired and other times it is replaced. Rotable and long lead-time parts together account for about 25% of the inventory value. Their high cost and low volume make them candidates for centralization. Therefore, our research focuses on these two types of inventory.

2.3 Lead-time for repairs

Ideally every repairable part would be repaired during gate 2. If this were the case there would be no need for rotable inventory. The original part would be repaired and put back on the same engine. However, many repair units are not able to complete repairs within gate 2. We looked at lead-time data for all parts repaired during the last three years and found that over 50% of the parts were late (arrived after gate 2) and late parts were late by an average of 17 days. This data is broken down by year in Table 3.

Table 3: Percentage of Late Parts and Average Days Late for Repairs

	Dates	% of Parts Late	Average Days Late
Year 1	08/07 – 07/08	41.6%	14
Year 2	08/08 – 07/09	42.2%	12
Year 3	08/09 – 07/10	65.2%	21
Overall	08/07 – 07/10	51.1%	17

The percentage of late parts is actually increasing over time. The significant increase in Year 3 is likely because of the other engine centers coming on line and therefore this performance will likely improve once the sites become more mature.

However, even only considering Year 1 and 2, performance is still substantially outside of network. Given this, our research assumes that performance is constant in the short term and does not focus on performance improvements.

2.4 Organizational Structure

In this section, we will describe the organizational structure elements that are relevant to this research. We will begin with a brief history of Pratt & Whitney and its business model. From there we will describe the elements that are unique to the three engine centers we are studying. Finally, we will discuss the impacts of Pratt & Whitney's strong entrepreneurial culture.

Overview of Pratt & Whitney and the CFM56[2] Engine

Pratt & Whitney built their first engine in 1925 in Hartford, CT. In addition to production of commercial and military engines, they provide engine repair and overhaul for their line of products as well as the CFM56 and V2500 engines. An aircraft maintenance repair and overhaul market study by Glasgow International Airport states "The engine manufacturers have increasing[ly] sought to raise their share of the engine overhaul market as it is a valuable source of substantial additional revenue and profit."

This research has focused on the three engine centers that overhaul the CFM56 engine models. Pratt & Whitney purchased the Norway Engine Center from Braathens in June 2000 and began repairing CFM56 engines. This was the first time Pratt & Whitney had overhauled an engine not made (at least in part) by Pratt & Whitney. Two new engine centers, the Shanghai Engine Center (SEC) and the

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[2] CFM56 is a registered trademark of CFM International.

Turkey Engine Center (TEC) opened as joint ventures with China Eastern Airlines and Turkish Technic respectively in 2009.

Entrepreneurial System

The general managers and material managers at each of the engine centers in the Pratt & Whitney network make decisions independently. They are each independently responsible for the financial success of their respective facilities. While this is true among all Pratt & Whitney engine centers, there is also additional complexity because two of the engine centers we looked at are joint ventures. A partner company is not necessarily incentivized to work with other sites in the Pratt & Whitney system. In addition, the processes for transferring material and information are unclear and complex. Today, the engine centers rarely share material and it is only in urgent situations on an ad hoc basis. In fact, it will be discussed in Chapter Three that we assume no lateral transshipment because of the weakness of these processes. The aftermarket materials management group at headquarters is primarily focused on financial metrics. They are responsible for the total rollup of inventory across all of the engine centers. These factors make centralization of inventory challenging. However Pratt & Whitney does have experience making planning decisions centrally through the Spares group.

Spares is the central distribution group for new parts. They serve both internal and external customers. They are responsible for forecasting and sourcing for new Pratt & Whitney spare parts demand. Spares maintains a high service level, which allows the engine centers to minimize their inventory level for new parts. However, there is no central planning or forecasting done for repair parts. Every repair unit has its own service level and lead-time. It should be noted that while this group is an excellent example of pooling inventory in Pratt & Whitney, it only provides Pratt & Whitney material and therefore does not service the three CFM56 engine centers.

Chapter 3 Future State

In this chapter we develop approaches to optimize rotable and long lead-time inventory. We begin with a literature review of the relevant research around both new and repairable low usage, high cost parts. The next section will then describe the single-echelon, single part rotable model we have developed. The rotable model determines the optimal inventory level for a part family for a given service level. The final section discusses an approach for long lead-time inventory. This model is also single-echelon, single part using an (S-1, S) ordering policy. The results of each model will be described in Chapter Four.

3.1 Inventory optimization literature review

Maintenance, repair and overhaul (MRO) parts are typically characterized by low demand and high value. These parts are either repairable or consumable. In our research, inventory of repairable parts is referred to as rotable and consumables are known as long lead-time parts. There has been extensive research into MRO inventory modeling, particularly as it applies to military operations. Nahmias (1981) provides a comprehensive review of the literature on multi-echelon MRO modeling. Sherbrooke (1968) pioneers the field with the METRIC model. The METRIC model is the first multi-echelon inventory model to optimize system-wide stock levels. This model is improved upon by Graves (1985). His model is shown to more closely estimate true values of backorders. A comparison of these two models is provided by Sherbrooke (1986).

There have been a number of significant research findings regarding multi-echelon modeling. Cohen (1986) and Alfredsson (1999) both build on METRIC by including lateral transshipment and emergency resupply. A heuristic is provided by Cagler (2004) to minimize system-wide holding costs subject to a minimum repair time. This was developed through research with an electronic machine manufacturer. The difference in equipment fill rate and part fill rate for a time-based customer service level agreement is investigated by Caggiano (2007). All of the above models use the (S-1, S) ordering policy. Moinzadeh (1986) proposes a batch ordering policy in the case of high set-up and shipping costs relative to holding cost. Wong's (2005) research centers on stocking location in his model looking at airline companies pooling spare parts inventory.

In addition, a few single-echelon models should be mentioned. Muckstadt (1980) compares multi-echelon and single-echelon models, finding that multi-echelon models are superior for time based service levels. Kukreja (2001) considers part families in which a higher-grade part can substitute for a like lower-grade part. They use network modeling considering parts as locations and substitutions as shipments. Single-echelon demand pooling through lateral transshipment is modeled by Wong (2005).

The research to date assumes that when a part is needed, it is needed immediately. This is generally true for MRO parts where one part could be causing an entire machine to be out of service. However, this is not the case for repair material in the engine overhaul business.

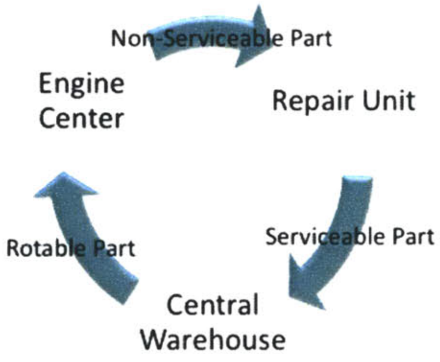
Pratt & Whitney has built material sourcing and repair time into their overhaul process (gate 2). This allotted time for repair has a substantial impact on the amount of inventory needed. Our model assumes that the gate 2 duration is fixed and that there is no benefit to improving it for a given part.

For example, if gate 2 is 20 days long then a part will need to be available on day 20 of gate 2 so that gate 3 (reassembly) can begin. This is a simplification since the duration of gate 2 varies depending on the section of the engine. Parts that come off the engine first and go back on last have a longer time for gate 2 than other parts. Using the simple example, if a part arrives in 15 days instead of 20 there is no benefit since the part will have to wait for all the other parts to arrive and gate 3 to begin.

3.2 Rotable Inventory Model

This is a multi-echelon closed loop system with demand happening at the engine center level, parts being sent to the repair units for repair, and a central warehouse sending exchange inventory to the engine centers as needed.

Figure 3: Repair Process with Exchange



The rotable inventory model uses an (S-1, S) order policy with both demand and lead-time assumed to be normally distributed. While the lead-time data is clearly normally distributed based on the large mean and n, the demand data is less clear. Many of these parts are very low usage and therefore may be represented better by a Poisson distribution. A general test of distribution is to calculate the coefficient of variation (C_v) of the parts. The C_v of nearly all parts were well above 0.5 which suggests that the Normal Distribution assumption holds. The (S-1, S) inventory model is appropriate when the usage is low and the value of the inventory is high. This model assumes that the order quantity is always one, thus only the reorder point (S-1) is needed to describe the policy. The reorder point of a part family is calculated for a given service level.

Assumptions

The model assumes repairs lead-times are independent and identically distributed and that this is a closed loop system with no consumption. The closed loop system assumption is not strictly true, however consumption is rare and only happens if the part is incorrectly identified as repairable in the initial inspection.

While this is a multi-echelon system, the model considers only a single-echelon approach because of two assumptions:

- Lateral transshipments are not allowed, and
- Transshipment lead-times are negligible

It is assumed that parts cannot be shipped between engine centers because this is currently how the engine centers operate. Lateral transshipments happen rarely today and there are not processes in place to facilitate these types of transactions. In Chapter Five we discuss the implications of lifting this assumption.

Transshipment lead-times are ignored because all shipments are by air and in nearly all cases, demand is known enough in advance that the part will arrive before it is needed. As discussed in the previous section, parts are not needed immediately. The engine center has the entire duration of gate 2 to receive the part back from repair or receive a rotable part from inventory.

In general parts are held in multiple levels to improve the speed of availability and reduce down time. This is unnecessary in this situation because of the negligible transshipment times and because there is an allotted time to source material (gate 2). Therefore we consider only two cases: inventory held at the engine center level, with each engine center holding inventory independently and all inventory held at the central warehouse. In the first case, the engine centers are independent and each is treated as single-echelon. In the second case, the central warehouse is treated as one large engine center with the aggregate demand of all three engine centers, also single-echelon. In each case the model operates the same. A comparison of the results is described in Chapter Four.

The model assumes that demand and lead-time are distributed across a part family rather than a specific part. A part family could consist of one or as many as 20 comparable parts. This assumption is dependent on the parts being interchangeable. While in general one might want to use a specific part, a substitution would be made if necessary[3].

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[3] Parts are often modified to improve performance. A part family would consist of a set of these modified parts. Therefore they can be assumed interchangeable. This assumption is not strictly correct because the modification may affect other parts in the engine. For example one part of part family A may require that the engine also use a particular part from part family B. However the assumption of interchangeable

Formulas

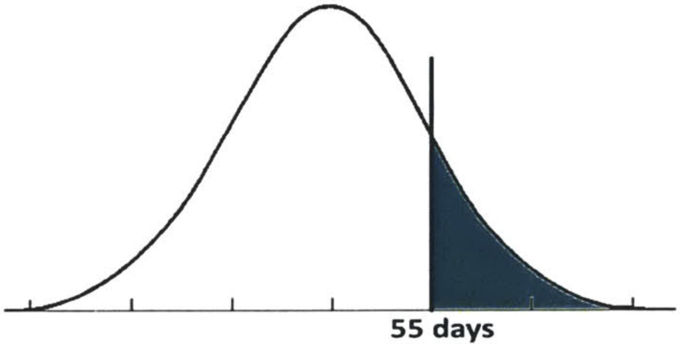
The model inputs include:

- Number of shop visits per day
- Expected demand per shop visit
- Variance of demand per shop visit
- Expected lead-time per repair
- Variance of lead-time per repair
- Network in days (allotted time for repair or gate 1 and gate 2 time)

If a repaired part is completed within the Network then the part is returned to the engine center and an exchange never takes place and no inventory is needed. Therefore we are only interested in those parts that return after the Network.

Specifically the model is based on the excess lead-time - the number of days the part takes to return after the Network.

Figure 4: Excess Lead-Time Distribution Example



In the example shown in Figure 4 above, the Network is 55 days. If the part returns within 55 days, it is on time and no inventory is needed. If the part returns after the network, say in 60 days then inventory is needed to buffer against this 5-day

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parts within part families is the industry norm given the high value and the low demand of parts.

delay. The model determines the distribution of the area to the right of the Network (shaded in blue in Figure 4), assuming this distribution is normal. The output of the model is the reorder point of each part family. From this a total inventory value is calculated.

$z = \frac{n-\mu}{\sigma}$ where n is the Network days, μ is the expected lead-time, and σ is the standard deviation of lead-time

Expected Excess Lead-Time

$$E[LT_e] = \sigma(f_\mu(z) - zF_\mu(-z))$$

where f_μ is the unit normal density function and F_μ is the standard normal cumulative function

Variance of Excess Lead-Time

$$Var[LT_e] = \sigma^2[-zf_\mu(z) + (z^2+1)F_\mu(-z)] - E[LT_e]^2$$

This mean and variance is then used to determine the reorder point (ROP).

Expected Demand during Excess Lead-Time

$$E[D_e] = E[LT_e] * E[D]$$

where D is the daily demand

Variance of Demand during Excess Lead-Time

$$Var[D_e] = Var[D] * E[LT_e] + E[D]^2 * Var[LT_e]$$

Reorder Point

$$ROP = E[D_e] + z_{SL} * \sigma[D_e]$$

where z_{SL} is the safety factor that corresponds to the desired service level

Detailed development of the equations used is found in the Rotable Model Calculation section in the appendices of this thesis.

An Excel spreadsheet is used to perform the calculations. Details on the data used for each input is below.

- **Number of shop visits per day** was taken from the official forecast for 2011. However, this is a parameter that users can change easily to allow for what-if-scenarios.
- **Expected demand per shop visit and variance** was calculated using 3 years of historical data. 3 years was necessary because of the low usage of most part families. Analysis showed demand per shop visit to be stable over time. Demand was calculated per purchase order with most part families having a single quantity per purchase order. For those part families with multi-quantities per purchase order the average quantity was used – known as units per equipment (UPE).
- **Expected lead-time per repair and variance** was calculated using the same time frame and source as demand. Lead-time was calculated from induction (the start of gate 1) through the receipt of the material from the repair unit or the exchange (the end of gate 2).
- **Network in days (allotted time for repair or gate 1 and gate 2 time)** is a standard Pratt & Whitney metric. However, a parameter was designed into the model interface to allow for what-if-scenarios. This is discussed further in the parameters section below.

Parameters

In addition to providing analysis for the business case described in Chapter Four, this model allows Pratt & Whitney to perform what-if scenarios around a number of parameters. Figure 5 shows the user interface of the model.

Figure 5: Rotable Inventory Model Interface

SV per Month
3 5.00
5 5.00
7 5.00

SL 0.95
Days 10.00
Swaps 2.00

Inventory Value
$31,491,391

The shop visits per month can be changed by engine model (-3, -5 or -7). In addition to being able to adjust the service level a user can also adjust the days from the network and the swap factor. The days from network (denoted as "Days" in Figure 5) is the adjustment from the network. For example Days =10 for a network of 55 days would correspond to a total TAT goal 65 days. Likewise Days = -10 would correspond to a total TAT goal of 45 days.

The Swap Factor is a unique to Pratt & Whitney operations. When a part does not return from repair on time there may actually be two choices – use a part from rotable inventory as we have been describing throughout this paper or in some cases the engine center could "borrow" the part from an engine earlier in the process (this is known as a swap). Performing a swap depends on a serviceable part being available on an earlier engine and on the customers of the engines being amenable. Swaps are preferable in some cases since they do not require inventory sitting on the shelf (therefore allowing smaller rotable inventory investment). However there is an additional cost of time and personnel in performing the swaps that is not accounted.

In practice swaps are common in the engine centers but the exact frequency is unknown. We do know that it is highly sensitive to the number of engines in the engine center and the customer mix. The swap factor has been added to the model in order to represent Pratt & Whitney operations to the full extent possible. The model decreases the reorder point by the swap factor.

Reorder Point with Swap Factor (F)

$$ROP = \begin{cases} E[D_e] + z_{SL} * \sigma[D_e] - F & if(E[D_e] + z_{SL} * \sigma[D_e]) > F \\ 0 & otherwise \end{cases}$$

This is a simplistic calculation and we assume a swap factor = 0 for the purposes of our analysis.

The simple user interface allows users to run many different potential scenarios. Pratt & Whitney is currently using this model to look at the impact on rotable inventory for a given range of networks. There is also discussion of using the model to aid in cost estimations for campaigns – that is bidding for large contracts of work.

Summary of Rotable Inventory Model

In the engine overhaul business material sourcing time is built into the process. This means that material is not needed immediately but rather after some specified amount of time. This feature is central to the rotable inventory model, which determines the optimal inventory level to buffer against parts out for repair.

The model determines the mean and variance of the excess lead-time – the portion of the lead-time that occurs after the specified time allotted. The excess lead-time is used to determine the optimal reorder point.

In the next section, we will look at another type of inventory – long lead-time parts.

3.3 Long Lead-Time Inventory Model

Long lead-time parts are new or serviceable replacement parts for parts that cannot be repaired. The lead-time for these parts is by definition longer than gate 2. The only supplier for new parts replacement parts is CFMI. CFMI (a joint venture between GE Aviation and Snecma) builds and supports CFM56 engines. It is assumed that the lead-time for parts from CFMI cannot be improved because the supplier is a direct competitor with the engine centers and does not have an incentive to improve. Lead-time performance could improve by sourcing serviceable rather than new material, dependent on serviceable material availability. For the purposes of this model we assume lead-time is stable.

There are approximately 300 parts that are categorized as long lead-time. The average demand for a long lead-time part is less than 1 per shop visit. The average price for a long lead-time part is $20,000.

Unlike the rotable inventory model, this is not a closed-loop system – parts are consumed. This is an (S-1, S) ordering system with constant lead-time and a Poisson demand distribution. The reorder point of a part is calculated for a given service level.

Assumptions

The model assumes constant lead-time using published lead-time data from the supplier because actual lead-time performance is not known. We were able to collect a small sample of actual lead-time data to validate published lead-times.

The model considers only a single-echelon system because it is assumed that lateral transshipments are not allowed and transshipment lead-times are negligible.

See the assumptions section of the rotable inventory model for a discussion of these assumptions.

The lead-time for these parts is defined as the time from the end of gate 1 to the beginning of gate 3 (so gate 2 only). This is different from the definition used in the rotable inventory model because we are using supplier lead-time, which would not include engine teardown (gate 1).

Formulas

The model inputs include:

- Lead-time (LT); the days from ordering until the part arrives
- Network (n); the days allotted for gate 2
- Expected shop visits per day (SV)
- Expected demand per shop visit (D/SV)
- Service Level (SL); the required fill rate

If the part arrives within the network (n) then the part is on time and no inventory is needed.

Daily Demand

$$D = D/SV * SV$$

Average Demand During Excess Lead-Time

$$\lambda = \begin{cases} D*(LT-n) & for \quad LT > n \\ 0 & for \quad LT \leq n \end{cases}$$

Reorder Point (R)

$$\frac{\Pr(R+1|\lambda)}{\Pr_{\le}(R+1|\lambda)} = SL$$

where Pr is the probability and $\Pr_{\le}$ is the cumulative probability.

To solve for the reorder point, we created a macro that increases R incrementally by 1 until the service level is achieved.

Examples

Below are two examples of the long lead-time inventory model. These examples highlight the differences between a relatively high usage part (D/SV>5) and a very low usage part (D/SV<0.2).

Table 4: Part A Example - High Usage Part

Part	LT (days)	n (days)	SV (visits/day)	D/SV (pieces)	SL (%)
A	30	22	0.5	10	95

The excess lead-time (LT – n) is 8 days. Therefore the inventory level needs to account for an 8-day gap to achieve the required service level. The reorder point for part A is 45 pieces, with a calculated service level of 95.5%.

Table 5: Part B Example - Low Usage Part

Part	LT (days)	n (days)	SV (visits/day)	D/SV (pieces)	SL (%)
B	30	22	0.5	.12	95

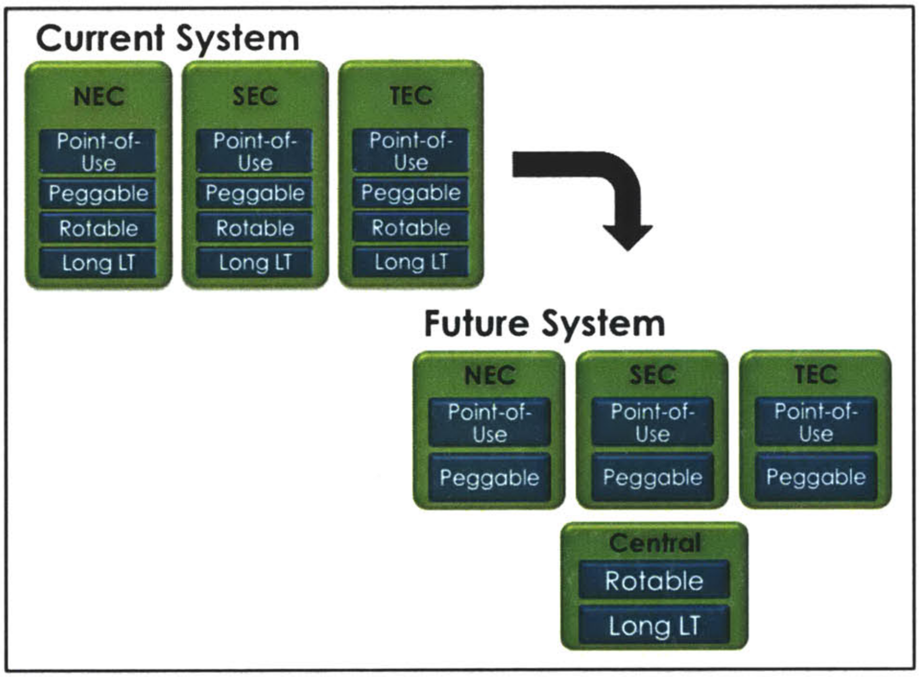
For part B, the reorder point is 2 pieces. For this example the calculated service level is 98.9%, well above the required 95%. This is due to the integer restriction of the reorder point. If the reorder point were 1, the service level would be only 92.8%. When $\lambda \leq .05$, no inventory is held because the service level is achieved with the parts on order.

The model calculates the reorder point for each of the 300 parts and outputs the total inventory value for the optimal inventory level. We use this to establish the future state. In Chapter Four, we use this model to compare the inventory value in a centralized system and a decentralized system.

Chapter 4 Business Case for Centralization

This chapter presents the business case for establishing a central warehouse for the three engine centers. We propose that this warehouse be located in Dallas, Texas, as Pratt & Whitney already has established space, infrastructure, and personnel. The impact of this decision on transportation costs is discussed in detail in Section 4.3. In addition, we propose that the planning and procurement for the central warehouse be managed separately from that of the engine centers.

Figure 6: Proposed Process Change



4.1 Why Centralize?

Currently the engine centers independently hold their inventory and make their inventory decisions. While this is true for all engine centers this a particularly precarious process for the three CFM56 engine centers. The engine centers that service Pratt & Whitney engines order their new replacement parts from Spares – a Pratt & Whitney organization. Spares maintains a high service level and creates a buffer for the entire system against long lead-times and back-orders. Spares therefore serves as a safety net through:

1. Minimizing the number of long lead-time parts
2. Expediting parts if a repair part is delayed
3. Accepting returns if demand at an engine center is overestimated

For the Norway, Turkey, and Shanghai engine centers, inventory is supplied directly from a competitor. This means that they are at an increased risk of parts (both repaired and replacement) arriving after gate 2 and of increasing their inventory ad hoc because they have no outlet to reduce unneeded inventory. In addition to serving as a buffer, a central warehouse would benefit the entire system through improved planning and procurement procedures.

Planning

Creating a central warehouse allows Pratt & Whitney to aggregate demand across the three engine centers. Because the product mix is similar across the engine centers and the inventory consists of low-usage, high-cost parts, aggregated demand significantly reduces the needed inventory. This reduction is described in detail in Section 4.2. Aggregated demand also reduces the risk of excess and obsolete inventory since the risk can now be shared across the three sites.

An engine center's focus is on getting serviceable engines out the door within the allotted time. A central planning group would be focused on developing demand forecasting and inventory optimization tools and processes to ensure a high service-level. This would provide a much needed service to the engine centers. Spares and the IMT[4] group have both proven the value of a central planning group. These two groups have been successful because of their reputations for providing great customer service. It is critical that a central planning group maintain a high service-level. If the engine centers do not trust the central warehouse they will maintain their own safety stock and create a duplicate supply of inventory, negatively impacting the total inventory value of the system.

Procurement

The largest profit margins in the engine overhaul business are from the sale of replacement parts, either new or serviceable. AeroStrategy's report presented by Stewart (2006) shows that material makes up 62% of the engine overhaul cost structure, with repair of parts representing an additional 13%. Because the CFM56 is not a Pratt & Whitney engine, it is much more profitable to use serviceable material rather than new. It is also more cost effective for the customer, creating more customer value and in many cases has a shorter lead-time.

Pratt & Whitney has a group responsible for sourcing serviceable material – the Commercial Serviceable Assets group (CSA). While they do source material for the 3 CFM56 engine centers the relationship is not strong. Centralizing inventory would create a single group with which to work. In addition, the central warehouse would be co-located with the CSA group in Texas, further encouraging cooperation.

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[4] IMT is the Integrated Management Team. IMT is a group in Materials Management that manages strategic rotable material for specific customers. They do not currently hold any CFM56 material.

The next sections in this chapter compare the various costs under the decentralized system with the proposed centralized system.

4.2 Inventory Holding Cost

The amount of inventory required is dependent on the number of shop visits of each engine model seen at each engine center. The following table shows the shop visits used in our analysis. This data has been disguised, as the actual number of shop visits expected in 2011 is confidential.

Table 6: Projected Shop Visits for 2011

	NEC	TEC	SEC	Total
CFM56-3	25	15	15	55
CFM56-5	50	20	20	90
CFM56-7	25	15	15	55
Total	100	50	50	200

The rotable inventory model is quite insensitive to engine model type; meaning that the total inventory value changes very little as shop visits are moved from one engine model to another.

Rotable Inventory

Using the model described in Chapter Three, we determine the value of the optimal inventory under each system. For comparison, we look at the amount of inventory needed for a 55-day Network and a 75-day Network, the 55-day Network being the industry standard and Pratt & Whitney's goal. The 75-day Network, while not the actual demonstrated TAT, is a stand-in to serve as a baseline comparison. This allows us to protect proprietary metrics and still have a base case for discussion.

Determining the optimal inventory for the centralized system is straightforward. We enter the projected shop visits for next year; assume a swap factor of zero and a 95% service level. The model gives us the total inventory value shared across the three engine centers as well as the optimal inventory level for each part family. This total value is shown in Table 7 below.

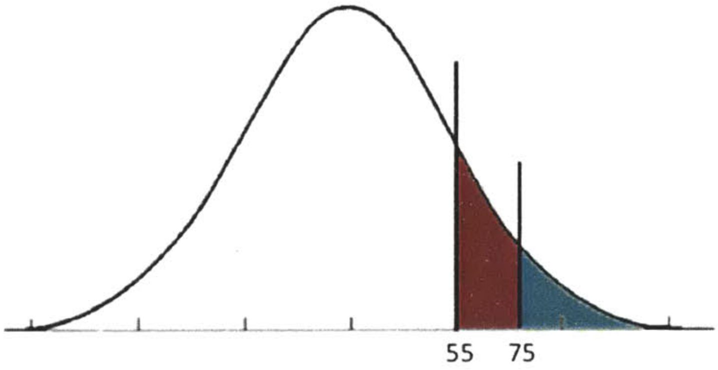
For the decentralized system, the demand is not shared across sites. We use the model to determine the optimal inventory level for each part family at each site using the same service level and swap factor but the number of shop visits for that particular engine center. We then sum the inventory value for each site to determine the system inventory value. This total value is shown in Table 7 below.

Table 7: Rotable Inventory Model Output Comparison

(in Millions)	Optimal Inventory Level		Savings	
Network	Decentralized	Centralized	Inventory Value	Holding Cost (18%)
75-day	$56.7	$37.9	$18.8	$3.4
55-day	$98.2	$67.5	$30.7	$5.5

Reducing the network from 75 days to 55 days requires a substantial investment in additional inventory. For the decentralized system, the inventory requirement goes from $57 million to $98 million. Whereas for the centralized system, it moves from $38 million to $68 million. As the network decreases, the acceptable lead-time (or gate 1 and gate 2 allotted time) goes down and the excess lead-time goes up (shaded regions in Figure 7).

Figure 7: Illustration of Change in Excess Lead-Time



This increase in excess lead-time increases the amount of safety stock needed. It also increases the product mix as some part families that do not require inventory at the higher network will require stock at the lower network.

Regardless of the network, the inventory requirement for a centralized system is strictly less than the inventory requirement of a decentralized system. This is because aggregating demand creates a large benefit to systems that have:

1. Similar product mix in each site
2. Parts that are largely high-cost, low-usage

The centralized system requires $30.7 million less than the decentralized system for the 55-day network. This is a 30% reduction in inventory investment; and at an 18% interest rate this is a savings in holding cost of $5.5 million annually.

While not impacted by the type of system used, reducing the TAT results in a work-in-progress (WIP) inventory reduction. Moving from a 75-day TAT to a 55-day TAT would reduce the number of engine in the system at a given time, reducing the accumulated value of inventory in progress, WIP. This reduction can be calculated using Little's Law:

Little's Law

$$I = RT$$
$$WIP = ArrivalRate * TAT$$
$$WIP = \frac{200 engines}{365 days} * 75 days$$

We assume a WIP inventory value of $1 million per engine and the results of our calculations are in Table 8.

Table 8: WIP Calculations

	Engines	**WIP Inventory Value (millions)**
75 days	41.1	$41.1
55 days	30.1	$30.1
Reduction	11	$11

It is important to note that a decrease in network does not necessarily ensure a decrease in TAT. The network is the TAT goal. This analysis assumes that the additional inventory investment would be immediately available and that there are no other barriers to TAT reduction. However, assuming the TAT reduction is realized there is a gain in inventory of $11 million and at 18% interest rate, an annual savings in holding cost of $2 million.

To focus on our proposed system change, we compare the baseline (75-day network, decentralized system) against our proposed system (55-day, centralized system).

Table 9: Comparison of Inventory Holding Costs

75-day Network	*Decentralized System*	*Centralized System*
Inventory Stock Level	$56.7 million	$37.9 million
WIP Level	$41.1 million	$41.1 million
Total Inventory Level (Inventory Stock + WIP)	$97.8 million	$79.0 million
Total Inventory Holding Cost (Total Inventory * 0.18)	**$17.6 million**	**$14.22 million**
55-day Network	*Decentralized System*	*Centralized System*
Inventory Stock Level	$98.2 million	$67.5 million
WIP Level	$30.1 million	$30.1 million
Total Inventory Level (Inventory Stock + WIP)	$128.3 million	$97.6 million
Total Inventory Holding Cost (Total Inventory * 0.18)	**$23.09 million**	**$17.57 million**

Going from a 75-day network to a 55-day network without changing the inventory system would increase the inventory holding cost by $5.5 million annually. However, with the proposed change, we find that the inventory value to be essentially equivalent despite the 20 day improvement in the network. With a 55-day network, the potential savings is a ($23.09M - $17.57M) = $5.52 reduction in annual holding cost. This does not consider additional other costs such as transportation, which will be discussed in Section 4.3.

Long Lead-Time Inventory

The long lead-time model assumes a constant gate 2 lead-time of 30 days. 30 days being the published lead-time to receive material from the supplier. This means that for a 75-day (20,35,20) network, no long lead-time inventory is needed; the gate 2 allotted time (35 days) is greater than 30 days. For a 55-day (15,20,20) network, the gate 2 allotted time is assumed to be 20 days and inventory is needed to buffer against demand during this 10-day period (lead-time – gate 2).

We find the optimal inventory level for the centralized system by entering the total number of shop visits (200) for all 3 sites. For the decentralized system, we find the optimal inventory for each site and then sum to find the total inventory value. The results are shown in Table 10.

Table 10: Long Lead-Time Inventory Model Output Comparison

(in Millions)	**Optimal Inventory Level**		**Savings**	
Network	**Decentralized**	**Centralized**	**Inventory Value**	**Holding Cost (18%)**
55-day	$3.8	$2.5	$1.3	$0.2

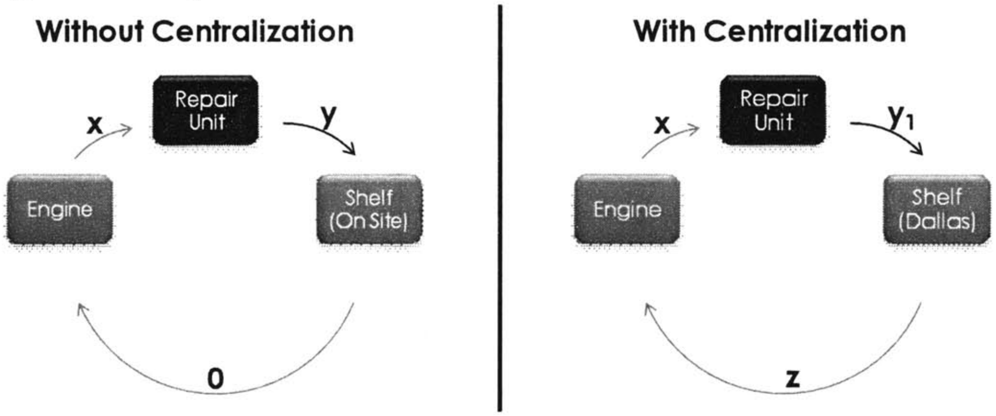
Centralizing this inventory results in a 34% reduction in inventory value. Given the relatively low inventory volume, this is only a roughly $200,000 savings in holding cost.

At this point, we do not recommend centralizing long lead-time inventory. The little savings available from the reduced holding cost would be offset by an increase in transportation cost. In addition, managing this type of inventory in the same warehouse as rotable inventory would add considerable complexity. Long lead-time material is consumable while rotable material is a closed-loop. This difference alone means that many of the processes would be different for each inventory type.

4.3 Transportation Cost

The transportation costs for the current and the proposed system are illustrated in Figure 8 below. All parts are air shipped and repair units are located throughout the world.

Figure 8: Transportation Costs



The costs without centralization are the cost from the engine center to the repair unit and back (x + y in the figure above).

The costs associated with centralization are the costs:

- From the engine center to the repair unit (x)
- From the repair unit to the central warehouse (y_1)
- From the central warehouse to the engine center (z)

Assuming that the difference between y and y_1 is negligible, the additional cost for centralization is z – the cost from the central warehouse to the engine center.

Based on the expected demand during excess lead-time for each part family, there will be approximately 2000 exchanges in 2011 for a 55-day network. Expected lane costs for each route were obtained. Additional transportation costs for a centralized system would be $1.3 million based on an average weight per exchange of 300lbs. If the weight were to average 500lbs the additional cost would be $2.2 million.

With a network of 55-days, changing to a centralized system would cost an additional $1.3 million in annual transportation cost. However, staying with the decentralized system would cost an additional $5.5 million in annual holding cost; tying up an additional $41.5 million in capital with duplicate inventory at the sites. In addition to the many qualitative benefits discussed at the beginning of this chapter, there is a $4.2 million cost advantage to centralization.

Chapter 5 Conclusions

This research was conducted over a six-month period on site at Pratt & Whitney. Out of this research we have not only developed a robust inventory model that Pratt & Whitney's Materials Management group is currently using for planning purposes, but also determined the benefits of centralizing inventory. This paper concludes with this final chapter looking at specific implementation recommendations, remaining questions, and the research findings.

5.1 Implementation recommendations

To realize the full benefit of centralization all rotable material should be held centrally, and therefore none should be kept at the engine centers. Given the entrepreneurial culture and the joint venture relationships, this will only be acceptable to the engine centers if they have confidence that the material will be available when needed. To assure this confidence, the central warehouse must treat the engine centers as the customers and provide the needed customer service. This customer service entails maintaining a sufficiently high service-level (assumed to be 95% throughout this research) and maintaining clear and efficient processes. Spares is an excellent example of the kind of organization that would inspire trust. Many of the procedures can be replicated from them.

Some shift in culture would facilitate implementation. Currently, in Pratt & Whitney many hold the view that "all inventory is bad." This thinking is limiting because it ignores the benefits of flexibility and responsiveness that comes with inventory. While in a perfect world the processes would be consistent such that the repaired parts would always return on time, when dealing with a large number of repair units and a mix of repairs this ideal is not realizable. This is not to say that no effort should be made to improve performance.

This analysis assumed that the lead-times are constant over time. However the best implementation of the central warehouse would be to couple centralization with process improvements to shorten lead-time and its variance. The shorter and more consistent the TAT of repairs are, the less inventory that will need to be held. We have shown that for rotable material centralization is preferable even if lead-time is shorter. In addition, centralization can facilitate process improvements by aggregating lead-time data for all three engine centers and by serving as the customer to the repair units and holding them accountable, something the engine centers do not have the capacity to do fully now.

Maintaining excellent customer service for the engine centers and coupling centralization with process improvement will help ensure that Pratt & Whitney realizes the benefits associated with implementing a central warehouse.

5.2 Remaining questions

The rotable inventory model optimizes part families individually. Additional savings could be found through a system-wide optimization. For example, rather than determining buffer stock such that every part family has a 95% service level, a system-wide optimization could determine the buffer stock such that the overall system has a 95% service level. The more expensive and slower moving parts could have a lower service level and the less expensive parts a higher, which would likely have a cost savings.

In addition, further research could investigate the potential benefits of lateral transshipments – shipment from one engine center to another. In Chapter Four we discuss the transportation cost as a reason that we do not currently recommend centralizing long lead-time inventory. This type of inventory might benefit more from lateral transshipments. Lateral transshipments would allow demand to be aggregated without realizing the full transportation costs of centralization.

Finally, swaps are very roughly estimated in this model. More research could be done to understand the process of swaps in the engine center and a more sophisticated way to account for this phenomenon.

5.3 Research findings

A number of broad implications come out of this research:

1. The benefits of stratifying inventory
2. The importance of centralization under certain parameters
3. The concept of excess lead-time in modeling inventory levels

The Benefits of Stratifying Inventory

Pratt & Whitney had a general understanding of the different types of inventory in an engine center. But without a clear definition and strategy for each type there is no straightforward way to evaluate the inventories' performance. For example, Pratt & Whitney spent a lot of effort focused on point-of-use material reduction, when in actuality given the low value of these parts, they are being managed quite effectively and effort would be better spent elsewhere. Without a clear stratification of inventory type, it is difficult to identify where improvement resources should be placed.

In addition to highlighting the areas of concern, stratifying inventory allows a company to establish inventory type-specific strategies. The service level for a low cost, high volume part is likely to be different than the service level of a high cost low usage part.

The Importance of Centralization Under Certain Parameters

One strategy dependent on inventory type is centralization. The decision to centralize is based on balancing the decreased inventory holding cost with the increased transportation cost that results from centralizing inventory. As the value of the product out paces the cost of transportation, centralization becomes beneficial. Additionally as the volume of the parts decreases the more advantage there is to aggregating demand through centralization.

The Concept of Excess lead-time in Modeling Inventory Levels

The engine overhaul business is unique for a number of reasons. One important distinction of the business process is the network. In most business cases, a part would need to be available immediately. In this business, Pratt & Whitney has all of gate 2 to source and repair material. This creates the need to differentiate between the lead-time and the excess lead-time. The derivations of the formulas used are in the appendix of this paper.

Pratt & Whitney is currently using the rotable inventory model to understand the amount and mix of inventory needed under a variety of scenarios, such as shorter turn-around-times or increased shop visits. This model is also useful to the sales group when bidding on large contracts as it provides an estimate for rotable inventory costs.

In conclusion, it has been shown that holding optimal inventory levels has a positive effect on turn around time. Additionally, centralizing this inventory when the parts are high value and low usage reduces the inventory value while maintaining the service level.

Rotable Model Calculations

This model calculates the reorder point (ROP) for a given service level (SL). This is for material sent out for repair. If the repaired part returns within the Network (n) days then demand is satisfied. If the repaired part returns in $n+1$ or more days then demand is satisfied by safety stock or is unsatisfied.

Demand (D) is Normally distributed with parameters μ and σ.

Lead-time (LT) is Normally distributed with parameters μ and σ.

Inputs include:

Number of shop visits / day (SV)

Expected demand per shop visit ($E[D/SV]$)

Variance of demand per shop visit ($Var[D/SV]$)

Expected Lead-time (μ)

Variance of lead-time (σ^2)

Network days (n)

Calculations:

Expected daily demand

$$E[D] = E[D/SV] * SV$$

Variance of daily demand

$$Var[D] = Var[D/SV] * SV$$

$$z = \frac{n-\mu}{\sigma}$$

Expected excess lead-time is the mean lead-time over n days

$$E[LT_e] = \int_{x=n}^{\infty}(x-n)f(x)dx$$

$$= \sigma\int_{x=z}^{\infty}(x-z)f_u(x)dx$$
$$= \sigma\int_{x=z}^{\infty}xf_u(x) - zf_u(x)dx$$
$$= \sigma[-\int_{x=z}^{\infty}-xf_u(x)dx - z\int_{x=z}^{\infty}f_u(x)dx]$$

$\frac{df_u(x)}{dx} = -xf_u(x),$ *thus*

$$E[LT_e] = \sigma[-[f_u(x)|_z^{\infty} - z[F_u(x)|_z^{\infty}]$$

$= \sigma(f_u(z) - zF_u(-z))$ *where f_u is the unit normal density function and F_u is the standard normal cumulative function*

Variance of excess lead-time

$$Var[LT_e] = E[LT_e^2 - E[LT_e]^2$$

$$= \int_{x=n}^{\infty}(x-n)^2f(x)dx - E[LT_e]^2$$
$$= \sigma^2\int_{x=z}^{\infty}(x-z)^2f_u(x)dx - E[LT_e]^2$$
$$= \sigma^2[\int_{x=z}^{\infty}x^2f_u(x) - 2zxf_u(x) + z^2f_u(x)dx] - E[LT_e]^2$$
$$= \sigma^2[\int_{x=z}^{\infty}-(f_u(x) - x^2f_u(x)) + f_u(x) - 2zxf_u(x) + z^2f_u(x)dx] - E[LT_e]^2$$
$$= \sigma^2[-1\int_{x=z}^{\infty}f_u(x) - x^2f_u(x)dx + 2z\int_{x=z}^{\infty}-xf_u(x)dx + (z^2+1)\int_{x=z}^{\infty}f_u(x)dx] - E[LT_e]^2$$

$\frac{df_u(x)}{dx} = -xf_u(x)$

$\frac{dxf_u(x)}{dx} = f_u(x) - x^2f_u(x),$ *thus*

$$Var[LT_e] = \sigma^2[-xf_u(x) + 2zf_u(x)|_z^{\infty} + (z^2+1)F_u(x)|_z^{\infty} - E[LT_e]^2$$

$$= \sigma^2[-[-zf_u(z) + 2zf_u(z)] + (z^2+1)(F_u(\infty) - F_u(z))] - E[LT_e]^2$$
$$= \sigma^2[-zf_u(z) + (z^2+1)F_u(-z)] - E[LT_e]^2$$

Expected demand during excess LT

$$E[D_e] = E[LT_e] * E[D]$$

Variance of demand during excess LT

$$Var[D_e] = Var[D] * E[LT_e] + E[D]^2 * Var[LT_e]$$

Reorder Point

$ROP = E[D_e] + z_{SL}\sigma_{D_e}$ *where* z_{SL} *is the safety factor corresponding to a desired service level*

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