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A Unified Theory of Tobin's q, Corporate Investment, Financing, and Risk Management

PATRICK BOLTON, HUI CHEN, and NENG WANG*

ABSTRACT

We propose a model of dynamic investment, financing, and risk management for financially constrained firms. The model highlights the central importance of the endogenous marginal value of liquidity (cash and credit line) for corporate decisions. Our three main results are: 1) investment depends on the ratio of marginal q to the marginal value of liquidity, and the relation between investment and marginal q changes with the marginal source of funding; 2) optimal external financing and payout are characterized by an endogenous double-barrier policy for the firm's cash-capital ratio; and 3) liquidity management and derivatives hedging are complementary risk management tools.

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When firms face external financing costs, they must deal with complex and closely intertwined investment, financing, and risk management decisions. How to formalize the interconnections among these margins in a dynamic setting and how to translate the theory into day-to-day risk management and real investment policies remains largely to be determined. Questions such as how corporations should manage their cash holdings, which risks they should hedge and by how much, or the extent to which holding cash is a substitute for financial hedging are not well understood.

Our goal in this article is to propose the first elements of a tractable dynamic corporate risk management framework — as illustrated in Figure 1 — in which cash inventory, corporate investment, external financing, payout, and dynamic hedging policies are characterized simultaneously for a "financially constrained" firm. We emphasize that risk management is not just about financial hedging; instead, it is tightly connected to liquidity management via daily operations. By bringing these different aspects of risk management into a unified framework, we show how they interact with and complement each other.

Insert Figure 1 About Here

The baseline model we propose introduces only the essential building blocks, which are: i) the workhorse neoclassical q model of investment[1] featuring constant investment opportunities as in Hayashi (1982); ii) constant external financing costs, which give rise to a corporate cash inventory problem as in Miller and Orr (1966); and iii) four basic financial instruments: cash, equity, line of credit, and derivatives (for example, futures). This parsimonious model already captures many situations that firms face in practice and yields a rich set of prescriptions.

With external financing costs the firm's investment is no longer determined by equating the marginal cost of investing with marginal q, as in the neoclassical Modigliani and Miller (1958) (MM) model (with no fixed adjustment costs for investment). Instead, investment of a financially constrained firm is determined by the *ratio of marginal q to the marginal cost of financing*:

$$\text{marginal cost of investing} = \frac{\text{marginal } q}{\text{marginal cost of financing}}.$$

When firms are flush with cash, the marginal cost of financing is approximately one, so that this equation is approximately the same as the one under MM. But when firms are close to financial distress, the marginal cost of financing, which is endogenous, may be much larger than one so that optimal investment may be far lower than the level predicted under MM. A key contribution of our article is to analytically and quantitatively characterize the marginal value of cash to a financially constrained firm as a function of the firm's investment opportunities, cash holding, leverage, external financing costs, and hedging opportunities.[2]

An important result that follows from the first-order condition above is that the relation between marginal q and investment differs depending on whether cash or credit is the marginal source of financing. When the marginal source of financing is cash, both marginal q and investment increase with the firm's cash holdings, as more cash makes the firm less financially constrained. In contrast, when the marginal source of financing is the credit line, we show that marginal q and investment move in opposite directions. On the one hand, investment decreases with leverage, as the firm cuts investment to delay incurring equity issuance costs. On the other hand, marginal q increases with the firm's leverage, because an extra unit of capital helps relax the firm's borrowing constraint by lowering the debt-to-capital ratio, and this effect becomes increasingly more important as leverage rises. Thus, there is no longer a monotonic relation between investment and marginal q in the presence of a credit line, and average q can actually be a more robust indicator for investment.

A second key result concerns the firm's optimal cash inventory policy. Much of the empirical literature on firms' cash holdings tries to identify a *target cash inventory* for a firm by weighing the costs and benefits of holding cash.[3] The idea is that this target level helps determine when a firm should increase its cash savings and when it should dissave.[4] Our analysis, however, shows that the firm's cash inventory policy is much richer, as it involves a combination of a *double-barrier policy* characterized by a single variable, the cash-capital ratio, and the continuous management of cash reserves in between the barriers through adjustments in investment, asset sales, as well as the firm's hedging positions. While this double-barrier policy is not new (it goes back to the inventory model of Miller and Orr (1966) in corporate finance), our model provides substantial new insight on

how the different boundaries depend on factors such as the growth rate and volatility of earnings, financing costs, cash holding costs, as well as the dynamics of cash holdings in between these boundaries. Besides cash inventory management, our model can also give concrete prescriptions for how a firm should choose its investment, financing, hedging, and payout policies, which are all important parts of dynamic corporate financial management.

For example, when the cash-capital ratio is higher, the firm invests more and saves less, as the marginal value of cash is smaller. When the firm is approaching the point where its cash reserves are depleted, it optimally scales back investment and may even engage in asset sales. This way the firm can postpone or avoid raising costly external financing. Since carrying cash is costly, the firm optimally pays out cash at the endogenous upper barrier of the cash-capital ratio. At the lower barrier the firm either raises more external funds or closes down. The firm optimally chooses not to issue equity unless it runs out of cash. Using internal funds (cash) to finance investment defers both the cash-carrying costs and external financing costs.[5] Thus, with a constant investment/financing opportunity set our model generates a dynamic pecking order of financing between internal and external funds. The stationary cash inventory distribution from our model shows that firms respond to financing constraints by optimally managing their cash holdings so as to stay away from financial distress situations most of the time.

A third new result is that our model integrates two channels of risk management, one via a state-noncontingent vehicle (cash), the other via state-contingent instruments (derivatives). In the presence of external financing costs, firm value is sensitive to both idiosyncratic and systematic risk. To limit its exposure to systematic risk, the firm can engage in dynamic hedging via derivatives (such as oil or currency futures). To mitigate the impact of idiosyncratic risk, it can manage its cash reserves by modulating its investment outlays and asset sales, and also by delaying or moving forward its cash payouts to shareholders. Financial hedging (derivatives) and liquidity management (cash, investment, financing, payout) thus play complementary roles in risk management. When dynamic hedging involves higher transactions costs, such as tighter margin requirements, we also show that the firm reduces its hedging positions and relies more on cash for risk management.

Only a handful of theoretical analyses examine firms' optimal cash, investment, and risk management policies. A key first contribution is by Froot, Scharfstein, and Stein (1993), who develop a static model of a firm facing external financing costs and risky investment opportunities.[6] Subsequent contributions on dynamic risk management focus on optimal hedging policies and abstract away from corporate investment and cash management. Notable exceptions include Mello, Parsons, and Triantis (1995) and Morellec and Smith (2007), who analyze corporate investment together with optimal hedging. Mello and Parsons (2000) study the interaction between hedging and cash management, but do not model investment. None of these models, however, considers external financing or payout decisions. Our dynamic risk management problem uses the same contingent-claim methodology as in the dynamic capital structure/credit risk models of Fischer, Heinkel, and Zechner (1989) and Leland (1994), but unlike these theories we explicitly model the wedge between a firm's internal and external financing and the firm's cash accumulation process. Our model extends these latter theories by introducing capital accumulation and thus integrates the contingent-claim approach with the dynamic investment/financing literature.

Our model also provides new and empirically testable predictions on investment and financing constraints. Fazzari, Hubbard, and Petersen (1988) (FHP) are the first to use the sensitivity of investment to cash flow (controlling for q) as a measure of a firm's financing constraints. Kaplan and Zingales (1997) provide an important critique on FHP and successors from both a theoretical (using a static model) and an empirical perspective. Recently, there is growing interest in using dynamic structural models to address this empirical issue, which we discuss next.

Gomes (2001) and Hennessy and Whited (2005, 2007) numerically solve discrete-time dynamic capital structure models with investment for financially constrained firms. They allow for stochastic investment opportunities and have no adjustment costs for investment.[7] However, these studies do not model cash accumulation and do not consider how cash inventory management interacts with investment and dynamic hedging policies. Hennessy, Levy, and Whited (2007) characterize an investment first-order condition for a financially constrained firm that is related to ours, but they consider a model with quadratic equity issuance costs, which leads to a fundamentally different cash

management policy from ours. Using a model related to Hennessy and Whited (2005, 2007), Riddick and Whited (2009) show that saving and cash flow can be negatively related after controlling for q, because firms use cash reserves to invest when receiving a positive productivity shock.[8]

In contrast to the impressive volume of work studying how adjustment costs affect investment,[9] very few analytical results are available on the impact of external financing costs on investment. Our model fills this gap by exploiting the simplicity of a framework that is linearly homogeneous in cash and capital, and for which a complete analytical characterization of the firm's optimal investment and financing policies, as well as its dynamic hedging policy and its use of credit lines, is possible. In terms of methodology, our paper is also related to Decamps et al. (2008), who explore a continuous-time model of a firm facing external financing costs. Unlike our setup, their firm only has a single infinitely lived project of fixed size, and hence they do not consider the interaction of the firm's real and financial policies.

The paper most closely related to ours is DeMarzo et al. (2010), henceforth DFHW. Both our paper and DFHW consider models of corporate investment that integrate dynamic agency frictions into the neoclassic q theory of investment (for example, Hayashi (1982)). The approach taken in DFHW is more microfounded around an explicit dynamic contracting problem with moral hazard, where investors dynamically manage the agent's continuation payoff based on the firm's historical performance. The key state variable in their dynamic contracting problem is the manager's continuation payoff.[10] Their dynamic contracting framework endogenizes the firm's financing constraints in a similar way to ours, even though firm value in their framework is expressed as a function of a different state variable. One key difference, however, between the two models is in the dynamics of the state variable measuring financial slack. In DFHW, the manager's equilibrium effort choice affects the volatility of financial slack but not directly the drift. In our model, in contrast, the manager directly influences the drift but not the volatility of the dynamics of financial slack. As a result, investment is monotonically linked to marginal q in DFHW, while investment is linked to the ratio between marginal q and the endogenous marginal value of financial slack in our model. This distinction leads to different implications on corporate investment, financing policies, and payout

to investors in the two models.

The remainder of the paper proceeds as follows. Section I sets up our baseline model. Section II presents the model solution. Section III continues with quantitative analysis. Sections IV and V extend the baseline model to allow for financial hedging and credit line financing. Section VI concludes.

I. Model Setup

We first describe the firm's physical production and investment technology. Next, we introduce the firm's external financing costs and its opportunity cost of holding cash. Finally, we state firm optimality.

A. Production Technology

The firm employs physical capital for production. The price of capital is normalized to unity. We denote by K and I the level of capital stock and gross investment, respectively. As is standard in capital accumulation models, the firm's capital stock K evolves according to

$$dK_t = (I_t - \delta K_t)\, dt, \quad t \geq 0, \tag{1}$$

where $\delta \geq 0$ is the rate of depreciation.

The firm's operating revenue at time t is proportional to its capital stock K_t, and is given by $K_t dA_t$, where dA_t is the firm's revenue or productivity shock over time increment dt. We assume that after accounting for systematic risk the firm's cumulative productivity evolves according to

$$dA_t = \mu dt + \sigma dZ_t, \quad t \geq 0, \tag{2}$$

where Z is a standard Brownian motion under the risk-neutral measure.[11] Thus, productivity shocks are assumed to be *i.i.d.*, and the parameters $\mu > 0$ and $\sigma > 0$ are the mean and volatility of

the risk-adjusted productivity shock dA_t. This production specification is often referred to as the "AK" technology in the macroeconomics literature.[12]

The firm's incremental operating profit dY_t over time increment dt is given by

$$dY_t = K_t dA_t - I_t dt - G(I_t, K_t)dt, \quad t \geq 0, \tag{3}$$

where $G(I, K)$ is the additional adjustment cost that the firm incurs in the investment process. We may interpret dY_t as cash flows from operations. Following the neoclassical investment literature (Hayashi (1982)), we assume that the firm's adjustment cost is homogeneous of degree one in I and K. In other words, the adjustment cost takes the form $G(I, K) = g(i)K$, where i is the firm's investment capital ratio ($i = I/K$) and $g(i)$ is an increasing and convex function. While our analyses do not depend on the specific functional form of $g(i)$, for simplicity we adopt the standard quadratic form

$$g(i) = \frac{\theta i^2}{2}, \tag{4}$$

where the parameter θ measures the degree of the adjustment cost. Finally, we assume that the firm can liquidate its assets at any time. The liquidation value L_t is proportional to the firm's capital, $L_t = lK_t$, where $l > 0$ is a constant.

Note that these classic AK production technology assumptions, plus the quadratic adjustment cost and the liquidation technology, imply that the firm's investment opportunities are constant over time. Without financing frictions, the firm's investment-capital ratio, average q and marginal q are therefore constant over time. We intentionally choose such a simple setting in order to highlight the dynamic effects of financing frictions, keeping investment opportunities constant. Moreover, these assumptions allow us to deliver the key results in a parsimonious and analytically tractable way.[13] See also Eberly, Rebelo, and Vincent (2009) for empirical evidence in support of the Hayashi homogeneity assumption for the upper-size quartile of Compustat firms.

B. Information, Incentives, and Financing Costs

Neoclassical investment models (Hayashi (1982)) assume that the firm faces frictionless capital markets and that the Modigliani and Miller (1958) theorem holds. In reality, however, firms often face important external financing costs due to asymmetric information and managerial incentive problems. Following the classic writings of Jensen and Meckling (1976), Leland and Pyle (1977), and Myers and Majluf (1984), a large empirical literature seeks to measure these costs. For example, Asquith and Mullins (1986) find that the average stock price reaction to the announcement of a common stock issue is -3% and the loss in equity value as a percentage of the size of the new equity issue is -31%. Calomiris and Himmelberg (1997) estimate the direct *transactions costs* that firms face when they issue equity. These costs are also substantial. In their sample the mean transactions costs, which include underwriting, management, legal, auditing, and registration fees as well as the firm's selling concession, are 9% of an issue for seasoned public offerings and 15.1% for initial public offerings.

We do not explicitly model information asymmetries and incentive problems. Rather, to be able to work with a model that can be calibrated, we directly model the costs arising from information and incentive problems in reduced form. Thus, in our model, we summarize the information, incentive, and transactions costs that a firm incurs whenever it chooses to issue external equity by a fixed cost Φ and a marginal cost γ. Together these costs imply that the firm will optimally tap equity markets only intermittently, and when doing so it raises funds in lumps, consistent with observed firm behavior.

To preserve the linear homogeneity of our model, we further assume that the firm's fixed cost of issuing external equity is proportional to capital stock K, so that $\Phi = \phi K$. In practice, external costs of financing scaled by firm size are likely to decrease with firm size. With this caveat in mind, we point out that there are conceptual, mathematical, and economic reasons for modeling these costs as proportional to firm size. First, by modeling the fixed financing costs proportional to firm size, we ensure that the firm does not grow out of the fixed costs.[14] Second, the information and

incentive costs of external financing may to some extent be proportional to firm size. Indeed, the negative announcement effect of a new equity issue affects the firm's entire capitalization. Similarly, the negative incentive effect of a more diluted ownership may also have costs that are proportional to firm size. Finally, this assumption allows us to keep the model tractable, and generates stationary dynamics for the firm's cash-capital ratio.

Having said that, a weakness of our model is that it will be misspecified as a structural model of firms' outside equity issue decisions. The model is likely to work best when applied to mature firms and worst when applied to start-ups and growth firms, as in reality small firms are not scaled-down versions of mature firms. Sharper quantitative predictions of the effects of external financing costs would require extending the model to a two-dimensional framework with both capital and cash as state variables. However, the main qualitative predictions of our current model are likely to be robust to this two-dimensional extension. In particular, the endogenous marginal value of cash will continue to play a critical role in determining corporate investment and other financial decisions.

We denote by H_t the firm's cumulative external financing up to time t and hence by dH_t the firm's incremental external financing over time interval $(t, t + dt)$. Similarly, let X_t denote the cumulative costs of external financing up to time t, and dX_t the incremental costs of raising incremental external funds dH_t. The cumulative external equity issuance H and the associated cumulative costs X are stochastic controls chosen by the firm. In the baseline model of this section, the only source of external financing is equity.

We next turn to the firm's cash inventory. Let W_t denote the firm's cash inventory at time t. In our baseline model with no debt, if the firm's cash is positive, the firm survives with probability one. However, if the firm runs out of cash ($W_t = 0$), it has to either raise external funds to continue operating, or liquidate its assets. If the firm chooses to raise external funds, it must pay the financing costs specified above. In some situations the firm may prefer liquidation, for example, when the cost of financing is too high or when the return on capital is too low. Let τ denote the firm's (stochastic) liquidation time. If $\tau = \infty$, then the firm never chooses to liquidate.

The rate of return that the firm earns on its cash inventory is the risk-free rate r minus a

carry cost $\lambda > 0$ that captures in a simple way the agency costs that may be associated with *free cash* in the firm.[15] Alternatively, the cost of carrying cash may arise from tax distortions. Cash retentions are tax disadvantaged as interest earned by the corporation on its cash holdings is taxed at the corporate tax rate, which generally exceeds the personal tax rate on interest income (Graham (2000) and Faulkender and Wang (2006)). The benefit of a payout is that shareholders can invest at the risk-free rate r, which is higher than $(r - \lambda)$, the net rate of return on cash within the firm. However, paying out cash also reduces the firm's cash balance, which potentially exposes the firm to current and future underinvestment and future external financing costs. The tradeoff between these two factors determines the optimal payout policy. We denote by U_t the firm's cumulative (nondecreasing) payout to shareholders up to time t, and by dU_t the incremental payout over time interval dt. Distributing cash to shareholders may take the form of a special dividend or a share repurchase.[16]

Combining cash flow from operations dY_t given in (3) with the firm's financing policy given by the cumulative payout process U and the cumulative external financing process H, the firm's cash inventory W evolves according to the following cash accumulation equation:

$$dW_t = dY_t + (r - \lambda)\, W_t dt + dH_t - dU_t, \tag{5}$$

where the second term is the interest income (net of the carry cost λ), the third term dH_t is the cash inflow from external financing, and the last term dU_t is the cash outflow to investors, so that $(dH_t - dU_t)$ is the net cash flow from financing. This equation is a general accounting identity, where dH_t, dU_t, and dY_t are endogenously determined by the firm.

The firm's financing opportunities are time-invariant in our model, which is not realistic. However, as we will show, even in this simple setting the interactions of fixed/proportional financing costs with real investment generate several novel and economically significant insights.

Firm optimality. The firm chooses its investment I, payout policy U, external financing policy H,

and liquidation time τ to maximize shareholder value defined below:

$$\mathbb{E}\left[\int_0^\tau e^{-rt}\left(dU_t - dH_t - dX_t\right) + e^{-r\tau}\left(lK_\tau + W_\tau\right)\right]. \tag{6}$$

The expectation is taken under the risk-adjusted probability. The first term is the discounted value of net payouts to shareholders and the second term is the discounted value from liquidation. Optimality may imply that the firm never liquidates. In that case, we have $\tau = \infty$. We impose the usual regularity conditions to ensure that the optimization problem is well posed. Our optimization problem is most obviously seen as characterizing the benchmark for the firm's efficient investment, cash inventory, dynamic hedging, payout, and external financing policy when the firm faces external financing and cash-carrying costs.

C. The Neoclassical Benchmark

As a benchmark, we summarize the solution for the special case without financing frictions, in which the Modigliani-Miller theorem holds. The firm's first-best investment policy is given by $I_t^{FB} = i^{FB}K_t$, where[17]

$$i^{FB} = r + \delta - \sqrt{(r+\delta)^2 - 2\left(\mu - (r+\delta)\right)/\theta}. \tag{7}$$

The value of the firm's capital stock is $q^{FB}K_t$, where q^{FB} is Tobin' s q,

$$q^{FB} = 1 + \theta i^{FB}. \tag{8}$$

Three observations are in order. First, due to the homogeneity property in production technology, marginal q is equal to average (Tobin's) q, as in Hayashi (1982). Second, gross investment I_t is positive if and only if the expected productivity μ is higher than $r + \delta$. With $\mu > r + \delta$ and hence positive investment, installed capital earns rents. Therefore, Tobin's q is greater than unity due to adjustment costs. Third, idiosyncratic productivity shocks have no effect on investment or

firm value. In the next section, we analyze the problem of a financially constrained firm.

II. Model Solution

When the firm faces costs of raising external funds, it can reduce future financing costs by retaining earnings (hoarding cash) to finance its future investments. Firm value then depends on two natural state variables, its stock of cash W and its capital stock K. Let $P(K, W)$ denote firm value. We show that firm decision-making and firm value depend on which of the following three regions it finds itself in: i) an external funding/liquidation region, ii) an internal financing region, and iii) a payout region. As will become clear below, the firm is in the external funding/liquidation region when its cash stock W is less than or equal to an endogenous *lower barrier* $\underline{W}$. It is in the payout region when its cash stock W is greater than or equal to an endogenous *upper barrier* $\overline{W}$. And it is in the internal financing region when W is in between $\underline{W}$ and $\overline{W}$. We first characterize the solution in the internal financing region.

A. Internal Financing Region

In this region, firm value $P(K, W)$ satisfies the following Hamilton-Jacobi-Bellman (HJB) equation:

$$rP(K,W) = \max_{I} \; (I - \delta K) P_K + [(r-\lambda)W + \mu K - I - G(I,K)]P_W + \frac{\sigma^2 K^2}{2} P_{WW}. \qquad (9)$$

The first term (the P_K term) on the right side of (9) represents the marginal effect of net investment $(I - \delta K)$ on firm value $P(K, W)$. The second term (the P_W term) represents the effect of the firm's expected savings on firm value, and the last term (the P_{WW} term) captures the effect of the volatility of cash holdings W on firm value.

The firm finances its investment out of the cash inventory in this region. The convexity of the physical adjustment cost implies that the investment decision in our model admits an interior

solution. The investment-capital ratio $i = I/K$ then satisfies the following first-order condition:

$$1 + \theta i = \frac{P_K(K, W)}{P_W(K, W)}. \tag{10}$$

With frictionless capital markets (the MM world) the marginal value of cash is $P_W = 1$, so that the neoclassical investment formula obtains: $P_K(K, W)$ is the marginal q, which at the optimum is equal to the marginal cost of adjusting the capital stock $1 + \theta i$. With costly external financing, on the other hand, equation (10) captures both real and financial frictions. The marginal cost of adjusting physical capital $(1 + \theta i)$ is now equal to the ratio of marginal q, $P_K(K, W)$, to the marginal cost of financing (or equivalently, the marginal value of cash), $P_W(K, W)$. Thus, the more costly the external financing (the higher P_W), the less the firm invests, ceteris paribus.

A key simplification in our setup is that the firm's two-state optimization problem can be reduced to a one-state problem by exploiting homogeneity. That is, we can write firm value as

$$P(K, W) = K \cdot p(w), \tag{11}$$

where $w = W/K$ is the firm's cash-capital ratio, and then reduce the firm's optimization problem to a one-state problem in w. The dynamics of w can be written as

$$dw_t = (r - \lambda)w_t dt - (i(w_t) + g(i(w_t))dt + (\mu dt + \sigma dZ_t) - w_t(i(w_t) - \delta)dt. \tag{12}$$

The first term on the right-hand side is the interest income net of cash-carrying costs. The second term is the total flow cost of (endogenous) investment (capital expenditures plus adjustment costs). While most of the time we have $i(w_t) > 0$, the firm may sometimes want to engage in asset sales (set $i(w_t) < 0$) in order to replenish its stock of cash and thus delay incurring external financing costs. The third term is the realized revenue per unit of capital (dA). Finally, the fourth term reflects the impact of changes in capital stock K_t on the cash-capital ratio. In accounting terms, this equation provides the link between the firm's income statement (source and use of funds) and

its balance sheet.

Instead of solving for firm value $P(K,W)$, we only need to solve for the firm's value-capital ratio $p(w)$. Note that marginal q is $P_K(K,W) = p(w) - wp'(w)$, the marginal value of cash is $P_W(K,W) = p'(w)$, and $P_{WW} = p''(w)/K$. Substituting these terms into (9) we obtain the following ordinary differential equation (ODE) for $p(w)$:

$$rp(w) = (i(w) - \delta)\left(p(w) - wp'(w)\right) + ((r-\lambda)w + \mu - i(w) - g(i(w)))\,p'(w) + \frac{\sigma^2}{2}p''(w). \quad (13)$$

We can also simplify the first-order condition (10) to obtain the following equation for the investment-capital ratio $i(w)$:

$$i(w) = \frac{1}{\theta}\left(\frac{p(w)}{p'(w)} - w - 1\right). \quad (14)$$

Using the solution $p(w)$ and substituting for this expression of $i(w)$ in (12), we obtain the equation for the firm's optimal accumulation of w.

To completely characterize the solution for $p(w)$, we must also determine the boundary $\underline{w}$ at which the firm raises new external funds (or closes down), the target cash-capital ratio after issuance (i.e., how much to raise), and the boundary $\overline{w}$ at which the firm pays out cash to shareholders.

B. Payout Region

Intuitively, when the cash-capital ratio is very high, the firm is better off paying out the excess cash to shareholders to avoid the cash-carrying cost. The natural question is how high the the cash-capital ratio needs to be before the firm pays out. Let $\overline{w}$ denote this endogenous payout boundary. Intuitively, if the firm starts with a large amount of cash ($w > \overline{w}$), then it is optimal for the firm to distribute the excess cash as a lump sum and bring the cash-capital ratio w down to $\overline{w}$. Moreover, firm value must be continuous before and after cash distribution. Therefore, for $w > \overline{w}$, we have the following equation for $p(w)$:

$$p(w) = p(\overline{w}) + (w - \overline{w}), \quad w > \overline{w}. \quad (15)$$

Since the above equation also holds for w close to $\overline{w}$, we may take the limit and obtain the following condition for the endogenous upper boundary $\overline{w}$:

$$p'(\overline{w}) = 1. \tag{16}$$

At $\overline{w}$ the firm is indifferent between distributing and retaining one dollar, so that the marginal value of cash must equal one, which is the marginal cost of cash to shareholders. Since the payout boundary $\overline{w}$ is optimally chosen, we also have the following "super contact" condition (see, for example, Dumas (1991)):

$$p''(\overline{w}) = 0. \tag{17}$$

C. External Funding/Liquidation Region

When the firm's cash-capital ratio w is less than or equal to the lower barrier $\underline{w}$, the firm either incurs financing costs to raise new funds or liquidates. Depending on parameter values, it may prefer either liquidation or refinancing by issuing new equity. Although the firm can choose to liquidate or raise external funds at any time, we show that it is optimal for the firm to wait until it runs out of cash, that is, $\underline{w} = 0$. The intuition is as follows. First, because investment incurs convex adjustment costs and the production is an efficient technology (in the absence of financing costs), the firm does not want to prematurely liquidate. Second, in the case of external financing, cash within the firm earns a below-market interest rate $(r - \lambda)$, while there is also time value for the external financing costs. Since investment is smooth (due to convex adjustment cost), the firm can always pay for any level of investment it desires with internal cash as long as $w > 0$. Thus, without any benefit for early issuance, it is always better to defer external financing as long as possible. The above argument highlights the robustness of the pecking order between cash and external financing in our model. With stochastic financing cost or stochastic arrival of growth options, the firm may time the market by raising cash in times when financing costs are low. See Bolton, Chen, and Wang (2011).

When expected productivity μ is low and/or the cost of financing is high, the firm will prefer liquidation to refinancing. In that case, because the optimal liquidation boundary is $\underline{w} = 0$, firm value upon liquidation is $p(0)K = lK$. We therefore have

$$p(0) = l. \tag{18}$$

If the firm's expected productivity μ is high and/or its cost of external financing is low, then it is better off raising costly external financing than liquidating its assets when it runs out of cash. To economize fixed issuance costs ($\phi > 0$), firms issue equity in lumps. With homogeneity, we can show that the total equity issue amount is mK, where $m > 0$ is endogenously determined as follows. First, firm value is continuous before and after equity issuance, which implies the following condition for $p(w)$ at the boundary $\underline{w} = 0$:

$$p(0) = p(m) - \phi - (1 + \gamma)\, m. \tag{19}$$

The right side represents the firm value-capital ratio $p(m)$ minus both the fixed and the proportional costs of equity issuance, per unit of capital. Second, since m is optimally chosen, the marginal value of the last dollar raised must equal one plus the marginal cost of external financing, $1 + \gamma$. This gives the following smooth pasting boundary condition at m:

$$p'(m) = 1 + \gamma. \tag{20}$$

D. Piecing the Three Regions Together

To summarize, for the liquidation case, the complete solution for the firm's value-capital ratio $p\,(w)$ and its optimal dynamic investment policy is given by: i) the HJB equation (13); ii) the investment-capital ratio equation (14); and iii) the liquidation condition (18) and payout boundary conditions (16) and (17).

Similarly, when it is optimal for the firm to refinance rather than liquidate, the complete solution for the firm's value-capital ratio $p(w)$ and its optimal dynamic investment and financing policy is given by: i) the HJB equation (13); ii) the investment-capital ratio equation (14); iii) the equity issuance boundary condition (19); iv) the optimality condition for equity issuance (20); and v) the endogenous payout boundary conditions (16) and (17). Finally, to verify that refinancing is indeed the firm's global optimal solution, it is sufficient to check that $p(0) > l$.

III. Quantitative Analysis

We now turn to quantitative analysis of the baseline model. For the benchmark case, we set the mean and volatility of the risk-adjusted productivity shock to $\mu = 18\%$ and $\sigma = 9\%$, respectively, which are in line with the estimates of Eberly, Rebelo, and Vincent (2009) for large U.S. firms. The risk-free rate is $r = 6\%$. The rate of depreciation is $\delta = 10.07\%$. These parameters are all annualized. The adjustment cost parameter is $\theta = 1.5$ (see Whited (1992)). The implied first-best q in the neoclassical model is $q^{FB} = 1.23$, and the corresponding first-best investment-capital ratio is $i^{FB} = 15.1\%$. We next set the cash-carrying cost parameter to $\lambda = 1\%$. The proportional financing cost is $\gamma = 6\%$ (see Altinkihc and Hansen (2000)) and the fixed cost of financing is $\phi = 1\%$, which jointly generate average equity financing costs that are consistent with the data. Finally, for the liquidation value we take $l = 0.9$ (as suggested in Hennessy and Whited (2007)). Table I summarizes all the key variables and parameters in the model.

Insert Table I About Here

Before analyzing the impact of costly external equity financing, we first consider the special case in which the firm is forced to liquidate when it runs out of cash. While this is an extreme form of financing constraint, it may be the relevant constraint in a financial crisis.

Case I: Liquidation. Figure 2 plots the solution in the liquidation case. In Panel A, the firm's value-capital ratio $p(w)$ starts at $l = 0.9$ (liquidation value) when its cash balance is equal to zero,

is concave in the region between zero and the endogenous payout boundary $\overline{w} = 0.22$, and becomes linear (with slope one) beyond the payout boundary ($w \geq \overline{w}$). In Section II, we show that the firm will never liquidate before its cash balance hits zero. Panel A of Figure 2 provides a graphic illustration of this result, where $p(w)$ lies above the liquidation value $l + w$ (normalized by capital) for all $w > 0$.

Insert Figure 2 About Here

Panel B of Figure 2 plots the marginal value of cash $p'(w) = P_W(K, W)$. The marginal value of cash increases as the firm becomes more constrained and liquidation becomes more likely. It also confirms that firm value is concave in the internal financing region ($p''(w) < 0$). The external financing constraint makes the firm hoard cash today in order to reduce the likelihood that it will be liquidated in the future, which effectively induces "risk aversion" for the firm. Consider the effect of a mean-preserving spread of cash holdings on the firm's investment policy. Intuitively, the marginal cost from a smaller cash holding is higher than the marginal benefit from a larger cash holding because the increase in the likelihood of liquidation outweighs the benefit from otherwise relaxing the firm's financing constraints. It is the concavity of the value function that gives rise to the demand for risk management. Also, note that the marginal value of cash reaches a value of 30 as w approaches zero. An extra dollar of cash is thus worth as much as \$30 to the firm in this region. This is because more cash helps keep the firm away from costly liquidation, which would permanently destroy the firm's future growth opportunities. Such high marginal value of cash highlights the importance of cash in periods of extreme financing frictions, which is what we have witnessed in the recent financial crisis.

Panel C plots the investment-capital ratio $i(w)$ and illustrates underinvestment due to the extreme external financing constraints. Optimal investment by a financially constrained firm is always lower than first-best ($i^{FB} = 15.1\%$), but especially when the firm's cash inventory w is low. When w is sufficiently low the firm will disinvest by selling assets to raise cash and move away from the liquidation boundary. Note that disinvestment is costly not only because the firm is

underinvesting but also because it incurs physical adjustment costs when lowering its capital stock. For the parameter values we use, asset sales (disinvestments) are at the annual rate of over 60% of the capital stock when w is close to zero! The firm tries very hard not to be forced into liquidation. Even at the payout boundary, the investment-capital ratio is only $i(\overline{w}) = 10.6\%$, about 30% lower than the first-best level i^{FB}. On the margin, the firm is trading off the cash-carrying costs with the cost of underinvestment. It will optimally choose to hoard more cash and invest more at the payout boundary when the cash-carrying cost λ is lower.

Finally, we consider a measure of investment-cash sensitivity given by $i'(w)$. Taking the derivative of investment-capital ratio $i(w)$ in (14) with respect to w, we get

$$i'(w) = -\frac{1}{\theta}\frac{p(w)p''(w)}{p'(w)^2} > 0. \tag{21}$$

The concavity of p ensures that $i'(w) > 0$ in the internal financing region, as shown in Panel D of Figure 2. Remarkably, the investment-cash sensitivity $i'(w)$ is not monotonic in w. In particular, when the cash holding is sufficiently low, $i'(w)$ actually increases with the cash-capital ratio. Formally, the slope of $i'(w)$ depends on the third derivative of $p(w)$, for which we do not have an analytical characterization.

Clearly, certain liquidation when the firm runs out of cash is an extreme form of financing constraint, which is why the marginal value of cash can be as high as \$30, and asset sales as high as an annual rate of 60%, when the firm runs out of cash. An important insight from this scenario, however, is that an extreme financing constraint causes the firm to hold more cash, defer payout, and cut investment aggressively even when its cash balances are relatively low. Remarkably, all of these actions help the firm stay away from states of extreme financing constraints most of the time, as our simulations below show.

Case II: Refinancing. Next we consider the setting in which the firm is allowed to issue equity. Figure 3 displays the solutions for both the case with fixed financing costs ($\phi = 1\%$) and the case without ($\phi = 0$). Observe that at the financing boundary $\underline{w} = 0$, the firm's value-capital ratio

$p(w)$ is strictly higher than l, so that external equity financing is preferred to liquidation under this model parameterization. Compared with the liquidation case, we find that the endogenous payout boundary (marked by the solid vertical line on the right) is $\overline{w} = 0.19$ when $\phi = 1\%$, lower than the payout boundary for the case of liquidation ($\overline{w} = 0.22$). Not surprisingly, firms are more willing to pay out cash when they can raise new funds in the future. The firm's optimal *return cash-capital ratio* is $m = 0.06$, marked by the vertical line on the left in Panel A. Without fixed costs ($\phi = 0$), the payout boundary drops further to $\overline{w} = 0.14$ and the firm's return cash-capital ratio is zero, as the firm raises just enough funds to keep w above zero.

Insert Figure 3 About Here

Figure 3 Panel B plots the marginal value of cash $p'(w)$, which is positive and decreasing between zero and $\overline{w}$, confirming that $p(w)$ is strictly concave in this case. Conditional on issuing equity and having paid the fixed financing cost, the firm optimally chooses the return cash-capital ratio m such that the marginal value of cash $p'(m)$ is equal to the marginal cost of financing $1 + \gamma$. To the left of the return cash-capital ratio m, the marginal value of cash $p'(w)$ lies above $1 + \gamma$, reflecting the fact that the fixed cost component in raising equity increases the marginal value of cash. When the firm runs out of cash, the marginal value of cash is around 1.7, much higher than $1 + \gamma = 1.06$. This result highlights the importance of fixed financing costs: even a moderate fixed cost can substantially raise the marginal value of cash in the low-cash region.

As in the liquidation case, the investment-capital ratio $i(w)$ is increasing in w and reaches the peak at the payout boundary $\overline{w}$, where $i(w) = 11\%$. Higher fixed cost of financing increases the severity of financing constraints, therefore leading to more underinvestment. This is particularly true in the region to the left of the return cash-capital ratio m, where the investment-capital ratio $i(w)$ drops rapidly. Asset sales go down quickly ($i'(w) > 10$) when w moves away from zero. This is because both asset sales and equity issuance are very costly. Removing the fixed financing costs greatly alleviates the underinvestment problem, where both the marginal value of cash and the investment-capital ratio become essentially flat except for very low w.

Average q, marginal q, and investment. We now turn to the model's predictions for average and marginal q. We take the firm's *enterprise value* – the value of all the firm's marketable claims minus cash, $P(K,W)-W$ – as our measure of the value of the firm's capital stock. Average q, denoted by $q_a(w)$, is then the firm's enterprise value divided by its capital stock:

$$q_a(w) = \frac{P(K,W)-W}{K} = p(w) - w. \tag{22}$$

First, average q increases with w. This is because the marginal value of cash is never below one, so that $q_a'(w) = p'(w) - 1 \geq 0$. Second, average q is concave provided that $p(w)$ is concave, in that $q_a''(w) = p''(w)$.

In our model where external financing is costly, marginal q, denoted by $q_m(w)$, is given by

$$q_m(w) = \frac{d\left(P(K,W)-W\right)}{dK} = p(w) - wp'(w) = q_a(w) - \left(p'(w)-1\right)w, \tag{23}$$

where $q_a(w) = p(w) - w$ (see equation (22)). An increase in capital stock K has two effects on the firm's enterprise value. First, the larger the capital stock, the higher the enterprise value. This is the standard average q channel, where $q_a(w) = p(w) - w$. Second, increasing capital stock mechanically lowers the cash-capital ratio $w = W/K$ for $W > 0$, thus making the firm more constrained. The wedge between marginal q and average q, $-\left(p'(w)-1\right)w$, reflects this effect of financing constraints on firm value. With $p'(w) > 1$ and $w > 0$, that wedge is negative, and marginal q is smaller than average q. Third, both marginal q and average q increase with w for $w > 0$, because $p(w)$ is strictly concave. Finally, as a special case, under MM, $p'(w) = 1$ and hence average q equals marginal q.

Figure 4 plots the average and marginal q for the liquidation case, the refinancing case with no fixed costs ($\phi = 0$), and the refinancing case with fixed costs ($\phi = 1\%$). The average and marginal q are below the first-best level, $q^{FB} = 1.23$, in all three cases, and they become lower as external financing becomes more costly.

Insert Figure 4 About Here

Stationary distributions of w, $p(w)$, $p'(w)$, $i(w)$, average q, and marginal q. We next investigate the stationary distributions for the key variables tied to optimal firm policies in the benchmark case with refinancing ($\phi = 1\%$).[18] We first simulate the cash-capital ratio under the physical probability measure. To do so, we calibrate the Sharpe ratio of the market portfolio $\eta = 0.3$, and assume that the correlation between the firm's technology shocks and the market return is $\rho = 0.8$. Then, the mean of the productivity shock under the physical probability is $\hat{\mu} = 0.20$. Figure 5 shows the distributions for the cash-capital ratio w, the value-capital ratio $p(w)$, the marginal value of cash $p'(w)$, and the investment-capital ratio $i(w)$. Since $p(w), p'(w)$, and $i(w)$ are all monotonic in this case, the densities for their stationary distributions are connected with that of w through (the inverse of) their derivatives.

Insert Figure 5 About Here

Strikingly, the cash holdings of a firm are relatively high most of the time, and hence the probability mass for $i(w)$ is concentrated at the highest values in the relevant support of w, while $p'(w)$ concentrates mostly around unity. Thus, the firm's optimal cash management policies appear to be effective at shielding itself most of the time from the states with the most severe financing constraints and underinvestment.

Table II reports the mean, median, standard deviation, skewness, and kurtosis for w, $i(w)$, $p'(w)$, $q_a(w)$, $q_m(w)$, and $i'(w)$. Not surprisingly, all these variables have skewness. The marginal value of cash and the investment-cash sensitivity are positively skewed, while the remaining variables have negative skewness. Interestingly, the kurtosis for $i(w)$, $p'(w)$, $q_a(w)$, and $i'(w)$ are especially large, in contrast to their small standard deviations. The *fat tail* from the distribution of cash holdings is dramatically magnified due to the highly nonlinear relation between these variables and w.

Insert Table II About Here

Empirical research on corporate cash inventory focuses mostly on firms' average holdings. As is apparent from Table II, average cash holdings provide an incomplete and even misleading picture

of firms' cash management, investment, and valuation. The same is true for empirical estimates of the marginal value of cash and investment-cash sensitivity, which are generally interpreted as capturing how financially constrained a firm is. Even though the median and the mean of $p'(w)$ are close to one (and those of $i'(w)$ close to zero), and even though both distributions have rather small standard deviations, the kurtosis is huge for both, indicating that the firm could become severely financially constrained.

The impact of these low probability yet severe financing constraint states is evident. The mean and median of $q_a(w)$ are 1.16, which is about 5% lower than $q^{FB} = 1.23$, the average q for a firm without external financing costs. Similarly, the mean and median of $i(w)$ is 0.104, which is about 31% lower than $i^{FB} = 0.151$, the investment-capital ratio for a firm without external financing costs. Therefore, simply looking at the first two moments for the marginal value of cash or investment-cash sensitivity provides a highly misleading description of firms' financing constraints. Firms endogenously respond to their financing constraints by adjusting their cash management and investment policies, which in turn reduces the time variation in investment, marginal value of cash, etc. However, the impact of financing constraints remains large on average.

The analysis above highlights that, by providing a more complete picture of firms' capital expenditures and cash holdings over the time series and in the cross-section, this model helps us better understand the empirical patterns of cash holdings. The model's ability to match the empirical distributions can be further improved if we allow for changing investment and financing opportunity sets and firm heterogeneity.

IV. Dynamic Hedging

In addition to cash inventory management, the firm can also reduce its cash flow risk through financial hedging (for example, using options or futures contracts). Consider, for example, the firm's hedging policy using market index futures.[19] Let F_t denote the futures price on the market

index at time t. Under the risk-adjusted probability, F_t evolves according to

$$dF_t = \sigma_m F_t dB_t, \tag{24}$$

where σ_m is the volatility of the aggregate market portfolio, and B_t is a standard Brownian motion that is partially correlated with firm productivity shocks driven by the Brownian motion Z_t, with correlation coefficient ρ.

Let ψ_t denote the hedge ratio, that is, the position in the market index futures (the notional amount) as a fraction of the firm's total cash W_t. Futures contracts often require that the investor hold cash in a margin account, which is costly. Let κ_t denote the fraction of the firm's total cash W_t held in the margin account ($0 \leq \kappa_t \leq 1$). In addition to the cash-carrying cost in the standard interest-bearing account, cash held in this margin account also incurs the additional flow cost ϵ per unit of cash. We assume that the firm's futures position (in absolute value) cannot exceed a constant multiple π of the amount of cash $\kappa_t W_t$ in the margin account.[20] That is, we require

$$|\psi_t W_t| \leq \pi \kappa_t W_t. \tag{25}$$

As the firm can costlessly reallocate cash between the margin account and its regular interest-bearing account at any time, the firm will optimally hold the minimum amount of cash necessary in the margin account. That is, provided that $\epsilon > 0$, optimality implies that the inequality (25) holds as an equality. When the firm takes a hedging position in the index future, its cash balance evolves as follows:

$$dW_t = K_t \left(\mu dA_t + \sigma dZ_t\right) - \left(I_t + G_t\right) dt + dH_t - dU_t + (r - \lambda) W_t dt - \epsilon \kappa_t W_t dt + \psi_t \sigma_m W_t dB_t. \tag{26}$$

Next, we investigate the special case in which there are no margin requirements for hedging.

A. Optimal Hedging with No Frictions

With no margin requirement ($\pi = \infty$), the firm carries all its cash in the regular interest-bearing account and is not constrained in the size of the index futures position ψ. Since firm value $P(K, W)$ is concave in W (i.e., $P_{WW} < 0$), the firm will completely eliminate its systematic risk exposure via dynamic hedging. The firm thus behaves in exactly the same way as the firm in our baseline model except that the firm is only subject to idiosyncratic risk with volatility $\sigma\sqrt{1-\rho^2}$. It is easy to show that the optimal hedge ratio ψ is constant in this case,

$$\psi^*(w) = -\frac{\rho\sigma}{w\sigma_m}. \tag{27}$$

Thus, for a firm with size K, the total hedge position is $|\psi W| = (\rho\sigma/\sigma_m)K$, which linearly increases with firm size K.

B. Optimal Hedging with Margin Requirements

Next, we consider the important effects of margin requirements and hedging costs. We show that financing constraints fundamentally alter hedging, investment/asset sales, and cash management. The firm's HJB equation now becomes

$$\begin{aligned} rP(K,W) = \max_{I,\psi,\kappa} \; & (I - \delta K)\, P_K(K,W) + ((r-\lambda)W + \mu K - I - G(I,K) - \epsilon\kappa W)\, P_W(K,W) \\ & + \frac{1}{2}\left(\sigma^2 K^2 + \psi^2\sigma_m^2 W^2 + 2\rho\sigma_m\sigma\psi W K\right) P_{WW}(K,W), \end{aligned} \tag{28}$$

subject to

$$\kappa = \min\left\{\frac{|\psi|}{\pi}, 1\right\}. \tag{29}$$

Equation (29) indicates that there are two candidate solutions for κ (the fraction of cash in the margin account): one interior solution and one corner solution. If the firm has sufficient cash, so that its hedging choice ψ is not constrained by its cash holding, the firm sets $\kappa = |\psi|/\pi$. This

choice of κ minimizes the cost of the hedging position subject to meeting the margin requirement. Otherwise, when the firm is short of cash, it sets $\kappa = 1$, thus putting all its cash in the margin account to take the maximum feasible hedging position: $|\psi| = \pi$.

The direction of hedging (long ($\psi > 0$) or short ($\psi < 0$)) is determined by the correlation between the firm's business risk and futures return. With $\rho > 0$, the firm will only consider taking a short position in the index futures as we have shown. If $\rho < 0$, the firm will only consider taking a long position. Without loss of generality, we focus on the case in which $\rho > 0$, so that $\psi < 0$.

We show that there are three endogenously determined regions for optimal hedging. First, consider the cash region with an interior solution for ψ (where the fraction of cash allocated to the margin account is given by $\kappa = -\psi/\pi < 1$). The first-order condition with respect to ψ is

$$\frac{\epsilon}{\pi} W P_W + \left(\sigma_m^2 \psi W^2 + \rho \sigma_m \sigma W K\right) P_{WW} = 0.$$

Using homogeneity, we may simplify the above equation and obtain

$$\psi^*(w) = \frac{1}{w}\left(\frac{-\rho\sigma}{\sigma_s} - \frac{\epsilon}{\pi}\frac{p'(w)}{p''(w)}\frac{1}{\sigma_s^2}\right). \tag{30}$$

Consider next the low-cash region. The benefit of hedging is high in this region ($p'(w)$ is high when w is small). The constraint $\kappa \leq 1$ is then binding, hence $\psi^*(w) = -\pi$ for $w \leq w_-$, where the endogenous cutoff point w_- is the unique value satisfying $\psi^*(w_-) = -\pi$ in (30). We refer to w_- as the maximum-hedging boundary.

Finally, when w is sufficiently high, the firm chooses not to hedge, as the net benefit of hedging approaches zero while the cost of hedging remains bounded away from zero. More precisely, we have $\psi^*(w) = 0$ for $w \geq w_+$, where the endogenous cutoff point w_+ is the unique solution of $\psi^*(w_+) = 0$ using equation (30). We refer to w_+ as the zero-hedging boundary.

We now provide quantitative analysis for the impact of hedging on the firm's decision rules and firm value. We choose the following parameter values: $\rho = 0.8$, $\sigma_m = 20\%$, $\pi = 5$ (corresponding

to 20% margin requirement), and $\epsilon = 0.5\%$. The remaining parameters are those for the baseline case. These parameters and the key variables for the hedging case are also summarized in Table I.

Figure 6 Panel A shows the hedging policy. We focus on the costly hedging case (depicted in solid lines). For sufficiently high cash balances ($w > w_+ = 0.11$), the firm chooses not to hedge at all because the marginal benefit of hedging is smaller than the marginal cost of hedging. The hedge ratio reaches the maximally allowed level, $\psi = -5$ in our example, for sufficiently low w ($w < w_- = 0.07$). This constant hedge ratio implies that the dollar amount of hedging goes to zero as w goes to zero, that is, when hedging is most valuable, the firm will be significantly constrained in its hedging capacity.[21] We also see that the firm's hedging position $\psi(w)$ increases with w in the middle region ($0.07 < w < 0.11$). Finally, we note that the firm optimally scales back its hedging position and as a result faces higher volatility when hedging becomes more costly.

Insert Figure 6 About Here

With hedging, the firm holds less cash, pays out to shareholders earlier, and raises less cash each time it issues equity. As shown in Figure 6 Panels B and D, as long as it is not too constrained, the firm also invests more and has a lower marginal value of cash (for the same cash-capital ratio) with hedging. Panel C shows that firm value also rises with hedging. The firm's cash management policies affect the dynamics of cash holdings, which ultimately determines the dynamics of hedge ratios. For instance, under costly hedging, the stationary distribution of cash holdings will be similar to Figure 5, which implies that the firm will not engage in financial hedging most of the time.

For sufficiently low cash, the firm may underinvest more when hedging becomes less costly (see Panel B). This might appear surprising, as Froot, Scharfstein, and Stein (1993) show that hedging mitigates underinvestment in a static model. However, in our dynamic model, hedging raises the firm's going concern value. At times of severe financing constraints, it is optimal to aggressively scale back investment in the short run in order to better manage risk and preserve firm value for the long run.

Specifically, as equation (14) shows, investment depends on the ratio $p(w)/p'(w)$. When w becomes sufficiently small, not only is $p(w)$ higher with hedging, but also the marginal value of cash $p'(w)$, as shown in Panel D. There are two reasons hedging *increases* $p'(w)$ for low w. First, as Panel A shows, the margin constraint is binding when w is sufficiently low. Thus, each extra dollar of cash can be put in the margin account, which helps move the firm closer to optimal (frictionless) hedging. Second, when the volatility of cash flow is reduced through hedging, an extra dollar of cash becomes more effective in helping the firm avoid issuing equity (especially when cash is low). When w is sufficiently small, the effect of hedging on the marginal value of cash can exceed its effect on firm value, which leads to lower investment. In summary, hedging can lead to more demand for precautionary savings, and hence more underinvestment for a severely financially constrained firm.

Many existing models of risk management focus on financial hedging while treating the operations of the firm as exogenous. Our analysis highlights a central theme that cash management, financial hedging, and asset sales are integral parts of dynamic risk management. It is therefore important to examine these different ways of managing risk in a unified framework.

How much value does hedging add to the firm? We answer this question by computing the net present value (NPV) of optimal hedging to the firm for the case with costly margin requirements. The NPV of hedging is calculated as follows. First, we compute the *cost of external financing* as the difference in Tobin's q under the first-best case and under Case II without hedging. Second, we compute the *loss in adjusted present value* (APV), which is the difference in Tobin's q under the first-best case and under Case II with costly margin. Then the difference between the costs of external financing and the loss in APV is simply the value created through hedging. On average, when measured relative to Tobin's q under hedging with a costly margin, the cost of external financing is about 6%, and the loss in APV is about 5%, so that the NPV of costly hedging is on the order of 1%, a significant creation of value to say the least for a purely financial operation.

V. Credit Line

Our baseline model of Section I can also be extended to allow the firm to access a credit line. This is an important extension to consider, as in practice many firms are able to secure such lines, and for these firms, access to a credit line is an important alternative source of liquidity.

We model the credit line as a source of funding the firm can draw on at any time it chooses up to a limit. We set the credit limit to a maximum fraction of the firm's capital stock, so that the firm can borrow up to cK, where $c > 0$ is a constant. The logic behind this assumption is that the firm must be able to post collateral to secure a credit line and the highest quality collateral does not exceed the fraction c of the firm's capital stock. We may thus interpret cK to be the firm's short-term debt capacity. For simplicity, we treat c as exogenous in this paper. It is straightforward to endogenize the credit line limit by assuming that banks charge a commitment fee on the unused part of the credit line. We investigate this case in the Internet Appendix. We also assume that the firm pays a constant spread α over the risk-free rate on the amount of credit it uses. Sufi (2009) shows that on average a firm pays 150 basis points over LIBOR on its credit lines, leading us to set $\alpha = 1.5\%$.

Since the firm pays a spread α over the risk-free rate to access credit, it will optimally avoid using its credit line before exhausting its internal funds (cash) to finance investment. As long as the interest rate spread α is not too high, credit line will be less expensive than external equity, so the firm also prefers to first draw on the line before tapping equity markets.[22] Our model thus generates a pecking order among internal funds, credit lines, and external equity financing.

When a credit line is the marginal source of financing ($w < 0$), $p(w)$ solves the following ODE:

$$rp(w) = (i(w) - \delta)\left(p(w) - wp'(w)\right) + ((r+\alpha)w + \mu - i(w) - g(i(w)))\, p'(w) + \frac{\sigma^2}{2} p''(w). \quad (31)$$

Next, when cash is the marginal source of financing ($w > 0$), as in the baseline model, $p(w)$ satisfies the ODE (13). As for the boundary conditions, when the firm exhausts its credit line before issuing

equity, the boundary conditions for the timing and amount of equity issuance are similar to those given in Section II. That is, we have $p(-c) = p(m) - \phi - (1+\gamma)(m+c)$ and $p'(m) = 1+\gamma$. At the payout boundary, the same conditions (16) and (17) hold here. Finally, $p(w)$ is continuous and smooth everywhere, including at $w = 0$, which gives two additional boundary conditions.

Figure 7 describes the effects of credit line. First, having access to a credit line increases $p(w)$ by lowering the cost of financing. Second, the firm hoards significantly less cash when it has access to a credit line: the payout boundary $\overline{w}$ drops from 0.19 to 0.08 when the credit line increases from $c = 0$ to $c = 20\%$. Third, with the credit line ($c = 20\%$), the firm raises $c + m = 0.1$ per unit of capital when it exhausts its credit line. Without a credit line ($c = 0$), the firm issues $m = 0.06$ per unit of capital when it runs out of cash. Thus, for our parameter choices, the firm with a credit line issues more equity. Fourth, with the credit line, the marginal value of cash at $w = 0$ is $p'(0) = 1.01$, which is substantially lower than $p'(0) = 1.69$ without credit line ($c = 0$).

Insert Figure 7 About Here

A credit line also substantially mitigates the firm's underinvestment problem (see Panel C of Figure 7). With the credit line ($c = 20\%$), the firm's investment-capital ratio when it runs out of cash ($w = 0$) is $i(0) = 11.7\%$, which is substantially higher than $i(0) = -21.4\%$ (indicating significant asset sales) when the firm is about to run out of cash and has no access to a line of credit. Even when the firm has exhausted its credit line ($w = -20\%$), it engages in much less costly asset sales: $i(-c) = -7.9\%$. The investment-cash sensitivity $i'(w)$ is also substantially lower when the firm has access to a credit line. For example, $i'(0)$ decreases from 11.8 to 0.27 when the firm's credit line is increased from zero to 0.2.

Next, we turn to the effect of liquidity (cash and credit) on average q, marginal q, and investment. The left panel of Figure 8 plots the firm's marginal q and average q for two otherwise identical firms: one with a credit line ($c = 20\%$) and the other without a credit line ($c = 0$). We see that average q still increases with the cash-capital ratio w in both the credit and the cash regions, as $q_a'(w) = p'(w) - 1 \geq 0$, where the inequality follows from the financing constraints ($p'(w) \geq 1$).

Insert Figure 8 About Here

Unlike average q, marginal q is no longer monotonic in w. Its relation with w now depends on whether the marginal source of financing is cash or credit. This can be seen from the following:

$$q'_m(w) = -p''(w)w.$$

Firm value is concave in both the cash and credit line regions ($p''(w) < 0$). Thus, marginal q increases in w in the cash region ($w > 0$) and decreases in w in the credit region ($w < 0$).

The different behavior of marginal q in these two regions is due to the opposite effects of increasing K on financial constraints. When the firm uses its internal cash to finance investment, increasing K lowers the cash-capital ratio w, making the firm more constrained. When the firm is on the borrowing margin, increasing K makes w less negative, hence lowering firm leverage and relaxing its financing constraint.

Recall that in the cash region, marginal q is always lower than average q as seen in (23). However, marginal q is higher than average q in the credit region ($w < 0$). When the debt capacity mechanism is sufficiently strong, marginal q in the credit region can be even larger than the first-best q.[23]

The above results have crucial implications for the relationship between q and investment. It is sometimes argued that when there are no fixed costs of investment, marginal q is a more accurate measure than average q for the firm's investment opportunities.[24] This is indeed true in the MM world, but it is not generally valid when firms face financing constraints, even in the absence of fixed costs for investment. The right panel of Figure 8 shows that while the investment-capital ratio $i(w)$ and average q increase with w in the credit region, marginal q decreases with w. To understand this result, first recall that the concavity of $p(w)$ implies that average q and investment move in the same direction with w in both regions. However, marginal q increases with $|w|$ for $w < 0$ due to the debt capacity mechanism we sketched out earlier. Therefore, average q rather than marginal q can be a more robust predictor of investment in the presence of credit lines.

VI. Conclusion

We propose a tractable and operational model that demonstrates how the presence of external financing costs influences a firm's optimal investment, financing, and risk management policies. The marginal value of liquidity, which is endogenous, plays a central role in these corporate decisions. We show that investment depends on the ratio of marginal q to the marginal value of liquidity; the optimal cash inventory policy takes the form of a double-barrier policy; and liquidity management and financial hedging are complementary risk management tools.

We see a few directions for related future research. The recent financial crisis suggests that market financing conditions can change abruptly for reasons orthogonal to the firm's own characteristics. Campello, Graham, and Harvey (2010a) and Campello et al. (2010b) find that the financially constrained firms planned deeper cuts in investment and spending, burned more cash, drew more credit from banks, and also engaged in more asset sales in the recent financial crisis. By enriching our model proposed in this paper to accommodate credit supply/external financing shocks, we can analyze and provide a more accurate description of the effect of a financial crisis on corporate financial policies. Bolton, Chen, and Wang (2011) extends our model to allow for time-varying investment and financing opportunities. This richer model produces predictions that are broadly in line with the empirical findings in the above papers. Additionally, our model links a firm's risk premium to cash,[25] which has been extended by Bolton, Chen, and Wang (2011) to study the endogenous link between cash holdings and equity returns in the cross-section of firms. Another significant omission in this paper is long-term debt. We leave the study of the optimal maturity structure of debt to future research.

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Table I
Summary of Key Variables and Parameters

This table summarizes the symbols for the key variables used in the model and the parameter values in the benchmark case. For each upper-case variable in the left column (except K, A, and F), we use its lower case to denote the ratio of this variable to capital. All the boundary variables are in terms of the cash-capital ratio w_t.

Variable	Symbol	Parameters	Symbol	Value
		Baseline model		
Capital stock	K	Risk-free rate	r	6%
Cash holding	W	Rate of depreciation	δ	10.07%
Investment	I	Risk-neutral mean productivity shock	μ	18%
Cumulative productivity shock	A	Volatility of productivity shock	σ	9%
Investment adjustment cost	G	Adjustment cost parameter	θ	1.5
Cumulative operating profit	Y	Proportional cash-carrying cost	λ	1%
Cumulative external financing	H	Capital liquidation value	l	0.9
Cumulative external financing cost	X	Proportional financing cost	γ	6%
Cumulative payout	U	Fixed financing cost	ϕ	1%
Firm value	P	Market Sharpe ratio	η	0.3
Average q	q_a	Correlation between market and firm	ρ	0.8
Marginal q	q_m			
Payout boundary	$\overline{w}$			
Financing boundary	$\underline{w}$			
Return cash-capital ratio	m			
		Hedging		
Hedge ratio	ψ	Margin requirement	π	5
Fraction of cash in margin account	κ	Flow cost in margin account	ϵ	0.5%
Futures price	F	Market volatility	σ_m	20%
Maximum-hedging boundary	w_-			
Zero-hedging boundary	w_+			
		Credit line		
		Credit line limit	c	20%
		Credit line spread over r	α	1.5%

Table II
Moments From the Stationary Distribution of the Refinancing Case

This table reports the population moments for the cash-capital ratio (w), the investment-capital ratio ($i(w)$), the marginal value of cash ($p'(w)$), average q ($q_a(w)$), and marginal q ($q_m(w)$) from the stationary distribution in the case with refinancing ($\phi = 1\%$).

	Cash capital ratio w	Investment capital ratio $i(w)$	Marginal value of cash $p'(w)$	Average q $q_a(w)$	Marginal q $q_m(w)$	Investment-cash sensitivity $i'(w)$
mean	0.159	0.104	1.006	1.164	1.163	0.169
median	0.169	0.108	1.001	1.164	1.164	0.064
std	0.034	0.013	0.018	0.001	0.001	0.376
skewness	-1.289	-6.866	9.333	-8.353	-3.853	8.468
kurtosis	4.364	76.026	146.824	106.580	22.949	122.694

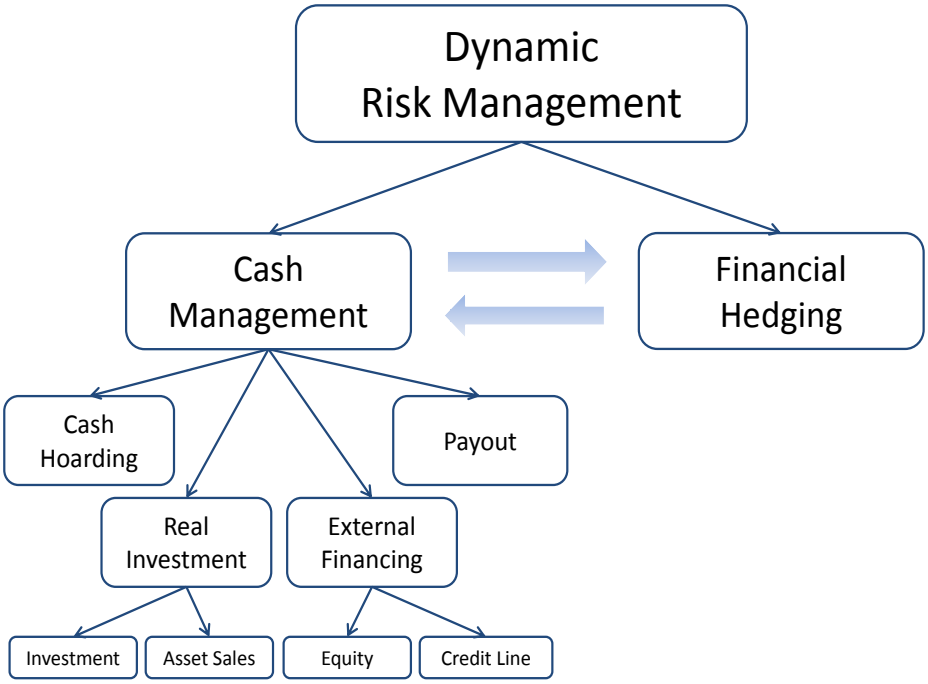

Figure 1. A unified framework for risk management.

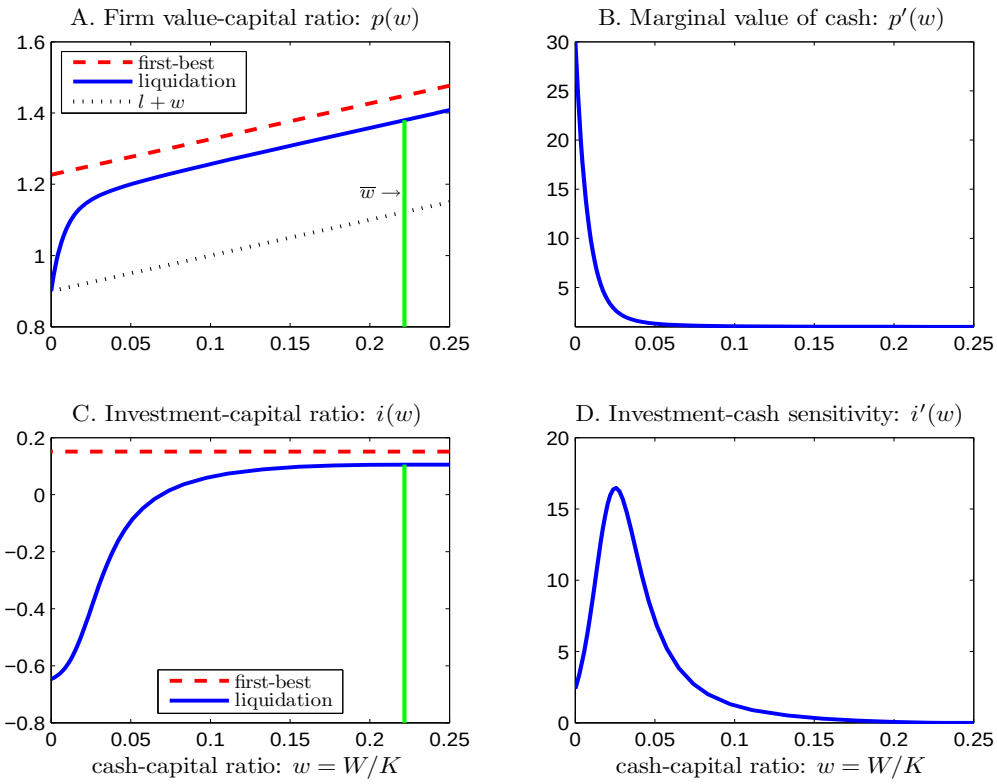

Figure 2. Case I – liquidation. This figure plots the solution for the case in which the firm has to liquidate when it runs out of cash ($w = 0$).

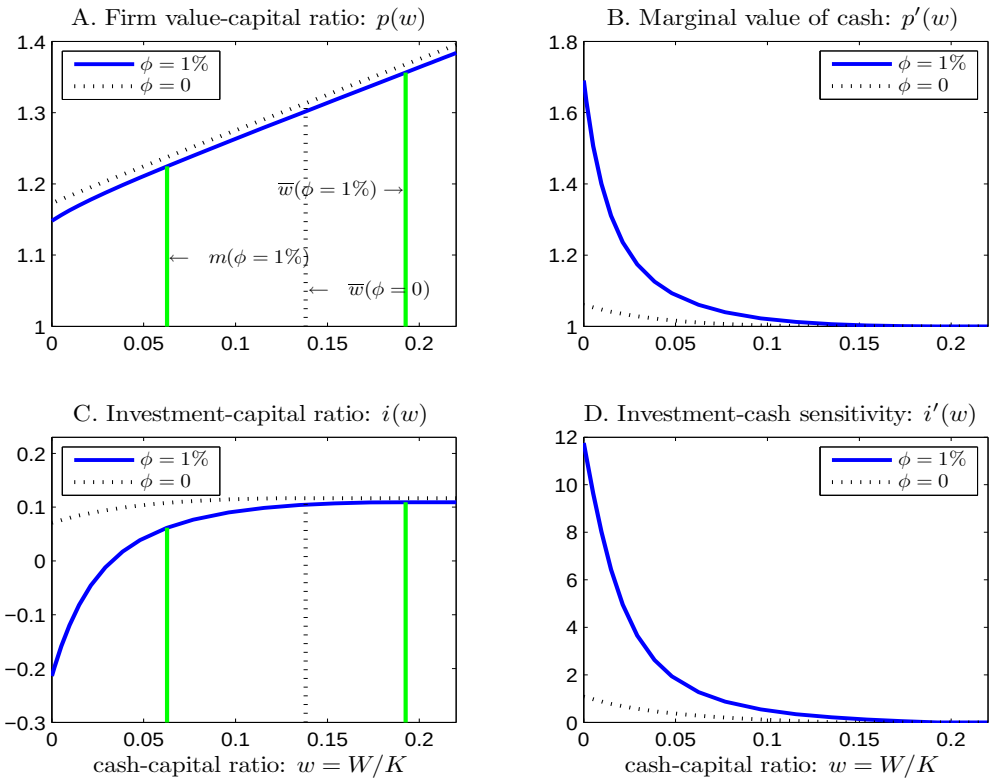

Figure 3. Case II – optimal refinancing. This figure plots the solution for the case of refinancing.

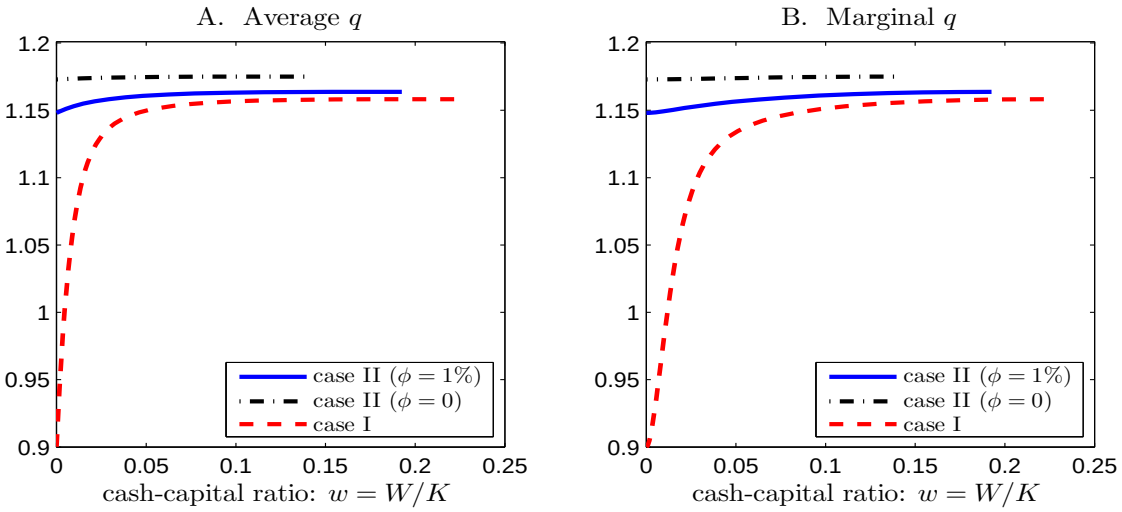

Figure 4. Average q and marginal q. This figure plots the average q and marginal q from the three special cases of the model. Case I is the liquidation case. Cases II and III are with external financing. The right end of each line corresponds to the respective payout boundary, beyond which both q_a and q_m are flat.

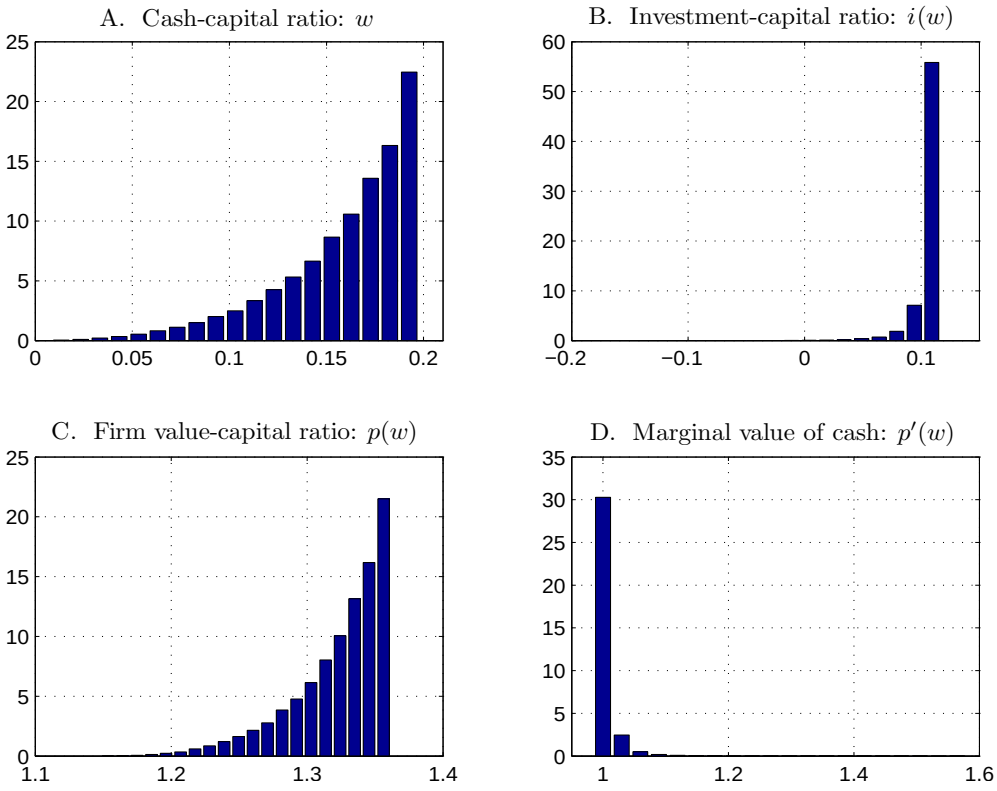

Figure 5. Stationary distributions in the case of refinancing. This figure plots the probability distribution functions (PDF) for the stationary distributions of cash holding, investment, firm value, and marginal value of cash for the case of refinancing with $\phi = 1\%$.

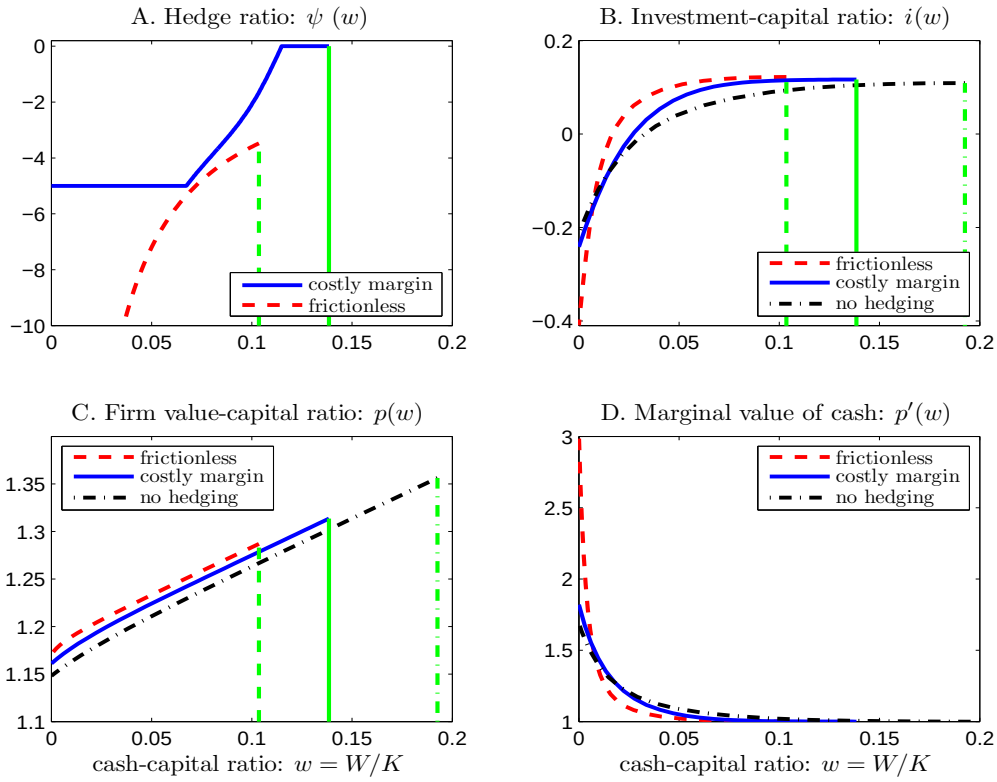

Figure 6. Optimal hedging. This figure plots the optimal hedging and investment policies, the firm value-capital ratio, and the marginal value of cash for Case II with hedging (with or without margin requirements). In Panel A, the hedge ratio for the frictionless case is cut off at -10 for display. The right end of each line corresponds to the respective payout boundary.

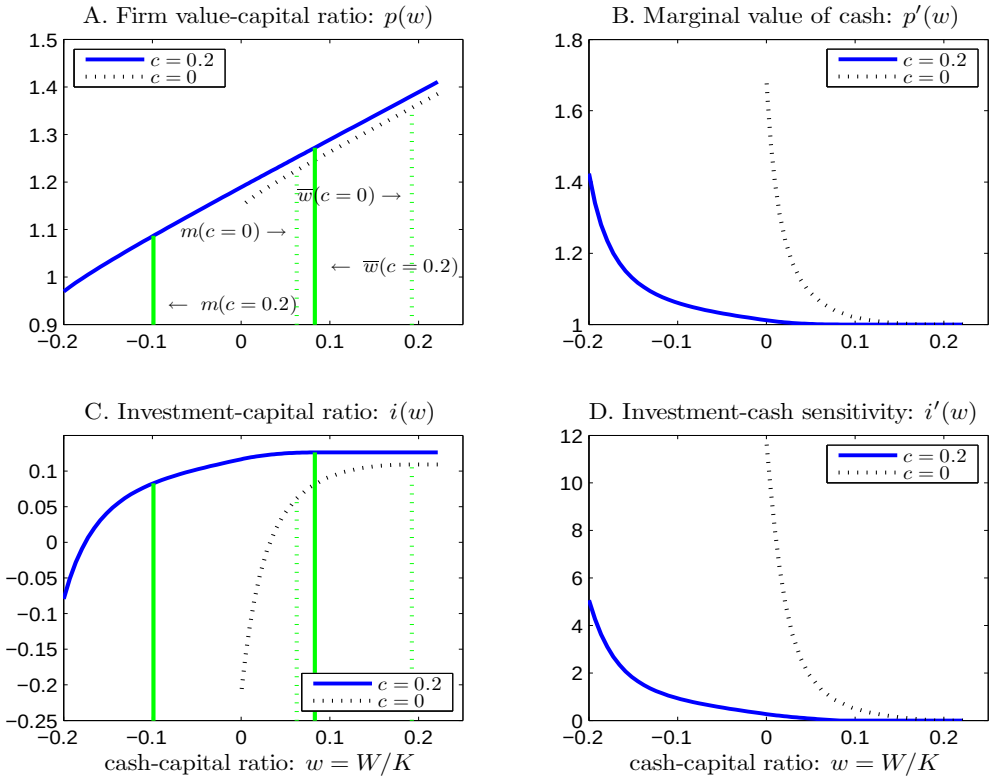

Figure 7. Credit line. This figure plots the model solution with credit lines and external equity. Each panel plots two scenarios: one without a credit line ($c = 0$) and the other with a credit line ($c = 20\%$). The spread on the credit line is $\alpha = 1.5\%$ over the risk-free rate r.

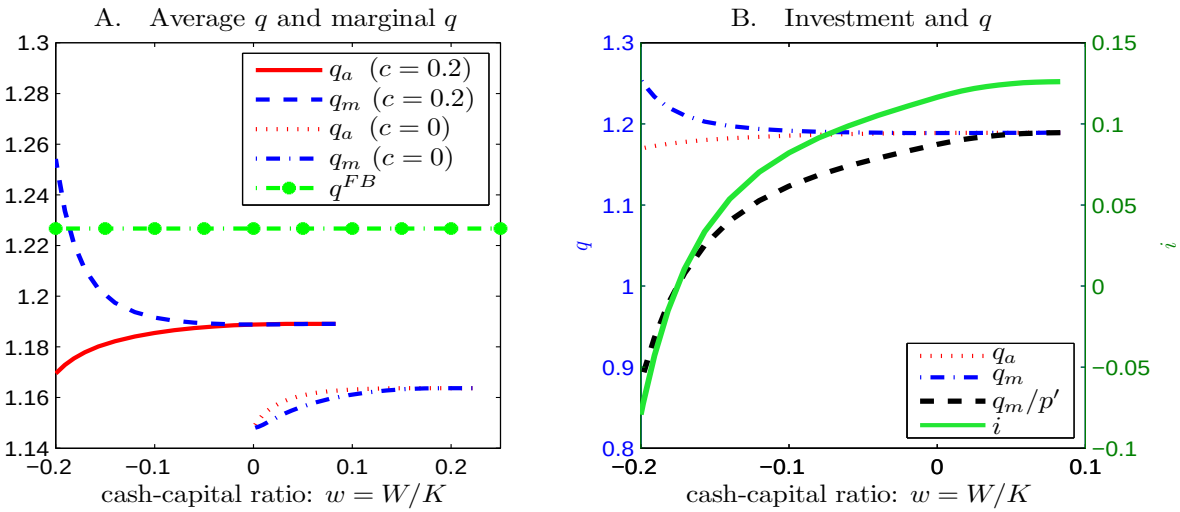

Figure 8. Investment and q with credit line. The left panel plots the average q (q_a) and marginal q (q_m) from the case with a credit line ($c = 0.2$) and without a credit line ($c = 0$) up to the respective payout boundaries. The right panel plots average q, marginal q, and the ratio of marginal q to the marginal value of liquidity (q_m/p') on the left axis, and the investment-capital ratio (i) on the right axis. These results are for the case with a credit line ($c = 0.2$).

Notes

[1]Brainard and Tobin (1968) and Tobin (1969) define the ratio between the firm's market value and the replacement cost of its capital stock as "Q" and propose that this ratio be used to measure the firm's incentive to invest in capital. This ratio has become known as Tobin's average Q. Hayashi (1982) provides conditions under which average Q is equal to marginal q. Abel and Eberly (1994) develop a unified q theory of investment in neoclassic settings. Lucas and Prescott (1971) and Abel (1983) are important early contributions.

[2]Hennessy, Levy, and Whited (2007) derive a related optimality condition for investment. As they assume that the firm faces quadratic equity issuance costs, the optimal policy for the firm in their setting is to either issue equity or pay out dividends at any point in time. In other words, the firm does not hold any cash inventory and does not face a cash management problem as in our setting.

[3]See Almeida, Campello, and Weisbach (2004, 2008), Faulkender and Wang (2006), Khurana, Martin, and Pereira (2006), and Dittmar and Mahrt-Smith (2007).

[4]Recent empirical studies find that corporations tend to hold more cash when their underlying earnings risk is higher or when they have higher growth opportunities (see, for example, Opler et al. (1999) and Bates, Kahle, and Stulz (2009)).

[5]This result is reminiscent of not prematurely exercising an American call option on a non-dividend-paying stock.

[6]See also Kim, Mauer, and Sherman (1998). Another more recent contribution by Almeida, Campello, and Weisbach (2008) extends the Hart and Moore (1994) theory of optimal cash holdings by introducing cash flow and investment uncertainty in a three-period model.

[7]Recently, Gamba and Triantis (2008) have extended Hennessy and Whited (2007) to introduce issuance costs of debt and hence obtain the simultaneous existence of debt and cash.

[8]In a related study, DeAngelo, DeAngelo, and Whited (2011) model debt as a transitory financing vehicle to meet the funding needs associated with random shocks to investment opportunities.

[9]See Caballero (1999) for a survey on this literature.

[10]The manager's continuation payoff gives the agent's present value of his future payments, discounted at his own rate. Interestingly, it can be interpreted as a measure of distance to liquidation/refinancing and can be linked to financial slack via (non-unique) financial implementation.

[11]We assume that markets are complete and are characterized by a stochastic discount factor Λ_t, which follows $\frac{d\Lambda_t}{\Lambda_t} = -rdt - \eta d\widehat{B}_t$, where $\widehat{B}_t$ is a standard Brownian motion under the physical measure $\mathbb{P}$ and η is the market price of risk (the Sharpe ratio of the market portfolio in the CAPM). Then, $B_t = \widehat{B}_t + \eta t$ will be a standard Brownian motion under the risk-neutral measure $\mathbb{Q}$. Finally, Z_t is a standard Brownian motion under $\mathbb{Q}$, and the correlation

between Z_t and B_t is ρ. The mean productivity shock under $\mathbb{P}$ is thus $\hat{\mu} = \mu + \eta\rho\sigma$.

[12]Cox, Ingersoll, and Ross (1985) develop an equilibrium production economy with the "AK" technology. See Jones and Manuelli (2005) for a recent survey.

[13]Nonconvex adjustment costs and a decreasing returns to scale production function would substantially complicate the analysis and do not permit a closed-form characterization of investment and financing policies.

[14]Indeed, this is a common assumption in the investment literature. See Cooper and Haltiwanger (2006) and Riddick and Whited (2009), among others. If the fixed cost is independent of firm size, it will not matter when firms become sufficiently large in the long run.

[15]This assumption is standard in models with cash. For example, see Kim, Mauer, and Sherman (1998) and Riddick and Whited (2009). If $\lambda = 0$, the firm will never pay out cash since keeping cash inside the firm incurs no costs but still has the benefits of relaxing financing constraints. If the firm is better at identifying investment opportunities than investors, we have $\lambda < 0$. In that case, raising funds to earn excess returns is potentially a positive NPV project. We do not explore cases in which $\lambda \leq 0$.

[16]A commitment to regular dividend payments is suboptimal in our model. We exclude any fixed or variable payout costs, which can be added to the analysis.

[17]For the first-best investment policy to be well defined, the following parameter restriction is required: $(r+\delta)^2 - 2\left(\mu - (r+\delta)\right)/\theta > 0$.

[18]We conduct additional analysis of the effects of various parameters on the distributions of cash and investment in the Internet Appendix, which is available on the Journal of Finance website at http://www.afajof.org/supplements.asp.

[19]In this analysis we only consider hedging of systematic risk by the firm. In practice, firms also hedge idiosyncratic risk by taking out insurance contracts. Our model can be extended to introduce insurance contracts, but this is beyond the scope of this article.

[20]For simplicity, we abstract from any variation in margin requirements, so that π is constant.

[21]Rampini and Viswanathan (2010) argue that financially constrained firms engage in less risk management due to collateral constraints. The margin requirement in our model leads to a similar prediction.

[22]When α is high and equity financing costs (ϕ, γ) are low, the firm may not exhaust its credit line before accessing external equity markets. For our parameter values, we find that the pecking order results apply between the credit line and external equity.

[23]This result does not appear in contracting models such as DFHW due to the manager's voluntary participation constraints.

[24]Caballero and Leahy (1996) show that average q can be a better proxy for investment opportunities in the presence of fixed costs of investment.

[25]We investigate how firm beta varies with cash holdings in the Internet Appendix.