Holographic Foam, Dark Energy and Infinite Statistics

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Abstract

Quantum fluctuations of spacetime give rise to quantum foam. Consistency with black hole physics then arguably dictates that the foam is of holographic type. Applied to cosmology, the holographic model requires the existence of unconventional (dark) energy/matter. We argue that dark energy is composed of an enormous number of inert “particles” of extremely long wavelength. Moreover, these particles necessarily obey infinite statistics in which all representations of the particle permutation group can occur. By our estimate, for every boson or fermion in the universe in the present cosmic era, there could be as many as $\sim 10^{31}$ “particles” obeying infinite statistics.

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I. INTRODUCTION

According to folklore, there are two kinds of statistics: Fermi-Dirac statistics for identical particles of half-integral spin and Bose-Einstein statistics for identical particles of integral spin. (There are also generalizations of these statistics known as para-Fermi and para-Bose statistics. But it is far less well-known that there is a third kind of particle statistics, known as infinite statistics, that is consistent with the general principles of quantum field theory. A collection of particles obeying infinite statistics can be in any representation of the particle permutation group: compare this with the rule that fermions (bosons) can only be in a totally antisymmetric (symmetric) state. While there are plenty of examples of fermions and bosons, there is no empirical evidence for particles of infinite statistics — until now perhaps. In this Letter, we will argue that actually infinite statistics should be the most familiar kind of statistics. For every fermion or boson, there could be as many as \(~10^{31}\) “particles” obeying infinite statistics nowadays in the observable universe.

We will apply our argument to the cosmology derived from the so-called holographic model of spacetime foam. The outline of this Letter is as follows. In the following sections, partly for completeness, we will first explain the logics behind the holographic quantum foam. Applied to cosmology, the holographic model predicts that the cosmic energy is of critical density, and the cosmic entropy is the maximum allowed by the holographic principle. We will also discuss the random-walk model which predicts a coarser spatial resolution in the mapping of spacetime geometry than the holographic model. Existing archived data on quasars from the Hubble Space Telescope can be used to rule out the random-walk model the demise of which, coupled with the fact (see below) that ordinary matter composed of fermions and/or bosons maps out spacetime only to the accuracy corresponding to the random-walk model, can be used to infer the existence of unconventional energy and/or matter (independent of recent cosmological observations). The main part of our argument is given in Section IV: there we will show that, in the framework of holographic foam cosmology, positivity of entropy requires the “particles” (or bits) constituting dark energy to obey infinite statistics. We give some concluding remarks in the final section.
II. HOLOGRAPHIC QUANTUM FOAM

Conceivably spacetime, like everything else, is subject to quantum fluctuations. As a result, spacetime is “foamy” at small scales, giving rise to a microscopic structure of spacetime known as quantum foam, also known as spacetime foam, and entailing an intrinsic limitation $\delta l$ to the accuracy with which one can measure a distance $l$. In principle, $\delta l$ can depend on both $l$ and the Planck length $l_P = \sqrt{\hbar G/c^3}$, the intrinsic scale in quantum gravity, and hence can be written as $\delta l \gtrsim l^{1-\alpha}l_P^\alpha$, with $\alpha \sim 1$ parametrizing the various spacetime foam models. (For related effects of quantum fluctuations of spacetime geometry, see Ref.[7].) In what follows, we will advocate the so-called holographic model corresponding to $\alpha = 2/3$, but we will also consider the (random walk) model with $\alpha = 1/2$ for comparison.

Let us first derive the holographic model[8, 9, 10] using an argument based on quantum computation[11, 12]. Since quantum fluctuations of spacetime manifest themselves in the form of uncertainties in the geometry of spacetime, the structure of spacetime foam can be inferred from the accuracy with which we can measure that geometry. Let us consider a spherical volume of radius $l$ over the amount of time $T = 2l/c$ it takes light to cross the volume. One way to map out the geometry of this spacetime region is to fill the space with clocks, exchanging signals with other clocks and measuring the signals’ times of arrival. This process of mapping the geometry is a sort of computation; hence the total number of operations (the ticking of the clocks and the measurement of signals etc) is bounded by the Margolus-Levitin theorem[13] in quantum computation, which stipulates that the rate of operations for any computer cannot exceed the amount of energy $E$ that is available for computation divided by $\pi\hbar/2$. A total mass $M$ of clocks then yields, via the Margolus-Levitin theorem, the bound on the total number of operations given by $(2Mc^2/\pi\hbar) \times 2l/c$. But to prevent black hole formation, $M$ must be less than $lc^2/2G$. Together, these two limits imply that the total number of operations that can occur in a spatial volume of radius $l$ for a time period $2l/c$ is no greater than $\sim (l/l_P)^2$. (Here and henceforth we neglect multiplicative constants of order unity, set $c = 1 = \hbar$ and will also set the Boltzmann constant equal to 1.) To maximize spatial resolution, each clock must tick only once during the entire time period. And if we regard the operations partitioning the spacetime volume into ”cells”, then on the average each cell occupies a spatial volume no less than $\sim l^3/(l^2/l_P^2) = ll_P^2$, yielding an average separation between neighboring cells no less than $l^{1/3}l_P^{2/3}$. This spatial separation
is interpreted as the average minimum uncertainty in the measurement of a distance \( l \), that is, \( \delta l \gtrsim l^{1/3}l_P^{2/3} \).

Parenthetically we can now understand why this quantum foam model has come to be known as the holographic model. Since, on the average, each cell occupies a spatial volume of \( l_P^3 \), a spatial region of size \( l \) can contain no more than \( l^3/(l_P^3) = (l/l_P)^2 \) cells. Thus this model corresponds to the case of maximum number of bits of information \( l^2/l_P^2 \) in a spatial region of size \( l \), that is allowed by the holographic principle, according to which, the maximum amount of information stored in a region of space scales as the area of its two-dimensional surface, like a hologram.

It will prove to be useful to compare the holographic model in the mapping of the geometry of spacetime with the one that corresponds to spreading the spacetime cells uniformly in both space and time. For the latter case, each cell has the size of \( (l_P^2)^{1/4} = l^{1/2}l_P^{1/2} \) both spatially and temporally, i.e., each clock ticks once in the time it takes to communicate with a neighboring clock. Since the dependence on \( l^{1/2} \) is the hallmark of a random-walk fluctuation, this quantum foam model corresponding to \( \delta l \gtrsim (l_P)^{1/2} \) is called the random-walk model. Compared to the holographic model, the random-walk model predicts a coarser spatial resolution, i.e., a larger distance fluctuation, in the mapping of spacetime geometry. It also yields a smaller bound on the information content in a spatial region, viz.,

\[
(\frac{l}{l_P})^2/(\frac{l}{l_P})^{1/2} = (\frac{l^2}{l_P^3})^{3/4} = (\frac{l}{l_P})^{3/2}.
\]

One remark is in order. The minimum \( \delta l \) just found for the holographic model corresponds to the case of maximum energy density \( \rho \sim (l/l_P)^{-2} \) for the region not to collapse into a black hole, i.e., the holographic model, in contrast to the random-walk model and other models, requires, for its consistency, the critical energy density which, in the cosmological setting, is \( (H/l_P)^2 \) with \( H \) being the Hubble parameter.

III. UNCONVENTIONAL (DARK) ENERGY/MATTER

The Planck length \( l_P \sim 10^{-33} \text{ cm} \) is so short that we need an astronomical (even cosmological) distance \( l \) for its fluctuation \( \delta l \) to be detectable. Let us consider light (with wavelength \( \lambda \)) from distant quasars or bright active galactic nuclei. Due to quantum fluctuations of spacetime, the wavefront, while planar, is itself “foamy”, having random fluctuations in phase \( \Delta \phi \sim 2\pi \delta l/\lambda \) as well as the direction of the wave vector given
by $\Delta \phi/2\pi$. In effect, spacetime foam creates a “seeing disk” whose angular diameter is $\sim \Delta \phi/2\pi$. For an interferometer with baseline length $D$, this means that dispersion will be seen as a spread in the angular size of a distant point source, causing a reduction in the fringe visibility when $\Delta \phi/2\pi \sim \lambda/D$.

Now we can use existing archived high-resolution data on quasars or ultra-bright active galactic nuclei from the Hubble Space Telescope to test the quantum foam models. Consider the case of PKS1413+135, an AGN for which the redshift is $z = 0.2467$. With $l \approx 1.2$ Gpc and $\lambda = 1.6 \mu$m, we find $\Delta \phi \sim 10 \times 2\pi$ and $10^{-9} \times 2\pi$ for the random-walk model and the holographic model of spacetime foam respectively. With $D = 2.4$ m for HST, we expect to detect halos if $\Delta \phi \sim 10^{-6} \times 2\pi$. Thus, the HST image only fails to test the holographic model by approximately 3 orders of magnitude.

However, the absence of a spacetime foam induced halo structure in the HST image of PKS1413+135 rules out convincingly the random-walk model. (In fact, the scaling relation discussed above indicates that all spacetime foam models with $\alpha \lesssim 0.6$ are ruled out by this HST observation.) This result has profound implications for cosmology. To wit, from the observed cosmic critical density in the present era (consistent with the prediction of the cosmology inspired by the holographic model of quantum foam) we deduce that $\rho \sim H_0^2/G \sim (R_H l_P)^{-2}$, where $H_0$ and $R_H$ are the present Hubble parameter and Hubble radius of the observable universe respectively. Treating the whole universe as a computer, one can apply the Margolus-Levitin theorem to conclude that the universe computes at a rate $\nu$ up to $\rho R_H^3 l_P^2$ for a total of $(R_H/l_P)^2$ operations during its lifetime so far. If all the information of this huge computer is stored in ordinary matter, then we can apply standard methods of statistical mechanics to find that the total number $I$ of bits is $(R_H^2/l_P^2)^{3/4} = (R_H l_P)^{3/2} \sim 10^{92}$. It follows that each bit flips once in the amount of time given by $I/\nu \sim (R_H l_P)^{1/2}$. On the other hand, the average separation of neighboring bits is $(R_H^3/l)^{1/3} \sim (R_H l_P)^{1/2}$. Hence, assuming that only ordinary matter exists to store all the information in the universe results in the conclusion that the time to communicate with neighboring bits is equal to the time for each bit to flip once. It follows that the accuracy to which ordinary matter maps out the geometry of spacetime corresponds exactly to the case of events spread out uniformly in space and time discussed above for the case of the random-walk model of spacetime foam. Succinctly, ordinary matter only contains an amount of information dense enough to map out spacetime at a level consistent with the random-
walk model. Observationally ruling out the random-walk model suggests that there must be other kinds of matter/energy with which the universe can map out its spacetime geometry to a finer spatial accuracy than is possible with the use of conventional matter. This line of reasoning then strongly hints at the existence of dark energy/matter, independent of the evidence from recent cosmological observations.

Moreover, the fact that our universe is observed to be at or very close to its critical energy density $\rho \sim (H/l_P)^2 \sim (R_H/l_P)^{-2}$ must be taken as solid albeit indirect evidence in favor of the holographic model because, as aforementioned, this model is the only model that requires the energy density to be critical. The holographic model also predicts a huge number of degrees of freedom for the universe in the present era, with the cosmic entropy given by $21 I \sim HR_H^3/l_P^3 \sim (R_H/l_P)^2 \sim 10^{123}$. Hence the average energy carried by each bit or “particle” is $\rho R_H^3/I \sim R_H^{-1}$. We also note that, with $(R_H/l_P)^2$ bits having performed $(R_H/l_P)^2$ operations, the overwhelming majority of the bits have had time to flip only about one time over the course of cosmic history. In other words, each “particle” has had only about one interaction.

IV. INFINITE STATISTICS

Now that we have concluded that there must be new kinds of energy/matter (other than ordinary matter), the crucial question is: what kinds of matter/energy can contain an amount of information dense enough to map out spacetime at a level consistent with the holographic model? To find that out, let us first consider a perfect gas of $N$ particles obeying Boltzmann statistics (which, rigorously speaking, is not a physical statistics but is still a useful statistics to work with) at temperature $T$ in a volume $V$. For the problem at hand, we take $V \sim R_H^3$, the size of the observable universe, $T \sim R_H^{-1}$, and very roughly $N \sim (R_H/l_P)^2$. We will also assume each particle has an effective mass $m \sim R_H^{-1}$ (coming from some sort of potential with which we are not going to concern ourselves). A standard calculation (using non-relativistic energy for each state) yields the partition function $Z_N = (N!)^{-1}(V/\lambda)^N$, where $\lambda = (2\pi/mT)^{1/2}$ is the thermal wavelength. With the free energy given by $F = -TlnZ_N = -NT[ln(V/N\lambda^3) + 1]$, we get, for the entropy of the system,

$$S = -(\partial F/\partial T)_{V,N} = N[ln(V/N\lambda^3) + 5/2]. \quad (1)$$
Note that, for the relativistic case with \( m = 0 \), the only changes in the above expressions are given by the substitution \( \lambda \rightarrow (\pi)^{2/3}/T \). With \( m \sim T \sim R_H^{-1} \), there is no significant qualitative difference between the non-relativistic and relativistic cases. Since \( V \sim \lambda^3 \), the entropy \( S \) in Eq. (1) becomes nonsensically negative unless \( N \sim 1 \) which is equally nonsensical since \( N \) should not be too different from \( (R_H/l_P)^2 \gg 1 \).

Intentionally we have calculated the entropy by applying the familiar Boltzmann statistics (with the correct Boltzmann counting factor), only to arrive at a contradictory result. But now the solution to this contradiction is pretty obvious: the \( N \) inside the log in Eq. (1) somehow must be absent. Then \( S \sim N \sim (R_H/l_P)^2 \) without \( N \) being small (of order 1) and \( S \) is non-negative as physically required. That would be the case if the “particles” were distinguishable and nonidentical! For in that case the Gibbs \( 1/N! \) factor is absent from the partition function \( Z_N \) and the entropy becomes

\[
S = N[\ln(V/\lambda^3) + 3/2].
\]  

We can add that, with or without the Gibbs factor, the internal energy is given by \( U = F + TS = (3/2)NT \).

Now the only known consistent statistics in greater than two space dimensions without the Gibbs factor \([23]\) is infinite statistics (sometimes called “quantum Boltzmann statistics”) \([2, 3, 4]\). Thus we have shown that the “particles” constituting dark energy obey infinite statistics. What is infinite statistics? Succinctly, a Fock realization of infinite statistics is provided by a \( q \) deformation of the commutation relations of the oscillators: \( a_k a_l^\dagger - qa_l a_k^\dagger = \delta_{kl} \) with \( q \) between -1 and 1 (the case \( q = \pm 1 \) corresponds to bosons or fermions). States are built by acting on a vacuum which is annihilated by \( a_k \). Two states obtained by acting with the \( N \) oscillators in different orders are orthogonal. It follows that the states may be in any representation of the permutation group. The statistical mechanics of particles obeying infinite statistics can be obtained in a way similar to the case of Boltzmann statistics, with the crucial difference that the Gibbs \( 1/N! \) factor is absent for the former. Infinite statistics can be thought of as corresponding to the statistics of identical particles with an infinite number of internal degrees of freedom, which is equivalent to the statistics of nonidentical particles since they are distinguishable by their internal states.

Infinite statistics appears to have one “defect”: a theory of particles obeying infinite statistics cannot be local \([4, 24]\). The expressions for the number operator, Hamiltonian,
etc., are both nonlocal and nonpolynomial in the field operators. The lack of locality may make it difficult to formulate a relativistic version of the theory; but it appears that a non-relativistic theory can be developed. Lacking locality also means that the familiar spin-statistics relation is no longer valid for particles obeying infinite statistics; hence they can have any spin. Thankfully, the TCP theorem and cluster decomposition have been shown to hold despite the lack of locality. 

Actually the lack of locality for theories of infinite statistics may have a silver lining. According to the holographic principle, the number of degrees of freedom in a region of space is bounded not by the volume but by the surrounding surface. This suggests that the physical degrees of freedom are not independent but, considered at the Planck scale, they must be infinitely correlated, with the result that the spacetime location of an event may lose its invariant significance. Since the holographic principle is believed to be an important ingredient in the formulation of quantum gravity, the lack of locality for theories of infinite statistics may not be a defect; it can actually be a virtue. Perhaps it is this lack of locality that makes it possible to incorporate gravitational interactions in the theory. Quantum gravity and infinite statistics appear to fit together nicely. This may be the reason why (charged, extremal) black holes appear to obey infinite statistics. Indirectly this may also explain why the holographic foam model has use for infinite statistics as we have just shown.

V. CONCLUSION

We have considered a perfect gas of “particles” of extremely long wavelength, obeying Boltzmann statistics (first in the conventional, then in the quantum version) in the Universe at temperature $T$. But we have seen that these “particles” have had interactions only of order one time on the average during the entire cosmic history. A question can be raised as to whether such an inert gas can come to thermal equilibrium at any well defined temperature. We do not have a good answer; but the fact that all these “particles”, though extremely inert, have a wavelength comparable to the observable Hubble radius may mean that they overlap significantly, and accordingly can perhaps share a (more or less) common temperature.

Another question concerns the sign of the pressure for this gas of “particles” and whether it is sufficiently negative to accelerate the expansion of the present Universe as has been
observed. Indeed the pressure for such a gas can be easily shown to be $P = (2/3)U/V$ and is blatantly positive. But this calculation is based on the simplifying assumption that the gas is perfect. Such a treatment may be sufficient for estimating the entropy, but is obviously inadequate to give the correct pressure. After all, as shown above, each “particle” has an energy comparable to $R^{-1}$. Such long-wavelength bits or “particles” carry negligible kinetic energy. Since pressure (energy density) is given by kinetic energy minus (plus) potential energy, a negligible kinetic energy means that the pressure of the unconventional energy is roughly equal to minus its energy density, leading to accelerating cosmic expansion. This scenario is very similar to that of quintessence, but it has its origin in local small scale physics – specifically, the holographic quantum foam!

Finally, is there any useful phenomenology that we can predict or use to explicitly check whether dark energy (and perhaps even dark matter) is composed of particles obeying infinite statistics? Since all those “particles” are so inert, we do not foresee any useful desktop experiments forthcoming soon that can shed light on the phenomenology of dark energy, a safer bet would be on cosmological observations (e.g., in connection with the scale-invariance of density fluctuations [26]). Further study is warranted.

In summary, we have suggested, in the framework of holographic foam cosmology, that dark energy is composed of $\sim 10^{123}$ extremely cold, inert, and long-wavelength “particles” obeying (necessarily) infinite statistics. By a staggering factor of $\sim 10^{123-92} = 10^{31}$, these “particles” far outnumber particles of the familiar Bose and Fermi statistics that we are all made of. Indeed we may be quite insignificant in the cosmic grand scheme. This is a most humbling realization.

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[14] Note that this result is not inconsistent with that found in X. Calmet, M. Graesser, and S. D. Hsu, Phys. Rev. Lett. 93, 211101 (2004).


[23] Recall that the Fermi statistics and Bose statistics give similar results as the conventional Boltzmann statistics at high temperature.
