Reply to: “Comment to: ‘Corrections to the fine structure constant in the spacetime of a cosmic string from the generalized uncertainty principle’ ”

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Abstract


In a recent Letter [1], we calculated the Bohr radius in the spacetime of a cosmic string. Our calculations to obtain the Bohr radius in the spacetime of a cosmic string were based on the quantization of the angular momentum in units of $\hbar$, i.e. $L_n = n\hbar$.

Based on G. de A. Marques’ comment in Ref.[2], the quantization of $L_n = n\hbar$ is correct only for a flat spacetime. In other words, in the presence of a cosmic string the quantization of the angular momentum must be considered as $L_{n(b)} = \frac{b}{2}\hbar$ where $b = 1 - \frac{4G\mu}{c^2}$ (see Ref. [2]). As pointed out in Eq.(7) of Ref.[2], in the weak field approximation we have

$$\frac{a_B}{\hat{a}_B} \approx 1 - \frac{G\mu}{c^2} \left( 8 + \frac{\pi}{4} \right), \quad (1)$$

where $a_B$ is the Bohr radius in the absence of a cosmic string and $\hat{a}_B$ is the Bohr radius in the presence of a cosmic string.

The purpose of this Reply is to correct calculations of Ref.[1] in accordance with the comment presented in Ref.[2]. Inserting Eq.(1) in the effective Planck constant $\hat{h}_{\text{eff}}$ in the spacetime of a cosmic string, as given in Eq.(30) of Ref.[1], one can easily obtain

$$\hat{h}_{\text{eff}} \approx \hbar \left[ 1 + \hat{\beta}^2 \left( \frac{me^2}{4\pi\epsilon_0 M^2_P G} \right)^2 \right], \quad (2)$$

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where we use

$$L_P = \frac{m e^2}{a_B} \left[ 1 - \frac{G\mu}{c^2 e^2} \left( \frac{8 + \pi}{4} \right) \right] \ll 1. \tag{3}$$

Eq.(3) is the corrected form of Eq.(31) in Ref.[1]. Substituting Eq.(2) of this Reply in Eq.(34) of Ref.[1], we are led to

$$\hat{\alpha}_{\text{eff}} \simeq \left( \frac{e^2}{4\pi \epsilon_0 \hbar c} - \frac{\pi G\mu}{4} \frac{e^2}{c^2 4\pi \epsilon_0 \hbar c} \right) \times \left[ 1 - \hat{\beta}^2 \left( \frac{m e^2}{4\pi \epsilon_0 M_P^2 G} \left[ 1 - \frac{G\mu}{c^2 e^2} \left( \frac{8 + \pi}{4} \right) \right] \right)^2 \right]. \tag{4}$$

This equation is the corrected form of Eq.(35) in Ref.[1]. One can rewrite Eq.(4) as

$$\hat{\alpha}_{\text{eff}} \simeq \left( \frac{e^2}{4\pi \epsilon_0 \hbar c} - \frac{\pi G\mu}{4} \frac{e^2}{c^2 4\pi \epsilon_0 \hbar c} \right) \times \left[ 1 - \hat{\beta}^2 \times 9.30 \times 10^{-50} \left[ 1 - 2 \times \left( \frac{8 + \pi}{4} \right) \times 10^{-6} \right] \right], \tag{5}$$

where we use $\left[ 1 - \left( \frac{8 + \pi}{4} \right) \frac{G\mu}{c^2 e^2} \right]^2 \simeq \left[ 1 - 2 \times \left( \frac{8 + \pi}{4} \right) \frac{G\mu}{c^2 e^2} \right]$ and $\frac{G\mu}{c^2 e^2} \sim 10^{-6}$.

It must be emphasized that Eq.(5) of this Reply is the corrected form of Eq.(36) in Ref.[1]. As given in Eq.(33) of Ref.[1], the fine structure constant $\hat{\alpha}$ in the spacetime of a cosmic string is defined by $\hat{\alpha} := \alpha \left( 1 - \frac{\pi G\mu}{4e^2} \right)$. Using Eq.(33) of Ref.[1] and Eq.(5) of this Reply, we obtain

$$\hat{\alpha}_{\text{eff}} \simeq \hat{\alpha} \left[ 1 - \hat{\beta}^2 \times 9.30 \times 10^{-50} \left[ 1 - 2 \times \left( \frac{8 + \pi}{4} \right) \times 10^{-6} \right] \right]. \tag{6}$$

Indeed, Eq.(6) is the corrected form of Eq.(37) of Ref.[1]. The only differences between Eq.(5) of this Reply and Eq.(36) of Ref.[1], and also between Eq.(6) of this Reply and Eq.(37) of Ref.[1], are due to the number 8 in the expression $\left( \frac{8 + \pi}{4} \right)$ as appears in Eqs.(5) and (6). In the weak field approximation, these differences have arisen from the quantization $L_{n(b)} = \frac{n}{b} \hbar$ where $b = 1 - \frac{4G\mu}{c^2}$ as pointed out in Ref.[2] and as considered in the calculations of this Reply.

Finally, it must be emphasized on the importance of this Reply because the author of the Comment [2] has obtained only the effects of the quantization of $L_{n(b)} = \frac{n}{b} \hbar$ on the Bohr radius in the spacetime of a cosmic string
and has not calculated the effects of this quantization on the main results of Ref. [1] which are the corrections to the fine structure constant in the spacetime of a cosmic string from the generalized uncertainty principle. Our calculations in this Reply complete the calculations presented in the Comment [2].

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References
