Husimi operator and Husimi function for describing electron’s probability distribution in uniform magnetic field derived by virtue of the entangled state representation *

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Abstract

For the first time we introduce the Husimi operator $\Delta_h(\gamma, \varepsilon; \kappa)$ for studying Husimi distribution in phase space $(\gamma, \varepsilon)$ for electron’s states in uniform magnetic field, where $\kappa$ is the Gaussian spatial width parameter. Using the Wigner operator in the entangled state representation $|\lambda\rangle$ (Hong-Yi Fan, Phys. Lett. A 301 (2002) 153; A 126 (1987) 145) we find that $\Delta_h(\gamma, \varepsilon; \kappa)$ is just a pure squeezed coherent state density operator $|\gamma, \varepsilon\rangle\kappa\langle\gamma, \varepsilon|$, which brings convenience for studying and calculating the Husimi distribution. We in many ways demonstrate that the Husimi distributions are Gaussian-broadened version of the Wigner distributions. Throughout our calculation we have fully employed the technique of integration within an ordered product of operators.

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1 Introduction

Since the discovery of quantum Hall effect [1]-[4], the motion of an electron in the presence of magnetic field has brought an upsurge of interest. The basic theory that underlies quantum Hall effect is the Landau energy-level [5]-[6]. In Ref. [7] we have introduced an entangled state representation $|\lambda\rangle$ to describe this system which brings much convenience, for a review we refer to Ref. [8]. This coincides with Dirac’s guidance in Ref. [9]: “When one has a particular problem to work out in quantum mechanics, one can minimize the labor by using a representation in which the representatives of the more important abstract quantities occurring in that problem are as simple as possible”. On the other hand, in quantum mechanics it is impossible to specify simultaneously the position $Q$ and the momentum $P$ of a particle due to Heisenberg uncertainty principle. Thus Wigner’s quantum phase-space distribution theory [10]-[12] is of increasing interest because it permits a direct comparison between classical and quantum dynamics. Following the idea of gauge-invariant Wigner operator proposed by Serimaa, Javanainen and Varro [13] we have constructed the corresponding Wigner operator and Wigner function theory for electrons’ states in the $|\lambda\rangle$ representation in Ref. [14], as well as established the corresponding tomographic theory which means the reconstruction of electron’s Wigner distribution from the tomographic data [15]. Let us briefly recall the original

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idea of Wigner function. Feynman [16] summarized it as posing the following question: If there is any density function $F_w(q,p)$ in quantum mechanics that satisfies

$$P(p) = \int_{-\infty}^{\infty} F_w(q,p) \, dq, \quad P(q) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F_w(q,p) \, dp,$$

where $P(q)$ [$P(p)$] is proportional to the probability for finding the particle at $q$ [at $p$ in momentum space]. The answer is

$$F_w(q,p) = \text{Tr}[\rho \, \Delta(q,p)] = \frac{1}{2\pi} \int_{-\infty}^{\infty} \left\langle q + \frac{v}{2} \right| \rho \left| q - \frac{v}{2} \right\rangle e^{-ivp} \, dv,$$  

where $\rho$ is a density operator, $|q\rangle$ is the eigenvector of the coordinates operator, $Q |q\rangle = q |q\rangle$, and $\Delta(q,p)$ is the single-mode Wigner operator. In the coordinate representation $\Delta(q,p)$ takes the form

$$\Delta(q,p) = \frac{1}{(2\pi)^2} \int_{-\infty}^{\infty} dv \, \exp [i(q - p) + iv(Q - q)] = \frac{1}{2\pi} \int_{-\infty}^{\infty} |q - \frac{v}{2}\rangle \langle q + \frac{v}{2}| e^{-ivp} \, dv,$$  

Eq. (1) indicates that $P(x)$ [$P(p)$] is the marginal distribution of $F_w(x,p)$. Using the technique of integration within ordered product (IWOP) of operators [17]-[18], we have performed the integral (3) to obtain an explicit operator [19]

$$\Delta(q,p) = \frac{1}{\pi} : e^{-(q-Q)^2-(p-P)^2} :,$$  

or

$$\Delta(q,p) \rightarrow \Delta(\alpha,\alpha^*) = \frac{1}{\pi} : \exp \left[ -2 (a^\dagger - \alpha^*) (a - \alpha) \right] :,$$  

where $\alpha = (q + ip)/\sqrt{2}$. $: :$ means normal ordering symbol, $Q = (a + a^\dagger)/\sqrt{2}, P = (a - a^\dagger)/(i\sqrt{2})$ is the momentum operator whose eigenvector is $|p\rangle$. It then follows from (4) that one-sided integral over the Wigner operator yields the pure position state density operator

$$\int_{-\infty}^{\infty} dp \, \Delta(q,p) = \frac{1}{\sqrt{\pi}} : e^{-(q-Q)^2} := |q\rangle \langle q|,$$  

and pure momentum state density operator

$$\int_{-\infty}^{\infty} dq \, \Delta(q,p) = \frac{1}{\sqrt{\pi}} : e^{-(p-P)^2} := |p\rangle \langle p|,$$  

respectively, so the marginal distribution of the Wigner function is $\int_{-\infty}^{\infty} dp \, \langle \psi | \Delta(q,p) | \psi \rangle = |\psi(q)|^2$ or $\int_{-\infty}^{\infty} dq \, \langle \psi | \Delta(q,p) | \psi \rangle = |\psi(p)|^2$, respectively. However, as many authors have pointed out that the Wigner function $F_w(q,p)$ is not a probability distribution since it may takes on both positive and negative values. To quickly see this we can use $D(\alpha) = \exp [\alpha a^\dagger - \alpha^* a]$, $N = a^\dagger a$, to express (5) as $\Delta(\alpha,\alpha^*) = \frac{1}{\pi} D(\alpha) (1)^N D^\dagger(\alpha)$. Let $D^\dagger(\alpha) |\psi\rangle = |\phi\rangle$, then from

$$\langle \psi | \Delta(\alpha,\alpha^*) | \psi \rangle = \langle \phi | (1)^N | \phi \rangle = \langle \phi | \sum_{n=0}^{\infty} |n\rangle \langle n| (1)^N \sum_{n=0}^{\infty} |n\rangle \langle n| \phi \rangle = \sum_{n=0}^{\infty} (-1)^n |\langle n| \phi \rangle|^2,$$  

where the existence of $(-1)^n$ implies that the Wigner distribution function itself is not a probability distribution due to $(-1)^n$ being both positive and negative. To overcome this shortcomings, the so-called Husimi distribution function $F_h(q,p,\kappa)$ is introduced [20], which is defined in a manner that
guarantees it to be non-negative and gives it a probability interpretation. Its definition is smoothing out the Wigner function by averaging over a "coarse graining" function,

\[ F_h(q, p, s) = \int_{-\infty}^{\infty} dq' dp' F_w(q', p') \exp \left[ -s (q' - q)^2 - \frac{(p' - p)^2}{s} \right], \]

where \( s \) is the Gaussian spatial width parameter, which determines the relative resolution in \( p \)-space versus \( q \)-space but is free to be chosen. It is understood that the Husimi density is given by the projection of the wave function \( \psi \) onto coherent states localized in phase space \((p, q)\) with a minimum product of the uncertainties \( \Delta P = \sqrt{\frac{\hbar}{2}}, \Delta Q = \sqrt{\frac{\hbar}{2}} \). In this sense \( s \) plays the role of squeezing-parameter. In Refs. [21]-[22] the Husimi operator which corresponds to Husimi function is introduced, which turns out to be a pure squeezed coherent state projector. An interesting question thus naturally arises: how to introduce Husimi functions of phase space for describing probability distribution of electron states in uniform magnetic field (UMF)? To our knowledge, such a question has not been posed in the literature before. As emphasized by Serimaa, Javanainen and Varro [13] that when one wants to establish phase space distribution theory for electron moving in UMF with the gauge potential \( \mathbf{\tilde{A}} = (-\frac{1}{2} B y, \frac{1}{2} B x, 0) \), electron’s canonical momentum operators \((p_x, p_y)\) (conjugate to electron’s coordinate operator \(x, y)\) should be replaced by its gauge-invariant kinetic momentum (in the units of \( h = c = 1, c \) denotes the speed of light), \( \Pi_x = p_x + e A_x, \Pi_y = p_y + e A_y \). Correspondingly, the Wigner operator for describing electrons’ motion in UMF should involve \( \Pi_x \) and \( \Pi_y \) as ingredient operators and therefore is gauge invariant. In Ref. [14] we have proposed Wigner operator in the entangled state representation (i.e. electron’s position representation, denoted by \(|\lambda\rangle\)). In this work we shall first introduce the Husimi operator \( \Delta_h (\epsilon, \gamma; \kappa) \) by using this Wigner operator. Remarkably, as one can see shortly later, that the Husimi operator \( \Delta_h (\epsilon, \gamma; \kappa) \) is just a pure squeezed coherent state density operator \(|\epsilon, \gamma; \kappa\rangle \langle \epsilon, \gamma| \) (the explicit form of \(|\epsilon, \gamma\rangle\rangle \kappa\) in Fock space can also be deduced, see Eq. (41) below), which brings much convenience to studying Husimi functions for various electron’s states. Thus a phase space Husimi distribution theory for electron moving in uniform magnetic field (UMF) can be successfully established. The work is arranged as follows: In Sec. 2 we briefly review the concise features of the normally ordered form of gauge invariant Wigner operator \( \Delta_B (\gamma, \epsilon) \) in expressing the marginal distribution probability in the \(|\lambda\rangle\) representation and its conjugate representation \(|\gamma\rangle\) (electron’s canonical momentum representation). In Sec. 3 we first introduce the Husimi operator \( \Delta_h (\epsilon, \gamma; \kappa) \) and then derive its normally ordered form, correspondingly, we introduce Husimi function for describing electron’s probability distribution. The marginal distributions of Husimi function turns out to be Gaussian-broadened version of the Wigner marginal distributions. We also notice that the Gaussian spatial width parameter can be related to the intensity of magnetic field. In Sec. 4 we introduce the two-mode squeezed coherent state \(|\gamma, \epsilon\rangle\rangle \kappa\), and show its capability of constituting a quantum mechanical representation, we then find that the pure state \(|\gamma, \epsilon \rangle\rangle \kappa\) is just the Husimi operator, so \(|\gamma, \epsilon\rangle\rangle \kappa\) is a good representation for illustrating the Husimi function. In Sec. 5 we further analyze physical explanation of Husimi function of electron’s states by calculating the uncertainty relation of electron’s position and momentum. In Sec. 6 we calculate the Husimi function of various electron’s states in a concise and neat way. In Sec. 7 we discuss squeezing of Husimi function by variation of magnetic field. In so doing, the Husimi function theory for describing of electron states in uniform magnetic field is established and the relationship between Husimi function and Wigner function is clearly illuminated.

2 Wigner operator in entangled state representation and its marginal distributions

The Hamiltonian for electron in UMF is \( H = (\Pi_x \Pi_+ + \frac{1}{2}) \Omega \), the ladder operators are related to electron’s kinetic momenta \((\Pi_x, \Pi_y)\), \( \Pi_\pm = \frac{\Pi_x \pm i \Pi_y}{\sqrt{2}} \), \( \Omega = \frac{e B}{M} \) is the cyclotron frequency, \( M \) is the
mass of electron. For the appropriate gauge-invariant Wigner operator [13]
\[ \Delta_B \left( \vec{k}, \vec{q} \right) = \frac{1}{(2\pi)^4} \int \int_{-\infty}^{\infty} d^2u d^2v \exp \left[ iu \left( \vec{P} - \vec{k} \right) + iv \left( \vec{Q} - \vec{q} \right) \right] , \] (10)
where \( \vec{k} = (k_1, k_2) , \quad \vec{q} = (q_1, q_2) , \quad \vec{P} = (\Pi_x, \Pi_y) , \quad \vec{Q} = (x, y) , \)
we have proved in Ref. [14] that \( \Delta_B \left( \vec{k}, \vec{q} \right) \) in the entangled state representation \( |\lambda\rangle \) [7]-[8] is expressed as (somehow similar in form to (3))
\[ \Delta_B (\gamma, \varepsilon) = \int \frac{d^2\lambda}{\pi^3} |\varepsilon^* - \lambda \rangle \langle \varepsilon^* + \lambda| e^{\gamma^* \lambda^* - \gamma \lambda} , \] (11)
where \( \gamma = \chi + i\sigma^* , \quad \varepsilon = \chi - i\sigma^* , \)
\[ \chi = \sqrt{\frac{M \Omega}{2}} (q_1 + iq_2) + i \sqrt{\frac{1}{2M \Omega}} (k_1 + ik_2) , \quad \sigma = \sqrt{\frac{1}{2M \Omega}} (k_1 - ik_2) , \]
the state \( |\lambda\rangle \) is
\[ |\lambda\rangle = \exp \left[ -\frac{1}{2} |\lambda|^2 - i\lambda \Pi_+ + \lambda^* K_+ + i\Pi_+ K_+ \right] |00\rangle , \quad \lambda = \lambda_1 + i\lambda_2 , \] (12)
here the vacuum state is annihilated by \( \Pi_+ |00\rangle = 0 , \quad K_- |00\rangle = 0 , \quad K_\pm \) are linear combination of guiding centers \( x_0 \) and \( y_0 \) [6][25],
\[ K_\pm = \sqrt{\frac{M \Omega}{2}} (x_0 \mp iy_0) , \]
\[ x_0 = x - \frac{\Pi_y}{M \Omega} , \quad y_0 = y + \frac{\Pi_x}{M \Omega} . \] (14)
Note that the above operators obey commutative relations,
\[ [\Pi_-, \Pi_+] = 1 , \quad [K_-, K_+] = 1 , \] (15)
\[ [K_\pm, \Pi_\pm] = 0 , \quad [x_0, \Pi_\pm] = 0 , \quad [y_0, \Pi_\pm] = 0 , \quad [x, y] = 0 , \]
\[ [x_0, y_0] = -\frac{i}{M \Omega} , \quad [\Pi_x, \Pi_y] = -iM \Omega , \]
\( |\lambda\rangle \) is named entangled state [15]. The motivation of introducing \( |\lambda\rangle \) lies in two aspects: Firstly, when magnetic field \( \vec{B} \) applies what we have operators physically describing the system at hand are the guiding centers and kinetic momenta. In other words, the dynamic variables in the Hamiltonian are \( \Pi_\pm \) , so the corresponding position eigenvector should be expressed by \( \Pi_\pm \) as well as \( K_\pm \). Secondly, \( |\lambda\rangle \) can conveniently describe the position of an electron in a uniform magnetic field, i.e. \( |\lambda\rangle \) satisfies the coordinate eigenvector equation
\[ (K_+ + i\Pi_-) |\lambda\rangle = \lambda^* |\lambda\rangle , \quad (K_- - i\Pi_+) |\lambda\rangle = \lambda |\lambda\rangle . \] (16)
Combining (12)-(16) yields
\[ x = \frac{1}{\sqrt{2M \Omega}} (K_+ + K_- - i\Pi_+ + i\Pi_-) , \quad y = \frac{i}{\sqrt{2M \Omega}} (K_+ - K_- + i\Pi_+ + i\Pi_-) , \] (17)
\[ x |\lambda\rangle = \sqrt{\frac{2}{M \Omega}} \lambda_1 |\lambda\rangle , \quad y |\lambda\rangle = -\sqrt{\frac{2}{M \Omega}} \lambda_2 |\lambda\rangle . \] (18)
Moreover, the Wigner operator expressed by (11) in \( |\lambda\rangle \) representation automatically includes the contribution form the magnetic field, this is another merit of introducing \( |\lambda\rangle \). The advantage of \( \Delta_B (\gamma, \varepsilon) \) also lies in that from (11) we can easily derive its marginal distributions. In fact, using the normally ordered form of \( |00\rangle \langle 00| = : \exp \left[ -\Pi_+ \Pi_- - K_+ K_- \right] : \) and the IWOP technique [17]-[18]
As (11) indicates, \( \chi \) and \( \langle d \rangle \), we can perform the integration in (11) to derive the normally ordered form of the Wigner operator \( \Delta_B (\gamma, \varepsilon) \)

\[
\Delta_B (\gamma, \varepsilon) = \int \frac{d^2 \lambda}{\pi^2} : \exp\{ -|\varepsilon^*|^2 - |\lambda|^2 - i(\varepsilon^* - \lambda) \Pi_+ + i(\varepsilon - \lambda^*) \Pi_- \\
+ (\varepsilon^* + \lambda) K_+ + i\Pi_+ K_+ - i\Pi_- K_- - \Pi_+ \Pi_- - K_+ K_- + \gamma^* \lambda^* - \gamma \lambda \} : \\
= \frac{1}{\pi^2} : \exp\{ -|\varepsilon^* - (K_+ + i\Pi_-)| [\varepsilon - (K_- - i\Pi_+)] \\
- [\gamma^* - (K_+ - i\Pi_-)] [\gamma - (K_- + i\Pi_+)] \} : .
\]

(19)

As (11) indicates, \( \chi = \frac{1}{2} (\gamma + \varepsilon) \), \( \sigma^* = \frac{1}{2\pi} (\gamma - \varepsilon) \), then (19) becomes

\[
\Delta_B (\gamma, \varepsilon) = \frac{1}{\pi^2} : \exp\{ -2(K_+ - \chi^*) (K_- - \chi) - 2(\Pi_+ - \sigma^*) (\Pi_- - \sigma) \} : ,
\]

(20)

which is a 2-dimensional generalization of Eq. (5), so (11) is a correct choice. Note that the normally ordered form of the projector \(|\lambda\rangle \langle \lambda|\) is

\[
|\lambda\rangle \langle \lambda| = : \exp\{ -[\lambda^* - (K_- - i\Pi_-)] [\lambda - (K_+ + i\Pi_-)] \} : ,
\]

(21)

with the completeness \( \int \frac{d^2 \lambda}{\pi^2} |\lambda\rangle \langle \lambda| = 1 \), so integrating (19) over \( d^2 \gamma \) and using (21) we see

\[
\pi \langle \psi | \int d^2 \gamma \Delta_B (\gamma, \varepsilon) |\psi\rangle = : \exp\{ -[\varepsilon^* - (K_+ + i\Pi_-)] [\varepsilon - (K_- - i\Pi_+)] \} : \\
= \langle \psi | \lambda \rangle \langle \lambda = \varepsilon^* | \psi \rangle = |\langle \psi | \lambda \rangle|^2 |\lambda = \varepsilon^* .
\]

(22)

\(|\langle \psi | \lambda \rangle|^2 \) is proportional to the probability for finding the electron with position value \( \left[ \sqrt{\frac{\lambda_1}{\pi \hbar}}, -\sqrt{\frac{\lambda_2}{\pi \hbar}} \right] \).

Note \( \langle \lambda \rangle \langle \lambda' \rangle = \pi \delta (\lambda - \lambda') \delta (\lambda^* - \lambda'^*). \) On the other hand, integrating (19) over \( d^2 \varepsilon \) leads to

\[
\pi \langle \psi | \int d^2 \varepsilon \Delta_B (\gamma, \varepsilon) |\psi\rangle = : \exp\{ -[\gamma^* - (K_+ - i\Pi_-)] [\gamma - (i\Pi_+ + K_-)] \} : \\
= \langle \psi | \zeta \rangle \langle \zeta = -\gamma^* | \psi \rangle = |\langle \psi | \zeta \rangle|^2 |\zeta = -\gamma^* .
\]

(23)

where we have defined the state vector \(|\zeta\rangle\) as

\[
|\zeta\rangle = \exp \left[ -\frac{1}{2} |\zeta|^2 - i\zeta \Pi_+ - \zeta^* K_+ - i\Pi_+ K_+ \right] |00\rangle, \zeta = \zeta_1 + i\zeta_2,
\]

(24)

and

\[
|\zeta\rangle \langle \zeta | = : \exp\{ -|\zeta|^2 - i\zeta \Pi_+ - \zeta^* K_+ - i\Pi_+ K_+ \\
+i\zeta^* \Pi_+ - \zeta K_- + i\Pi_- K_- - \Pi_+ \Pi_- - K_+ K_- \} : \\
= : \exp\{ -[\zeta - (i\Pi_- - K_+)] [\zeta^* - (-i\Pi_+ - K_-)] \} : .
\]

(25)

with the completeness \( \int \frac{d^2 \zeta}{\pi^2} |\zeta\rangle \langle \zeta | = 1 \). \(|\zeta\rangle\) is the common eigenvector of the canonical momenta \((P_x, P_y)\), which can be shown as the following. In fact, due to

\[
(i\Pi_- - K_+) |\zeta\rangle = \zeta |\zeta\rangle, \quad (K_- + i\Pi_+) |\zeta\rangle = -\zeta^* |\zeta\rangle.
\]

(26)
and using

\[ p_x = \sqrt{\frac{\Omega}{8}} \left[ \Pi_+ + \Pi_- + iK_+ - iK_- \right] = \frac{\Pi_x}{2} + \frac{\Omega}{2} y_0, \quad (27) \]

\[ p_y = \sqrt{\frac{\Omega}{8}} \left[ i\Pi_- - i\Pi_+ - K_+ + K_- \right] = \frac{\Pi_y}{2} - \frac{\Omega}{2} x_0, \]

we see

\[ p_x |\zeta\rangle = \sqrt{\frac{\Omega}{2}} \zeta_2 |\zeta\rangle, \quad p_y |\zeta\rangle = \sqrt{\frac{\Omega}{2}} \zeta_1 |\zeta\rangle. \quad (28) \]

Thus \(|\langle \psi | \zeta \rangle|^2\) in (23) is proportional to the probability for finding the electron with momentum value \((\sqrt{\frac{\Omega}{2}} \zeta_2, \sqrt{\frac{\Omega}{2}} \zeta_1)\). Combine (22) and (23) we see that the marginal distributions of the Wigner function for electron states are physical meaningful in the entangled state representation \(|\lambda\rangle\) (or \(|\zeta\rangle\)). This in turn explains that the Wigner operator \(\Delta_B (\gamma, \varepsilon)\) expressed in \(|\lambda\rangle\) representation is a convenient choice which possesses the correct statistical meaning. Note

\[ \int d^2\varepsilon \int d^2\gamma \Delta_B (\gamma, \varepsilon) = 1. \quad (29) \]

For a general theory of entangled Wigner function we refer to [23].

### 3 Husimi operator: normally ordered form; the marginal distributions of Husimi distribution function

In this section we want to introduce the Husimi function \(W_h (\gamma, \varepsilon; k)\) for describing electron’s probability distribution, the corresponding Husimi operator \(\Delta_h (\gamma, \varepsilon; k)\), in reference to Eq. (9), is defined as smoothing out \(\Delta_B (\gamma', \varepsilon')\) by averaging over a ”coarse graining” function,

\[ \Delta_h (\gamma, \varepsilon; k) = 4 \int d^2\gamma' d^2\varepsilon' \Delta_B (\gamma', \varepsilon') \exp \left[ -\kappa |\varepsilon - \varepsilon'|^2 - \frac{|\gamma - \gamma'|^2}{\kappa} \right], \quad (30) \]

where \(\kappa\) is the Gaussian spatial width parameter, which is free to be chosen, and \(W_h (\gamma, \varepsilon; k) = \langle \psi | \Delta_h (\gamma, \varepsilon, \kappa) | \psi \rangle\). Using (19) and the IWOP technique we perform the integration in (30),

\[
\Delta_h (\gamma, \varepsilon; k) = \frac{4}{\pi^2} \int d^2\gamma' d^2\varepsilon' : \exp\left\{-\frac{\kappa}{1+\kappa} \left[ \varepsilon^* - (K_+ - i\Pi_-) \right] \left[ \varepsilon - (K_- + i\Pi_+) \right] \right\} \exp\left\{-\kappa |\varepsilon - \varepsilon'|^2 - \frac{|\gamma - \gamma'|^2}{\kappa} \right\} \\
= \frac{4\kappa}{(1+\kappa)^2} : \exp\left\{-\frac{\kappa}{1+\kappa} \left[ \varepsilon^* - (K_+ - i\Pi_-) \right] \left[ \varepsilon - (K_- + i\Pi_+) \right] \right\} \exp\left\{-\kappa |\varepsilon - \varepsilon'|^2 - \frac{|\gamma - \gamma'|^2}{\kappa} \right\} \\
- \frac{1}{1+\kappa} \left[ \gamma^* - (K_+ - i\Pi_-) \right] \left[ \gamma - (K_- + i\Pi_+) \right] : , \quad (31)\]

which is the explicit normally ordered form of the Husimi operator. Using \(\gamma = \gamma_1 + i\gamma_2, \varepsilon = \varepsilon_1 + i\varepsilon_2\), (17) and (27) we can further change (31) into the form

\[
\Delta_h (\gamma, \varepsilon; k) = \frac{4\kappa}{(1+\kappa)^2} : \exp\left\{-\frac{\kappa}{1+\kappa} \left[ \varepsilon_1 - \sqrt{\frac{\Omega}{2}} x \right]^2 \right\} \exp\left\{-\frac{\kappa}{1+\kappa} \left[ \varepsilon_2 - \sqrt{\frac{\Omega}{2}} y \right]^2 \right\} \\
- \frac{1}{1+\kappa} \left[ \gamma_1 + \sqrt{2\Omega} p_y \right]^2 + \left[ \gamma_2 - \sqrt{2\Omega} p_x \right]^2 \right\} : . \quad (32)\]
Using (31) we perform the one-sided integration $d^2\gamma$ over $\Delta_h$,

$$
\int \frac{d^2\gamma}{\pi} \Delta_h (\gamma, \varepsilon; k) = \frac{4\kappa}{1 + \kappa} : \exp \left\{ -\kappa \left( \varepsilon^* - K_+ - i\Pi_- \right) \left( \varepsilon - K_- + i\Pi_+ \right) \right\} :.
$$

(33)

On the other hand, using the $|\lambda\rangle$ representation in (21) and $x|\lambda\rangle = \sqrt{\frac{2}{\Omega M}} \lambda_1 |\lambda\rangle$, $y|\lambda\rangle = -\sqrt{\frac{2}{\Omega M}} \lambda_2 |\lambda\rangle$ in (18) as well as the IWOP technique we can derive the operator identity

$$
\exp \left\{ g \left[ \left( s_1 - \sqrt{\frac{M\Omega}{2}} x \right)^2 + \left( s_2 - \sqrt{\frac{M\Omega}{2}} y \right)^2 \right] \right\}
= \int \frac{d^2\lambda}{\pi} \exp \left\{ - (1 - g) |\lambda\rangle^2 + \lambda (K_- - i\Pi_+ - gs) + \lambda^* (K_+ + i\Pi_- - gs^*) + g |\lambda\rangle^2 - (K_- - i\Pi_+) (K_+ + i\Pi_-) \right\}:
= \frac{1}{1 - g} : \exp \left\{ \frac{g}{1 - g} (s^* - K_+ - i\Pi_-) (s - K_- + i\Pi_+) \right\} : ,
$$

(34)

where $s = s_1 + is_2$. So (33) can be simplified as (identifying $-\kappa$ in (33) as $g$ in (34))

$$
\int \frac{d^2\gamma}{\pi} \Delta_h (\gamma, \varepsilon; \kappa) = 4\kappa e^{-\kappa \left[ (\varepsilon_1 - \sqrt{\frac{M\Omega}{2}} x)^2 + (\varepsilon_2 - \sqrt{\frac{M\Omega}{2}} y)^2 \right]},
$$

(35)

thus the marginal distribution of Husimi operator is a Gaussian operator with the factor $\kappa$. It then follows from (35), (22) and (18) the marginal distribution of Husimi function in "$\lambda$-direction",

$$
\int \frac{d^2\gamma}{\pi} W_h (\gamma, \varepsilon; k) = |\langle \psi | \Delta_h (\gamma, \varepsilon; \kappa) |\psi \rangle|
= 4\kappa |\langle \psi | \Delta_h (\gamma, \varepsilon; \kappa) |\psi \rangle|
= 4\kappa |\langle \psi | e^{-\kappa (|\lambda\rangle^2 + (s - K_-)^* (s - K_-))} |\lambda\rangle |\psi \rangle|
= 4\kappa \int \frac{d^2\lambda}{\pi} e^{-|\varepsilon - \lambda|^2} |\psi (\lambda) |^2 .
$$

(36)

Comparing (36) with (22) we see that (36) is a Gaussian-broadened version of the quantal position probability distribution $|\psi (\lambda) |^2$ (one marginal distribution of the Wigner function). Similarly, performing the one-sided integration $d^2\varepsilon$ over $\Delta_h$ in (32) leads to

$$
\int \frac{d^2\varepsilon}{\pi} \Delta_h (\gamma, \varepsilon; \kappa)
= \frac{4}{1 + \kappa} : \exp \left\{ -\frac{1}{1 + \kappa} \left[ \gamma^* - (K_+ - i\Pi_-) \right] \left[ \gamma - (i\Pi_+ + K_-) \right] \right\} : .
$$

(37)

From (25) and (28) as well as the IWOP technique we can prove another operator identity
where $v = v_1 + iv_2$. Thus Eq. (37) becomes (identifying $-1/\kappa$ in (37) as $g$ in (38))

$$
\int \frac{d^2 \zeta}{\pi} \Delta_h (\gamma, \varepsilon, \kappa) = \frac{4}{\kappa} e^{\frac{1}{\kappa} \left[ (\gamma_1 + \sqrt{\pi \kappa} \rho_+)^2 + (\gamma_2 - \sqrt{\pi \kappa} \rho_+)^2 \right]},
$$

(39)

so the another marginal distribution of (31) is also a Gaussian operator but with the factor $\frac{1}{\kappa}$. It then follows from (39) another marginal distribution of the Husimi function in ”\(\zeta\)–direction”

$$
\int \frac{d^2 \zeta}{\pi} W_h (\gamma, \varepsilon; k) = \langle \psi | \int \frac{d^2 \zeta}{\pi} \Delta_h (\gamma, \varepsilon; \kappa) | \psi \rangle
$$

(40)

which is a Gaussian-broadened version of the quantal momentum probability distribution $|\psi (\zeta)|^2$, (another Wigner marginal distribution (comparing with Eq. (23))). Therefore, an operator-representation theory which underlies the Husimi distribution of electron in UMF is established, and the Husimi function’s marginal distributions are clear.

4 The Husimi operator as a pure squeezed coherent state density operator

By noticing $|00 \rangle \langle 00| = : \exp[-\Pi_+ \Pi_- - K_+ K_-] :$ we observe that the normally ordered form of the Husimi operator $\Delta_h (\gamma, \varepsilon, \kappa)$ in (31) can be decomposed as

$$
\Delta_h (\gamma, \varepsilon, \kappa) = \frac{4 \kappa}{(1 + \kappa)^2} \exp \left\{ - \frac{1}{1 + \kappa} \left[ |\varepsilon|^2 + |\gamma|^2 - (\kappa \varepsilon + \gamma) K_+ + i (\kappa \varepsilon^* - \gamma^*) \Pi_+ - i (\kappa - 1) \Pi_+ K_+ \right] \right\}
$$

$$
\times : \exp[-\Pi_+ \Pi_- - K_+ K_-] : \exp \left\{ - \frac{1}{1 + \kappa} \left[ (\kappa \varepsilon^* + \gamma^*) K_- - i (\kappa \varepsilon - \gamma) \Pi_- + i (\kappa - 1) \Pi_- K_- \right] \right\}
$$

$$
|\gamma, \varepsilon\rangle_{\kappa} (\gamma, \varepsilon),
$$

(41)

where we have defined the new state

$$
|\gamma, \varepsilon\rangle_{\kappa} = \frac{2 \sqrt{\kappa}}{1 + \kappa} \exp \left\{ - \frac{1}{1 + \kappa} \left[ |\varepsilon|^2 + |\gamma|^2 \right] - (\kappa \varepsilon + \gamma) K_+ + i (\kappa \varepsilon^* - \gamma^*) \Pi_+ - i (\kappa - 1) \Pi_+ K_+ \right\} |00\rangle.
$$

(42)
Thus the Husimi operator $\Delta_k (\lambda, \zeta, \kappa)$ is just the pure state density operator $| \gamma, \varepsilon \rangle_\kappa \langle \gamma, \varepsilon |$, this is a remarkable result. It turns out that $| \gamma, \varepsilon \rangle_\kappa$ is a two-mode squeezed canonical coherent state because it obeys the eigenvector equations

$$
(K_- \cosh r + i\Pi_+ \sinh r) | \gamma, \varepsilon \rangle_\kappa = \frac{\sqrt{k}e + \gamma/k}{2} | \gamma, \varepsilon \rangle_\kappa
$$

(43)

and

$$
(\Pi_- \cosh r + iK_+ \sinh r) | \gamma, \varepsilon \rangle_\kappa = i \frac{\gamma^*/\sqrt{k} - \sqrt{k}e^*}{2} | \gamma, \varepsilon \rangle_\kappa
$$

(44)

where $\frac{1}{\sqrt{k}} = \tanh r$ is a squeezing parameter, $e^r = \frac{1}{\sqrt{k}}, \cosh r = \frac{1 + e^{2r}}{\sqrt{k}}$. The corresponding squeezing operator is

$$
S (r) = e^{i(xp_y + yp_x - r)} = \exp [i r (\Pi_+ K_+ + \Pi_- K_-)],
$$

(45)

(For a review of general squeezed state theory in quantum optics we refer to [24]). The disentangling of (45) is

$$
S (r) = \sec hr \exp [(i\Pi_+ K_+ \tanh r) \exp[(K_+ K_- + \Pi_+ \Pi_-) \ln \sec hr]] \times \exp (i\Pi_- K_- \tanh r).
$$

(46)

From (46), (14)-(15) we derive

$$
S^{-1} K_- S = K_- \cosh r + i\Pi_+ \sinh r, \quad S^{-1} \Pi_- S = \Pi_- \cosh r + iK_+ \sinh r, \quad S^{-1} K_+ S = K_+ \cosh r - i\Pi_- \sinh r, \quad S^{-1} \Pi_+ S = \Pi_+ \cosh r - iK_- \sinh r,
$$

(47)

and using (18) and (27) we have

$$
S^{-1} x S = \sqrt{k}x, \quad S^{-1} y S = \sqrt{k} y, \quad S^{-1} p_x S = p_x/\sqrt{k}, \quad S^{-1} p_y S = p_y/\sqrt{k}.
$$

(48)

(49)

In (19) we see that $\lambda$ denotes the eigenvalue of electron’s coordinates, so $S (r)$ has a natural representation in $| \lambda \rangle$ representation [25]

$$
S (r) = e^{-r} \int \frac{d^2 \lambda}{\pi} e^{-r \lambda} \langle \lambda |, \quad e^r = \frac{1}{\sqrt{k}},
$$

(50)

from $\langle \lambda | \lambda' \rangle = \pi \delta^{(2)} (\lambda - \lambda')$, $S (r) | \lambda \rangle = e^{-r} | e^{-r} \lambda \rangle$, so (50) embodies another merit of constructing the entangled state representation $| \lambda \rangle$. From the eigenvalue equations (19) we also see that the eigenvalue of $x$ and $y$ varies with $B$, since $\sqrt{\frac{1}{M^2}} = \frac{1}{\sqrt{eB}}$, so the variation of the magnetic field intensity $B$ is related to squeezing of electron’s orbit track. Thus the variation of Gaussian spatial width parameter $\sqrt{k}$ can also be interpreted as the change of magnetic field intensity $\sqrt{B}$. From (43)-(44) we notice that $| \gamma, \varepsilon \rangle_\kappa$ can be expressed as the result of the squeezing operator operating on the state $| \gamma, \varepsilon \rangle$, i.e.

$$
| \gamma, \varepsilon \rangle_\kappa = S^{-1} (r) | \gamma, \varepsilon \rangle,
$$

(51)

where

$$
| \gamma, \varepsilon \rangle \equiv \exp \left[\frac{1}{4} (k|e|^2 + |\gamma|^2) \right] + i \frac{i \gamma^*/\sqrt{k} - \sqrt{k}e^*}{2} \Pi_+ \left(\sqrt{k}e + \gamma/\sqrt{k} K_+ \right) |00\rangle
$$

(52)

is a normalized two-mode coherent state [25] for an electron in UMF, and we have dropped the inconsequential phase factor $\exp \left\{ \frac{\kappa}{4(1+\kappa)} (e^* \gamma - \gamma^* e) \right\}$ in the result of calculating $S^{-1} (r) | \gamma, \varepsilon \rangle$. 


5 Further explanation of the Husimi function

Using (52), (48) and (18) we see that in the state \(|\gamma = 0, \varepsilon = 0\rangle\) the variance of electron’s position \(x\) is

\[
(\Delta x)^2 \equiv \kappa \langle 0, 0 | x^2 | 0, 0 \rangle \kappa - (\kappa \langle 0, 0 | x | 0, 0 \rangle \kappa)^2 = \langle 0, 0 | S(r) x^2 S^{-1}(r) | 00 \rangle = \frac{1}{2M\Omega\kappa},
\]

while the variances of \(p_x\) is

\[
(\Delta p_x)^2 = \langle 00 | S(r) p_x^2 S^{-1}(r) | 00 \rangle = \frac{\kappa\Omega}{8} \langle 0, 0 | \Pi_+ + \Pi_- - iK_+ + iK_- | 00 \rangle = \frac{\kappa\Omega}{4}.
\]

On the other hand, \(|\gamma, \varepsilon\rangle\) is complete

\[
\frac{1}{4\pi} \int d^2\varepsilon \int d^2\gamma \langle \gamma, \varepsilon | \kappa \rangle \langle \gamma, \varepsilon | = 1,
\]

so the Husimi density

\[
\langle \psi | \Delta_h (\gamma, \varepsilon, \kappa) | \psi \rangle = |\langle \psi | \gamma, \varepsilon \rangle \kappa|^2
\]

is given by the projection of the wave function onto the squeezed coherent states localized in phase space with a minimum product of the uncertainties

\[
\Delta p_x = \sqrt{\frac{\kappa\Omega}{4}}, \quad \Delta x = \sqrt{\frac{1}{\kappa\Omega\kappa}}, \quad \Delta x\Delta p_x = \frac{1}{2}
\]

In this sense the Gaussian spatial width parameter \(\kappa = \frac{2\Delta p_x}{M\Omega\Delta x} = \frac{2\Delta p_x}{c^2\Delta \kappa}\) plays the role of squeezing-parameter (note that in the units of \(\hbar = c = 1\), \(\sqrt{2/\hbar}\) is the magnetic length.) Further, using (41) we can re-express the marginal distribution (40) of the Husimi function of electron’s quantum state \(|\psi\rangle\) as

\[
\int \frac{d^2\varepsilon}{\pi} W_h (\gamma, \varepsilon; k) = \int \frac{d^2\gamma}{\pi} |\kappa \gamma, \varepsilon \rangle \langle \gamma, \varepsilon | \psi \rangle|^2.
\]

We can also recast (36) as

\[
\int \frac{d^2\gamma}{\pi} W_h (\gamma, \varepsilon; \kappa) = \int \frac{d^2\gamma}{\pi} |\kappa \gamma, \varepsilon \rangle \langle \gamma, \varepsilon | \psi \rangle|^2.
\]

Eqs. (58) and (59) indicate the relationship between probability density of \(|\psi\rangle\) in the \(\kappa \langle \gamma, \varepsilon |\) representation and those in the entangled state \(\langle \lambda |\) representation.

6 Husimi functions of some electron’s states

Eq. (41) brings great convenience to calculate Husimi functions of various electron’s states. Using the two-mode coherent state’s completeness relation [25]-[27]

\[
\int \frac{d^2z_1 d^2z_2}{\pi^2} |z_1, z_2\rangle \langle z_1, z_2| = 1,
\]

where

\[
|z_1, z_2| = \langle 00 | \exp \left[ -\frac{1}{2} (|z_1|^2 + |z_2|^2) + z_1^* \Pi_+ + z_2^* K_+ \right],
\]

\[
\langle z_1, z_2 | \Pi_+ = \langle z_1, z_2 | z_1^*, \quad \langle z_1, z_2 | K_+ = \langle z_1, z_2 | z_2^*.
\]
and (42) we immediately have

\[
\langle z_1, z_2 \mid \gamma, \varepsilon\rangle_{\kappa} = \frac{2\sqrt{\kappa}}{1 + \kappa} e^{-((|z_1|^2 + |z_2|^2)/2)}
\]

\[
\times \exp\left\{ -\frac{1}{2(1 + \kappa)}\left[|\varepsilon|^2 + |\gamma|^2 + |\varepsilon'|^2 + |\gamma'|^2\right]\right\}
\]

\[
\times \int \frac{d^2z_1 d^2z_2}{\pi^2} \exp\left\{- |z_1|^2 - |z_2|^2 - \frac{1}{1 + \kappa} \left[ (\kappa\varepsilon^* + \gamma') z_2 - i (\kappa\varepsilon - \gamma') z_1 \right.ight.
\]

\[
+ i (\kappa - 1) z_1 z_2 - (\kappa\varepsilon + \gamma) z_2^* + i (\kappa\varepsilon^* - \gamma^*) z_1^* - i (\kappa - 1) z_1^* z_2^* \right\}
\]

\[
= \exp\left\{ -\frac{\kappa}{4}|\varepsilon' - \varepsilon|^2 - \frac{|\gamma' - \gamma|^2}{2\kappa} \right\}
\]

\[
\times \left\{ \frac{\kappa - 1}{4(1 + \kappa)} \left( \varepsilon^* \gamma' - \varepsilon' \gamma^* + \varepsilon \gamma' - \varepsilon' \gamma \right) \right\},
\]

(63)

where the third and fourth terms in the last exponential are all pure imaginary, so we immediately obtain the Husimi function of $|\varepsilon', \gamma\rangle_{\kappa}$,

\[
\kappa \langle \varepsilon', \gamma' \mid \Delta_{\kappa} (\varepsilon, \gamma, \kappa) \mid \varepsilon', \gamma'\rangle_{\kappa} = |\kappa \langle \varepsilon, \gamma \mid \varepsilon', \gamma'\rangle_{\kappa}|^2
\]

\[
= \exp\left\{ -\frac{\kappa}{2}|\varepsilon' - \varepsilon|^2 - \frac{|\gamma' - \gamma|^2}{2\kappa} \right\},
\]

(64)

which is also a Gaussian broadened function. Further, using (50)-(52) and (12) we have

\[
\langle \lambda \mid \gamma, \varepsilon\rangle_{\kappa} = \langle \lambda \mid S^{-1} (r) \mid \gamma, \varepsilon\rangle = \sqrt{\kappa} \langle \sqrt{\kappa} \lambda \mid \gamma, \varepsilon\rangle
\]

\[
= \sqrt{\kappa} \exp\left\{ -\frac{1}{4} \left( \kappa |\varepsilon|^2 + |\gamma|^2 / \kappa \right) - \frac{1}{2\kappa} \left| \lambda \right|^2 - \lambda^* \gamma^* - \kappa \varepsilon^* \right\}
\]

\[
+ \lambda^* \varepsilon + \gamma / \sqrt{\kappa} - \sqrt{\kappa} \varepsilon^* \gamma / \sqrt{\kappa} + \sqrt{\kappa} \varepsilon^* \right\}
\]

\[
= \sqrt{\kappa} \exp\left\{ -\frac{1}{2\kappa} \left( |\varepsilon|^2 + |\gamma|^2 \right) + \kappa \text{Re} (\lambda\varepsilon) + i \text{Im} (\lambda\gamma + i \text{Im} \varepsilon \gamma^*) \right\},
\]

(65)

so the Husimi function of the electron’s coordinate eigenstate $|\lambda\rangle$ is

\[
\langle \lambda \mid \Delta_{\kappa} (\gamma, \varepsilon, \kappa) \rangle_{\lambda} = \kappa \langle \sqrt{\kappa} \lambda \mid \gamma, \varepsilon\rangle^2 = \kappa \exp\{-\kappa|\lambda - \varepsilon^*|^2\},
\]

(66)

which is a Gaussian. This is in sharply contrast with the Wigner function of $|\lambda\rangle$ which can be calculated by using (11)

\[
\langle \lambda \mid \Delta_{\beta} (\gamma, \varepsilon) \rangle_{\lambda} = \langle \lambda \mid \int \frac{d^2\lambda'}{\pi^2} \left| \varepsilon^* - \lambda' \right> \left< \varepsilon^* + \lambda' \left| e^{\gamma^* \lambda'^* - \gamma \lambda'} |\lambda\rangle \right.ight.
\]

\[
= \int \frac{d^2\lambda'}{\pi} \delta^{(2)} (\lambda^* - \varepsilon^* + \lambda') \delta^{(2)} (\lambda^* - \varepsilon^* - \lambda') e^{\gamma^* \lambda'^* - \gamma \lambda'}
\]

\[
= \frac{1}{4\pi} \delta^{(2)} (\lambda - \varepsilon^*).
\]

(67)
where \( \varepsilon = \chi - i\sigma^* = \sqrt{\frac{M\Omega}{2}} (q_1 + iq_2) \), so

\[
\langle \lambda | \Delta_B (\gamma, \varepsilon) | \lambda \rangle = \frac{1}{4\pi} \delta \left( \lambda_1 - \sqrt{\frac{M\Omega}{2}} q_1 \right) \delta \left( \lambda_2 + \sqrt{\frac{M\Omega}{2}} q_2 \right),
\]

which is in consistent with Eq. (18). Comparing (66) and (67) and recall the limiting Gaussian-form of Delta function we can see again that Husimi function is the Gaussian-broadened version of Wigner function. Next we consider a Landau state,

\[
|n, m\rangle = \frac{\Pi^n K^n}{\sqrt{n!m!}} |00\rangle = \frac{1}{\sqrt{n!m!}} \frac{\partial^n}{\partial z_1^n} \frac{\partial^m}{\partial z_2^m} e^{z_1 \Pi_+ e \cdot z_2 K_+} |00\rangle |z_1 = z_2 = 0\]

where \( n, m = 0, 1, 2, \ldots \), from (62) we know

\[
\langle n, m | \gamma, \varepsilon \rangle_k = \frac{1}{\sqrt{n!m!}} \frac{\partial^n}{\partial z_1^n} \frac{\partial^m}{\partial z_2^m} \langle z_1, z_2 | \gamma, \varepsilon \rangle_k |z_1 = z_2 = 0\]

\[
= \frac{1}{\sqrt{n!m!}} \langle z_1, z_2 | 2\sqrt{\kappa} \exp\left\{ \frac{-1}{1 + \kappa} \kappa |\gamma|^2 \right\} \frac{\partial^n}{\partial z_1^n} \frac{\partial^m}{\partial z_2^m} \times \exp\left\{ \frac{-1}{1 + \kappa} \left[ -\left( \kappa \varepsilon + \gamma \right) z_2^* + i \left( \kappa \varepsilon^* - \gamma^* \right) z_1^* + i \left( \kappa - 1 \right) z_1^* z_2^* \right] \right\} |00\rangle |z_1 = z_2 = 0\]

\[
= \frac{1}{\sqrt{n!m!}} \frac{2\sqrt{\kappa}}{1 + \kappa} \exp\left\{ \frac{-1}{1 + \kappa} \kappa |\gamma|^2 \right\} \times \left( \frac{1 - \kappa}{1 + \kappa} \right)^{(m+n)/2} H_{m,n} \left( -\frac{\left( \kappa \varepsilon + \gamma \right)}{\sqrt{\kappa^2 - 1}}, \frac{-\left( \kappa \varepsilon^* - \gamma^* \right)}{\sqrt{\kappa^2 - 1}} \right).
\]

where \( H_{m,n} \) is two-variable Hermite polynomial[28] whose definition is

\[
H_{m,n}(x, y) = \sum_{l=0}^{\min(m,n)} \frac{m!n!(-1)^l}{l!(m-l)!(n-l)!} x^{m-l} y^{n-l},
\]

which is not a direct product of two independent single-variable Hermite polynomials. The generating function of \( H_{m,n}(x, y) \) is

\[
\sum_{m,n=0}^{\infty} \frac{z^m z^n}{m!n!} H_{m,n}(x, y) = \exp\{-zz' + zx + z'y\},
\]

so

\[
H_{m,n}(x, y) = \frac{\partial^n}{\partial z_1^n} \frac{\partial^m}{\partial z_2^m} e^{-zz' + zx + z'y} |z = z' = 0\]

\[
\langle n, m | \Delta_B (\gamma, \varepsilon; \kappa) | n, m \rangle = | \langle n, m | \gamma, \varepsilon \rangle_k |^2
\]

\[
= \frac{4\kappa}{n!m!} \frac{(1 - \kappa)}{(1 + \kappa)}^2 \exp\left\{ \frac{-\kappa |\gamma|^2}{1 + \kappa} \right\} \times \left| H_{m,n} \left( -\frac{\left( \kappa \varepsilon + \gamma \right)}{\sqrt{\kappa^2 - 1}}, \frac{-\left( \kappa \varepsilon^* - \gamma^* \right)}{\sqrt{\kappa^2 - 1}} \right) \right|^2.
\]

7 Squeezing of Husimi function by variation of magnetic field
In (48) we have mentioned that the variation of magnetic field intensity may cause squeezing of orbit track of electron’s motion. Let the corresponding squeezing operator is $S(\mu)$, in the $|\lambda\rangle$ representation it is expressed by (see Appendix)

$$S(\mu) = \int \frac{d^2 \lambda}{\pi \mu} |\lambda/\mu\rangle \langle \lambda|.$$  

(75)

Under the squeezing transform the Wigner operator changes

$$S(\mu) \Delta_B (\gamma, \varepsilon) S^{-1}(\mu) = \int \frac{d^2 \lambda}{\pi^3} \left[ \frac{\varepsilon^*}{\mu} - \lambda \right] \left\langle \frac{\varepsilon^*}{\mu} + \lambda \right| e^{i \mu(\gamma^* \lambda - \gamma \lambda)} = \Delta_B (\mu \gamma, \varepsilon/\mu).$$  

(76)

From (30) we see that the Husimi function of the lowest Landau state is

$$S(\mu) \Delta_h (\gamma, \varepsilon; k) S^{-1}(\mu) = 4 \int d^2 \gamma' d^2 \varepsilon' \Delta_B (\mu \gamma', \varepsilon'/\mu) \exp \left[ -\kappa |\varepsilon - \varepsilon'|^2 - \frac{|\gamma - \gamma'|^2}{\kappa} \right]$$

$$= \Delta_h (\mu \gamma, \varepsilon/\mu; k \mu^2).$$  

(77)

we again see the squeezing parameter $\mu$ is equivalent to the Gaussian broaden parameter $1/\sqrt{\kappa}$, (77) and (41) indicates

$$S(\mu)|\gamma, \varepsilon\rangle_{\kappa} = |\mu \gamma, \varepsilon/\mu\rangle_{\kappa^2}.  

(78)

From (31) we see the Husimi function of the lowest Landau state is

$$\langle 00| \Delta_h (\gamma, \varepsilon; k)|00\rangle = \frac{4\kappa}{(1 + \kappa)^2} \exp \left\{-\frac{\kappa}{1 + \kappa} |\varepsilon|^2 - \frac{1}{1 + \kappa} |\gamma|^2 \right\}.  

(79)

Using (41), (51), (77) and (79) we immediately obtain the Husimi function of squeezed Landau vacuum state,

$$\langle 00| S(\mu) \Delta_h (\gamma, \varepsilon; k) S^{-1}(\mu)|00\rangle = \langle 00| \Delta_h (\mu \gamma, \varepsilon/\mu; k \mu^2)|00\rangle = \frac{4\kappa \mu^2}{(1 + \kappa \mu^2)^2} \exp \left\{-\frac{\kappa}{\kappa \mu^2 + 1} |\varepsilon|^2 - \frac{\mu^2}{\kappa \mu^2 + 1} |\gamma|^2 \right\}.  

(80)

In summary, for the first time we have introduced the Husimi operator $\Delta_h (\gamma, \varepsilon; k)$ for electron in UMF, and shown $\Delta_h (\lambda, \zeta, \kappa) = |\lambda, \zeta\rangle_{\kappa} \langle \lambda, \zeta|$, i.e. the Husimi operator actually is a pure squeezed coherent state projector. The normally ordered form of Husimi operator are also derived which provides us with an operator version to examine various properties of the Husimi distribution. We have in many ways demonstrated that Husimi (marginal) distributions are Gaussian-broadened version of the Wigner (marginal) distributions. Throughout the paper we have fully employed the technique of integration within an ordered product of operators and the entangled state representation, each of them seems an efficient method for studying quantum statistical physics [30].

8 Appendix

Using (12) the IWOP technique we can derive $S(\mu)$’s normal ordering [29],

$$S(\mu) = \int \frac{d^2 \lambda}{\pi \mu} |\lambda/\mu\rangle \langle \lambda| = \int \frac{d^2 \lambda}{\pi \mu^2} \exp \left\{-\frac{1}{2} |\lambda|^2 \left( 1 + \frac{1}{\mu^2} \right) + \lambda \left( K_+ - i \frac{\Pi_+}{\mu} \right) \right\}$$

$$+ \lambda^* \left( K_+ + i \Pi_+ \right) + i \Pi_+ K_+ - i \Pi_- K_- - K_+ K_- - \Pi_+ \Pi_- \right\}:$$

$$= \frac{2\mu}{1 + \mu^2} \exp \left[ \frac{2\mu^2}{1 + \mu^2} \left( K_- - i \frac{\Pi_-}{\mu} \right) \left( K_+ + i \Pi_+ \right) - (K_- - i \Pi_-)(K_+ + i \Pi_+) \right]:$$

$$= \sec hf \exp \left\{ i \Pi_+ K_+ \tanh f \right\} \left[ (K_+ K_- + \Pi_+ \Pi_-) \ln \sec hf \exp \left( i \Pi_+ K_- \tanh f \right) \right],$$

where $f = \mu \lambda$, and $\Pi_\pm = \Pi_x \pm i \Pi_y$, $K_\pm = K_x \pm i K_y$. The exact form of $S(\mu)$ is given by (80).
where $\mu = e^{i\lambda}$, we can say that the classical dilation $\lambda \rightarrow \frac{\lambda}{\mu}$ maps into the squeezing operator $S(\mu)$.(75) again realizes Dirac’s statement that the symbolic method can “express the physical law in a neat and concise way”.

References


