A New Large Number Coincidence and the Maximum Number of Bits in the Universe
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ABSTRACT

Seven apparently distinct, pure numbers of order near $10^{121}$ are generated from terms associated with modern cosmology, the holographic conjecture and the computational capacity of the Universe. This new large number coincidence problem is then resolved, being shown to result from the physics of the standard cosmological model. However, the fact that the coincidence problem among pure numbers of order near $10^{121}$ occurs only in this already special epoch presents a distinct coincidence problem. That separate coincidence problem is solved by scaling the cosmological constant to the sixth power of the nucleon mass.

I. Introduction

It has been demonstrated that Dirac, Eddington and others were essentially correct to suggest that some physics was responsible for generating the suspiciously large gallery of pure numbers of order near $10^{40}$ (Funkhouser 2006). The coincidence problem among the large, pure numbers was found to be a result of the physics of the standard cosmological model. However, the fact that the Dirac-Eddington large-number coincidence occurs only in this same epoch in which the cosmic coincidence also occurs represents a distinct coincidence problem that is not resolved through modern cosmology. It was found that this separate coincidence problem would be explained if the vacuum energy density associated with the cosmological constant, $\Lambda$, were scaled to the characteristic gravitational energy density of the nucleon. That scaling law,

$$\frac{c^2}{G} \Lambda \sim \frac{G m_n^2}{l_n^4} \quad (1)$$

where $m_n$ and $l_n$ are respectively the mass and Compton wavelength of the nucleon, also would relate several other large-number coincidence problems including the Eddington-Weinberg relation (Funkhouser 2006). (Note that in this paper the cosmological constant has the unit inverse second-squared.) Scaling the cosmological constant to the sixth power of the nucleon mass (as in Eq. (1)) was originally proposed by Zel’dovich based on considerations of quantum field theory (Zel’dovich 1967). Mena Marugan and Carneiro also proposed that scaling law based on possible holographic limitations to the nucleon volume (Mena 2006).

With the discovery of the accelerating Universe and re-introduction of Einstein’s cosmological constant a new mystery concerning large pure numbers was initiated. The vacuum energy density $\varepsilon_\Lambda$ associated with the cosmological constant is nearly 122 orders of magnitude smaller than the energy density $\varepsilon_P$ associated with the Planck scale. The factor $10^{122}$ is particularly interesting because an ensemble of apparently distinct, pure numbers of similar order may be found among the terms associated with modern cosmology, the holographic conjecture and the computational capacity of the universe.

In this letter a new coincidence problem among pure numbers of order near $10^{121}$ is presented. The new coincidence problem is resolved using physics from the standard cosmological model. However, the fact that the new coincidence problem is generated only in this same epoch in which the cosmic coincidence and the Dirac-Eddington large-
number coincidence occur constitutes a distinct coincidence problem. It is shown that the proposed relationship in Eq. (1) would explain that coincidence.

II. The Pure Numbers of Order Near $10^{121}$

The first two large pure numbers of order near $10^{121}$ presented here are associated with modern cosmology and involve the cosmological constant. The ratio of the Planck energy density to the apparent vacuum density may be expressed explicitly as

$$\frac{\epsilon_p}{\epsilon_\Lambda} = \frac{3m_p c^2}{3 \Lambda c^2} = 1.7 \times 10^{122},$$

(2)

where $m_p$ and $l_p$ are the Planck mass and length, respectively, $G$ is Newton’s gravitation constant $c$ is the speed of light in a vacuum. This term is, by definition, proportional to $c^5 / (Gh\Lambda)$ where $h$ is the Planck constant. That pure number is a fundamental term and will be shown to represent the maximum number of bits ever available to our Universe. The other pure numbers of similar order will be reduced to this term.

Wesson has introduced a microphysical mass scale that is obtained from the cosmological constant. The mass $m_w = h\Lambda^{1/2}c^{-2}$ is meant to represent the smallest natural mass quantum. Mongan has shown that $m_w$ would be the mass associated with one bit of information in a holographic Universe (Mongan 2006). The mass of the observable Universe happens to be about $10^{120}$ times greater than the Wesson mass quantum:

$$\frac{M_0}{m_w} \approx 1.6 \times 10^{120}.$$

(3)

Note that the term $\left(\frac{Gm_w^2}{(hc)}\right)^{-1}$ is of order $10^{120}$, but this is because it also reduces to $c^5 / (Gh\Lambda)$.

The next two large numbers presented are taken from analyses of the information content of the observable Universe. According to the holographic conjecture the number $N_0$ of bits of information currently allowed in the observable Universe is one fourth of the surface area $A_0 = 4\pi R_0^2$ of the sphere whose radius is the cosmic particle horizon $R_0$, measured in Planck units. That pure number is given by

$$N_0 = \frac{\pi R_0^2}{l_p^3} \approx 2.0 \times 10^{122}.$$

(4)

Note that the similarity between Eq. (2) and Eq. (4) in itself represents an interesting coincidence. It just so happens that the epoch in which the cosmic coincidence occurs is also the epoch in which the number of available bits of information is of order near the ratio of the Planck density to the vacuum density. That coincidence will be explained in the following paragraphs.

Mena Marugan and Carneiro have suggested that the holographic principle may require that the number of characteristic nucleon volumes contained within the volume of the observable Universe be of order near the total number of bits of information (Mena 2006). In any case, the observable number of characteristic nuclear volumes is

$$\left(\frac{R_0}{l_n}\right)^3 \approx 9.3 \times 10^{122}$$

(5)

where $l_n$ is the Compton wavelength of the nucleon.
The next large pure number of interest to this discussion is taken from considerations of the computational power of the observable Universe. Lloyd has determined that the total number of computations \( N_c \) performed by the observable Universe is given by

\[
N_c = \frac{M_0 c^3 T_0}{h} = 8.7 \times 10^{120},
\]

where \( T_0 \) is the age of the Universe (Lloyd 2002). That \( N_c \) should be of similar order to the number of bits of information in the Universe \( N_0 \) in Eq. (4) is not obvious.

The final two large pure numbers of order near \( 10^{121} \) are

\[
\frac{GM_0^2}{hc} = 1.9 \times 10^{119}
\]

and

\[
\frac{R_0 M_0 c}{h} = 1.4 \times 10^{120}.
\]

The term in Eq. (7) is identical to the square of the ratio of the mass of the observable Universe to the Planck mass. In general, the square of the ratio of any mass \( M \) to the Planck mass \( m_p \) is identical to the ratio of the characteristic gravitational self-energy \( (GM^2/l_m) \) of that mass to its rest energy \( Mc^2 \) where \( l_m \) is the Compton wavelength associated with the mass. The term in Eq. (8) is the ratio of the cosmic particle horizon to the Compton wavelength associated with the mass of the observable Universe.

Insomuch as the large-number coincidence of Dirac and Eddington ever constituted a problem, the terms in Eqs. (2) – (8) also constitute a coincidence problem. The coincidence problem associated with the gallery of pure numbers of order near \( 10^{121} \) in Eqs. (2) – (8) is resolved in the following paragraph using a few physical scaling laws from the standard cosmological model. The first such scaling law follows from the Friedmann-Robertson-Walker-Lemaitre equation and the Raychaudhuri equation (Funkhouser 2006):

\[
\frac{GM}{R} \sim c^2
\]

where \( M \) is the observable mass of the Universe at any time in a matter-dominated Universe and \( R \) is the particle horizon at that same time. This relationship is still roughly satisfied in this era since vacuum-dominance has just begun in this age. The cosmic coincidence between the mass-energy and vacuum energy densities may be expressed as 
\( G\rho_0 \sim \Lambda \), where \( \rho_0 \) is the current density of matter. As a result of the cosmic coincidence and Eq. (9) the following scaling law is satisfied in this era:

\[
\Lambda \sim \frac{c^2}{R_0^2}.
\]

Another scaling law that is useful in resolving the new large-number coincidence

\[
H_0 \sim \sqrt[3]{\Lambda} \sim 1/T_0
\]

where \( H_0 \) is the current value of the Hubble parameter. Eq. (11) results from the FRLW equations, the Raychaudhuri Equation and the fact that the energy density of the cosmos is apparently vacuum-dominated (Funkhouser 2006).

Eqs. (9)-(11) resolve the new large-number coincidence problem as follows. Eq. (3) follows from Eq. (2), (9) and (10). Eq. (4) follows from Eqs. (2) and (10), which is to say that the number of available bits in the Universe is of order near the maximum
number that the Universe could ever contain. The maximum number of bits is achieved as the horizon approaches the critical de Sitter horizon \( R_\Lambda \sim c / \sqrt{\Lambda} \) and the number of available bits is \( (R_\Lambda / l_p)^2 \sim c^5 / (G \Lambda) \). Eq. (6) and Eq. (7) both follow from Eqs. (2), (9), (10) and (11). Eq. (8) follows from Eqs. (2), (9) and (10). The only pure number of order near \( 10^{121} \) that has not been physically linked to Eq. (2) is the term in Eq. (5). All that remains of the coincidence among pure numbers of order near \( 10^{121} \) is therefore

\[
\frac{\epsilon_p}{\epsilon_\Lambda} \sim \frac{R_0^3}{l_p^3},
\]

which no longer constitutes a compelling coincidence problem. This relationship does have an important role in a separate coincidence problem that is detailed in the following section.

II. The Triple Cosmic Coincidence Problem and the Cosmological Constant

There are now at least three apparently distinct coincidence problems among large numbers that are generated only in this epoch. The cosmic coincidence, the Dirac-Eddington large-number coincidence, and the new coincidence among pure numbers of order near \( 10^{121} \) all occur only in this epoch. It is reasonable to hypothesize that some physical connection is behind the coincidence of three such coincidence problems.

It was shown in Ref. (Funkhouser 2006) that the large number coincidence of Dirac and Eddington would follow from the cosmic coincidence if Eq. (1) represented a physical scaling. Curiously, that same scaling law explains why the new coincidence problem among terms of order near \( 10^{121} \) is satisfied in the same epoch as those other two coincidence problems.

If Eq. (1) could be considered a scaling law then the term in Eq. (5) would be physically linked to all of the other terms in Eqs. (2) – (8) since Eq. (5) follows from Eq. (1) and Eq. (10). This in itself is not important since the coincidence among pure numbers of order near \( 10^{121} \) was resolved without connecting Eq. (5) to the other terms. However, if Eq. (5) were the result of physics then each of the Dirac-Eddington numbers of order near \( 10^{40} \) would be physically related to each of the large pure numbers of order near \( 10^{121} \). This would happen because the cube-root of the term in Eq. (5) is \( (R_0/l_p) \), which is one of the pure numbers of order \( 10^{40} \) that follows from Eq. (1) (Funkhouser 2006). So, if Eq. (1) were a physical scaling law then the coincidence of pure numbers of order near \( 10^{121} \) would follow from the Dirac-Eddington large number coincidence, which would follow from the cosmic coincidence, thus resolving the triple-coincidence problem of large numbers.

References:
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