Expectations for LHC from Naturalness: Modified vs. SM Higgs Sector

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Abstract

Common lore has it that naturalness of electroweak breaking in the SM requires new physics (NP) at $\Lambda \lesssim 2 - 3$ TeV, hopefully within the reach of LHC. Moreover the Higgs should be light ($m_h \lesssim 219$ GeV) to pass electroweak precision tests (EWPT). However one should be prepared for “unexpected” (although admittedly unpleasant) results at LHC, i.e. no NP and/or a heavy Higgs. We revisit recent attempts to accommodate this by modifying the SM Higgs sector (using 2-Higgs-doublet models). We find that these models do not improve the naturalness of the SM, and so they do not change the expectations of observing NP at LHC. We also stress that a heavy SM Higgs would not be evidence in favour of a modified Higgs sector, provided certain higher order operators influence EWPT. On the other hand, we show that NP can escape LHC detection without a naturalness price, and with the pure SM as the effective theory valid at LHC energies, simply if the cut-off for top loops is slightly lower than for Higgs loops.

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1 Introduction

It is commonly assumed that the Hierarchy Problem of the Standard Model (SM) indicates the existence of New Physics (NP) beyond the SM at a scale $\Lambda \lesssim \text{few TeV}$, hopefully at the reach of LHC. The argument is well known and goes as follows:

In the SM (taken as the effective theory valid below $\Lambda$) the mass parameter $m^2$ in the Higgs potential

$$V = \frac{1}{2} m^2 h^2 + \frac{1}{4} \lambda h^4,$$

receives dangerous quadratically-divergent contributions [1]. At one-loop, and using a momentum cut-off regularization,

$$\delta_q m^2 = \frac{3}{64\pi^2} \Lambda^2 (3g^2 + g'^2 + 8\lambda - 8\lambda_t^2),$$

where $g, g', \lambda$ and $\lambda_t$ are the $SU(2) \times U(1)_Y$ gauge couplings, the Higgs quartic coupling and the top Yukawa coupling, respectively. The requirement of no fine-tuning between the above contribution and the tree-level value of $m^2$ sets an upper bound on $\Lambda$. E.g. for a Higgs mass $m_h = 115 - 200$ GeV,

$$\left| \frac{\delta_q m^2}{m^2} \right| \lesssim 10 \Rightarrow \Lambda \lesssim 2 - 3 \text{ TeV},$$

where we have implicitly used $m_h^2 = 2\lambda v^2$ and $v^2 = -m^2/\lambda$ (with $v = 246$ GeV). Obviously, if one is stricter about the largest acceptable size of $\delta_q m^2$, then $\Lambda^2$ decreases accordingly. E.g. requiring $|\delta_q m^2| \leq |m^2|$ implies $\Lambda \lesssim 1$ TeV, well inside LHC reach. These numbers are somewhat modified when one considers higher order corrections to eq. (2) (see sect. 2 and refs. [2–4]).

There are several reasons to consider possible departures from this simple SM scenario. First, the above upper bound on $\Lambda$ is generically in some tension with the experimental lower bounds on the suppression scale of higher order operators, derived from electroweak precision tests (EWPT) [5], which typically require $\Lambda \gtrsim 10$ TeV. This is the so-called Little Hierarchy problem. It should be kept in mind, however, that if the NP at $\Lambda \simeq \text{few TeV}$ is “clever enough”, it can still be consistent with EWPT. This is the case of Supersymmetric (SUSY) and Little Higgs models, although these scenarios have their own fine-tuning problems [4,6]. Second, one should be prepared to interpret the possible (though admittedly unpleasant) situation in which no NP is found at LHC, apart from the Higgs, in spite of the previous naturalness arguments.
based on the simple SM Higgs sector. Finally, it could happen that the Higgs found at LHC is beyond the range consistent with EWPT \( m_h \lesssim 186 - 219 \text{ GeV [7]} \), pointing out again to some departure from the ordinary SM Higgs sector.

One of the simplest modifications of the SM Higgs sector one can think of is the two Higgs doublet model (2HDM). In a series of recent papers, Barbieri et al. \[8–10\], have examined the capabilities of such scenario (in different settings) to address the previous questions. Their conclusions are that suitable 2HDMs might raise the scale of new physics above the LHC reach (keeping naturalness under control) with a light or heavy Higgs (depending on the model), in a way consistent with EWPT. Thus these models are claimed to have “improved naturalness” compared with the pure SM.

These models are very interesting for several reasons, namely its relative simplicity and the fact that they can arise from certain SUSY scenarios (e.g. those with low-scale SUSY breaking \[11,4\]), and likely from other kinds of models. Thus a closer look at them is appropriate. In particular, a more careful evaluation of the fine-tuning in these models and the subsequent comparison to the SM performance is needed. This is the first goal of this paper.

The second goal is to examine if the SM alone could be able to cope with the above-mentioned “unexpected” situations at LHC, i.e. a heavy SM Higgs and/or the possibility that LHC does not find any NP.

In section 2 we examine in detail the issue of naturalness of the pure SM (as low-energy effective theory). In particular we specify a sensible criterion to evaluate the degree of fine-tuning in the SM or in any other model (to allow a fair comparison). Also we discuss the role of radiative corrections, and compute the fine-tuning of the SM for different assumptions about \( m_h \) and \( \Lambda \). In section 3 we perform a similar analysis for the 2HDMs proposed in the literature with the aim of reducing the SM fine-tuning. We evaluate the fine-tuning and compare it with the SM one for each scenario. In section 4 we examine whether the SM alone could accommodate a heavy Higgs or a cut-off beyond the LHC reach with no fine-tuning price. Finally, in section 5 we present our conclusions. Besides, we collect in appendix A the contributions to the \( T\)-parameter in generic 2HDMs, applying them to the three cases discussed in section 3.
2 The Standard Model and the Scale of New Physics

In order to evaluate the improvement in naturalness (if any) of generic 2HDMs, or indeed of any alternative to the SM, it is necessary in the first place to perform an evaluation as rigorous as possible of the degree of fine-tuning of the SM itself for given values of the Higgs mass, \( m_h \), and the cut-off scale, \( \Lambda \).

First of all, eq. (2) can be renormalization-group (RG) improved, including leading-log corrections to all orders. In practice, this is taken care of by evaluating the couplings in eq. (2) at the cut-off scale, \( \Lambda \) [2]. Physically this makes good sense: one can think of \( \delta_q m^2 \) as a threshold correction from physics at \( \Lambda \) and it naturally depends on couplings at that scale. Hence

\[
m^2(\Lambda) = m_0^2 + \delta_q m^2 \bigg|_{\Lambda},
\]

where \( m_0^2 \) is the tree-level value of the mass parameter at the scale \( \Lambda \) and

\[
\delta_q m^2 \bigg|_{\Lambda} = \frac{3}{64\pi^2} \Lambda^2 \left[ 3g^2(\Lambda) + g'^2(\Lambda) + 8\lambda(\Lambda) - 8\lambda_t^2(\Lambda) \right].
\]

Second, for a fair comparison, one needs a sensible criterion to quantify the degree of fine-tuning, which should be applied to all the models. Here we follow the standard Barbieri and Giudice criterion [13]: we write the Higgs VEV as \( v^2 = v^2(p_1, p_2, \cdots) \), where \( p_i \) are the fundamental input parameters of the model under study, and define \( \Delta_{p_i} \), the fine tuning parameter associated to \( p_i \), by

\[
\frac{\delta M_Z^2}{M_Z^2} = \frac{\delta v^2}{v^2} = \Delta_{p_i} \frac{\delta p_i}{p_i},
\]

where \( \delta M_Z^2 \) (or \( \delta v^2 \)) is the change induced in \( M_Z^2 \) (or \( v^2 \)) by a change \( \delta p_i \) in \( p_i \). Roughly speaking \( 1/|\Delta_{p_i}| \) measures the probability of a cancellation among terms of a given size to obtain a result which is \( |\Delta_{p_i}| \) times smaller. Due to the statistical meaning of \( \Delta_{p_i} \), makes sense to define the total fine-tuning as [4]

\[
\Delta \equiv \left[ \sum_i \Delta_{p_i}^2 \right]^{1/2}.
\]

(Other definitions, such as \( \Delta = \text{Max}\{\Delta_{p_i}\} \), are possible and have been used in the literature.)

1 Although in many cases both definitions give very similar results (typically one single \( \Delta_{p_i} \) dominates \( \Delta \)) we believe definition (7) is more satisfactory conceptually. As an (extreme) example consider the case of an observable \( O \) that depends on a large number \( N \) of input parameters, say \( O = \sum_i \alpha_i p_i \) where \( \alpha_i \sim \mathcal{O}(1) \) with random signs and with the measured value of \( O \) and the natural values of the \( p_i \) being of the same order. In such case all \( \Delta_{p_i}^2 \sim 1 \) but the fine-tuning is \( \mathcal{O}(\sqrt{N}) \) (this example would correspond to a random walk where one expects such wandering of \( O \) away from 1).
In order to evaluate the fine-tuning $\Delta$ of the SM, one should first identify the relevant unknown parameters, $p_i$, to be plugged in eqs. (6) and (7). From eq. (2), we see that the most relevant ones are $\Lambda$ and $\lambda$ (the experimentally well-known couplings $g, g'$ and $\lambda_t$ have a negligible impact on the fine-tuning [14]). Therefore, we will have

$$\Delta \simeq \sqrt{\Delta_{\lambda}^2 + \Delta_{\Lambda}^2}. \quad (8)$$

As stressed in [10], it could happen that the NP that cancels the quadratically divergent corrections is different for the loops involving the top, the Higgs, etc. In that case, one should introduce different cut-offs:

$$\delta_q m^2 = \frac{3}{64\pi^2} \left[ (3g^2 + g'^2)\Lambda_g^2 + 8\lambda \Lambda_{\lambda_h}^2 - 8\lambda_t^2 \Lambda_t^2 \right]. \quad (9)$$

In this case one should consider $\Delta_{\Lambda_t}$ and $\Delta_{\Lambda_h}$ (the most relevant fine-tuning parameters) separately and use

$$\Delta \simeq \sqrt{\Delta_{\Lambda_t}^2 + \Delta_{\Lambda_h}^2 + \Delta_{\lambda}^2}. \quad (10)$$

Fig. 1 shows the naturalness upper bounds on $\Lambda$, derived for $\Delta_{\Lambda_t}, \Delta_{\Lambda_h} = 10$, as a function of $m_h$ (red and blue dashed lines). Notice that, keeping $\Lambda$ fixed, $\Delta_{\Lambda_t}$ decreases with increasing $m_h$ (or, equivalently, for fixed $\Delta_{\Lambda_t}$, the larger $m_h$, the larger may $\Lambda_t$ be). This follows trivially from $v^2 = -m^2/\lambda$ and the one-loop expression for $\delta_q m^2$, eq. (2). Then

$$\Delta_{\Lambda_t} \simeq \frac{3 \lambda^2 \Lambda_t^2}{4\pi^2 \lambda v^2} = \frac{3 \lambda^2 \Lambda_t^2}{2\pi^2 m_h^2}. \quad (11)$$

This fact has been used sometimes to suggest that a heavy Higgs behaves better for naturalness than a light one. A similar reasoning would indicate that $\Delta_{\Lambda_h}$ is independent of $m_h$:

$$\Delta_{\Lambda_h} \simeq \frac{-3 \lambda \Lambda_h^2}{4\pi^2 \lambda v^2} = \frac{-3 \Lambda_h^2}{4\pi^2 v^2}. \quad (12)$$

RG effects cause important deviations from this tree-level expectation because the ratio of $\lambda$'s that enters $\Delta_{\Lambda_h}$ is roughly $\lambda(\Lambda)/\lambda(m_h)$. Using the RGE for $\lambda$

$$\frac{d\lambda}{d \ln Q} = \frac{1}{16\pi^2} \left[ 24\lambda^2 + \frac{3}{8} (3g^4 + 2g^2g'^2 + g'^4) - 6\lambda^4 - 3\lambda(3g^2 + g'^2 - 4\lambda_t^2) \right], \quad (13)$$

it follows that, in the low $m_h$ range, $\lambda$ is small and runs to smaller values in the ultraviolet (UV), giving $\lambda(\Lambda)/\lambda(m_h) < 1$ and weakening the naturalness bound on $\Lambda_h$, as shown in fig. 1. In the high $m_h$ range, $\lambda$ is bigger at $m_h$ and increases in the UV so that $\lambda(\Lambda)/\lambda(m_h) > 1$, tightening the bound on $\Lambda_h$. The effect is stronger for larger $\Lambda.$
Figure 1: SM naturalness upper bounds on the NP scale for $\Delta = 10$ as a function of $m_h$ with $\Delta = \Delta_{A_t}$ (red dashed), $\Delta_{A_h}$ (blue dashed), $(\Delta_{A_t}^2 + \Delta_{A_h}^2)^{1/2}$ (solid green). The black lines show the corresponding bounds for a single cut-off, $\Lambda$: $\Delta = \Delta_{A}$ (black dashed), $(\Delta_{A}^2 + \Delta_{h}^2)^{1/2}$ (black solid).

The combined $\Delta = (\Delta_{A_t}^2 + \Delta_{A_h}^2)^{1/2}$ interpolates between both behaviours: it is dominated by $\Delta_{A_t}$ at low $m_h$ and by $\Delta_{A_h}$ at large $m_h$ (see the solid green line $\Delta = 10$ in Fig. 1). Notice that, although $\Lambda_t$ and $\Lambda_h$ are independent parameters, they are taken to be numerically equal in this figure.

On the other hand it is perfectly possible that the NP that cancels the quadratic divergence associated to the top loop is the same that cancels that of the Higgs loops. In other words, both could be different sectors of a single piece of NP, with $\Lambda_t \sim \Lambda_h$ (we discuss this in more detail in section 4). This can occur, for instance, when the NP corresponds to the supersymmetric partners of the SM particles, with all masses determined by a unique scale of SUSY breaking. Then one has to evaluate a single $\Delta_{A}$; the corresponding contour plot $\Delta_{A} = 10$ is shown by the black dashed line in Fig. 1. There we can notice a throat (sometimes called “Veltman’s throat”) at $m_h \sim 225$ GeV, that arises from an accidental cancellation between the various terms in eq. (2),
in particular between the top and Higgs contributions (which have opposite sign). This fact has been used since long ago to suggest a preferred mass for the Higgs [1]. However, at present the Higgs mass (and thus the $\lambda$ coupling) is unknown and one has to include $\Delta\lambda$ in the fine-tuning analysis [4]. The combined $\Delta = (\Delta_{\lambda}^2 + \Delta_{\lambda}^2)^{1/2} = 10$ bound is shown by the solid black line in Fig. 1, where one can see how the throat reduces to a bump. In summary, the solid lines of Fig. 1 show the degree of fine-tuning of the SM for given $\{\Lambda, m_h\}$ under the assumption of independent or correlated $\Lambda_t$ and $\Lambda_h$ cut-offs. Since the contributions to $\delta_q m^2$ proportional to $\Lambda_t^2$ and $\Lambda_h^2$ have opposite signs, assuming a unique cut-off gives a less severe fine-tuning $\Delta \sim a\Lambda^2 - b\Lambda^2$ (with $a, b > 0$) than for independent cut-offs, $\Delta \sim \sqrt{(a\Lambda_f^2)^2 + (b\Lambda_h^2)^2}$, see eq. (10). (The opposite will be the case for contributions with the same sign, simply because $|a + b| > \sqrt{a^2 + b^2}$.)

It is useful, for future comparisons, to show the naturalness bounds for $\Delta = 10$ and 100 in both cases (independent or correlated $\Lambda_t$ and $\Lambda_h$). These are given in Fig. 3.

Figure 2: SM fine-tuning associated to $\Lambda_h$ as a function of the Higgs mass for different values of the cut-off scale.

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\(\Delta_{\Lambda_h}\) vs. \(m_h\) (GeV)

- \(\Lambda = 10\) TeV
- \(\Lambda = 5\) TeV
- \(\Lambda = 3\) TeV

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\(\Delta = 10\) bound

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\(\Delta = 100\) bound
Figure 3: SM naturalness upper bounds on the NP scale for $\Delta = 10$ and 100 as a function of $m_h$ for independent $\Lambda_t$ and $\Lambda_h$ cut-offs: $\Delta = \Delta_{\Lambda_t}$ (red dashed), $\Delta_{\Lambda_h}$ (blue dashed), $(\Delta^2_{\Lambda_t} + \Delta^2_{\Lambda_h})^{1/2}$ (solid black). The lower and upper limits on $m_h$ (from LEP and EWPT) are also shown.

and Fig. 4 respectively, where we have also shown the present LEP lower limit on $m_h$ ($m_h \geq 114.4$ GeV [15]) and the upper bounds from EWPT, $m_h < 186$ GeV, or $m_h < 219$ GeV if the LEP lower limit is included in the fit (both numbers at 95% confidence-level) [7]. For later use we introduce as in [10] the notation $m_{EW}$ for such EWPT upper bound on $m_h$.

As is clear from Figs. 3 and 4, the main difference between correlated and independent cut-offs is the presence or absence of the Veltman’s throat (even if substantially reduced). Apart from this, both estimates of the fine-tuning show similar trends. Notice that choosing $\Lambda \sim 10$ TeV (to avoid problems with EWPT) requires $\Delta \gtrsim 100$. If $\Lambda \sim 2$ TeV one generically has problems with EWPT, but this depends on the particular sort of NP showing up at that scale: there could be no problem in the presence of some symmetry (e.g. custodial symmetry) or a loop suppression (e.g. due to $R$-parity or $T$-parity, etc.). In such cases the Little Hierarchy problem would be absent or softened (i.e. there would be no tension between the values of $\Lambda$ required by EWPT and...
Let us finally discuss a technical point concerning our evaluation of the various $\Delta_{p_i}$ in eq. (6). From $v^2 = -m^2/\lambda$

$$\frac{\delta v^2}{v^2} = \frac{\delta m^2}{m^2} - \frac{\delta \lambda}{\lambda}.$$  \hspace{1cm} (14)

Now, $\delta \lambda/\lambda$ is normally much smaller than $\delta m^2/m^2$. In addition, only $\delta m^2/m^2$ contains information about the quadratically divergent corrections, which are the source of the hierarchy problem on which we are focusing in this paper. So, to compute $\Delta_{p_i}$ in (6), we can approximate the tuning in $v^2$ by the tuning in $m^2$ using

$$\frac{\delta v^2}{v^2} \simeq \frac{\delta m^2}{m^2} \simeq \frac{\delta \{\delta q m^2\}}{m^2} \bigg|_{\Lambda},$$ \hspace{1cm} (15)

where we have evaluated $\delta m^2/m^2$ at the scale $\Lambda$ and used eq. (4). This procedure is accurate, simplifies the computation and, furthermore, it makes sense since the actual cancellation between the tree-level and the radiative contributions to $m^2$ occurs at the scale $\Lambda$.  

Figure 4: Same as in Fig. 3 but using a single cut-off $\Lambda$. $\Delta = \Delta_{\Lambda}$ (blue dashed lines), $(\Delta_{\Lambda}^2 + \Delta_{\Lambda}^3)^{1/2}$ (black solid lines).
3 Examples of Modified Higgs Sectors

In this section we revisit three modifications of the Higgs sector that have been recently proposed in the literature with the general aim of improving the naturalness of the SM.

3.1 The Inert Doublet Model (IDM)

Motivated by the reduction of $\Delta \Lambda_t$ for large $m_h$, Barbieri et al. have explored in [10] the possibility of reconciling a large $m_h$ with EWPT in the most economical way. The model, called the “Inert Doublet Model”, is a particular 2HDM with parity $H_2 \to -H_2$.\(^{(16)}\)

This makes $H_1$ the only Higgs field coupled to matter. The scalar potential is

$$V = \mu_1^2|H_1|^2 + \mu_2^2|H_2|^2 + \lambda_1|H_1|^4 + \lambda_2|H_2|^4 + \lambda_3|H_1|^2|H_2|^2$$

$$+ \lambda_4|H_1^\dagger H_2|^2 + \frac{\lambda_5}{2}[(H_1^\dagger H_2)^2 + \text{h.c.}]. \quad (17)$$

The next assumption is that the parameters of the potential are such that only $H_1$ develops a VEV. $H_2$ does not couple to fermions and does not get a VEV (thus the name “inert doublet”) but it has weak and quartic interactions, playing an active role in EWPT. The Higgs VEV, $v = \sqrt{2} \langle H_1^0 \rangle$, is given by the SM-like relation

$$v^2 = -\frac{\mu_1^2}{\lambda_1}. \quad (18)$$

We can write

$$H_1 = \left(\frac{G^+}{(v + h^0 + iG^0)/\sqrt{2}}\right), \quad H_2 = \left(\frac{H^+}{(H^0 + iA^0)/\sqrt{2}}\right). \quad (19)$$

where $G^+, G^0$ are the usual SM Goldstones, $h^0$ is the SM-like Higgs and $H^+, H^0$ and $A^0$ are charged, scalar and pseudoscalar extra “Higgs” states. The Higgs $h^0$ has a SM-like mass

$$m_{h^0}^2 = 2\lambda_1 v^2. \quad (20)$$

The states coming from the inert doublet $H_2$ have a common mass roughly given by $\mu_2^2$, with some splittings induced by electroweak symmetry breaking (EWSB):

$$m_f^2 = \mu_2^2 + \lambda_f v^2, \quad (21)$$
with \( I = \{ H^+, H^0, A^0 \} \) and

\[
\begin{align*}
\lambda_{H^+} &= \lambda_3 , \\
\lambda_{H^0} &= \lambda_3 + \lambda_4 + \lambda_5 , \\
\lambda_{A^0} &= \lambda_3 + \lambda_4 - \lambda_5 .
\end{align*}
\]

These mass splittings inside the \( H_2 \) multiplet break custodial symmetry and will play a crucial role in providing a contribution to \( \Delta T \) of the right size to compensate that coming from a heavy \( h^0 \). The SM Higgs gives a contribution to \( T \) that grows logarithmically with \( m_{h} \):

\[
T \sim -\frac{3}{8\pi \cos^2 \theta_w} \ln \left( \frac{m_{h}}{M_Z} \right)
\]

which for \( m_{h} \sim 400 - 600 \) GeV is excluded at more than 99.9% C.L. (The experimental value is \( T^{\exp} \simeq 0.1 \pm 0.15 \). The inert doublet provides an additional contribution (see appendix A)

\[
\Delta T \simeq \frac{1}{12\pi s_W^2 M_W^2} (m_{H^+} - m_A) (m_{H^+} - m_S) ,
\]

which, if \( \Delta T \simeq 0.25 \pm 0.1 \), compensates the too negative contribution (23) getting agreement with experiment. This requires [10]

\[
(m_{H^+} - m_A)(m_{H^+} - m_S) = M^2, \quad M = 120^{+20}_{-30} \text{ GeV} ,
\]

which seems to be a reasonable value for EWSB mass splittings. For later use, note that for \( \mu^2 \gg \lambda I \nu^2 \) the previous condition reads

\[
(\lambda_{H^+} - \lambda_{H^0})(\lambda_{A^0} - \lambda_{H^0}) \frac{\nu^4}{4\mu^2} = (\lambda_4^2 - \lambda_5^2) \frac{\nu^4}{4\mu^2} \simeq M^2 .
\]

This shows explicitly that the \( \Delta T \) contribution of the inert doublet decouples with increasing \( \mu^2 \), as it should. More importantly for our purposes, it also shows that, for large \( \mu^2 \), one needs large \( \lambda_4^2 - \lambda_5^2 \) to get the right value of \( \Delta T \).

This model is an interesting example of how to accommodate the possible detection of a heavy Higgs at the LHC. Moreover, the proponents of the model stress that having a heavy Higgs relaxes the SM Hierarchy Problem, since it allows for larger \( \Lambda_I \). The price would be that the NP responsible for the cancellation of the SM quadratic divergences could escape LHC (although we could observe a modified Higgs sector). We would
like to discuss these naturalness aspects here. As shown in sect. 2, a large $m_h$ does not necessarily imply less fine-tuning. In the SM context, although $\Delta\Lambda_c$ decreases with $m_h$, $\Delta\lambda_h$ increases, eventually dominating the total fine-tuning. This happens because of the RG increase of $\lambda$ from $m_h$ to $\Lambda$, where the quadratic radiative correction $\delta q m^2$ has to be computed. In the present model something similar is likely to take place: let us focus for the moment on the impact of $\delta q \mu^2$ on the fine-tuning. From eq. (18), the relevant mass parameter for EW breaking is $\mu^2$, which receives the quadratically-divergent radiative correction

$$\delta q \mu^2 = \frac{3}{64\pi^2} \left[ -8\lambda^2 + (3g^2 + g')^2 \Lambda^2 + 8\lambda_1 \Lambda H_1 + \frac{4}{3}(2\lambda_3 + \lambda_4)\Lambda H_2 \right] ,$$

where, following the assumption of ref. [10], we have allowed independent cut-offs for different contributions. Eq. (27) has a structure which is very similar to the SM eq. (5), with the role of $\lambda$ played by $\lambda_1$. The two main differences are the presence of the $\Lambda H_2$ term and the RGE for $\lambda_1$, which is now:

$$\frac{d\lambda_1}{d \ln Q} = \frac{1}{16\pi^2} \left[ 24\lambda^2 + \lambda_3^2 + (\lambda_3 + \lambda_4)^2 + \lambda_3^2 + \lambda_5^2 \right] \right] ,$$

The $\lambda_1$ and gauge terms in this RGE are as in the SM, but the additional terms cause $\lambda_1$ to grow with the scale more quickly than $\lambda$ in the SM [see eq. (13)]. Both differences contribute to produce a larger fine-tuning than in the SM for a given $m_h$ (although, of course, EWPT can now be under control for much higher $m_h$). The best case, taking both effects into account, will occur for small $\lambda_{3,4,5}$. On the other hand, the $\lambda_i$ couplings cannot be chosen at will. As stressed above, they have to be consistent with the desired $\Delta T$. Besides this, there are additional constraints to ensure the stability of the vacuum and the perturbativity of the $\lambda_i$ couplings [10].

Following the analysis performed in [10], we choose the IDM parameters as follows. First we remind the reader that the lightest state in $H_2$ is stable (thanks to the $H_2 \rightarrow -H_2$ symmetry) and is therefore a candidate for dark matter [10]. This forces one to choose $m_{H^+} > m_{H^0}, m_{A^0}$ so that this DM candidate is neutral. We follow [10] and call $m_L = \min\{m_{H^0}, m_{A^0}\}$ the mass of this lightest state and $m_{NL} \equiv m_L + \Delta m = \max\{m_{H^0}, m_{A^0}\}$. Once we choose $m_L$ and $\Delta m$, $m_{H^+}$ is determined by eq. (25). In fact, $\lambda_4$ and $\lambda_5$ are then fixed once the inert spectrum has been arranged in this way.
On the other hand, $\lambda_1$ is determined by $m_h$. Of the two remaining quartic Higgs couplings we take $\lambda_2 \simeq 0$ (its value is not really important) and we choose $\lambda_3$ such that $\lambda_L \equiv \lambda_3 + \lambda_4 - |\lambda_5|$ satisfies perturbativity and vacuum stability constraints [10]. For numerical work we then take $M = 120$ GeV, $\Delta m = 50$ GeV and $\lambda_L = -0.5$. This choice gives us a wide range for $m_L$ to agree with the needed $\Delta T$ and to meet all other constraints [10].

Fig. 5 shows the total fine-tuning $^3 \Delta = (\sum_{a=t,H_1,H_2} \Delta_{\lambda_a}^2 + \sum_i \Delta_{\lambda_i}^2)^{1/2}$ versus the cut-off $\Lambda$ (for simplicity we show the results when all the cut-offs are numerically equal) for $m_h = 400$ and 600 GeV (red and green solid lines) for the particular case $m_L = m_h$. Notice that the case with $m_h = 400$ GeV behaves better, as expected from the general trend discussed in section 2 (see Fig. 2). The violet dashed-dotted line corresponds to $m_h = 115$ (SM) and 219 GeV, using independent cut-offs.

$^3$ We evaluate $\Delta$ by approximating the tuning in $\nu^2$ by the tuning in $\mu^2$ in the spirit of eq. (15) for the SM.
to $m_h = 400$ GeV and $m_L = 150$ GeV. This is a case with small $\lambda_{3,4,5}$ leading to a fine-tuning that is sensibly smaller, in agreement with the above discussion. In fact, this line is close to the optimal situation in this scenario, see below. In any case, it is clear that reaching $\Lambda > 2$ TeV requires a substantial fine-tuning ($\Delta > 10$). To see if this model improves the situation over the SM one, we have also plotted the SM fine-tuning $\Delta = (\Delta_{\Lambda_T}^2 + \Delta_{\Lambda_h}^2 + \Delta_{\lambda}^2)^{1/2}$ as a band between the limit cases $m_h = 115$ GeV and 219 GeV (dashed black lines). Clearly, the situation of the IDM can be hardly considered an improvement in naturalness, especially if the SM Higgs is near its upper EWPT bound. This conclusion is strengthened if one assumes a universal cut-off for all the contributions in $\delta_q m^2$ and $\delta_q \mu_1^2$ (for the SM and IDM respectively). As discussed in sect. 2 this is a perfectly reasonable possibility. The corresponding fine-tuning curves are shown in Fig. 6, where the SM (for $m_h$ around 200 GeV) is in a much better position, even when the optimal choice $m_L = 150$ GeV is used for the IDM.

Our conclusion at this point is therefore that, although the IDM could offer an
appealing explanation for the hypothetical detection of a heavy Higgs, it does not improve the naturalness of EWSB with respect to the SM. This conclusion would be softened if i) one assumes independent cut-offs, or ii) if one requires $\Delta$ to be $O(1)$. Notice that in the latter case $\Lambda$ is smaller, and then the effect of the RG running on $\Delta_{\Lambda H}$ (which is harmful for the fine-tuning) is less important. Both assumptions were taken in ref. [10]. However, we believe that to require $\Delta = 1$ is excessively severe, especially taking into account that a small $\Lambda$ generically leads to problems with EWPT.

Let us now examine how the fine-tuning in $\mu_2^2$ changes with $m_L$: this is shown in Fig. 7 for $m_h = 400$ GeV. Naturalness clearly favours low values of $m_L$. This is because high $m_L$ means high $\mu_2$ and, in order to satisfy the $\Delta T$ constraint, larger values of $\lambda_4$ and/or $\lambda_5$ are needed [see eq. (26)]. This then enhances $\delta_q \mu_1^2$ [see eq. (27)] and thus $\Delta^{(\mu_1)}$. We can also see from this plot that choosing a large $m_h$ (and thus a large $\lambda_1$) limits the scale up to which we can maintain perturbativity. The dashed line shows this limit (for $\lambda_1 = 4\pi$).
The $\delta q \mu_1^2$ contribution is not the only source of potential fine-tuning in the IDM: $\delta q \mu_2^2$, which is given by

$$\delta q \mu_2^2 = \frac{3}{64\pi^2} \left[ (3g^2 + g'^2)\Lambda_y^2 + 8\lambda_2\Lambda_{H_2}^2 + \frac{4}{3}(2\lambda_3 + \lambda_4)\Lambda_{H_1}^2 \right], \quad (29)$$

can also be an additional source of (independent) fine-tuning. Let us call the corresponding fine-tunings $\Delta^{(\mu_1)}$ and $\Delta^{(\mu_2)}$. Notice from (21) that a small $m_L$ [the optimal choice for $\Delta^{(\mu_1)}$] typically requires small $\mu_2^2$. Then, the larger $\delta q \mu_2^2$, the more unnatural this possibility is. Fig. 7 shows $\Delta^{(\mu_2)}$, evaluated in the same way as $\Delta^{(\mu_1)}$. As expected, for $\Lambda$ fixed, $\Delta^{(\mu_2)}$ increases for decreasing $m_L$, which balances the behavior of $\Delta^{(\mu_1)}$. In fact one should multiply both fine-tunings, as they correspond to different quantities, but, even if one does not, it is clear that the improvement gained in $\Delta^{(\mu_1)}$ by going to small $m_L$ is lost by this additional source of fine-tuning in $\mu_2$.

From the previous discussion we finally conclude that i) although the IDM model is very interesting, it does not improve the naturalness of the SM; ii) the structure of the model requires an additional fine-tuning in $\mu_2^2$ of a size that can be similar to that required for a correct EW breaking (i.e. the tuning associated to $\mu_1^2$). This result is common in models with a structure more complicated than that of the SM. Normally, epicycles are penalized in naturalness estimates (which is somehow satisfactory). This happens e.g. in Little Higgs Models [6] and also here, though in a much less dramatic way. Therefore the possibility that the NP responsible for the cancellation of the dangerous quadratic divergences could escape LHC detection is similar in the IDM and in the SM. In the IDM case, however, a Higgs sector different from the SM one would be observed.

### 3.2 The Barbieri-Hall Model

This model (BH Model from now on), presented in ref. [9], is another particular version of a 2HDM aimed at improving the naturalness of the SM. The idea here is to maintain the lightest Higgs below the EWPT bound ($m_h \lesssim 219$ GeV), but coupling the top quark mainly to the heaviest Higgs, so that $\Delta_{\lambda_t}$ can be much smaller than in the SM.

The Higgs potential has the form (17), but now the parameters are chosen so that both $H_1$ and $H_2$ get VEVs: $(H_i^0) = v_i/\sqrt{2}$, with $v_1^2 + v_2^2 = (246$ GeV$)^2$. More precisely, the minimization conditions read

$$\mu_1^2 + \lambda_1 v_1^2 + \frac{1}{2} \tilde{\lambda} v_2^2 = 0,$$
\[ \mu_2^2 + \lambda_2 v_2^2 + \frac{1}{2} \tilde{\lambda} v_1^2 = 0, \]  

(30)

where \( \tilde{\lambda} \equiv \lambda_3 + \lambda_4 + \lambda_5 \). In addition, a discrete symmetry is imposed so that only \( H_2 \) couples to the up-quarks. The squared mass matrix for the two neutral Higgs bosons is

\[ \begin{pmatrix} 2\lambda_1 v_1^2 & \tilde{\lambda} v_1 v_2 \\ \tilde{\lambda} v_1 v_2 & 2\lambda_2 v_2^2 \end{pmatrix}. \]  

(31)

Assuming that the 22 entry is the largest and the off-diagonal entry is small, the two mass eigenvalues \( m_\pm^2 \) are

\[ m_+^2 \simeq 2\lambda_2 v_2^2, \quad m_-^2 \simeq 2\left( \lambda_1 - \frac{\tilde{\lambda}^2}{4\lambda_2} \right) v_1^2. \]  

(32)

The important point is that, in this case, the lightest and the heaviest neutral Higgs bosons are mainly along the \( h_1 \) and \( h_2 \) directions respectively: 

\[ h_- = \cos \alpha \ h_1 + \sin \alpha \ h_2, \quad h_+ = \cos \alpha \ h_2 - \sin \alpha \ h_1 \] 

with a small mixing angle:

\[ \alpha \approx \frac{-\tilde{\lambda}}{2\lambda_2 \tan \beta}, \]  

(33)

where \( \tan \beta = v_2/v_1 \). Therefore, the lightest Higgs, \( h_- \), has almost no coupling to the up-quarks, and in particular to the top. The quadratically divergent corrections to the \( \mu_i^2 \) mass parameters are

\[ \delta q \mu_1^2 = \frac{3\Lambda_t^2}{64\pi^2} \left[ (3g^2 + g'^2) + 8\lambda_1 + \frac{4}{3}(2\lambda_3 + \lambda_4) \right], \] 

\[ \delta q \mu_2^2 = \frac{3\Lambda_t^2}{64\pi^2} \left[ (3g^2 + g'^2) + 8\lambda_2 + \frac{4}{3}(2\lambda_3 + \lambda_4) - 8\lambda_t^2 \right], \]  

(34)

where we have taken a universal cut-off for simplicity, but each term can be multiplied by a different cut-off if desired. It is explicit from (34) that \( \lambda_t \) only affects \( \mu_2^2 \).

Let us focus first on the impact of \( \delta q \mu_2^2 \) on the fine-tuning, as the authors of [9] do. From eqs. (30) and the smallness of \( \alpha \), we see that \( v_2^2 = v^2 \sin^2 \beta \simeq -\mu_2^2/\lambda_2 \). So, the fine-tuning associated to \( \Lambda_t \) is given by

\[ \Delta_{\Lambda_t} \simeq \left| \frac{\Lambda_t}{\mu_2^2} \frac{\delta q \mu_2^2}{\delta \Lambda_t} \right| \Lambda_t \approx \frac{3\lambda^2}{4\pi^2} \frac{\Lambda_t^2}{\lambda_2 v_2^2} \simeq \frac{3\lambda^2}{2\pi^2} \frac{\Lambda_t^2}{m_+^2}. \]  

(35)

Hence, \( \Delta_{\Lambda_t} \) is suppressed for large \( m_+ \), even if \( h_- \) is light. This trick was used in ref. [9] to push \( \Lambda_t \) to quite high values: taking \( m_+ \simeq 500 - 1000 \text{ GeV} \), \( \Lambda_t \) can be as large as 2 TeV, even if one demands \( \Delta_{\Lambda_t} \leq 1 \).
The previous analysis, however, does not take into account the contribution $\Delta_{A_{H_2}}$ to the total fine-tuning. This effect was not considered in [9] but it is the one that puts the strongest constraint on the scale of NP. Due to the large size assumed for $m_+$, $\lambda_2$ is large and gets even larger in the UV, through RG running. As we have discussed above, the RG increase of $\lambda_2$ from $m_+$ to $\Lambda$ enhances the corresponding contribution to $\delta_{qH_2}$, and thus the value of $\Delta_{A_{H_2}}$. Actually, beyond some scale $\Lambda_{NP}$ not too far from the EW scale, $\lambda_2$ gets non-perturbative (say $\lambda_2 \geq 4\pi$). Beyond $\Lambda_{NP}$ the model enters a strong-coupling regime and eventually $\lambda_2$ reaches a Landau Pole at some higher scale. The perturbative limit on $\Lambda$ is, for some choices of the parameters, even below the previous estimates of $\Lambda$. In any case, we are interested in the cut-off scale that produces a total fine-tuning $\Delta \leq 10$ (or any other sensible value) and this typically will happen below $\Lambda_{NP}$.

Of course, the effect of the RGE for $\lambda_2$ depends on the values of other couplings, especially on $\lambda_{3,4,5}$. Explicitly,

$$
\frac{d\lambda_2}{d\ln Q} = \frac{1}{16\pi^2} \left[ 24\lambda_2^2 + \lambda_3^2 + (\lambda_3 + \lambda_4)^2 + \lambda_5^2 
+ \frac{3}{8}(3g^4 + 2g^2g'^2 + g'^4) - 6\lambda_4^4 - 3\lambda_2(3g^2 + g'^2 - 4\lambda_2^2) \right].
$$

(36)

Notice that the extra couplings always contribute to strengthen the RG increase of $\lambda_2$, so a good choice for fine-tuning purposes is to minimize their effect by taking their values as small as possible. This choice also minimizes the quadratically-divergent corrections, $\delta_{qH_1,2}$, as given by eqs. (34). Taking into account that the masses of the charged and the pseudoscalar Higgs are given by

$$
m_{H^+}^2 = -\frac{1}{2}(\lambda_4 + \lambda_5)v^2,
$$

$$
m_{A^0}^2 = -\lambda_5v^2,
$$

(37)

it seems that $\lambda_3 = \lambda_4 = 0$ might be an optimal choice. Then $\lambda_5$ must be negative, with a lower bound (in absolute size) given by the lower bound on $m_{H^+}$. For $\tan \beta = 0.8 - 1$ [9] this bound, coming from $b \to s\gamma$ constraints, is about $200 - 250$ GeV [16].

Using a unique cut-off, $\Lambda_t \equiv \Lambda_{H_t} \equiv \Lambda$, the naturalness upper bound on $\Lambda$, as a function of $m_+$ for $m_- = 115$ GeV is shown in Fig. 8, for $\Delta^{(\mu_2)} = \{[\Delta_{A_{H_2}}^{(\mu_2)}]^2 + \sum_{i}[\Delta_{A_i}^{(\mu_2)}]^2\}^{1/2} = 10$. The perturbativity limit, $\Lambda_{NP}$, is also shown. The origin of the lower bound $m_+ \gtrsim 310$ GeV in Fig. 8 is the following. From the mass matrix (31) it
follows that for a given value of $m_-$, there is a minimal value of $m_+$, which occurs for $\lambda_2 v_2^2 = \lambda_1 v_1^2$ and $|\tilde{\lambda}|$ as small as possible. More precisely

$$(m_+^2)_{\text{min}} = m_-^2 + 2|\tilde{\lambda}| v_1 v_2.$$  \hspace{1cm} (38)

In our case, with $\tan \beta = 1$ and $\lambda_3 = \lambda_4 = 0$ we get

$$(m_+^2)_{\text{min}} = m_-^2 + 2m^2_{H^+}.$$ \hspace{1cm} (39)

which, for $m_- = 115 \text{ GeV}$ and $m_{H^+} = 200 \text{ GeV}$ gives $m_+ \gtrsim 310 \text{ GeV}$. We have checked that with our choice of parameters $T$ is in agreement with the experimental value in all the range shown for $m_+$. In fact, for the largest values of $m_+$ a cancellation similar to that in the IDM (between $\Delta T_\parallel$ and $\Delta T_\perp$, see appendix A) is at work.

For comparative purposes we also show in Fig. 8 the naturalness bound in the SM for the same Higgs mass (see Fig. 3). Clearly, there is no substantial improvement with respect to the SM. Furthermore, one should take into account the fine-tuning
Figure 9: Same as in Fig. 8, but using $\lambda_4 = \lambda_5$ and $\hat{\lambda} = 0$. The two green dashed lines correspond to the $m_+$ upper bounds from EWPT.

associated to $\mu_2^2$, i.e. $\Delta^{(\mu_1)}$. Actually, it is clear from the same figure that, although $\Delta^{(\mu_2)}$ decreases with $m_+$ (for fixed $\Lambda$), $\Delta^{(\mu_1)}$ goes the opposite way. In fact, $\Delta^{(\mu_1)}$ is very restrictive, even more than $\Delta$ in the SM for $m_h = m_-$. The reasons for this behaviour are, first that having $\hat{\lambda} \neq 0$ forces $\lambda_1(m_h)$ to be larger than $\lambda$ for the same $m_- = 115$ GeV [see eq. (32)]. Second, the RG evolution of $\lambda_1$ is very different from that of $\lambda$ precisely because $H_1$ does not couple to the top quark: while in the SM $\lambda$ gets much weaker in the UV, $\lambda_1$ does not change much. These effects cause the $\lambda_1$ contribution in $\delta_q\mu_1^2$ to be much larger than in the SM for the same $m_- = 115$ GeV. The global situation is then similar to that in the IDM (see Fig. 7): the improvement gained in $\Delta^{(\mu_2)}$ by going to small $m_+$ is counterbalanced by the $\Delta^{(\mu_1)}$ fine-tuning. Again, one should multiply both fine-tunings, but even if we do not (i.e. if we are conservative) it is clear how models with more structure are penalized in naturalness considerations.

One can try to change the $\lambda_3 = \lambda_4 = 0$ assumption in order to relax the lower bound on $m_+$ and to reduce the fine-tuning. A convenient choice is to use $\lambda_4 = \lambda_5$. 
and adjust $\lambda_3$ to get $\bar{\lambda} = 0$. This allows to minimize the splitting between $m_+^2$ and $m_\tau^2$. The result for the tuning is shown in Fig. 9. By going to lower values of $m_+$ we access a region of parameter space where a Veltman-like cancellation takes place in $\Delta^{(\mu_2)}$ between the quadratically divergent contributions of the top and the Higgses. On the other hand, the tuning $\Delta^{(\mu_1)}$ is very similar to the SM one but slightly worse: now, in addition to the different RG-evolution of $\lambda_1$ explained above, $\lambda_3$ is not small and also contributes to $\delta_q\mu_1^2$. (In fact, one cannot make $\bar{\lambda} = 0$ with $\lambda_3 = 0$ due to the lower bound on $m_{H^+}$.) Once more it is clear that the improvement gained in $\Delta^{(\mu_2)}$ by going to small $m_+$ is counterbalanced by the $\Delta^{(\mu_1)}$ fine-tuning.

With $\lambda_4 = \lambda_5$ one has $m_A = m_{H^+}$ and the Higgs contribution to $T$ is very simple (see appendix A). The constraint from EWPT can then be written as [9]

$$m_+^2 < m_{EW}^2 \left[ \frac{m_{EW}^2}{m_-^2} \right]^{1/\tan^2\beta}. \quad (40)$$

For $m_{EW} = \{186, 219\}$ GeV and $m_- = 115$ GeV, this gives $m_+ < \{301, 417\}$ GeV and these bounds are represented as well in the figure.

Finally, one can calculate the fine-tuning using uncorrelated $\Lambda_t$ and $\Lambda_{H_i}$ cut-offs. This does not improve the naturalness since the accidental cancellation in $\Delta^{(\mu_2)}$ around $m_+ \sim 200$ GeV is now absent. In fact, according to the general discussion in section 1 we expect that the bound from $\Delta^{(\mu_2)}$ ($\Delta^{(\mu_1)}$) will be stronger (weaker) because $\delta_q\mu_2^2$ ($\delta_q\mu_1^2$) contains contributions of opposite (the same) sign. This is illustrated in Fig. 10, where the parameters of the model have been taken as for Fig. 9. In Fig. 10 we have used the same numerical values for $\Lambda_t$ and $\Lambda_{H_i}$, but one could take different values for the different cut-offs. Then, $\Lambda_t$ could be much higher than usual, as already discussed around eq. (35). So, the NP responsible for cancelling the top quadratic divergences, presumably strongly interacting, could be beyond LHC reach, as stressed in ref. [9]. However, the NP that compensates the large quadratic corrections associated to the Higgs sector itself should show up at much lower scales\(^4\).

In summary, this 2HDM shows that $\Lambda_t$ could be much larger than $\Lambda_H$ even if the light Higgs is within the experimentally preferred range. However, the global fine-tuning is not improved and we generically expect NP (beyond the 2HD sector) to be on the LHC reach.

\(^4\)In that circumstance, one should worry about how rigorous is it to use the SM as the effective theory between $\Lambda_H$ and $\Lambda_t$. These results at least indicate that the strongly coupled NP might appear at higher scales than other kinds of NP.
3.3 Twin Higgs Model

This model, originally proposed by Chacko, Goh and Harnik in ref. [17], postulates the existence of a mirror world: a $Z_2$ replica of the full SM. Calling $H_1$ the SM Higgs and $H_2$ its mirror copy, the Higgs sector of this model is a very particular kind of 2HDM with potential

$$V = \mu^2(|H_1|^2 + |H_2|^2) + \lambda(|H_1|^2 + |H_2|^2)^2 + \gamma(|H_1|^4 + |H_2|^4),$$

that respects the $Z_2$ parity but allows communication with the mirror world [18] through a mixed term $|H_1|^2|H_2|^2$. A discussion of the naturalness of EWSB in this model was performed by ref. [17] and later on by [8]. For $\gamma > 0$ and $\mu^2 < 0$ this potential has a minimum that breaks the electroweak symmetry with $\langle H_1^0 \rangle = \langle H_2^0 \rangle = v/\sqrt{2}$ [8]. Three of the four degrees of freedom of each doublet are eaten by the longitudinal components of the $W^\pm$ and $Z^0$ gauge bosons of our world and the mirror world. Two scalar degrees of freedom remain as physical Higgses, with a squared mass matrix that
reads
\[
\begin{pmatrix}
2(\lambda + \gamma)v^2 & 2\lambda v^2 \\
2\lambda v^2 & 2(\lambda + \gamma)v^2
\end{pmatrix},
\] (42)
with eigenvalues
\[
m_-^2 = 2\gamma v^2,
\]
\[
m_+^2 = 2(2\lambda + \gamma)v^2,
\] (43)
and eigenvectors \( h_{\pm}^0 = \text{Re}(H_1^0 \pm H_2^0) \). We see that the Higgs mass eigenstates are mixtures with 50% \( H_1 \) component and 50% \( H_2 \) component so that they have reduced couplings to matter and gauge bosons in our world. The eigenvector \( h_+ \) corresponds to the Higgs excitations along the breaking direction and therefore, concerning the minimization condition, \( m_+ \) plays a role similar to \( m_h \) in the SM (in fact, \( m_+^2 = -2\mu^2 \)). With \( \gamma = 0 \), the global \( SU(4) \) symmetry of the \(|H_1|^2 + |H_2|^2\) terms would result in a massless Goldstone boson in the direction transverse to the breaking, \( i.e. \ m_- = 0 \). Having \( \gamma \neq 0 \) with \( \gamma \ll \lambda \) avoids this and gives in a natural way a light Higgs in the spectrum [17].

Let us examine the structure of quadratically divergent corrections to the Higgs mass parameters in this model. As the \( Z_2 \) symmetry is not broken, both \( H_1 \) and \( H_2 \) receive the same corrections, given by
\[
\delta_q \mu^2 = \frac{\Lambda^2}{8\pi^2} \left[ \frac{3}{8} (3g^2 + g'^2) + 5\lambda + 3\gamma - 3\lambda_t^2 \right].
\] (44)
Notice that this formula assumes all the couplings in the mirror world take exactly the same values as in our world. We can write eq. (44) in terms of particle masses by writing \( 5\lambda + 3\gamma = (5m_+^2 + m_-^2)/(4v^2) \). Then we find a result very similar to the SM case with the replacement \( m_h^2 \to (5m_+^2 + m_-^2)/6 \). If we fix \( m_- \) to a low value, say \( m_- = 115 \) GeV, the quadratic correction in (44) as a function of \( m_+^2 \) behaves a bit better than the SM quadratic correction as a function of \( m_h^2 \) [8].

A second difference with respect to the SM behaviour comes from a different RG evolution of the couplings in this case. We should compare the RGE for \( (5\lambda + 3\gamma) \) in this model with that for \( (3\lambda) \) in the SM, see eq. (13) . Again we find a very similar result except for the replacement \( 4(3\lambda)^2 \to (16/5)(5\lambda + 3\gamma)^2 + 4(3\gamma)^2 \). For the case of interest, with \( \gamma \ll \lambda \), we therefore conclude that RG effects in the quadratic corrections of this model are a bit softer than in the SM.
Figure 11: Upper bound on the scale $\Lambda$ of New Physics from the requirement of less than 10% tuning of EWSB in the Twin Higgs model with $m_- = 115$ GeV. Shown by the dashed lines are the EWPT upper bounds on $m_+$. For comparison, the arrow marks the SM naturalness upper bound on $\Lambda$ for $m_h = m_-$. Finally, as happened in the 2HDMs discussed before (see appendix A), the constraints on the Higgs masses derived from EWPT are modified with respect to the SM ones. Now one has [8]

$$ m_+ m_- < m_{EW}^2, \quad (45) $$

where, as usual, $m_{EW} = \{186, 219\}$ GeV is the EWPT indirect bound on the SM Higgs mass.

As a result of the effects just discussed, this model is able to improve over the naturalness of the SM Higgs sector. Fig. 11 shows the upper bound on the scale of New Physics, $\Lambda$ (we limit our analysis to a unique cut-off), that follows from imposing that the fine-tuning in $\mu^2$ (calculated as discussed in previous sections) is smaller than 10% and choosing $m_- = 115$ GeV. For comparison, the SM bound for $m_h = 115$ GeV, i.e. $\Lambda < 1.32$ TeV, is also indicated. However, it is more instructive to compare the bound on $\Lambda$ as a function of $m_+$ with the SM curve as a function of $m_h$, Figs. 1 and 4.
Then we see that the current curve has a shape very similar to the SM one, but it is a slightly bit higher. Moreover, the range of $m_+$ compatible with EWPT is also wider, fully including the maximum of the curve. Nevertheless, the improvement with respect to the SM situation is not dramatic.

Let us now discuss the case in which one introduces a small breaking of the $Z_2$ symmetry by considering different masses for $H_1$ and $H_2$. More explicitly, we add to the potential (41) a term [8]

$$\delta V = m^2 (|H_1|^2 - |H_2|^2) .$$

(46)

With such modification, the minimum of the potential moves away from $\tan \theta \equiv \langle H_1^0 \rangle / \langle H_2^0 \rangle \equiv v_1/v_2 = 1$ (although we still have to keep $v_1 = 246$ GeV), with

$$\cos 2\theta = -\left(\frac{2\lambda + \gamma}{\gamma}\right) \frac{m^2}{\mu^2} .$$

(47)

The squared mass matrix for the two Higgses takes now the form

$$\begin{pmatrix} 2(\lambda + \gamma)v_1^2 & 2\lambda v_1 v_2 \\ 2\lambda v_1 v_2 & 2(\lambda + \gamma)v_2^2 \end{pmatrix} ,$$

(48)

with eigenvalues

$$m_-^2 \simeq 2\gamma(v_1^2 c_\theta^2 + v_2^2 s_\theta^2) ,$$

$$m_+^2 \simeq 2\lambda(v_1^2 + v_2^2) + 2\gamma(v_1^2 s_\theta^2 + v_2^2 c_\theta^2) ,$$

(49)

where we have expanded in $\gamma/\lambda$. The eigenvectors are defined as $h_- = \sqrt{2}\text{Re}(c_\alpha H_1^0 + s_\alpha H_2^0)$ and $h_+ = \sqrt{2}\text{Re}(-s_\alpha H_1^0 + c_\alpha H_2^0)$. From (48)

$$\tan 2\alpha = -\frac{\lambda}{\lambda + \gamma} \tan 2\theta .$$

(50)

For $\gamma \ll \lambda$, one has $\alpha \simeq -\theta$, so that $h_+$ is still aligned with the breaking direction and its mass is still of direct relevance for the naturalness of electroweak breaking (again $m_+^2 \simeq -2\mu^2$).

Before presenting the results for the fine-tuning in the case $m \neq 0$, notice that $\lambda$ and $\gamma$ in (48) can be obtained in terms of $m_+$ and $m_-$ as

$$\lambda = \pm \frac{1}{2v_1 v_2} \sqrt{(m_+^2 s_\theta^2 - m_-^2 c_\theta^2)(m_+^2 c_\theta^2 - m_-^2 s_\theta^2)} ,$$

$$\gamma = \frac{m_+^2 + m_-^2}{2(v_1^2 + v_2^2)} - \lambda .$$

(51)
Figure 12: Contour lines of the 10% naturalness upper bounds on the scale of New Physics $\Lambda$ (in TeV) in the Twin Higgs model with $m \neq 0$ and $m_- = 115$ GeV. The dashed lines show the EWPT upper bounds on $m_+$. It follows that, for fixed $m_+$, $m_-$ and $\theta$, there are two different solutions for $\lambda$ and $\gamma$ with different signs for $\lambda$ (the region $\lambda < 0$ is accessible provided $|\lambda| < \gamma/2$, to avoid an instability in the scalar potential). It can be shown that the best case for naturalness corresponds to $\lambda > 0$ and we restrict our analysis to that case.

We can also see from eqs. (51) that the parameter space is limited to the region $m_+ \geq m_- \text{Max}\{\tan \theta, 1/\tan \theta\}$. This is shown in Fig. 12 which corresponds to the case $m_- = 115$ GeV: the accessible parameter space lies inside the “fish” profile. For any $\theta$, the minimal value of $m_+$ corresponds to taking $\lambda \to 0$ in the mass matrix (48). In that limit, the mass matrix is diagonal, $H_1$ is the SM Higgs with mass $m^2_{h_1} = 2\gamma v^2_1$ and $H_2$ is the Higgs boson of the mirror world, with mass $m^2_{h'} = 2\gamma v^2_2$. In Fig. 12, this limit corresponds to the boundary of the allowed region of parameter space. Along the upper limit, with $\tan \theta > 1$ one has $m_+ \equiv m_h \geq m_- \equiv m_{h'}$. Therefore, $m_+$ plays the role of the mass of the SM Higgs boson along that line. For the lower limit of parameter space in Fig. 12, with $\tan \theta < 1$, one has instead $m_+ \equiv m_{h'} \geq m_- \equiv m_h$ and therefore, along
that line $m_+$ is simply the mass of the mirror Higgs, totally decoupled from our world (which has a Higgs mass fixed to $m_h = 115$ GeV). We have also marked in Fig. 12 the line $\theta = \pi/4$, which corresponds to $m = 0$.

The comments above are very useful to understand the behaviour of the fine-tuning associated to EWSB in the general case with $m \neq 0$. Before discussing them, let us remark that, in the case with $v_1 \neq v_2$ we are really interested in the tuning associated with electroweak breaking in our world, and therefore in the tuning necessary to get right $v_1$, which is fixed by the minimization condition

$$v_1^2 = -\frac{\mu^2}{2\lambda + \gamma} - \frac{m^2}{\gamma}.$$  \hspace{1cm} (52)

The upper bounds on $\Lambda$ coming from requiring less than 10% tuning are shown by Fig. 12 as contour lines (in TeV) in the parameter space \{m$_+$, $\theta$\} for $m_-= 115$ GeV. We can recognize the SM numbers (see Fig. 1 or 4) along the upper limit of the allowed region of parameter space, where $m_+$ is precisely the SM Higgs mass. Along the line $\theta = \pi/4$ we can recognize the numbers corresponding to the $m = 0$ case shown in Fig. 11. Finally, along the lower limit of the allowed parameter space we recover the upper bound $\Lambda \simeq 1.3$ TeV, corresponding to the SM case with $m_- = 115$ GeV. The plot also shows the constraint on $m_+$ and $m_-$ from EWPT (see appendix A), which reads [8]

$$m_+^2 < m_{EW}^2 \left[ \frac{m_{EW}^2}{m_-^2} \right]^{1/\tan^2 \alpha}.$$  \hspace{1cm} (53)

Equation (53) generalizes (45) for the case of $m \neq 0$. The region above these lines (corresponding to the two cases $m_{EW} = 186$ GeV and 219 GeV) is disfavored. From this plot we conclude again that, even though the upper bound on the scale of New Physics can be higher than in the SM (the best point, giving $\Lambda \simeq 5$ TeV, corresponds to $m_+ \sim 215$ GeV and $\theta \sim 5\pi/16$), the global effect is never dramatic.

4 The SM and the Hierarchy Problem Revisited

We have learned from the previous examples that it is not easy to improve significantly the naturalness of the SM by complicating the Higgs sector. Such complication could be justified, however, to explain the hypothetical detection at LHC of a heavy Higgs or the non-detection of strongly coupled NP able to cancel the top quadratic contributions
to the Higgs mass. In this section we take a different view and explore the possibilities of the pure SM, as effective theory, to accommodate such possibilities.

First, let us briefly consider the case of a heavy SM Higgs, i.e. well above the range allowed from EWPT (which require $m_h < 186 - 219$ GeV at 95% c.l.). As is well known, a heavy Higgs would conflict mainly with the measured value of the $T$ parameter, see eq. (23), so that new physics beyond the SM is needed to reconcile theory and experiment. In the IDM [10] discussed in subsect. 3.1 such extra physics comes from the extension of the Higgs sector (to a particular 2HDM). However, the same NP that cancels the SM quadratic contributions to the Higgs mass parameter could fix $\Delta T$ [10]. E.g. it is well known that a non-renormalizable operator

$$\frac{\kappa_6^2}{\Lambda^2} \left| H^\dagger D_\mu H \right|^2,$$  \hspace{1cm} (54)

where $\kappa_6^2$ is a dimensionless coefficient and $\Lambda$ is the scale of NP, gives a contribution to $T$

$$\Delta T \simeq -\frac{\kappa_6^2 \alpha_{em}^2}{2 \alpha_{em}^2 \Lambda^2} \approx -\kappa_6^2 \left( \frac{2 \text{ TeV}}{\Lambda} \right)^2,$$  \hspace{1cm} (55)

that could move the final value of $T$ inside the experimental range. E.g. for $m_h = 400$ GeV, this requires $\kappa_6^2 > 0$ and $\Lambda$ in the range $\kappa_6 \times 4$ TeV, something perfectly consistent with the naturalness upper bounds on $\Lambda$ (see solid lines for $\Delta = 10$ in Figs. 1–4 of sect. 2). In that case, no hint of NP, apart from higher order operators, would be left below $\Lambda$, e.g. the spectrum of the effective theory would be just the SM one, with no modified Higgs sector.

Let us now consider the possibility that LHC does not find any NP able to cancel the top (or the other) quadratic contributions to the Higgs mass parameter, or perhaps the most extreme (and unpleasant) situation that, apart from the Higgs itself, no NP is found at all. In particular we want to know if that situation could be consistent (with no fine-tunings) with just the SM as the effective theory.

Assuming a unique cut-off for all the quadratically divergent loops, the fine-tuning status of the SM was shown in Fig. 1 (upper solid line). Actually, once the Higgs mass is measured one should not include $\Delta \lambda$ in the fine-tuning analysis. In that future time, the curve describing the fine-tuning will be the black dashed line of Fig. 1, which shows a deep throat around $m_h = 225$ GeV. So, if in the future the Higgs happens to have a mass in the region $\sim 225 \pm 25$ GeV, the NP could easily escape detection with no fine-tuning price. Such change of conclusion might seem paradoxical (why should we wait...
Figure 13: SM naturalness upper bound on the scale of NP for $\Delta = 10$ with cut-offs correlated as $\Lambda_t^2 = \zeta \Lambda_h^2$ for different values of $\zeta$.

until a future measurement for such an statement?), but indeed is not strange at all and is characteristic of fine-tuning arguments: being based on statistical considerations, the conclusions may vary according to our partial (and time-dependent) knowledge of the relevant parameters in the problem. At present, however, we should establish the fine-tuning estimates using our actual knowledge of the physical parameters, so the upper solid line of Fig. 1 gives the present status. Still we see that the cut-off could lie as far as 3-4 TeV for $m_h$ in the 200 GeV region, which might allow the NP to escape LHC detection (depending on what NP it is). A Higgs in that region would be still consistent with EWPT (even if marginally), so this is certainly a possibility to keep in mind. Moreover, as discussed above, higher order operators like (54) can improve the consistency with EWPT.

However, we could face in the future the situation that a light Higgs, say $m_h \simeq 150$ GeV, is discovered at the LHC, but no other signal of NP is found. Would this necessarily represent a fine-tuned situation? Probably, the most pleasing answer would be
“yes”, since it would imply that this undesirable situation is unlikely. Nevertheless, the answer could be different. As discussed in sect. 2, it is perfectly possible that the cut-offs of all quadratically divergent contributions are correlated, namely when they arise from the same piece of NP (as can occur when the NP is SUSY). But notice that this does not mean that all cut-offs should be exactly equal. Again SUSY is a good example: squark masses (in particular stop masses) do not need to be equal to the Higgsino masses: the former are determined by the high energy soft masses plus RG effects, while the second depend on the $\mu$ parameter (which can be correlated with, but is not necessarily equal to, $m_{\text{soft}}$). Therefore it is likely that $\Lambda_t$ and $\Lambda_h$ are correlated but not equal, say $\Lambda_t^2 = \zeta \Lambda_h^2$ with some proportionality factor $\zeta = \mathcal{O}(1)$, which depends on the unknown NP. This should be plugged in eq. (2) to re-evaluate $\delta_q m^2$ and the fine-tuning $\Delta$. Obviously, varying $\zeta$, even slightly, shifts the value of $\lambda$ (and thus of $m_h$) where the approximate cancellation of the quadratic contributions takes place. Consequently, the position of Veltman’s throat changes, as shown in Fig. 13, where the cut-off plotted is always the smallest one, i.e. $\text{Min} \{\Lambda_h, \Lambda_t\}$. Interestingly, taking a modest $\zeta = 1/2$ the throat is around $m_h = 150$ GeV, with $\Lambda \sim 3$ TeV. In this situation NP could escape LHC detection with no fine-tuning. It is remarkable how correlated, but slightly different, cut-offs change in a physically significant way the ordinary expectations about the (approximate) cancellation of the quadratically divergent contributions. This can be crucial for the detection or non-detection of NP at LHC. As argued above, this situation is perfectly possible and makes less compelling the ordinary hierarchy-problem argument to expect NP showing up at LHC. This is also in line with the conclusions of ref. [4] in the sense that fine-tuning arguments can only be applied reliably when a concrete example of NP is assumed. Although the general bound of eq. (3) is usually valid (and even conservative), $\mathcal{O}(1)$ factors in that equation are crucial for the visibility of NP at LHC.

One may also wonder about the validity of using the SM as the effective theory at scales between the smallest and the largest cut-off. E.g. if $\Lambda_h > \Lambda_t$, how reliable is it to evaluate Higgs-loops within the SM above $\Lambda_t$? This question was already raised in the context of the BH model, and briefly discussed in footnote 3. Of course, the answer depends on the particular sort of NP entering at $\Lambda_t$. In any case, this kind of analyses at least indicates that the dangerous radiative corrections may (approximately) cancel for different values of $m_h$, depending of the characteristics of the NP.
We can make a more precise statement by using the approach of ref. [4]: modelling the NP as a set of new particles with (possibly) $h$-dependent masses, the cancellation of the SM quadratically divergent corrections requires

$$\left. \frac{\partial^2 \text{Str} M^2}{\partial h^2} \right|_{h=0} \equiv \sum_a \frac{N_a}{2} \left. \frac{\partial^2 m_a^2}{\partial h^2} \right|_{h=0} + \sum_b \frac{N_b}{2} \left. \frac{\partial^2 m_b^2}{\partial h^2} \right|_{h=0} = 0,$$

where $m_a, N_a, m_b, N_b$ are the masses and multiplicities (with negative sign for fermions) of the SM (NP) states. Then, the logarithmic and finite contributions of NP to $m^2$ are given, in the $\overline{MS}$ scheme, by

$$\delta_{\text{NP}}^{\overline{MS}} m^2 = \sum_b \frac{N_b}{32\pi^2} \left[ \left. \frac{\partial^2 m_b^2}{\partial h^2} \right|_{h=0} \left( \log \frac{m_b^2}{Q^2} - 1 \right) + \left. \left( \frac{\partial m_b^2}{\partial h} \right)^2 \log \frac{m_b^2}{Q^2} \right|_{h=0} \right],$$

where $Q$ is the renormalization scale, to be identified with a high-energy cut-off scale, $\Lambda_{\text{HE}}$, which sets the limit of validity of the NP description (it could be as large as $M_{\text{Pl}}$). Quantitatively, these contributions are roughly similar to the SM quadratically-divergent one, eq. (2), replacing $m_b \rightarrow \Lambda$. Hence, the naive estimate (2) is reasonable [and even conservative due to the logarithmically-enhanced terms in (57)], but we can expect different ”cut-offs” associated to the different $m_b$ masses, couplings and RG effects in (57).

## Summary and Conclusions

Generic arguments based on the size of quadratically-divergent contributions to the Higgs mass (Big Hierarchy problem) imply the existence of New Physics (NP) beyond the SM at a scale $\Lambda \lesssim \text{few TeV}$. The precise size of $\Lambda$ depends on the degree of fine-tuning one is willing to tolerate: for 10% fine-tuning and a Higgs mass $m_h = 115 - 200$ GeV, then $\Lambda \lesssim 2 - 3$ TeV, hopefully within the reach of LHC.

There are some reasons to consider possible departures from this simple SM scenario.

a) The above upper bound on $\Lambda$ is generically in some tension with the experimental lower bounds on the suppression scale of higher order operators, derived from electroweak precision tests (EWPT), which typically require $\Lambda \gtrsim 10$ TeV (Little Hierarchy problem). This tension would be relaxed if the upper bound on $\Lambda$ could be pushed up to the $\mathcal{O}(10)$ TeV region, which would imply that the NP responsible for the
cancellation of the quadratic divergences would escape LHC detection. Actually, one should be prepared to face the possibility (admittedly unpleasant) that no NP apart from the Higgs is found at LHC, in spite of naturalness arguments.

b) It could happen that the Higgs found at LHC is beyond the range consistent with EWPT ($m_h < \sim 186 - 219$ GeV).

Both cases may be interpreted as pointing out to some departure from the ordinary SM Higgs sector. One of the simplest and best motivated modifications of the SM Higgs sector one can think of is the two Higgs doublet model (2HDM). Several recent works [8–10] have examined the capabilities of such scenarios to address the previous questions. These 2HDMs include: the “Inert Doublet Model” (IDM), the “BH Model” and the Twin-Higgs Model. The conclusions of [8–10] are that these models can raise the scale of new physics above the LHC reach (keeping naturalness under control) with a light or heavy Higgs (depending on the model), in a way consistent with EWPT. Then the NP responsible for the cancellation of the SM quadratic divergences could escape LHC, although we could observe a modified Higgs sector. Thus these models are claimed to “improve the naturalness” of the pure SM.

Our first goal in this paper has been to perform a careful and fair comparison of the naturalness of these modified-Higgs-sector models with that of the SM. This requires a sensible criterion to quantify the degree of fine-tuning, which should be applied to all the models and computed as rigorously as possible (including e.g. radiative effects). Our conclusions are that, generically, these models do not improve the naturalness of the SM, i.e. they are not able to push significantly the NP cut-off beyond the SM estimate. Actually, in many cases the naturalness is worsened rather than improved. This is in part because the structure of the models normally requires additional fine-tunings of a size similar to that required for a correct EW breaking. This result is common in models with a structure more complicated than that of the SM. Normally, epicycles are penalized in naturalness estimates (which is somehow satisfactory).

This does not mean that these models are not interesting. They are well motivated and show how a LHC phenomenology different from the pure SM expectations could take place. E.g. the IDM shows explicitly how a heavy Higgs can be accommodated in a way consistent with EWPT. On the other hand, not all the models are on the same footing with respect to naturalness. The Twin-Higgs model behaves the best; in fact, it is the only one able to improve the SM naturalness, though not dramatically.
Our second goal in the paper has been to examine if the SM alone could be able to cope with the above-mentioned “unexpected” situations at LHC: i) a heavy SM Higgs (well above the range allowed from EWPT, $m_h < 186-219$ GeV at 95% c.l.) and/or ii) the possibility that LHC does not find any NP able to cancel the quadratic contributions to the Higgs mass, or perhaps the most extreme (and unpleasant) situation that, apart from the Higgs itself, no NP is found at all.

Regarding question i), it is well-known that a heavy Higgs is especially harmful for the $T$–parameter. However, these dangerous contributions to $T$ could be compensated by the same NP which is responsible for the cancellation of the quadratic contributions to $m_h$. I.e. there is no need to modify the Higgs sector of the effective theory below the scale of NP (as the IDM does), although of course that is a possibility. We have emphasized that higher dimension operators suppressed by a scale $\Lambda$ consistent with the naturalness bound can do the job. In that case, no hint of NP, apart from higher order operators, would be left below $\Lambda$, e.g. the spectrum of the effective theory would be just the SM one, with no modified Higgs sector.

Regarding question ii) we have shown first that if $m_h$ lies in the 200 GeV region, $\Lambda$ can be as large as 3-4 TeV, which might allow the NP to escape LHC detection (depending on what NP it is). This is because at $m_h \sim 225$ GeV there is an accidental cancellation between the top and Higgs loop-contributions to the Higgs mass (assuming the same cut-off for both kinds of loops, i.e. $\Lambda_t = \Lambda_h$). Consequently the fine-tuning is suppressed and $\Lambda$ can be larger (this is the so-called “Veltman’s throat”, visible in Fig. 1). A Higgs in that region would still be consistent with EWPT (even if marginally), and so this is certainly a possibility. Moreover, as mentioned above, higher order operators can improve the consistency with EWPT.

If the Higgs is lighter, say $m_h \lesssim 150$ GeV, the cancellation is not efficient and $\Lambda$ falls below 2 TeV, probably within LHC reach (see again Fig. 1). However, this prospect changes if the different cut-offs are correlated but not exactly equal. Parametrizing $\Lambda_t^2 = \zeta \Lambda_h^2$ (where $\zeta$ depends on the kind of NP), we have shown that varying $\zeta$, even slightly, shifts the value of $m_h$ where the approximate cancellation of the quadratic contributions takes place. Consequently, the position of Veltman’s throat changes, as shown in Fig. 13. E.g. taking a modest $\zeta = 1/2$ the throat is around $m_h = 150$ GeV, with $\Lambda \sim 3$ TeV. In this situation NP could escape LHC detection with no fine-tuning. This makes less compelling the ordinary hierarchy-problem argument to expect NP
showing up at LHC. Although the general bound of eq. (3) is usually valid (and even conservative), $O(1)$ factors in that equation (e.g. arising from taking $\Lambda_t \neq \Lambda_h$, as mentioned) are crucial for the visibility of NP at LHC.

To conclude, it is not easy to improve the naturalness of the SM by modifying the Higgs sector. Moreover, although the general arguments give good prospects to expect NP within the reach of LHC, it may anyway happen that NP escapes LHC detection without a fine-tuning price while leaving the pure SM as the sole effective theory valid at LHC energies.

Appendix A

In this Appendix we recall the well known expression [19] for the $\Delta T$ contribution from two Higgs doublets and then particularize it to the three different scenarios discussed in section 3. The rotation angle $\alpha$ between the CP-even neutral Higgs fields $h_1$ and $h_2$ and the mass eigenstates $h_-$ and $h_+$ (with $m_- \leq m_+$) is defined by

$$\begin{bmatrix} h_- \\ h_+ \end{bmatrix} = \begin{pmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{pmatrix} \begin{bmatrix} h_1 \\ h_2 \end{bmatrix}, \tag{A.1}$$

while $\beta$ is simply given by $\tan \beta = v_2/v_1$ as usual. The Higgs contribution to the $T$ parameter can be conveniently written as the sum of two different pieces, one SM-like contribution related to excitations along the breaking direction, $h_\parallel \equiv h_1 \cos \beta + h_2 \sin \beta$, and one additional correction related to excitations along the transverse direction $h_\perp \equiv -h_1 \sin \beta + h_2 \cos \beta$. The explicit expressions are as follows. For the parallel contribution

$$\Delta T_\parallel = \sin^2(\alpha - \beta) \Delta T_{SM}(m_+) + \cos^2(\alpha - \beta) \Delta T_{SM}(m_-), \tag{A.2}$$

where

$$\Delta T_{SM}(m) = \frac{1}{4\pi s_W^2 M_W^2} \left[ A(m, M_W) + \frac{M_W^2 m^2}{m^2 - M_W^2} \log \frac{m^2}{M_W^2} - (M_W \rightarrow M_Z) \right], \tag{A.3}$$

and

$$A(x, y) \equiv \frac{1}{8} \left( x^2 + y^2 - \frac{2x^2 y^2}{x^2 - y^2} \log \frac{x^2}{y^2} \right). \tag{A.4}$$

For later use we also recall that, for not very hierarchical $x/y$, one can use the approximation [10]

$$A(x, y) \simeq \frac{1}{6} (x - y)^2. \tag{A.5}$$
In addition, for large $m$ one has

$$\Delta_{SM}(m) \simeq \text{Const.} - \frac{3}{8\pi \cos^2 \theta_w} \ln \frac{m}{M_Z}, \quad (A.6)$$

so that the EWPT constraint on $\Delta T$ is a constraint on $\log m$.

The transverse correction is

$$\Delta T_\perp = \frac{1}{4\pi s_W^2 M_W^2} \left\{ A(m_A, m_{H^+}) + \sin^2(\alpha - \beta) [A(m_{H^+}, m_-) - A(m_A, m_-)] 
+ \cos^2(\alpha - \beta) [A(m_{H^+}, m_+) - A(m_A, m_+)] \right\}, \quad (A.7)$$

where $m_A$ is the mass of the pseudoscalar Higgs and $m_{H^+}$ the mass of the charged Higgs.

### A.1 Inert Doublet Model

In this model one has $\beta = 0$ and $\alpha = 0$ (when $m_- = m_{h_1}$) or $\alpha = \pi/4$ (when $m_+ = m_{h_1}$). To treat both cases simultaneously call $m_h$ the mass along $h_1$ and $m_S$ the mass along $h_2$. We then get

$$\Delta T_\parallel = \Delta T_{SM}(m_h),$$

$$\Delta T_\perp = \frac{1}{4\pi s_W^2 M_W^2} \left[ A(m_A, m_{H^+}) + A(m_{H^+}, m_S) - A(m_A, m_S) \right], \quad (A.8)$$

and using (A.5)

$$\Delta T_\perp \simeq \frac{1}{12\pi s_W^2 M_W^2} (m_{H^+} - m_A)(m_{H^+} - m_S). \quad (A.9)$$

### A.2 Barbieri-Hall Model

In the particular case with small mixing angle $\alpha$ one gets

$$\Delta T_\parallel = \sin^2 \beta \Delta T_{SM}(m_+) + \cos^2 \beta \Delta T_{SM}(m_-),$$

$$\Delta T_\perp = \frac{1}{4\pi s_W^2 M_W^2} \left[ A(m_A, m_{H^+}) + \sin^2 \beta [A(m_{H^+}, m_-) - A(m_A, m_-)] 
+ \cos^2 \beta [A(m_{H^+}, m_+) - A(m_A, m_+)] \right]. \quad (A.10)$$

For $m_A \simeq m_{H^+}$ one can neglect $\Delta T_\perp$. In that case the EWPT bound on $\Delta T$ is a constraint on $\sin^2 \beta \log m_+ + \cos^2 \beta \log m_-.$

If $\alpha$ is not small, one should use the general formulae (A.2) and (A.7). It is still true that $\Delta T_\perp$ is negligible for $m_A \simeq m_{H^+}.$
A.3 Twin Higgs

In this case only $H_1$ is a $SU(2)_L$ Higgs doublet but one can still use the general formulas (A.2) and (A.7) simply setting $\beta = 0$ and $m_A = m_{H^+} = 0$ ($A^0$ and $H^+$ would in fact be Goldstone bosons coming from $H_2$). One then gets

$$
\Delta T_\parallel = \sin^2 \alpha \Delta T_{SM}(m_+) + \cos^2 \alpha \Delta T_{SM}(m_-),
$$

$$
\Delta T_\perp = 0,
$$

(A.11)

and EWPT constrain the combination $\sin^2 \alpha \log m_+ + \cos^2 \alpha \log m_-.$

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