A Simplified Dynamics Block Diagram for a Four-Axis Stabilized Platform

Author:

Hendrik Daniël Mouton [a]

University of Cape Town, Rondebosch, Cape Town, South Africa, 7701

Abstract:

It is relatively straight-forward to derive comprehensive dynamics block diagrams for two- and three-axis balanced stabilized platforms. But trying to do it for four and higher order systems is getting rapidly such an amount of work, and the resulting block diagrams so extensive, that sensible simplification from the start if feasible could be very beneficial. Those terms that are going to be insignificantly small relative to the other terms should be left out right from the start when deriving the equations. In this paper a method is shown to simplify the dynamics model of stabilized platforms, with the four-axis platform as the main example. The relevant equations are derived for the four-axis example, the block diagram is compiled from the equations after adding simple control loops around the dynamics model to get to the final expanded block diagram, and then some simulation results are shown. The method is validated by showing that the simulation results for a three-axis system are exactly the same for the comprehensive and simplified block diagrams, using the same method. It is proposed that it is therefore reasonable to expect similar findings for the four-axis case.

[a] Associate Professor H D Mouton, Mechanical Engineering, UCT, hennie.mouton@uct.ac.za

Introduction:

Various sorts of cameras could typically be mounted on stabilized platforms to give much improved quality images. It would provide stabilization against rotational movement of the base as for example naturally experienced during linear movement of the base (land craft or aircraft or overhead sporting camera mounts). References [1] and [2] give very good overall views of major issues in stabilized tracking design. Designing such platforms requires proper dynamics modelling of the mechanical configurations. It is not difficult to find the typical relevant equations to represent the dynamics – references [3] and [4] are typical examples, but converting them to sensible block diagrams to assist in simulations and to more easily “see” what is happening is not that readily available. So this paper contains block diagrams, being derived from basic dynamics equations such as given in reference [5], but the main purpose of the paper is to demonstrate a method to develop simplified but still valid block diagrams.

A three-axis example:

To demonstrate the possible validity of the simplification method to be given in this paper, it is first demonstrated with a three-axis stabilized platform. The derivation of the equations for the three-axis case is not going to be given here – only the definitions, the reasoning for ignoring certain terms, a block diagram including control loops and then simulation results. The block diagram will clearly indicate the comprehensive and simplified models. The results will show that for a suitable test scenario the simplified dynamics model gives the same results as the comprehensive model. If the latter results are indeed very similar, it will be assumed to be valid to apply the same method of simplification to four- and more-axis stabilized platforms.

Of course different configurations are possible for the three-axis case. But the demonstration is going to be done on a specific chosen example. The assumption is that other configurations will give similar results.

An outer to inner yaw-pitch-roll configuration was chosen. It is assumed that the control loops in the end will provide good stabilization of the platform in roll, yaw and pitch as made possible by the three axes.

Nomenclature for the three-axis case:

From outer to inner, these are the Euler angles and subscripts to be used:

- 'B' will be used as subscript for the base.
- ψ is the yaw angle between the base and the outer yaw gimbal – 'yaw' will be used as subscript.
- θ is the pitch angle between the yaw gimbal and the pitch gimbal – 'pit' will be used as subscript.
- ϕ is the roll angle between the pitch gimbal and the platform to be stabilized – 'rol' will be used as the subscript.

ω denotes inertial angular rate.

x, y and z correspond basically to the axes about which roll, pitch and yaw rotations happen respectively.

The following picture should clarify the configuration and the symbols.

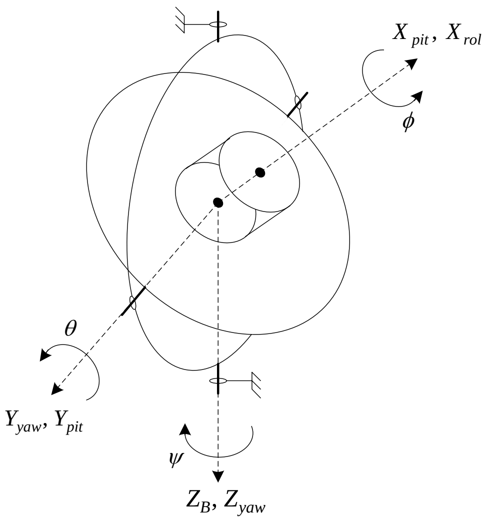


Figure 1. Three-axis configuration

Small/large signal distinction for three axes:

Because of the assumptions above, the following distinction between potentially large and small signals can be made.

Table 1. Small/large signals for three-axis configuration

	ω_{xrol}	ω_{yrol}	ω_{zrol}	ω_{xpit}	ω_{ypit}	ω_{zpit}	ω_{xyaw}	ω_{yyaw}	ω_{zyaw}
Small	√	√	√		√	√			√
Large				√			√	√	

Because it is assumed that the stabilization is going to be good, disturbance torques can be ignored that contain multiplication with any of the signals in this table that are classified as ‘Small’.

Block diagram and simulation results for three axes:

This leads to comprehensive and simplified block diagrams. The simplified block diagram is the same as the comprehensive one but with all blocks coloured with grey omitted – see the first block diagram to follow.

By looking at the increase in disturbance torques when modelling the dynamics of the three axes from the inner axis to the outer axis as represented in the comprehensive block diagram, an idea can be formed how expanded comprehensive four- and five-axis platforms models would be.

Rather simple stabilization loops are added in the block diagram to follow, with $\frac{{\omega_g}^2}{s^2+2\zeta_g\omega_g s+{\omega_g}^2}$ representing feedback from gyros mounted on the platform, measuring ω_{xrol} , ω_{yrol} and ω_{zrol} . A proportional controller and one lead compensator close each loop. The command inputs to the three loops are zero because the intention of the loops is to control ω_{xrol} , ω_{yrol} and ω_{zrol} at zero despite the base motions ω_{xB} , ω_{yB} and ω_{zB} applied to the system. They are applied one after the other but overlapping as shown in the first graph below. Their amplitudes are 1.0 rad/s and their frequencies linearly increase from 1.0 Hz to 50.0 Hz. In the later graphs they will be shown on top of one another simply because many other signals must be shown on the same graphs.

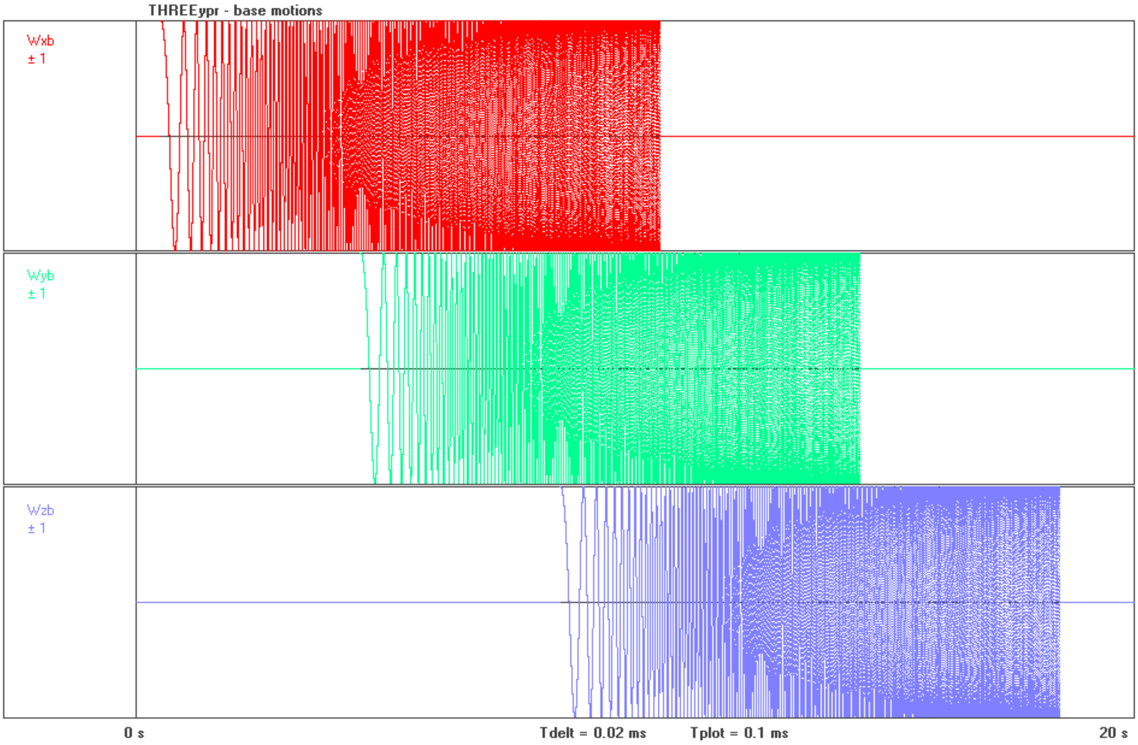


Figure 2. Applied base motions

The amplitudes of the measurements ω_{xrol}, ω_{yrol} and ω_{zrol} are indications of the quality of stabilization – the smaller these signals the better the stabilization. ω_{zrol} is significantly larger than ω_{xrol} and ω_{yrol} as can be expected from such a three-axis system. It will later be seen that the four-axis system provides much better stabilization with basically the same control loops, but that aspect is not the focus of this paper.

The initial yaw, pitch and roll angles, ψ, θ and ϕ, were set at 45° to prevent some disturbance torques to be very small because of small angles. If some disturbance torques were small because of small angles it would have defied the validation of the simplification proposed here.

Remember: In the block diagram below the simplified block diagram is the same as the comprehensive one but with all blocks coloured with grey omitted.

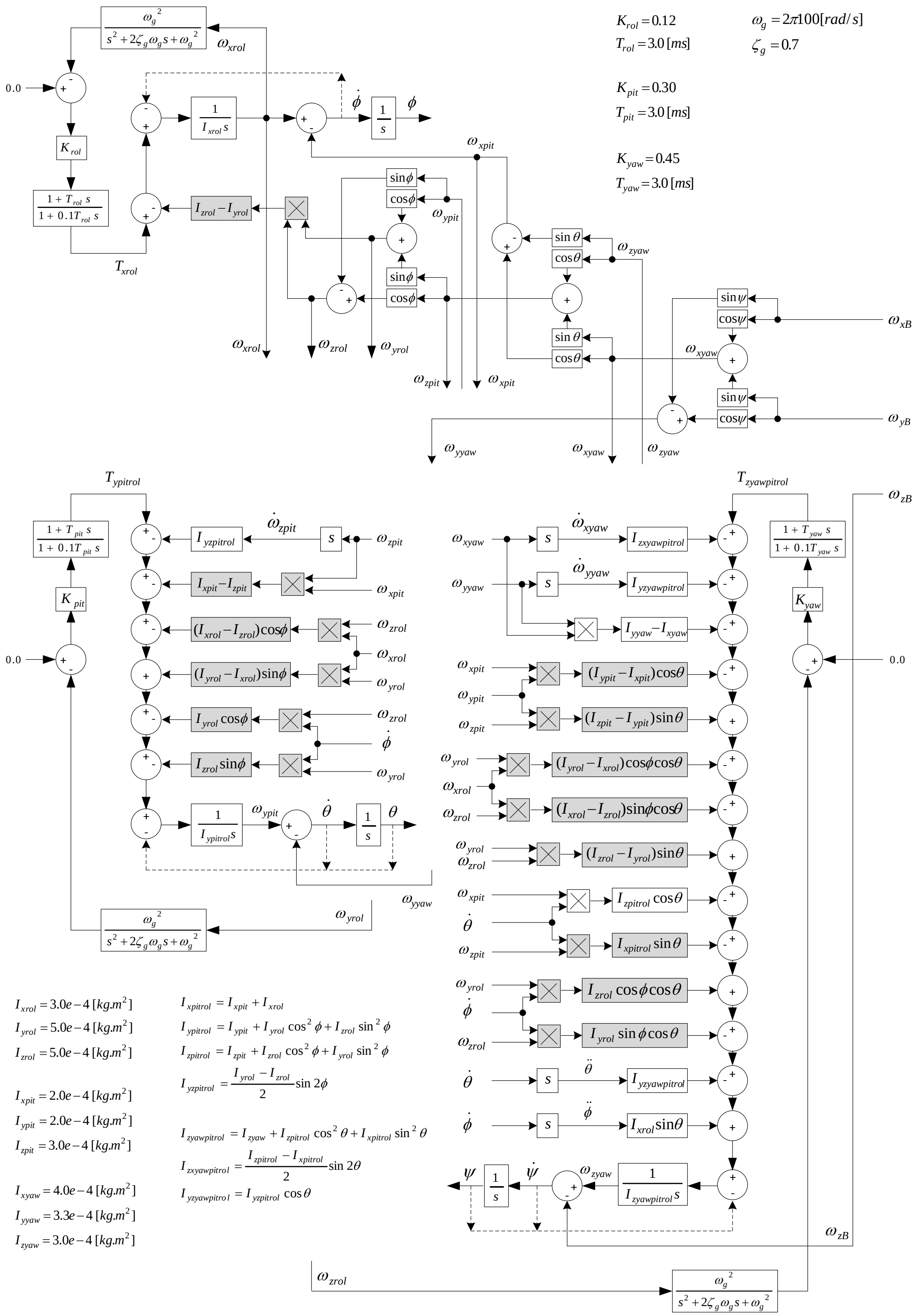


Figure 3. Comprehensive and simplified block diagram of three-axis yaw-pitch-roll

The results for the comprehensive and simplified models are shown below, demonstrating that the proposed simplifications are valid since the results are identical.

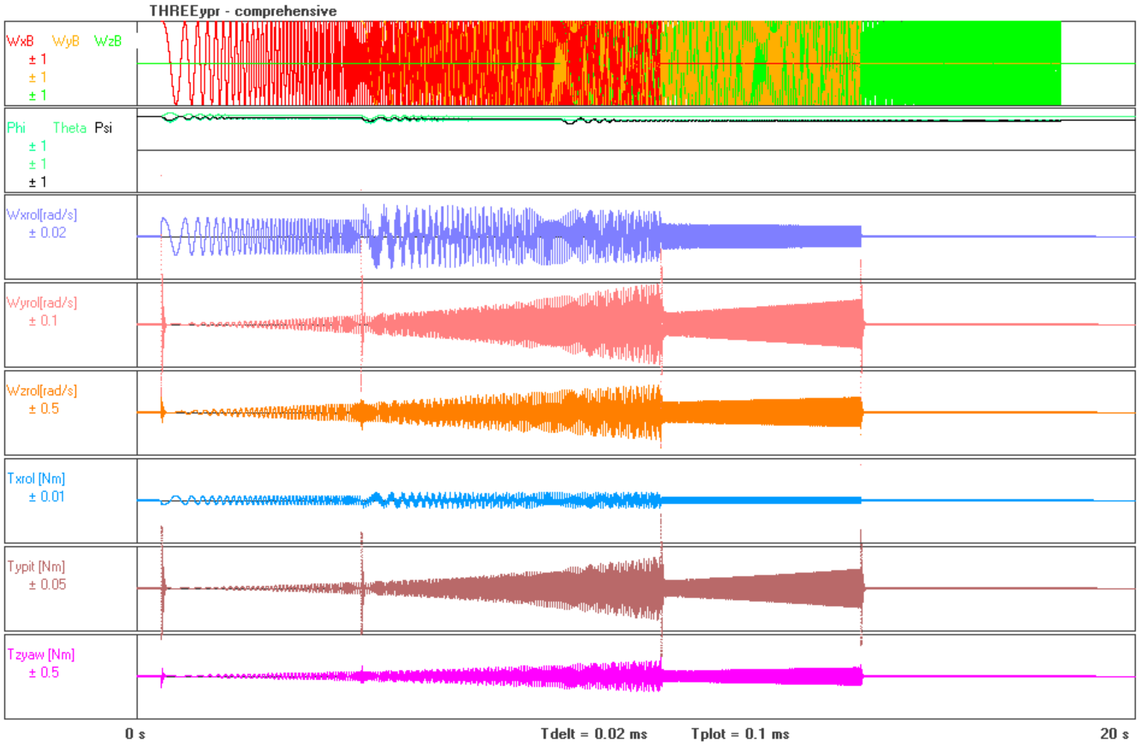


Figure 4. Comprehensive three-axis graph results

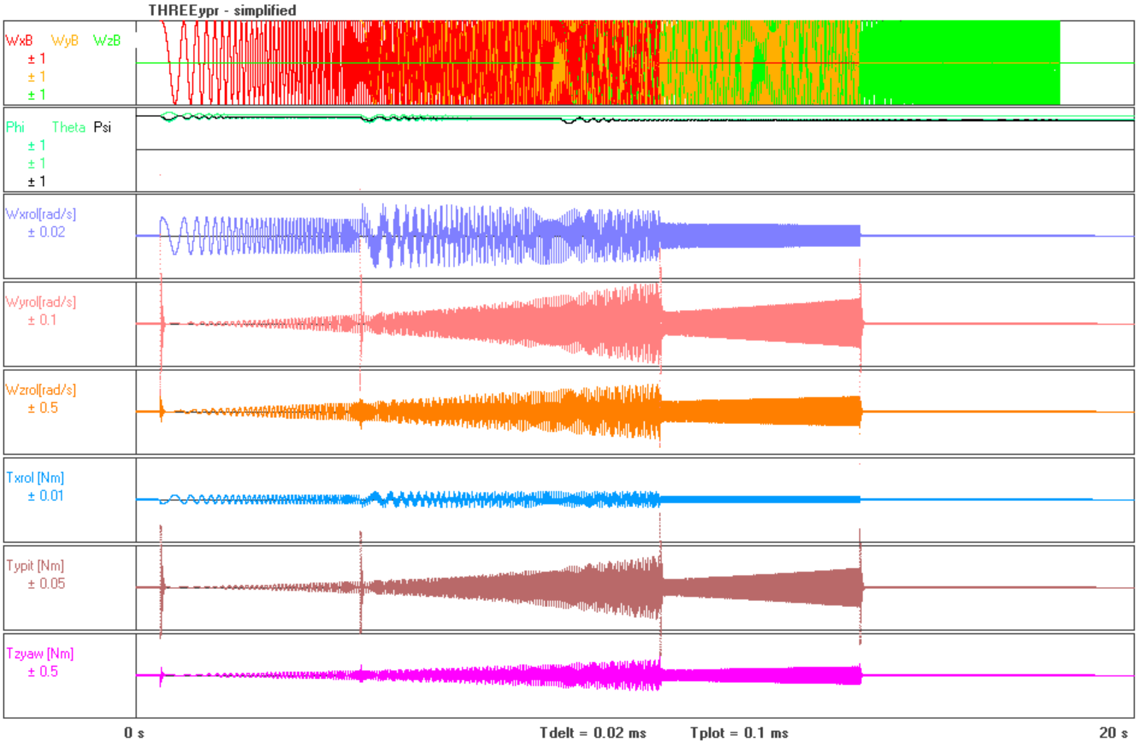


Figure 5. Simplified three-axis graph results

The chosen four-axis example:

Like for all axes, different configurations are also possible for the four-axis case. The principle to be discussed here is applicable to all configurations.

An outer to inner yaw-pitch-yaw-roll configuration was chosen. It is assumed that the control loops in the end will provide good stabilization of the platform in roll, yaw and pitch as made possible by the three inner axes. The outer yaw axis is only for rough alignment – typically not controlled by an inertial rate loop like the inner three axes, but by a relative rate loop ensuring that the inner yaw gimbal angle remains relatively small.

Nomenclature for the four-axis case:

From outer to inner, these are the Euler angles and subscripts to be used:

- 'B' will be used as subscript for the base.
- β is the yaw angle between the base and the outer yaw gimbal – 'Yaw' will be used as subscript.
- θ is the pitch angle between the outer yaw gimbal and the pitch gimbal – 'pit' will be used as subscript.
- ψ is the yaw angle between the pitch gimbal and the inner yaw gimbal – 'yaw' will be used as subscript.
- ϕ is the roll angle between the inner yaw gimbal and the platform to be stabilized – 'rol' will be used as the subscript.

ω denotes inertial angular rate.

x, y and z correspond basically to the axes about which roll, pitch and yaw happen respectively.

The following picture should clarify the configuration and the symbols.

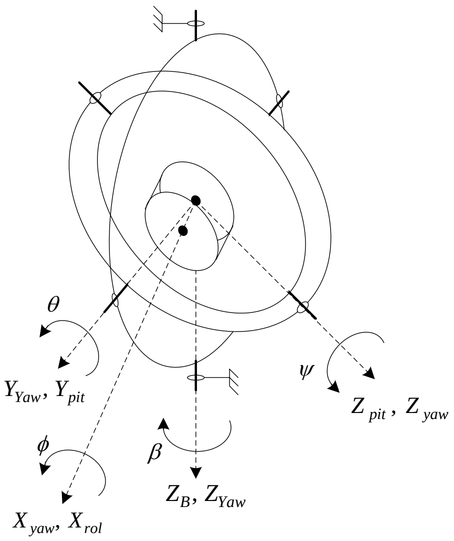


Figure 6. Four-axis configuration

Small/large signal distinction for four axes:

Because of the assumptions above, the following distinction between potentially large and small signals can be made.

Table 2. Small/large signals for four-axis configuration

	ω_{xrol}	ω_{yrol}	ω_{zrol}	ω_{xyaw}	ω_{yyaw}	ω_{zyaw}	ω_{xpit}	ω_{ypit}	ω_{zpit}	ω_{xYaw}	ω_{yYaw}	ω_{zYaw}
Small	√	√	√		√	√		√				
Large				√			√		√	√	√	√

Because it is assumed that the stabilization is going to be good, disturbance torques can be ignored that contain the terms that were classified as 'Small' in the tables above.

Derivation of equations for the four axes:

Apply $\underline{T} = \frac{d}{dt_a}(\underline{W}) + \underline{\omega_a} \times \underline{W}$ with $\underline{T}$ the net torque and $\underline{W} = \underline{\underline{I}} \cdot \underline{\omega}$ to the platform (rol), the inner yaw gimbal (yaw), the pitch gimbal (pit) and the outer yaw gimbal (Yaw).

$\frac{d}{dt_a}$ refers to the derivative w.r.t. a non-inertial axes system.

$\underline{\omega_a}$ is the inertial angular rate vector of the non-inertial axes system.

$\underline{\omega}$ is the inertial angular rate vector of the body under consideration.

$\underline{\underline{I}}$ is the Moment of Inertia matrix of the body under consideration.

<u>To the platform ('rol')</u>:

$$\underline{W} = \begin{bmatrix} I_{xrol} & 0 & 0 \\ 0 & I_{yrol} & 0 \\ 0 & 0 & I_{zrol} \end{bmatrix} \cdot \begin{bmatrix} \omega_{xrol} \\ \omega_{yrol} \\ \omega_{zrol} \end{bmatrix} \qquad \underline{\omega_a} = \begin{bmatrix} \omega_{xrol} \\ \omega_{yrol} \\ \omega_{zrol} \end{bmatrix}$$

It is assumed that the symmetry is such that the inertia matrix can be represented in the diagonal form.

$$\therefore \underline{T_{rol}} = \begin{bmatrix} T_{xrol} \\ T_{yrol} \\ T_{zrol} \end{bmatrix} = \begin{bmatrix} I_{xrol}\dot{\omega}_{xrol} \\ I_{yrol}\dot{\omega}_{yrol} \\ I_{zrol}\dot{\omega}_{zrol} \end{bmatrix} + \begin{bmatrix} \omega_{yrol}\omega_{zrol}\left(I_{zrol} - I_{yrol}\right) \\ \omega_{zrol}\omega_{xrol}\left(I_{xrol} - I_{zrol}\right) \\ \omega_{xrol}\omega_{yrol}\left(I_{yrol} - I_{xrol}\right) \end{bmatrix}$$

$$\therefore T_{xrol} \approx I_{xrol}\dot{\omega}_{xrol} \qquad (1)$$

$$T_{yrol} \approx I_{yrol}\dot{\omega}_{yrol}$$

$$T_{zrol} \approx I_{zrol}\dot{\omega}_{zrol}$$

<u>To the inner yaw gimbal ('yaw')</u>:

$$\underline{W} = \begin{bmatrix} I_{xyaw} & 0 & 0 \\ 0 & I_{yyaw} & 0 \\ 0 & 0 & I_{zyaw} \end{bmatrix} \cdot \begin{bmatrix} \omega_{xyaw} \\ \omega_{yyaw} \\ \omega_{zyaw} \end{bmatrix} \qquad \underline{\omega_a} = \begin{bmatrix} \omega_{xyaw} \\ \omega_{yyaw} \\ \omega_{zyaw} \end{bmatrix}$$

It is assumed that the symmetry is such that the inertia matrix can be represented in the diagonal form.

$$\therefore \underline{T_{yaw}} = \begin{bmatrix} T_{xyaw} \\ T_{yyaw} \\ T_{zyaw} \end{bmatrix} = \begin{bmatrix} I_{xyaw}\dot{\omega}_{xyaw} \\ I_{yyaw}\dot{\omega}_{yyaw} \\ I_{zyaw}\dot{\omega}_{zyaw} \end{bmatrix} + \begin{bmatrix} \omega_{yyaw}\omega_{zyaw}\left(I_{zyaw} - I_{yyaw}\right) \\ \omega_{zyaw}\omega_{xyaw}\left(I_{xyaw} - I_{zyaw}\right) \\ \omega_{xyaw}\omega_{yyaw}\left(I_{yyaw} - I_{xyaw}\right) \end{bmatrix}$$

$$\therefore T_{xyaw} \approx I_{xyaw}\dot{\omega}_{xyaw}$$

$$T_{yyaw} \approx I_{yyaw}\dot{\omega}_{yyaw}$$

$$T_{zyaw} \approx I_{zyaw}\dot{\omega}_{zyaw}$$

$\underline{T_{yaw}}$ is the net torque applied to the inner yaw gimbal.

Call the torque applied to this gimbal from the outside $\underline{T_{yawrol}}$

$\therefore \underline{T_{yaw}} = \underline{T_{yawrol}} - \underline{T_{rol}} \quad \therefore \underline{T_{yawrol}} = \underline{T_{yaw}} + \underline{T_{rol}}$

Now take the roll angle ϕ between the 'yaw' and 'rol' axes into account:

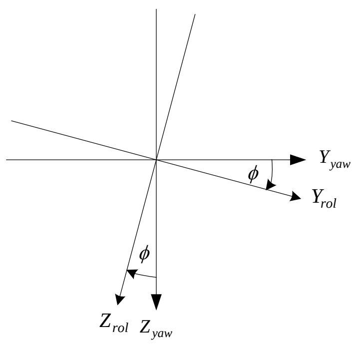


Figure 7. Roll angle transformation

$$\therefore \underline{T_{yawrol}} = \begin{bmatrix} T_{xyawrol} \\ T_{yyawrol} \\ T_{zyawrol} \end{bmatrix} = \begin{bmatrix} T_{xyaw} + T_{xrol} \\ T_{yyaw} + T_{yrol}\cos\phi - T_{zrol}\sin\phi \\ T_{zyaw} + T_{zrol}\cos\phi + T_{yrol}\sin\phi \end{bmatrix}$$

$$\therefore T_{xyawrol} \approx I_{xyaw}\dot{\omega}_{xyaw} + I_{xrol}\dot{\omega}_{xrol}$$

$$T_{yyawrol} \approx I_{yyaw}\dot{\omega}_{yyaw} + I_{yrol}\dot{\omega}_{yrol}\cos\phi - I_{zrol}\,\dot{\omega}_{zrol}\sin\phi$$

$$T_{zyawrol} \approx I_{zyaw}\dot{\omega}_{zyaw} + I_{zrol}\dot{\omega}_{zrol}\cos\phi + I_{yrol}\,\dot{\omega}_{yrol}\sin\phi$$

The following are the equations for the angular rates and accelerations:

$$\begin{aligned} \omega_{xrol} &= \omega_{xyaw} + \dot{\phi} \\ \omega_{yrol} &= \omega_{yyaw}\cos\phi + \omega_{zyaw}\sin\phi \\ \omega_{zrol} &= \omega_{zyaw}\cos\phi - \omega_{yyaw}\sin\phi \end{aligned} \tag{2}$$

$$\begin{aligned} \therefore \dot{\omega}_{xrol} &= \dot{\omega}_{xyaw} + \ddot{\phi} \\ \dot{\omega}_{yrol} &= \dot{\omega}_{yyaw}\cos\phi + \dot{\omega}_{zyaw}\sin\phi + \dot{\phi}\omega_{zrol} \\ \dot{\omega}_{zrol} &= \dot{\omega}_{zyaw}\cos\phi - \dot{\omega}_{yyaw}\sin\phi - \dot{\phi}\omega_{yrol} \end{aligned}$$

Substitute $\dot{\omega}_{xrol}$, $\dot{\omega}_{yrol}$ and $\dot{\omega}_{zrol}$ in $T_{xyawrol}$, $T_{yyawrol}$ and $T_{zyawrol}$.

$$\therefore T_{xyawrol} \approx \dot{\omega}_{xyaw}\left(I_{xyaw} + I_{xrol}\right) + \ddot{\phi}I_{xrol}$$

$$T_{yyawrol} \approx \dot{\omega}_{yyaw}\left(I_{yyaw} + I_{yrol}\cos^2\Phi + I_{zrol}\sin^2\phi\right) + \dot{\omega}_{zyaw}\left(I_{yrol} - I_{zrol}\right)\sin\phi\cos\phi + \dot{\phi}\omega_{zrol}I_{yrol}\cos\phi + \dot{\phi}\omega_{yrol}I_{zrol}\sin\phi$$

$$T_{zyawrol} \approx \dot{\omega}_{zyaw}\left(I_{zyaw} + I_{zrol}\cos^2\Phi + I_{yrol}\sin^2\phi\right) + \dot{\omega}_{yyaw}\left(I_{yrol} - I_{zrol}\right)\sin\phi\cos\phi -$$
$$\dot{\phi}\omega_{yrol}I_{zrol}\cos\phi + \dot{\phi}\omega_{zrol}I_{yrol}\sin\phi$$

Define:

$$I_{xyawrol} = I_{xyaw} + I_{xrol} \tag{3}$$

$$I_{yyawrol} = I_{yyaw} + I_{yrol}\cos^2\phi + I_{zrol}\sin^2\phi \tag{4}$$

$$I_{zyawrol} = I_{zyaw} + I_{zrol}\cos^2\phi + I_{yrol}\sin^2\phi \tag{5}$$

$$I_{yzyawrol} = \frac{I_{yrol} - I_{zrol}}{2}\sin 2\phi \tag{6}$$

$$\therefore T_{xyawrol} \approx \dot{\omega}_{xyaw}I_{xyawrol} + \ddot{\phi}I_{xrol}$$

$$T_{yyawrol} \approx \dot{\omega}_{yyaw}I_{yyawrol} + \dot{\omega}_{zyaw}I_{yzyawrol} + \dot{\phi}\omega_{zrol}I_{yrol}\cos\phi + \dot{\phi}\omega_{yrol}I_{zrol}\sin\phi$$

$$T_{zyawrol} \approx \dot{\omega}_{zyaw}I_{zyawrol} + \dot{\omega}_{yyaw}I_{yzyawrol} - \dot{\phi}\omega_{yrol}I_{zrol}\cos\phi + \dot{\phi}\omega_{zrol}I_{yrol}\sin\phi \tag{7}$$

<u>To the pitch gimbal ('pit')</u>:

$$\underline{W} = \begin{bmatrix} I_{xpit} & 0 & 0 \\ 0 & I_{ypit} & 0 \\ 0 & 0 & I_{zpit} \end{bmatrix} \cdot \begin{bmatrix} \omega_{xpit} \\ \omega_{ypit} \\ \omega_{zpit} \end{bmatrix} \qquad \underline{\omega_a} = \begin{bmatrix} \omega_{xpit} \\ \omega_{ypit} \\ \omega_{zpit} \end{bmatrix}$$

It is assumed that the symmetry is such that the inertia matrix can be represented in the diagonal form.

$$\therefore \underline{T_{pit}} = \begin{bmatrix} T_{xpit} \\ T_{ypit} \\ T_{zpit} \end{bmatrix} = \begin{bmatrix} I_{xpit}\dot{\omega}_{xpit} \\ I_{ypit}\dot{\omega}_{ypit} \\ I_{zpit}\dot{\omega}_{zpit} \end{bmatrix} + \begin{bmatrix} \omega_{ypit}\omega_{zpit}\left(I_{zpit} - I_{ypit}\right) \\ \omega_{zpit}\omega_{xpit}\left(I_{xpit} - I_{zpit}\right) \\ \omega_{xpit}\omega_{ypit}\left(I_{ypit} - I_{xpit}\right) \end{bmatrix}$$

$$\therefore T_{xpit} \approx I_{xpit}\dot{\omega}_{xpit}$$

$$T_{ypit} = I_{ypit}\dot{\omega}_{ypit} + \omega_{zpit}\omega_{xpit}\left(I_{xpit} - I_{zpit}\right)$$

$$T_{zpit} \approx I_{zpit}\dot{\omega}_{zpit}$$

$\underline{T_{pit}}$ is the net torque applied to the pitch gimbal.

Call the torque applied to this gimbal from the outside $\underline{T_{pityawrol}}$.

$$\therefore \underline{T_{pit}} = \underline{T_{pityawrol}} - \underline{T_{yawrol}} \qquad \therefore \underline{T_{pityawrol}} = \underline{T_{pit}} + \underline{T_{yawrol}}$$

Now take the yaw angle ψ between the 'pit' and 'yaw' axes into account:

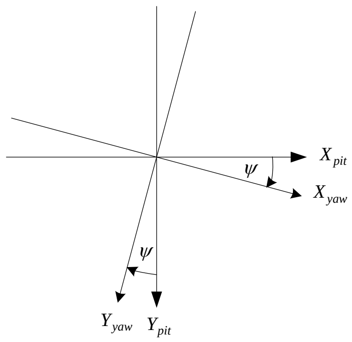


Figure 8. Yaw angle transformation

$$\therefore \underline{T_{pityawrol}} = \begin{bmatrix} T_{xpityawrol} \\ T_{ypityawrol} \\ T_{zpityawrol} \end{bmatrix} = \begin{bmatrix} T_{xpit} + T_{xyawrol}\cos\psi - T_{yyawrol}\sin\psi \\ T_{yrol} + T_{yyawrol}\cos\psi + T_{xyawrol}\sin\psi \\ T_{zpit} + T_{zyawrol} \end{bmatrix}$$

$$\therefore T_{xpityawrol} \approx I_{xpit}\dot{\omega}_{xpit} + \left(\dot{\omega}_{xyaw}I_{xyawrol} + \ddot{\phi}I_{xrol}\right)\cos\psi -$$
$$\left(\dot{\omega}_{yyaw}I_{yyawrol} + \dot{\omega}_{zyaw}I_{yzyawrol} + \dot{\phi}\omega_{zrol}I_{yrol}\cos\phi + \dot{\phi}\omega_{yrol}I_{zrol}\sin\phi\right)\sin\psi$$
$$\approx I_{xpit}\dot{\omega}_{xpit} + \left(\dot{\omega}_{xyaw}I_{xyawrol} + \ddot{\phi}I_{xrol}\right)\cos\psi -$$
$$\left(\dot{\omega}_{yyaw}I_{yyawrol} + \dot{\omega}_{zyaw}I_{yzyawrol}\right)\sin\psi$$

with $\dot{\phi}\omega_{yrol}$ and $\dot{\phi}\omega_{zrol}$ ignored.

$$T_{ypityawrol} \approx I_{ypit}\dot{\omega}_{ypit} + \omega_{zpit}\omega_{xpit}\left(I_{xpit} - I_{zpit}\right) +$$
$$\left(\dot{\omega}_{yyaw}I_{yyawrol} + \dot{\omega}_{zyaw}I_{yzyawrol} + \dot{\phi}\omega_{zrol}I_{yrol}\cos\phi + \dot{\phi}\omega_{yrol}I_{zrol}\sin\phi\right)\cos\psi +$$
$$\left(\dot{\omega}_{xyaw}I_{xyawrol} + \ddot{\phi}I_{xrol}\right)\sin\psi$$
$$\approx I_{ypit}\dot{\omega}_{ypit} + \omega_{zpit}\omega_{xpit}\left(I_{xpit} - I_{zpit}\right) +$$
$$\left(\dot{\omega}_{yyaw}I_{yyawrol} + \dot{\omega}_{zyaw}I_{yzyawrol}\right)\cos\psi + \left(\dot{\omega}_{xyaw}I_{xyawrol} + \ddot{\phi}I_{xrol}\right)\sin\psi$$

with $\dot{\phi}\omega_{yrol}$ and $\dot{\phi}\omega_{zrol}$ ignored.

$$T_{zpityawrol} \approx I_{zpit}\dot{\omega}_{zpit} +$$
$$\dot{\omega}_{zyaw}I_{zyawrol} + \dot{\omega}_{yyaw}I_{yzyawrol} - \dot{\phi}\omega_{yrol}I_{zrol}\cos\phi + \dot{\phi}\omega_{zrol}I_{yrol}\sin\phi$$
$$\approx I_{zpit}\dot{\omega}_{zpit} + \dot{\omega}_{zyaw}I_{zyawrol} + \dot{\omega}_{yyaw}I_{yzyawrol}$$

with $\dot{\phi}\omega_{yrol}$ and $\dot{\phi}\omega_{zrol}$ ignored.

The following are the equations for the angular rates and accelerations:

$$\omega_{xyaw} = \omega_{xpit}\cos\psi + \omega_{ypit}\sin\psi$$

$$\omega_{yyaw} = \omega_{ypit}\cos\psi - \omega_{xpit}\sin\psi \tag{8}$$

$$\omega_{zyaw} = \omega_{zpit} + \dot{\psi}$$

$$\therefore \dot{\omega}_{xyaw} = \dot{\omega}_{xpit}\cos\psi + \dot{\omega}_{ypit}\sin\psi + \dot{\psi}\omega_{yyaw}$$

$$\dot{\omega}_{yyaw} = \dot{\omega}_{ypit}\cos\psi - \dot{\omega}_{xpit}\sin\psi - \dot{\psi}\omega_{xyaw}$$

$$\dot{\omega}_{zyaw} = \dot{\omega}_{zpit} + \ddot{\psi}$$

Substitute $\dot{\omega}_{xyaw}$, $\dot{\omega}_{yyaw}$ and $\dot{\omega}_{zyaw}$ in $T_{xpityawrol}$, $T_{ypityawrol}$ and $T_{zpityawrol}$.

$$\begin{aligned}
\therefore T_{xpityawrol} \approx\ & I_{xpit}\dot{\omega}_{xpit} + \\
& (\dot{\omega}_{xpit}\cos\psi + \dot{\omega}_{ypit}\sin\psi + \dot{\psi}\omega_{yyaw})I_{xyawrol}\cos\psi - \\
& (\dot{\omega}_{ypit}\cos\psi - \dot{\omega}_{xpit}\sin\psi - \dot{\psi}\omega_{xyaw})I_{yyawrol}\sin\psi - \\
& (\dot{\omega}_{zpit} + \ddot{\psi})I_{yzyawrol}\sin\psi + \ddot{\phi}I_{xrol}\cos\psi \\
=\ & \dot{\omega}_{xpit}\left(I_{xpit} + I_{xyawrol}\cos^2\psi + I_{yyawrol}\sin^2\psi\right) + \\
& \dot{\omega}_{ypit}\left(I_{xyawrol} - I_{yyawrol}\right)\sin\psi\cos\psi - \dot{\omega}_{zpit}I_{yzyawrol}\sin\psi + \\
& \dot{\psi}\omega_{yyaw}I_{xyawrol}\cos\psi + \dot{\psi}\omega_{xyaw}I_{yyawrol}\sin\psi - \ddot{\psi}I_{yzyawrol}\sin\psi + \ddot{\phi}I_{xrol}\cos\psi
\end{aligned}$$

$$\begin{aligned}
T_{ypityawrol} \approx\ & I_{ypit}\dot{\omega}_{ypit} + \omega_{zpit}\omega_{xpit}\left(I_{xpit} - I_{zpit}\right) + \\
& (\dot{\omega}_{ypit}\cos\psi - \dot{\omega}_{xpit}\sin\psi - \dot{\psi}\omega_{xyaw})I_{yyawrol}\cos\psi + \\
& (\dot{\omega}_{xpit}\cos\psi + \dot{\omega}_{ypit}\sin\psi + \dot{\psi}\omega_{yyaw})I_{xyawrol}\sin\psi + \\
& (\dot{\omega}_{zpit} + \ddot{\psi})I_{yzyawrol}\cos\psi + \ddot{\phi}I_{xrol}\sin\psi \\
=\ & \dot{\omega}_{ypit}\left(I_{ypit} + I_{yyawrol}\cos^2\psi + I_{xyawrol}\sin^2\psi\right) + \\
& \dot{\omega}_{zpit}I_{yzyawrol}\cos\psi + \dot{\omega}_{xpit}\left(I_{xyawrol} - I_{yyawrol}\right)\sin\psi\cos\psi + \omega_{zpit}\omega_{xpit}\left(I_{xpit} - I_{zpit}\right) - \\
& \dot{\psi}\omega_{xyaw}I_{yyawrol}\cos\psi + \dot{\psi}\omega_{yyaw}I_{xyawrol}\sin\psi + \ddot{\psi}I_{yzyawrol}\cos\psi + \ddot{\phi}I_{xrol}\sin\psi
\end{aligned}$$

$$\begin{aligned}
T_{zpityawrol} \approx\ & I_{zpit}\dot{\omega}_{zpit} + \left(\dot{\omega}_{zpit} + \ddot{\psi}\right)I_{zyawrol} + \left(\dot{\omega}_{ypit}\cos\psi - \dot{\omega}_{xpit}\sin\psi - \dot{\psi}\omega_{xyaw}\right)I_{yzyawrol} \\
=\ & \dot{\omega}_{zpit}\left(I_{zpit} + I_{zyawrol}\right) - \dot{\omega}_{xpit}I_{yzyawrol}\sin\psi + \dot{\omega}_{ypit}I_{yzyawrol}\cos\psi - \\
& \dot{\psi}\omega_{xyaw}I_{yzyawrol} + \ddot{\psi}I_{zyawrol}
\end{aligned}$$

Define:

$$I_{xpityawrol} = I_{xpit} + I_{xyawrol}\cos^2\psi + I_{yyawrol}\sin^2\psi \tag{9}$$

$$I_{ypityawrol} = I_{ypit} + I_{yyawrol}\cos^2\psi + I_{xyawrol}\sin^2\psi \tag{10}$$

$$I_{zpityawrol} = I_{zpit} + I_{zyawrol} \tag{11}$$

$$I_{xypityawrol} = \frac{I_{xyawrol} - I_{yyawrol}}{2}\sin 2\psi \tag{12}$$

$$I_{yzpityawrol} = I_{yzyawrol}\cos\psi \tag{13}$$

$$I_{zxpityawrol} = I_{yzyawrol}\sin\psi \tag{14}$$

$$\therefore T_{xpityawrol} \approx \dot{\omega}_{xpit} I_{xpityawrol} + \dot{\omega}_{ypit} I_{xypityawrol} - \dot{\omega}_{zpit} I_{zxpityawrol} + \dot{\psi}\omega_{yyaw} I_{xyawrol}\cos\psi + \dot{\psi}\omega_{xyaw} I_{yyawrol}\sin\psi - \ddot{\psi} I_{yzyawrol}\sin\psi + \ddot{\phi} I_{xrol}\cos\psi$$

$$T_{ypityawrol} \approx \dot{\omega}_{ypit} I_{ypityawrol} + \dot{\omega}_{zpit} I_{yzpityawrol} + \dot{\omega}_{xpit} I_{xypityawrol} + \omega_{zpit}\omega_{xpit}\left(I_{xpit} - I_{zpit}\right) - \dot{\psi}\omega_{xyaw} I_{yyawrol}\cos\psi + \dot{\psi}\omega_{yyaw} I_{xyawrol}\sin\psi + \ddot{\psi} I_{yzyawrol}\cos\psi + \ddot{\phi} I_{xrol}\sin\psi \quad (15)$$

$$T_{zpityawrol} \approx \dot{\omega}_{zpit} I_{zpityawrol} - \dot{\omega}_{xpit} I_{zxpityawrol} + \dot{\omega}_{ypit} I_{yzpityawrol} - \dot{\psi}\omega_{xyaw} I_{yzyawrol} + \ddot{\psi} I_{zyawrol}$$

<u>To the outer yaw gimbal (Yaw)</u>:

$$\underline{W} = \begin{bmatrix} I_{xYaw} & 0 & 0 \\ 0 & I_{yYaw} & 0 \\ 0 & 0 & I_{zYaw} \end{bmatrix} \cdot \begin{bmatrix} \omega_{xYaw} \\ \omega_{yYaw} \\ \omega_{zYaw} \end{bmatrix} \qquad \underline{\omega_a} = \begin{bmatrix} \omega_{xYaw} \\ \omega_{yYaw} \\ \omega_{zYaw} \end{bmatrix}$$

It is assumed that the symmetry is such that the inertia matrix can be represented in the diagonal form.

$$\therefore \underline{T_{Yaw}} = \begin{bmatrix} T_{xYaw} \\ T_{yYaw} \\ T_{zYaw} \end{bmatrix} = \begin{bmatrix} I_{xYaw}\dot{\omega}_{xYaw} \\ I_{yYaw}\dot{\omega}_{yYaw} \\ I_{zYaw}\dot{\omega}_{zYaw} \end{bmatrix} + \begin{bmatrix} \omega_{yYaw}\omega_{zYaw}\left(I_{zYaw} - I_{yYaw}\right) \\ \omega_{zYaw}\omega_{xYaw}\left(I_{xYaw} - I_{zYaw}\right) \\ \omega_{xYaw}\omega_{yYaw}\left(I_{yYaw} - I_{xYaw}\right) \end{bmatrix}$$

$\underline{T_{Yaw}}$ is the net torque applied to the outer yaw gimbal.

Call the torque applied to this gimbal from the outside $\underline{T_{Yawpityawrol}}$.

$$\therefore \underline{T_{Yaw}} = \underline{T_{Yawpityawrol}} - \underline{T_{pityawrol}} \qquad \therefore \underline{T_{Yawpityawrol}} = \underline{T_{Yaw}} + \underline{T_{pityawrol}}$$

Now take the pitch angle θ between the 'Yaw' and 'pit' axes into account:

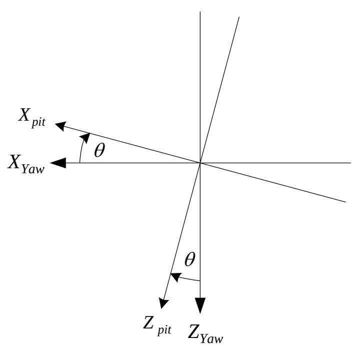


Figure 9. Pitch angle transformation

$$\therefore \underline{T_{Yawpityawrol}} = \begin{bmatrix} T_{xYawpityawrol} \\ T_{yYawpityawrol} \\ T_{zYawpityawrol} \end{bmatrix} = \begin{bmatrix} T_{xYaw} + T_{xpityawrol}\cos\theta + T_{zpityawrol}\sin\theta \\ T_{yYaw} + T_{ypityawrol} \\ T_{zYaw} + T_{zpityawrol}\cos\theta - T_{xpityawrol}\sin\theta \end{bmatrix}$$

$\therefore T_{zYawpityawrol} \approx I_{zYaw}\dot{\omega}_{zYaw} + \omega_{xYaw}\omega_{yYaw}\left(I_{yYaw} - I_{xYaw}\right) + \left(\dot{\omega}_{zpit}I_{zpityawrol} - \dot{\omega}_{xpit}I_{zxpityawrol} + \dot{\omega}_{ypit}I_{yzpityawrol} - \dot{\psi}\omega_{xyaw}I_{yzyawrol} + \ddot{\psi}I_{zyawrol}\right)\cos\theta - \left(\dot{\omega}_{xpit}I_{xpityawrol} + \dot{\omega}_{ypit}I_{xypityawrol} - \dot{\omega}_{zpit}I_{zxpityawrol} + \dot{\psi}\omega_{yyaw}I_{xyawrol}\cos\psi + \dot{\psi}\omega_{xyaw}I_{yyawrol}\sin\psi - \ddot{\psi}I_{yzyawrol}\sin\psi + \ddot{\phi}I_{xrol}\cos\psi\right)\sin\theta$

$\approx I_{zYaw}\dot{\omega}_{zYaw} + \omega_{xYaw}\omega_{yYaw}\left(I_{yYaw} - I_{xYaw}\right) + \left(\dot{\omega}_{zpit}I_{zpityawrol} - \dot{\omega}_{xpit}I_{zxpityawrol} + \dot{\omega}_{ypit}I_{yzpityawrol} - \dot{\psi}\omega_{xyaw}I_{yzyawrol} + \ddot{\psi}I_{zyawrol}\right)\cos\theta - \left(\dot{\omega}_{xpit}I_{xpityawrol} + \dot{\omega}_{ypit}I_{xypityawrol} - \dot{\omega}_{zpit}I_{zxpityawrol} + \dot{\psi}\omega_{xyaw}I_{yyawrol}\sin\psi - \ddot{\psi}I_{yzyawrol}\sin\psi + \ddot{\phi}I_{xrol}\cos\psi\right)\sin\theta$

with $\dot{\psi}\omega_{yyaw}$ ignored.

The following are the equations for the angular rates and accelerations:

$$\omega_{xpit} = \omega_{xYaw}\cos\theta - \omega_{zYaw}\sin\theta$$
$$\omega_{ypit} = \omega_{yYaw} + \dot{\theta} \qquad (16)$$
$$\omega_{zpit} = \omega_{zYaw}\cos\theta + \omega_{xYaw}\sin\theta$$

$$\therefore \dot{\omega}_{xpit} = \dot{\omega}_{xYaw}\cos\theta - \dot{\omega}_{zYaw}\sin\theta - \dot{\theta}\omega_{zpit}$$
$$\dot{\omega}_{ypit} = \dot{\omega}_{zYaw} + \ddot{\theta}$$
$$\dot{\omega}_{zpit} = \dot{\omega}_{zYaw}\cos\theta + \dot{\omega}_{xYaw}\sin\theta + \dot{\theta}\omega_{xpit}$$

Substitute $\dot{\omega}_{xpit}$, $\dot{\omega}_{ypit}$ and $\dot{\omega}_{zpit}$ in $T_{zYawpityawrol}$.

$$\begin{aligned} \therefore T_{zYawpityawrol} \approx\ & I_{zYaw}\dot{\omega}_{zYaw} + \omega_{xYaw}\omega_{yYaw}\left(I_{yYaw} - I_{xYaw}\right) + \\ & \left(\dot{\omega}_{zYaw}\cos\theta + \dot{\omega}_{xYaw}\sin\theta + \dot{\theta}\omega_{xpit}\right)I_{zpityawrol}\cos\theta + \\ & \left(\dot{\omega}_{zYaw}\cos\theta + \dot{\omega}_{xYaw}\sin\theta + \dot{\theta}\omega_{xpit}\right)I_{zxpityawrol}\sin\theta - \\ & \left(\dot{\omega}_{xYaw}\cos\theta - \dot{\omega}_{zYaw}\sin\theta - \dot{\theta}\omega_{zpit}\right)I_{zxpityawrol}\cos\theta - \\ & \left(\dot{\omega}_{xYaw}\cos\theta - \dot{\omega}_{zYaw}\sin\theta - \dot{\theta}\omega_{zpit}\right)I_{xpityawrol}\sin\theta + \\ & \left(\dot{\omega}_{yYaw} + \ddot{\theta}\right)I_{yzpityawrol}\cos\theta - \left(\dot{\omega}_{yYaw} + \ddot{\theta}\right)I_{xypityawrol}\sin\theta - \\ & \dot{\psi}\omega_{xyaw}\left(I_{yzyawrol}\cos\theta + I_{yyawrol}\sin\psi\sin\theta\right) + \\ & \ddot{\psi}\left(I_{zyawrol}\cos\theta + I_{yzyawrol}\sin\psi\sin\theta\right) - \ddot{\phi}I_{xrol}\cos\psi\sin\theta \\ = & \ \dot{\omega}_{zYaw}\left(I_{zYaw} + I_{zpityawrol}\cos^2\theta + I_{zxpityawrol}\sin 2\theta + I_{xpityawrol}\sin^2\theta\right) - \\ & \dot{\omega}_{xYaw}\left[I_{zxpityawrol}(\cos^2\theta - \sin^2\theta) + \left(I_{xpityawrol} - I_{zpityawrol}\right)\sin\theta\cos\theta\right] + \\ & \dot{\omega}_{yYaw}\left(I_{yzpityawrol}\cos\theta - I_{xypityawrol}\sin\theta\right) + \end{aligned}$$

$$\omega_{xYaw}\omega_{yYaw}\left(I_{yYaw} - I_{xYaw}\right) +$$
$$\dot{\theta}\omega_{xpit}\left(I_{zpityawrol}\cos\theta + I_{zxpityawrol}\sin\theta\right) +$$
$$\dot{\theta}\omega_{zpit}\left(I_{zxpityawrol}\cos\theta + I_{xpityawrol}\sin\theta\right) -$$
$$\dot{\psi}\omega_{xyaw}\left(I_{yzyawrol}\cos\theta + I_{yyawrol}\sin\psi\sin\theta\right) +$$
$$\ddot{\theta}\left(I_{yzpityawrol}\cos\theta - I_{xypityawrol}\sin\theta\right) +$$
$$\ddot{\psi}\left(I_{zyawrol}\cos\theta + I_{yzyawrol}\sin\psi\sin\theta\right) - \ddot{\phi}I_{xrol}\cos\psi\sin\theta$$

Define:

$$I_{zYawpityawrol} = I_{zYaw} + I_{zpityawrol}\cos^2\theta + I_{zxpityawrol}\sin 2\theta + I_{xpityawrol}\sin^2\theta \quad (17)$$
$$I_{zxYawpityawrol} = I_{zxpityawrol}\cos 2\theta + \frac{I_{xpityawrol} - I_{zpityawrol}}{2}\sin 2\theta \quad (18)$$
$$I_{yzYawpityawrol} = I_{yzpityawrol}\cos\theta - I_{xypityawrol}\sin\theta \quad (19)$$

$$\therefore T_{zYawpityawrol} \approx \dot{\omega}_{zYaw}I_{zYawpityawrol} - \dot{\omega}_{xYaw}I_{zxYawpityawrol} + \dot{\omega}_{yYaw}I_{yzYawpityawrol} +$$
$$\omega_{xYaw}\omega_{yYaw}\left(I_{yYaw} - I_{xYaw}\right) +$$
$$\dot{\theta}\omega_{xpit}\left(I_{zpityawrol}\cos\theta + I_{zxpityawrol}\sin\theta\right) +$$
$$\dot{\theta}\omega_{zpit}\left(I_{zxpityawrol}\cos\theta + I_{xpityawrol}\sin\theta\right) -$$
$$\dot{\psi}\omega_{xyaw}\left(I_{yzyawrol}\cos\theta + I_{yyawrol}\sin\psi\sin\theta\right) +$$
$$\ddot{\theta}\left(I_{yzpityawrol}\cos\theta - I_{xypityawrol}\sin\theta\right) +$$
$$\ddot{\psi}\left(I_{zyawrol}\cos\theta + I_{yzyawrol}\sin\psi\sin\theta\right) - \ddot{\phi}I_{xrol}\cos\psi\sin\theta \quad (20)$$

Now take the yaw angle β between the ‘B’ and ‘Yaw’ axes into account:

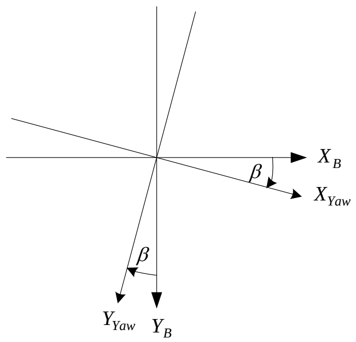


Figure 10. Outer yaw angle transformation

$$\omega_{xYaw} = \omega_{xB}\cos\beta + \omega_{yB}\sin\beta$$
$$\omega_{yYaw} = \omega_{yB}\cos\beta - \omega_{xB}\sin\beta \quad (21)$$
$$\omega_{zYaw} = \omega_{zB} + \dot{\beta}$$

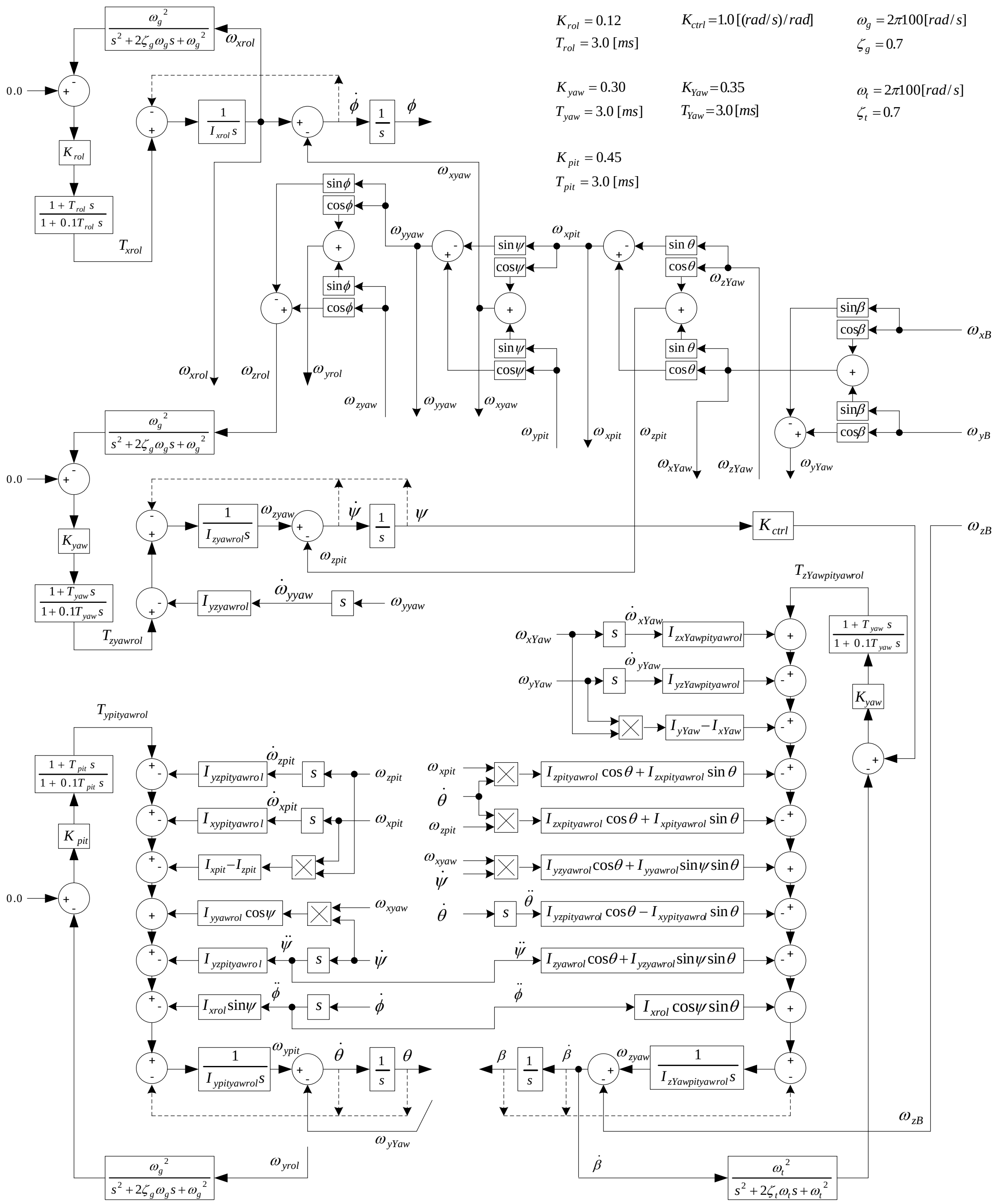
Block diagram and simulation results for four axes:

From equations (1) to (21) and with the addition of friction, the following block diagram can be compiled.

Rather simple stabilization loops are added in the block diagrams, with $\frac{{\omega_g}^2}{s^2+2\zeta_g\omega_g s+{\omega_g}^2}$ representing feedback from gyros mounted on the platform, measuring ω_{xrol} , ω_{zrol} and ω_{yrol} . A proportional controller and one lead compensator close each loop. The outer yaw gimbal has a tachometer, represented by $\frac{{\omega_t}^2}{s^2+2\zeta_t\omega_t s+{\omega_t}^2}$, feeding back the relative angular rate $\dot{\beta}$ and also a proportional controller and one lead compensator. It is driven by the inner yaw gimbal angle ψ through a gain K_{ctrl} to effectively keep ψ at a small angle therefore aiding good stabilization.

The size of the measurements ω_{xrol} , ω_{yrol} and ω_{zrol} is an indication of the quality of stabilization – the smaller these signals the better the stabilization. They can be seen in the graphs below to be much smaller than the previous three-axis case as expected.

The initial Yaw, pitch and roll angles β, θ and ϕ were set at 45° to prevent some disturbance torques to be very small because of small angles.



$I_{xrol} = 3.0e-4\,[kg.m^2]$ $\quad I_{xpit} = 4.0e-4\,[kg.m^2]$

$I_{yrol} = 5.0e-4\,[kg.m^2]$ $\quad I_{ypit} = 3.0e-4\,[kg.m^2]$

$I_{zrol} = 5.0e-4\,[kg.m^2]$ $\quad I_{zpit} = 3.3e-4\,[kg.m^2]$

$I_{xyaw} = 2.0e-4\,[kg.m^2]$ $\quad I_{xYaw} = 5.0e-4\,[kg.m^2]$

$I_{yyaw} = 3.0e-4\,[kg.m^2]$ $\quad I_{yYaw} = 4.4e-4\,[kg.m^2]$

$I_{zyaw} = 2.0e-4\,[kg.m^2]$ $\quad I_{zYaw} = 4.0e-4\,[kg.m^2]$

$$I_{xyawrol} = I_{xyaw} + I_{xrol}$$
$$I_{yyawrol} = I_{yyaw} + I_{yrol}\cos^2\phi + I_{zrol}\sin^2\phi$$
$$I_{zyawrol} = I_{zyaw} + I_{zrol}\cos^2\phi + I_{yrol}\sin^2\phi$$
$$I_{yzyawrol} = \frac{I_{yrol} - I_{zrol}}{2}\sin 2\phi$$

$$I_{xpityawrol} = I_{xpit} + I_{xyawrol}\cos^2\psi + I_{yyawrol}\sin^2\psi$$
$$I_{ypityawrol} = I_{ypit} + I_{yyawrol}\cos^2\psi + I_{xyawrol}\sin^2\psi$$
$$I_{zpityawrol} = I_{zpit} + I_{zyawrol}$$
$$I_{xypityawrol} = \frac{I_{xyawrol} - I_{yyawrol}}{2}\sin 2\psi$$
$$I_{yzpityawrol} = I_{yzyawrol}\cos\psi$$
$$I_{zxpityawrol} = I_{yzyawrol}\sin\psi$$

$$I_{zYawpityawrol} = I_{zYaw} + I_{zpityawrol}\cos^2\theta + I_{zxpityawrol}\sin 2\theta + I_{xpityawrol}\sin^2\theta$$
$$I_{zxYawpityawrol} = I_{zxpityawrol}\cos 2\theta + \frac{I_{xpityawrol} - I_{zpityawrol}}{2}\sin 2\theta$$
$$I_{yzYawpityawrol} = I_{yzpityawrol}\cos\theta - I_{xypityawrol}\sin\theta$$

Figure 11. Simplified block diagram of four-axis yaw-pitch-yaw-roll

The simulation result is the following.

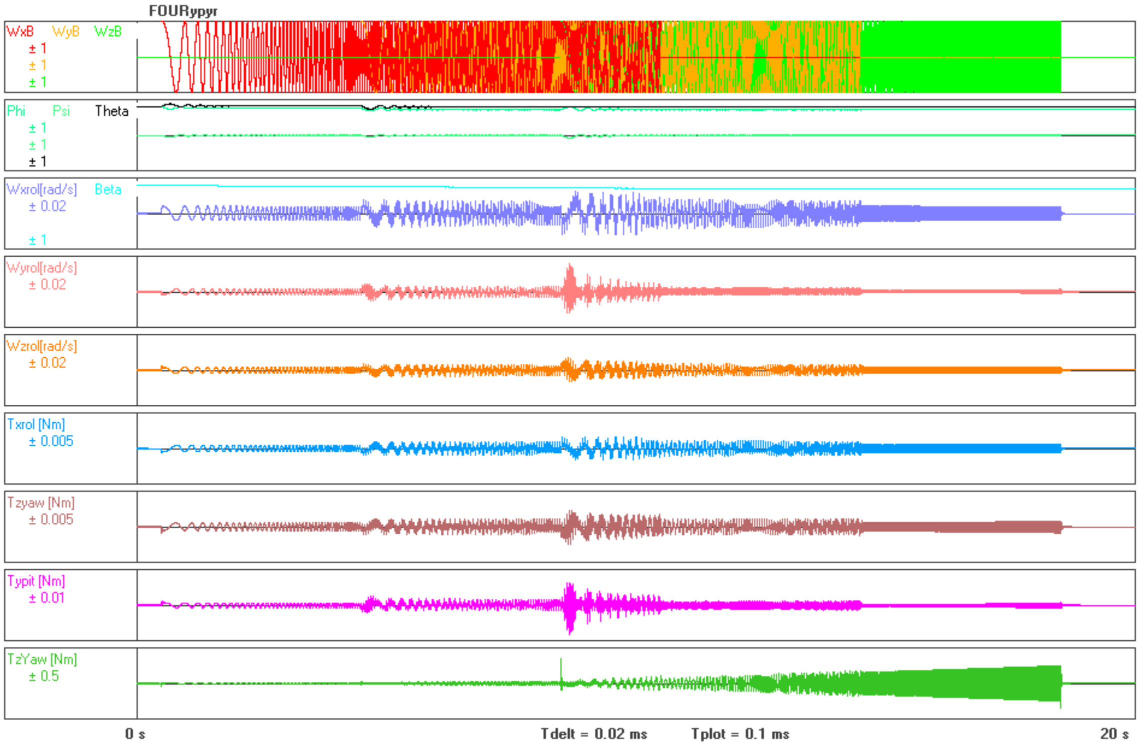


Figure 12. Simplified four-axis graph results

As could be expected, ω_{xrol} , ω_{yrol} and ω_{zrol}, which are indications of the quality of stabilization, are much smaller than for the three-axis case.

Conclusions:

Based on the demonstration with the three-axis example where the comprehensive and simplified models were compared and found to give identical results, and the reasoning for ignoring certain terms when deriving the simplified equations and model, it is concluded that the proposed simplification method is valid also for more-axis, like the four-axis configuration.

The saving in the amount of work to be done to derive models for four- and higher number of axes platforms by using this method is substantial as can be appreciated a bit by comparing the comprehensive and simplified three-axis block diagrams. Actually the amount of work to be saved will increase dramatically as the number of axes is increased.

Summary:

In this paper a method was shown to simplify the dynamics model of stabilized platforms, with the four-axis platform as the example vehicle. The relevant simplified equations for a specific four-axis configuration were derived, the resulting block diagram was given, and then some simulation results were shown after adding simple control loops around the dynamics model.

The validity of the simplification method was demonstrated by showing that identical simulation results were achieved for a comprehensive and simplified three-axis model example.

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