Implementing Control Charts to Corporate Financial Management

MARTIN KOVARIK
Tomas Bata University in Zlin
Department of Statistics
and Quantitative Methods
Nam. T. G. Masaryka 5555, 760 01 Zlin
CZECH REPUBLIC
m1kovarik@fame.utb.cz

LIBOR SARGA
Tomas Bata University in Zlin
Department of Statistics
and Quantitative Methods
Nam. T. G. Masaryka 5555, 760 01 Zlin
CZECH REPUBLIC
sarga@fame.utb.cz

Abstract: In the paper, corporate financial management using statistical process control (SPC), especially Shewhart's control charts operating with the constant mean, control charts with non-constant mean, and process capability indices will be introduced. The center line, UCL and LCL for the control charts will be defined with the regulated process not allowed to cross the UCL and LCL boundaries. Altman's model (the so called Z-score), the most popular corporate financial stability index, will be used. We will demonstrate benefits of SPC on two case studies: the first will focus on corporate financial flow control, the second will include six companies. Special types of control charts, i.e., CUSUM and EWMA, will be discussed due to their mean shift sensitivity and practical applications demonstrated on additional two case studies. The results prove control charts can be successfully implemented not only in manufacturing processes but in corporate financial management as well.

Key–Words: Altman's Z-score, Statistical Process Control, Shewhart's control charts, process capability indices, EWMA control chart, CUSUM control chart

1 Introduction

Statistical financial flow management deals with corporate cash flow control. Monitoring cash flow, companies can avoid losses from undelivered goods, bad financial investments, etc. We have selected Altman's model for financial analysis which should be done once per year. For the purposes of Case Study 1 (section 5.1), we will present monthly values for an unspecified company, Case Study 2 (section 5.2) will describe situation in six unspecified companies also using monthly data. We will conclude the article by describing dynamic control charts together with practical examples (Case Study 3 and 4 in sections 5.3 and 5.4, respectively).

Prediction models of corporate financial problems constitute a way to evaluate health of a company using aggregated number (index) which attempts to include all the financial analysis components, i.e., profitability, liquidity, indebtedness, and capital structure with each having its own weight. The weights, based on empirical research, represent components' importance in the financial health. Many models use them for predicting financial problems, e.g., Beaver's test, Edmister's analysis, Altman's test, Tamar's risk index, ZCR coefficient, Lis' index, Taffler's index, Springate-Gordon's index, Fulmer's index, IN 95 index, and IN index in a form of numerical intervals. We describe some of the most widely-used models in the following part [1, 2].

Altman's Z-score was proposed in 1968 as a prognostic index for solvency based on discriminant analysis of 60 companies listed on the NYSE at the time. The aim of the article is to introduce a tool for bankruptcy prediction or more precisely future liquidity problems [2].

2 Z-score and Indices

2.1 General Z-score and Indices

Altman's Z-Score is based on discriminant analysis principle. General notation of a discriminant function is [2]:

$$\begin{aligned} Z &= a_1X_1 + a_2X_2 + a_3X_3 \\ &\quad + a_4X_4 + a_5X_5 + a_6X_6, \end{aligned} \tag{1}$$

where a_i is a discriminant coefficient, $i = 1, 2, 3, \ldots, 6$, and X_i is a discriminant variable, $i = 1, 2, 3, \ldots, 6$. The latter is identical for all Z-Score variations [2].

$$x_1 = \frac{\text{working capital}}{\text{total assets}} \tag{2}$$

$$x_2 = \frac{\text{revenue after tax} + \text{ retained earnings}}{\text{total assets}} \tag{3}$$

$$x_3 = \frac{\text{EBIT}}{\text{total assets}} \quad (4)$$

$$x_4 = \frac{\text{market share value}}{\text{total debts}} \quad (5)$$

$$x_5 = \frac{\text{returns}}{\text{total assets}} \quad (6)$$

$$x_6 = \frac{\text{undertakings after deadline}}{\text{returns}}. \quad (7)$$

2.2 Z-Score Model for Joint-Stock Companies

Z-Score model for joint-stock companies is the original Altman's Z-Score model tested and constructed for the U.S. companies. However, parameters for Czech companies differ significantly from American ones and model's informative value is therefore very low. We will denote Z-model for joint-stock companies as Z_1-Score [1]. It is defined as [2]:

$$Z_1 = 1.2X_1 + 1.4X_2 + 3.3X_3 + 0.6X_4 + 1.0X_5 + 0.0X_6. \quad (8)$$

X_6 is equal to zero, and the formula is thus simplified to:

$$Z_1 = 1.2X_1 + 1.4X_2 + 3.3X_3 + 0.6X_4 + 1.0X_5. \quad (9)$$

Companies belong to the one of the following intervals according to their Z_1-Score:

- $Z_1 > 2.99 \rightarrow$ Safe Zone: financially strong,
- $Z_1 \in \langle 1.81; 2.98 \rangle \rightarrow$ Grey Zone: small financial problems,
- $Z_1 < 1.80 \rightarrow$ Distress Zone: serious financial problems.

Healthy companies in good condition belong to the Safe Zone while those in the Distress Zone face serious challenges and bankruptcy risk. Grey Zone companies have problems but it is hard to determine if the situation will get better or not [3].

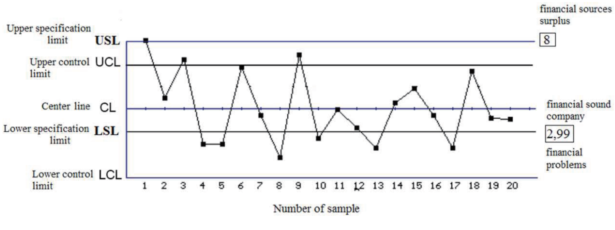

Figure 1: Altman's Index for Joint-Stock Companies in the Shewhart's Concept [Own work]

2.3 Z-Score for Czech Economy

Insolvency is very important in the Czech corporate ecosystem. X_6 was thus added to the original Z_1-Score model whose disadvantage is a small number of bankrupted companies in the sample size on which it can be tested. Z-Score model tailored for Czech economy will be denoted as Z_1_CZ [2]. It is of the following form [2]:

$$Z_1\_CZ = 1.2X_1 + 1.4X_2 + 3.3X_3 + 0.6X_4 + 1.0X_5 + 1.0X_6. \quad (10)$$

Z_1_CZ's classification intervals are identical to Z_1.

2.4 Z-Score Model for Other "Non-Joint-Stock" Companies

Criticism of the Z_1-Score model for non-joint-stock companies appeared after 1968. Its modification was later based on changes in the X_4 index. Other coefficients also changed together with the classification criteria. The new model was published in 1983 and will be denoted as Z_2-Score [2]. It is of the form [2]:

$$Z_2 = 0.717X_1 + 0.847X_2 + 3.107X_3 + 0.420X_4 + 0.992X_5 + 0.000X_6. \quad (11)$$

Discriminant coefficient of X_6 is equal to zero and the model is thus simplified to:

$$Z_2 = 0.717X_1 + 0.847X_2 + 3.107X_3 + 0.420X_4 + 0.992X_5. \quad (12)$$

Classification intervals were also modified:

- $Z_2 > 2.90 \rightarrow$ Safe Zone,
- $Z_2 \in \langle 1.23; 2.90 \rangle \rightarrow$ Grey Zone,
- $Z_2 < 1.23 \rightarrow$ Distress Zone.

Grey zone in Z_2 is wider than in Z_1 (see section 2.2).

2.5 Z-Score Model for Non-Manufacturing Companies and Emerging Markets

The variation of the index published in 1995 does not include X_5 so that the influence of industries exhibited by variables in X_5 is minimized. All coefficients for X_1 to X_4 were changed, and the model is useful for industrial comparison with different kinds of assets financing. It is denoted as Z_3-Score [2]:

$$Z_3 = 6.86X_1 + 3.26X_2 + 6.72X_3 + 1.05X_4. \quad (13)$$

Its classification intervals are:

- $Z_3 > 2.60 \rightarrow$ Safe Zone,
- $Z_3 \in \langle 1.10; 2.60 \rangle \rightarrow$ Grey Zone,
- $Z_3 < 1.10 \rightarrow$ Distress Zone.

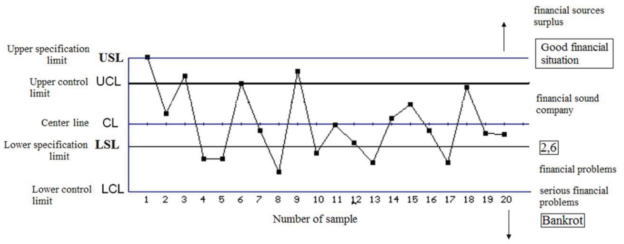

Figure 2: Updated Model for Non-Manufacturing, Trading and Emerging Companies in the Shewhart's concept [Own work]

According to Altman, it is good enough for prediction and can successfully anticipate bankruptcy two years before its appearance. Distant future result are, however, statistically insignificant. More variations have been derived from the original Altman's model, for example Springate–Gordon's and Fulmer's model (see sections 2.6 and 2.7).

2.6 Springate–Gordon's Model

Based on the Altman's model and tested on data from 40 companies, 19 proportional indices were initially considered. Discriminant analysis chose four and the model was defined as [6]:

$$S = 1.03X_1 + 3.07X_2 + 0.66X_3 + 0.4X_4, \quad (14)$$

where

$$x_1 = \frac{\text{net working capital}}{\text{property}} \quad (15)$$

$$x_2 = \frac{\text{EBIT}}{\text{property}} \quad (16)$$

$$x_3 = \frac{\text{EBT}}{\text{short-term undertakings}} \quad (17)$$

$$x_4 = \frac{\text{returns}}{\text{property}}. \quad (18)$$

If $S < 0.862$, financial problems should be expected, and the company is classified as "failed".

2.7 Fulmer's Model

The model is suitable for small companies. Originally, 40 indices were analyzed on data gained from 40 companies, half of which were successful and the other half failed. It is defined as [5]:

$$F = 5.528X_1 + 0.212X_2 + 0.073X_3 + 1.270X_4 + 0.120X_5 + 2.335X_6 + 0.575X_7 + 1.083X_8 + 0.894X_9 - 6.075, \quad (19)$$

where

$$x_1 = \frac{\text{retained earnings}}{\text{property}} \quad (20)$$

$$x_2 = \frac{\text{returns}}{\text{property}} \quad (21)$$

$$x_3 = \frac{\text{EBT}}{\text{capital}} \quad (22)$$

$$x_4 = \frac{\text{cash flow}}{\text{total debt}} \quad (23)$$

$$x_5 = \frac{\text{total debt}}{\text{property}} \quad (24)$$

$$x_6 = \frac{\text{short-term undertakings}}{\text{property}} \quad (25)$$

$$x_7 = \text{property} \quad (26)$$

$$x_8 = \frac{\text{net working capital}}{\text{total undertakings}} \quad (27)$$

$$x_9 = \frac{\text{EBIT}}{\text{cost interest}}. \quad (28)$$

In case $F < 0$, the corporation should expect financial problems in the future.

3 Statistical Process Control: Shewhart's Control Charts

Statistical process control is one way to effectively use statistical methods for corporate financial flow management. Variability of financial flow must be respected: if the same type of calculation is used, same results will not be obtained – values from the Altman's model. Control charts consist of a center line (CL) placed at a reference value, the upper control limit (UCL) and the lower control limit (LCL), also called action limits. UCL and LCL are boundaries of random cause of process variability, and a decision rule for process control. CL, UCL and LCL are plotted in software, e.g., QC Expert. When the process is "in control", 99.73 % of sample data lies between UCL and LCL [8].

Engineering limits are usually given by upper specification limit (USL) and lower specification limit (LSL) according to the Altman's model. For the above-mentioned examples, $USL = 8$ and $LSL = 2.99$. If the value is lower than 2.99, the company has financial problems, if the value is higher than 8, there is a financial surplus [3].

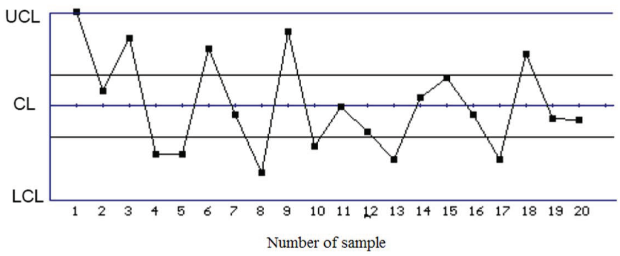

Figure 3: Shewhart's Control Chart [Own work]

W. A. Shewhart proposed the concept of basic control charts, graphical aids to separate identifiable causes from random causes in process variability. Control chart construction has mathematical and statistical basis. Classic control charts belong to a class of charts without memory: actual value does not include previous ones. It is suitable for identifying sporadic mistakes/deviations in the process, i.e., deviations higher than 2σ [12].

To build a Shewhart's Control Chart:

- Step 1: We first choose part of the process to be analyzed and prepare data.
- Step 2: Based on data from Step 1, we calculate a statistical model represented by sample mean and sample standard deviation. We test statistical conditions for Shewhart's control chart use.
- Step 3: A control chart is constructed on the basis of parameters from Step 2, i.e., sample mean and sample standard deviation. Center line, upper and lower control limits are plotted to the chart.
- Step 4: Data from the selected process is plotted to the constructed chart. We focus on "strange cases" which signalize unexpected variations in process behavior. A basic "strange case" is when UCL or LCL crosses the line.
- Step 5: "Strange cases" are registered and their root causes investigated. When found, precautions can be devised and implemented [7, 9].

4 Other Types of Control Charts

4.1 CUSUM

CUSUM control charts are based on cumulative sums. Introduced by Page in 1954, their main advantage is very quick detection of relatively small process mean shifts which is significantly quicker than in Shewhart's control charts. Sequential sums of deviations from μ_0 are used for their construction. If μ_0 is the population mean target value and X_j sample mean, a CUSUM control chart is constructed as:

$$S_i = \sum_{j=1}^{i} (X_j - \mu_0), \tag{29}$$

plotting variables of the same type. The process is called random walk [7].

4.2 CUSUM for Individual Values and Samples Means from Normally-Distributed Data

Values of x_i are independent, sampled from identical normal distribution $N(\mu, \sigma^2)$ with known population mean and standard deviation σ. We assume logical subgroups with the same volume n.

CUSUM C_n for $n = 1$ (individual values) is then:

- on a base of ordinal scale:

$$C_n = \sum_{j=1}^{i} (x_j - \mu), \tag{30}$$

- on a base of normal distribution where $\mu = 0$ and $\sigma = 1$:

$$U_j = \frac{(x_j - \mu)}{\sigma}, \tag{31}$$

$$S_n = \sum_{i=1}^{j} U_i. \tag{32}$$

C_n is approximately identical to S_n, measured in units of standard deviation σ. Equation for C_n can be written recurrently:

$$C_0 = 0 \tag{33}$$
$$C_n = C_{n-1} + (x_n - \mu); \tag{34}$$

and identically for S_n:

$$S_0 = 0 \tag{35}$$
$$S_n = S_{n-1} + U_n \tag{36}$$

Suppose the observed variable's distribution $N(\mu, \sigma^2)$ changes to $N(\mu + \delta, \sigma^2)$ for integer t (at a certain moment). Population mean μ will then exhibit a shift of δ starting at (m, C_m) which grows linearly with slope δ. The shifts can be more complex but CUSUM control charts can detect it nevertheless [7, 11].

4.3 Process Capability

Process capability indices (PCIs) can be divided into two groups: those measuring process' potential capabilities, and those measuring its actual capabilities. The former determine how capable a process is when certain conditions are met - essentially, if mean of the process' natural variability is centered to the target of engineered specifications. The latter do not require a centered process to be accurate [5].

Process capability analysis assumes:

- the process is statistically under control (determined from the control chart),
- data is normally distributed (tested using histograms and/or tests of normality, e.g., chi-square test, Kolmogorov–Smirnov test, Shapiro–Wilk test, etc.

The most frequently used PCIs are C_p and C_{pk}. They measure processes potential and actual capabilities to consistently produce non-defective products within control limits. If either $C_p \geq 1.33$ or $C_{pk} \geq 1.33$, the process is considered potentially/actually capable [7].

4.4 EWMA

EWMA (Exponentially-Weighted Moving Average) dynamic control charts are used when the following conditions are met:

- observations are not independent and positively autocorrelated,
- mean is not constant and changes slowly.

A sudden change in mean will cause control limit violation. These dynamic charts provide not only information about the "in control" process but also about its dynamic development. As we mentioned, only data which is not independent with positive autocorrelation can be considered. If the measured observations are influenced by previous ones, we can conclude they are dependent. A special case of such dependence is a so-called autocorrelation of the first degree when linear. If there is positive autocorrelation in data, smaller values follow smaller values and higher values follow higher values. Data tends to preserve its original values. A process is unstable in case of negative autocorrelation: higher values follow smaller values and smaller values follow higher values [13].

Suppose that we measure values $x_1, x_2, x_3, \ldots$ for variable X in a process. We use one-step predictions to construct CL, UCL, and UCL for the control chart. The predictions are determined from $\hat{x_k} = x_{k-1} + \lambda e_k$ for $k = 1, 2, 3, \ldots$ where the initial prediction value $\hat{x_0}$ is equal the target value of μ_0. Parameter λ (level of "forgetting") is calculated by minimizing $\sum_{k=1}^{n} e_k^2$, n is equal to the number of measured observations for a regulated variable and is recommended to be greater than 50. If one-step prediction error values of e_k for optimal λ are not correlated and normally-distributed, CL_k, UCL_k, and LCL_k for the EWMA dynamic control chart are calculated from the following equations [7]:

$$CL_k = \hat{x}_{k-1}, \tag{37}$$
$$LCL_k = \hat{x}_{k-1} - \hat{\sigma_p} u_{1-\frac{\alpha}{2}}, \tag{38}$$
$$LCL_k = \hat{x}_{k-1} + \hat{\sigma_p} u_{1-\frac{\alpha}{2}}, \tag{39}$$
$$\hat{\sigma}_p^2 = \frac{1}{n-1} \sum_{k=1}^{n} e_k^2, \tag{40}$$

where $\hat{\sigma}_p^2$ is e_k's standard deviation while values of e_k are determined for an optimal λ [5, 10].

5 Case Studies

The aim of this part is to analyze whether it is possible to use control charts for financial flow management. Their limits can be calculated using Altman's index to determine if the company is in good financial standing. When process' stability is corrupted, it is necessary to look for a root cause of why the index is low or high. Application of statistical process control methods can indicate changes in financial flows before they become a serious threat.

5.1 Case Study 1: Calculating SPC for a Single Company

Table 1: Financial data for Case Study 1 [Own work]

Rank	Value	Rank	Value
1	3.578	11	4.280
2	3.953	12	3.577
3	4.288	13	3.855
4	4.191	14	3.605
5	3.129	15	3.700
6	3.039	16	3.415
7	3.525	17	3.535
8	4.595	18	3.455
9	3.915	19	3.210
10	3.757	20	3.355

The data in Tab. 1 consists of 20 values spanning two years from the balance sheet of an unspecified company. It was instated to the Altman's model

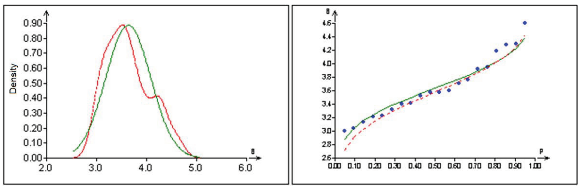

Figure 4: Test of Normality: Density Estimation (histogram) and Quantile Chart [Own work]

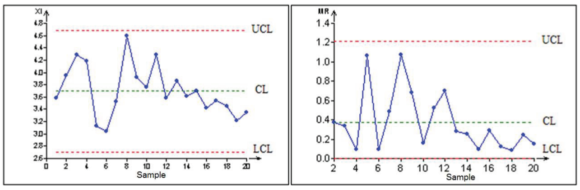

Figure 5: Control charts: x-individual (left) and R (right) [Own work]

formula, CL, UCL, and LCL are listed in Tab. 2. We subsequently constructed a special control chart for individual x_i; the most common value was in the range from 3 to 4, i.e., the company has no serious financial problems and is quite healthy.

The tolerance levels are $LSL = 8$ and $USL = 2.99$. MR is calculated from two neighboring values, therefore $n = 2$ and $d_2 = 1.128, D_3 = 0, D_4 = 3.267, \sigma = 0.4177$. We then use equations 29, 30, and 31 for quantifying CL, UCL, and LCL for x-individual control chart, and equations 37, 38 and 39 for CL, UCL, and LCL for control chart R (Fig. 5).

Table 2: Control limits calculations [Own work]

x-individual	R
$CL = 3.6978$	$CL = 0.3724$
$UCL = 4.6884$	$UCL = 1.2168$
$LCL = 2.7022$	$LCL = 0$

We need to test data normality (Fig. 4), comparing theoretical distribution shapes with actual ones for our data, before we can begin constructing the control charts. Red and green distributions should be very close for the data to be normally distributed, a prerequisite obviously met in Fig. 4. The analysis therefore proved data normality, and we can construct the control charts.

Table 3: Capability indices calculations [Own work]

capability index C_p	$C_p = 1.999$
financial flow stability C_{pk}	$C_{pk} = 0.5730$

C_p equals 1.999, i.e., the company is doing well

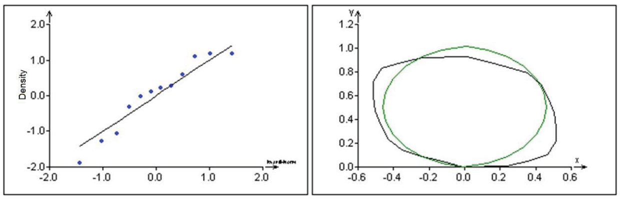

Figure 6: Test of Normality: Q–Q chart and circle plot [Own work]

and is financially sound. C_{pk}, however, was calculated to be 0.5730 which means financial flows are not under control. The x-individual control chart shows declining tendency towards the -2σ boundary, a warning before the control limit is crossed.

5.2 Case Study 2: Calculating SPC for 6 Companies

Source data in Tab. 4 originates from balance sheets of six companies, observed monthly. We again instate them into the Altman's model; the results are listed in Tab. 5. Tolerance limits: $LSL = 8$ and $USL = 2.99, \sigma = 0.4177$.

Table 4: Source data for six companies [Own work]

Rank	X_1	X_2	X_3	X_4	X_5	X_6
1	3.255	3.215	3.426	3.129	3.217	3.511
2	3.436	3.888	3.366	3.273	4.036	3.327
3	3.012	4.164	4.326	3.792	3.430	3.600
4	3.292	3.576	3.686	3.235	3.601	3.673
5	3.155	4.347	3.081	4.221	3.262	3.769
6	3.705	4.214	3.427	3.466	3.424	3.188
7	3.330	3.407	3.334	3.126	3.541	3.527
8	3.235	4.742	3.766	3.759	3.433	3.383
9	3.201	3.993	3.430	3.910	3.633	3.396
10	3.980	3.836	3.580	3.394	2.453	3.204
11	4.118	3.519	3.495	3.945	3.243	3.191
12	3.486	4.482	3.336	3.644	3.573	3.741

Using equations 29, 30, and 31, we get CL, UCL, and LCL for x-mean control chart, and equations 37, 38 and 39 to quantify CL, UCL, and LCL for control chart R. We need to test normality before constructing control charts by means of exploratory analysis, depicted in Fig. 6.

Q–Q graph for normally-distributed data without outliers is line-shaped, for normally-distributed data with outliers the line is deformed with ending points outside the line. Circle plot provides visual evaluation of data normality based on skewness and kurtosis. A green circle (ellipse) is optimal for normal distribu-

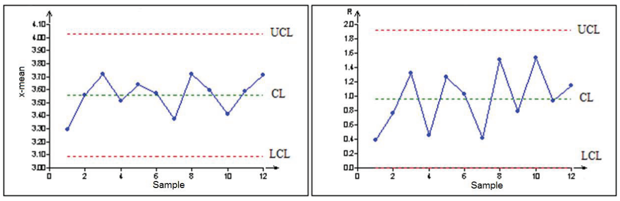

Figure 7: Control charts: x-mean (left) and R (right) [Own work]

tion, the black represents source data. Both are identical in case of normal distribution. The exploratory data analysis therefore proved normally-distributed data. We can now construct the control charts x-mean and R (Fig. 7).

Table 5: Control limits calculations [Own work]

x-mean	R
$CL = 3.5568$	$CL = 0.9596$
$UCL = 4.0277$	$UCL = 1.9231$

Table 6: Calculations of capability indices [Own work]

Capability index C_p	$C_p = 6.007$
Stability of financial flow C_{pk}	$C_{pk} = 1.3592$

C_p equals 6.007, the companies were doing well during the observed period and are financially sound. C_{pk} is equal to 1.3592: financial flows are under control with no financial problems based on the sample data. Variability of results among the six companies lie within UCL and LCL control limits, i.e., process variability does not cross the control lines.

5.3 Case Study 3: CUSUM

- $\mu_0 = 10, n = 1, \sigma = 1,0,$
- We would like to detect a shift 1.0: $\sigma = 1.0\,(1.0) = 1.0$, $(d = 1,0)$,
- Process mean is out of control: $\mu_1 = 10 + 1 = 11$,
- $K = \frac{d}{2} = \frac{1}{2}$ and $H = 5, \sigma = 5$ (recommended),
- Equations for C_i^+ and C_i are:

$$\begin{aligned} C_i^+ &= \max\left[0, x_i - 10.5 + C_{i+1}^+\right] \\ C_i^- &= \left[0, 10.5 - x_i + C_{i-1}^-\right] \end{aligned}$$

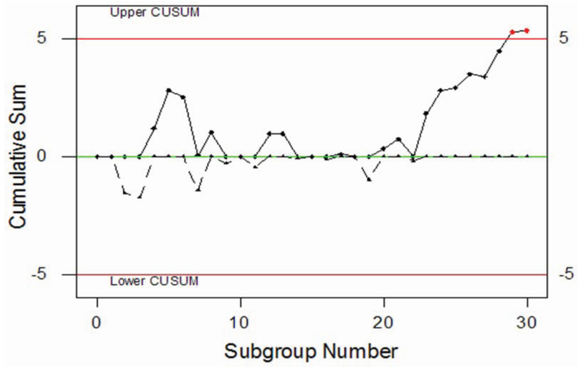

Figure 8: CUSUM control chart for case study 3 [Own work]

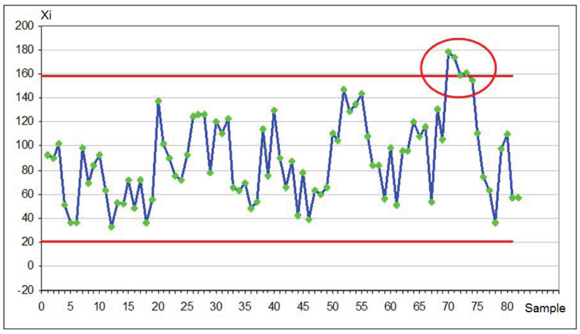

Figure 9: Shewhart's control chart [Own work]

The CUSUM chart (Fig. 8) shows the process is out of control. In the following step, a root cause (causes) should be looked for, precaution(s) implemented and CUSUM control chart plotted again. If the process was adjusted, it could be useful to estimate its mean caused by the shift.

Figures 9 and 10 graphically compare CUSUM and Shewhart's control charts.

The example practically demonstrates sensitivity of the CUSUM control chart compared with the Shewhart's control chart for sample means. It does not detect deviations (Fig. 9) in lower values while CUSUM

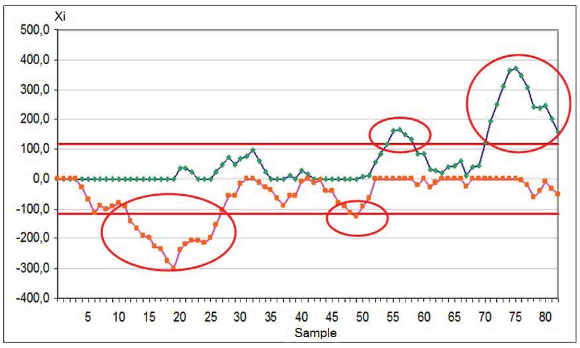

Figure 10: CUSUM control chart [Own work]

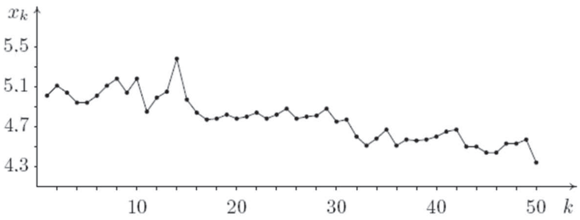

Figure 11: Lineplot [Own work]

control chart detects process mean deviation around subgroup 20 (Fig. 10). Shewhart's control chart also does not detect the shift in upper values (around subgroup 56) but only the big shift around subgroup 70.

5.4 Case Study 4: EWMA

The financial data for a company are provided (millions of CZK) in Tab. 7. The initial value of $x = 5.00$.

Table 7: Source data for EWMA control chart construction [Own work]

k	x_k	k	x_k	k	x_k	k	x_k
1	5.01	16	4.84	31	4.77	46	4.44
2	5.11	17	4.77	32	4.60	47	4.53
3	5.04	18	4.78	33	4.51	48	4.53
4	5.12	19	4.82	34	4.58	49	4.57
5	4.94	20	4.78	35	4.67	50	4.34
6	5.01	21	4.80	36	4.51		
7	5.11	22	4.84	37	4.57		
8	5.18	23	4.78	38	4.56		
9	5.04	24	4.82	39	4.57		
10	5.18	25	4.88	40	4.60		
11	4.85	26	4.78	41	4.65		
12	4.99	27	4.80	42	4.67		
13	5.05	28	4.81	43	4.50		
14	5.38	29	4.88	44	4.50		
15	4.97	30	4.75	45	4.44		

Fig. 11 shows values to be declining, i.e., a constant process mean is not present. Next, we analyze if there exists an autocorrelation of the first degree by constructing a correlation chart between x_k and x_{k+1} for $k = 1, 2, 3, \ldots, 49$. The result is depicted in Fig. 12.

The scatterplot is ellipse-shaped and the ellipse's main axis forms an acute angle with the x axis. Based on this observation, we can conclude there is a significant first-degree positive autocorrelation between x_k and x_{k+1}. Exact coefficient of autocorrelation equals 0.850. i.e., strong, statistically-significant positive autocorrelation. We will construct the EWMA

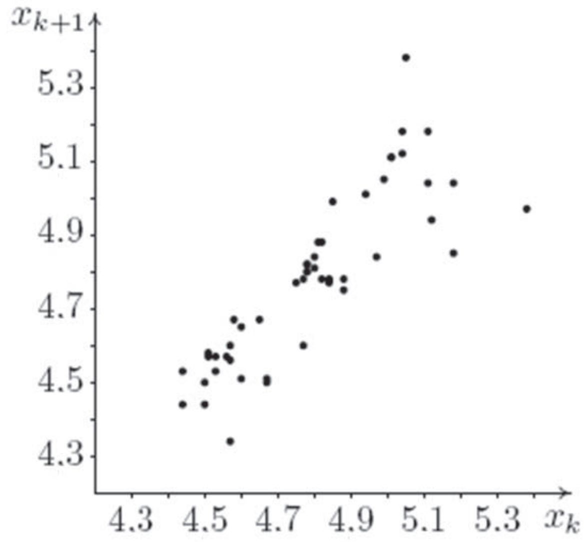

Figure 12: Correlation chart between x_k and x_{k+1} (scatterplot) [Own work]

Table 8: $S(\lambda)$ values [Own work]

λ	$S(\lambda)$	λ	$S(\lambda)$
0.4	0.663138	0.47	0.658893
0.5	0.659108	0.48	0.658852
0.6	0.665795	0.49	0.658926

dynamic control chart and compute predicted values of $\hat{x_k}$ for empirically-selected values of λ in interval $\langle 0; 1 \rangle$. Starting value was set to $\mu_0 = 5.00$. We then determine the value of $S(\lambda) = \sum_{k=1}^{n} e_k^2$ (the sum of error squares) for selected λ. The calculation for $\lambda = 0.48$, which was found to be optimal, is shown in Tab. 8. $S(\lambda)$ is parabolic with one extreme (minimum).

Tab. 8 shows λ parameter found in the first and the third column, and $S(\lambda)$ in the second and the fourth column. The values of $S(\lambda)$ for $0.4, 0.5$, and 0.6 are given in the first two columns. The function's minimum is between 0.4 and 0.5; we set λ equal to $0.47, 0.48$ and 0.49, repeated the process, and found $S(\lambda)$ minimum equals 0.48. The EWMA control chart is depicted in Fig. 13 where LCL and UCL are plotted as full black lines, and CL as a dashed line. Inputs of the controlled attribute are represented by dots.

The process is under control with all observations inside the control band except for sample 14 which may have been caused by process instability or inaccurate measurement.

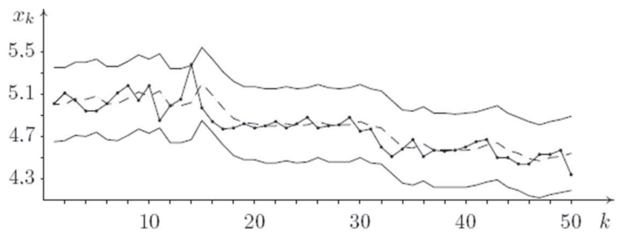

Figure 13: The EWMA control chart [Own work]

6 Conclusion

The article dealt with control charts applications in financial data. This kind of data is very sensitive to mean shifting, strong autocorrelation also appears very often. We therefore focused on CUSUM and EWMA dynamic control charts. Versatility of control charts not only in manufacturing but also in managing financial stability of cash flows throughout the paper was highlighted. A refined identification of the type of intervention affecting the process will allow users to effectively track sources of out-of-control situations, an important step in eliminating special causes of variation.

Autocorrelated observations mainly arise under automated data collection schemes, typically controlled by software which can be upgraded to handle SPC functions. Under such integrated scheme, usefulness of the proposed procedure will be optimized. We would recommend a properly-designed time series control charts as control charts for individual measurements in wide range of applications. These are almost perfectly non-parametric (distribution-free) procedures.

Traditional statistical process control (SPC) schemes, such as Shewhart's and CUSUM control charts assume data collected from processes to be independent. However, the assumption has been challenged as it has been found data is serially correlated in many practical situations. Performance of traditional control charts deteriorates significantly under autocorrelation, and monitoring of forecasted errors after appropriate time series model has been fitted to a process has been proposed to compensate. The method is intuitive as autocorrelation can be accounted for by the underlying time series model while the residual component captures process' independent random errors. Traditional SPC schemes can be applied to monitor residuals.

Subsequent work on this problem can be broadly classified into two themes: time series models based and model-free. For the former, three general approaches have been proposed: those which monitor residuals, those based on direct observations, and those based on new statistics. Their brief account is presented in this chapter.

The time series model-based approach is easy to understand and effective in some situations. However, it requires identifying appropriate time series model from a set of initial in-control data. This may not be easy to establish in practice and may be too complicated to practicing engineers. Hence, the model-free approach has recently attracted much attention.

We are of the opinion SPC control charts provide an alternative to traditional instruments of financial control. We attempted to prove it in this article where we have demonstrated how SPC can be effectively implemented to corporate financial management, and the different types of control charts which exhibit varying sensitivity to process mean shifts. Specifically, Shewhart's and CUSUM control charts were compared, and it was concluded the latter is suitable for processes in which small mean shifts need to be detected early on. Possible future expansion of our work is application of SPC into other areas where data-driven real-time statistical control is warranted.

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