The de facto standard method for the study of two-particle channels from lattice QCD consists in first extracting the finite-volume spectrum from lattice correlators using the variational method and then relating it to the infinite-volume scattering phase shifts using the finite-size formula introduced by Lüscher or one of its generalizations. This approach has been applied successfully to a large number of two-particle channels, paving the way for a complete ab-initio description of hadron physics. The HAL QCD method is a more recent addition to this field and proposes an alternative way to extract the scattering phase shifts of a two-particle system from lattice QCD simulations. Wave function-like correlators are computed on the lattice which can be related to the scattering phase shifts in infinite volume. The energy-dependence of these wave functions is modeled by a non-local kernel through the Schrödinger equation and this kernel is approximated from lattice input, leading to predictions for the scattering phase shifts in the whole elastic region.

A part of this thesis is dedicated to the numerical application of the HAL QCD method to various meson-meson channels. In particular, we study the pion-pion channel in the isospin $I = 1$ and $I = 2$ channels. For $I = 2$, the HAL QCD method allows us to
extract the scattering phase shifts from simulations at a pion mass of $m_\pi = 700$ MeV. The $I = 1$ P-wave channel contains the rho meson and is particularly challenging. The HAL QCD method is found to face severe difficulties in this channel which only allows us to acquire a qualitative understanding of the interaction. We also study several charmed meson-meson channels which have been predicted to host tetraquark bound states by some quark model calculations. No bound state is found in the pion mass range $m_\pi = 410 \sim 700$ MeV and the quark-mass dependence of the results hints at the absence of bound states at the physical point.

Another part of this thesis is dedicated to improve the theoretical tools available for the study of two-particle channels in lattice QCD. We first show how the HAL QCD method can be extended to treat the interaction above the inelastic threshold for both coupled two-particle channels and channels with more than two particles. We then go on to propose two new methods which address some of the criticisms of the HAL QCD method while retaining its core ideas. The effective potential method is a new way to extract the finite-volume spectrum for two-particle channels. It extends the variational method to rectangular correlation matrices which are used to parameterize an effective Hamiltonian operator. The finite-volume spectrum is then related to the spectrum of this operator. The kernel approximation method is an alternative to the HAL QCD method based on a careful study of the finite-volume effects. It relies on the same wave function-like correlators in finite volume, which we rigorously relate to the infinite-volume Bethe-Salpeter kernel. The properties of this kernel are then used to study the energy-dependence of the correlators as well as the mixing of their angular momentum components by the cubic group. This leads to a well-defined strategy to extract the scattering phase shifts. Our two newly proposed methods are compared numerically to the previously available ones and are found to be both correct and efficient.