# 論文の内容の要旨

## 論文題目: Properties and phenomenology of non-Abelian vortices in dense QCD

(高密度QCDにおける非可換量子渦の性質と現象論)

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QCD matter at high densities and low temperatures is expected to be a color superconductor. At extremely high densities, the ground state is the color-flavor locked (CFL) phase, that exhibits color superconductivity as well as superfluidity. It is known that QCD matter in the CFL phase hosts topologically stable vortices. Important feature of these vortices is that they have internal degrees of freedom, which are bosonic and fermionic modes. These modes are localized only around the core of vortices, and they propagate along the vortices. The bosonic modes are called orientational zero modes, which are a kind of Nambu-Goldstone modes. The symmetry group of QCD is $G = U(1)_{\mathrm{B}} \times SU(3)_{\mathrm{C}} \times SU(3)_{\mathrm{L}} \times SU(3)_{\mathrm{R}}$, where $U(1)_{\mathrm{B}}$ is the baryon number symmetry, $SU(3)_{\mathrm{C}}$ the local color symmetry, and $SU(3)_{\mathrm{L(R)}}$ is the symmetry to rotate the flavor of left(right)-handed quarks. In the CFL phase, $G$ is spontaneously broken down to the color-flavor locked symmetry $H = SU(3)_{\mathrm{C+L+R}} \equiv SU(3)_{\mathrm{C+F}}$, apart from discrete symmetry. In the presence of a vortex, the symmetry group $H$ is further broken down to $H' = SU(2) \times U(1)$ only around the core. The orientational zero modes are the Nambu-Goldstone modes associated with the symmetry breaking $H \to H'$ and they are the coordinates that parametrize the coset space $H/H' \simeq \mathbb{C}P^2$. On the other hand, the fermionic modes are "Majorana" fermions as a consequence of the particle-hole symmetry of the effective Hamiltonian. There is one interesting feature in the case of vortices in the CFL phase, which is that there appear multiple Majorana fermions in one vortex. This results in a unique structure in the non-Abelian statistics of vortices as discussed later. This thesis mainly consists of two investigations on the non-Abelian vortices in the CFL phase. The first topic is the interaction between the vortices and quasiparticles in the CFL phase. The latter topic is the properties of non-Abelian statistics of vortices with multiple Majorana fermions.

Firstly, we discuss the interaction of vortices with quasiparticles, such as $U(1)_{\mathrm{B}}$ phonons, gluons, CFL mesons, and photons. It is necessary to determine the interaction to discuss physical phenomena such as scattering or radiation of quasiparticles by vortices. We can also investigate the interaction between vortices using vortex-quasiparticle interaction, since the intervortex force is mediated by quasiparticles.

The interaction Lagrangian between vortices and phonons and gluons is derived via the dual formation. While the phonons are blind to the orientations of vortices and only couple to the position of a vortex, it turns out the interaction with gluons is dependent on the orientation of a vortex. This gives rise to an orientation-dependent interaction energy between two vortices. The orientation-dependent interaction works in such a way to reduce the total color flux of two interacting vortices.

We discuss the interactions of vortices with CFL mesons. The CFL mesons are the Nambu-Goldstone associated with the chiral symmetry breaking in the CFL phase, whose dynamics is described by the chiral Lagrangian. We extend the chiral Lagranginan, and derive the Lagrangian of mesons under the background of a vortex solution. The Lagrangian incorperates the modification of meson propagation or interaction in the presence of a vortex.

We also investigate the interaction of vortices with photons and its phenomenological consequences. The electromagnetic property of vortices can be phenomenologically important as it could lead to some observable effects. The orientational zero modes localized on vortices are charged with respect to $U(1)_{\rm EM}$ symmetry. The Lagrangian that includes the interaction term of photons and orientational modes is determined by symmetry consideration as

$$\mathcal{L}_{g\mathbb{C}P^2} = \sum_{\alpha=0,3} C_\alpha \left[\mathcal{D}^\alpha \phi^\dagger \mathcal{D}_\alpha \phi + (\phi^\dagger \mathcal{D}^\alpha \phi)(\phi^\dagger \mathcal{D}_\alpha \phi)\right], \tag{1}$$

where the covariant derivative is defined by $\mathcal{D}_\alpha \phi = \left(\partial_\alpha - ie\sqrt{6}A_\alpha T_8\right)\phi$, and $T_8 = \frac{1}{\sqrt{6}}\mathrm{diag}(-2,1,1)$ is the generator of $U(1)_{\rm EM}$ group in our choice of basis. Based on this interaction, we discuss the scattering of photons off a vortex. The scattering cross section of photons per unit length of a vortex is obtained as

$$\frac{d\sigma}{dz} = \frac{\left(12C_3 e^2 f(\phi)\right)^2 \eta^2}{8\pi}\lambda = 288\pi \left(C_3 \alpha \eta f(\phi)\right)^2 \lambda, \tag{2}$$

for photons whose electric fields are parallel to the vortex. Here, $\eta$ is a constant of order unity, $\alpha$ is the fine structure constant, and $f(\phi)$ is a function of $\phi$ defined as $f(\phi) \equiv \phi^\dagger (T_8)^2 \phi + (\phi^\dagger T_8 \phi)^2$, and $\lambda$ is the wavelength of the incident photon. On the other hand, if a photon's electric field is perpendicular to the vortex, the photon is not scattered.

Based on the properties stated above, we discuss the optical property of a vortex lattice. The rotating CFL matter is expected to be threaded with quantum vortices along the axis of rotation, resulting in the formation of a vortex lattice. This is basically the same phenomenon as when one rotates atomic superfluids. If CFL matter exists inside the core of a rotating dense stars, there should be a lattice of vortices in the core. It is shown that a lattice of vortices serves as a polarizer of photons, because of the interaction of orientational modes with the photons. Suppose that a linearly polarized photon is incident on a vortex lattice as shown in Fig. 1. If the electric field of the photon is parallel to the vortices, it induces currents along the vortices, which results in the attenuation of the

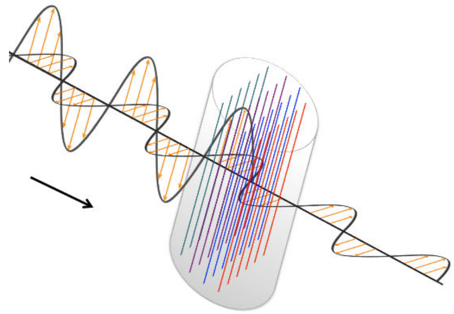

Figure 1: Photons entering a vortex lattice.

photon. On the other hand, waves with electric fields perpendicular to the vortices are not affected. This is exactly what a polarizer does. A lattice passes electromagnetic waves of a specific polarization and blocks waves of other polarizations. This phenomenon, resulting from the electromagnetic interaction of vortices, may be useful for finding observational evidence of the existence of CFL matter.

Secondly, we analyze the non-Abelian statistics of vortices, which is brought about by the existence of Majorana fermions inside vortices. There has been considerable interest recently in zero-energy fermion modes trapped inside vortices in superconductors. Vortices in a chiral $p$-wave superconductor are endowed with non-Abelian statistics because of the zero-energy Majorana fermions inside them. Excitations which obey non-Abelian statistics are called non-Abelian anyons. They are expected to form the basis of topological quantum computations and have been investigated intensively.

A vortex in the CFL phase has been also shown to have Majorana zero modes inside the core. What is more, in the case of a CFL vortex, there are multiple Majorana zero modes. The number of zero modes is closely related to the winding number of a vortex, which is evident from the index theorem for this system. For example, a vortex with minimal winding number has three Majorana fermions inside them. In this thesis, the non-Abelian statistics of vortices with multiple Majorana fermions is investigated and shown to have a novel structure. We show that the transformation matrix of the states under the exchange of two vortices can be written as the tensor product of two matrices. One matrix is identical to the exchange matrix for vortices with a single Majorana fermion in each core, that is found by Ivanov, and the other matrix is shown to be a generator of the Coxeter group, which is a symmetry group of certain polytopes in high dimensions.

We consider the exchange statistics of vortices, each of which traps an odd number $(N)$ of Majorana fermions. Let $\gamma_k^a$ be the Majorana fermion operator localized to $k$-th vortex with $a = 1 \cdots N$. They are self-conjugate, $(\gamma_k^a)^\dagger = \gamma_k$, and satisfy the Clifford algebra, $\{\gamma_k^a, \gamma_l^b\} = 2\delta^{ab}\delta_{kl}$. We refer to the anticlockwise exchange of $k$-th and $k+1$-th vortex as $T_k$. Under the operation $T_k$, the fermion operators $\gamma_k^a$ are transformed as

$$T_k : \begin{cases} \gamma_k^a \;\;\;\, \to \gamma_{k+1}^a \\ \gamma_{k+1}^a \to -\gamma_k^a \end{cases} \qquad \text{for all } a\,, \tag{3}$$

with the rest $\gamma_\ell^a$ $(\ell \neq k,\, k+1)$ unchanged. We can explicitly construct the operator that induced the transformations above in terms of the fermion operators as

$$\tau_k^{[N]} \equiv \prod_{a=1}^{N} \exp\left(\frac{\pi}{4}\gamma_{k+1}^a \gamma_k^a\right) = \prod_{a=1}^{N} \frac{1}{\sqrt{2}}\left(1+\gamma_{k+1}^a\gamma_k^a\right) . \tag{4}$$

The operator defined above indeed induces the appropriate transformation on $\gamma_k^a$ as

$$\tau_k^{[N]}\gamma_k^a(\tau_k^{[N]})^{-1} = \gamma_{k+1}^a , \tag{5}$$

$$\tau_k^{[N]}\gamma_{k+1}^a(\tau_k^{[N]})^{-1} = -\gamma_k^a , \tag{6}$$

$$\tau_k^{[N]}\gamma_\ell^a(\tau_k^{[N]})^{-1} = \gamma_\ell^a \qquad (\ell \neq k,\ k+1) . \tag{7}$$

It is shown that the exchange operator $\tau_k^{[N]}$, generating the exchange of two neighboring vortices, can be factorized into two parts as

$$\tau_k^{[N]} = \sigma_k^{[N]} h_k^{[N]}. \tag{8}$$

The operator $h_k^{[N]}$ is essentially equivalent to the exchange operator introduced by Ivanov, that corresponds to the case $N = 1$. If it is expressed in terms of the $SO(N)$ singlet Majorana operator $\overline{\gamma}_k$ defined by

$$\overline{\gamma}_k \equiv \frac{1}{N!} \mathrm{e}^{i\frac{\pi}{4}(N-1)} \epsilon^{a_1 a_2 \cdots a_N} \gamma_k^{a_1}\gamma_k^{a_2}\cdots\gamma_k^{a_N} , \tag{9}$$

then it has the same form as the exchange operator $\tau_k$ in the case of the single Majorana fermion, i.e. $h_k^{[N]} = \exp\left(\frac{\pi}{4}\overline{\gamma}_{k+1}\overline{\gamma}_k\right)$. On the other hand, the other operator $\sigma_k^{[N]}$ is a generator of a Coxeter group, which is shown by checking that $\sigma_k^{[N]}$ satisfies the Coxeter relations of the type $A_{2m-1}$ (the symmetric group $S_{2m}$) for $n = 2m$ vortices. We also discuss relation between the operator decomposition and the tensor-product structure in the Matrix representation. We show that the factorization of the exchange operators results in the tensor-product structure in its matrix representation in a suitable basis.