VELOCITY FIELD IN HYDRAULIC JUMPS AT LARGE REYNOLDS NUMBERS: DEVELOPMENT OF AN ARRAY OF TWO DUAL-TIP PHASE-DETECTION PROBES

AUTHORS: Hang WANG and Hubert CHANSON
HYDRAULIC MODEL REPORTS

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Velocity Field in Hydraulic Jumps at Large Reynolds Numbers: Development of an Array of Two Dual-Tip Phase-detection Probes

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Experimental hydraulic jump in a horizontal rectangular channel with flow direction from left to right (Top) and phase-detection probe configuration for four-point measurement with two dual-tip probes viewed in elevation (Bottom)
Abstract

The measurement of turbulent velocity in highly aerated flow is difficult because of the presence of air bubbles. It is even more challenging for hydraulic jumps with relatively high Froude numbers, because of the large-scale, three-dimensional turbulence and self-sustained instabilities. While most previous studies were focused on the description of flow characteristics in the longitudinal-vertical plane, very limited experimental data are available to date in terms of the transverse motions including the instantaneous velocity fluctuations. This study presents a new method aimed to characterise the three-dimensional velocity field in hydraulic jumps using a four-sensor phase-detection probe array. Besides the longitudinal velocity and turbulence intensity measured in both positive and negative velocity flow regions, a characteristic instantaneous transverse velocity component was derived together with a measure of its fluctuations. The transverse velocity component characterised the three-dimensional nature of turbulent structures, although the time-averaged flow pattern was two-dimensional and the average transverse velocity was zero. The corresponding velocity fluctuation was found to be one order of magnitude smaller than the longitudinal velocity fluctuation. While the longitudinal fluctuation was significantly affected by the large-scale flow instabilities, the instantaneous transverse velocity fluctuation was thought to be free of the effects of unsteady, pseudo-periodic motions. The time-averaged velocities and turbulence intensities were also measured in directions with some angle from the longitudinal direction and were expected to be a combination of longitudinal and transverse components. The uncertainties related to the correlation analysis and flow complexities were discussed. The present data provided some guidelines for the use of phase-detection probe array and correlation signal processing techniques in complex turbulent two-phase flows.

Keywords

Air-water flow; Velocity measurement; Velocity turbulence; Flow recirculation; Phase-detection conductivity probe; Four-point measurement; three-dimensional flow structure; transverse velocity fluctuation; Reynolds stress; Correlation analysis; Physical modelling.
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C  time-averaged void fraction defined as volume of air per unit volume of air and water;
C(+) time-averaged void fraction measured with normal phase-detection probe positions with probe sensors pointing upstream;
C(-) time-averaged void fraction measured with reversed phase-detection probe positions with probe sensors pointing downstream;
C_{\text{max}} local maximum time-averaged void fraction in the turbulent shear region;
c instantaneous void fraction;
d_{1} inflow water depth (m) immediately upstream of the jump toe;
d_{2} downstream water depth (m);
F bubble count rate (Hz) defined as the number of bubbles or water droplets per second;
F(+) bubble count rate (Hz) measured with normal phase-detection probe positions with probe sensors pointing upstream;
F(-) bubble count rate (Hz) measured with reversed phase-detection probe positions with probe sensors pointing downstream;
F_{\text{max}} maximum bubble count rate (Hz) in the turbulent shear region;
F_{\text{sec}(+)} secondary maximum bubble count rate (Hz) in the recirculation region measured with normal phase-detection probe positions with probe sensors pointing upstream;
F_{\text{sec}(-)} secondary maximum bubble count rate (Hz) in the recirculation region measured with reversed phase-detection probe positions with probe sensors pointing downstream;
F_{i}^{r} Froude number: Fr_{i} = \frac{V_{i}}{g \times d_{i}^{1/2}};
g gravity acceleration (m/s^{2}): g = 9.80 \text{ m/s}^{2} in Brisbane, Australia;
h upstream sluice gate opening (m);
L_{r} jump roller length (m);
L_{X} longitudinal integral turbulent length scale (m);
L_{Z} transverse integral turbulent length scale (m);
Q flow rate (m^{3}/s);
R_{ii}(\tau) normalised auto-correlation function;
R_{\text{max}}(\tau) maximum auto-correlation coefficient;
R_{ij}(\tau) normalised cross-correlation function;
(R_{ij})_{\text{max}} maximum cross-correlation coefficient;
Re Reynolds number: Re = \frac{\rho \times V_{i} \times d_{i}^{1/2}}{\mu};
T statistical air-water interfacial travel time (s) between two phase-detection probe tips;
T_{0.5} characteristic time lag (s) in the auto-correlation function for which R_{ij}(T_{0.5}) = 0.5;
T_{i} auto-correlation time scale (s);
T_{ij} cross-correlation time scale (s);
T_{X} longitudinal integral turbulent time scale (s);
T_{Z} transverse integral turbulent time scale (s);
[T]_{ij} statistical air-water interfacial travel time (s) between phase-detection probe tips i and j, i,j = 1,2,3,4;
Tu air-water interfacial turbulence intensity;
Tu_{X} longitudinal turbulence intensity;
Tu_{X}(+) longitudinal turbulence intensity measured with normal phase-detection probe positions with probe sensors pointing upstream;
Tu_{X}(-) longitudinal turbulence intensity measured with reversed phase-detection probe positions with probe sensors pointing downstream;
U free-stream velocity (m/s) in the supercritical flow measured with Prandtl-Pitot tube;
V air-water interfacial velocity (m/s);
V_{ij} air-water interfacial velocity (m/s) measured between phase-detection probe tips i and j, i,j = 1,2,3,4;
\( V_{\text{max}} \) maximum air-water interfacial velocity (m/s) in the turbulent shear region;

\( V_{\text{recirc}} \) average recirculation velocity (m/s) in the recirculation region;

\( V_x \) longitudinal component of air-water interfacial velocity (m/s);

\( V_x(+) \) longitudinal interfacial velocity (m/s) measured with normal phase-detection probe positions with probe sensors pointing upstream;

\( V_x(-) \) longitudinal interfacial velocity (m/s) measured with reversed phase-detection probe positions with probe sensors pointing downstream;

\( V_z \) characteristic instantaneous transverse interfacial velocity (m/s);

\( V_z(+) \) characteristic transverse interfacial velocity (m/s) measured with normal phase-detection probe positions with probe sensors pointing upstream;

\( V_z(-) \) characteristic transverse interfacial velocity (m/s) measured with reversed phase-detection probe positions with probe sensors pointing downstream;

\( V_1 \) average inflow velocity (m/s): \( V_1 = Q/(W \times d_1) \);

\( v' \) standard deviation of instantaneous velocity (m/s);

\( v_{ij}' \) standard deviation of instantaneous velocity (m/s) between phase-detection probe tips i and j, i,j = 1,2,3,4;

\( v_{ij}'(+) \) standard deviation of instantaneous velocity (m/s) between probe tips i and j measured with normal phase-detection probe positions with probe sensors pointing upstream;

\( v_{ij}'(-) \) standard deviation of instantaneous velocity (m/s) between probe tips i and j measured with reversed phase-detection probe positions with probe sensors pointing downstream;

\( v_z' \) standard deviation of transverse instantaneous velocity (m/s);

\( W \) channel width (m);

\( x \) longitudinal distance (m) from the upstream sluice gate;

\( x_1 \) longitudinal position (m) of jump toe;

\( Y_{V_{\text{max}}} \) vertical position (m) of the maximum velocity in the turbulent shear region;

\( Y_{0.5} \) vertical position (m) where the average velocity is half maximum: \( Y_{0.5} = y(V = V_{\text{max}}/2) \);

\( Y_{90} \) vertical position (m) where the void fraction is 0.9;

\( y \) vertical distance (m) above the channel bed;

\( y^* \) vertical position (m) of local minimum void fraction between the turbulent shear region and recirculation region;

\( z \) transversal distance (m) from the channel centreline;

\( \Delta x \) longitudinal separation distance (m) between dual-tip phase-detection probe tips;

\( \Delta x_{ij} \) longitudinal separation distance (m) between phase-detection probe tips i and j, i,j = 1,2,3,4;

\( \Delta z \) transversal separation distance (m) between dual-tip phase-detection probe tips;

\( \Delta z_{ij} \) transversal separation distance (m) between phase-detection probe tips i and j, i,j = 1,2,3,4;

\( \Theta \) diameter (m);

\( \eta \) water elevation above the channel bed (m);

\( \mu \) dynamic viscosity of water (Pa\cdot s);

\( \rho \) water density (kg/m\(^3\));

\( \tau \) time lag (s);

\( \tau_{0.5} \) characteristic time lag (s) in the cross-correlation function for which \( R_{xx'}(T+\tau_{0.5}) = (R_{xx'})_{\text{max}}/2 \);
1. INTRODUCTION

1.1 Introduction of hydraulic jump

A hydraulic jump is a sudden transition from a supercritical flow to a subcritical flow (Montes 1998). It is commonly encountered in natural waterways and hydraulic structures with presence of barrier structures or a rapid expansion of flow cross-sectional area (Hager 1992). Figure 1.1 shows a hydraulic jump forming at the downstream foot of Chinchilla weir (QLD Australia). The arrows indicate the inflow direction, and the curved impingement perimeter is highlighted by the depth discontinuity and the rough free-surface immediately downstream of the impingement point (i.e. jump toe). Because of the presence of the vortical and recirculating flow structures, this turbulent flow region downstream of the jump toe is called the jump roller (Long et al. 1991).

![Hydraulic jump](image)

Figure 1.1 – Hydraulic jump formed at the end of Chinchilla weir with a curved impingement perimeter. Red arrows indicate the upstream impinging flow direction. Photography by Hubert Chanson in November 1997 at Chinchilla QLD Australia

The pattern of a hydraulic jump may vary significantly with the flow conditions. For a horizontal rectangular channel, the flow pattern is primarily characterised by the inflow Froude number (Henderson 1966, Liggett 1994):

\[
Fr_1 = \frac{V_1}{\sqrt{g \times d_1}}
\]  

(1.1)

where \( V_1 \) is the average inflow velocity, \( d_1 \) is the inflow depth, \( g \) is the gravity acceleration and the subscript 1 refers to the upstream flow conditions. A hydraulic jump with an inflow Froude number \( Fr_1 > 4 \) to 5 is typically associated with a breaking free-surface of the jump roller, visible eddy structures, flow recirculation, air entrainment and a large rate of energy dissipation (Montes 1998). Figure 1.2 illustrates an experimental hydraulic jump with \( Fr_1 = 7.5 \). A large amount of air is entrained into the roller at the jump toe (singular aeration) and through the roller surface (interfacial aeration). Some large-scale vortical structures are visualised by the entrained air bubbles. The bubbles and vortices are advected downstream and vanished in the tailwater as the flow de-aerated and turbulence dissipated.

In Figure 1.2 the upstream and downstream conjugate depths and the longitudinal range of the jump roller are also marked. Dimensional analysis and physical data indicated that the conjugate depth ratio \( d_2/d_1 \), relative roller length \( L_r/d_1 \) and average roller surface profile were simply functions of the
inflow Froude number $F_{r1}$ (Chanson 2011, Wang 2014). For a horizontal rectangular channel with smooth bed and sidewalls, the conjugate depth ratio $d_2/d_1$ can be deduced from the momentum equations (Bélanger 1841):

$$\frac{d_1}{d_2} = \frac{1}{2} \sqrt{1+8 \times F_{r1}^2} - 1$$  \hspace{1cm} (1.2)

while the jump roller length $L_r$ and roller surface profile are mostly given by empirical data, for example in Wang & Chanson (2015b):

$$\frac{L_r}{d_1} = 6 \times (F_{r1} - 1) \hspace{1cm} \text{for} \hspace{0.5cm} 2 < F_{r1} < 10 \hspace{1cm} (1.3)$$

$$\frac{\eta - d_1}{d_2 - d_1} = \left( \frac{x - x_1}{L_r} \right)^{0.537} \hspace{1cm} \text{for} \hspace{0.5cm} 3.8 < F_{r1} < 10, \hspace{0.5cm} 0 < (x - x_1)/L_r < 1 \hspace{1cm} (1.4)$$

Herein the roller length $L_r$ is defined as the longitudinal distance over which the time-averaged water elevation increases monotonically, and $\eta$ is the time-averaged water elevation above the invert. Further empirical and computational expressions of $L_r$ and $\eta$, other than Equations (1.3) and (1.4), were proposed by Hager et al. (1990) (1), Valiani (1997), Murzyn & Chanson (2009b), Chanson (2011) and Richard & Gavrilyuk (2013).

In addition, dimensional considerations showed the significance of the Reynolds number for physical modelling of a turbulent shear flow (Chanson & Chachereau 2013). The Reynolds number may be defined as:

$$Re = \frac{\rho \times V \times d_1}{\mu}$$  \hspace{1cm} (1.5)

where $\rho$ and $\mu$ are respectively the water density and dynamic viscosity. A number of air-water flow properties and characteristic turbulent scales showed noticeable correlation with the Reynolds number (Chanson & Gualtieri 2008, Murzyn & Chanson 2008, Chanson & Chachereau 2013, Wang 2014).

---

1 Note the different definition of jump roller length by Hager et al. (1990).
1.2 Review of velocity and turbulence measurements in hydraulic jumps

A hydraulic jump flow is highly turbulent and highly aerated. The characterisation of turbulence is of significant importance for the understanding of flow regimes and practical applications, e.g. to enhance fluid mixing and energy dissipation. However, the presence of large number of air bubbles adversely affects the use of most traditional velocity measurement instruments such as laser Doppler velocimetry (LDV), acoustic Doppler velocimetry (ADV) and particle image velocimetry (PIV). These instruments are designed for monophase flow, and their application in hydraulic jump studies was restricted to very weak jumps with small Froude numbers and a low aeration level. For example, Svendsen et al. (2000) used LDV on hydraulic jumps for $F_{R1} < 1.6$. Liu et al. (2004) and Mignot & Cienfuegos (2010) used micro ADV with highest Froude numbers being 3.3 and 2 respectively. Hornung et al. (1995) applied PIV measurements on travelling hydraulic jumps with a range of Froude numbers from 2 to 6. Lennon & Hill (2006) used PIV on stationary jumps for $1.4 < F_{R1} < 3$. The main limitation of the operation of these instruments in bubbly flow is related to the obscuration in the laser/acoustic beam paths caused by the air-water interfaces, and the disturbance on bubble distribution and diffusion by the tracer particles. In addition, special treatment may be required to distinguish the tiny bubbles from the tracer particles (Boyer et al. 2002). Wanieliski et al. (2001) used air bubbles directly as the tracer particles during LDV measurements of high Froude number hydraulic jumps, but the bubble size was restricted between 1 and 500 μm. Mossa & Tolve (1998) applied a flow visualisation technique to investigate the air concentration across vertical sections of jump roller by evaluating the grey levels of the images. This technique was developed by Leandro et al. (2012) to the two-dimensional flow field and the results were compared to direct phase-detection probe data (see below). A disadvantage of bubble tracking and imaging techniques is that the visualised plane lies in the lateral boundary layer next to the channel sidewall where the bubble diffusion and velocity field may be distorted.

In the past decades, the largest number of and most successful air-water flow measurements in hydraulic jumps were conducted with intrusive phase-detection probes. The needle-shaped probe sensor is designed to pierce the bubbles and droplets, and the air-water interfaces are detected based upon the different electrical resistivity or optical reflectivity between air and water (Cartellier 1992, Chanson 2002). Besides the local void fraction and bubble count rate measured by a single needle sensor, the simultaneous sampling of two sensors of a dual-tip probe enables derivation of time-averaged air-water interfacial velocity and corresponding turbulence intensity. A number of physical data demonstrated that, in high-velocity free-surface flows, the gas-liquid flows behave as a quasi-homogenous mixture within the flow region with void fraction less than 90%, and the two phases travel with a nearly identical velocity, the slip velocity being negligible (Rao & Kobus 1971, Cain & Wood 1981a,b, Wood 1991, Chanson 1997). Therefore, the phase-detection probe provides an approach to the velocity and turbulence quantification in strong hydraulic jumps. The present study was performed based upon this technique, with the primary focus on the characterisation of three-dimensional flow structure and turbulence field.

The turbulence measurement in hydraulic jumps dated back to 1950s when Rouse et al. (1959) modelled hydraulic jumps with air flows in a jump-shaped wind tunnel and measured turbulence characteristics using hot-film anemometers. Resch & Leutheusser (1972a,b) measured in air-water flows the velocity fluctuations in longitudinal and transverse directions with hot-film anemometers. Particularly, they investigated for both fully- and partially-developed inflow conditions, indicating distinctive difference in terms of turbulence development due to the extent of boundary layer growth on the bottom of the incident flow. Chanson & Bratthberg (2000) measured the time-averaged void fraction and interfacial velocity using conductivity needle probes. The typical void fraction and velocity distributions are sketched in Figure 1.3 for partially-developed inflow conditions. Their results were reproduced by most following studies either using conductivity probes (Kucukali & Chanson 2008, Murzyn & Chanson 2009a, Wang & Chanson 2014) or using

From the successive two-point detection of air-water interfaces with a dual-tip probe, the turbulence intensity was derived based upon correlation analysis of the probe signals under several key assumptions (Chanson & Toombes 2002). The validity of the direct application of this technique in the upper roller is however arguable because of the development of large-scale turbulent structures and inhomogeneous turbulent field. Wang et al. (2014) discussed the impact of the large-scale flow instabilities on the turbulence characterisation by applying a triple decomposition technique to the signal processing. Their results showed a high turbulence level in the jump roller, which were comparable to the very-limited turbulence intensity data available for hydraulic jumps with medium to high Froude numbers (Resch & Leutheusser 1972b, Babb & Aus 1981).

Limited attention has been devoted to the study of three-dimensional flow structures in hydraulic jump, partially because the time-averaged flow pattern can be reasonably treated as two-dimensional. However, people do observe instantaneous three-dimensional structures in the roller. The only water velocity fluctuation measurement in the transverse direction that the authors are aware of to date was reported by Resch & Leutheusser (1972b). Further transverse integral turbulent scales were evaluated by means of correlation analyses, including the free-surface length and time scales measured by Murzyn et al. (2007) and Chachereau & Chanson (2011b) and the bubbly vortical turbulent scales by Zhang et al. (2013) and Wang et al. (2014). The results of Wang et al. (2014) indicated different dimensions of coherent bubbly structures in the longitudinal and transverse directions in the region immediately downstream of the impingement point. The magnitude and vertical distributions of the transverse integral turbulent length scales were found similar in two types of free-surface air-water flow: i.e. hydraulic jump and stepped spillway flows (Wang 2014).

1.3 Objectives and report structure

The aim of the present work is to characterise the turbulent velocity field in hydraulic jump using a new phase-detection probe array configuration. An array of four sensors was used to detect velocity components in a horizontal plane. A characteristic instantaneous transverse velocity component and the corresponding transverse velocity fluctuation were derived, presented along with a discussion of the validity and uncertainties of this method. Besides the normal probe orientation with the needle
sensors aligned against the inflow direction, the probes were reversed in the upper recirculation region to examine the impact of the reversing flow on the air-water interface detection. The significance of this work includes:

- new method to characterise the complex velocity and turbulence field in highly aerated flow with three-dimensional flow structures;
- the latest experimental data of transverse velocity fluctuations and turbulent scales in strong hydraulic jump;
- detailed velocity distributions in different zones of the jump roller.

Following the introduction to the background of this study, the instrumentation and data processing techniques are described. The experimental results are presented in two parts: the first part encompasses the basic air-water flow properties, time-averaged longitudinal velocity distributions, and a brief summary of turbulence intensity, integral turbulent scales and triple decomposition technique. The second part is focused on the four-point measurement and transverse velocity characterisation, followed by the discussion of uncertainties of this research. Some relevant data processing and analysis are implemented in the appendices for reference.
2. FACILITY AND INSTRUMENTATION

2.1 Experimental facility

The experimental channel was a 3.2 m long, 0.5 m wide rectangular channel built with a smooth, horizontal HDPE bed and 0.4 m high glass sidewalls (Fig. 1.2). Water was discharged into the channel from an upstream head tank (Fig. 2.1). The head tank was equipped with a series of baffles and flow straighteners, followed by an undershoot gate, the rounded edge of the gate (Ø = 0.3 m) inducing a horizontal impinging flow without contraction. The position of the hydraulic jump was controlled by an overshoot gate located at the downstream end of the channel. The flow rate was measured with a Venturi meter in the supply pipeline that fed the head tank, with an expected accuracy of ±2%.

The phase-detection probes were mounted on a trolley system. The vertical position of the probes was monitored using a MitutoyoTM digimatic scale unit with an accuracy of ±0.01 mm. In the present study, y is the vertical coordinate and z is the transverse coordinate (Fig. 2.1).

![Figure 2.1 – Sketch of experimental channel and key parameters of flow conditions.](image)

2.2 Instrumentation

The clear water depths were measured with a pointer gauge. The accuracy of inflow depth measurement was determined to be between 0.2 mm (accuracy of the pointer gauge) and the inflow surface roughness which was a function of the Froude number.

The main instrument was an array of phase-detection conductivity probes. Also known as resistivity probe, an intrusive phase-detection probe discriminates between air and water phases based upon the different electrical resistance of air and water. Figure 2.2A shows a dual-tip phase-detection probe equipped with two identical needle sensors of different lengths. The design and the manufacturing of the probes were done at the University of Queensland. Each needle sensor has a metallic core wire (Ø 0.25 mm silver wire herein (2)) insulated from the outer conductive coat layer (Ø 0.8 mm stainless steel needle tube herein). The cross-section on the needle tip is exposed, and the core wire and the outer needle are electrically connected when the tip is in water. Once an air bubble is pierced by the sensor tip, the circuit is cut off by the large electrical resistance of air, resulting in a voltage drop in the output signal (Fig. 2.2B). The two sensors were mounted parallel.

2 Both silver and platinum wires were tested as the material of core wire. No difference was observed in terms of measurement results.
in an x-z plane, with a transverse separation Δz and a longitudinal distance between the sensor tips Δx, as illustrated in Figure 2.2A. The leading and trailing sensors were ideally designed to pierce an air-water interface one after another. In practice, the signals of both sensors were often analysed statistically to identify the average time lag between the successive detection of the same bubbly flow structure by the sensors. Both sensors were excited simultaneously by an electronic system (Ref. UQ82.518) designed with a response time less than 10 μs. In the present study, all probe sensors were sampled at 20 kHz for 45 s at each single measurement location.

Some comparative velocity data were collected using a Prandtl-Pitot tube in the non-aerated flow region underneath the jump roller. The Prandtl-Pitot tube (Dwyer™ Series 160) had 3 mm outer diameter with a 1 mm stagnation tapping on the tube tip and eight 0.5 mm piezometric tappings around the tube.

![Image of dual-tip phase-detection probe](image1)

![Image of voltage output](image2)

(A) Top view of a dual-tip phase-detection probe  (B) Voltage output of a dual-tip phase-detection probe

Figure 2.2 – Dual-tip phase-detection probe sensors and typical voltage output signal in bubbly flow.

### 2.3 Array of phase-detection sensors and four-point measurements

The use of two dual-tip phase-detection probes enabled a four-point simultaneous measurement of the air-water flow. Figures 2.3A and 2.3B illustrate two probe array configurations with the probes placed side by side. The two dual-tip probes were identically designed with symmetrical sensor positions (Δx = 6.5 mm, Δz = 1.86 mm). The four needle sensors, numbered from 1 to 4, were located within the same x-z plane: i.e., the sensor tips were at identical vertical elevation y above the invert. The leading tips had the same longitudinal positions and were separated by a transverse distance Δz_{12}. The first series of four-point measurements was conducted with the probe configuration in Figure 2.3A. The sensor layout in Figure 2.3B was later adopted to avoid interference of the longer sensor in the path between the shorter sensor of the same probe and the longer sensor of the side-by-side probe. The results in this report are mainly presented for the second configuration (Fig. 2.3B), and some comparisons are developed between the two configurations.

For each dual-tip probe, the longitudinal time-averaged velocity \( V_x \) and the velocity fluctuation \( v'_x \) were calculated following a series of previous studies (Chanson & Brattberg 2000, Chanson 2010, etc.). Considering a much shorter sampling duration, it was possible to derive some transverse interfacial velocity component using a similar correlation analysis of the signals of two leading sensors. Basically, for a given leading tip separation Δz_{12}, the four-point measurement was expected to provide the turbulence characteristics between any two measurement points (tips 1 to 4), hence a 4×4 matrix for each parameter (see next chapter). Further, with a series of measurements using various separation distances Δz_{12}, the transverse integral turbulent length and time scales \( L_Z \) and \( T_Z \) were obtained. The integral turbulent scale results were first presented by Wang et al. (2014) and relevant data processing is enclosed in Appendix A. Herein the sensor separation distance Δz_{12} was
selected to be slightly larger than the typical turbulent length scale \( L_z \) in the shear flow region of jump roller. In this region, \( L_z \) was found between 4 and 8 mm depending on the longitudinal distance from the toe (Wang et al. 2014). Accordingly the leading tip separation was set at \( \Delta z_{12} = 9.0 \) mm for Configuration I (Fig. 2.3A) and \( \Delta z_{12} = 10.0 \) mm for Configuration II (Fig. 2.3B). Such values were about 5 to 10 times larger than the majority of bubble sizes in the shear region.

![Figure 2.3 - Phase-detection probe configuration for four-point measurement with two dual-tip probes (view in elevation).](image)

(A) Configuration I  
(B) Configuration II

Figure 2.3 – Phase-detection probe configuration for four-point measurement with two dual-tip probes (view in elevation).

![Figure 2.4 - Photographs of phase-detection probe arrays sketched in Figure 2.3.](image)

(A) Configuration I, \( \Delta z_{12} = 9.0 \) mm  
(B) Configuration II, \( \Delta z_{12} = 10.0 \) mm

Figure 2.4 – Photographs of phase-detection probe arrays sketched in Figure 2.3.

### 2.4 Experimental flow conditions

Air-water flow measurements were performed for two hydraulic jumps with the same Froude number \( Fr_1 = 7.5 \) but different inflow aspect ratios \( h/W = 0.04 \) and 0.06, where \( h \) is the upstream gate opening and \( W \) is the channel width (\( W = 0.5 \) m). The corresponding Reynolds numbers were \( Re = 6.8 \times 10^4 \) and \( 1.4 \times 10^5 \). The longitudinal jump toe position \( x_1 \) was set at \( x_1 = 41.5 \times h \) downstream of the gate. Such an inflow length corresponded to a partially-developed inflow condition with a relative boundary layer thickness \( \delta/d_1 \approx 0.7 \) at the jump toe, \( \delta \) and \( d_1 \) being respectively the inflow boundary layer thickness and water depth at \( x = x_1 \). The measurements were undertaken in five vertical cross-sections along the centreline of jump roller. The flow conditions and the longitudinal distance from each cross-section to the jump toe are summarised in Table 2.1, where \( Q \) is the flow rate and \( V_1 \) is the average inflow velocity.
Table 2.1 – Flow conditions and longitudinal measurement locations.

<table>
<thead>
<tr>
<th>Q [m$^3$/s]</th>
<th>h [m]</th>
<th>$x_1$ [m]</th>
<th>$d_1$ [m]</th>
<th>$V_1$ [m]</th>
<th>Fr$_1$</th>
<th>Re</th>
<th>($x-x_1$)/$d_1$ [-]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0347</td>
<td>0.02</td>
<td>0.83</td>
<td>0.021</td>
<td>3.37</td>
<td>7.5</td>
<td>6.8×10$^4$</td>
<td>4.03</td>
</tr>
<tr>
<td>0.0705</td>
<td>0.03</td>
<td>1.25</td>
<td>0.033</td>
<td>4.27</td>
<td>7.5</td>
<td>1.4×10$^5$</td>
<td>3.79</td>
</tr>
</tbody>
</table>

Note: Q: flow rate; h: upstream gate opening; $x_1$: longitudinal jump toe position; $d_1$: inflow depth; $V_1$: average inflow velocity; Fr$_1$: inflow Froude number; Re: Reynolds number.
3. DATA PROCESSING OF PHASE-DETECTION PROBES AND PROBE ARRAY

3.1 Presentation

Post-processing and analyses of the phase-detection probe signal provide a large amount of information on the air-water flow and turbulence characteristics. A typical signal segment is shown in Figure 2.2B, consisting of 7 to 8 bubbles detected by each sensor within 0.04 s. For a sampling duration of 45 s, the number of detected bubbles or droplets varied from less than 400 to over 8000. The derivation of basic air-water flow properties from the raw signal involves a single threshold technique. Applying a 50% threshold between the air and water voltage levels, the signal is converted to a binary dataset of instantaneous void fraction c with c = 0 for water and c = 1 for air.

The time-averaged void fraction C is thus given by

\[ C = \frac{1}{N} \sum_{i=1}^{N} c_i \]

The bubble count rate F is defined as the number of air-water or water-air interfaces per second. The characterisation of further air-water flow properties such as bubble chord time and bubble clustering may be found in Chanson & Carosi (2007), Wang et al. (2015) and Felder & Chanson (2015).

On the other hand, most velocity and turbulence properties are calculated based upon auto-correlation and cross-correlation analyses of the raw signals. The most relevant data processing techniques are introduced below, with further details available in the Appendices A to C together with some techniques of lesser relevance, and are referred to in this report. These include the derivation of integral turbulent scales (Appendix B) and the application of triple decomposition technique (Appendix C).

3.2 Time-averaged velocity and turbulence intensity

Given the distance between two measurement points, the time-averaged velocity of the air-water interfaces is derived from the mean interfacial travel time over this distance (Herringe & Davis, 1974). A cross-correlation between the phase-detection signals collected at these two points yields a typical correlation function as sketched in Figure 3.1A, where \( R_{ij} \) is the cross-correlation coefficient and \( \tau \) is the time lag. The mean interfacial travel time equals the time lag \( T \) of the maximum cross-correlation coefficient: i.e. \( R_{ij}(\tau = T) = (R_{ij})_{\text{max}} \) (Fig. 3.1A). For a dual-tip phase-detection probe with the sensor tips aligned in the longitudinal direction, the time-averaged longitudinal velocity is:

\[ V_x = \frac{\Delta x}{T} \quad \text{(3.1)} \]

In Figure 3.1A, the time lag \( T > 0 \) implies a positive longitudinal velocity. The negative velocity in the recirculation region would correspond to \( T < 0 \).

Figure 3.1B sketches a typical auto-correlation function of the signal of a single sensor tip. The shapes of the auto- and cross-correlation functions provide additional further information on the turbulent field (Chanson & Toombes 2002). The turbulence intensity \( \text{Tu} = v'/V \) is estimated within some key assumptions. First, it is assumed that the successive detection of air-water interfaces by the phase-detection probe is a true random process, thus the auto- and cross-correlation functions follow a Gaussian distribution (Chanson & Toombes 2002). This assumption yields the standard deviations of the auto- and cross-correlation functions being 0.851×\( T_{0.5} \) and 0.851×\( \tau_{0.5} \) respectively, where \( T_{0.5} \) and \( \tau_{0.5} \) are time lags over which the auto-correlation coefficient and cross-correlation coefficient decrease from maximum to half of the maximum: i.e. \( R_{ij}(T_{0.5}) = (R_{ij})_{\text{max}}/2 = 0.5 \), \( R_{ij}(T+\tau_{0.5}) = (R_{ij})_{\text{max}}/2 \) (Fig. 3.1A). Physical observations showed that the approximation was
reasonable in the high-velocity flow regions with large void fraction thus the time lag $\tau$ was small to moderate (Carosi & Chanson 2006). Second, it is assumed that the number of air-water interfaces $n$ was infinitely large, and the average interfacial travel time $T$ satisfies that:

$$\frac{1}{T} \sqrt{\sum_{i=1}^{n} (t_i - T)^2} = \sqrt{\sum_{i=1}^{n} \left( \frac{t_i - T}{t_i} \right)^2}$$

(3.2)

where $t_i (i = 1, \ldots, n)$ is the instantaneous interfacial travel time. A multiplication by $n^{-0.5}$ on each side of Equation (3.2) yields the true turbulence intensity $v'/V$ on the right hand side, while the turbulence level $T_u$ is derived from the left hand side based upon the first assumption above (Chanson & Toombes 2002):

$$T_u = 0.851 \times \frac{\sqrt{T_{0.5}^2 - T_u^2}}{T}$$

(3.3)

A more general form of Equation (3.3) is based on the integral auto- and cross-correlation time scales $T_{ii}$ and $T_{ij}$ (Fig. 3.1) (Felder & Chanson 2014):

$$T_u = \frac{\sqrt{2}}{\sqrt{\pi} \times T} \sqrt{\frac{T_{ii}}{(R_{ij\text{max}})^2} - T_{ii}^2}$$

(3.4)

The derivation of Equation (3.4) is detailed in Appendix A. Note that $T_u$ is measured as a spatial-averaged turbulence intensity between two phase-detection probe sensors. A variation of the separation distance between probe tips results in different cross-correlation function shapes hence different estimate of turbulence intensity $T_u$. A larger probe tip separation gives smaller estimate of $T_u$, and the difference is more significant for low-velocity flows (Wang 2014).

![Figure 3.1](image)

Figure 3.1 – Definition sketch of normalised correlation functions of dual-tip phase-detection probe signals

The calculation of time-averaged velocity and turbulence intensity using Equations (3.1), (3.3) and (3.4) is based upon the cross-correlation analysis which provides reliable information in the flow region with void fraction no less than 5% and a medium to high velocity. A very small number of bubbles or very low velocity would result in bias in correlation function hence inaccurate estimate of interfacial travel time. Consequently, the phase-detection probe often failed to provide accurate velocity and turbulence intensity next to the channel bed where the void fraction was close to zero.
and in the upper shear layer where the instantaneous velocity fluctuated between positive and negative, with an average velocity close to zero. Further in the free-surface recirculation region, the reversing flow was disturbed by the probe support structure before it was detected by the probe sensors. The probe wake might affect adversely the recirculation velocity measurement. Herein, besides the velocity measurements using probes of normal orientation (i.e. with probe sensors pointing upstream, Fig. 3.2A), the recirculation velocity was also measured with reversed probes pointing to downstream direction (Fig. 3.2B). The accuracy of velocity data next to the channel bed was assessed with a comparison to Prandtl-Pitot tube data (Fig. 3.2C). The Prandtl-Pitot tube was suitable in the less aerated region with constant flow direction parallel to the invert.

In the present study, the phase-detection probe signals were sampled at 20 kHz for 45 s per tip. All signals were evenly divided into fifteen non-overlapping segments before the correlation calculation was applied. The average auto- and cross-correlation functions were taken over the fifteen segments to minimise any bias of the characteristic time scales.

![Figure 3.2](image)

**Figure 3.2 – Velocity and turbulence measurement using phase-detection probes and Prandtl-Pitot tube in different regions of hydraulic jump roller.**

### 3.3 Four-point measurements

Considering an array of two dual-tip probes as sketched in Figure 2.3, a basic correlation analysis of simultaneously sampled signals yields a correlation tensor:

\[
R_{ij}(\tau) = \begin{bmatrix}
R_{11} & R_{12} & R_{13} & R_{14} \\
R_{21} & R_{22} & R_{23} & R_{24} \\
R_{31} & R_{32} & R_{33} & R_{34} \\
R_{41} & R_{42} & R_{43} & R_{44}
\end{bmatrix}
\]

(3.5)

where \( \tau \) is the time lag, \( i,j = 1,2,3,4, \ i \neq j \). \( R_{ii} \) is an auto-correlation function and \( R_{ij} \) is a cross-correlation function, with \( R_{ij} = -R_{ji} \). The corresponding correlation time scale matrix is deduced:

\[
T_{ij} = \begin{bmatrix}
T_{11} & T_{12} & T_{13} & T_{14} \\
T_{21} & T_{22} & T_{23} & T_{24} \\
T_{31} & T_{32} & T_{33} & T_{34} \\
T_{41} & T_{42} & T_{43} & T_{44}
\end{bmatrix}
\]

(3.6)

where

\[
T_{ij} = \int_{\tau(R_{ij} = 0)}^{\tau(R_{ij} = (R_{ij})_{\text{max}})} R_{ij}(\tau) d\tau
\]

(3.7)
Note a property of the auto-correlation function: \((R_{ii})_{max} = 1\) and \(\tau(R_{ii}) = (R_{ii})_{max} = 0\). For the probe array configuration shown in Figure 2.3, the time scales \(T_{11}, T_{22}, T_{33}\) and \(T_{44}\) are independent of the longitudinal and transverse separation distances. The time scales \(T_{13}\) and \(T_{24}\) are functions of the longitudinal separation distances \((\Delta x_{13}, \Delta x_{24})\), while the time scales \(T_{12}, T_{34}, T_{23}\) and \(T_{14}\) are functions of the transverse separation distances \((\Delta z_{12}, \Delta z_{34}, \Delta z_{23}, \Delta z_{14})\).

Given the distance \((\Delta x_{ij}^2 + \Delta z_{ij}^2)^{1/2}\) between sensors \(i\) and \(j\), the time-averaged velocity \(V_{ij}\) is:

\[
V_{ij} = \frac{\sqrt{\Delta x_{ij}^2 + \Delta z_{ij}^2}}{[T]_{ij}}
\]  

(3.8)

where \([T]_{ij}\) is the average interfacial travel time between the sensors \(i\) and \(j\). \([T]_{ij}\) is given by the time lag of maximum cross-correlation coefficient \((R_{ij})_{max}\). Based upon physical and geometrical considerations, six characteristic velocities are obtained altogether, following the relationships:

\[
V_{13} \approx V_{24} \approx V_x
\]

(3.9)

\[
V_{12} \approx V_{34} \approx V_z
\]

(3.10)

\[
V_{23} = \sqrt{V_{13}^2 + V_{12}^2} \approx V_{14} = \sqrt{V_{24}^2 + V_{12}^2} \approx \sqrt{V_x^2 + V_z^2}
\]

(3.11)

where \(V_x\) and \(V_z\) are the time-averaged interfacial velocity components in the \(x\)- and \(z\)-directions respectively, with \(x\) the longitudinal direction and \(z\) the horizontal transverse direction.

The turbulence fluctuations can be derived from the shape of the auto- and cross-correlation functions. Based upon Equation (3.4), the standard deviation of the interfacial velocity may be calculated as:

\[
v_{ij}' = \sqrt{\frac{2\pi(\Delta x_{ij}^2 + \Delta z_{ij}^2)}{\pi [T]_{ij}}} \times \sqrt{\left(\frac{T_{ij}}{(R_{ij})_{max}}\right)^2 - T_{ii}^2}
\]

(3.12)

A total of six characteristic velocity fluctuations may be estimated. For a quasi-two-dimensional flow, the following relationships may hold:

\[
v_{13}' \approx v_{24}' \approx V_x'
\]

(3.13)

\[
v_{12}' \approx v_{34}' \approx V_z'
\]

(3.14)

\[
v_{23}' = \sqrt{v_{13}'^2 + v_{12}'^2 + 2\times(v_{13}' \times v_{12}')^2} \approx v_{14}' = \sqrt{v_{24}'^2 + v_{12}'^2 + 2\times(v_{24}' \times v_{12}')^2}
\]

(3.15)

Table 3.2 summarises the proposed characteristic time-averaged interfacial velocities and velocity fluctuations.
Table 3.2 – Characteristic time-averaged interfacial velocities and velocity fluctuations measured with an array of two dual-tip phase-detection probes.

<table>
<thead>
<tr>
<th>Term</th>
<th>Equation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_{13}$</td>
<td>$V_{13} = \frac{\Delta x_{13}}{[T]_{13}}$</td>
<td>Longitudinal interfacial velocity</td>
</tr>
<tr>
<td>$V_{24}$</td>
<td>$V_{24} = \frac{\Delta x_{24}}{[T]_{24}}$</td>
<td>Longitudinal interfacial velocity</td>
</tr>
<tr>
<td>$V_{12}$</td>
<td>$V_{12} = \frac{\Delta z_{12}}{[T]_{12}}$</td>
<td>Transverse interfacial velocity</td>
</tr>
<tr>
<td>$V_{34}$</td>
<td>$V_{34} = \frac{\Delta z_{34}}{[T]_{34}}$</td>
<td>Transverse interfacial velocity</td>
</tr>
<tr>
<td>$V_{23}$</td>
<td>$V_{23} = \sqrt{\frac{\Delta x_{23}^2 + \Delta z_{23}^2}{[T]_{23}}}$</td>
<td></td>
</tr>
<tr>
<td>$V_{14}$</td>
<td>$V_{14} = \frac{\sqrt{\Delta x_{14}^2 + \Delta z_{14}^2}}{[T]_{14}}$</td>
<td></td>
</tr>
</tbody>
</table>

$V_{13}'$ | $v_{13}' = \frac{2 \times \Delta x_{13}^2}{\sqrt{\pi \times [T]_{13}}} \times \sqrt{\frac{T_{13}}{(R_{13})_{max}}^2 - T_{11}^2}$ | Longitudinal interfacial velocity fluctuation |

$V_{24}'$ | $v_{24}' = \frac{2 \times \Delta x_{24}^2}{\sqrt{\pi \times [T]_{24}}} \times \sqrt{\frac{T_{24}}{(R_{24})_{max}}^2 - T_{22}^2}$ | Longitudinal interfacial velocity fluctuation |

$V_{12}'$ | $v_{12}' = \frac{2 \times \Delta z_{12}^2}{\sqrt{\pi \times [T]_{12}}} \times \sqrt{\frac{T_{12}}{(R_{12})_{max}}^2 - T_{11}^2}$ | Transverse interfacial velocity fluctuation |

$V_{34}'$ | $v_{34}' = \frac{2 \times \Delta z_{34}^2}{\sqrt{\pi \times [T]_{34}}} \times \sqrt{\frac{T_{34}}{(R_{34})_{max}}^2 - T_{33}^2}$ | Transverse interfacial velocity fluctuation |

$V_{23}'$ | $v_{23}' = \frac{2 \times (\Delta x_{23}^2 + \Delta z_{23}^2)}{\sqrt{\pi \times [T]_{23}}} \times \sqrt{\frac{T_{23}}{(R_{23})_{max}}^2 - T_{22}^2}$ |                                  |

$V_{14}'$ | $v_{14}' = \frac{2 \times (\Delta x_{14}^2 + \Delta z_{14}^2)}{\sqrt{\pi \times [T]_{14}}} \times \sqrt{\frac{T_{14}}{(R_{14})_{max}}^2 - T_{11}^2}$ |                                  |

**Calculation of characteristic transverse velocity component**

For signals consisting of a sufficiently large amount of sample points, the cross-correlation function $R_{12}(\tau)$ between the leading tip signals exhibits a maximum at zero time lag: i.e. $[T]_{12} \approx 0$. This corresponds to the detection of the longitudinal interface convection at the same longitudinal positions, i.e. $\Delta x_{12} = 0$. For a relatively long sampling duration (e.g. 45 s), the statistical analysis hardly gives any information of the instantaneous transverse motion in a quasi-two-dimensional flow. On the other hand, a small signal segment may be able to reflect some instantaneous transverse interface motion. Such a time interval should be comparable to or slightly larger than the time scale of the transverse interface motion. While a too small time interval might not cover a sufficient amount of air-water interfaces, a too large interval would contain too many interfaces...
belonging to various motions and give an average transverse velocity being infinitely large ($[T]_{12} \approx 0$ in Equation (3.8)).

Herein a time interval 0.2 s was selected to investigate the transverse interfacial motion. Figure 3.3 shows a cross-correlation function between two 0.2 s leading tip signal segments. The correlation coefficient $R_{12}$ is plotted as a function of the ratio of transverse sensor separation $\Delta z_{12} = 9$ mm to time lag $\tau$. The peaks in the correlation function might indicate some characteristic transverse velocities. The local maximum correlation coefficient was picked for every 0.5 m/s velocity bin between $\Delta z_{12}/\tau = -5$ and 5 m/s, as marked by arrows in Figure 3.3. For the entire 45 s signal array, a total of fifty non-overlapping 0.2 s signal segments were analysed, giving 120 to 300 characteristic velocities. Figure 3.4A shows the probability distributions of these transverse velocities for different transverse sensor separations $\Delta z_{12}$ (\(^3\)). The probability density functions followed closely the normal distribution, with the average being zero. Some small transverse velocities were only recorded for small distances $\Delta z_{12}$ because of the small size or short "lifetime" of the bubbly structures moving or oscillating transversely. These small turbulent structures were not detected by both sensors when $\Delta z_{12}$ was larger than their largest transverse displacement. For a given $\Delta z_{12}$, the median transverse velocity amplitude $|V_z|$ was considered. It might reflect the typical instantaneous velocity or velocity fluctuation magnitude between the given distance $\Delta z_{12}$. Figure 3.4B shows $|V_z|$ at the elevation of maximum bubble count rate as a function of $\Delta z_{12}$, indicating a high correlation between $\Delta z_{12}$ and $|V_z|$. Further the results are not independent of the selection of time interval (herein 0.2 s).

Figure 3.5 shows the transverse velocity component $|V_z|$ obtained for a variety of time interval 0.05 s, 0.1 s, 0.2 s, 0.4 s and 0.8 s. Four data sets are plotted for different flow conditions and measurement locations. All data showed a monotonically increase in transverse velocity component with increasing time interval. Therefore, it is important to note that the results in the present study were obtained for a transverse sensor separation $\Delta z_{12} = 10$ mm (Fig. 2.3B) and a time interval selection of 0.2 s.

\[ \Delta z_{12}/\tau (\text{m/s}) \]
\[ R_{12} \]

-2 -1.5 -1 -0.5 0 0.5 1 1.5 2
-0.2 -0.1 0 0.1 0.2 0.3 0.4

Figure 3.3 – Cross-correlation function between two 0.2-s leading tip signals as a function of the ratio of transverse tip separation to time lag; Arrows indicating the maximum peaks in every 0.5 m/s interval – Flow conditions: $Q = 0.0347$ m\(^3\)/s, $x_1 = 0.83$ m, $d_1 = 0.0206$ m, $Fr_1 = 7.5$, $Re = 6.8 \times 10^4$; $x-x_1 = 0.25$ m, $y = Y_{Fmax} = 0.036$ m; $\Delta z_{12} = 9$ mm.

\(^3\) The smallest $\Delta z_{12}/W = 7.1 \times 10^{-3}$ was achieved using a specially designed dual-tip probe with two sensors of identical length, where the channel width $W = 0.5$ m.
(A, Left) – Probability density functions of characteristic transverse velocity for different sensor tip separation distances – Flow conditions as in Figure 3.2.
(B, Right) – Median transverse velocity amplitude for different leading sensor tip separations; data at \((x-x_1)/d_1 = 12.5\), \(y = Y_{F_{\text{max}}}\) for three Reynolds numbers with \(Fr_1 = 7.5\).

Figure 3.4 – Effects of transverse separation distances between probe sensor tips on characteristic instantaneous transverse velocity \(|V_z|\).

Figure 3.5 – Effects of selected time interval of small signal segments used for transverse velocity calculation.
4. RESULTS (1) VELOCITY AND TURBULENCE CHARACTERISTICS IN LONGITUDINAL DIRECTION

4.1 Flow pattern and basic air-water flow properties

For an inflow Froude number $F_{r1} = 7.5$, the hydraulic jump was characterised with a marked jump roller. While the impinging flow sustained a relatively high velocity beneath the roller, flow recirculation took place next to the free-surface, with spray and splashing projected in air. A shear layer formed between the bottom boundary layer ($V_x > 0$) and the reversing flow region ($V_x < 0$) due to the large velocity gradient from positive to negative. A substantial amount of air was entrained into the shear layer at the jump toe. The formation of the shear layer and the entrainment of air were observed together with successive downstream ejection of large-size vortices. Bubbles were advected in the vortices and interacted with turbulent structures of different length and time scales.

Figures 4.1A to 4.1D show a series of photographs of the jump roller side every three seconds. The flow direction was from left to right, and the red arrows indicate the instantaneous position of jump toe observed next to the side wall. Within the roller region, the convection of large-size vortices and the recirculating motion next to the free-surface were visualised by the large amount of entrained bubbles. The path of the large vortices was considered as the shear layer centreline where the void fraction was high and the velocity field was unsteady. Note the longitudinal oscillations of jump toe position and the associated roller surface deformation. The upstream shifting of the jump toe was associated with the process of the downstream water body releasing its potential energy. This process was balanced by the conservation of momentum, for which the conjugate depth ratio $d_2/d_1$ was determined by the inflow Froude number $F_{r1}$ for a given channel geometry and flow resistance (Chanson 2012). The unsteadiness was also linked with the pseudo-periodic formation of large vortices in the shear layer, which were further coupled with the entrapment of air pockets at the toe and the flow bulking. The dimensionless jump toe oscillation amplitude and frequency were affected by the Froude and Reynolds numbers (Wang & Chanson 2015).

Visual observations also suggested some instantaneous three-dimensional flow structures. For example, the transverse impingement perimeter at the jump toe exhibited some wave patterns fluctuating with an oscillating mean jump toe position (Zhang et al. 2013), also seen in travelling jumps (Leng & Chanson 2015). Figure 4.2 presents a top view of the roller, showing an instantaneous jump toe position shifting towards downstream on one side of the centreline and towards upstream on the other side (red arrows). Simply, there was instantaneous mass and momentum exchange in the transverse direction, although any transverse motion would be zero on average for a sufficiently long period of time.

The time-averaged void fraction $C$ and bubble count rate $F$ were measured with the phase-detection conductivity probes. For two dual-tip probes mounted side-by-side, the measurements were performed with a probe leading tip aligned on the channel centreline ($z/W = 0$) and the other probe leading tip separated by $\Delta z_{12} = 10$ mm in the transverse direction (Fig. 2.3B). The results from both leading sensors are plotted in terms of time-averaged void fraction $C$ in Figure 4.3 and dimensionless bubble count rate $F \times d_1/V_1$ in Figure 4.4. Figures 4.3A and 4.4A show the results for the smaller Reynolds number $Re = 6.8 \times 10^4$, and Figures 4.3B and 4.4B for the larger Reynolds number $Re = 1.4 \times 10^5$. The roller free-surface is plotted at $y = Y_{90}$ where $C = 0.9$. 

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Figure 4.1 – Side view of hydraulic jump roller – Flow conditions: \( Q = 0.0347 \, \text{m}^3/\text{s}, \, x_1 = 0.83 \, \text{m}, \, d_1 = 0.0206 \, \text{m}, \, Fr_1 = 7.5, \, Re = 6.8 \times 10^4; \) flow from left to right.

Figure 4.2 – Top view of transverse jump toe perimeter with red arrows indicating instantaneous impingement point shifting – Flow conditions identical to Fig. 4.1.
Figure 4.3 – Time-averaged void fraction distribution measured with two phase-detection probe leading sensors at $z = 0$ and $z = 10$ mm.

Figure 4.4 – Bubble count rate distribution measured with two phase-detection probe leading sensors at $z = 0$ and $z = 10$ mm.
Both time-averaged void fraction and bubble count rate distributions showed typical shapes reported by a number of previous studies (Chanson & Brattberg 2000, Kucukali & Chanson 2008, Murzyn & Chanson 2009a, Chachereau & Chanson 2011, Wang & Chanson 2015a,b). The results from both probe leading sensors were almost identical. The void fraction showed a local maximum $C_{\text{max}}$ close to the shear layer centreline corresponding to the air entrainment at the jump toe and the advection of highly-aerated vortices. The dimensionless bubble count rate exhibited a maximum $F_{\text{max}} d_1/V_1$ at a lower elevation corresponding to the presence of largest number of bubbles, which was related to both local void fraction and turbulent shear stress level. In the upper free-surface region, the monotonically increasing void fraction distribution reflected an interfacial air-water exchange. The bubble count rate distribution presented a secondary peak for $C \approx 0.3$ to 0.5. Please note that the bubble count rate was underestimated herein in the recirculation region because the use of two phase-detection probes next to each other generated a large wake at the sensor tips. Flow was decelerated in the wake region and small bubbles tended to merge into large ones.

The local maximum void fraction $C_{\text{max}}$ and bubble count rate $F_{\text{max}}$ in the shear layer decreased with increasing distance from the jump toe. Their longitudinal decay is plotted in Figures 4.5 and 4.6 respectively. The present data are compared with the empirical correlations of Wang (2014). While the maximum void fraction was determined by the Froude number at a given longitudinal position, with negligible effect of Reynolds number, the maximum bubble count rate was a function of both Froude and Reynolds numbers. The present results suggested slower longitudinal de-aeration and dissipation of the shear layer compared to the results of Wang (2014). Overall, the time-averaged void fraction and bubble count rate results indicated distinctive difference in terms of air transport and turbulence development between the turbulent shear region and free-surface recirculation region.

![Figure 4.5](image1.png)  
**Figure 4.5 (Left)** – Longitudinal decay of local maximum void fraction $C_{\text{max}}$ in shear layer – Comparison with empirical correlation of Wang (2014).

![Figure 4.6](image2.png)  
**Figure 4.6 (Right)** – Longitudinal decay of dimensionless maximum bubble count rate $F_{\text{max}} d_1/V_1$ in shear layer – Comparison with empirical correlations of Wang (2014).

### 4.2 Time-averaged longitudinal velocity in jump roller

The time-averaged longitudinal velocity results $V_x/V_1$ are presented in Figure 4.7A for $Re = 6.8 \times 10^4$ and in Figure 4.7B for $Re = 1.4 \times 10^5$. The subscript $x$ refers to the longitudinal component, and we use $V_x(+) \text{ and } V_x(-)$ to denote respectively the results given by normal and reversed probe orientations (Fig. 3.2A and 3.2B). Both datasets were obtained on the channel centreline ($z/W = 0$).
and at 10 mm transversely apart \((z/W = 0.02)\). The Prandtl-Pitot tube data measured next to the bottom on the centreline are plotted for comparison.

For both flow conditions, the time-averaged longitudinal velocity showed a positive velocity region in the lower part of the roller, with a maximum velocity close to the bottom, and a negative velocity region in the upper part of the roller. Due to the limitation of the cross-correlation analysis, there was a band area in the upper shear layer with absence of meaningful velocity data. The time-averaged velocity changed from positive to negative with increasing elevation across this band. The band thickness was estimated to be roughly about \(d_1\) and its vertical position increased with increasing water depth and the enlargement of vortical structures that were advected in the shear layer. The raw dual-tip probe signals suggested frequent switches of velocity direction within this area, leading to a small mean velocity \(V_x \sim 0\) but a large turbulence intensity defined by \(T_{u_x} = \frac{v_x}{V_x}\) (Wang & Chanson 2015b). The velocities measured at slightly separated transverse positions were almost identical. Comparison between the data recorded with probes of opposite orientations did not show major differences, though a close check of the recirculation velocity distributions suggested less data scattering for the results given by the reversed probe sensors \((V_x(-))\) in Fig. 4.7). It implied limited impact of phase-detection probe orientation on velocity measurement in the recirculation region, and the data scattering caused by the probe interference was acceptable even for a reversing flow. However, this conclusion only applies to the free-surface recirculation region where the velocity was moderate to small and the gravity force dominated rather than shear stress. The results measured with the phase-detection probe were close to those given by the Prandtl-Pitot tube in the lower shear region. With a small tube diameter, the Prandtl-Pitot tube was able to provide more details of the boundary layer development next to the bed. Some small discrepancy between the phase-detection probe data and Prandtl-Pitot tube data might be related to the system uncertainties because the data were not collected in the same hydraulic laboratory (4).

One useful characteristic value is the maximum velocity \(V_{\text{max}}\) in the lower shear layer. The maximum velocity \(V_{\text{max}}\) decreased from the free-stream velocity of the impinging supercritical flow at the jump toe along the longitudinal direction. Wang (2014) proposed an empirical relationship to describe the longitudinal decay of \(V_{\text{max}}\):

\[
V_{\text{max}} = U \exp \left( - \frac{1}{5\times(F_r-1)} \times \frac{x-x_1}{d_1} \right) \quad \text{for} \ 3.8 < F_r < 10 \quad (4.1)
\]

where \(U\) is the upstream free-stream velocity, \(U \approx 1.1 \times V_1\) (Chanson & Brattberg 2000, Wang 2014). Equation (4.1) is compared with the present data in Figure 4.8 for \(F_r = 7.5\), showing a good agreement. Figure 4.8 also includes correlations given by Murzyn & Chanson (2009a), Chanson (2010) and Wang (2014) for a broader range of Froude numbers.

On the other hand, the recirculation velocity showed some quasi-uniform distribution across the upper roller. The depth-averaged recirculation velocity \(V_{\text{recirc}} < 0\) is plotted in Figure 4.9 and no longitudinal variation is seen. For the given Froude number \(F_r = 7.5\), the ratio of average recirculation velocity to inflow velocity was \(V_{\text{recirc}}/V_1 = 0.34\), close to the finding of Wang (2014) for the same Froude number and slightly smaller than the average data of Chanson (2010) for \(5.1 < F_r < 11.2\). Limited previous data suggested a slight decrease in \(V_{\text{recirc}}/V_1\) with increasing Froude number for \(3 < F_r < 10\) (Chachereau & Chanson 2011, Wang 2014).

---

4 The Prandtl-Pitot tube data were collected in Seddon Building (82C#) at UQ in 2011. The phase-detection probe data were collected in Advanced Engineering Building (49#) at UQ in 2014 in the same channel.
Figure 4.7 – Time-averaged interfacial velocity distribution measured with two dual-tip phase-detection probes at $z = 0$ and $z = 10$ mm and with normal and reversed probe orientation in recirculation region – Comparison with Prandtl-Pitot tube data next to channel bed.
The vertical position $Y_{V_{\text{max}}}$ corresponding to the maximum velocity $V_{\text{max}}$ were found to increase slowly with increasing distance from the toe (Fig. 4.10). Such a position might characterise the upper limit of the developing boundary layer next to the channel bed. The location of zero-crossing of velocity profile, $y(V=0)$, was estimated at the centre of the "blind band" where physical velocity data were unavailable. The results are also included in Figure 4.10, suggesting a linear increase of the zero-velocity elevation along the roller.

Given the characteristic velocities $V_{\text{max}}$ and $V_{\text{recirc}}$, the elevations $Y_{V_{\text{max}}}$ and $y(V=0)$ and a no-slip condition next to the bottom, the time-averaged velocity profile might be predicted with a wall jet equation (Rajaratnam 1965, Chanson & Brattberg 2000, Chanson 2010):
\[
V = \left( \frac{y}{Y_{\text{max}}} \right)^N
\]

for \( \frac{y}{Y_{\text{max}}} < 1 \) (4.2)

\[
\frac{V-V_{\text{recirc}}}{V_{\text{max}}-V_{\text{recirc}}} = \exp \left( -\frac{1}{2} \times 1.765 \times \left( \frac{y-Y_{\text{max}}}{Y_{0.5}} \right)^2 \right)
\]

for \( \frac{y}{Y_{\text{max}}} > 1 \) (4.3)

where \( N \) is a constant between 6 and 10, and \( Y_{0.5} \) is another characteristic elevation where \( V = V_{\text{max}}/2 \). Equations (4.2) and (4.3) imply a self-similar velocity distribution independent of Froude or Reynolds number. Figure 4.11 compares Equation (4.3) with all experimental data. Note that Equation (4.3) only applies to the flow region where a marked roller is seen with negative free-surface flow.

Figure 4.11 – Self-similar velocity profile in hydraulic jump roller with reversing free-surface flow – Comparison with Equation (4.3).

4.3 Turbulence intensity and integral turbulent scales

The turbulence intensity \( \text{TU}_x = v_x/V_x \) was calculated using Equation (3.3). The results measured at two transverse locations (\( z/W = 0 \) and 0.02) are presented in Figure 4.12 for two flow conditions. In the upper recirculation region, the data \( \text{TU}_x(+) \) given by normal phase-detection probe positions with upstream-pointing sensors are compared with the data \( \text{TU}_x(-) \) given by reversed probes with downstream-pointing sensors. The overall data distributions were consistent with previous studies using similar instrumentation (Murzyn & Chanson 2009a, Chachereau & Chanson 2011, Wang & Chanson 2014). That is, in the lower turbulent shear flow, the turbulence intensity increased with increasing elevation from the bottom, whereas it decreased along the longitudinal direction as the turbulence was dissipated downstream. The magnitude of \( \text{TU}_x \) was found mainly between 1 and 2.5 in this region. Such values were high for random velocity turbulence, even with consideration to the large Froude and Reynolds numbers. Indeed, the contribution of slower flow motions associated with self-sustained flow instabilities could not be neglected (Wang et al. 2014). Wang et al. (2014) assessed the contribution of the slow fluctuating motions, including the longitudinal jump toe oscillations, vertical water depth fluctuations, and formation and advection of large vortices, by applying a triple decomposition technique. Their study showed high-frequency component of the
decomposed signal, corresponding to the "true" velocity turbulence, between 0.5 and 1.5 in the turbulent shear region close to the toe, which also decreased along the longitudinal direction. The low-frequency signal component reflected significant effect of the large-scale flow motions. Details of the triple decomposition data processing are given in Appendix C.

In the recirculation region next to the free-surface, the turbulence intensities were high with large data scatter, because of the effects of jump translations and free-surface deformations (Fig. 4.12). After a signal decomposition, the turbulence intensities derived from the high-frequency signal component remained at a similar magnitude of those in the shear flow region (Wang et al. 2014). The contribution of the low-frequency component was thus significant. Although the existence of low-frequency component was mainly attributed to the fluctuating nature of hydraulic jump, a comparison between the results given by opposite probe orientations \( (T_{u,x}(+)) \) and \( \tilde{T}_{u,x}(-) \) in Fig. 4.12 suggests that the intrusive probe disturbance in the reversing flow also had some effects. The wake effects induced by the probe support structures at the phase-detection sensors were avoided by applying a reversed probe position, yielding smaller and less scattered turbulence intensity data. Overall, the present study showed that the disturbance of the phase-detection probes affected adversely the turbulence characterisation in the jump roller, though the impact on time-averaged velocity measurement was relatively limited.

Some characteristic turbulent scales could be further derived from the correlation analysis following the corresponding assumptions. One was the integral turbulent length scale characterising the typical size of coherent vortical structures in which the air-water interfaces were advected. Herein only the definition and experimental results of the longitudinal integral turbulent length scale \( L_X \) are presented. Further information of the transverse integral turbulent length scale \( L_Z \) and integral turbulent time scales \( T_Z \) and \( T_X \) is provided in Appendix B for reference.

The longitudinal integral turbulent length scale \( L_X \) was derived from a series of measurements at a given location, using dual-tip phase-detection probes with different longitudinal separation distances \( \Delta x \) between two sensor tips. The correlation between the simultaneous signals of the two probe sensors decreased with an increasing separation distance \( \Delta x \). When the separation distance was larger than the size of the largest coherent structures, the correlation coefficient was expected to drop to zero. An integration of the maximum correlation coefficient as a function of the sensor separation distance yielded:

\[
L_X = \frac{\Delta x (R_{ij})_{\text{max}} - 0}{\int_0^{(R_{ij})_{\text{max}}} \Delta x d(\Delta x)}
\]  

(4.4)

where \( (R_{ij})_{\text{max}} \) is the maximum cross-correlation coefficient between the phase-detection probe signals. The length scale \( L_X \) is a measure of the typical longitudinal dimension of coherent bubbly structures. Figure 4.13 shows the typical distributions of \( L_X/d_1 \) in five cross-sections on the roller centreline. The dimensionless length scale \( L_X/d_1 \) is shown typically between 0.2 and 0.8 in the shear flow region and between 0.6 and 1.6 in the free-surface region. These are about one order of magnitude larger than the typical bubble sizes in the two flow regions respectively (Chanson 2010, Wang 2014). The longitudinal length scale exhibited a local maximum at some position above the boundary layer and below the path of large-size vortices in the shear layer. The local maximum length scale decreased rapidly as the shear flow was decelerated and turbulence was dissipated. In contrast, the length scale increased with increasing longitudinal distance in the free-surface region. Maximum length scales were shown close to the positions of 50% void fraction. These positions were close to the time-averaged water elevations measured by acoustic displacement meters (Murzyn & Chanson 2009b, Wang 2014), and might be considered as a pseudo-interface between bubbly flow \( (C < 0.5) \) and spray region above \( (C > 0.5) \).
Figure 4.12 – Turbulence intensity distribution measured with two dual-tip phase-detection probes at z = 0 and z = 10 mm and with normal and reversed probe orientation in recirculation region.
4.4 Impact of flow reversal on void fraction and bubble count rate measurements

The phase-detection conductivity probes were designed to be used in air-water flows with the needle sensors aligned against the approaching flow direction (Chanson & Toombes 2002). In the present study, measurements were performed with upstream-pointing probes through full cross-sections of jump roller. Taking into account the flow recirculation in the upper free-surface region, complementary measurements were conducted in this region with reversed probe positions to maximise the measurement accuracy (Fig. 3.2B). The setup with reversed probes was only meaningful in the recirculation region where $V_x < 0$. The time-averaged velocity data showed little impact of probe orientation on velocity measurement in the recirculation region, while the turbulence intensity data suggested some improvement in data quality when the probes were reversed next to the free-surface. Herein a discussion is further developed in terms of void fraction and bubble count rate results.

Figures 4.14A and 4.14B compare respectively the time-averaged void fraction and bubble count rate given by opposite probe orientations at a full cross-section. In the turbulent shear region where the time-averaged velocity was mostly positive ($^5$), the sensors pointing to upstream was able to record a local maximum void fraction $C_{\text{max}}$ and a maximum bubble count rate $F_{\text{max}}$. With increasing elevation from the channel bed, both void fraction and bubble count rate (herein $C(\text{+})$ and $F(\text{+})$) increased first to the local maxima then decreased to local minima. These bell-shape distributions were not recorded with a reversed phase-detection probe with the sensors pointing to downstream. Instead, monotonically increasing void fraction and bubble count rate profiles were obtained with $C(\text{-}) < C(\text{+})$ and $F(\text{-}) < F(\text{+})$ for $y < y^*$, $y^*$ being the characteristic elevation of the local minimum void fraction. The failure in void fraction and bubble count rate measurements with reversed probes was largely related to the presence of probe support structure that disturbed the streamwise air-water interface convection at upstream of the sensors. The probe itself induced a wake where the bubble break-up and coalescence processes were changed. Such a wake effect was strong in the shear flow region because of the high velocity and large momentum. In addition, with the needle sensors pointing downstream, a large proportion of bubbles slipped past the sensor tips instead of being pierced. The miss of such bubbles from the interface detection also contributed to the underestimate of void fraction and bubble count rate.

$^5$ The boundary between turbulent shear region and free-surface recirculation region, i.e. $y = y^*$ corresponding to the local minimum void fraction, was not consistent with the boundary between positive and negative velocity regions, with $y^* > y(V=0)$. 
Figure 4.14 – Comparison between basic air-water flow properties measured with phase-detection probes pointing upstream and downstream – Flow conditions: \( Q = 0.0347 \, \text{m}^3/\text{s}, \, x_1 = 0.83 \, \text{m}, \, d_1 = 0.0206 \, \text{m}, \, Fr_1 = 7.5, \, Re = 6.8 \times 10^4; \, (x-x_1)/d_1 = 24.3 \).

In the free-surface recirculation region \((y > y^*)\), identical time-averaged void fraction was obtained with the opposite probe orientations, i.e. \(C(-) = C(+)\). The trend of bubble count rate distribution was reproduced by reversed probes, but the results showed consistently \(F(-) > F(+)\) for \(y > y^*\). The underestimate of bubble count rate in the reversing flow with normal probe position was linked to the wake effect caused by the probe support structures. The use of a probe array enhanced the wake effect and induced secondary vortices at the probe tips. Figure 4.15 demonstrates the difference between \(F(+)\) and \(F(-)\) by comparing the secondary maximum bubble count rates \(F_{sec}(+)\) and \(F_{sec}(-)\) at various longitudinal positions. It can be seen that, for the given Froude number \(Fr_1 = 7.5\), the secondary maximum bubble count rate was underestimated by 40% to 50% with the normal side-by-side probe setup in the reversing flow. The findings indicated that air-water flow measurements must be performed with phase-detection probes against the flow direction in a high-speed flow region, while the data quality should be carefully controlled if flow recirculation occurs next to the free-surface.

Figure 4.15 – Comparison between secondary maximum bubble count rate next to free-surface measured with phase-detection probes of opposite orientations.
5. RESULTS (2) FOUR-POINT AIR-WATER FLOW MEASUREMENTS AND TRANSVERSE VELOCITY FLUCTUATIONS

5.1 Characteristic transverse velocity component

Although the classical hydraulic jump is traditionally treated as a two-dimensional flow, large-scale turbulent eddies are visible with instantaneous three-dimensional vortices. The use of two dual-tip phase-detection probes enabled a four-point simultaneous measurement of the air-water flow, and the turbulent characteristics might be derived in the transverse direction. Figure 5.1A shows a photograph of the probe array with two symmetrically designed probes placed side by side. The four needle sensors were located within the same x-z plane, and the leading and trailing sensor tips had the same longitudinal positions respectively. The configuration in Figure 5.1A was modified as in Figure 5.1B to avoid disturbance of the leading sensors on the transverse velocity measurements. Figure 5.1C sketches the velocity components measured between sensor tips numbered from 1 to 4. The dimensions of the probe array were specified in Figure 2.3, i.e.: \( \Delta x_{13} = \Delta x_{24} = 6.5 \text{ mm}, \Delta z_{13} = \Delta z_{24} = 1.86 \text{ mm}, \Delta z_{12} = 10 \text{ mm} \). The experimental data reported in this chapter were collected with this probe configuration (Fig. 5.1B and 5.1C) unless otherwise specified.

(A) Configuration I, view in elevation

(B) Configuration II, view in elevation
The time-averaged interfacial velocity was deduced between two phase-detection probe sensors as:

\[ V_{ij} = \sqrt{\frac{\Delta x_{ij}^2 + \Delta z_{ij}^2}{[T]_{ij}}} \]  

(5.1)

where \( \Delta x_{ij} \) and \( \Delta z_{ij} \) are respectively the longitudinal and transverse distances between the sensor tips i and j (i,j = 1,2,3,4, Fig. 5.1C), and \([T]_{ij}\) is the average interfacial travel time given by time lag of maximum cross-correlation coefficient between the corresponding sensor signals. Assuming \( \Delta z_{13} \ll \Delta x_{13} \), Equation (5.1) yields the longitudinal interfacial velocity \( V_{13} = \Delta x_{13}/[T]_{13} \), which is equivalent to Equation (3.1) for one dual-tip probe.

Along the main flow direction, the time-averaged longitudinal velocity component was measured with a satisfying accuracy given an appropriately large number of samples (i.e. air-water interfaces) and a medium to high velocity with a constant flow direction. The results were presented in Section 4.2. On the other hand, for the transverse velocity component, the time-averaged two-dimensional flow pattern suggested a zero average transverse velocity. Compared to the 45 s sampling duration which was sufficiently long to reflect the average interfacial motions, smaller signal segments were required to capture the instantaneous motions. Herein 0.2 s was selected with consideration of the physical distance between the probe tips. It is acknowledged that the 0.2 s long sample encompassed a relatively small number of detected interfaces, and the cross-correlation function might be biased and lacked accuracy. Therefore, repeated selection of non-overlapping signal segments was required to minimise potential errors. The method to determine the characteristic transverse velocity component \( |V_z|\) is described in Section 3.2 and the uncertainties are discussed in Section 6.

The four-point velocity measurements were performed for two flow conditions with the same Froude number \( F_{r1} = 7.5 \) but different Reynolds numbers \( Re = 6.8 \times 10^4 \) and \( Re = 1.4 \times 10^5 \). The corresponding inflow aspect ratios were \( h/W = 0.04 \) and \( 0.06 \) respectively (Table 2.1). The transverse separation distance between the leading probe tips was \( \Delta z_{12} = 10 \) mm for the probe array configuration in Figure 5.1C. Measurements with reversed probe positions were complemented in the free-surface recirculation region.

Figures 5.2 and 5.3 present the transverse velocity amplitude results \( |V_z|/V_1 \) at five vertical cross-sections for each flow. Herein \( |V_z| \) is a median transverse velocity amplitude. The instantaneous transverse velocity could be in either +z or -z direction, the average being zero. The longitudinal velocity component \( V_1/V_1 \) is also plotted for comparison. Figures 5.2 and 5.3 show similar profile shapes between the transverse and longitudinal velocity components in the shear flow and recirculating flow regions respectively. In the positive velocity region \( (V_x > 0) \), the transverse velocity amplitude \( |V_z| \) also reached a maximum at the boundary layer edge. In the recirculation

(C) Sketch of Configuration II and velocity components, view in elevation
Figure 5.1 – Four-point phase-detection probe array configurations – Flow direction from left to right for all figures.
region ($V_x < 0$), $|V_z|$ was relatively uniform and the results were almost independent of the phase-detection probe orientation ($|V_z(+)| \approx |V_z(-)|$). The ratio of transverse to longitudinal velocity amplitudes was mostly between $|V_z|/|V_x| = 0.4$ and 0.5. That is, for a physical measurement with length scale $\sim 10^{-2}$ m and a time scale no larger than 0.2 s, a typical velocity of instantaneous transverse motion of air-water interfaces was about half of the time-averaged longitudinal velocity.

The derivation process implied that $|V_z|$ was a function of both the sensor separation distance $\Delta z$ and the duration of the signal segment (herein 0.2 s for a transverse distance $\Delta z_{12} = 10$ mm). For a selected signal duration, an increase in sensor separation distance yielded larger transverse velocity results because it performed a filtering on the small velocity components. Figure 5.4 shows the transverse velocity results given by different distances $\Delta z_{12}/W$ between the leading probe tips. The characteristic transverse turbulent length scale of the bubbly structures was suggested in an order of $0.1 \times d_1$ to $d_1$ (Wang et al. 2014), comparable to the smallest distances $\Delta z_{12}/W = 0.0071$ and 0.018 (i.e. $\Delta z_{12}/d_1 = 0.17$ and 0.44) in Figure 5.4. A too large sensor separation was thought to hardly capture the transverse motion of a coherent structure thus the data might be of less interest. By comparison, the measurement of time-averaged longitudinal velocity was independent of the longitudinal sensor separation $\Delta x$.

![Figure 5.2](image-url) - Characteristic instantaneous transverse velocity component $|V_z|/V_1$ compared with longitudinal velocity component $V_x/V_1$ – Flow conditions: $Q = 0.0347$ m$^3$/s, $x_i = 0.83$ m, $d_1 = 0.0206$ m, $Fr_1 = 7.5$, $Re = 6.8 \times 10^4$. 
Geometrical and physical considerations may suggest that:

\[ V_{23} = \sqrt{V_{13}^2 + V_{12}^2} \approx V_{14} = \sqrt{V_{24}^2 + V_{12}^2} \approx \sqrt{V_x^2 + V_z^2} \quad (5.2) \]

The relationships are sketched in Figure 5.1C. The velocity vectors \( V_{23} \) and \( V_{14} \) were calculated based upon the data of longitudinal and transverse velocity components for the given transverse distance \( \Delta z_{12} = 10 \text{ mm} \). The calculated results were compared with the data measured directly between the corresponding phase-detection sensors. Figure 5.5 plots the calculation results (Eq. (5.2)) against the measurement results. The bias of the point distributions from the 1:1 line was attributed to the scattering velocity data measured between probe sensors 1 and 4 or between sensors 2 and 3. Although the measurement of \( V_{14} \) and \( V_{23} \) with the probe configuration in Figure 5.1C was not interfered by the other sensors, the data quality deteriorated significantly compared to the measurement of longitudinal velocity. This suggested that aligning the phase-detection sensors along the main flow direction was essential for a high-quality velocity and turbulence measurement, and any marked deviation from the main flow direction might lead to a deterioration of data quality. In addition, it is noteworthy that \( V_x \) in Equation (5.2) was a time-averaged velocity whereas the \( V_z \)
was a characteristic instantaneous velocity. Therefore, their combination should be considered at the smaller time scale, i.e. 0.2 s in Equation (5.2), compared to the directly measured velocities derived from 45 s long samples. Nevertheless, the data showed calculation and measurement results in the same order of magnitude. Direct comparisons between the vertical profiles of the calculation and measurement results are given in Appendix D for all cross-sections.

Figure 5.4 – Variation in transverse velocity results with sensor separation distance for a fixed signal segment duration 0.2 s – Comparison with the absolute longitudinal velocity magnitude.

Figure 5.5 – Comparison between velocity vectors measured with phase-detection sensors and calculated with Equation (5.2).
5.2 Transverse turbulent fluctuations

The velocity standard deviation $v'_{z} = v'_{12}$ was calculated using Equation (3.12) for the given characteristic transverse velocity $|V_{z}| = |V_{12}|$. The results are presented at three cross-sections in Figures 5.6A to 5.6C in the form of relative velocity fluctuation to the local time-averaged longitudinal velocity $v'_{12}/|V_{z}|$. The cross-sections and flow conditions are specified in Table 5.1. For each cross-section, the data are compared with the longitudinal velocity fluctuations $Tu_{x} = v'_{13}/|V_{z}|$ and $v'_{24}/|V_{z}|$.

Table 5.1 – Flow conditions for and locations of transverse turbulent fluctuation measurements.

<table>
<thead>
<tr>
<th>Q [m$^{3}$/s]</th>
<th>h [m]</th>
<th>$x_{1}$ [m]</th>
<th>$d_{l}$ [m]</th>
<th>$V_{1}$ [m]</th>
<th>Fr$_{1}$ [-]</th>
<th>Re [-]</th>
<th>(x-$x_{1}$)/$d_{l}$ [-]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0347</td>
<td>0.02</td>
<td>0.83</td>
<td>0.0206</td>
<td>3.37</td>
<td>7.5</td>
<td>6.8$\times$10$^{4}$</td>
<td>12.14</td>
</tr>
<tr>
<td>0.0705</td>
<td>0.03</td>
<td>1.25</td>
<td>0.033</td>
<td>4.27</td>
<td>7.5</td>
<td>1.4$\times$10$^{5}$</td>
<td>11.36 22.73</td>
</tr>
</tbody>
</table>

Figure 5.6 shows relative transverse velocity fluctuations $v'_{12}/|V_{z}|$ typically between 0.02 and 0.5, compared to the longitudinal turbulence intensities mostly larger than 1. Despite the data scattering, the distributions of transverse velocity fluctuations tended to follow an increasing trend from the turbulent shear region to the free-surface recirculation region. The data was scattered because the cross-correlation functions given by the small signal segments (0.2 s) often contained random and biased peaks, which might introduce errors to the estimate of maximum correlation coefficient $(R_{ii})_{\text{max}}$ and correlation time scales $T_{ii}$ and $T_{ii}$ in Equation (3.12). Please note that the present data represented the fluctuation of the characteristic transverse velocity component $|V_{12}|$ rather than the fluctuation of all instantaneous transverse velocity samples for which the time-averaged value was zero. Resch & Leutheusser (1972b) measured both longitudinal and transverse velocity fluctuations in the roller using a double V-shaped hot-film probe for Fr$_{1} = 6$. Their data showed a marked maximum transverse velocity fluctuation $v'_{z}/V_{1} \approx 0.15$ in the turbulent shear layer, $V_{1}$ being the average inflow velocity. This was quantitatively comparable to the present results, though the former dealt with continuous water velocity while the present study recorded the velocity of consecutive air-water interfaces. However, Resch & Leutheusser (1972b) measured much smaller longitudinal velocity fluctuations with the maximum $v'_{x}/V_{1} \approx 0.3$ in the shear layer. It was believed that their data reflected the fast velocity fluctuations, whereas the present data were a combination of fast velocity turbulence and slow, pseudo-periodic motions of the flow. A decomposition of the present signals yielded a turbulence intensity linked with the fast velocity fluctuations around 1. Implicitly the agreement in transverse velocity fluctuation results might suggest little impact of the slow, pseudo-periodic motions of the flow in the transverse direction. Lastly, a comparison between Figures 5.6B and 5.6C at different longitudinal positions shows that the magnitude of transverse velocity turbulence decreased along downstream positions.

A direct comparison between the transverse and longitudinal velocity fluctuations is shown in Figure 5.7 in the form of $v'_{z}/v'_{x}$. For the tested flow conditions (Table 5.1), most data points scattered between 0.01 and 0.3, with a median value of $v'_{z}/v'_{x} = 0.05$. Overall, with the influence of flow instabilities in the longitudinal direction, the present results indicated transverse velocity fluctuations in one to two orders of magnitude smaller than the longitudinal velocity fluctuations, with larger velocity fluctuations in the upper roller than in the lower shear flow.

The data in Figure 5.6 showed that:

$$v'_{13} \approx v'_{24} \approx v'_{x}$$  \hspace{1cm}(4.1)
Figure 5.6 – Comparison between transverse and longitudinal velocity fluctuations.
For the quasi-two-dimensional flow, further relationships between the velocity fluctuation components suggested:

\[
\begin{align*}
\nu_{23}' &= \sqrt{\nu_{13}'^2 + \nu_{12}'^2 + 2\times (\nu_{13}' \times \nu_{12}')} \approx \nu_{14}' = \sqrt{\nu_{24}'^2 + \nu_{12}'^2 + 2\times (\nu_{24}' \times \nu_{12}')} \\
\end{align*}
\]

(5.4)

Physically, the terms \(\nu_{23}'^2\) and \(\nu_{14}'^2\) are proportional to a combination of normal and tangential Reynolds stresses. Thus the results might provide an indirect means to estimate the tangential Reynolds stress component:

\[
(\nu_x \times \nu_z)' \approx \frac{1}{2} \times (\nu_{23}'^2 - \nu_{12}'^2 - \nu_{z}'^2)
\]

(5.5)

where \(\nu_{23}'\) (or \(\nu_{14}'\)) was measure between two phase-detection sensors separated by a longitudinal distance \(\Delta x\) and transverse distance \(\Delta z\).

Figure 5.8 presents the dimensionless Reynolds stresses for the corresponding velocity fluctuations. That is, the transverse normal Reynolds stress \(\rho \times (1-C) \times \nu_{23}'^2/(\rho \times \nu_1^2/2)\), the longitudinal normal Reynolds stress \(\rho \times (1-C) \times \nu_{13}'^2/(\rho \times \nu_1^2/2)\), and the tangential Reynolds stress \(\rho \times (1-C) \times (\nu_1 \times \nu_z)'/(\rho \times \nu_1^2/2)\), where \(\rho\) is the density of water and \(C\) is the void fraction of air-water flow. The present data showed dimensionless normal Reynolds stress larger than 1 in the longitudinal direction, and typically smaller than 0.1 in the transverse direction. The tangential Reynolds stress was given in the same order of magnitude as the longitudinal normal stress. The tangential stress was yielded unusually large with Equation (5.5) because the terms \(\nu_{23}'\) (or \(\nu_{14}'\)) measured apart from the main flow direction were sometimes large, scattered and physically meaningless. The unsuccessful measurement of \(\nu_{23}'\) (or \(\nu_{14}'\)) was associated with the complex flow structures that led to broadened and sometimes biased cross-correlation functions between the probe signals. For comparison, Resch & Leutheusser (1972b) measured the Reynolds stresses of water phase using hot-film probes, and their results showed dimensionless tangential stress \((\nu_1 \times \nu_z)' / \nu_1^2\) in the order of \(10^{-2}\), whereas the longitudinal stress \(\nu_z'^2 / \nu_1^2\) was in the order of \(10^{-1}\). The validity of such a means for Reynolds stress estimate may be further tested in more organised air-water flows with lesser flow recirculation and instabilities.
Figure 5.8 – Dimensionless normal and tangential Reynolds stresses.

(A) Fr\(_1\) = 7.5, Re = 6.8×10\(^4\), (x-x\(_1\))/d\(_1\) = 12.14

(B) Fr\(_1\) = 7.5, Re = 1.4×10\(^5\), (x-x\(_1\))/d\(_1\) = 11.36

(C) Fr\(_1\) = 7.5, Re = 1.4×10\(^5\), (x-x\(_1\))/d\(_1\) = 22.73
6. DISCUSSION: EXPERIMENTAL UNCERTAINTIES

6.1 Uncertainties related to facilities and instrumentation

Relatively large flow rates were required in the present study to achieve the designed inflow conditions (Table 2.1). The Venturi meter provided an accuracy of 2% for the flow rate measurement, and the discharge was double-checked by measuring the head tank water depth. The accuracy of the determination of Froude number depended largely upon the inflow depth measurement. While the point gauge measured the smooth water depth \(d_1\) with errors less than 0.2 mm, the inflow depth uncertainty was associated with the relative free-surface roughness for large flow rates with high inflow turbulence level. Herein the inflow depth uncertainty was estimated to be less than 5% for the largest flow rate \(Q = 0.0705 \text{ m}^3/\text{s}\), for which the inflow free-surface breaking was visible.

For the experimental setup and measurement, the most noticeable uncertainty was related to the determination of longitudinal jump toe position \(x_1\), which further affected the relative measurement location \((x-x_1)/d_1\) in the roller. The jump toe oscillated around a mean position which was sensitive to any resistance introduced by intrusive instruments. The oscillation amplitude increased with increasing inflow depth. The influence of the instationary jump toe position was carefully assessed by comparing the air-water flow properties with previous data collected in the same flume. The comparison showed satisfying consistency in terms of all flow properties that were functions of the longitudinal position. The vertical measurement location \(y/d_1\) was monitored with a digital scale, the error (~ 0.01 mm) being negligible. Compared to the uncertainties of the experimental setup, the error in air-water flow measurement associated with the phase-detection conductivity probe design was considered to be small and negligible. It is noteworthy that the micro-size bubbles with length scales smaller than the sensor tip diameter (0.25 mm) could not be detected. The contribution of these small particles to the flow aeration and turbulence modulation was not taken into account.

The width of the experimental flume was 0.5 m. It was not clear whether the finite channel width would affect the flow transverse motions on the centreline.

6.2 Uncertainties related to four-point measurements and data processing method

The measurement of velocity and turbulence properties between two points using the phase-detection probes had some limitations linked with the statistical nature of cross-correlation technique. The application of cross-correlation technique required a relatively high interfacial velocity with a constant flow direction. This was not satisfied in some flow regions in hydraulic jump roller, leading to bias in time-averaged velocity, turbulence intensity and turbulent length and time scales.

A new method to characterise the velocity fluctuations other than in the longitudinal direction was presented. The simultaneous use of two dual-tip phase-detection probes provided the possibility to quantify the transverse interfacial velocity components in the instantaneously three-dimensional flow. Consider the phase-detection probe array sketched in Figure 5.1C, while the velocity components \(V_{13}, V_{24}, V_{23}\) and \(V_{14}\) were calculated as time-averaged velocities over a 45 s sampling duration, the transverse component \(V_{12}\) was derived from a number of much shorter (0.2 s) sample segments and characterised some instantaneous velocity fluctuations rather than the average (the average transverse velocity being zero). The uncertainties using this method were however relatively large due to the turbulent nature of the flow and the statistical consideration in data processing. First, the small signal segment only contained a limited number of air-water interfaces, and the corresponding cross-correlation function might exhibit random and biased peaks (e.g. Fig. 3.2). The bias could not be minimised using segment-averaged correlation functions because the
average velocity was zero. Instead, characteristic velocities corresponding to local peak correlation coefficients were collected for each signal segment and the median velocity amplitude $|V_z|$ was considered. There was some chance that the selected $|V_z|$ was not a real physically-meaningful data. Moreover, the shape of the correlation function peak was usually not a smooth bell-shape, thus the estimate of correlation time scales might be inaccurate. As a result, errors might be derived from the calculation of velocity fluctuation using:

$$ \nu_j' = \sqrt{\frac{2\pi (\Delta x_j^2 + \Delta z_j^2)}{\pi \sigma_j}} \times \left( \frac{T_{ij}}{(R_{ij})_{\text{max}}} \right)^2 - T_{ij}^2 $$

(6.1)

The data scattering can be seen in Figure 5.6 and the consequent uncertainties were also reflected in the calculation of Reynolds stresses (Fig. 5.8).

Second, the detection of air-water interfaces recorded a combination of all interface motions, both longitudinally and transversely. The 0.2 s time interval was selected to filter the longitudinal components and to best reflect the possible transverse motions. However, there was no means to justify whether the characteristic peaks in correlation functions (e.g. Fig. 3.2) corresponded actually to a longitudinal or transverse motion. For a small-size signal segment, it was also possible that a maximum correlation coefficient was given by random detection of irrelevant bubbles. The filtering of phase-detection probe signals yielded flow properties dependent upon the size of signal segment (herein 0.2 s) as well as the physical sensor separation distance (herein $\Delta z = 10$ mm). Any change in signal segment size or sensor separation distance would affect the derived instantaneous transverse velocity fluctuations. Therefore, more physical data and/or theoretical consideration are required to support and justify the selection of these parameters.

### 6.3 Uncertainties related to relative sensor positions in phase-detection probe array

Two phase-detection probe array configurations were applied in the present study, as sketched in Figure 2.3. After a preliminary measurement with Configuration I (Fig. 2.3A), Configuration II (Fig. 2.3B) was introduced to avoid the disturbance of leading sensors to the diagonal transport of air-water interfaces between the leading and trailing sensors of two probes. Improvement in data quality can be seen in terms of the diagonal velocity results ($V_{23}$ and $V_{14}$ in Fig. 5.1C) in the high-speed positive flow region. Figure 6.1 presents a comparison between the diagonal velocities measured with both probe array configurations. Both datasets reflected the trend of time-averaged velocity distribution, and less data scatter was obtained using Configuration II in the shear flow region. The data given by Configuration I was not able to depict the boundary layer development next to the bottom. The data scattering in the positive flow region might be attributed to the interference of probe tip 1 in Figure 2.3 (also in Fig. 5.1) in the path of interfacial transport between tips 2 and 3. In the free-surface recirculation region, both datasets were scattered because of the complex interaction between the reversing flow and intrusive phase-detection probes.

The transverse instantaneous velocity characteristics were also obtained for Configuration I between two leading probe tips ($\Delta z_{12} = 9$ mm), including the velocity amplitude $|V_x|$ and fluctuation $\nu_x'$. The results were expected to be close to those given by Configuration II ($\Delta z_{12} = 10$ mm), because the measurements were not affected by the positions of the trailing probe tips. This can be seen Figure 6.2 where the characteristic transverse velocity amplitude and fluctuations are compared between the two configurations. Interestingly, the dataset also showed good agreements in the upper free-surface region where the relative sensor positions were expected to have effects on the reversing flow measurements.
Figure 6.1 – Diagonal interfacial velocities $V_{23}$ and $V_{14}$ measured with Configurations I and II – Flow conditions: $Q = 0.0347 \text{ m}^3/\text{s}$, $x_1 = 0.83 \text{ m}$, $d_1 = 0.0206 \text{ m}$, $Fr_1 = 7.5$, $Re = 6.8 \times 10^4$; $(x-x_1)/d_1 = 12.14$.

Figure 6.2 – Transverse velocity amplitude and fluctuation measured with Configurations I and II and compared to time-averaged longitudinal velocity and turbulence intensity – Flow conditions: $Q = 0.0347 \text{ m}^3/\text{s}$, $x_1 = 0.83 \text{ m}$, $d_1 = 0.0206 \text{ m}$, $Fr_1 = 7.5$, $Re = 6.8 \times 10^4$; $(x-x_1)/d_1 = 12.14$. 

(A) Transverse velocity amplitude

(B) Transverse velocity fluctuation
Four-point air-water flow measurements were performed in hydraulic jumps using a phase-detection probe array with four phase-detection sensors. The four probe needle sensors were placed in a horizontal x-z plane, with the two leading sensors and two trailing sensors at respective same longitudinal positions. The auto-correlation of each sensor signal and cross-correlation between the signals of any two sensors yielded the correlation tensor as a function of time lag, thus providing statistical turbulence properties measured between any two sensor tips. The results encompassed the longitudinal components of time-averaged interfacial velocity and turbulence intensity, which were well-recorded in a series of previous studies. The longitudinal velocity field was carefully depicted with opposite probe orientations in the impinging shear flow and recirculating free-surface flow regions, as well as with complementary Prandtl-Pitot tube data next to the channel bed. Self-similarity profiles were shown for the time-averaged longitudinal velocity, highlighting several characteristic velocities and elevations. The probe orientation had little impact on the time-averaged velocity measurement, while a reversed probe in the recirculation region gave smaller velocity fluctuations. With the four-point probe sensor array, it was further possible to derive the velocity and turbulence intensity components in a direction with an angle tan⁻¹(Δz/Δx) from the longitudinal direction, Δx and Δz being the longitudinal and transverse separation distances between the sensor tips. When Δx = 0, the turbulence properties were obtained in the transverse direction.

The statistical cross-correlation analysis had inherent limitations in abstracting information of particle motions other than along the main flow direction. Instead of the time-averaged transverse velocity that equalled zero, a characteristic instantaneous transverse velocity was derived based upon a number of small signal segments. Such a transverse velocity component was the result of a signal filtering for a given length scale (i.e. sensor separation distance) and a time scale (i.e. duration of signal segment). It was expected to provide a measure of the instantaneous transverse motion velocity in the bubbly flow. For a length scale ~10⁻² m and a time scale no larger than 0.2 s, the typical velocity of instantaneous transverse interface motions was estimated at 40% to 50% of the time-averaged longitudinal velocity. The corresponding transverse velocity fluctuations were one to two orders of magnitude smaller than the longitudinal turbulence intensity, with dimensionless transverse velocity fluctuations v²/\bar{V} ≈ 0.02 to 0.5. The substantial difference was linked to the fact that the large-scale, low-frequency motions of the flow had considerable influence on the quantification of longitudinal velocity turbulence, while the effects on transverse velocity fluctuations seemed to be limited. The direct measurement of turbulence intensity in a direction apart from the longitudinal direction was however lack of satisfying accuracy because of the limitation of correlation analysis. The errors were further reflected in the estimate of tangential Reynolds stress.

With the statistical nature of correlation analysis, the present method faced challenges when applied in turbulent flow with large-scale flow instabilities, strong anisotropy and flow recirculation. Nevertheless, the present study provided new experimental data depicting the three-dimensional velocity field in relatively strong hydraulic jumps and some guidelines for the use of phase-detection probes in such complex air-water flows. A future study may include a triple correlation analysis to examine the relationship between longitudinal and transverse velocity components. A measurement of vertical velocity may be also of interest.
8. ACKNOWLEDGEMENTS

The authors thank Dr Frédéric Murzyn (ESTACA, France), and Professor Daniel Bung (FH Aachen University of Applied Sciences, Germany) for their review and comments that enhanced the report. They further thank Jason Van Der Gevel and Stewart Matthews (The University of Queensland) for their technical assistance in the laboratory work. They acknowledge the financial support of the Australian Research Council (Grant DP120100481).
APPENDIX A. TURBULENCE INTENSITY ESTIMATE

The turbulence intensity $T_u$ was estimated based upon a simultaneous two-point measurement of the air-water interfacial velocity field. This appendix presents the derivation of the turbulence intensity $T_u$ based upon the correlation analyses of the dual-tip phase-detection probe signals. Relevant information was published in Chanson & Toombes (2001,2002), Felder & Chanson (2012,2014). This technique was validated for a two-point measurement along the main flow direction, with reasonably high flow velocity and void fraction. Occurrence of large-scale turbulence such as a periodic production of large eddies may lead to biased results.

A correlation analysis of the dual-tip phase-detection probe signals gives typical correlation functions as sketched in Figure A.1. Figure A.1A is the typical cross-correlation function $R_{ij}(\tau)$ between the signals of two probe sensors, and Figure A.1B is the typical auto-correlation function $R_{ii}(\tau)$ of the leading probe sensor signal, $\tau$ being the time lag. Assuming that the successive detections of bubbles by the probe sensors are true random processes, the cross-correlation function $R_{ij}(\tau)$ is a Gaussian distribution (Chanson & Toombes 2002):

$$R_{ij}(\tau) = (R_{ij})_{\text{max}} \times \exp \left( -\frac{1}{2} \left( \frac{\tau - T}{\sigma_{ij}} \right)^2 \right) \quad (A.1)$$

where $\sigma_{ij}$ is the standard deviation of the cross-correlation function, the subscript $i$ refers to the leading tip signal and the subscript $j$ to the trailing tip signal.

\begin{align}
\text{(A) Cross-correlation function} & \\
\text{(B) Auto-correlation function} & \\
\end{align}

Figure A.1 – Definition sketch of normalised correlation functions of dual-tip phase-detection probe signals

Figure A.1A defines a cross-correlation time scale $T_{ij}$ as the integration of cross-correlation function from the maximum to the first zero-crossing:

$$T_{ij} = \int_0^{\tau(R_{ij} = 0)} R_{ij}(\tau)d\tau \quad (A.2)$$

After simplification the cross-correlation time scale $T_{ij}$ becomes:

$$T_{ij} = (R_{ij})_{\text{max}} \times \frac{\sqrt{\pi}}{2} \times \sigma_{ij} \quad (A.3)$$
Similarly, define the auto-correlation time scale $T_{ii}$ as (Fig. A.1B):

$$ T_{ii} = \int_{0}^{\tau(R_{ii} = 0)} R_{ii}(\tau) d\tau \quad (A.4) $$

If the auto-correlation function is a Gaussian distribution, $T_{ii}$ becomes:

$$ T_{ii} = \sqrt{\frac{\pi}{2}} \cdot \sigma_{ii} \quad (A.5) $$

where $\sigma_{ii}$ is the standard deviation of the auto-correlation function. The turbulence intensity $T_u$ is defined as the ratio of the velocity standard deviation to the time-averaged velocity: $T_u = v'/V$.

When the velocity is measured with a dual-tip phase-detection probe, the standard deviation of the interfacial velocity equals:

$$ v'^2 = \frac{1}{n} \sum_{k=1}^{n} (v_k - V)^2 = \frac{V^2}{n} \sum_{k=1}^{n} (t_k - T)^2 $$

(A.6)

where $v_k$ is the instantaneous velocity data equal to $\Delta x/t_k$, $V$ is the time-averaged velocity ($V = \Delta x/T$), $n$ is the number of interfaces, $t_k$ is the interface travel time data and $T$ is the travel time for which the cross-correlation function is maximum. With an infinitely large number $n$ of interfaces, an extension of the mean value theorem for definite integrals may be used as $1/t_k^2$, and $(t_k - T)^2$ are positive and continuous functions over the interval $k = (1, n)$ (Spiegel 1974). The result implies that there exists at least one characteristic travel time $t'$ satisfying $t_1 \leq t' \leq t_n$ such that:

$$ \left( \frac{v'}{V} \right)^2 = \frac{1}{n} \frac{1}{t'^2} \sum_{k=1}^{n} (t_k - T)^2 = \frac{\sigma_t^2}{t'^2} $$

(A.7)

where $\sigma_t$ is the standard deviation of the interface travel time. If the intrinsic noise of the probe signal is uncorrelated to the turbulent velocity fluctuations with which the bubbles are convected, the standard deviation of the cross-correlation function $\sigma_{ij}$ satisfies:

$$ \sigma_{ij}^2 = \sigma_{ii}^2 + \sigma_t^2 \quad (A.8) $$

The turbulent intensity becomes:

$$ \frac{v'}{V} = \frac{\sqrt{\sigma_{ij}^2 - \sigma_{ii}^2}}{t'} $$

(A.9)

Assuming that $t' \sim T$, the turbulent intensity $v'/V$ equals:

$$ T_u = \frac{v'}{V} = \frac{\sqrt{\sigma_{ij}^2 - \sigma_{ii}^2}}{T} $$

(A.10)

Replacing Equations (A.3) and (A.5) into Equation (A.10), the turbulent intensity may be expressed as:
\[
Tu = \frac{\sqrt{2}}{\sqrt{\pi} T} \times \sqrt{\frac{T_{ij}^2}{(R_{ij}^*)_{\text{max}}} - T_{ii}^2} \quad (A.11)
\]

A simplified form of Equation (A.11) is also commonly used. Defining \( T_{0.5} \) the time scale for which the normalised auto-correlation function equals 0.5 and \( \tau_{0.5} \) the characteristic time for which: \( R_{ij}(T+\tau_{0.5}) = R_{ij}(T)/2 \) (Fig. A.1), the standard deviations of the auto- and cross-correlation functions equal: \( \sigma_{ii} = T_{0.5}/1.175 \), \( \sigma_{ij} = \tau_{0.5}/1.175 \). Then Equation (A.10) yields (Chanson & Toombes 2002):

\[
Tu = 0.851 \times \frac{\sqrt{T_{0.5}^2 - \tau_{0.5}^2}}{T} \quad (A.12)
\]

The assumption that both auto- and cross-correlation functions follow closely a Gaussian distribution was justified by physical observations for small to moderate time lags \( \tau \), namely, in moderate to high speed flows (e.g. Carosi & Chanson 2006). For large time lags, the "tail" of the data differs from the normal distribution: e.g., the first zero-crossing \( (R_{ij} = 0) \) being observed for a finite value. The above development may however be amended by assuming:

\[
R_{ij}(\tau) = (R_{ij})_{\text{max}} \times \exp \left( -\frac{1}{2} \left( \frac{\tau-T}{\sigma_{ij}} \right)^2 \right) \quad \text{for } R_{ij} > \xi \times (R_{ij})_{\text{max}} \quad (A.13)
\]

with \( 0 < \xi < 1 \). The results yield:

\[
T_{ij} = (R_{ij})_{\text{max}} \times \Delta \times \sqrt{\pi \times \sigma_{ij}} \quad (A.14)
\]

\[
Tu = \frac{1}{\Delta \times \sqrt{2 \times \pi T}} \times \sqrt{\frac{T_{ij}^2}{(R_{ij})_{\text{max}}} - T_{ii}^2} \quad (A.15)
\]

where:

\[
\Delta = \frac{1}{\sqrt{2 \times \pi}} \int_0^\infty \exp \left( -\frac{t^2}{2} \right) \times dt \quad (A.16)
\]

\[
\frac{1}{\sqrt{2 \times \pi}} = \exp \left( -\frac{\tau_{0.5}^2}{2} \right) \quad (A.17)
\]

Basically \( \Delta = 0.5 \) for \( \xi = 0 \), and further results are listed in Table A.1 (Spiegel 1974).
Table A.1 – Gaussian function distribution: relationship between $\xi$ and $\Delta$.

<table>
<thead>
<tr>
<th>$\xi$</th>
<th>$\Delta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0$</td>
<td>$0.50$</td>
</tr>
<tr>
<td>$0.01$</td>
<td>$0.4988$</td>
</tr>
<tr>
<td>$0.10$</td>
<td>$0.484$</td>
</tr>
<tr>
<td>$0.20$</td>
<td>$0.4637$</td>
</tr>
<tr>
<td>$0.30$</td>
<td>$0.4364$</td>
</tr>
<tr>
<td>$0.50$</td>
<td>$0.380$</td>
</tr>
</tbody>
</table>
APPENDIX B. INTEGRAL TURBULENT LENGTH AND TIME SCALES IN AIR-WATER FLOWS

The integral turbulent length and time scales constitute some comprehensive characterisation of the turbulent scales in the air-water flow (Chanson 2007, Chanson & Carosi 2007). The integral length scales $L_X$ and $L_Z$ characterise the longitudinal and transverse sizes of the relatively large vortical structures advecting the air bubbles and interacting with the air-water interfaces. The integral time scales $T_X$ and $T_Z$ quantify the associated time scales. The longitudinal integral turbulent scales were measured herein based upon a number of experiments with identical setup and flow conditions but different longitudinal separation distances $\Delta x$ between two phase-detection sensors. This was achieved in the present study using several dual-tip phase-detection probes with different lengths of needle sensors (Fig. B.1A), expanding the earlier findings of Chanson (2007) and Zhang et al. (2013). The longitudinal integral length and time scales were calculated as:

$$L_X = \int_0^{\Delta x(R_{ij})_{max} = 0} (R_{ij})_{max} \times d(\Delta x)$$  \hspace{1cm} (B.1)

$$T_X = \frac{1}{L_X} \times \int_0^{\Delta x(R_{ij})_{max} = 0} (R_{ij})_{max} \times T_{ij} \times d(\Delta x)$$  \hspace{1cm} (B.2)

where $R_{ij}$ is the cross-correlation function between the dual-tip probe sensors, $(R_{ij})_{max}$ is the maximum cross-correlation coefficient which is a decreasing function of the sensor separation distance, and $T_{ij}$ is the cross-correlation time scale given by an integration of the cross-correlation function from maximum to the first zero-crossing (Fig. B.1). The transverse integral turbulent scales were measured with repeated experiments with different transverse separations $\Delta z$ between the sensors. The length and time scales were defined as:

$$L_Z = \int_0^{\Delta z(R_{ij})_{max} = 0} (R_{ij})_{max} \times d(\Delta z)$$  \hspace{1cm} (B.3)

$$T_Z = \frac{1}{L_Z} \times \int_0^{\Delta z(R_{ij})_{max} = 0} (R_{ij})_{max} \times T_{ij} \times d(\Delta z)$$  \hspace{1cm} (B.4)

Herein the smallest non-zero $\Delta z$ that was physically achievable was given by a phase-detection probe with identical length of sensors ($\Delta z = 0.87$ mm). The larger separation distances were achieved using two side-by-side dual-tip probes, with $\Delta z$ being the separation between leading sensor tips (Fig. B.1B). Figure B.2A shows a series of correlation functions $R_{ij}$ for different longitudinal separations $\Delta x$ for the same flow conditions and measurement position. The variation of maximum correlation coefficients $(R_{ij})_{max}$ with $\Delta x$ are illustrated in Figure B.2B. The cross-correlation function equalled to the auto-correlation function of the leading sensor signal for $\Delta x = \Delta z = 0$, hence $(R_{ij})_{max} = (R_{ii})_{max} = 1$. The zero-crossing $\Delta x((R_{ij})_{max} = 0)$ and $\Delta z((R_{ij})_{max} = 0)$ were estimated based upon the decreasing trends of maximum correlation coefficients (Fig. B.2B). Table B.1 lists the longitudinal and transverse sensor separations $\Delta x$ and $\Delta z$ for the measurements of integral turbulent scales, the flow conditions being $Fr_1 = 7.5$ and $Re = 6.8 \times 10^4$. 
Longitudinal probe sensor separations $\Delta x_1 < \Delta x_2$  
Transverse probe sensor separations $\Delta z_1 < \Delta z_2$

Figure B.1 – Sketches of typical cross-correlation functions and time scales for different longitudinal and transverse probe separations.

Figure B.2 – Variation of cross-correlation function and maximum correlation coefficient with longitudinal separation distances between phase-detection probe sensors – Flow conditions: $Q = 0.0347 \text{ m}^3/\text{s}$, $d_1 = 0.0206 \text{ m}$, $x_1 = 0.83 \text{ m}$, $F_{r1} = 7.5$, $Re = 6.8 \times 10^4$; $x-x_1 = 0.25 \text{ m}$, $y = 0.04 \text{ m}$.

Table B.1 – Separation distances between two phase-detection probe sensor tips for the measurement of longitudinal and transverse integral turbulent scales with flow conditions $F_{r1} = 7.5$ and $Re = 6.8 \times 10^4$.

<table>
<thead>
<tr>
<th>Turbulent properties</th>
<th>$\Delta x$ [mm]</th>
<th>$\Delta z$ [mm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L_X$, $T_X$</td>
<td>2.57, 5.0, 7.25, 9.28, 13.92, 29.68</td>
<td>2.0</td>
</tr>
<tr>
<td>$L_Z$, $T_Z$</td>
<td>0</td>
<td>0.9, 3.6, 9.0, 17.1, 27.0, 36.6, 49.2, 92.0</td>
</tr>
</tbody>
</table>
Figure B.3A shows the vertical distributions of longitudinal integral turbulent length scale $L_X$ at five streamwise positions for the given flow conditions ($Fr_1 = 7.5$, $Re = 6.8 \times 10^4$). The transverse integral turbulent length scale $L_Z$ is shown in Figure B.3B for the same flow. In Figure B.3A, the vertical data distribution showed two characteristic peaks, one in the lower shear region and the other next to the free-surface. The upper maximum was related to the large-scale surface waves, while the lower maximum was linked to the intense turbulent structures advecting air bubbles at high velocity in the shear flow. The maximum integral length scale in shear region decreased with increasing streamwise distance, the largest value being about $0.8 \times d_1$ close to the jump toe. Such a characteristic length scale was in an order of magnitude of $10^{-3}$ to $10^{-2}$ m, smaller than the observed typical size of large eddies but larger than the scale of bubble clusters. It might imply that the coherent turbulent structures in the shear flow consisted of a number of bubble clusters and were advected as a part of the large vortex.

Figure B.3B presents transverse integral length scales in the same order of magnitude as the longitudinal scales. It indicated that turbulent structures of comparable sizes developed both along and perpendicular to the main flow direction. However, the data profile exhibited some different shapes within a short distance at downstream of the jump toe. On the one hand, no maximum transverse length scale was seen in the shear region, and the values were slightly smaller than the longitudinal ones at the same position. It corresponded to a main direction of momentum convection along the streamline. On the other hand, the length scale was larger in the transverse direction than in the longitudinal direction next to the free-surface. This reflected some local quasi-two-dimensional recirculating flow structures with axes parallel to the transverse direction. The transverse length scale data were consistent with the earlier studies of Chanson (2007) and Zhang et al. (2013). A direct comparison between $L_X$ and $L_Z$ in the same cross-section can be seen in Figure B.4.

(A) Longitudinal integral turbulent length scale
(B) Transverse integral turbulent length scale

Figure B.3 – Vertical distributions of longitudinal and transverse integral turbulent length scales at five longitudinal positions on the channel centreline – Flow conditions: $Q = 0.0347$ m$^3$/s, $d_1 = 0.0206$ m, $x_1 = 0.83$ m, $Fr_1 = 7.5$, $Re = 6.8 \times 10^4$. 
Figure B.4 – Comparison between longitudinal and transverse integral turbulent length scales; flow conditions the same as in Fig. B.3.

The transverse integral length scales are compared with the data of Chanson (2007) and Zhang et al. (2013) in Figure B.5. The data are plotted relative to the characteristic elevation $Y_{90}$ where $C = 0.9$ to facilitate the comparison. Despite the different Froude numbers and intake aspect ratios between the experiments, the transverse integral turbulent length scales $L_Z/Y_{90}$ exhibited similar distributions within the turbulent shear region. Figure B.5 also includes some datasets of transverse integral turbulent length scales obtained on stepped spillways (Chanson & Carosi 2007, Felder & Chanson 2009). The measurements on the stepped spillway were performed in several cross-sections of the skimming flow normal to the pseudo-bottom formed by the step edges, with $y = 0$ at the step edge. Interestingly, no marked difference was shown between the hydraulic jumps and stepped spillway flows in terms of the transverse turbulent length scale $L_Z/Y_{90}$, though the turbulent flow patterns and air entrainment mechanisms were substantially different. Both hydraulic jump and stepped spillway flows were commonly treated as quasi-two-dimensional, nevertheless, the existence of three-dimensional turbulent structures was well observed. The comparison indicated that the transverse length scales of the turbulent structures were similar in the two types of open channel flows.

Figure B.5 – Vertical distribution of transverse integral turbulent length scale close to jump toe – Comparison with data of Chanson (2007) and Zhang et al. (2013) in hydraulic jump and data of Chanson & Carosi (2007) and Felder & Chanson (2009) in stepped spillway flows.
Typical longitudinal and transverse integral turbulent time scales $T_X$ and $T_Z$ are presented in Figures B.6A and B.6B respectively, and $T_X$ and $T_Z$ are compared in Figure B.7. Similar distributions were seen between both time scales, with the longitudinal scale slightly larger than the transverse scale. The integral time scales were in an order of $10^{-3}$ s in the lower shear region and of $10^{-2}$ s next to the free-surface. No longitudinal variation was shown in the shear flow, while longitudinal increase in time scales was seen next to the free-surface region. The results gave a statistic measure of the characteristic time scales for a range of advective turbulent structure sizes.

(A) Longitudinal integral turbulent time scale
(B) Transverse integral turbulent time scale

Figure B.6 (Left) – Vertical distributions of longitudinal and transverse integral turbulent time scales at five longitudinal positions on the channel centreline – Flow conditions: $Q = 0.0347$ m$^3$/s, $d_1 = 0.0206$ m, $x_1 = 0.83$ m, $Fr_1 = 7.5$, $Re = 6.8 \times 10^4$.

Figure B.7 – Comparison between longitudinal and transverse integral turbulent time scales; flow conditions the same as in Fig. B.6.
APPENDIX C. APPLICATION OF TRIPLE DECOMPOSITION TECHNIQUE TO AIR-WATER TURBULENT FLOWS

In a hydraulic jump, the macroscopic and microscopic turbulent flow properties existed at the same time within a wide range of time scales. Detailed air-water flow measurements using intrusive phase-detection probes enabled turbulence characterisation of the bubbly flow, although the velocity fluctuation was not a truly random process because of the existence of low-frequency, pseudo-periodic fluctuating motions of the jump roller. These motions encompassed longitudinal oscillations of jump toe position, vertical fluctuations of water depth, and formation and downstream advection of large-size vortices. They all introduced large-scale non-randomness into the air-water interface motions and led to unusually large turbulence intensities measured at a fixed location. Felder & Chanson (2014) observed similar unsteady flow patterns in the air-water flows on a pooled stepped spillway. They developed a triple decomposition technique for non-stationary air-water flows and were able to identify the true turbulent properties of the flow (Felder 2013). We applied the triple decomposition technique to the hydraulic jump flow to quantify the turbulent flow contributions linked to the fast and slow fluctuating velocity components. Herein the triple decomposition technique is introduced in details and the relevant experimental results can be found in Wang et al. (2014).

For the triple decomposition data processing, the voltage signal of the phase-detection probe was decomposed into an average component, a low-frequency component corresponding to the slow fluctuations and a high-frequency component corresponding to the fast turbulent motions. The frequency thresholds between the signal components were identified based upon experimental investigations on free-surface dynamics, spectral analysis of instantaneous void fraction signals and some sensitivity studies. Table C.1 summarises some characteristic frequencies of the large-scale flow instabilities, including free-surface fluctuations, jump toe oscillations and formation of large-size vortices. Typical frequency ranges were documented from 0.8 to 4 Hz for free-surface fluctuations and from 0.4 to 2 Hz for jump toe oscillations and large vortex formations and advections. Altogether the relevant experimental studies suggested a frequency range between 0.4 to 4 Hz for the pseudo-periodic motions, and the findings applied to a wide range of flow conditions.

Table C.1 – Characteristic frequency ranges of pseudo-periodic motions in hydraulic jump.

<table>
<thead>
<tr>
<th>Motions of flow</th>
<th>Reference</th>
<th>Method</th>
<th>Frequency [Hz]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Free-surface fluctuations</td>
<td>Murzyn &amp; Chanson (2009b)</td>
<td>ADM measurement</td>
<td>0.8 to 4.0</td>
</tr>
<tr>
<td></td>
<td>Chachereau &amp; Chanson (2011b)</td>
<td>ADM measurement</td>
<td>1.6 to 3.9</td>
</tr>
<tr>
<td></td>
<td>Wang &amp; Chanson (2015a)</td>
<td>ADM measurement</td>
<td>1.2 to 3.7</td>
</tr>
<tr>
<td>Jump toe oscillations</td>
<td>Chanson (2006)</td>
<td>visual observation</td>
<td>0.6 to 2.0</td>
</tr>
<tr>
<td></td>
<td>Murzyn &amp; Chanson (2009b)</td>
<td>visual observation</td>
<td>0.5 to 0.8</td>
</tr>
<tr>
<td></td>
<td>Chanson (2010)</td>
<td>visual observation</td>
<td>0.4 to 0.8</td>
</tr>
<tr>
<td></td>
<td>Richard &amp; Gavrilyuk (2013)</td>
<td>numerical simulation</td>
<td>0.2 to 1.1</td>
</tr>
<tr>
<td></td>
<td>Wang &amp; Chanson (2015b)</td>
<td>visual observation</td>
<td>0.7 to 1.4</td>
</tr>
<tr>
<td></td>
<td>Wang &amp; Chanson (2015b)</td>
<td>ADM measurement</td>
<td>0.5 to 1.3</td>
</tr>
<tr>
<td>Large vortex advections</td>
<td>Chanson (2010)</td>
<td>visual observation</td>
<td>0.4 to 1.1</td>
</tr>
<tr>
<td></td>
<td>Wang &amp; Chanson (2015a)</td>
<td>visual observation</td>
<td>0.4 to 1.4</td>
</tr>
</tbody>
</table>

Note: ADM: acoustic displacement meters.

Further spectral analysis of the raw voltage output was conducted. The energy density function of signal reflected the detection of air-water interfaces. Figure C.1 presents a power spectral density function of the raw signal at the elevation of maximum void fraction $Y_{C_{max}}$ in the shear flow,
indicating some characteristic frequencies at 0.4, 10.7 and 216 Hz. The characteristic frequencies indicated a higher frequency range between 10.7 and 216 Hz corresponding to the detection of most air bubbles, while the impacts of flow instabilities was reflected in a range between 0.4 and 10.7 Hz. For most flow conditions with $3.8 < Fr_1 < 10$ and $3.5 \times 10^4 < Re < 1.6 \times 10^5$, these characteristic frequencies were seen at about 0.3 to 0.5, 10 to 15 and above 100 Hz depending upon the position in jump roller.

Figure C.1 – Power spectral density function of raw phase-detection probe signal – Flow conditions: $Q = 0.0347 \, m^3/s$, $d_1 = 0.0206 \, m$, $x_1 = 0.83 \, m$, $Fr_1 = 7.5$, $Re = 6.8 \times 10^4$; $(x-x_1)/d_1 = 12.5$, $y/d_1 = 2.8$.

Overall, both experimental investigations and spectral analysis suggested the frequencies of slow fluctuations in an order of magnitude of $10^{-1}$ to 1 Hz. Herein the lower and upper cut-off frequencies of the slow fluctuations were set at 0.33 and 10 Hz respectively. The selection was supported by a spectral analysis of instantaneous interfacial velocity fluctuations and a sensitivity study of cut-off frequencies by Felder (2013) for a similar hydraulic jump configuration. With the selected frequency thresholds, the average, low-frequency and high-frequency signal components were respectively obtained using low-pass, band-pass and high-pass filtering of raw phase-detection probe signal. Figure C.2 gives an example of the raw and filtered signals.

The decomposition of the instantaneous void fraction:

$$c = \bar{c} + c' + c''$$  \hspace{1cm} (C.1)

eyielded decomposed time-averaged void fraction given by the mean component of raw signal, i.e.

$$\bar{C} = \frac{1}{n} \sum_{i}^{n} \bar{c} \approx C$$  \hspace{1cm} (C.2)

$$C' = \frac{1}{n} \sum_{i}^{n} c' \approx 0$$  \hspace{1cm} (C.3)

$$C'' = \frac{1}{n} \sum_{i}^{n} c'' \approx 0$$  \hspace{1cm} (C.4)
Herein the symbol with overbar refers to the decomposed term of mean signal component, and the symbols with single and double prime respectively stand for the low-frequency and high-frequency terms. For the selected frequency ranges, most bubble count rates equalled to the high-frequency component, i.e. $F \approx F''$, $\bar{F} \approx F' \approx 0$.

Since the turbulence properties were deduced from the correlation functions of synchronous raw signals, the decomposition of turbulence properties were based upon the decomposition of the corresponding correlation functions. The decomposition of the correlation functions was a linear process (Felder & Chanson 2014). That is, the decomposed correlation functions were proportional to the correlation functions calculated for the filtered signal components. No time-averaged components appeared in the decomposition results, and the cross-correlation between the low-
frequency and high-frequency components were negligible. Therefore, the auto- and cross-correlation functions were expressed as:

\[ R_{ii}(\tau) \approx R_{ii}'(\tau) + R_{ii}''(\tau) \]  
\[ R_{ij}(\tau) \approx R_{ij}'(\tau) + R_{ij}''(\tau) \]

where \( R_{ii}' \) and \( R_{ij}' \) are respectively proportional to the auto- and cross-correlation functions of the low-frequency (band-pass filtered) signals, and \( R_{ii}'' \) and \( R_{ij}'' \) are proportional to the auto- and cross-correlation functions of the high-frequency (high-pass filtered) signals. Figure C.3 shows a set of cross-correlation functions for the raw signal and its low-frequency and high-frequency components, indicating \( R_{ij} \approx R_{ij}'+R_{ij}'' \).

![Cross-correlation functions](image)

Figure C.3 – Raw and decomposed cross-correlation functions of phase-detection signals; flow conditions and measurement location the same as in Fig. C.2.

Since the correlation functions were decomposed linearly, the time-averaged air-water interfacial velocity deduced from the low-frequency and high-frequency signal components were respectively:

\[ V' = \frac{\Delta x}{T'} - V \]  
\[ V'' = \frac{\Delta x}{T''} \approx V \]

Experimental results showed that \( V \) and \( V'' \) were about identical, while the distribution of \( V' \) was relatively scattered. The close agreement between \( V \) and \( V'' \) was linked to the sequential detection of air-water interfaces with two phase-detection probe sensors being a high-frequency process (Felder & Chanson 2014). The data of \( V' \) was less accurate because of the less accurate estimate of \( T' \) with a broad, flat peak in the decomposed cross-correlation function (Fig. C.3).

The auto-correlation time scale could be written as:

\[ T_{ii} \approx T_{ii}'+T_{ii}'' \]
where

\[ T_{ii}^\tau = \int_{0}^{\pi(R_i' = 0)} R_{ii}'(\tau) \times d\tau \quad (C.10) \]

\[ T_{ii}'' = \int_{0}^{\pi(R_i'' = 0)} R_{ii}''(\tau) \times d\tau \quad (C.11) \]

and the cross-correlation time scale became:

\[ T_{ij} \approx T_{ij}^\tau + T_{ij}'' \quad (C.12) \]

\[ T_{ij}^\tau = \int_{0}^{\pi(R_{ij} = 0)} R_{ij}'(\tau) \times d\tau \quad (C.13) \]

\[ T_{ij}'' = \int_{0}^{\pi(R_{ij}'' = 0)} R_{ij}''(\tau) \times d\tau \quad (C.14) \]

The corresponding turbulent time/length scales could be developed based upon their definitions given by Equations (B.1) to (B.4).

Particularly, the turbulence intensities of the low- frequency and high-frequency motions were expressed as:

\[ Tu' = 0.851 \times \sqrt{\frac{\tau_{0.5}^{i2} - T_{0.5}^{i2}}{T''}} \quad (C.15) \]

\[ Tu'' = 0.851 \times \sqrt{\frac{\tau_{0.5}^{i2} - T_{0.5}^{i2}}{T''}} \quad (C.16) \]

Note that the validity of Equations (C.15) and (C.16) was not theoretically justified due to the non-linearity of the decomposition, but their application to experimental data showed that the decomposition of turbulence intensity was possible and yielded (Felder & Chanson 2014):

\[ Tu \approx Tu' + Tu'' \quad (C.17) \]

Further justification of the decomposition of turbulence intensity was given by Felder & Chanson (2014) by comparing stepped spillway flows with and without instabilities. Their study demonstrated comparable turbulence intensities deduced from the raw signal of the stable flow and the high-frequency signal component of the instable flow, thus the high-frequency signal component gave agreeable turbulence intensity with absence of the impact of flow instabilities.

Detailed experimental data derived from the triple decomposition technique were reported in Wang et al. (2014). The successful application of triple decomposition technique to the hydraulic jumps highly improved the quality of turbulence characterisation.
APPENDIX D. VELOCITY VECTORS WITH AN ANGLE FROM LONGITUDINAL DIRECTION IN HYDRAULIC JUMPS

For a four-point air-water flow measurement using a phase-detection probe array as sketched in Figure D.1, the velocity at which air-water interfaces travelled from Sensor 1 to Sensor 4 could be derived from a cross-correlation analysis between the signals of Sensors 1 and 4:

\[
V_{14} = \frac{\sqrt{\Delta x_{24}^2 + \Delta z_{12}^2}}{[T]_{14}} \tag{D.1}
\]

where \(\Delta x_{24}\) and \(\Delta z_{12}\) equal respectively the longitudinal and transverse distances between the sensor tips 1 and 4, and \([T]_{14}\) is the time lag of maximum cross-correlation function between the sensor signals. The velocity given by Equation (D.1) was a time-averaged velocity. The velocity vector should also satisfy the relationship:

\[
V_{14} = \sqrt{V_{24}^2 + V_{12}^2} \tag{D.2}
\]

where \(V_{24}\) and \(V_{12}\) are the longitudinal and transverse velocity components respectively. The longitudinal component \(V_{24}\) was given as a time-averaged velocity by the signals of Sensors 2 and 4, whereas the transverse component \(V_{12}\) was only available as a characteristic instantaneous velocity selected based upon a number of small signal intervals because the average transverse velocity was zero. The selection of \(V_{12}\) is described in Section 3.2 and the experimental data are presented in Section 5.1. As a result, the velocity \(V_{14}\) given by Equation (D.2) should also be a characteristic amplitude of instantaneous velocity.

Herein the velocity \(V_{14}\) calculated with Equation (D.1) was compared with that directly measured with phase-detection probe sensors 1 and 4 (Fig. D.2 and D.3). Similarly, comparisons are also made for velocity \(V_{23}\) (Fig. D.1). Note that signs were added to be consistent with the longitudinal velocity direction. The measurements were performed for two flow conditions with the same Froude number \(F_{r1} = 7.5\) but different Reynolds numbers \(Re = 6.8 \times 10^4\) and \(1.4 \times 10^5\) (at five cross-sections for each flow). In the legend of Figures D.1 and D.2, we use (+) to denote the data obtained with normal probe positions with the sensors pointing upstream, and use (-) to denote the data obtained with reversed probe positions with the sensors pointing downstream in the recirculation region. Overall, the results showed more scattered data distributions for the time-averaged velocities directly measured with two sensors that were not aligned along the main flow direction. The calculated instantaneous velocities showed expected distributions with \(V_{14} \approx V_{23}\).

![Figure D.1 – Phase-detection probe array configuration for four-point air-water flow measurement.](image-url)
Figure D.2 – Vertical distributions of velocity vectors $V_{14}$ and $V_{23}$ for $Fr_1 = 7.5$, $Re = 6.8 \times 10^4$ – Comparison between data given by Equations (D.1) and (D.2).
Figure D.3 – Vertical distributions of velocity vectors $V_{14}$ and $V_{23}$ for $Fr_1 = 7.5$, $Re = 1.4 \times 10^5$ – Comparison between data given by Equations (D.1) and (D.2).

(A) $(x-x_1)/d_1 = 3.79$

(B) $(x-x_1)/d_1 = 7.58$

(C) $(x-x_1)/d_1 = 11.36$

(D) $(x-x_1)/d_1 = 17.06$

(E) $(x-x_1)/d_1 = 22.73$
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