

Default and Renegotiation in PPP Auctions*

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Abstract

The winners of auctions for PPP contracts, especially for major infrastructure projects such as highways, often enter financial distress, requiring the concession to either be re-allocated or re-negotiated. We build a simple model to identify the causes and consequences of such problems. In the model, firms bid toll charges for a fixed-term highway concession, with the lowest bid winning the auction. The winner builds and operates the highway for the fixed concession period. Each bidder has a privately known construction cost and there is common uncertainty regarding the level of demand that will result for the completed highway. Because it is costly for the Government to re-assign the concession, it is exposed to a hold-up problem, which bidders can exploit through the strategic use of debt. Each firm chooses its financial structure to provide optimal insurance against downside demand risk: the credible threat of default is used to extort an additional transfer payment from the Government. We derive the optimal financial structure and equilibrium bidding behaviour and show that (i) the auction remains efficient, but (ii) bids are lower than they would be if all bidders were cash financed, and (iii) the more efficient the winning firm, the more likely it is to require a Government bail-out

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and the higher the expected transfer it extracts from the Government. We discuss potential resolutions of this problem, including the use of Least-Present-Value-of-Revenue (LPVR) auctions.

1 Introduction

Public-Private Partnerships (PPPs) are increasingly used to provide infrastructure services and other public goods. Thomsen (2005) reports that worldwide investment in PPPs in the early 1990s had reached \$131 billion whereas the World Bank PPP database suggests that their total value reached nearly \$1.2 trillion dollars globally as of 2006. The popularity of PPPs seems to rest, at least among development circles, on their perceived ability to shift risks from the public to the private sector. However, the implications of this shift in risk are not well understood.

Bracey and Moldovan (2007) point out that about 50 percent of PPPs never even reach the financing stage and, of those that do, about 50 percent are renegotiated during the building or implementation phases. This suggests that the winners of PPP contracts, especially for major infrastructure projects such as highways, frequently enter financial distress, requiring the concession to either be re-allocated or re-negotiated. Very often, these issues are due to revenue falling short of expectations.

There are many examples of PPPs for the construction and operation of highways that failed due to lower than expected demand. One such example is the extension of the M1 Motorway in Hungary. Once the M1 was completed, it became clear that the project was at risk of default as traffic and toll revenues were only half the amount forecast by investors, lenders and the Hungarian government. The final outcome was the renationalisation of the project. A successor PPP contract to build the M5 highway from Budapest to Serbia also ran into trouble once realised demand was lower than expected. The outcome of renegotiation was the subsidisation of the toll by transfers from the government to the concessionaire.¹ Examples in Australia include the cross city tunnel in Sydney² and the Clem 7 tunnel in Brisbane.³

Our main objective in the present paper is to better understand the prevalence of bail-outs under such arrangements. In our model, firms bid for a concession to build and operate a highway⁴ in a first-price, sealed-bid auction.⁵ Each firm has privately known construction costs, but all face common

¹See Bracey and Moldovan (2007).

²See <http://www.tollroadsnews.com/node/1742>.

³See <http://www.theaustralian.com.au/archive/business-old.end-of-the-road-as-rivercity-motorway-sinks-owing-13bn/story-e6frg9i6-1226011606877>

⁴We assume throughout the paper that construction is bundled with operation in the tender process. Iossa and Martimort (2008) and Martimort and Pouyet (2008) examine whether such bundling is optimal and the resulting implications for contract design.

⁵As Engel, Fischer and Galetovic (2001) point out, highway franchises are typically auctioned on the basis of a fixed term with lowest toll bid winning (or fixed toll and lowest concession length).

uncertainty about future demand for the highway. Bids take the form of the toll to be charged and the firm with the lowest bid is granted the concession. Bidders also decide how to finance the up-front construction costs conditional on winning the contract.

A key feature of the model is the strategic use of debt-financing to “hold up” the Government. To be clear, in our model there is no underlying reason for firms to use debt to finance construction; firms are not cash constrained. Instead, the only reason why firms might use debt finance is to force the government to renegotiate. Indeed, we show that in low demand states, the winning firm threatens bankruptcy. Provided its debt is not too high, the Government has an incentive to renegotiate and provide a transfer to the financially distressed firm, rather than bear the cost of re-allocating the concession.

Our model suggests that default and re-negotiation are natural outcomes of PPP auctions. However, this does not result in an inefficient allocation process. A bidder’s optimal capital structure depends on its cost type, and the possibility of renegotiation causes firms to bid more aggressively than in an unlimited liability setting, but the equilibrium bidding function is still monotone in the firm’s cost type. This implies that the auction allocates the contract to the lowest cost firm, but the bids may appear unrealistically attractive to a Government which fails to anticipate the subsequent hold-up.

The most surprising result is that (for our demand specification) the severity of the hold-up problem is increasing in the efficiency of the winning firm: more efficient firms are bailed out more often – and extract a higher expected transfer from Government – than less efficient firms.

We note that there are reasons other than demand uncertainty that can lead to renegotiation. These include lack of compliance with agreed-upon terms and departure from expected outcomes (Guasch, 2004), and departure from contract terms due to the existence of transaction costs, inherent contractual incompleteness, or the imperfect ability of governments to enforce contracts (Williamson, 1985; Masten and Saussier, 2000; Bajari and Tadelis, 2001; Grossman and Hart, 1986; and Guasch et al., 2006). Guasch et al. (2007, 2008) examined over 1,000 concession contracts in Latin America and found that the conditions conducive to renegotiations are a combination of contract characteristics, regulatory environment and economic shocks.

The rest of the paper is organised as follows. Section 2 describes a version of the model in which there are only two possible demand states. This has the virtue of simplicity, so we can solve explicitly for the equilibrium capital structure of firms and the equilibrium (symmetric) bidding function, but the environment is restrictive. In the two-state world, all firms have the same level of debt, all face the risk-free interest rate, and the likelihood and size

of any bail-out is independent of the winning firm's cost type. None of these features survive the extension to a continuum of states, which is covered in Section 3. Section 4 discusses methods for ameliorating the hold-up problem, including the use of Least-Present-Value-of-Revenue (LPVR) auctions.

1.1 Related literature

We discuss connections with the PPP literature in the final section, but our analysis also has precursors in the literature on auctions. It is a descendant of Spulber (1990), who studied procurement auctions in which the winning bidder may default on performance if information revealed after the auction results in cost overruns. Spulber focusses on the potential for bid pooling – a “race to the bottom” in which inefficient firms bid low in the expectation of performing only in the most favourable circumstances, forcing more efficient firms to do likewise.⁶ He notes that renegotiation may restore efficiency.

In a similar vein, Zheng (2001) sets out to explain the outcomes of FCC spectrum auctions, in which winning bidders declared bankruptcy and defaulted on the payment of their bids. In his model, bidders differ in cash constraints. All face a common interest rate to borrow. If this rate is low enough, the most cash-constrained bidder wins and therefore defaults with high probability. Intuitively, if the interest rate is very low (say zero), bidders with less cash on hand have an advantage, as they have less to lose when they default. This allows them to out-bid their cashed-up rivals.

In an important extension of Zheng's work, Rhodes-Kropf and Viswanathan (2005) allow cash-constrained bidders to finance their bids. When each bidder has a private cash constraint but equal access to a competitive finance market, they prove that allocative inefficiencies continue to arise.

Unlike these models, our bidders are not cash constrained and there is no efficiency issue in equilibrium. Rather, we focus on the strategic use of debt to undermine the transfer of risk from the public to the private partner. Our model is also tailored to incorporate typical features of highway concession auctions: there is a long-run post-auction relationship between the seller (Government) and the winning bidder, construction takes place before revenue is earned, and bids are toll rates for the completed highway.

Finally, the strategic choice of debt in our model parallels that of a regulated firm in Spiegel and Spulber (1994, 1997). There, a regulated firm takes on debt so that the regulator sets a higher regulated price to reduce the likelihood of default. As in our model, these authors assume that the regulator (Government) bears an exogenous (unmodelled) cost of default. However, in

⁶See also Board (2007), who studies a common value environment in which bidders receive private signals about value.

our model, the firm chooses both debt and price, and in equilibrium these choices are made to increase the probability of insolvency (absent a bail-out), not decrease it. We also endogenise the cost of debt, whereas this cost is exogenous in Spiegel and Spulber.

2 A two-state model

2.1 Set-up

We consider a first-price, sealed-bid auction for a highway concession of fixed duration. Bidders have heterogeneous and privately known construction costs, though all are assumed to be able to complete the construction within the same timeframe. For simplicity, we assume that firms have common (zero) operating costs once the road is built. Each firm bids the toll it will charge for use of the road while it holds the concession. The lowest bid wins, with ties resolved using uniform randomisation. The winner builds the road then operates it for the specified fixed period, charging the toll it bid at the auction. For technical convenience, we ignore discounting; that is, the interest rate on risk-free debt is zero.

Toll revenue is uncertain at the time of bidding. More specifically, demand for the completed road will be $q = \theta - p$ for some $\theta \in \{\theta_L, \theta_H\}$, with

$$0 < \theta_L < \theta_H \leq 2\theta_L.$$

(The restriction $\theta_H \leq 2\theta_L$ ensures that firms will never charge a price which exceeds the “choke price” in state θ_L .) Let $\pi \in (0, 1)$ be the probability of θ_H , such that expected toll revenue is $p(\bar{\theta} - p)$, where $\bar{\theta} = \pi\theta_H + (1 - \pi)\theta_L$. Bidders are risk-neutral.

Given the absence of discounting, we model the situation as three discrete stages: the auction stage, the construction stage, and the operation stage – the entire post-construction concession is collapsed into a single period. The value of θ is realised at the start of the third stage. Firms decide whether or not to default on their obligations to creditors after observing θ . We assume that all revenue generated in the third stage (i.e., over the life of the concession) is available to creditors in the event of default.⁷

⁷In practice, an insolvent firm will default early in the concession so creditors may not have access to all future revenue streams – bankruptcy costs may reduce the residual value of the concession below the present value of the revenue stream. This will affect the optimal capital structure of firms, since debt will be more expensive than equity financing. Firms who do not face cash constraints will only borrow for strategic reasons. We plan to explore the consequences of this alternative bankruptcy assumption – and cash constraints – in future work.

Firm i has construction cost c_i , drawn randomly (and independently of other firms' costs) according to the common distribution F , defined on the support $[\underline{c}, \bar{c}]$, where

$$0 \leq \underline{c} < \bar{c} < \left(\frac{\bar{\theta}}{2}\right)^2.$$

We assume that F is strictly increasing and differentiable, with associated density f . Note that $(\bar{\theta}/2)^2$ is the *ex ante* expected revenue of a monopolist.

A critical component of our model is the financing of construction costs. These costs are incurred prior to revenue being earned and must be financed by the concessionaire. This privatisation of finance is a standard feature of PPP arrangements. As a benchmark, we first analyse the model under the assumption that the firm funds all construction costs using cash/equity (i.e., is exposed to unlimited liability for losses on the project). Following that we consider a more realistic scenario in which the firm may choose a mixture of debt and equity financing, and the cost of funds is endogenously determined.

2.2 Cash financing (unlimited liability)

Suppose there exists a symmetric and differentiable bidding strategy $p(c_i)$, with $p' > 0$ so that the firm with the lowest construction cost wins the auction. Since p is differentiable and type \bar{c} expects to cover its costs at the monopoly price, we must also have

$$p(c) \leq \frac{\bar{\theta}}{2}$$

for all $c \in [\underline{c}, \bar{c}]$. To see this, notice that a firm that bids above $\bar{\theta}/2$ could strictly increase both its probability of winning and its expected revenue conditional on winning by bidding $\bar{\theta}/2$ instead. The latter bid would also guarantee a strictly positive payoff.

If the equilibrium bidding strategy is efficient, then c_i solves:

$$\max_c [p(c) (\bar{\theta} - p(c)) - c_i] (1 - F)^{n-1}(c)$$

Defining $b(c, \bar{\theta}) = p(c) (\bar{\theta} - p(c))$ and $\bar{F}(c) = 1 - F(c)$ this becomes

$$\max_c [b(c, \bar{\theta}) - c_i] \bar{F}^{n-1}(c)$$

Since $p(c) \leq \bar{\theta}/2$ for all c , $b(\cdot)$ is strictly increasing in c . By standard arguments (see, for example, Menezes and Monteiro, 2004, Chapter 3, for

the case of first-price auctions) $b(\bar{c}, \bar{\theta}) = \bar{c}$ and for any $c < \bar{c}$,

$$b(c, \bar{\theta}) = - \int_c^{\bar{c}} z \frac{d\bar{F}^{n-1}(z)}{\bar{F}^{n-1}(c)}.$$

Recall that the highest type, \bar{c} , can cover its costs at the monopoly price, so all firm types are willing to participate in the auction.

Let X denote the random variable corresponding to the lowest of $n - 1$ random draws from F . Then

$$H(z) = \Pr[X \leq z] = 1 - \bar{F}^{n-1}(z)$$

is the distribution function for X and we may write

$$b(c, \bar{\theta}) = \int_c^{\bar{c}} z \frac{dH(z)}{[1 - H(c)]} = \mathbb{E}[X \mid X > c].$$

Thus, the winning firm's surplus is the difference between its own cost and the expected value of the next lowest cost; a common result in the literature on first-price auctions.

It is obvious that the function $b(c, \bar{\theta})$ is strictly increasing in c . Since

$$b_c(c, \bar{\theta}) = (\bar{\theta} - 2p(c)) p'(c),$$

it follows that $p'(c)$ has the same sign as $b_c(c, \bar{\theta})$, so p is also strictly increasing in c as was assumed.

To find an explicit expression for $p(c)$, we let $\Lambda(c) = \mathbb{E}[X \mid X > c]$ and solve

$$p(c) (\bar{\theta} - p(c)) = \Lambda(c).$$

Taking the smaller root, we have

$$p(c) = \frac{\bar{\theta}}{2} - \sqrt{\left(\frac{\bar{\theta}}{2}\right)^2 - \Lambda(c)}.$$

Figure 1 illustrates.

Note that the two-state assumption plays no role in the construction of this benchmark equilibrium. Only the expected value $\bar{\theta}$ appears in the calculations. The same results would be obtained for any assumption about the distribution of θ . In particular, Figure 1 continues to describe the “unlimited liability” benchmark for the model with a continuum of states analysed in Section 3.⁸

⁸Hansen (1988) also solves a bidding model in which firms bid prices at which they will

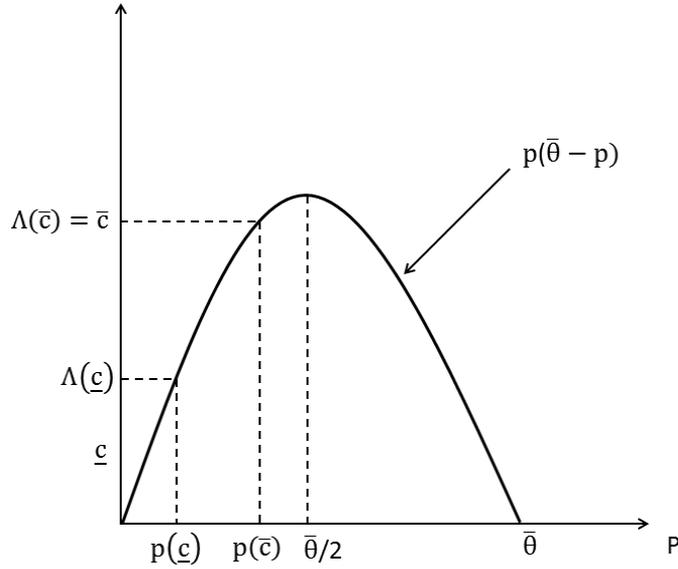


Figure 1: Equilibrium with cash financing

2.3 Endogenous financing and strategic default

Consider a firm i that uses both cash (equity) K_i and debt $D_i = c_i - K_i$ to fund the construction phase. We assume that firms face no cash constraint, so each can choose any level of K_i , and all have access to the same competitive market for debt. This implies that firms are distinguishable only by their construction costs.

Since firms face no cash constraint, they will hold debt only for strategic reasons in our model. Note that we do not allow firms to take on more debt than is necessary to finance their construction costs.

Assume that there is a symmetric equilibrium bidding strategy $\tilde{p}(c_i)$ and define

$$\tilde{b}(c, \theta) = \tilde{p}(c) (\theta - \tilde{p}(c)).$$

We assume that $\tilde{b}(c, \theta_H) \geq c$ for all $c \in [\underline{c}, \bar{c}]$ to ensure the winning firm is always solvent in the high demand state. However, we allow for the possibility that toll revenue may not cover construction costs when demand is low.

supply goods to a market. In Hansen's set-up, firms all have zero fixed costs but positive and heterogeneous marginal costs, while our firms have positive and heterogeneous fixed costs (i.e., the costs of construction) but all have zero marginal costs (i.e., zero maintenance costs). As Hansen's analysis indicates, our assumption of zero marginal costs greatly simplifies the derivation of the equilibrium bids.

We further assume that financing is not revealed as part of the tender process: for example, the winning firm might finalise its finance only after the auction.⁹ This assumption is necessary to ensure that costs remain private information, since costs may be inferred from D_i .

If firm i wins the auction and $D_i > 0$, it may choose to default if realised revenue is too low. Let us suppose that the winning bidder is always solvent in the high-demand state (θ_H). If firm i borrows at the risk-free rate (zero), it is insolvent in the low demand state if $\tilde{b}(c_i, \theta_L) < D_i$.¹⁰ In the absence of a Government bail-out, an insolvent firm will default and make a loss equal to

$$-(c_i - D_i).$$

In the event of default, the Government re-assigns the concession and pays the original creditors $\tilde{b}(c_i, \theta_L)$ from the proceeds of the sale – that is, we assume the Government bears all transaction costs associated with re-assigning the concession.^{11, 12} On top of these transaction costs, the Government also faces costs from disruption to road-users during the transition and associated political costs.¹³ Indeed, the international evidence suggests that

⁹In practice, financing arrangements are often revealed as part of a PPP tender. If so, borrowing must be consistent with the cost type imputed from the toll bid. A firm that wishes to deviate from the equilibrium bid function must tailor its financing to the implied construction cost, rather than the actual cost. As we show below, a firm's debt is based on its toll bid and does not depend on its construction cost directly (the latter only affects K_i). Hence, a firm that deviates from equilibrium will have the incentive to borrow as if it really were the type it is pretending to be. However, it may need to demonstrate a cash facility that is greater (if it bids above the equilibrium toll) or lower (if it bids below the equilibrium toll) than it will actually need. In the latter case, the firm's owners will simply raise more cash *ex post* if it wins the auction. The former case is more problematic, as the excess cash becomes liable to seizure by creditors if the firm defaults on its loan. This will act as an extra disincentive to bid above the equilibrium toll – the firm's payoff function will be kinked at its equilibrium toll, with the binding constraint being the one that prevents the firm from reducing its bid below the equilibrium toll rate. We plan to explore these extensions to the model in future work.

¹⁰Note that $\tilde{b}(c, \theta_L)$ depends on c only through $\hat{p}(c)$ so solvency is observable to the bank and the Government.

¹¹The main point of the paper requires only that the Government face some positive cost of re-assigning the concession. This is what creates the potential for hold-up by the concessionaire. The hold-up potential would still exist even if the creditors bear part of the bankruptcy costs. However, as noted above, this would complicate the analysis by raising the cost of debt financing relative to cash.

¹²Alternatively, the government could allow the concessionaire to increase the toll. This would result in similar qualitative results although we would need to take into account the reduction in demand (for a given realisation of the state of nature) that would follow the increase in toll.

¹³Dahdal (2010) argues that the failure of the cross-city tunnel in Sydney was of one

renegotiation is more prevalent than outright failure. For example, Guasch (2004) found that 55 per cent of Latin American transport concessions from his 1985-2000 dataset were renegotiated. Similarly, 76 per cent of the Latin American contracts in the water and sanitation industry studied by Guasch and Straub (2009) were subject to renegotiation. These considerations support the view that governments prefer to renegotiate than to replace the concessionaire.

We therefore assume that the Government places value $\bar{T} \in (0, \underline{c})$ on keeping the current concession-holder in place. This assumption is similar to that Spiegel and Spulber (1994) where a regulator faces an exogenous cost if the regulated firm is allowed to go bankrupt.

Thus, if demand turns out to be low and

$$0 < D_i - \tilde{b}(c_i, \theta_L) \leq \bar{T},$$

the Government and the concessionaire have an incentive to renegotiate. Under this renegotiation, the Government pays

$$D_i - \tilde{b}(c_i, \theta_L)$$

to clear the firm's debts (thereby validating our assumption that the firm borrows at the risk-free rate) plus an additional transfer $t \geq 0$. This gives surplus

$$\bar{T} - \left(D_i - \tilde{b}(c_i, \theta_L) \right) - t$$

to the Government and surplus t to the firm. The Nash bargaining solution for t is

$$t = \alpha \left[\bar{T} - \left(D_i - \tilde{b}(c_i, \theta_L) \right) \right],$$

where α is the firm's bargaining strength.

The firm therefore anticipates payoff

$$\alpha \left[\bar{T} - \left(D_i - \tilde{b}(c_i, \theta_L) \right) \right] - K_i = \alpha \left[\bar{T} + \tilde{b}(c_i, \theta_L) \right] + (1 - \alpha) D_i - c_i \quad (1)$$

in state θ_L . In state θ_H there is no default (by assumption), so the firm's payoff is $\tilde{b}(c_i, \theta_H) - c_i$. Since the firm's expected payoff is therefore strictly

of the most politically damaging issues undermining the credibility of the government: *“Issues such as competence, value for money, putting the interests of private contracting parties ahead of the public have all fed into an image of an out of touch and commercially incompetent government – even if in reality such a negative reputation is not entirely warranted.”* The state government took several measures to boost the concessionaire's revenues (e.g., restricting traffic in areas adjacent to the tunnel) in an attempt to avoid its bankruptcy.

increasing in D_i , it will maximise its leverage subject to the constraint

$$0 \leq D_i - \tilde{b}(c_i, \theta_L) \leq \min \left\{ \bar{T}, c_i - \tilde{b}(c_i, \theta_L) \right\}.$$

This constraint is necessary to ensure that default is a credible threat (the first inequality), that the Government is willing to renegotiate ($D_i - \tilde{b}(c_i, \theta_L) \leq \bar{T}$) and that the firm does not borrow more than it needs ($D_i \leq c_i$). The following assumption ensures that the latter constraint is never binding.

Assumption 0. The Government is never willing to bail out a firm that is 100% debt financed: $\bar{T} < c - \tilde{b}(c, \theta_L)$ for all $c \in [\underline{c}, \bar{c}]$.

Note that this assumption refers to the equilibrium bidding function, so can only be verified once we have computed the equilibrium bidding function. However, is clear that it will be satisfied provided $\bar{T} < \underline{c}$ and θ_L is sufficiently low, since $\tilde{b}(c, \theta_L) \rightarrow 0$ as $\theta_L \rightarrow 0$.

Under Assumption 0, the firm chooses

$$D_i = \tilde{b}(c_i, \theta_L) + \bar{T}$$

and receives payoff $\tilde{b}(c_i, \theta_L) + \bar{T} - c_i < 0$ in state θ_L . Each firm will choose to be strategically leveraged in order to “hold up” the Government in the low demand state if the firm wins the bid. Its debt will be set at a level such that all of \bar{T} is required to clear its debts when $\theta = \theta_L$ (given its bid). If debt were higher than this the Government would not bail out the firm. If debt were lower, the Government would implicitly secure the debt, but the firm could only negotiate for fraction α of the residual $\bar{T} - [D_i - \tilde{b}(c_i, \theta_L)]$. Firms use debt to extort all of \bar{T} in state θ_L and thereby partially insure themselves against losses when demand is low, courtesy of the tax-payer.

Next, we analyse the bidding strategies. Assume that \tilde{p} is differentiable and strictly increasing. These assumptions will be verified *ex post*.

If firm i bids $\tilde{p}(c)$, it will borrow $\tilde{b}(c, \theta_L) + \bar{T}$ and receive equilibrium payoff

$$\tilde{b}(c, \bar{\theta}) - c_i + (1 - \pi)\bar{T}$$

conditional on winning the auction. Letting

$$\tilde{\beta}(c) = \tilde{b}(c, \bar{\theta}) + (1 - \pi)\bar{T},$$

c_i must solve

$$\max_{c \in [\underline{c}, \bar{c}]} \left[\tilde{\beta}(c) - c_i \right] \bar{F}^{n-1}(c).$$

Provided $\tilde{\beta}' > 0$, it follows by standard arguments that

$$\tilde{\beta}(c) = \int_c^{\bar{c}} z \frac{dH(z)}{[1 - H(c)]} = \mathbb{E}[X \mid X > c].$$

Clearly $\tilde{\beta}' > 0$ as assumed. We therefore deduce that

$$\tilde{b}(c, \bar{\theta}) = \mathbb{E}[X \mid X > c] - (1 - \pi)\bar{T}$$

for all $c \in [\underline{c}, \bar{c}]$. In other words, firms bid more aggressively (demand lower expected toll revenue) than they would under cash financing due to their ability to hold-up the Government and partially offset their losses in state θ_L .

We may again find an explicit functional form for $\tilde{p}(c)$ by solving

$$p(\bar{\theta} - p) = \Gamma(c)$$

where

$$\Gamma(c) = \mathbb{E}[X \mid X > c] - (1 - \pi)\bar{T}.$$

Thus

$$\tilde{p}(c) = \frac{\bar{\theta}}{2} - \sqrt{\left(\frac{\bar{\theta}}{2}\right)^2 - \Gamma(c)}.$$

Note that \tilde{p} is differentiable and strictly increasing, as we assumed. Assuming $\Gamma(\underline{c}) \geq 0$, the construction of the equilibrium bidding function is illustrated in Figure 2.

The potential to hold-up the Government makes firms bid more aggressively than in the model with cash financing – compare Figures 1 and 2. Debt is used strategically to extort transfer \bar{T} from the Government in the low demand state. Less efficient firms will be more highly leveraged, since $\tilde{b}(c, \theta_L)$ is increasing in c , but the equilibrium bidding function is strictly increasing, so the auction allocates to the most efficient firm.¹⁴

This simple two-state model is useful for illustrating the hold-up problem faced by the Government, but delivers implausibly stark predictions. The winning firm always threatens default in state θ_L and always obtains a bail-out of exactly \bar{T} from the Government. Moreover, all firms borrow at the risk-free rate and a firm's equilibrium level of debt is independent of π and θ_H .

¹⁴We assumed in this analysis that firms are solvent in the high demand state. It is conceivable that some bidders may prefer a higher level of debt, so that they are bailed out in state θ_H and bankrupt in state θ_L , which will therefore alter bidding behaviour. We will allow for this possibility in the analysis of the continuous state space model.

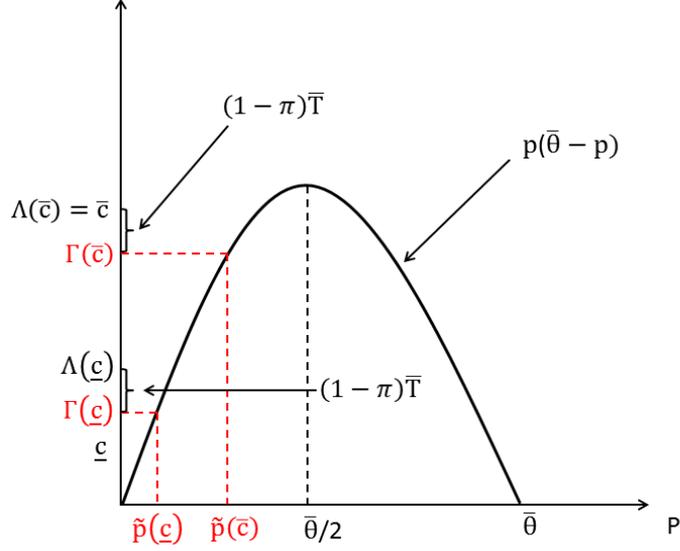


Figure 2: Equilibrium with optimal leverage

A more realistic scenario emerges if we allow θ to take on a continuum of values. In this case, different types will renegotiate in different contingencies. Firm i will set its debt equal to $\tilde{b}(c_i, \theta) + \bar{T}$ for *some* state θ , but this θ may depend on i . In states that are worse than θ , firm i will declare bankruptcy and the Government will not offer a bailout, while in states better than θ the firm will either pay its debts or else renegotiate for the Government to pay them plus an additional transfer. If there is a non-zero probability of bankruptcy, debt servicing costs will rise above the risk-free rate.

Given its construction costs, each firm decides on its optimal bid and its optimal level of debt, which in turn determine the contingencies in which it is bankrupt and in which it renegotiates. These contingencies may depend on the firm's cost type. The nature of this dependence is not obvious *a priori*. In the next section, we explore it further by analysing a model with a continuum of states.

3 A continuum of states

Suppose now that θ is distributed on $[\theta_L, \theta_H]$ according to the differentiable and strictly increasing distribution function G , with $G' = g$. Let

$$\bar{\theta} = \int_{\theta_L}^{\theta_H} \theta dG(\theta).$$

We continue to assume $\theta_H \leq 2\theta_L$ so that equilibrium tolls do not exhaust demand in any state. In particular, no firm will rationally bid more than $\theta_H/2$.

3.1 Optimal leverage

If firm i plans to bid p , it will choose an associated financial structure which maximises its expected payoff contingent on winning the auction. Suppose it sets its debt level at $D \in [0, c_i]$. Then its expected profit contingent on being the winning bidder is

$$\begin{aligned} -(c_i - D) &+ \int_{\Theta_0(p, D)} [p(\theta - p) - [1 + r(p, D)] D] dG(\theta) \\ &+ \int_{\Theta_1(p, D)} \alpha [\bar{T} + p(\theta - p) - [1 + r(p, D)] D] dG(\theta) \end{aligned}$$

where $r(p, D)$ is the interest rate on the firm's debt,

$$\Theta_0(p, D) = \{\theta \in [\theta_L, \theta_H] \mid [1 + r(p, D)] D \leq p(\theta - p)\}$$

is the set of contingencies in which the firm is solvent and

$$\Theta_1(p, D) = \{\theta \in [\theta_L, \theta_H] \mid 0 < [1 + r(p, D)] D - p(\theta - p) \leq \bar{T}\}$$

is the set of contingencies in which the firm holds up the Government. Note that the optimal level of debt is independent of c_i given the bid p , though the latter will obviously depend on the firm's cost type in equilibrium.

If θ is such that

$$\begin{aligned} [1 + r(p, D)] D - p(\theta - p) &> \bar{T} \\ \Leftrightarrow \theta &< \frac{[1 + r(p, D)] D + p^2 - \bar{T}}{p} \end{aligned}$$

the firm declares bankruptcy and receives $-(c_i - D)$. These are the only contingencies in which the bank is not paid in full. This occurs with probability

$$G\left(\frac{[1 + r(p, D)] D + p^2 - \bar{T}}{p}\right),$$

so $r(p, D)$ is the solution (in r) to

$$D = (1+r) D \left[1 - G \left(\frac{(1+r) D + p^2 - \bar{T}}{p} \right) \right] + \int_{\theta_L}^{[(1+r)D+p^2-\bar{T}]/p} p(\theta - p) dG(\theta).$$

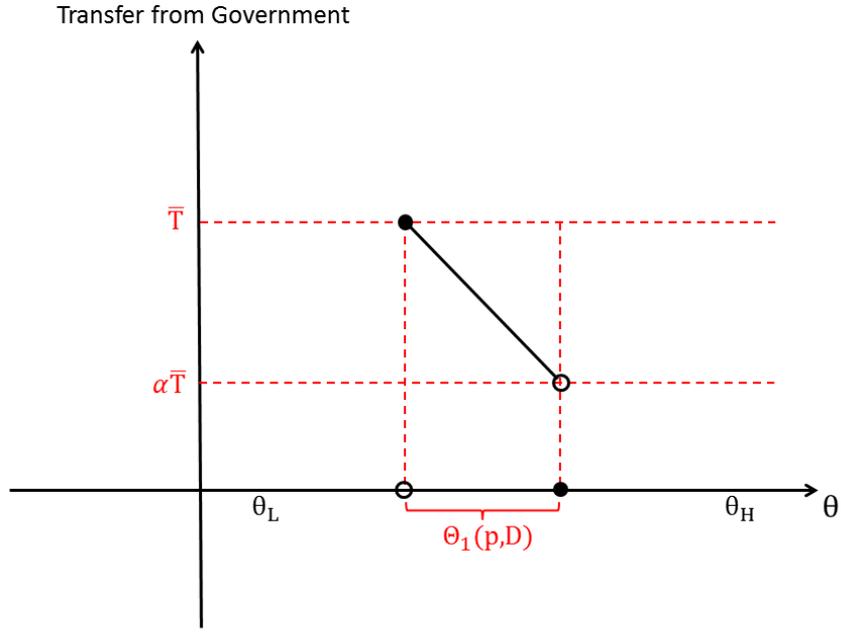


Figure 3: Government transfer as a function of θ

Given that the bank makes D in expected value, the firm's expected profit is easily calculated as

$$p(\bar{\theta} - p) - c_i + (\text{expected payment from the Government}).$$

Therefore, the firm chooses D to maximise its expected transfer payment from the Government, which is

$$\begin{aligned} & \int_{\Theta_1(p,D)} [[1+r(p,D)] D - p(\theta - p)] + \alpha (\bar{T} - [[1+r(p,D)] D - p(\theta - p)]) dG(\theta) \\ &= \int_{\Theta_1(p,D)} \alpha \bar{T} + (1 - \alpha) [[1+r(p,D)] D - p(\theta - p)] dG(\theta) \quad (2) \end{aligned}$$

That is, for each state $\theta \in \Theta_1(p, D)$ the Government pays the amount needed to clear the firm's debts, plus an additional transfer of

$$\alpha (\bar{T} - [[1 + r(p, D)] D - p(\theta - p)])$$

which is determined by the bargaining power of the firm.

Figure 3 illustrates the optimisation problem of the firm. It plots the integrand in (2) as a function of the demand state. Note that this integrand is linear in θ and varies between \bar{T} and $\alpha\bar{T}$ as θ varies over the range $\Theta_1(p, D)$. We also observe from the integrand in (2) that the slope of the transfer function is $-(1 - \alpha)p$ when $\theta \in \Theta_1(p, D)$. Hence, the length of the interval $\Theta_1(p, D)$ is \bar{T}/p .

The firm chooses D to maximise the expected value of the transfer function in Figure 3. Varying D will shift the interval $\Theta_1(p, D)$ but will not alter its length. We may therefore characterise the firm's decision as the optimal choice of the left-hand end point for Θ_1 , denoted by z in Figure 4. Given the choice of z , the implied choice for D is found by solving

$$[1 + r(p, D)] D - p(z - p) = \bar{T} \quad (3)$$

In other words, we may think of the firm choosing the state ($\theta = z$) in which all of \bar{T} is needed to clear its debts. This is the lowest demand state in which it will be bailed out by the Government. If $\theta < z$ the firm goes bankrupt as it is too expensive to bail out. If

$$z < \theta < z + \frac{\bar{T}}{p}$$

the firm is insolvent but the gap between its toll revenue and $(1 + r)D$ is less than \bar{T} . The firm acquires fraction α of this difference through the renegotiation process. If $\theta \geq z + (\bar{T}/p)$, the firm is solvent.

Therefore, rather than choosing D to maximise (2), we can think of the firm choosing $z \in [\theta_L, \theta_H]$ to maximise

$$\int_z^{\min\{z + (\bar{T}/p), \theta_H\}} [\bar{T} - (1 - \alpha)p(\theta - z)] dG(\theta) \quad (4)$$

as illustrated in Figure 4.¹⁵ If $z + (\bar{T}/p) > \theta_H$ then debt repayments exceed

¹⁵There is one additional constraint on the choice of z :

$$z \leq p + \frac{(1 + r)c_i - \bar{T}}{p}.$$

This ensures that $D \leq c_i$. It is analogous to Assumption 0 from the two-state model. It needs to be checked after computing the equilibrium bidding function. For the uniform case solved below, this constraint will be satisfied provided θ_L is low enough (as for the two-state model).

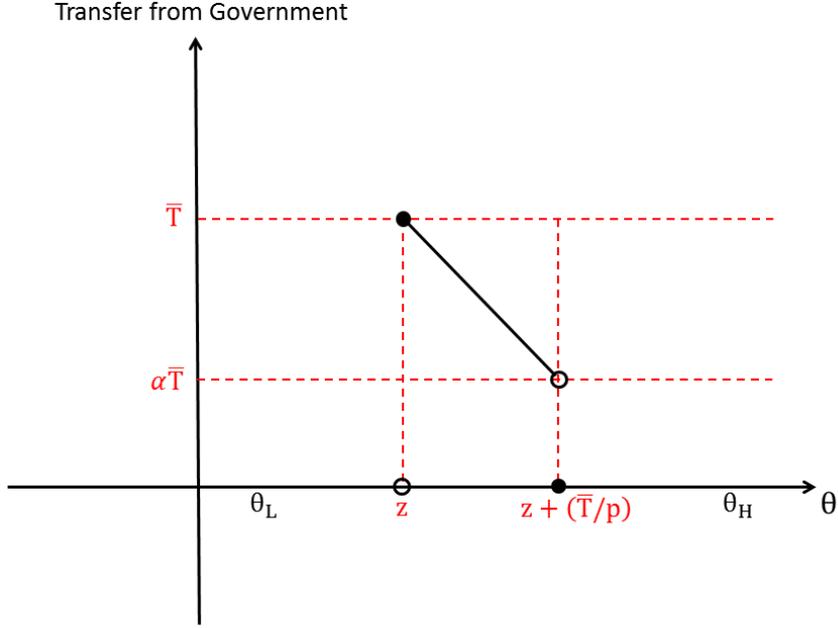


Figure 4: Optimal choice of z

revenue in every state. We assume that banks are still willing to lend in anticipation of the additional contribution from Government.

As in the two-state case, the firm's optimal leverage depends on its cost type only through its bid – this is clear from Figure 4 and (3). Note that a *lower* value of p *increases* the expected transfer from Government for any $z \in [\theta_L, \theta_H]$. This follows from the fact that decreasing the value of p will weakly increase the Government transfer at every θ – see Figure 5. It follows that the *higher* a firm bids in the auction, the *lower* its expected transfer from the Government (conditional on winning).¹⁶ Thus, if the equilibrium bid function is strictly increasing, more efficient firms extort higher expected transfers from the public purse. We will shortly verify that the symmetric equilibrium bid function is indeed strictly increasing when θ is uniformly distributed. In this case, observing large bail-outs is not evidence that the PPP auction failed to select an efficient provider. Indeed, it is evidence of the opposite.

The intuition for this result – that firms who bid lower expect a higher payment from the Government – is also discernible from Figure 5. Firms

¹⁶This can also be established by applying the Envelope Theorem to (4).

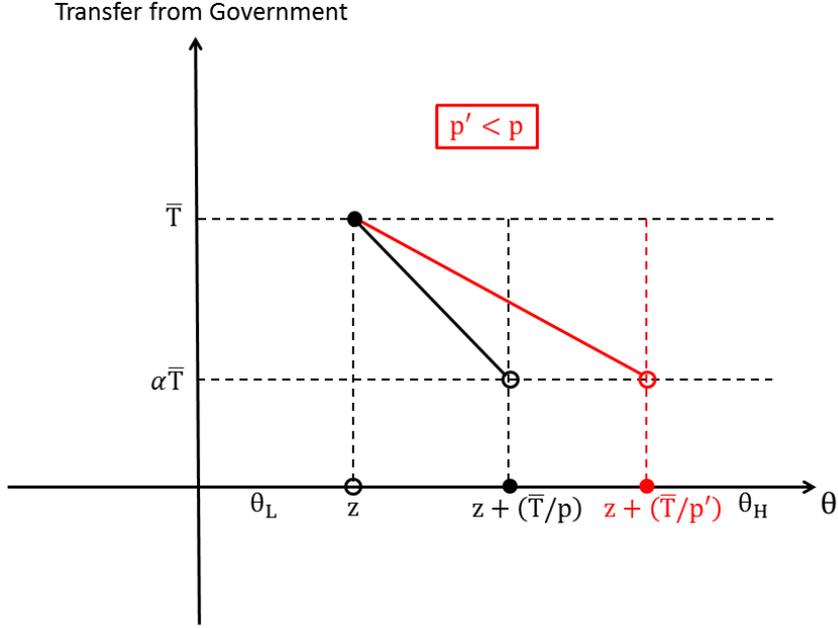


Figure 5: Effect on the transfer from reducing the toll ($p' < p$), for fixed z

choose their capital structure by choosing a state (z) in which all of \bar{T} is needed to clear their debts. For states $\theta \in (z, z + (\bar{T}/p)]$, revenue is higher so not all of \bar{T} is required for debt repayment. The firm can only obtain fraction α of the remainder through bargaining. Once revenue is high enough to repay all debt, the firm's claims on the Government vanish. Thus, the expected payment that the firm can obtain depends on how quickly revenue rises with θ . The faster revenue increases with the state, the lower the expected transfer to the firm. For our demand structure, lower prices reduce the rate at which revenue increases with θ (see Figure 6).

From the geometry of Figure 4 it is clear that a firm which aims to maximise its payment from the Government would choose $p = 0$ so that toll revenue is the same in each state (i.e., zero). This firm would borrow exactly \bar{T} so that it is bailed out in every state (hence it pays the risk-free rate of interest) and extracts \bar{T} with probability one.¹⁷ Choosing $p = 0$ would also maximise the firm's chances of winning the auction. However, it cannot be an equilibrium for all firms to bid $p = 0$, since we assumed that $\bar{T} < \underline{c}$ so

¹⁷The firm effectively sets $z = \theta_L$.

the winning firm would make a loss with certainty.¹⁸ There is a trade-off between maximising revenue from hold-up and maximising toll revenue, and the appropriate balance must be struck at a strictly positive toll. However, since the hold-up incentive reinforces the incentive to set p low to win the auction, we expect more aggressive bidding than in the absence of strategic leverage (i.e., when $\bar{T} = 0$).

In summary, given its toll bid p , the winning firm chooses its debt level D to maximise the expected transfer from Government. In choosing its bid, the firm trades off three effects: the probability of winning, the toll revenue and the (D -maximised) Government transfer. This contrasts with the more familiar two-way trade-off – between the probability of winning and the toll revenue – when bids are cash financed. It also leads naturally to the following typology of equilibria: “low toll” equilibria in which all types bid so low that renegotiation is certain and all profit is from transfers, “high toll” equilibria in which all types are solvent with positive probability, and “mixed equilibria” in which some (high cost) types derive all profit from negotiated transfers and other (low cost) types have two sources of expected revenue: tolls and transfers. Note that a “low toll” equilibrium can only arise if $\alpha > 0$, otherwise the Government transfer is just enough to pay debt in every bail-out state and the firm earns zero profit.

We also observe that the effect of p on transfer revenue is not smooth, creating the potential for a non-differentiability in the equilibrium bidding function for “mixed equilibria”. The derivative of the expected transfer with respect to p will change discontinuously at the p value satisfying

$$z + \frac{\bar{T}}{p} = \theta_H \quad \Leftrightarrow \quad p = \frac{\bar{T}}{\theta_H - z}.$$

At this p value, the expected transfer will rise more slowly as p falls than it falls as p rises: see Figure 5.¹⁹

¹⁸Even without this assumption, the firm could make no more than zero profit from such a strategy. It would receive \bar{T} from the Government in every state and this would be just enough to cover its debts.

¹⁹The first-order condition for a local maximum of (4) is

$$\begin{aligned} (1 - \alpha)p \left[G \left(\min \left\{ z + \frac{\bar{T}}{p}, \theta_H \right\} \right) - G(z) \right] - \bar{T}g(z) \\ + \mathbb{I} \left[z + \frac{\bar{T}}{p} < \theta_H \right] (\alpha \bar{T}) g \left(z + \frac{\bar{T}}{p} \right) = 0 \end{aligned} \quad (5)$$

or else

$$(1 - \alpha)p [G(\theta_H) - G(z)] \in \left[\bar{T}g(z) - (\alpha \bar{T}) g \left(z + \frac{\bar{T}}{p} \right), \bar{T}g(z) \right] \quad (6)$$

It remains to determine the optimal choice of z for each $p > 0$ and hence to determine the equilibrium bidding function. To keep the analysis as simple as possible, we consider a special case of our model.

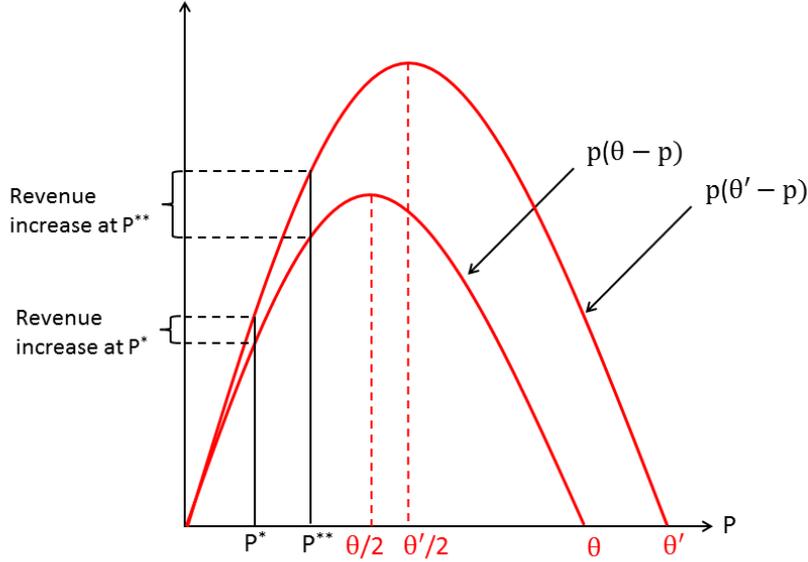


Figure 6: Revenue increase ($\theta \rightarrow \theta'$) versus toll

3.2 The uniform case

Suppose that θ is uniformly distributed on $[\theta_L, \theta_H]$. For this case, it is easy to compute the optimal debt level for each bid p .

Lemma 1 Let $p > 0$ be given. If θ is *uniformly* distributed, then it is optimal for the firm to choose $z = \theta_L$.

Proof It is evident from Figure 4 that the firm is indifferent about which $z \in [\theta_L, \theta_H - (\bar{T}/p)]$ to choose.²⁰ If $\theta_H - (\bar{T}/p) < \theta_L$ then the firm strictly prefers to set $z = \theta_L$. It is therefore without loss of generality to suppose that all firms choose $z = \theta_L$.

where $\mathbb{I}[\cdot]$ is an indicator function that takes value one if the specified condition holds and zero otherwise. Condition (6) is the FOC for a maximum at the “kink” in (4).

²⁰Assuming that $p \leq \theta_L$.

It follows that creditors are exposed to zero default risk, so $r = 0$ and debt satisfies

$$D = \bar{T} + p(\theta_L - p).$$

Note that firms which bid higher toll rates have a (weakly) lower probability of receiving a Government bail-out,²¹ as well as a lower expected payment from the Government.

A firm that bids p (and chooses its debt level optimally) receives an expected payment from the Government equal to

$$\begin{aligned} & \left[\bar{T} - \frac{1}{2}(1 - \alpha) \min \{ \bar{T}, p(\theta_H - \theta_L) \} \right] \left(\min \left\{ \frac{\bar{T}}{p(\theta_H - \theta_L)}, 1 \right\} \right) \\ &= \begin{cases} \frac{(1+\alpha)\bar{T}^2}{2p(\theta_H - \theta_L)} & \text{if } \bar{T} < p(\theta_H - \theta_L) \\ \bar{T} - \frac{1}{2}(1 - \alpha)p(\theta_H - \theta_L) & \text{if } \bar{T} \geq p(\theta_H - \theta_L) \end{cases} \end{aligned} \quad (7)$$

Next, we characterise equilibrium behaviour. Given the non-differentiability in (7), we must be careful about assuming differentiability of the equilibrium bidding function. There are two cases in which differentiability is plausible: (i) in a *low toll* equilibrium where bids are strictly increasing in c and bounded above by $\bar{T}/(\theta_H - \theta_L)$, and (ii) in a *high toll* equilibrium where bids are strictly increasing in c and bounded below by $\bar{T}/(\theta_H - \theta_L)$.

3.2.1 A low toll equilibrium

Note that in a low toll equilibrium all of the winning firm's profit is obtained from exploiting the hold-up problem. If no type bids above $\bar{T}/(\theta_H - \theta_L)$, then the winning bidder is bailed out in every state.

Suppose that \hat{p} is a differentiable and strictly increasing equilibrium bidding function, with $\hat{p}(\bar{c}) \leq \bar{T}/(\theta_H - \theta_L)$. Let

$$\hat{b}(c, \theta) = \hat{p}(c)(\theta - \hat{p}(c)).$$

If firm i bids $\hat{p}(c)$ and chooses its debt level optimally, its expected payoff (conditional on winning the auction) will be

$$\hat{b}(c, \bar{\theta}) - c_i + \bar{T} - \frac{1}{2}(1 - \alpha)\hat{p}(c)(\theta_H - \theta_L).$$

We therefore define

$$\hat{\beta}(c) = \hat{b}(c, \bar{\theta}) + \bar{T} - \frac{1}{2}(1 - \alpha)p(\theta_H - \theta_L)$$

²¹Hence a (weakly) higher probability of solvency.

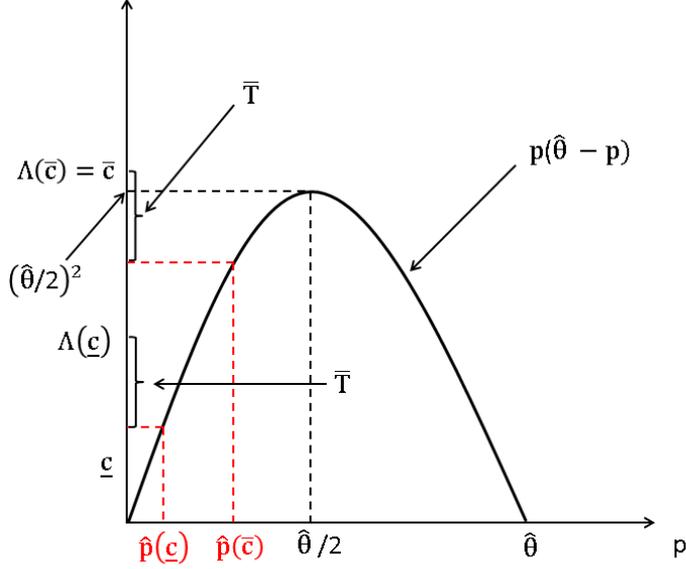


Figure 7: Low toll equilibrium with uniform θ distribution

such that, for an efficient bidding mechanism, c_i solves

$$\max_{c \in [\underline{c}, \bar{c}]} \left[\hat{\beta}(c) - c_i \right] \bar{F}^{n-1}(c).$$

By a similar argument to that used previously, we deduce $\hat{\beta}(c) = \mathbb{E}[X \mid X > c]$ and hence

$$\hat{b}(c, \bar{\theta}) = \mathbb{E}[X \mid X > c] - \bar{T} + \frac{1}{2}(1 - \alpha)\hat{p}(c)(\theta_H - \theta_L)$$

for all $c \in [\underline{c}, \bar{c}]$.

To find $\hat{p}(c)$, it is necessary to solve

$$\begin{aligned} p(\bar{\theta} - p) + \bar{T} - \frac{1}{2}(1 - \alpha)p(\theta_H - \theta_L) &= \Lambda(c) \\ \Leftrightarrow p(\hat{\theta} - p) &= \Lambda(c) - \bar{T} \end{aligned} \quad (8)$$

where $\hat{\theta} = \hat{\pi}\theta_H + (1 - \hat{\pi})\theta_L$ and

$$\hat{\pi} = \pi - \frac{1}{2}(1 - \alpha).$$

We can solve (8) provided $\hat{\theta} \geq 0$ and

$$\left(\frac{\hat{\theta}}{2}\right)^2 \geq \bar{c} - \bar{T} \quad (9)$$

(i.e., $\hat{\theta} \geq 2\sqrt{\bar{c} - \bar{T}}$) – see Figure 7. The solution is valid provided

$$\hat{p}(\bar{c}) \leq \frac{\bar{T}}{(\theta_H - \theta_L)} \quad (10)$$

(the “low toll” condition). From Figure 7 we see that conditions (9) and (10) will be met, for a given value of $\bar{T}/(\theta_H - \theta_L)$, provided $\bar{c} - \bar{T}$ and $\hat{\theta}$ are sufficiently low. The latter requires that θ_L is low, $\pi(\theta_H - \theta_L)$ is low or α is low (i.e., weak demand or weak bargaining power for the concessionaire). In particular, a low toll can partially offset the effects of a weak bargaining position for the firm – recall the discussion of Figure 6.

3.2.2 High toll equilibrium

Let \hat{p} be a differentiable and strictly increasing equilibrium bidding function, with $\hat{p}(\underline{c}) \geq \bar{T}/(\theta_H - \theta_L)$. Since no type bids below $\bar{T}/(\theta_H - \theta_L)$ and \hat{p} is strictly increasing, the winning firm is solvent with positive probability. If firm i bids $\hat{p}(c)$, its expected payoff (conditional on winning the auction) is

$$\hat{b}(c, \bar{\theta}) = c_i + \frac{(1 + \alpha)\bar{T}^2}{2\hat{p}(c)(\theta_H - \theta_L)}.$$

Note that the presence of a hold-up problem gives stronger incentives to lower the toll price than in the low-toll equilibrium. Once the toll reaches $\bar{T}/(\theta_H - \theta_L)$, marginal (hold-up) incentives for further toll reductions are constant at

$$\frac{1}{2}(1 - \alpha)(\theta_H - \theta_L).$$

Proceeding similarly to the previous section, we have

$$\hat{b}(c, \bar{\theta}) = \mathbb{E}[X \mid X > c] - \frac{(1 + \alpha)\bar{T}^2}{2\hat{p}(c)(\theta_H - \theta_L)}$$

for all $c \in [\underline{c}, \bar{c}]$, with $\hat{p}(c)$ the solution to

$$p(\bar{\theta} - p) + \frac{(1 + \alpha)\bar{T}^2}{2p(\theta_H - \theta_L)} = \Lambda(c)$$

$$\Leftrightarrow p(p^2 - \bar{\theta}p + \Lambda(c)) = \frac{2(\theta_H - \theta_L)}{(1 + \alpha)\bar{T}^2} \quad (11)$$

The roots of $p^2 - \bar{\theta}p + \Lambda(c)$ are

$$\frac{\bar{\theta}}{2} \pm \sqrt{\left(\frac{\bar{\theta}}{2}\right)^2 - \Lambda(c)},$$

so we have the situation depicted in Figure 8. A local maximum must occur on a downward sloping portion of the cubic, so the middle of the three intersections in Figure 8 is the only viable candidate for the equilibrium bid of type c .

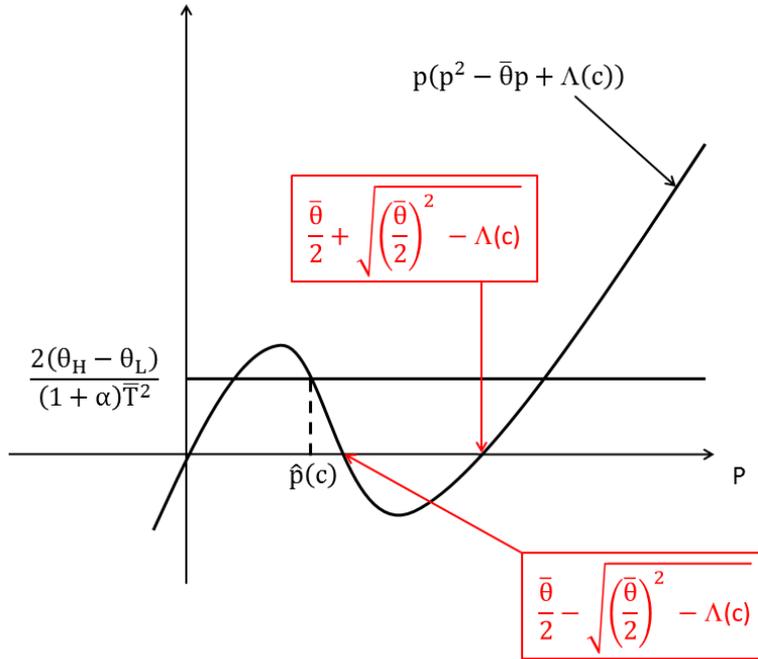


Figure 8: High toll equilibrium

Note (Figure 9) that \hat{p} is strictly increasing in c as assumed. It follows that a high toll equilibrium exists only if the local maximum value of the cubic

$$p(p^2 - \bar{\theta}p + \Lambda(c)) \quad (12)$$

is at least

$$\frac{2(\theta_H - \theta_L)}{(1 + \alpha)\bar{T}^2},$$

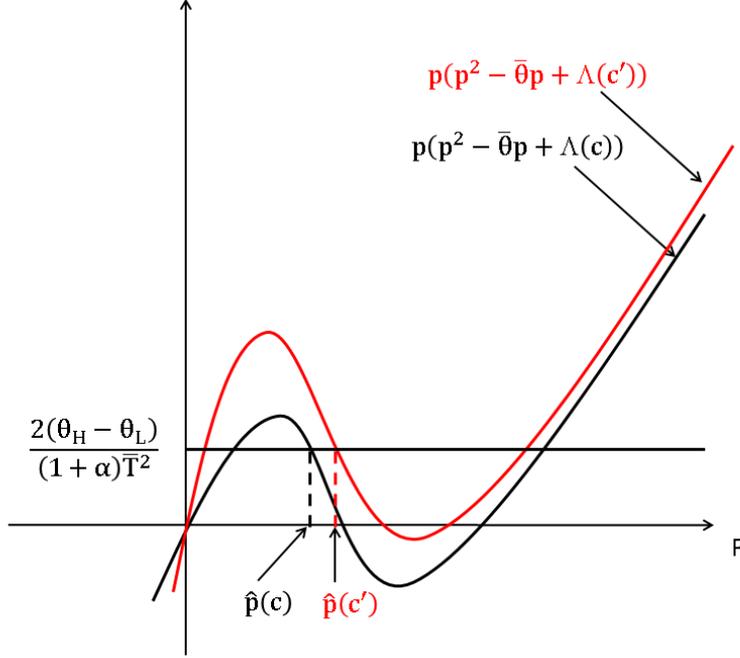


Figure 9: High toll equilibrium ($c' > c$)

and $\hat{p}(\underline{c}) \geq \bar{T}/(\theta_H - \theta_L)$ (the “high toll” condition). See Figure 10. To satisfy these conditions, it suffices, for a given value of $\bar{T}/(\theta_H - \theta_L)$, to choose $\bar{\theta}$ high enough so that the local maximum of (12) occurs (at or) above $p = \bar{T}/(\theta_H - \theta_L)$, and $(1 + \alpha)\bar{T}$ high enough that

$$\frac{2(\theta_H - \theta_L)}{(1 + \alpha)\bar{T}^2}$$

is below the value of (12) at its local maximum. That is, we require strong demand, high returns from hold-up and a strong bargaining position for the firm.

It should be noted that there is one further necessary condition for equilibrium existence. The middle solution to (11) must not be dominated by a price at or below $\bar{T}/(\theta_H - \theta_L)$. A bid in the latter range guarantees that the firm will win the auction (in a high toll equilibrium) and be bailed out in every state. Thus, we require that

$$\max_{p \leq \bar{T}/(\theta_H - \theta_L)} p(\hat{\theta} - p) + \bar{T} - c \leq [\Lambda(c) - c] \bar{F}^{n-1}(c)$$

for all $c \in [\underline{c}, \bar{c}]$. This will be so provided $\bar{T}/(\theta_H - \theta_L)$ is sufficiently low.

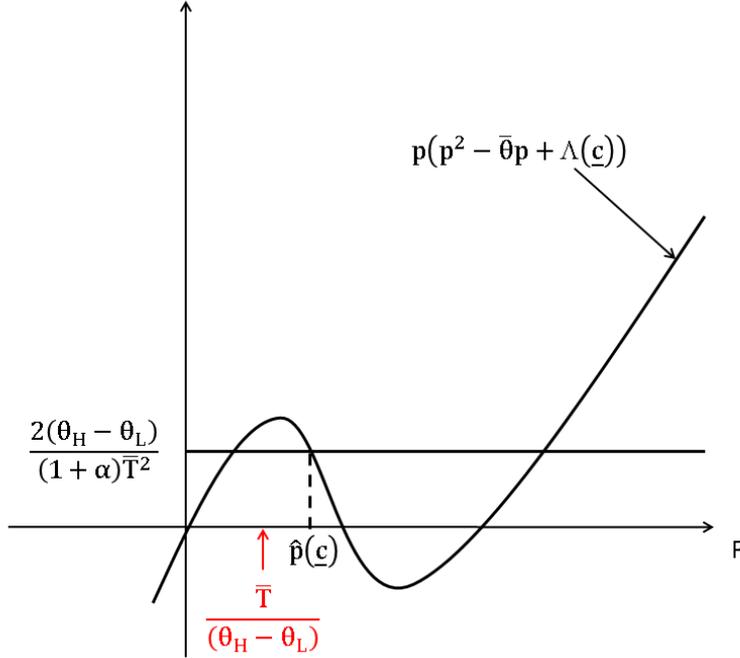


Figure 10: Existence of high toll equilibrium

3.2.3 Mixed equilibrium

The model with a uniform θ distribution also admits equilibria that are mixtures of the “low toll” and the “high toll” variety. In these “mixed” equilibria, the bidding function is non-differentiable at a toll equal to $\bar{T}/(\theta_H - \theta_L)$. It resembles the “low toll” equilibrium below this value and the “high toll” equilibrium above.

3.3 Summary

Firms choose debt levels strategically in order to hold-up the Government, so the winning firm will threaten default with positive probability. Under our demand structure, a firm that bids lower is able to extract a higher expected bail-out from the Government. Provided the equilibrium bidding function is strictly increasing, this means that more efficient firms make higher demands on the public purse (and hence charge very low tolls).

We computed the symmetric equilibrium bid function when θ is uniformly distributed and verified that it is strictly increasing. In this case, firms never actually go bankrupt, but each firm i credibly threatens default in an interval

of states of the form $[\theta_L, \theta_i^*]$ for some $\theta_i^* \in (\theta_L, \theta_H]$. When $\theta_i^* = \theta_H$ firm i threatens default in all states, so its return on equity comes entirely from Government transfers. Depending on parameters, we may observe a “low toll” equilibrium, in which toll bids are so low that $\theta_i^* = \theta_H$ for every firm; a “high toll” equilibrium in which $\theta_i^* < \theta_H$ for every firm; or a “mixed equilibrium” in which more efficient firms are solvent in some states while less efficient firms threaten default in all states.

Weak demand or weak concessionaire bargaining power encourage “low toll” equilibria, while strong demand or strong bargaining power encourage “high toll” equilibria. The link between demand and tolls is natural, though the correlation between bargaining strength and equilibrium toll levels may seem counter-intuitive. The latter arises because toll reductions are a partial substitute for bargaining strength. Under our demand structure, lower tolls allow the firm to credibly threaten bankruptcy in more states and thereby increase the returns from hold-up.

3.4 Variations and extensions

Let us briefly discuss the robustness of these conclusions.

The linear demand structure is not critical. Figure 6 reveals the properties of demand that drive the result that more efficient winners extract a higher expected bail-out. The key property is that higher demand states (i.e., higher values of θ) increase the sensitivity of revenue to price increases. Any demand structure in which the unknown demand parameter has this effect should produce qualitatively similar features. Since more efficient firms bid lower tolls, their revenue rises less rapidly with θ so they require larger bail-outs over a larger range of demand states.

Alternative θ distributions will obviously change the equilibrium in complex ways. However, one change is easily predicted and is in the direction of greater realism. With a Uniform distribution for θ , all firms choose debt levels such that they are bailed out in the worst demand state (θ_L) – none is allowed to fail. With a non-Uniform distribution – especially one with low density in the neighbourhood of θ_L – this is unlikely to be the case. Instead, firms will face some positive probability of going bankrupt at the optimal level of debt. Hence they will pay above the risk-free rate on their debt, and this rate is likely to vary depending on the firm’s toll bid. Note, however, that this will not alter the fact that more efficient winners extract a higher expected bail-out. This is clear from Figure 4. More efficient firms, who bid lower tolls, can always achieve a higher expected transfer from the Government.

We have assumed a first-price auction structure throughout, as this con-

forms more closely with practice (World Bank, 2008). Nothing of significance would change if we assumed a second-price structure – but nor would we obtain any significant simplification. Under a second-price structure each firm would bid so that its total revenue – tolls plus transfers – matches its construction cost. But the complicated process of “inverting” this total revenue to obtain the implied toll bid would be the same as in the present analysis.²²

There is also another reason to think that Governments might prefer first-price auctions. Hansen (1988) showed that these may yield lower expected supply prices than second-price auctions. His model differs from ours, but the same conclusion is easily shown, at least for the benchmark case of cash-financed bidding. From Figure 1 we see that the toll is a strictly increasing and *convex* function of revenue. Let τ denote this function. In a first-price auction, when a type c firm wins it charges a toll equal to

$$\tau(\mathbb{E}[X \mid X > c]) \tag{13}$$

where X is the random variable corresponding to the lowest of $n - 1$ random draws from the construction cost distribution F . In a second-price auction, if type c wins it charges an expected toll equal to

$$\mathbb{E}[\tau(X) \mid X > c]$$

which is higher than (13) by Jensen’s inequality.

Finally, one might consider introducing a reserve price into the auction. Of course, firms already bid unrealistically low tolls in anticipation of additional transfers. A reserve price might still be considered, either to exclude firms whose implied costs are too high (relative, say, to a public sector comparator), or to further reduce the toll charged by the winning firm. However, the gains from the latter must now be offset against the increase in the expected transfer that will be paid when the toll is reduced. Hold-up is likely to dull the incentives to use reserve prices to manage procurement costs.

4 Discussion

We have presented a model in which to study the equilibrium financial structure and equilibrium bids of firms competing for a PPP contract (highway concession). Our motivation was to better understand the prevalence of default and renegotiation in such PPPs. Faced with such evidence, one might

²²However, if we were to allow firms to differ in maintenance costs as well as (or instead of) construction costs, then the analysis in Hansen (1988) suggests that a second-price structure *would* yield significant simplification.

reasonably question whether the PPP auction has awarded the contract to the most efficient firm (cf, Spulber, 1990; Zheng, 2001). Our results suggest that there is no reason to question the efficiency of the auction. Bidders use debt strategically to partially insure against low demand states, by credibly threatening default and triggering a re-negotiation. Bids are therefore lower than under unlimited liability, but the contract is still awarded to the most efficient firm. The most surprising result is that probability and size of the bail-out is increasing in the efficiency of the winning firm.

These insights complement the existing PPP literature and provide a theoretical framework to understand the relationship between the PPP tender, high leverage ratios and default in PPP contracts. This relationship is highlighted, for example, by a recent report published by the Department of Infrastructure and Transport (2012) that surveys the empirical literature on PPPs for toll road concessions. The report links overbidding (that is, bids which appear to over-estimate the value of the contract), and the subsequent renegotiation or default, to a number of characteristics, including financial structuring focused on high leverage and debt maximisation.

Our results then offer a number of implications for PPP design. First, they reinforce doubts about the ability of the Government to effectively transfer risk to the private partner. In this respect, our paper complements the work of Engel, Fischer and Galetovic (2001, 2008) who propose a novel auction mechanism to provide better demand-side risk management and to mitigate the need to renegotiate contracts. These authors introduce the notion of a variable term concession implemented through a Least-Present-Value-of-Revenue (LPVR) auction. Under this mechanism, the auction allocates the contract to the bidder who bids the lowest present value of expected revenue. The Government specifies the toll for each demand state before bids are taken. The duration of the contract is endogenously determined, since the contract specifies that the concession remain in place until the firm recovers its bid. The LPVR auction thus fully insures the winning bidder, which ensures efficient risk-sharing, since bidders are risk-averse in the model of Engel, Fischer and Galetovic. Bids then simply reflect construction cost.

Engel, Fischer and Galetovic (EFG) assume that is possible to find a mechanism that avoids further renegotiation by fully transferring the risk of default to the Government. In the absence of commitment, the Government must ensure non-negative PVR in every state, not just in expectation. The Government effectively renegotiates in advance: setting the toll and concession duration as a function of the state, so there is no need to re-negotiate *ex post*.

Thus, EFG consider a more complete contract than in our model. We assume that it is politically infeasible for Governments to commit to transfers

or toll increases in advance. Since most highway concessions have in fact been allocated on the basis of very incomplete contracts – bids are typically non-contingent tolls or concession durations – it is important to understand the properties of such PPP auctions, and the interaction of financing choices and renegotiation in the outcomes of such contractual arrangements. The present paper is a first step in this direction.

A second implication for PPP design emerging from this paper relates to the desirability to restrict bidders' financial structure. High equity requirements can ameliorate the hold-up problem – by forcing debt levels down so that default is not a credible threat – but at the cost of reducing participation from cash-constrained bidders. Although such restrictions on the financial structure of bidders might sound intrusive, there are parallels both in regulation and taxation. Regulated firms are usually allowed a rate of return on capital based on a capital structure that is deemed efficient, and which may vary from the regulated firm's actual capital structure. Similarly, there are thin capitalisation rules that are designed to prevent firms from issuing too much debt in a particular tax jurisdiction to take advantage of the tax deductibility of interest.

Finally, the analysis above suggests that there might be room for auction design to mitigate the incentives to choose debt strategically and to minimise the prevalence of renegotiation. A first step is to consider auctions in the LPVR spirit, in which firms bid the present value of revenue and choose the toll *ex post*. The analysis of such auctions, with endogenous financing, are a useful subject for future research.

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