Choice Set Generation Algorithm Suitable for Measuring Route Choice Accessibility

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A new algorithm that generated a set of paths between a pair of origin–destination nodes in a transportation network for the purpose of generating a measure of accessibility on the level of route choice was designed, developed, and tested. The proposed algorithm incorporated the well-known issue of path overlap in the process of generating the path choice set. This algorithm fit naturally into the class of iterative penalty-based *K*th-shortest-path algorithms; in this class the link penalty terms are designed to reflect the amount of overlap between the paths already generated. With the proposed algorithm, paths were generated in order of decreasing utility and corrected by a path size correction factor; it was thus highly efficient in the sense that a comparatively small number of paths could result in a broad spectrum of desirable choices. The algorithm was developed in response to the Valencia paradox, which arose from using logsums from the existing algorithm for choice set generation as a route-level accessibility measure for the bicycle network in San Francisco, California. The Valencia paradox occurs when an accessibility measure decreases following an improvement to actual network accessibility. A detailed case study demonstrated the effectiveness of the proposed algorithm in minimizing this kind of paradoxical result and generating a route-level accessibility measure suitable for making fine-grained planning decisions.

The concept of accessibility is widely used in travel demand modeling as a way of communicating detailed level-of-service information from one level of a model to another. Accessibility information from mode choice and destination choice models, often in the form of logsums [as formulated by Ben-Akiva and Lerman in 1985 (*1*) and commonly used as in work by Bradley et al. in 2010 (*2*)], can be used to inform other higher-level models such as auto ownership or total travel demand.

A new choice set generation algorithm for route choice models and a resulting measurement of network accessibility are proposed here. The algorithm was implemented into the bicycle route choice component of the SF-CHAMP regional activity–based model (*3*). The resulting accessibility measure captures both preference heterogeneity and marginally suboptimal paths and is suitable both for feeding back into other components of the SF-CHAMP model and for accessibility analysis in its own right. The algorithm substantially mitigates a significant impediment to using a route choice logsum as an accessibility measure, namely, that the process of correcting for overlapping paths in the path size logit model commonly used for route choice model estimation can lead to logsums that decrease despite network performance improvements (*4*).

The rest of this paper is organized as follows. First, there is a discussion of the motivation behind using a network-based accessibility measure as opposed to a single best path. Second, the previous route choice implementation is described and the Valencia paradox is introduced. Then a review of route choice methodologies is presented. Next the path size penalty algorithm (PSPA) is introduced as a proposed solution to the Valencia paradox, followed by a case study application demonstrating PSPA's usefulness in planning decisions and then a few comments about the algorithm's computational needs.

MOTIVATION AND BACKGROUND

Network-Based Accessibility

The goal in measuring accessibility at the level of route choice is to capture network effects that are not apparent from other aspects of the model. Specifically, the research team believes that a more resilient network that offers multiple high-utility routes for many origin–destination (O-D) pairs offers value over a network with fewer feasible routes. The team believes that it is desirable for the model to reflect both the heterogeneity of choices valued by different users and the increased utility that comes from improvements made not only to the highest-utility routes but to secondary routes as well. All of these goals can be served by using route choice–level accessibilities as part of the model feedback.

Historically, the role of a route choice model within an activity-based model has been solely to measure a single best path and to ignore marginally less optimal options despite great levels of variability among individual preferences. Hood et al. found significant variations in route preferences among both user types and travel purpose (*5*). One approach to incorporating route-level information into an activity-based model is the nonlabeled mode approach described by Stratton et al., in which route choice preferences are simulated on an individual level in a combined mode choice and route choice model (*6*). The current research takes a different approach by generating a route-level accessibility measure that can then be fed back into other model components such as a traditional mode choice model.

Furthermore, a single-path measurement of accessibility for a specific mode cannot capture the potential network effects, or the value of having multiple good options. Specifically, a more resilient network that offers multiple high-utility routes for many O-D pairs should be preferred over a network with fewer feasible routes for a number

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of reasons. First, absent a network that performs identically 100% of the time, different routes will be preferred at different times on the basis of their performance at any given time. (For example, if there is construction on Street A, users will value the existence of Street B as an alternative.) Second, although most network algorithms assume that users have perfect knowledge of all of their available options and will pick the optimal one, it is likely that a not-insignificant number of users will pick a marginally suboptimal route either because they do not have perfect information or because they cannot perceive the difference. Failure to calculate the performance of the group of likely routes results in performance calculations that are blind to the conditions that users are actually experiencing. Finally, network resilience has system-level value in and of itself (*7*) and can allow the network to function acceptably even when it has undergone a severe disruption such as a natural disaster.

Previous Implementation

The SF-CHAMP activity-based model currently includes a bicycle route choice model described in detail by Hood et al. (*5*). A brief overview of some of the model features that will be relevant to the current research follows. The model was developed from revealed-preference data collected by the CycleTracks smartphone app (*8*). The model is a path size logit model (about which more will be said in the next section) and model estimation used choice sets generated by a doubly stochastic algorithm as formulated by Bovy and Fiorenzo-Catalano (*9*) and described by Hood et al. (*5*). Relevant variables in the bicycle utility equation include the distance traveled on links with each of four types of bicycle infrastructure (none, separated bicycle paths, bike lanes, and bike routes); the elevation gain on the link (nonnegative, e.g., downhill is equivalent to flat); whether the cyclist is traveling the wrong way down a one-way link; and the number of turns. The precise utility equation is

$$V_{i,n} = \beta_0 X_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3 + \beta_r X_r + \beta_w X_w + \beta_T T \quad (1)$$

The coefficient values and corresponding variables for one class of users are given in the following list:

β_0 = –1,
β_1 = –0.57,
β_2 = –0.49,
β_3 = –0.92
β_r = –59.14,
β_w = –4.02,
β_T = –0.11,
X_0 = distance (miles) on links with no bike facility,
X_1 = distance (miles) on links with a Class 1 bike facility (separated path),
X_2 = distance (miles) on links with a Class 2 bike facility (bike lane),
X_3 = distance (miles) on links with a Class 3 bike facility (bike route),
X_r = rise (miles) on each link, nonnegative,
X_w = distance (miles) on wrong-way links, and
T = number of turns.

The accessibility (A) for an individual n of an O-D pair (o, d) is

$$A_{n,o,d} = \log \sum_{i \in C_n} e^{U_{i,n}} \quad (2)$$

where $U_{i,n}$ is the perceived utility associated to path i by individual n and C_n is the choice set generated for individual n by the route choice model.

Although the initial version of the bicycle route choice model performs well on the whole, sensitivity testing of the accessibility measure revealed some paradoxical results on a fine-grained level. Specifically, the model's sensitivity to bike lanes was tested along Valencia Street, in San Francisco, California. This facility, created in the late 1990s, has since become one of the most heavily used bicycle facilities in the city. Researchers compared the calculated bicycle accessibility of a location in downtown San Francisco (marked with a star in Figure 1) from all origins within the city, with and without the bike lanes. Although most origins near the improved facility reflected the improvement in the form of improved accessibility, certain origins actually showed a small but noticeable decrease in accessibility. This phenomenon, dubbed the "Valencia paradox," is shown in Figure 1. Increased accessibility is shown in shades of gray, with darker shades indicating a larger increase. Decreased accessibility is shown as dots of varying sizes, with larger dots indicating a larger decrease. Valencia Street is the north–south street marked with a heavy black line.

Initial investigation of the paradox revealed that neither filtering the choice sets, as suggested by Bovy and Fiorenzo-Catalano (*9*), nor using a different variant of the path size correction factor, as suggested by Bekhor et al. (*10*), resolved the paradox. Further investigation revealed that the root of the paradox lay in the diversity of the choice sets produced by the doubly stochastic algorithm. Figure 2 shows the choice set of 96 paths found by the doubly stochastic algorithm, both with and without the Valencia bike lanes, for the origin with the largest decrease in accessibility. Without the Valencia bike lanes, the doubly stochastic algorithm finds a wide variety of paths, since no one path stands out as considerably more attractive than others. However, with the Valencia bike lanes, the doubly stochastic algorithm finds far fewer paths. Plainly, the reason for this finding is not because there are fewer routes available with the construction of the new facility but rather because the utility of Valencia Street in particular is now so much better than that of surrounding streets that the doubly stochastic algorithm primarily finds routes that use it. Even when the doubly stochastic algorithm is given a larger variance in its coefficient randomizations, the paradox persists.

The proposed choice set generation algorithm in this study largely resolves this paradox by finding the set of routes that has the maximum possible logsum, including the path size correction term. In other words, the algorithm picks the high-utility paths with the knowledge of the existing path overlaps with the paths already in the set. By incorporating the path size correction into the path search algorithm itself, the algorithm does not surprise the modelers with significant accessibility decreases due to path overlap penalties applied after the choice set is generated. Thus, the logsums generated by this algorithm form an accessibility measure that is suitable for making even fine-grained planning decisions.

PATH OVERLAP CORRECTIONS AND ROUTE CHOICE SET GENERATION METHODS

A necessary assumption in using logit formulations for modeling discrete choices is that the error terms are independent and identically distributed. This assumption is violated when it comes to modeling route choice behavior since paths typically have overlaps and consequently are not necessarily independent. A famous example

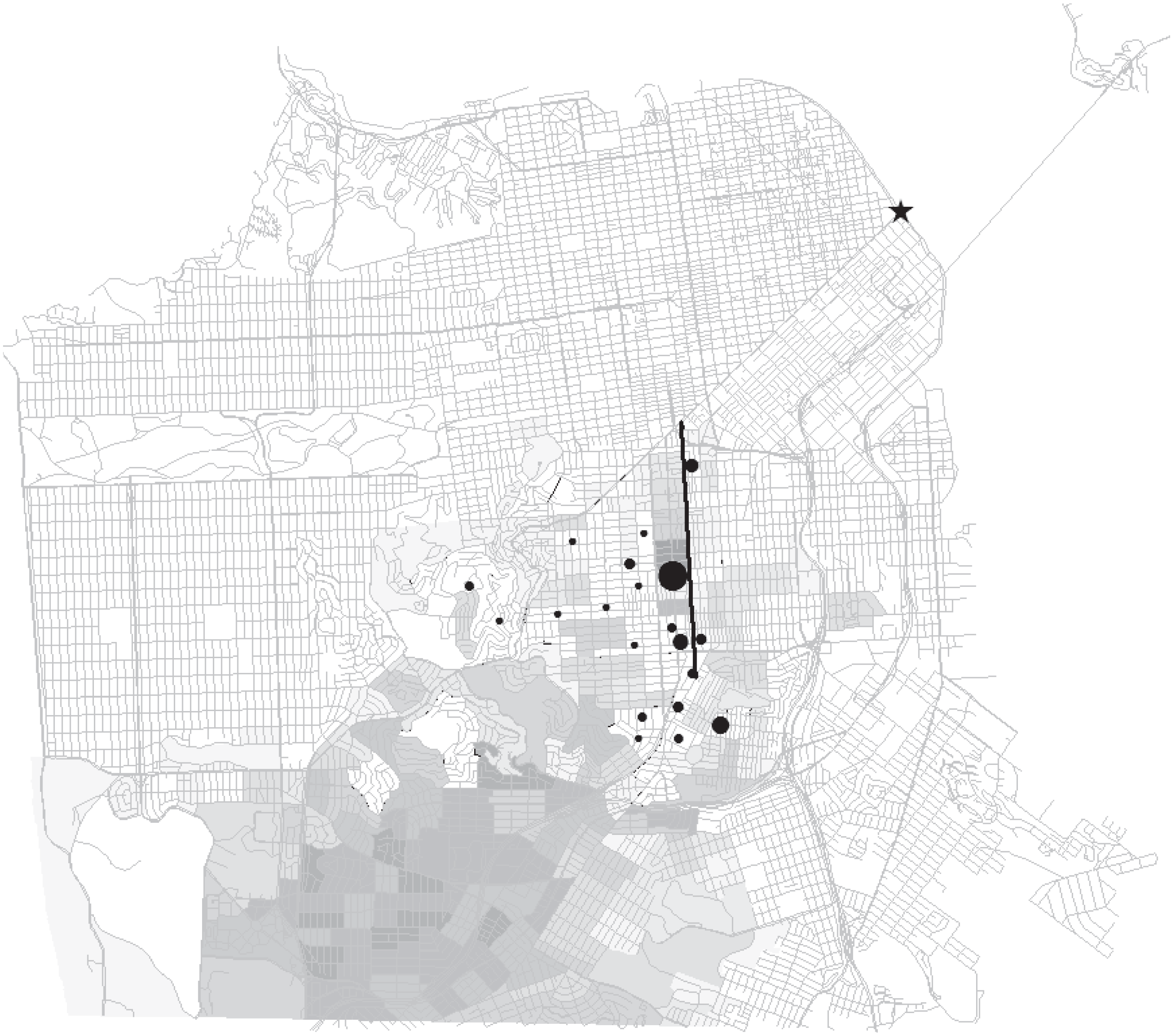


FIGURE 1 Change in accessibility: choice sets generated by doubly stochastic algorithm.

that explains the effect of path overlaps on the perceptions of the travelers uses the three possible paths shown in Figure 3*a.*

With an origin at Node O, destination at Node D, and an intermediate Node I, there are in theory three possible paths imaginable between O and D. However, the perceptions of travelers change when the overlapping distance (or impedance) of link (O, I) changes. One extreme case is when the distance of link (O, I) is small (Figure 3*b*). In this case travelers perceive three independent paths between O and D. The other extreme case (Figure 3*c*) is when the distance of link (O, I) is large. In this case travelers tend to perceive only two independent paths.

This issue has been treated in route choice modeling by introducing correction factors to the utility of overlapping paths. In order to better model the perceptions of travelers, several correction factors have been proposed to reflect the significance of path overlaps on the utility of alternatives. Cascetta et al. proposed a route choice formulation (C-logit) with several potential forms of a commonality factor to adjust the utility of overlapping alternative paths (*11*). Ben-Akiva and Ramming presented the path size logit model. Similar to the C-logit model, the path size logit model adjusts the utility of alternative routes by a path size correction term (*12*). As the amount by which a path overlaps with other alternative paths increases, the path size correction decreases to adjust the utility of the overlapping path. Bovy et al. present a more theoretically appealing path size calculation (PSC), presented in the next section (*13*). A thorough survey of existing approaches to route choice modeling with overlap considerations may be found elsewhere (*14–16*).

There are several choice set generation methods in the literature that aim to generate a realistic set of possible routes that a traveler would actually perceive. These existing methods fall into two general categories: deterministic and stochastic. The deterministic approaches, including the *K*th shortest path, link elimination, and link penalty, are

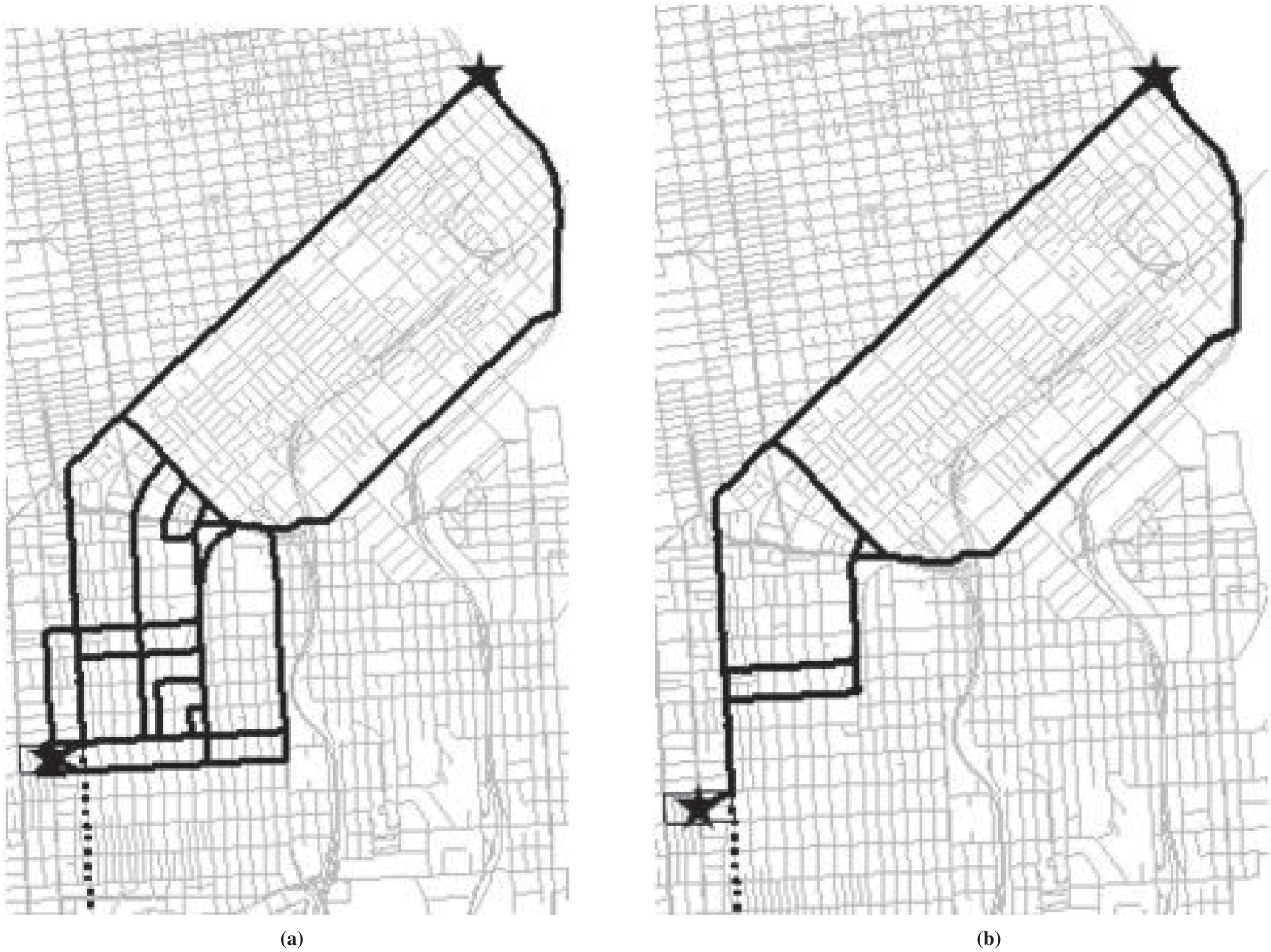


FIGURE 2 Paths found by doubly stochastic algorithm (*a*) without and (*b*) with Valencia bike lanes.

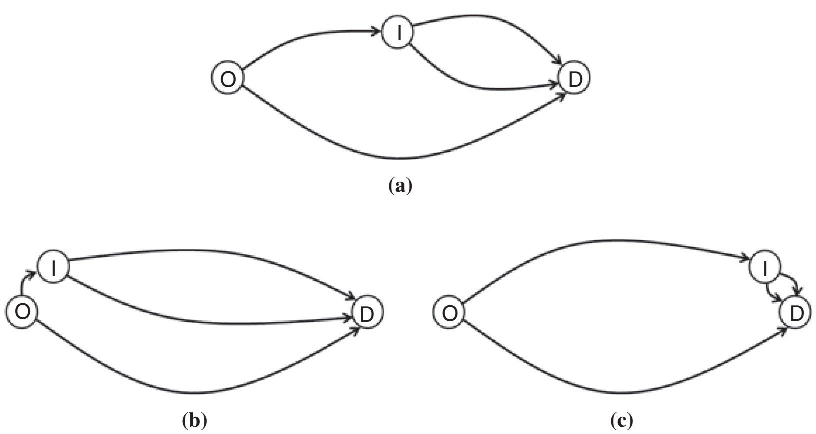


FIGURE 3 Example of path overlaps: (*a*) three possible paths between O and D, (*b*) small distance of link (O, I), and (*c*) large distance of link (O, I).

all based on successive or iterative shortest-path calculations. These methods benefit from the latest advances in fast shortest-path calculations; however, important shortcomings have been identified for each of them. For instance, the link elimination method iteratively finds the shortest path, removes all or some of the links of that path from the network, and then finds a new shortest path until a choice set of the desired size is generated. In this method essential links such as initial access links or bridges may be eliminated from the network after they appear in an identified path; this approach causes unreasonable paths or even infeasibility in the form of network discontinuity. In link penalty approaches, instead of eliminating links from the network, the algorithm when it searches for new paths increases the cost of links of already identified paths via a defined penalty term. Existing link penalty methods typically do not have a theoretically reliable definition for penalty terms. If the penalty terms are too small, the generated paths will end up being very similar or redundant, and the method would be computationally expensive. However, if large penalty terms are chosen, the generated paths could become unrealistic in the same fashion as in the link elimination methods. The current proposed algorithm fits into the link penalty category but is based on a theoretically reliable expression for the penalty terms.

The literature also contains several stochastic approaches to generating the route choice set, such as the doubly stochastic algorithm cited in the previous section. Frejinger et al. (*17*) and Flötteröd and Bierlaire (*18*) have developed a choice set generation algorithm in which paths are chosen as a sample with desired distribution from the full universe of potential paths. This algorithm, as implemented in Bioroute (http://transp-or.epfl.ch/bioroute), was also implemented and tested in this research. However, testing revealed that it would not be a feasible alternative to either the doubly stochastic or the proposed PSPA algorithm for the purpose of generating a route-level accessibility measure because the algorithm ran very slowly on the San Francisco bicycle network, and the generated paths were either too similar or very unrealistic when the algorithm was tuned to get a reasonable distribution from which to sample. The research team's assessment is that although the method appears to work well for generating choice sets for the purpose of model estimation, the choice sets it generates are not useful for the purpose of creating a route choice accessibility measure.

A thorough survey of existing approaches to route choice set generation may be found in the dissertation of Frejinger (*19*).

PATH SIZE PENALTY ALGORITHM

The proposed PSPA in this research generates a set of path choices between an O-D pair in a transportation network, with consideration of path size correction factors for the path overlaps in the generated set. PSPA fits into the category of iterative penalty-based *K*th-shortest path algorithms. PSPA differs from other algorithms in this class in that the proposed penalty term is applied to the link costs before each iteration of the shortest-path calculation. The penalty term is designed to reflect the amount of path overlaps among the paths already generated. The penalty term is applied at each iteration, and paths are selected with the knowledge of the existing path overlaps in the already generated subset of paths.

The proposed penalty term is designed according to the PSC factor proposed by Bovy et al. to account for path overlaps in a path size logit utility expression (*13*). In this proposed model, U_{in} is calculated as follows:

$$U_{in} = V_{in} + \mathrm{PSC}_i + \varepsilon_{in} \tag{3}$$

where

V_{in} = deterministic part of utility, calculated on basis of attributes of path i and individual n;
ε_{in} = random part; and
PSC_i = path size correction factor to account for path overlaps between links of path i and other paths in choice set.

PSC_i is calculated for each path as follows:

$$\mathrm{PSC}_i = -\frac{1}{\mu}\sum_{a\in\Gamma_i}\frac{l_a}{L_i}\ln\sum_{j\in C_n}\delta_{aj} \tag{4}$$

where

Γ_i = set of all links in path i,
l_a = length of link a,
L_i = length of path i,
δ_{aj} = link-path incidence factor (equals 1 if link a is in path j and 0 otherwise), and
μ = scale factor (equals 1 in this research).

By defining PSC_i as in Equation 4, the penalty increases (i.e., the perceived utility decreases) more when the degree of overlap is high among the links in path i and other paths in the choice set.

With a vector of logit utility coefficients $B = [\beta 1, \beta 2, \ldots, \beta k]$ and the matrix of link attribute variables $X_{(k,m)}$, where m is the number of links in the network and k is the number of utility attributes for each link, the systematic utility of each path i, V_i, can be written as follows:

$$V_i = \sum_{a\in\Gamma_i}(BX)_a \tag{5}$$

Next, by defining the generalized cost function $c(a)$ for link a as $c(a) = (BX)_a$, a simple shortest-path calculation (i.e., minimum-cost path calculation) can generate the path with the optimal systematic utility V_i between any arbitrary O-D pair in the transportation network.

This research proposes a penalty term $p(a)$ that adds to the generalized cost $c(a)$ and directs the shortest-path calculation to identify the path that has the largest corrected utility. This penalty term $p(a)$ can be calculated as follows:

$$p(a) = \frac{l_a}{\mu L}\ln\sum_{j\in C_t}\delta_{aj} \tag{6}$$

where

t = index of iteration for PSPA,
C_t = set of generated paths between origin and destination at end of iteration t,
l_a = length (i.e., cost) of link a, and
L = length (i.e., cost) of current shortest path.

With the generalized link cost $c(a)$ and penalty term $p(a)$ the pseudocode for generating T paths is as follows:

Initialization. $t := 1$; $C_t := \varnothing$ or empty set; $L :=$ shortest distance (minimum-cost) path between origin and destination; set the link cost according the generalized cost $c(a)$. The initial penalty term $p(a)$ is 0;

Step 1 (Iteration t). Find the shortest (minimum-cost) path between the origin and destination pair and assign it to path i;

Step 2. $C_t := C_t \cup \{i\}$;

Step 3. For all the links a in path i, update $p(a)$ on the basis of Equation 6; and

Step 4. $t := t + 1$; if $t < T$, go to Step 1, otherwise stop.

The important feature of the PSPA is that by using this algorithm, the generated path i at iteration t is the path with the largest utility, including the path size correction with respect to the paths found during iterations 1 to $i - 1$. Therefore, with the PSPA, the paths are generated in decreasing order of their corrected utility. With this desired character, the modelers are capable of generating a set of N best path choices by running the PSPA for N iterations ($T = N$). However, the path size correction (for each link a in path i) based on penalty term $p(a)$ at iteration $i < T$ is not exactly equal to the PSC_i factor calculated on the basis of the whole set of paths after all T paths are generated because the PSPA is performing on the basis of just the knowledge of the overlaps in the paths generated in previous iterations and not on the basis of the final set of paths. As a result the PSPA is considered to be an approximate algorithm.

CASE STUDY

With the new choice set algorithm in place, the next issue to be addressed is that of using it to generate an accessibility measure. As before, the accessibility for every O-D pair is given in Equation 2.

Figure 4 again shows the difference in accessibility to downtown San Francisco with and without the Valencia Street bike lanes, with the five-path PSPA now used to generate the choice sets. The number and size of accessibility decreases are considerably less; the

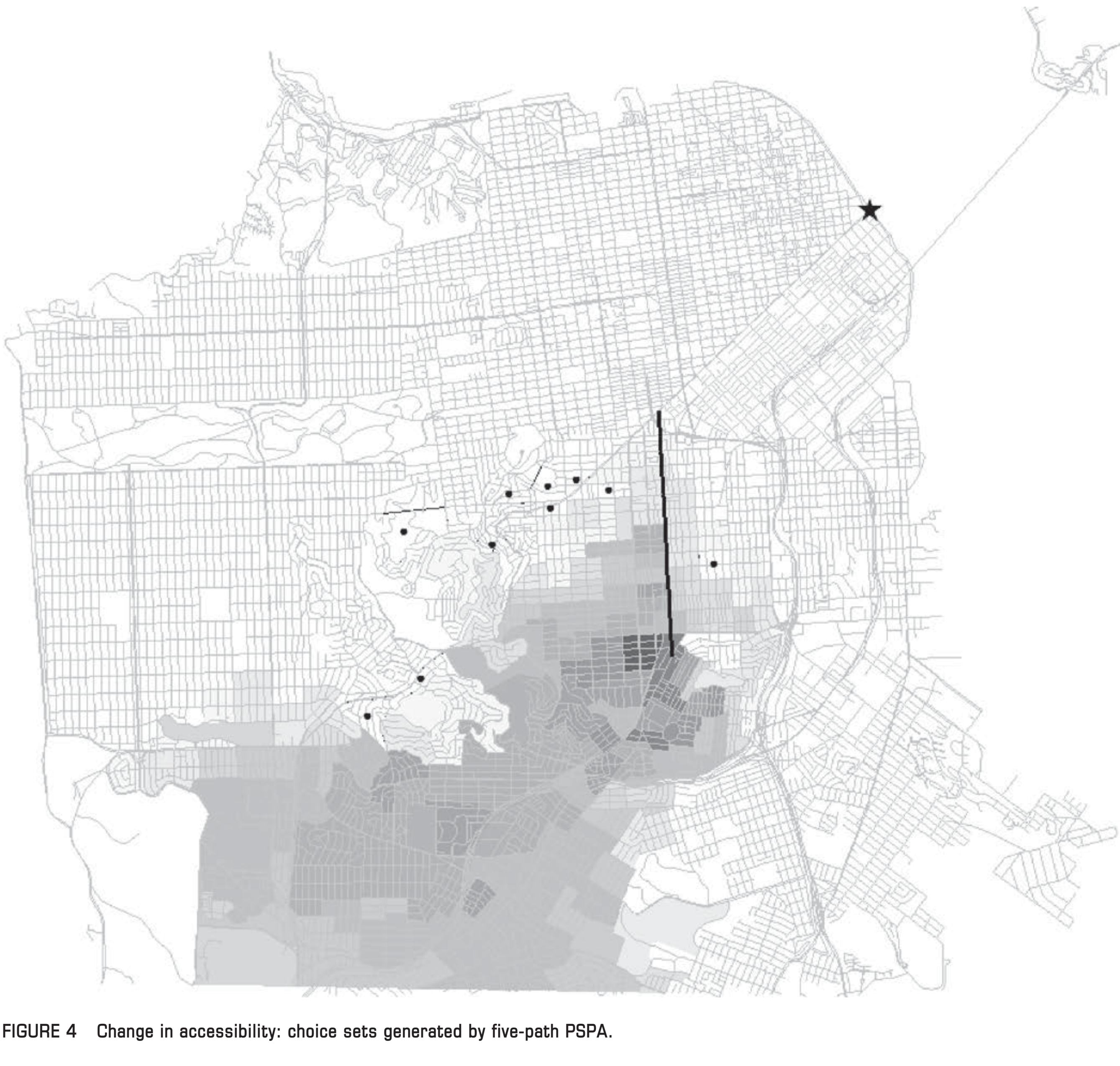


FIGURE 4 Change in accessibility: choice sets generated by five-path PSPA.

(a)

(b)

FIGURE 5 Paths found by five-path PSPA (*a*) without and (*b*) with Valencia bike lanes.

Valencia paradox, though not completely absent, is a much smaller phenomenon. Also, only five paths are needed for each O-D pair in contrast to the 96 paths used with the DS algorithm. Figure 5 shows the actual paths for the same O-D pair as in Figure 3, generated both with and without the bike lane. The diversity of paths in the choice sets is much more comparable between the scenarios than in the double stochastic choice sets.

As previously observed, the PSPA generates paths in order of decreasing utility and incorporates the path size correction. As a result, the PSPA can generate accessibility information with a relatively small number of paths, and the additional practical information gained by generating additional paths becomes small quite quickly. An example of this phenomenon can be seen in Figures 6 and 7, which show the same accessibility comparison as that in Figures 4 and 5, but with 10 paths in each choice set. The 5-path version contains nearly the same information as the 10-path version, despite the latter's consuming twice the computing resources.

The value of measuring accessibility by an aggregate logsum rather than by a single best path can also be observed via this case study. Figures 8 and 9 show the difference in accessibility as calculated by a one-path PSPA, equivalent to simply finding the singular best path for every O-D pair. Although this version of the accessibility measure does capture some difference in utility with versus without the Valencia bike lanes, that difference is not as strong as seen in the five-path PSPA. This finding reinforces the research hypothesis that making fine-grained planning decisions can be assisted by using an accessibility measure incorporating information from multiple paths and not simply from the single best path.

COMPUTATIONAL CONSIDERATIONS

The first version of the PSPA was written in C++ and used a heap structure to implement a Dijkstra shortest-path algorithm to search for minimum-cost paths. To perform a five-path search on the San Francisco bicycle network, which contains approximately 1,000 origins and destinations, 10,000 nodes, and 35,000 links, the algorithm took approximately one week to run on a 4-core 2-GHz Opteron. The long run time, though partially attributable to a comparatively slow machine, indicates a potential drawback with an algorithm that must run separate shortest-path searches for each O-D pair rather than performing one-to-all shortest-path searches.

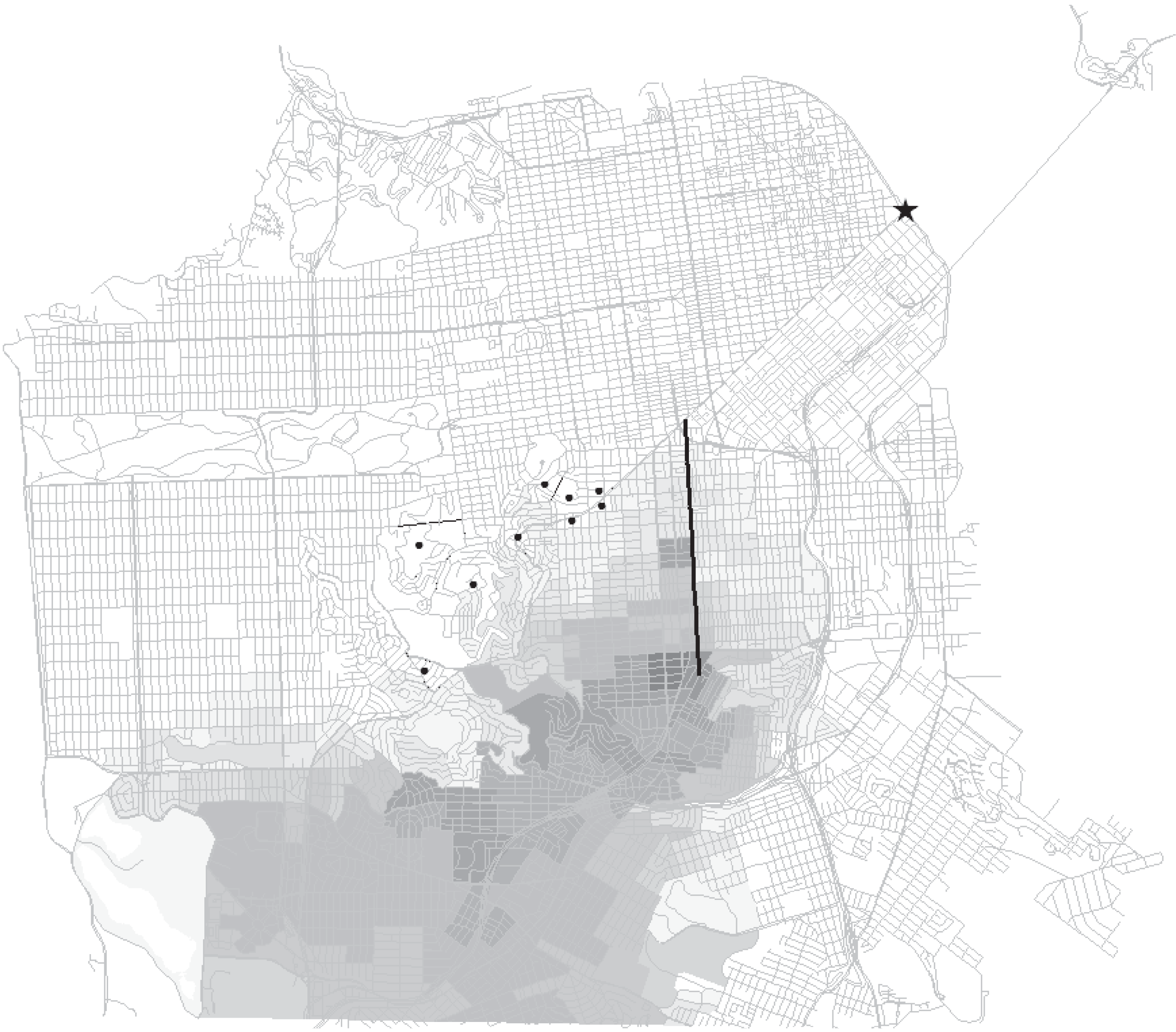
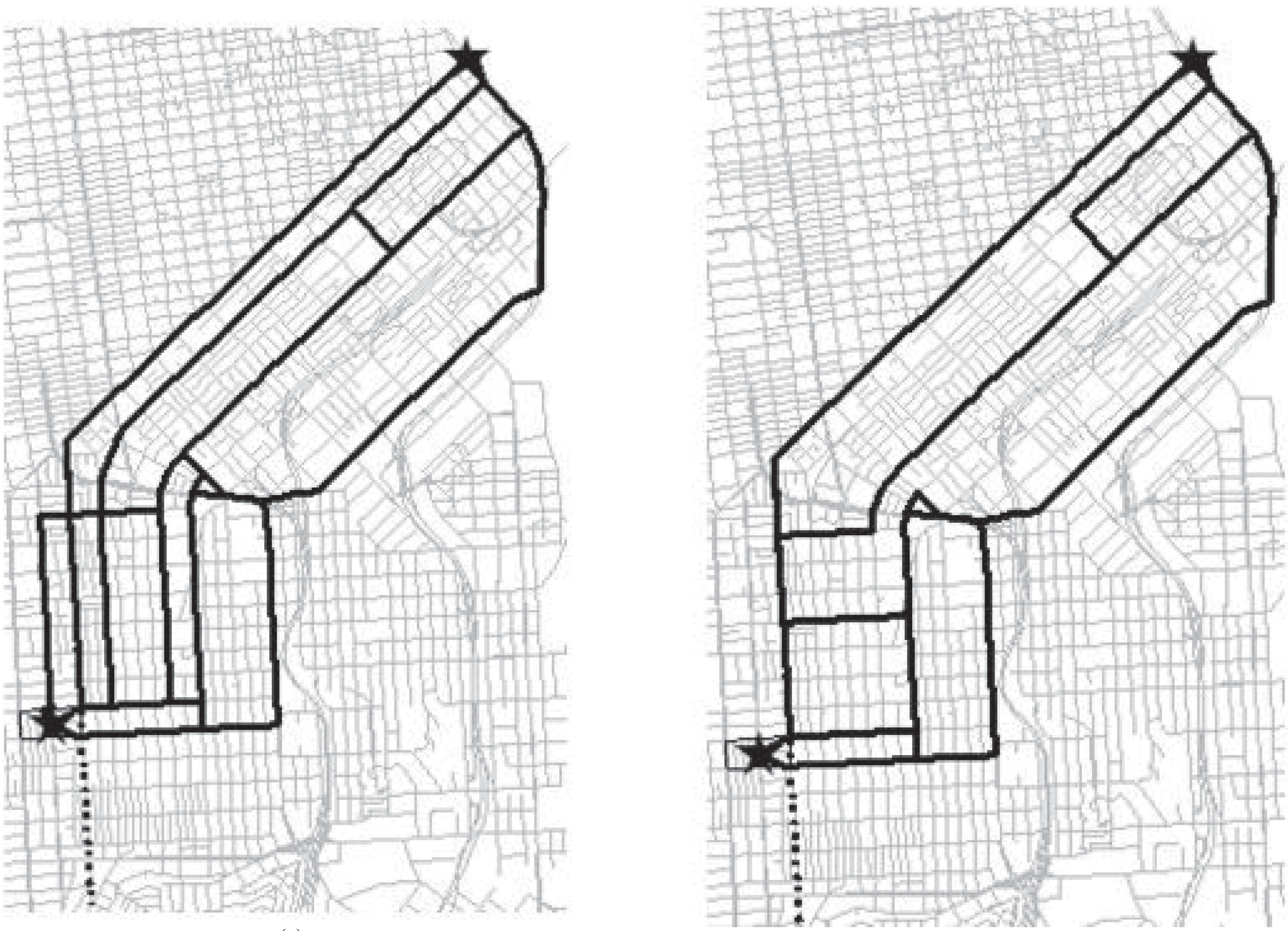


FIGURE 6 Change in accessibility: choice sets generated by 10-path PSPA.



(a) (b)

FIGURE 7 Paths found by 10-path PSPA (*a*) without and (*b*) with Valencia bike lanes.

FIGURE 8 Change in accessibility: only single best path.

(a) (b)

FIGURE 9 Single best path (*a*) without and (*b*) with Valencia bike lanes.

The second version of the PSPA, now in final development, is written in Python with the graph-tool module (http://projects.skewed.de/graph-tool), whose shortest-path searches are implemented in C++ via the Boost libraries. The algorithm runs in the cloud on a cluster of 32-core Intel Xeon E5-2680 machines. On a cluster of 10 such machines, Version 2 performs a full five-path search in less than an hour. The considerable increase in speed can be attributed to a combination of the speed of the graph-tool–Boost algorithms, faster machines, and parallelization to use multiple machines.

CONCLUSIONS AND FUTURE WORK

This study explored and addressed an important challenge that arises when route choice logsums are used as an accessibility measure. The proposed PSPA and accompanying penalty term were derived from theoretical features of path-size correction methods. In addition, the algorithm and the choice sets it generates were tested with real network applications via the San Francisco bicycle network. Experiments with this network show that the proposed PSPA decreases the existing paradoxical instances in both size and frequency of appearance. In addition, these experiments verify that the algorithm's desired feature of generating the paths in order of corrected utility improves the efficiency of computation when the choice set is generated.

One possible direction of future research is to apply the PSPA to other real-size networks; this application could shed more light on the performance of the PSPA in practice. Another possible direction to expand this research is to use the PSPA for the purpose of estimating the parameters of a route choice model in addition to the accessibility measure that was the motivation for this research. Since the PSPA requires an initial route choice model with defined parameters, an iterative framework might be used that starts with a random, yet reasonable initial set of route choice parameters and generates the path choice set based on those initial parameters. Then the generated choice set could be fed back to the parameter estimation and the process iterated until desired criteria are met.

Another future avenue of research could be to explore the possibility of generating an algorithm that is capable of performing in a one-to-all or all-to-one manner as opposed to the proposed PSPA, which currently runs one-to-one. This research could significantly improve the computational efficiency of the algorithm, especially when it is dealing with large-scale transportation networks.

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