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ORIGINAL ARTICLE

Creating opportunities to learn in mathematics education: a sociocultural perspective

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Abstract The notion of 'opportunities to learn in mathematics education' is open to interpretation from multiple theoretical perspectives, where the focus may be on cognitive, social or affective dimensions of learning, curriculum and assessment design, issues of equity and access, or the broad policy and political contexts of learning and teaching. In this paper, I conceptualise opportunities to learn from a sociocultural perspective. Beginning with my own research on the learning of students and teachers of mathematics, I sketch out two theoretical frameworks for understanding this learning. One framework extends Valsiner's zone theory of child development, and the other draws on Wenger's ideas about communities of practice. My aim is then to suggest how these two frameworks might help us understand the learning of others who have an interest in mathematics education, such as mathematics teacher educator-researchers and mathematicians. In doing so, I attempt to move towards a synthesis of ideas to inform mathematics education research and development.

Keywords Opportunities to learn · Sociocultural perspectives · Teacher development · Community of practice · Valsiner · Zone theory · Mathematics teacher educators

Background

Interest in 'creating opportunities to learn' underpins a wide range of research in mathematics education, whether the focus is on cognitive, social or affective dimensions of learning, curriculum and assessment design, issues of equity and access, or the broad policy and political contexts of learning and teaching (e.g. see Atweh et al. 2011; Gutiérrez and Boero 2006; Perry et al. 2012). In this paper, as well as considering *how* opportunities to learn can be created, I want to pose a second question: *Who* has opportunities to learn? Many mathematics education researchers are interested in students' mathematics learning or in the professional learning of teachers of mathematics, but there are others who might 'learn in mathematics education'. Here, I am

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referring specifically to university-based mathematics teacher educator-researchers and mathematicians, whose opportunities to learn have received much less attention in the research literature. I will argue that we need to know more about the professional preparation of mathematics teacher educators and how they continue to learn throughout their careers. I will also contend that creating opportunities for collaboration between mathematics educators and mathematicians can lead to productive interdisciplinary learning about how to improve mathematics education. The approach I will take to explore, both the 'how' and 'who' of creating opportunities to learn in mathematics education, involves reflecting on the development of my own research interests and the theoretical tools I have used to understand learning. However, this reflection is also future-oriented because it leads to discussion of new research questions and challenges.

The theoretical standpoint that informs this reflection draws on sociocultural perspectives on learning. Lerman (1996) defined sociocultural approaches to mathematics teaching and learning as involving 'frameworks which build on the notion that the individual's cognition originates in social interactions…and therefore the role of culture, motives, values, and social and discursive practices are central, not secondary' (p. 4). Sociocultural perspectives on learning grew from the work of Vygotsky in the early 20th century. One of the key claims of Vygotsky's (1978) theoretical approach concerns the social origins of higher mental functions, and he introduced the concept of the zone of proximal development (ZPD) to explain how a child's interaction with an adult or more capable peer awakens mental functions that are yet to mature. A second theme at the heart of Vygotsky's theoretical approach is a reliance on a genetic or developmental method: in other words, to understand mental phenomena, it is necessary to study the process of growth and change rather than the product of development. Early studies that applied Vygotsky's ideas in educational settings tended towards a literal view of learning as internalisation of this interchange between child and adult, but more sophisticated interpretations began to emerge in later research that attended to cultural practices and institutional contexts, and the role of personal histories, beliefs and values in shaping teaching and learning interactions. Thus, this line of inquiry takes a *change* perspective and focuses on the relationship between the individual and the environment. One example of how later researchers extended Vygotsky's original conceptualisation of the ZPD is provided by Valsiner's (1997) zone theory of child development, which introduced two additional 'zones' to incorporate the social setting and the goals and actions of participants. Valsiner's work has been used in mathematics education to study opportunities to learn experienced by school students and teachers (e.g. Blanton 2005; Hussain et al. 2009).

Vygotsky was also one of several theorists who influenced the development of a *practice* perspective within sociocultural research, such as the concept of situated learning in a community of practice (Lave and Wenger 1991). Although this concept arose from studying informal learning in apprenticeship and other out-of-school contexts, community of practice models have been fruitfully applied in mathematics education research focused on school classrooms and teacher professional learning (e.g. Gómez 2002; Graven 2004).

The purpose of this paper is to consider how the two lines of sociocultural inquiry identified above, the *change* perspective based on Valsiner's (1997) zone theory and the *practice* perspective informed by Wenger's (1998) ideas about communities of practice, can help build more integrated theories for understanding and creating opportunities to learn in mathematics education. The first part of the paper outlines some results from my own research that applied each of these perspectives to interpret students' and teachers' learning. The second part extends each perspective to new research domains and other learners. Zone theory is proposed as a framework for studying the learning and development of mathematics teacher educator-researchers, and a community of practice perspective is suggested as a means of examining learning through 'boundary encounters' between communities of mathematics educators and mathematicians. The paper concludes with some reflections on this proposed research agenda.

A community of practice interpretation of students' and teachers' learning in mathematics education

Creating learning opportunities for students

My early research, carried out in the mid-1990s, was motivated by questions about what specific actions a teacher might take to create a culture of mathematical inquiry in a secondary school mathematics classroom (Goos 2004). This seemed to be an important question at a time when curriculum reforms in Australia and elsewhere were placing increased emphasis on mathematical reasoning, problem solving and communication (e.g. Australian Education Council 1991; National Council of Teachers of Mathematics 1989). I was attracted to sociocultural themes, evident in research that demonstrated a clear shift away from viewing mathematics learning as acquisition towards understanding learning as participation in the discursive and cultural practices of a community (Sfard 1998). I used the concept of a community of inquiry to help me understand how one particular teacher structured learning activities and social interactions to develop his students' mathematical thinking. This investigation, carried out over two school years, focused on the detailed practices through which so-called reform approaches were enacted in classrooms.

From analysis of my classroom observation field notes and video-recordings as well as interviews with the teacher and students, I developed a set of five statements that reflected the teacher's assumptions about mathematics teaching and learning:

(1) Mathematical thinking is an act of sense-making, and rests on the processes of specialising, generalising, conjecturing and convincing.
(2) The processes of mathematical inquiry are accompanied by habits of individual reflection and self-monitoring.
(3) Mathematical thinking develops through teacher scaffolding of the processes of inquiry.
(4) Mathematical thinking can be generated and tested by students through participation in equal-status peer partnerships.
(5) Interweaving of familiar and formal knowledge helps students to adopt the conventions of mathematical communication.

I justified each of these statements with evidence from the data corpus, comprising teacher actions underpinned by each assumption and student actions in appropriating the teacher's mathematical attitudes and pedagogical expectations. Together, the assumptions and actions represented a synthesis of evidence from the study as a whole to show how the teacher created a culture of mathematical inquiry.

Creating learning opportunities for teachers

As satisfying as this study proved to be in identifying how the teacher created learning opportunities for his students, it still left me feeling unconvinced about the theoretical framework I had devised for explaining what made the classroom a community. My research interests had shifted towards teacher education, and researchers were starting to invoke Lave and Wenger's (1991) notion of community as a context for teachers' learning (e.g. Graven 2004). Wenger (1998) used community of practice as a point of entry into a broader conceptual framework in which learning was conceived as participating 'in the *practices* of social communities and constructing *identities* in relation to these communities' (p. 4, original emphasis). These ideas proved useful in researching the professional socialisation of beginning teachers. One of my research questions asked how communities of practice are formed in a pre-service teacher education programme and sustained after graduation and entry into the profession (Goos and Bennison 2008). This research was prompted, in part, by the unanticipated ways in which my pre-service students used the course bulletin board as an online space for professional discussions during and after their university programme.

A significant aspect of the study was an examination of the assumption that a 'virtual' community of practice will create opportunities for teachers to learn. In teacher education research, this is a premise that is not always tested to discover whether such a community really exists or what it is actually achieving. Wenger's three dimensions of practice—mutual engagement, joint enterprise and shared repertoire—were used to analyse more than 1,500 messages posted to the Yahoo Groups bulletin boards over almost 2 years in order to characterise the activities of the community and trace its emergent structure.

My analysis showed that the Yahoo Groups bulletin board created emergent, rather than pre-determined, opportunities for these pre-service teachers to learn in mathematics education, in keeping with Wenger's perspective on learning as an informal and tacit process. However, community of practice models are perhaps not well suited to analysing the role of a teacher educator who deliberately sets out to ensure that certain types of learning occur. Encouraged by an earlier experiment with using Valsiner's zone theory in teacher education (Goos et al. 1994), I began to apply this theory more systematically to understand relationships between learning, teaching and the contexts in which teachers develop their pedagogical identities.

A zone theory interpretation of students' and teachers' learning in mathematics education

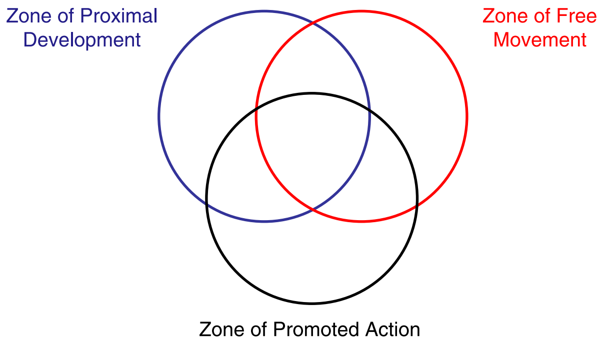
Valsiner viewed the zone of proximal development (ZPD) as a set of possibilities for development that are in the process of becoming realised as individuals negotiate their relationship with the learning environment and the people in it. He extended Vygotsky's original conceptualisation of the ZPD by proposing the existence of two additional zones: the zone of free movement (ZFM) and the zone of promoted action

(ZPA). The ZFM structures an individual's access to different areas of the environment, the availability of different objects within an accessible area and the ways the individual is permitted or enabled to act with accessible objects in accessible areas. The ZPA comprises activities, objects or areas in the environment in respect of which the individual's actions are promoted. The ZPA can include areas that are currently outside the ZFM as well as those that are inside; thus, the actions being suggested, while possible, may seem 'forbidden' at the present time. The ZFM and ZPA are dynamic and inter-related, forming a ZFM/ZPA complex that is constantly being re-organised by adults in interactions with children. However, a key claim of Valsiner's theory is that children are active participants in their own development: they can change the environment in order to achieve their emerging goals. Thus, the process of development is neither completely random nor fully determined; instead, it is directed, or 'canalised', along a set of possible pathways jointly negotiated by the child in interaction with the environment and other more mature people. One way to represent the relationship between the zones is shown in Fig. 1.

Although Valsiner's (1997) theory is intended to explain child development, he noted that the ZFM/ZPA complex is also observable in the context of education, both formal and informal. He provided classroom examples to show how teachers can set up narrow or expansive ZFM/ZPA systems, with different implications for the choices allowed to students in completing set tasks. Consider an example from my own experience in researching the integration of digital technologies into mathematics teaching and learning (Fig. 2). The teacher set up an expansive zone of free movement that included classroom access to various forms of technologies for learning mathematics, but then only offered learning activities—a zone of promoted action—that required students to use technology in routine ways, as a replacement for pen and paper calculations. In Fig. 2, this situation is represented by a large circle for the ZFM, with a much smaller circle inside this region showing the ZPA. The ZFM/ZPA complex set up by the teacher limited students to exploring only some aspects of the broader field of action available in the classroom.

Two different approaches to zone theory are evident in the mathematics education research literature, one of which defines the zones from the perspective of the student-as-learner and the other from the perspective of the teacher-as-learner.

Fig. 1 Representation of relationships between Valsiner's zones



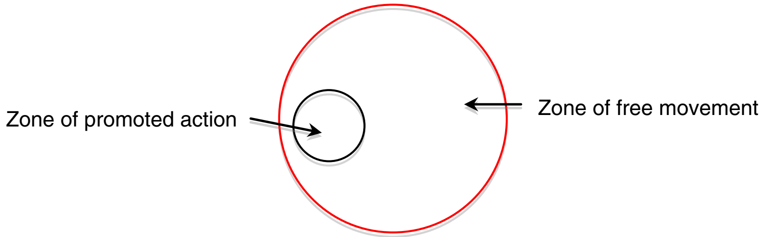


Fig. 2 Example of a ZFM/ZPA complex

Zone theory approach #1: focus on student-as-learner

A teacher's instructional choices about what to promote and what to allow in the classroom establish a ZFM/ZPA complex that characterises the learning opportunities experienced by students. This approach was taken by Blanton et al. (2005), who compared the ZFM/ZPA complexes organised by three mathematics and science teachers in their respective classrooms as a means of revealing these teachers' understanding of student-centred inquiry. They found that two of the teachers created the appearance of promoting discussion and reasoning when their teaching actions did not allow students these experiences. Approach #1 is thus useful for explaining apparent contradictions between the types of learning that teachers claim to promote and the learning environment they actually allow students to experience.

Zone theory approach #2: focus on teacher-as-learner

Valsiner (1997) argued that zone theory is applicable to any human developmental phenomena where the environment is structurally organised, and thus, it seems reasonable to extend the theory to the study of teacher learning and development in structured educational environments. Hussain et al. (2009) proposed a partially developed extension of Valsiner' theory in a study of teachers who participated in a professional development programme that introduced them to collaborative learning approaches in primary school mathematics. The analysis initially focused on how the teachers created new ZFM/ZPAs for their students (student-as-learner), but the intervention process led to a parallel transformation in the teachers' ZFMs as they restructured their relationships with students and other mathematical objects in the classroom (possible extension to teacher-as-learner).

My own approach to the use of zone theory goes even further in its explicit focus on the teacher-as-learner. Re-interpreting the zones from this perspective, the teacher's zone of proximal development becomes a set of possibilities for development of new knowledge, beliefs, goals and practices created by the teacher's interaction with the environment, the people in it and the resources it offers. The zone of free movement structures the teacher's environment, or professional context; so that elements of the ZFM could include perceptions of students (behaviour, motivation, abilities and socio-economic background), access to resources and teaching materials, curriculum and assessment requirements, and organisational structures and cultures of the school. While the zone of free movement suggests which teaching actions are *permitted*, the zone of promoted action can be interpreted as activities offered via teacher education programmes, formal professional development or informal interaction with colleagues that *promote* certain teaching approaches. It is worth noting here that pre-service teachers develop under the influence of two distinct ZFM/ZPAs that do not necessarily coincide—one provided by their university programme and the other by their supervising teacher during the practicum.

In previous studies, I have found Approach #2 helpful for analysing alignments and tensions between teachers' knowledge and beliefs, their professional contexts and the professional learning opportunities available to them in order to understand why they might embrace or reject teaching approaches promoted by teacher educators (Goos 2005, 2009). One part of this research programme has been investigating factors that influence how beginning teachers who have graduated from a technology-rich pre-service programme integrate digital technologies into their practice. Table 1 maps onto each of Valsiner's zones a range of factors known to influence teachers' use of technology in mathematics classrooms. Note that this mapping is not intended to define the zones with such precision as to contradict Valsiner's view of 'bounded indeterminacy' in relation to developmental trajectories, since pathways of development are constrained rather than determined. Instead, the adaptation of zone theory provides a way of studying the formation of ZFM/ZPA complexes that support or hinder teachers' learning.

A case study of teacher-as-learner

Consider the case of Adam, a beginning teacher who participated in the research referred to above (more fully discussed by Goos 2005). Adam completed his practice teaching sessions at a school that had been designated a Centre of Excellence in mathematics and technology, with government funding to resource all classrooms in the mathematics building with computers, Internet access, data projectors, graphics calculators and data loggers. New mathematics syllabuses additionally mandated the use of computers or graphics calculators in teaching and assessment programmes. We

Table 1 Factors affecting teachers' use of technology

Valsiner's zones	Factors influencing teachers' use of digital technologies
Zone of proximal development (possibilities for developing new teacher knowledge, beliefs, goals and practices)	Mathematical knowledge Pedagogical content knowledge Skill/experience in working with technology General pedagogical beliefs
Zone of free movement (structures teachers' access to different areas of the environment, availability of different objects within an accessible area, and ways the teacher is permitted or enabled to act with accessible objects in accessible areas)	Perceptions of students (e.g. motivation, behaviour, socio-economic status, abilities) Access to resources (time, hardware, software, teaching materials) Technical support Curriculum and assessment requirements Organisational structures and cultures
Zone of promoted action (activities, objects or areas in the environment in respect of which the teacher's actions are promoted)	Pre-service teacher education Professional development Informal interaction with teaching colleagues

could say that this environment offered an expansive zone of free movement enabling integration of digital technologies into mathematics teaching. Adam's supervising teacher, who was the Director of the Centre of Excellence, also encouraged him to use any form of technology that was available for promoting students' mathematics learning. The practicum environment therefore organised a ZFM/ZPA complex that both *promoted* and *permitted* technology integration: this situation is represented by the large overlap between the ZFM and ZPA circles in Fig. 3. The ZFM/ZPA complex offered by the practicum school was also likely to canalise Adam's development along a pathway towards new, technology-enriched pedagogical knowledge and practices, as indicated by the large overlap between Adam's ZPD and the ZFM/ZPA complex in Fig. 3.

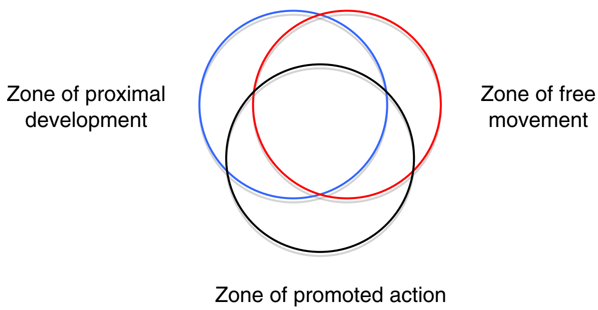
After graduation, Adam was employed in the same school but experienced a different set of constraints. Because not all classes could be scheduled in the well-equipped mathematics building, Adam had to teach some of his lessons in other classrooms without computers, data projectors or Internet access. Now that he was a full-time staff member of the school, he discovered that many of the other mathematics teachers were sceptical about using technology. Some of these teachers accused Adam of not teaching in the 'right' way. He, in turn, disagreed with their teaching approaches, which in his view betrayed negative perceptions about students:

> You do an example from a textbook, start at Question 1(a) and then off you go. And if you didn't get it – it's because you're dumb, it's not because I didn't explain it in a way that reached you.

Adam was now in a difficult situation that required him to defend his instructional decisions while negotiating professional relationships with other teachers, some of whom did not share his beliefs about teaching and learning. In these circumstances, technology-rich teaching seemed to be neither universally *permitted* (ZFM) nor consistently *promoted* (ZPA). Nevertheless, in his first year of full-time teaching, Adam continued to expand his teaching repertoire with digital technologies, often preferring to work with graphics calculators as portable tools that could be used in any classroom. He said that he saw technology as a means of giving students access to tasks that build mathematical understanding, and in this, he claimed to have been influenced by the university pre-service course and the teacher who had been his practicum supervisor.

It is not possible to explain Adam's appropriation of technology over this period of time by just 'adding up' the positive and negative influences listed in Table 1. A zone theory analysis would argue that Adam was an active agent in his own development in

Fig. 3 Adam's zone configuration during the practicum



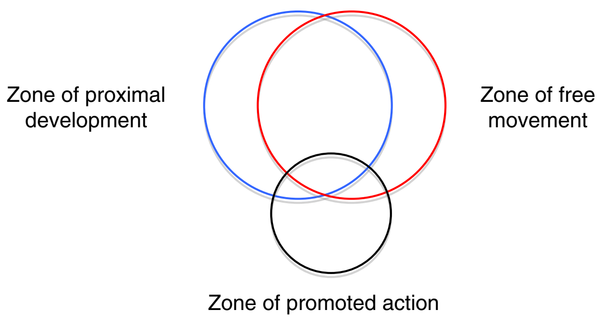
two distinctive ways. First, he interpreted his technology-rich ZFM as affording his preferred teaching approach, despite subtle hindrances in the distribution of technology resources throughout the school. He also decided to pay attention only to those aspects of the mathematics department's ZPA that were consistent with teaching approaches promoted by the university pre-service course. Figure 4 represents his situation in the year following graduation as still offering a generous zone of free movement, but with a restricted zone of promoted action that no longer overlaps so strongly with the ZFM because the teaching approaches Adam observed did not take full advantage of the school's technology-rich environment.

In his second year of teaching, Adam was transferred to a school where there was even more limited access to computer laboratories and only one class set of graphics calculators. None of the mathematics teachers were interested in using technology, and they preferred the same kind of teacher-centred, textbook-oriented teaching approaches as some of his colleagues in his previous school. The situation in his new school is represented in Fig. 5 by a more restricted zone of free movement and a similarly small zone of promoted action, which nevertheless overlaps considerably with the ZFM because the technology-free teaching actions promoted are aligned with the almost technology-free professional environment.

My role now, as a teacher educator-researcher, is to influence Adam's interpretation of the ZFM/ZPA complex to maintain his sense of personal agency. I encouraged him to view the single class set of graphics calculators as an opportunity he could exploit, simply because he was the only teacher who wanted to use them. I also supported him in increasing his involvement in the local mathematics teacher professional association where I hoped he would find a ZPA external to the school that would nurture his potential for further development.

This zone framework had proven useful for analysing research on teacher change conducted by other mathematics educators (Goos and Geiger 2010), which illustrates its broad applicability across research contexts. Part of this analysis involved asking what is learned by the teacher educator-researchers who work with mathematics teachers (see also Goos et al. 2010; 2011; 2012). This is the point where I want to move from past experience to propose new challenges for researching opportunities to learn in mathematics education.

Fig. 4 Adam's zone configuration during his first year of teaching



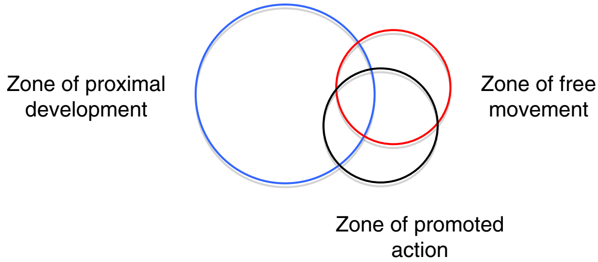


Fig. 5 Adam's zone configuration during his second year of teaching

The learning and development of mathematics teacher educator-researchers

Until recently, there has been little attention given to research on teacher educators (Robinson and McMillan 2006) and there are few published studies of the development of mathematics teacher educators. In one example, Clark et al. (2009a, 2009b) examined their respective practices and development as teacher educators across three different cultural contexts, using a personal narrative and 'researcher as subject' framework. In the other, Rhodes et al. (2009) described a professional development activity for mathematics teacher educators, involving observation and analysis of a laboratory class conducted by another teacher educator. Even (2008) noted that neglect of the education of mathematics teacher educators, by comparison to that of mathematics teachers, mirrors earlier research in mathematics education that focused more on students' learning than on teachers' learning. Thus, the processes by which mathematics teacher educators learn—whether informally, by engaging in teacher education practice, or formally, in programmes designed to educate educators—have not been systematically investigated (Llinares and Krainer 2006).

Theoretical approaches found in existing studies of teacher educator development largely draw on the notion of reflective practice. In mathematics education, Tzur (2001) and Krainer (2008) provided reflective self-studies of their own developmental trajectories, tracing their experiences as mathematics learners, teachers, teacher educators and mentors of fellow mathematics teacher educators to identify critical events and experiences that advanced their professional knowledge and practice. Reflection has also been a tool used in meta-studies where mathematics teacher educators analysed their own learning as part of a larger teacher professional development project (e.g. Diezmann et al. 2007; Even 2008; Zaslavsky and Leikin 2004). Reflective practice is claimed to lead to greater awareness of the personal theories motivating one's practice. However, because sociocultural theories take into account the settings in which practice develops, this perspective may have more to offer to those who wish to study the complexity of social practices and situations that engender learning in teacher educators.

Zone theory approach #3: focus on teacher-educator-as-learner

The theoretical approach I propose for studying opportunities to learn in mathematics teacher education extends the zone framework outlined in the previous section. There, I showed how it could be applied in two connected layers:

(1) when the *student is the learner*: the ZPD represents possibilities for the student's development, and the teacher creates classroom ZFM/ZPA complexes that canalise the student's learning; and
(2) when the *teacher is the learner*: the ZPD represents possibilities for the teacher's development, and the ZFM/ZPA complexes that canalise the teacher's learning are created by a range of factors within the teacher's professional environment. At this second layer, a teacher educator may come into the picture by promoting certain teaching actions, that is, by offering a ZPA for the teacher-as-learner.

What if we imagined a third layer, with the teacher educator as the learner and the ZPD representing possibilities for teacher educator development? I have represented these three layers in Fig. 6 to show distinctions between the ZPD, ZFM and ZPA at each layer. The arrows connecting the layers via ZPAs and ZPDs are meant to indicate that those who teach—including those who teach teachers—are also learners. However, the characteristics of teacher educators' zones of proximal development, zones of free movement and zones of promoted action, and the nature of interactions between the zones, are unclear because of the lack of previous research in this area.

Taking a zone theory perspective on teacher educator development gives rise to a new set of research questions. How do our professional contexts as teacher educators structure our interactions with prospective and practising teachers (ZFM)? What activities and areas of the professional environment do we access that promote certain approaches to educating teachers (ZPA)? How do the ZFM/ZPA complexes thereby created canalise our learning and development as mathematics teacher educator-researchers, and how do we negotiate these pathways for development throughout our careers? What does mathematics teacher educator learning look like (ZPD)?

Analysis of a zone of free movement for mathematics teacher educator-researchers might consider the following:

(1) characteristics of our teacher education students, such as their mathematical knowledge and their beliefs about mathematics teaching and learning;

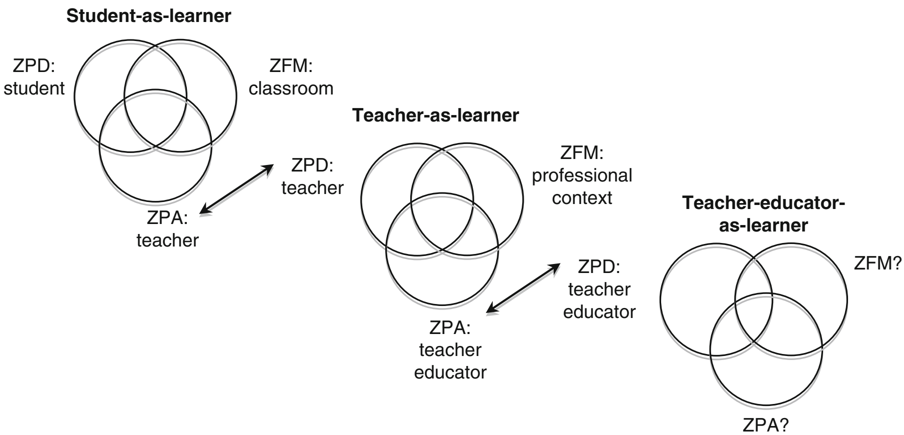


Fig. 6 Three layers of application of zone theory

(2) structural characteristics of teacher education programmes, such as class sizes, modes of delivery and the balance between courses focusing on general pedagogy, mathematics content and mathematics teaching methods;
(3) the extent to which curriculum and assessment requirements are influenced by professional accreditation authorities;
(4) organisational structures that limit the time available for teaching of methods courses;
(5) challenges in finding suitable practicum placements for prospective teachers; and
(6) university cultures that values research above teaching.

Regarding the zone of promoted action, particular approaches to teacher education are promoted through both our research and our practice. Some researchers have represented mathematics teacher educators' learning as a life-long process of growth through practice. For example, Zaslavsky and Leikin (2004) presented a three-layered hierarchical model of learning, where each successive layer contains the knowledge of mathematics learners, mathematics teachers and mathematics teacher educators, respectively. A recursive relationship exists between the layers as each form of knowledge operates and reflects on knowledge in the layer beneath. There is also space for a fourth layer representing the knowledge of educators of mathematics teacher educators. Tzur's (2001) self-analysis of his own growth as a teacher educator is an example of how an individual moves through these four layers of learning mathematics, learning to teach mathematics, learning to teach mathematics teachers and learning to mentor fellow mathematics teacher educators.

Mathematics teacher educators are also well positioned to learn from their research with teachers, even though this learning is often left unacknowledged and unarticulated (Jaworski 2001). A current project provides a striking example of the potential for this type of research to stimulate teacher educators' learning. The aim of the project is to provide an evidence base for improving university-based mathematics teacher education (see Callingham et al. 2011). One of the research team's assumptions is that developing pedagogical content knowledge (PCK) is central to teacher education courses, even though we accept that this concept is not easy to define and even harder to measure. Notwithstanding our reservations about using surveys to investigate pre-service teachers' PCK, we set about designing items that for pragmatic reasons could be administered online and scored automatically (see Beswick and Goos 2012, for our findings concerning primary pre-service teachers' PCK). As a project team, we had lengthy debates and sometimes heated arguments about what aspects of PCK to incorporate into survey items, what kind of choices to include as possible answers and which answers were 'better' than others. These discussions not only advanced our own understanding of PCK but also caused us to question the different emphases we gave to aspects of PCK in our respective teacher education courses. One of the members of this research team wrote a conference paper about the dilemmas we faced in designing the surveys and the sense of exhilaration we experienced from the rare experience of having conversations about our work as mathematics teacher educators (Chick 2011). Our project provided a glimpse of what mathematics teacher educator learning might look like, but because this was unanticipated, we did not fully capture or analyse our own learning. One of the significant outcomes of this research is the realisation that the nature of 'PCK for mathematics teacher educators' deserves further investigation.

Zone theory is useful because it allows us to study the processes of *change* as individuals negotiate their relationships with the environment and the material and human resources it offers. In this section, I have explained how zone theory could be extended to inform new research on the learning and development of mathematics teacher educators. However, a different theoretical lens and a different unit of analysis are needed if we are interested in examining the learning that results from interdisciplinary interactions between mathematics teacher educators and other professionals with an interest in mathematics education. Here, a *practice* perspective is more appropriate because it makes available the theoretical concepts of community and boundary for investigating these interactions.

Opportunities to learn across disciplinary boundaries in mathematics education

It is generally accepted that the preparation of prospective teachers of mathematics needs to include development of mathematics content knowledge as well as pedagogical content knowledge. The Teacher Education and Development Study in Mathematics (TEDS-M), an international comparative study of the competencies of mathematics teachers in 16 countries at the end of their training, collected outcome measures for mathematics content knowledge and pedagogical content knowledge for pre-service primary teachers as well as information about opportunities to learn—defined in terms of the content and teaching methods experienced during teacher education. Knowledge outcomes differed significantly between participating countries and between teacher education programmes within countries, with opportunities to learn within these programmes found to be highly relevant to development of these two types of knowledge for teaching (Blömeke et al. 2012).

One question that arises from such studies is—who are the teacher educators? Some answers were provided by a survey of a sample of participants in the 15th conference of the International Commission on Mathematical Instruction (ICMI-15), which focused on the professional education and development of teachers of mathematics. Amongst the 21 countries and country regions included in the sample, mathematicians tended to teach mathematics content courses while mathematics teacher educators taught the mathematics pedagogy courses (Tatto et al. 2010). While there may be questions about who is better placed to help prospective teachers acquire the knowledge they need for teaching mathematics, it has been argued that both mathematicians and mathematics teacher educators have an important role to play (Hodgson 2001). However, despite calls for greater dialogue between education and mathematics academics in Australian universities (Brown 2010), there have been few instances of productive collaboration in the design and delivery of pre-service mathematics teacher education programmes. I suspect that Australia is not unique in this regard. These observations, coupled with recent positive experience of working with mathematicians on teaching-related projects, lead me to wonder about opportunities to learn across disciplinary boundaries in mathematics education. How might such opportunities be recognised or created, theorised and studied? To sketch out a possible answer, I return to Wenger's (1998) work on communities of practice.

Boundary encounters between communities of practice

Wenger (1998) describes the three defining characteristics of communities of practice as mutual engagement of participants, negotiation of a joint enterprise and development of a shared repertoire of resources for creating meaning. Because communities of practice evolve over time, they also have mechanisms for maintenance and inclusion of new members. Based on this description, one can accept that mathematicians, mathematics teachers and mathematics teacher educator-researchers would claim membership of distinct, but related, communities of professional practice. This commonsense conclusion is confirmed by more rigorous analyses of differences between the epistemologies and values of these communities (e.g. Geiger and Goos 2006; Goldin 2003). Although communities of practice have 'insiders' and 'outsiders', they are not completely isolated from other practices or from the rest of the world. There are various ways in which communities may be connected across the boundaries that define them.

Wenger (1998) writes of boundary encounters as discrete events that give people a sense of how meaning is negotiated within another practice. The most fleeting of these is the one-on-one conversation between individuals from two communities to help advance the boundary relationship. For example, a mathematics teacher educator might telephone a mathematics teacher who is supervising the practicum experience of one of her pre-service students to discuss problems that the student is encountering at school. A more enriching instance of the boundary encounter involves immersion in another practice through a site visit. For example, a mathematician might visit a school to speak to students and teachers about careers in mathematics. However, both of these cases involve only one-way connections between different practices. A two-way connection can be established when delegations comprising several participants from each community are involved in an encounter, such as in the recent Australian Curriculum, Assessment and Reporting Authority's (ACARA) consultation sessions that brought together delegations of school teachers, university mathematicians and mathematics educators to provide feedback on the development of the *Australian curriculum: Mathematics*. Wenger suggests that if 'a boundary encounter – especially of the delegation variety – becomes established and provides an ongoing forum for mutual engagement, then a practice is likely to start emerging' (p. 114). Such boundary practices then become a longer-term way of connecting communities in order to coordinate perspectives and resolve problems.

While boundary practices might evolve spontaneously, they can also be facilitated by brokering. Wenger (1998) notes that the job of brokering is complex because it involves translating, coordinating and aligning the perspectives of different communities of practice. Most importantly, it requires the ability 'to cause learning by introducing into a practice elements of another' (p. 109). Bouwma-Gearhart et al. (2012) identified brokering as one of the key interdisciplinary strategies for improving pre-service teacher education in the science, technology, engineering and mathematics (STEM) disciplines in US research universities. In their study, successful interdisciplinary collaborations recognised that relationships evolve through a number of stages, from mutual suspicion of each other's discipline, towards awareness of and respect for other types of knowledge and research expertise, and that participants are at different stages of understanding about pedagogical issues and familiarity with the language and concepts of each other's disciplines. Brokering was crucial to building these collaborations. Successful brokers connected the disciplinary paradigms; they were able to speak the specialised language of mathematics, as well as translate the language and concepts of education research into forms that mathematician academics could understand and use. Bouwma-Gearhart et al. found successful brokers in both mathematics and education schools and faculties and with doctoral qualifications in either of these disciplines. What mattered was their ability to understand and coordinate the expertise that academics from all disciplines could contribute to the task of improving pre-service teacher education.

Several sites offer potential for productive boundary practices involving two-way connections between communities of mathematics teacher educators and mathematicians. These include the development of school mathematics curricula and the pre-service preparation of teachers, as discussed above. A sociocultural research agenda informed by a *practice* perspective might have the following aims:

(1) to develop a theory of boundary relations between the communities of mathematics teacher educators and mathematicians, based on Wenger's notions of boundary encounters and boundary practices;
(2) to design, enact and analyse different types of boundary practices that connect these communities of practice;
(3) to analyse the role of brokers who seek to connect professional communities; and
(4) to examine the processes of learning through exchange of expertise across disciplinary boundaries.

The design of tasks or units of study for pre-service teacher education that develop both mathematical content knowledge and pedagogical content knowledge may provide a productive context for promoting boundary practices. For example, a further unanticipated outcome of the mathematics teacher education research project described in the previous section was that mathematicians became interested in PCK and its relationship with mathematical content knowledge. This interest was discovered when project team members—mathematics teacher educators—gave presentations on the design of PCK survey items at conferences on undergraduate mathematics teaching (e.g. Goos 2011). Current Australian government initiatives, such as the Enhancing the Training of Mathematics and Science Teachers Program (Office for Learning and Teaching 2013), also encourage collaboration between faculties and schools of mathematics and education in order to integrate the content and pedagogical expertise of mathematicians and mathematics teacher educators. Although it is too early to know whether such initiatives will foster new boundary practices that connect the perspectives of these two communities, there is clear potential for research into the nature of interdisciplinary learning about how to improve mathematics education.

Concluding comments

The notion of 'opportunities to learn in mathematics education' gives rise to a multitude of questions and is open to interpretation from multiple theoretical perspectives. I have framed this idea as two questions: *Who* has opportunities to learn? *How* are these opportunities created? By following two lines of sociocultural inquiry, drawing, respectively, on the *change* perspective provided by zone theory and the *practice* perspective offered by community of practice concepts, I traced out a past and possible future research trajectory that considers these questions. Two things are important in this future research agenda. First, we need to know more about the professional formation of mathematics teacher educator-researchers. Calls for improvements to mathematics education are implicitly based on the assumption that well-prepared mathematics teacher educators are available, who can foster change in teachers' practices (Zaslavsky and Leikin 2004). However, as Loughran (2006) points out, the transition from school teacher to teacher educator is a struggle for most who take on this role. Being a good teacher educator requires more than being a good teacher. It also involves being able to articulate a knowledge of practice that is both informed by theory and firmly grounded in particular contexts, to create learning experiences that are both constructive and discomforting, to display both the authority and uncertainty that comes from experience in teaching and to model the risk taking and messiness associated with learning about teaching. If teaching is complex and demanding, then teaching about teaching is even more so. Yet little is known about how mathematics teacher educators are prepared for this role, how they continue to learn and develop throughout their careers and how their professional formation influences their teacher education practices and identities. Understanding this learning is essential if mathematics teacher educators are to contribute effectively to improving the quality of mathematics teacher education programmes

I would also argue that improvements to mathematics education – involving, for example, curriculum development, teacher preparation, and supporting student learning of mathematics as they transition from school to university – would benefit from productive collaboration between the professional communities that have an interest in such issues. Creating opportunities to learn across interdisciplinary boundaries may lead to new understanding of how to integrate the mathematical and pedagogical expertise of community members to enrich mathematics education.

A second notable aspect of my proposed research agenda is a desire to synthesise ideas to create integrated theories about mathematics learning and teaching. Bishop (2010) observed that, as a research community, we are strong on analysis but weak on synthesis, and he called for more integrated research development. The application of zone theory and community of practice concepts to learners other than students and teachers is a small step in this direction.

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