Forecasting Sovereign Default Risk with Merton’s Model

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Abstract
Gray, Merton, and Bodie (2007) adapt Merton's (1974) structural model for corporations to make it applicable to sovereign countries that have issued both local and foreign currency debt. We apply this model to eight emerging markets. The model underestimates sovereign credit spreads, and often assigns a near-zero probability of default in contrast to CDS spreads. We do find, however, a strong time-series correlation between the model implied credit spreads and the market CDS spreads. In addition we show that the most important determinant of the distance-to-default is the exchange rate volatility. Recent changes in exchange rate volatility can predict sovereign CDS spreads.

Keywords: Sovereign credit risk; structural model; emerging debt.
1. Introduction

In Merton's (1974) structural model the equity of a firm is modelled as a call option on the market value of the assets with strike price equal to the debt of the firm. As such the model provides a theoretical relationship between equity and corporate bond prices. Gray et al. (2007) adapt Merton's model to apply it at the aggregate level to the sovereign balance sheet. The basic idea is that the local currency debt can be seen as the equity of a sovereign given the ability of a country to print new money. And the foreign currency debt is similar to the corporation's debt because the country cannot easily create more foreign currency and practice shows that foreign currency debt is senior to local currency liabilities (Sims, 1999).

In this study we provide an empirical analysis of the sovereign structural model for eight emerging countries: Brazil, Hungary, Mexico, Philippines, Poland, South Africa, South Korea and Turkey. Our primary goal is to see how well the sovereign structural model explains the empirical data.

For corporations the empirical success of structural models has been measured in three different ways. First, for example Eom et al. (2004) look at the model's absolute accuracy in providing bond prices and CDS spreads. Also capital structure arbitrage studies belong to this category. The credit spreads generated from the Merton model are generally too low especially for corporations with an A credit rating or even higher. These model spreads can be close to zero, whereas market spreads can be hundreds of basis points. One possible reason is model misspecification, and there have been many attempts to improve upon Merton’s model, such as allowing for default before maturity. Another important reason is that structural models focus on the probability of default. Elton et al. (2001), and Colin-Dufresne et al. (2001) both show that the probability of default can only explain 25 percent of credit spreads, and the other 75 percent can to a large extent be attributed to a common systematic risk factor.

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1 The model is not applicable to countries in the European Monetary Union (EMU) for two reasons. First, it is very difficult for these countries to print new money. Second, these countries generally have access to a liquid debt market and hence in most cases do not need foreign currency debt.

2 Capital structure arbitrage is a strategy that attempts to make use of deviations between model and market prices, see for example Yu (2006), Duarte et al. (2007) and Schaefer and Strebulaev (2008). Since in our case model spreads are always lower, we would need to go short in CDS and short in local debt or buy put options on local debt, neither of which is feasible.

Second, rather than looking at spread levels several studies look at spread changes.\textsuperscript{4} Avramov et al. (2007), for example, argue that structural models are empirically successful if they motivate variables that capture the variation in spread changes. Both Avramov et al. and Zhang et al. (2009) find that equity volatility, which plays an important role in Merton's model, explains around fifty percent of the total variation in corporate CDS spreads.

Finally, several studies look at forecasting spread changes. Gebhardt et al. (2005) find that equity returns predict corporate bond returns of the same firm. They attribute this to the debt market being less informational efficient than the equity market, among others due to stocks being more actively traded than corporate bonds. Bharath and Shumway (2008) show that the Merton distance to default measure can predict the actual default probability.

In our empirical study for the sovereign structural model we contribute to all three aforementioned types of empirical studies. First, we also find that the model of Gray et al. predicts too low credit spreads. In fact the average model spread for the eight countries from April 2002 to February 2010 is just 0.3 basis points, compared to 220 basis points for the market CDS spreads. Only in the recent crisis and in country specific crises we see larger spreads but these are still much smaller than market spreads. Hence in terms of pricing accuracy the model is inadequate.

Second, the correlation between contemporaneous changes in model spreads and market CDS spreads averages 34 percent for the eight emerging market countries. We find an even stronger relationship between exchange rate volatility and sovereign CDS spreads with an average correlation of 48 percent. The exchange rate enters the model because all inputs need to be denominated in the same currency. Hence the local currency debt is converted into the foreign currency using the exchange rate. As a result the exchange rate volatility plays an important role, similar to that of equity volatility in Merton's model. Carr and Wu (2007) argue that the credit quality of a country is linked to the volatility of its currency because they are both positively linked to a country specific risk factor. They find that CDS spreads of Mexico and Brazil co-vary with their respective currency option implied volatilities.

\textsuperscript{4} Related, Schaefer and Strebulaev (2008) look at the predicted relationship between equity and bonds. They find that structural models provide quite accurate predictions of the sensitivity of corporate bond returns to changes in the value of the equity.
Finally, we look at forecasting spread changes. We test two simple trading strategies. The first strategy reduces possible influences of economic fundamentals and global risk appetite, found to be important drivers of sovereign risk by Baek et al. (2005) and Longstaff et al. (2011). Specifically we take simultaneously offsetting long and short CDS positions in two countries based on the recent changes in their respective exchange rate volatilities. The model predicts that an increase in the exchange rate volatility will increase the likelihood that the market value of the sovereign assets will drop below the default barrier. Hence the CDS spread will increase. We go long in the CDS of the country for which the exchange rate volatility has relatively increased. We find a high Sharpe ratio of 0.64 for this strategy. We find a Sharpe ratio of 0.66 if we base the strategy on (relative) changes in model implied spreads instead of exchange rate volatilities. The second strategy we consider is a directional strategy. We predict the CDS spreads of a country using past changes in the exchange rate volatility or changes in the model spreads for that country. Also this strategy has a good performance with a Sharpe ratio of 0.66 when using past exchange rate volatility changes, and 0.41 when using past changes in model spreads. Hence both results support the structural model approach for sovereigns. Our results also imply that the market fails to incorporate all fundamentals in CDS spreads in a timely way, in contrast to the more liquid currency market.

The remainder of this document is as follows. Section 2 describes the structural model for sovereign countries. Section 3 provides details on the data. The results are presented in Section 4. Finally, Section 5 concludes.
2. Methodology

*Merton's model for corporates*

We first present the Merton model for corporates in the same way as we will present the Gray et al. (2007) model for sovereigns. This will make it easier to compare the two models. Table 1 shows in a simplistic way the balance sheet of a corporation.

<table>
<thead>
<tr>
<th>Assets</th>
<th>Liabilities</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cash</td>
<td>Corporate bonds</td>
</tr>
<tr>
<td>Inventories</td>
<td>Equity</td>
</tr>
<tr>
<td>Accounts receivable</td>
<td>Accounts payable</td>
</tr>
<tr>
<td>Non-current assets (property etc)</td>
<td>Provisions, deferred tax etc</td>
</tr>
</tbody>
</table>

The claim of the debt holders on the corporation’s assets is considered to be senior to the claim of the equity holders. Hence the value of the equity is equal to the residual value of the corporation’s assets after the promised payments on debt have been made. In Merton's model the value of the equity is modeled as an implicit call option on the market value, \( A \), of the corporation's assets, with strike price equal to the debt, \( B \). Assuming \( A \) follows a log-normal distribution, the value of the equity, \( E \), can be expressed as

\[
E = AN(d_1) - Be^{-rT}N(d_2),
\]

where

\[
d_1 = \frac{\ln(A/B) + (r + \frac{1}{2}\sigma_A^2)T}{\sigma_A\sqrt{T}}, \quad d_2 = d_1 - \sigma_A\sqrt{T}.
\]

Here \( \sigma_A \) is the volatility of the market value of the assets, \( r \) is the risk-free rate of interest, and \( T \) is the maturity of the debt. The risk-free bond, \( B \), is in practice more complex with corporations issuing multiple bonds with different maturities and coupon payments. Theoretically a corporation defaults when the value of its assets is less than the promised payments on the debt. In practice corporations default at much higher asset levels. Therefore the distress barrier lies somewhere between the book
value of the corporation's short-term debt and the book value of the total debt. The KMV model uses an empirical rule to construct the distress barrier. In particular they use the rule of thumb that $B$ is equal to the short-term debt plus fifty percent of the long-term debt, see for example Crouhy et al. (2000).

Following Ito's lemma a second equation links the equity volatility to the asset volatility,

$$\sigma_E = \frac{A}{E} \sigma_A N(d_1)$$  \hspace{1cm} (2)$$

Using the equity market to get $E$ and $\sigma_E$ the two equations (1) and (2) can be solved for the two unknowns, $A$ and $\sigma_A$.

*Adaptation of Merton's model to sovereigns*

Gray et al. (2007) adapt Merton's model for corporations to make it applicable to sovereign countries. They start with formulating the combined balance sheet of the government and the central bank, see Table 2. The assets of the sovereign country consist of four items: (i) Foreign currency reserves including actual reserves and contingent reserves from international financial institutions such as the IMF and other governments; (ii) The net fiscal assets which equal the government's budget surplus or deficit on taxes and revenues minus expenditures; (iii) Credit to other sectors such as the corporate, financial and household sector; and (iv) Other public assets such as the equity of public enterprises.

The liabilities also contain four items: (i) (Implicit) financial guarantees to the so-called too-big-to-fail entities as we have seen in the government support and bailouts of large financial institutions during the financial crisis of 2008 and 2009; (ii) Foreign currency debt which is issued by the public sector and denominated in a foreign currency; (iii) Local currency debt which is issued by the public sector and denominated in the local currency; and (iv) The monetary base which is related to the money supply in the country's economy.
Gray et al. (2007) argue that the balance sheet of the country’s public sector and the balance sheet of a corporation show important similarities in both structure and priority of the claims. Local currency debt and the monetary base are called local currency liabilities and have certain features similar to the equity of a corporation. The public sector controls the money supply and therefore they have the option to repay their local currency debt by creating more domestic currency. However, expansion of the money supply can cause inflation which lowers the real value of the payments to the local currency debt holders. This is similar to equity of corporations because excessive issue of shares dilutes existing holders’ claims and reduces the price per share on the balance sheet of a corporation.

The foreign currency debt is analogous to the risky debt of a corporation because here the public sector cannot easily create more foreign currency since excessively creating domestic currency will lower the demand of the domestic currency and hence depreciate the foreign exchange rates. Moreover, Sims (1999) considers foreign currency debt to be senior to local currency liabilities because in stress situations most governments prefer to inflate local currency debt instead of defaulting on foreign currency debt.

The sovereign Contingent Claims Approach (CCA) model is similar to the corporate CCA model with local currency liabilities modeled as an implicit call option on the country’s assets and foreign currency debt modeled as a distress barrier minus an implicit put option on the country’s assets. The distress barrier is based on the book value of the foreign currency debt using the empirical rule that it is equal to the short-term debt plus fifty percent of the long-term debt.

An important feature of the sovereign CCA model is that the balance sheet items are measured in one currency unit, commonly a “hard” currency such as the US Dollar or the Euro, with the corresponding risk-free interest rate.

Essentially, the sovereign CCA model is the same as the corporate CCA model. We get two equations with two unknowns, the market value of the assets ($A$) and the volatility of the market value of the assets ($\sigma_A$). The market value of the

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**Table 2: Combined balance sheet of the government and the central bank**

<table>
<thead>
<tr>
<th>Assets</th>
<th>Liabilities</th>
</tr>
</thead>
<tbody>
<tr>
<td>Foreign reserves</td>
<td>Guarantees</td>
</tr>
<tr>
<td>Net fiscal assets</td>
<td>Foreign currency debt</td>
</tr>
<tr>
<td>Credit to other sectors</td>
<td>Local currency debt</td>
</tr>
<tr>
<td>Other public assets</td>
<td>Monetary base</td>
</tr>
</tbody>
</table>
equity of the corporation is replaced by the market value of the local currency liabilities ($LCL$), the equity volatility by the volatility in $LCL$ ($\sigma_{LCL}$), and the distress barrier ($B_f$) is based on the book value of the foreign currency debt. We then get the following two equations:

$$LCL = AN(d_1) - B_f e^{-rT} N(d_2)$$  \hspace{1cm} (3)

$$\sigma_{LCL} = \frac{A}{LCL} \sigma_A N(d_1)$$  \hspace{1cm} (4)

where

$$d_1 = \frac{\ln(A / B_f) + (r + \frac{1}{2} \sigma_A^2)T}{\sigma_A \sqrt{T}}, \quad d_2 = d_1 - \sigma_A \sqrt{T}.$$  

The foreign exchange rate is indirectly an important input parameter because it largely influences the value and volatility of the assets and the local currency liabilities. All input parameters are directly observable except for the market value and volatility of the assets of the sovereign which, therefore, can be estimated from equations (3) and (4).

We will apply the model to emerging markets. There are several reasons why the sovereign CCA model is not applicable to developed countries. First, developed countries have direct access to large and liquid international markets to issue debt in their domestic currency and that is why developed countries have no or only a small amount of foreign currency debt. Therefore, the foreign distress barrier of most developed countries will be zero. Moreover, countries from the Economic and Monetary Union (EMU) have very limited control over the money supply of the European Central Bank (ECB) and therefore the analogy between local currency liabilities and equity disappears. This leads us to the conclusion that the sovereign CCA model is best suited for emerging countries with a considerable amount of foreign currency debt.

There are a few important risk measures that can be derived from the model output. First, the risk-neutral probability of default of the sovereign is equal to $N(-d_2)$. Second, the distance to default is $d_2$. Finally, to find the model-implied credit spread we first need to find the current value of the risky debt with promised payments $B_f$. The value of the risky debt at time $T$ is exactly the same as $B_f$ minus the payoff of a
put option on the assets, with exercise price equal to $B_f$. Since the liabilities of the sovereign are equal to the assets of the sovereign, we could also derive the value of the risky debt $D_f$ by subtracting the value of the local currency liabilities from the assets:

$$D_f = A - LCL$$

Then the yield-to-maturity of the risky debt is

$$y = \frac{\ln(B_f / D_f)}{T}$$

and the model-implied credit spread is equal to the difference between this yield and the risk-free rate, $r$,

$$s = y - r$$

(5)

3. Data

We apply the model of Gray et al. (2007) to eight emerging (EM) countries: Brazil, Hungary, Mexico, the Philippines, Poland, South-Africa, South-Korea and Turkey. This choice is based on the liquidity of the emerging currencies (apart from the Philippines they are all part of the eight most liquidly traded EM currencies), the quality of CDS data (the Czech Republic was replaced by the Philippines for this reason), and their importance in terms of outstanding foreign currency debt (Brazil and South-Korea are the largest markets).

We obtain daily data from April 2002 to February 2010 on exchange rates, 5-year CDS spreads and 5-year government bond yields for the discount factor in the model, assuming a 5-year horizon to match the CDS maturity. We use central bank websites, government reports and Bloomberg to collect data on the monetary base, and (to compute) the market value of local currency bonds (fixed, floating and inflation-linked bonds). We find a close match between the Bloomberg computed market values and the government reported values. The local currency liabilities, $LCL$, is the sum of the monetary base and the market value of the local debt. For all countries we express these local liabilities in US dollars.

5 April 2002 is the starting point of CDS data for the Philippines. For the other countries the data are available for a longer history, but not included in our study.
For the default barrier, $B_f$, we need the book value of foreign currency debt. Here we also include debt denominated in the local currency but linked to a foreign currency. We label all debt maturing within one year as short-term, and the remainder as long-term. The default barrier is then set equal to one time the short-term debt plus fifty percent of the long-term debt. $B_f$ is also expressed in US Dollar terms.

The final input parameter is the volatility of $LCL$. The volatility is defined as the standard deviation of daily returns of $LCL$ over the past three months giving it both some stability but also being able to timely react to new information. Note that this volatility will depend on the volatility in the monetary base, the volatility of the market value of the local debt, and the exchange rate volatility. As one would expect the main driver of the volatility is the exchange rate volatility. One discomforting aspect of the volatility of $LCL$ is that there are jumps due to the redemption or issuance of new local debt. For computing the volatility of $LCL$ we try to remove such jumps when they are connected to redemptions or new issuance by excluding daily changes of $LCL$ larger than 25%.

Finally we obtain CDS spread data from Bloomberg. Table 3 shows the average and standard deviation of the CDS spreads, the amount of $LCL$, $B_f$ and the standard deviation of the exchange rates (FX volatility). The average CDS spread of the eight countries is 220 basis points, with Brazil having the largest spreads and Poland the smallest. Brazil is also the country of these eight with on average the most local and hard currency debt. The South-African Rand has the largest currency volatility.

| Table 3: Average and standard deviation spreads, debt and FX volatility |
|---------------------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|
|                           | Brazil                         | Hungary                        | Mexico                          | Philippines                     | Poland                          | South-Africa                    | South-Korea                    | Turkey                          | Avg                             |
| CDS spread (bp)           | Avg 543.1                      | 93.6                           | 142.2                           | 322.9                           | 60.0                            | 132.7                          | 81.0                           | 386.7                           | 220.3                           |
|                           | St dev 750.6                   | 126.0                          | 97.0                            | 143.1                           | 68.9                            | 94.9                           | 85.5                           | 284.1                           | 206.3                           |
| LCL bln USD               | Avg 532.2                      | 67.3                           | 267.2                           | 47.3                            | 154.9                           | 134.7                          | 270.2                          | 175.3                           | 206.1                           |
|                           | St dev 244.5                   | 21.4                           | 64.4                            | 20.2                            | 49.0                            | 27.6                           | 94.4                           | 73.0                            | 74.3                            |
| $B_f$ bln USD             | Avg 95.7                       | 11.7                           | 65.6                            | 16.8                            | 25.3                            | 5.2                            | 3.2                            | 35.6                            | 32.4                            |
|                           | St dev 12.2                    | 2.5                            | 3.4                             | 2.2                             | 8.8                             | 0.5                            | 1.0                            | 5.7                             | 4.5                             |
| FX volatility             | Avg 15.2%                      | 13.1%                          | 8.6%                            | 5.5%                            | 12.3%                           | 17.2%                          | 8.6%                           | 13.5%                           | 11.7%                           |
|                           | St dev 8.4%                    | 5.7%                           | 5.3%                            | 1.9%                            | 6.2%                            | 5.8%                           | 8.6%                           | 6.0%                            | 6.0%                            |

Note: Sample averages and standard deviations CDS spread (basis points), local currency liabilities in bln USD ($LCL$), default barrier in bln USD ($B_f$) and the annualized FX volatility from April 2002 - February 2010. The final column is the equal weighted average of the eight countries. In February 2010 the eight countries cover about 50% of the index weight of the JP Morgan EMBI global index for USD debt and more than 75% of the index weight of the JP Morgan GBI-EM index for local currency debt.
We observe a positive correlation of 41 percent between the ratio of the average default barrier level $B_f$ and the average local currency liability level $LCL$ with the average CDS spread. Such a correlation makes sense in the model of Gray et al. since a relatively large amount of foreign currency debt $B_f$ means a higher default risk for which investors should be compensated with a higher CDS spread.

4. Results
Following the literature on corporate credits we conduct several analyses for sovereign debt. In Section 4.1 we report on the level of the model implied CDS spreads. The contemporaneous correlations of CDS spread changes and variables based on the structural model are discussed in Section 4.2. Finally, the outcomes of various trading strategies based on the structural model are presented in Section 4.3.

4.1. The level of model-implied credit spreads
Figure 1 shows as an example the market CDS spreads, model-implied spreads and the rolling 3-month exchange rate volatility for Brazil. The model-implied spread is often (near) zero, in contrast to the market CDS spread. Only during the crisis in Brazil in 2002 and the global crisis in 2008 the model-implied spread has a more sizeable value but still very small compared to the market CDS spreads. We had to multiply them by 100 in the graph to make them visible. The spikes in market CDS spreads coincide with spikes in exchange rate volatility, the most important input parameter of the sovereign structural model in terms of dynamics. It is also noteworthy that market CDS spreads were much larger in the 2002 Brazil crisis than during the 2008 global crisis. In contrast the exchange rate volatility and (hence) the model-implied spread reached new highs in 2008. For the other countries we see similar patterns with model-implied spreads generally way too low and only sizeable (but still relatively small) when market CDS spreads are extreme. This in part is due to the high distance-to-defaults the model generates.

Table 4 shows the average model-implied spreads. The average model-implied spread is just 0.3 basis points, compared to 220 basis points for the market CDS spreads. This is a similar finding to that of Eom et al. (2004) for Merton’s model for corporations. Hence similar to existing studies on structural models for corporations (see e.g. Eom et al., 2004) we have trouble to match point estimates of spreads from the sovereign structural model to those observed in the CDS market: model-implied...
spreads are too low. We therefore focus on the dynamics in CDS spreads: to what extent can they be explained or predicted by the model?

Table 4: Average and standard deviation model-implied spreads

<table>
<thead>
<tr>
<th></th>
<th>Brazil</th>
<th>Hungary</th>
<th>Mexico</th>
<th>Philippines</th>
<th>Poland</th>
<th>South-Africa</th>
<th>South-Korea</th>
<th>Turkey</th>
<th>Avg</th>
</tr>
</thead>
<tbody>
<tr>
<td>CDS spread (bp) Avg</td>
<td>543.1</td>
<td>93.6</td>
<td>142.2</td>
<td>322.9</td>
<td>60.0</td>
<td>132.7</td>
<td>81.0</td>
<td>386.7</td>
<td>220.3</td>
</tr>
<tr>
<td>St dev</td>
<td>750.6</td>
<td>126.0</td>
<td>97.0</td>
<td>143.1</td>
<td>68.9</td>
<td>94.9</td>
<td>85.5</td>
<td>284.1</td>
<td>206.3</td>
</tr>
<tr>
<td>Model spread (bp) Avg</td>
<td>154.9</td>
<td>20.5</td>
<td>6.3</td>
<td>0.0</td>
<td>13.9</td>
<td>8.1</td>
<td>0.1</td>
<td>39.8</td>
<td>30.5</td>
</tr>
<tr>
<td>St dev</td>
<td>557.7</td>
<td>81.3</td>
<td>40.0</td>
<td>0.0</td>
<td>78.9</td>
<td>50.9</td>
<td>0.5</td>
<td>141.4</td>
<td>118.8</td>
</tr>
</tbody>
</table>

Note: Sample averages and standard deviations CDS spread (basis points) and model spread (basis points) multiplied by a factor of 100 from April 2002 - February 2010. We apply the Gray et al. (2007) model in equations (3) and (4) on a daily basis, and then use equation (5) to compute the model spreads. The final column is the equal weighted average of the eight countries.

Figure 1: CDS spreads, model-implied spreads and FX volatility for Brazil

Note: We apply the Gray et al. (2007) model in equations (3) and (4) on a daily basis to Brazil. The graph shows the market 5-year CDS spreads and the model-implied spreads (times 100) using equation (5) on the left y-axis, and the rolling 3-months volatility of the spot BRL/USD exchange rate on the right y-axis.
4.2 Correlations between CDS spread changes and structural variables

We first look at the correlations between 1-month and 3-month changes in model inputs and outputs with 1-month and 3-month changes in market CDS spreads. The results are shown in Table 5. As expected, the changes in the distance-to-default, \( d_2 \) from equation (2), are negatively correlated with the changes in CDS spreads. A larger distance-to-default implies a lower probability of default and hence a lower CDS spread. Of course, the changes in the model-implied probability of default, \( N(-d_2) \), are positively correlated with changes in CDS spreads. The positive correlation between changes in exchange rate volatility and CDS spreads is also logical as an increase in the volatility makes it more likely the distance-to-default can be bridged and hence increases the probability of default, all else equal.

Looking at Panel A and B in Table 5 it is clear that correlations are rising when increasing the window from 1 to 3 months implying noise in the shorter windows and possibly a delayed response of either model-variables or market CDS spreads. We will investigate the latter in the next section. We get the highest correlation at 48 percent between 3-month changes in exchange rate volatility and 3-month changes in CDS spreads. Hence a substantial amount of the change in CDS spreads can be explained by changes in volatility. The 31 percent correlation between 1-month changes in CDS spreads and changes in volatility is similar to the 36 percent correlation found in Avramov et al. (2007) for 1-month changes in corporate bond spreads and changes in idiosyncratic volatility.

Table 5: Correlations of CDS spread changes with model inputs and outputs

<table>
<thead>
<tr>
<th>Panel A: 1-month changes</th>
<th>Brazil</th>
<th>Hungary</th>
<th>Mexico</th>
<th>Philippines</th>
<th>Poland</th>
<th>South-Africa</th>
<th>South-Korea</th>
<th>Turkey</th>
<th>Avg</th>
</tr>
</thead>
<tbody>
<tr>
<td>( d_2 )</td>
<td>-0.15</td>
<td>-0.25</td>
<td>-0.21</td>
<td>-0.09</td>
<td>-0.19</td>
<td>-0.29</td>
<td>-0.14</td>
<td>-0.27</td>
<td>-0.20</td>
</tr>
<tr>
<td>( N(-d_2) )</td>
<td>0.25</td>
<td>0.20</td>
<td>-0.08</td>
<td>0.27</td>
<td>0.09</td>
<td>0.09</td>
<td>0.01</td>
<td>0.33</td>
<td>0.14</td>
</tr>
<tr>
<td>( \sigma_{FX} )</td>
<td>0.29</td>
<td>0.44</td>
<td>0.31</td>
<td>-0.02</td>
<td>0.32</td>
<td>0.33</td>
<td>0.53</td>
<td>0.28</td>
<td>0.31</td>
</tr>
<tr>
<td>Spread</td>
<td>0.16</td>
<td>0.16</td>
<td>-0.10</td>
<td>0.26</td>
<td>0.08</td>
<td>0.07</td>
<td>0.00</td>
<td>0.26</td>
<td>0.11</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Panel B: 3-month changes</th>
<th>Brazil</th>
<th>Hungary</th>
<th>Mexico</th>
<th>Philippines</th>
<th>Poland</th>
<th>South-Africa</th>
<th>South-Korea</th>
<th>Turkey</th>
<th>Avg</th>
</tr>
</thead>
<tbody>
<tr>
<td>( d_2 )</td>
<td>-0.24</td>
<td>-0.38</td>
<td>-0.31</td>
<td>-0.19</td>
<td>-0.35</td>
<td>-0.44</td>
<td>-0.30</td>
<td>-0.38</td>
<td>-0.32</td>
</tr>
<tr>
<td>( N(-d_2) )</td>
<td>0.58</td>
<td>0.43</td>
<td>0.12</td>
<td>0.27</td>
<td>0.23</td>
<td>0.36</td>
<td>0.32</td>
<td>0.66</td>
<td>0.37</td>
</tr>
<tr>
<td>( \sigma_{FX} )</td>
<td>0.44</td>
<td>0.68</td>
<td>0.40</td>
<td>0.11</td>
<td>0.57</td>
<td>0.52</td>
<td>0.71</td>
<td>0.39</td>
<td>0.48</td>
</tr>
<tr>
<td>Spread</td>
<td>0.43</td>
<td>0.38</td>
<td>0.11</td>
<td>0.27</td>
<td>0.22</td>
<td>0.36</td>
<td>0.32</td>
<td>0.61</td>
<td>0.34</td>
</tr>
</tbody>
</table>

Note: Correlations between the change in 5-year CDS spreads and (i) the change in the model-implied distance-to-default, \( d_2 \); (ii) the change in the model-implied probability of default, \( N(-d_2) \); (iii) the change in the 3-month exchange rate volatility, \( \sigma_{FX} \); and (iv) the model-implied spread from equation (5). In the upper panel we look at 1-month changes, in the lower panel at 3-month changes. The sample period is March 2002 to February 2010 resulting in 94 monthly observations. Overlapping data are used for 3-month changes, every time shifting time by one month.
4.3 Trading strategies

4.3.1 Relative strategies
In the trading strategies we will predict the relative movement of the CDS spreads for each pair of countries. Reason for a relative strategy is that we want to play the dynamics predicted by the Gray et al. model whilst hedging ourselves against a market-wide spread widening or tightening as a consequence of global (event) risk (see Pan and Singleton, 2008) or changing global economic conditions. The correlation between monthly changes of CDS spreads between countries is 56% on average and positive for all 28 pairs.

For the direction of the trade we look at 3-month changes in model-implied spreads and exchange rate volatility. For example, if the exchange rate volatility in Brazil has declined by more than in Hungary, we expect the Brazil CDS spread to decline relative to the CDS spread for Hungary. We therefore take a short position in the Brazilian CDS (selling default protection), and a long position in the Hungarian CDS (buying default protection).

Ben Dor et al. (2007a; 2007b) show that the risk of a CDS contract depends on the level of the spread times the duration. Given that we enter 5-year CDS contracts for both countries and hence durations are approximately equal we simply scale the positions by the CDS spreads. If, for example, the CDS spread is 300 basis points for Brazil and 50 basis points for Hungary this scaling implies the position in the CDS for Hungary is 6 times as large as the one for Brazil. Added benefit is that the periodic spread payments add up to zero as we have a matching long and short position. Hence the profit of the trade only depends on the spread changes in both countries, and we create a zero-investment strategy. In equation form we get the following expression for the 1-month return in month $t$ from the opposing CDS positions in countries $i$ and $j$,

\[
\text{Return}^j_i \approx D_{i,t} \cdot \text{bet}_{i,t} \left[ \frac{CDS^i_t - CDS^i_{t-1}}{CDS^i_{t-1}} - \frac{CDS^j_t - CDS^j_{t-1}}{CDS^j_{t-1}} \right]
\]  (6)
where $D$ is the duration (5 years), $bet_{t-1}$ is a scaling factor related to the total bet size, and $CDS_t$ is the CDS spread in month $t$ in country $i$. For the success ratio and Sharpe ratio the total bet size does not matter. The risk-scaling to match the risks (and payments) of the two CDS positions is reflected in the denominators in equation (6).

For the bet size we can take simple long/short positions ($bet_{t-1} = \pm 1$), but alternatively we can let the bet size depend on the magnitude of the changes in the predictive variables over time. The larger the change, the more spread movement we can expect. Hence we set the bet size proportional to the $z$-score, i.e. computing the current difference in 3-month changes of model spreads or exchange rate volatilities, deducting the 5-year average and dividing by the 5-year standard deviation.

**Strategy based on model spreads**

Table 6 shows the results for the CDS strategy based on changes in model spreads. The results are quite strong, with a positive result for each group of 7 country pairs as well as for the portfolio of 28 unique country pairs. The Sharpe ratio is 0.66 for the portfolio with a return of 0.98% per annum and a standard deviation of 1.48% per annum. Note that return and standard deviation are scalable by increasing or decreasing the bet size. Using long/short positions instead of $z$-scores gives an Sharpe ratio of 0.36. It is therefore important to consider the size of the (relative) model credit spread changes. The strong results prove that the sovereign structural model of Gray et al. is a nice step towards better understanding sovereign credit risk. Furthermore the results show that the market does not fully incorporate information from the aggregate sovereign balance sheets into the market CDS spreads.

**Table 6: CDS strategy based on changes in model spreads**

<table>
<thead>
<tr>
<th>Country</th>
<th>Brazil</th>
<th>Hungary</th>
<th>Mexico</th>
<th>Philippines</th>
<th>Poland</th>
<th>South-Africa</th>
<th>South-Korea</th>
<th>Turkey</th>
<th>Portfolio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Return (%)</td>
<td>0.82</td>
<td>0.80</td>
<td>0.65</td>
<td>1.80</td>
<td>1.79</td>
<td>1.00</td>
<td>0.62</td>
<td>0.36</td>
<td>0.98</td>
</tr>
<tr>
<td>Stdev (%)</td>
<td>2.22</td>
<td>2.17</td>
<td>1.75</td>
<td>3.01</td>
<td>2.26</td>
<td>1.14</td>
<td>2.53</td>
<td>2.55</td>
<td>1.48</td>
</tr>
<tr>
<td>Sharpe</td>
<td>0.37</td>
<td>0.37</td>
<td>0.37</td>
<td>0.60</td>
<td>0.79</td>
<td>0.88</td>
<td>0.24</td>
<td>0.14</td>
<td>0.66</td>
</tr>
</tbody>
</table>

Note: Performance statistics from the strategy trading market CDS country spreads based on the relative changes in model spreads. At the end of each month we compute the 3-month change in the model spread for each country. For each country pair we compute the difference in the 3-month changes, as well as the 5-year average and the 5-year standard deviation of these differences. The resulting $z$-score is used as bet size in equation (6). The movements in the CDS spreads in the next month ($t$) are used to compute the performance of the trade using equation (6). We follow this strategy from April 2002 to February 2010. The table shows in each country column the annualized return, standard deviation and Sharpe ratio from the average returns of the 7 possible pairs. The final column does the same for all 28 unique country pairs.
Figure 2: Cumulative CDS strategy returns based on model-implied spreads

Note: Cumulative performance from the strategy trading market CDS country spreads based on the relative changes in model spreads. At the end of each month we compute the 3-month change in the model spread for each country. For each country pair we compute the difference in the 3-month changes, as well as the 5-year average and the 5-year standard deviation of these differences. The resulting z-score is used as bet size in equation (6). The movements in the CDS spreads in the next month (t) are used to compute the performance of the trade using equation (6). We follow this strategy from April 2002 to February 2010. The figure shows the cumulative portfolio performance based on all 28 unique country pairs.

Figure 2 shows the cumulative performance of the portfolio of 28 unique country pairs. This graph shows that most of the performance is obtained in the aftermath of the September 2008 default of Lehman Brothers. On the one hand this is positive for an EM debt investor, as this period caused a severe drawdown for local and foreign currency sovereign debt, emerging credits and emerging currencies. On the other hand the strategy does not add performance consistently throughout time. We believe that this is due to often having near zero model spreads. We, therefore, next look at a similar strategy but now based on exchange rate volatilities.

Strategy based on exchange rate volatilities

Table 7 shows the results for the CDS strategy based on changes in exchange rate volatilities. Also these results are quite strong, with again a positive result for each group of 7 country pairs as well as for the portfolio of 28 unique country pairs. The Sharpe ratio is 0.64 for the portfolio with a return of 0.77% per annum and a standard deviation of 1.19% per annum. With long/short bets the Sharpe ratio is 0.33. Hence again the size of the volatility changes is important.

Figure 3 shows the cumulative performance of the portfolio of 28 unique country pairs. In contrast to Figure 2 we see that the strategy based on exchange rate
volatility is making its performance more gradually over time, although also here crises aid the performance. Most performance is generated until September 2003 hence including the Brazilian crisis of 2002 and since November 2007 hence soon after the start of the credit crisis mid 2007.

Not surprisingly given the different times the two strategies based on model CDS spreads and exchange rate volatilities perform (the correlation is just 11%) the equally weighted combination of the two strategies gives a Sharpe ratio of 0.87. Using as an alternative past 3-month changes in LCL (local debt expressed in USD) gives a Sharpe ratio of 0.47, similar to Gebhardt et al.'s (2005) finding that the equity market leads the corporate bond market.

Table 7: CDS strategy based on changes in exchange rate volatilities

<table>
<thead>
<tr>
<th>Brazil</th>
<th>Hungary</th>
<th>Mexico</th>
<th>Philippines</th>
<th>Poland</th>
<th>South-Africa</th>
<th>South-Korea</th>
<th>Turkey</th>
<th>Portfolio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Return (%)</td>
<td>1.08</td>
<td>0.07</td>
<td>0.61</td>
<td>1.82</td>
<td>0.57</td>
<td>1.23</td>
<td>0.05</td>
<td>0.77</td>
</tr>
<tr>
<td>Stdev (%)</td>
<td>2.40</td>
<td>1.72</td>
<td>1.06</td>
<td>2.19</td>
<td>1.55</td>
<td>2.09</td>
<td>2.02</td>
<td>2.15</td>
</tr>
<tr>
<td>IR</td>
<td>0.45</td>
<td>0.04</td>
<td>0.57</td>
<td>0.83</td>
<td>0.37</td>
<td>0.59</td>
<td>0.02</td>
<td>0.36</td>
</tr>
</tbody>
</table>

Note: Performance statistics from the strategy trading market CDS country spreads based on the relative changes in exchange rate volatilities. At the end of each month we compute the 3-month change in the 3-month exchange rate volatility (computed based on the standard deviation of daily returns) for each country. For each country pair we compute the difference in the 3-month changes, as well as the 5-year average and the 5-year standard deviation of these differences. The resulting z-score is used as bet size in equation (6). The movements in the CDS spreads in the next month \( t \) are used to compute the performance of the trade using equation (6). We follow this strategy from April 2002 to February 2010. The table shows in each country column the annualized return, standard deviation and Sharpe ratio from the average returns of the seven possible pairs. The final column does the same for all 28 unique country pairs.

Figure 3: Cumulative CDS strategy returns based on exchange rate volatilities

Note: Cumulative performance from the strategy trading market CDS country spreads based on the relative changes in exchange rate volatilities. At the end of each month we compute the 3-month change in the 3-month exchange rate volatility for each country. For each country pair we compute the difference in the 3-month changes, as well as the 5-year average and the 5-year standard deviation of these differences. The resulting z-score is used as bet size in equation (6). The movements in the CDS spreads in the next month \( t \) are used to compute the performance of the trade using equation (6). We follow this strategy from April 2002 to February 2010. The figure shows the cumulative portfolio performance based on all 28 unique country pairs.
4.3.2 Directional strategies

The directional strategy intends to profit from the movement of the CDS spread of an individual country. Contrary to the relative strategy, the directional strategy is not hedged against a market-wide spread widening or tightening and the return of the directional strategy depends on the carry of CDS contract.

For the direction of the trade we look at 3-month changes in model-implied spreads and exchange rate volatility. For example, if the exchange rate volatility in Brazil has declined we expect the Brazilian CDS spread to decline as well. We therefore take a short position in the Brazilian CDS contract (selling default protection).

To make the bets in terms of risk comparable across the 8 countries we again scale by the CDS spread. In equation form we then get the following expression for the 1-month return in month $t$ from the CDS position in country $i$,

$$\text{Return}_{t}^{i} \approx \frac{\text{bet}_{t-1}}{\text{CDS}_{t-1}^{i}} \left[ D_{t-1} (\text{CDS}_{t}^{i} - \text{CDS}_{t-1}^{i}) - \frac{\text{CDS}_{t-1}^{i}}{12} \right]$$

(7)

where $D$ is the duration (5 years), $\text{bet}_{t-1}$ is a scaling factor related to the total bet size, and $\text{CDS}_{t}^{i}$ is the CDS spread in month $t$ in country $i$. For the success ratio and Sharpe ratio the total bet size does not matter. Because the size of the position is adjusted for the level of the CDS spread, the carry return is the same for each month and each country: the bet size divided by 12.

Strategy based on model spreads

Table 8 shows the results for the CDS strategy based on changes in model spreads. The results are reasonably strong, with a positive result for each country as well as for the portfolio. The Sharpe ratio is 0.41 for the portfolio with a return of 1.71% per annum and a standard deviation of 4.18% per annum. The information ratio is somewhat lower than the result for the relative strategy, but the standard deviation is almost three times as high, confirming the larger risk exposure. Finally, the carry has a negligible impact on results.
Table 8: CDS directional strategy based on changes in model spreads

<table>
<thead>
<tr>
<th>Country</th>
<th>Return (%)</th>
<th>Stdev (%)</th>
<th>Sharpe</th>
</tr>
</thead>
<tbody>
<tr>
<td>Brazil</td>
<td>1.09</td>
<td>2.87</td>
<td>0.38</td>
</tr>
<tr>
<td>Hungary</td>
<td>4.98</td>
<td>11.07</td>
<td>0.45</td>
</tr>
<tr>
<td>Mexico</td>
<td>0.27</td>
<td>4.36</td>
<td>0.06</td>
</tr>
<tr>
<td>Philippines</td>
<td>-1.32</td>
<td>3.78</td>
<td>-0.35</td>
</tr>
<tr>
<td>Poland</td>
<td>7.78</td>
<td>15.83</td>
<td>0.49</td>
</tr>
<tr>
<td>South-Africa</td>
<td>-0.06</td>
<td>4.84</td>
<td>-0.01</td>
</tr>
<tr>
<td>South-Korea</td>
<td>2.18</td>
<td>8.50</td>
<td>0.26</td>
</tr>
<tr>
<td>Turkey</td>
<td>-1.25</td>
<td>4.50</td>
<td>-0.28</td>
</tr>
<tr>
<td>Portfolio</td>
<td>1.71</td>
<td>4.18</td>
<td>0.41</td>
</tr>
</tbody>
</table>

Note: Performance statistics from the strategy trading market CDS based on the change in model spreads. At the end of each month we compute the 3-month change in the model spread. We compute the average and the standard deviation of these 3-month changes in the past 5 years. The resulting z-score is used as bet size in equation (7). The movement in the CDS spread in the next month \((t)\) is used to compute the performance of the trade using equation (7). We follow this strategy from April 2002 to February 2010. The table shows in each country column the annualized return, standard deviation and Sharpe ratio. The final column is the average of the eight countries.

Figure 4: Cumulative CDS strategy returns based on model-implied spreads

Note: Cumulative performance from the strategy trading market CDS based on the changes in model spreads. At the end of each month we compute the 3-month change in the model spread. We compute the average and the standard deviation of these 3-month changes in the past 5 years. The resulting z-score is used as bet size in equation (7). The movement in the CDS spread in the next month \((t)\) is used to compute the performance of the trade using equation (7). We follow this strategy from April 2002 to February 2010. The figure shows the cumulative portfolio performance based on the average of the eight countries.

Figure 4 shows the cumulative performance of the portfolio of 8 countries which is very similar to the relative strategy based on the model spread shown in Figure 2 in the sense that basically all the performance is made during the recent crisis.
Strategy based on exchange rate volatilities

Table 9 shows the results for the CDS strategy based on changes in the exchange rate volatility. These results are stronger, with a positive result for every country as well as for the portfolio. The Sharpe ratio is 0.66 for the portfolio with a return of 2.38% per annum and a standard deviation of 3.63% per annum.

Figure 5 shows the cumulative performance of the portfolio of 8 countries. Also in this case the performance is the highest during the recent crisis, but in contrast to the strategy based on model spreads performance is also positive prior to the crisis.

Table 9: CDS directional strategy based on changes in exchange rate volatilities

<table>
<thead>
<tr>
<th></th>
<th>Brazil</th>
<th>Hungary</th>
<th>Mexico</th>
<th>Philippines</th>
<th>Poland</th>
<th>South-Africa</th>
<th>South-Korea</th>
<th>Turkey</th>
<th>Portfolio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Return (%)</td>
<td>1.67</td>
<td>2.36</td>
<td>2.27</td>
<td>2.07</td>
<td>3.29</td>
<td>2.61</td>
<td>3.37</td>
<td>1.44</td>
<td>2.38</td>
</tr>
<tr>
<td>Stdev (%)</td>
<td>4.30</td>
<td>4.87</td>
<td>5.66</td>
<td>3.17</td>
<td>7.47</td>
<td>4.32</td>
<td>7.48</td>
<td>4.03</td>
<td>3.63</td>
</tr>
<tr>
<td>IR</td>
<td>0.39</td>
<td>0.48</td>
<td>0.40</td>
<td>0.65</td>
<td>0.44</td>
<td>0.60</td>
<td>0.45</td>
<td>0.36</td>
<td>0.66</td>
</tr>
</tbody>
</table>

Note: Performance statistics from the strategy trading market CDS based on the changes in the exchange rate volatility. At the end of each month we compute the 3-month change in the exchange rate volatility (computed based on the standard deviation of daily returns in the past 3 months). We also compute the average and standard deviation of these changes in the past 5 years. The resulting z-score is used as bet size in equation (7). The movement in the CDS spread in the next month \((t)\) is used to compute the performance of the trade using equation (7). We follow this strategy from April 2002 to February 2010. The table shows in each country column the annualized return, standard deviation and Sharpe ratio. The final column is the average of the eight countries.

Figure 5: Cumulative CDS strategy returns based on exchange rate volatilities

Note: Cumulative performance from the strategy trading market CDS country spreads based on the changes in the exchange rate volatility. At the end of each month we compute the 3-month change in the exchange rate volatility (computed based on the standard deviation of daily returns in the past 3 months). We also compute the average and standard deviation of these changes in the past 5 years. The resulting z-score is used as bet size in equation (7). The movement in the CDS spread in the next month \((t)\) is used to compute the performance of the trade using equation (7). We follow this strategy from April 2002 to February 2010. The figure shows the cumulative portfolio performance based on the average of the eight countries.
5. Conclusion

We provide the first empirical study into the Gray et al. (2007) structural model for sovereigns. We show that the structural approach for emerging countries that issue both local and foreign currency denominated debt has its merits. The model outcomes such as the distance-to-default, the default probability, and spreads are strongly correlated with market CDS spreads. Furthermore we show that exchange rate volatility has an important role in the model, similar to that of equity volatility in Merton’s structural model for corporations.

We build strategies based on the model spreads and exchange rate volatilities that show a very strong performance. This indicates that the market does not fully price in the sovereign balance sheet information into CDS spreads. Furthermore these strategies work especially well during crises, when EM debt investors suffer losses in local and foreign currency denominated debt for both sovereigns and corporations. Hence the strategy provides a nice diversification to EM debt investors.

For further research it will be interesting to look for improvements to the Merton framework applied to sovereigns. Similar to empirical results for the Merton model applied to corporations we find that the pricing is inaccurate. Model implied spreads are often near-zero and on average too small, also during crisis periods. In addition, according to Reinhart and Rogoff (2009) default probabilities probably depend much more on the overall level of debt than the amount of external debt. A different model may incorporate the default risk from a large amount of local debt.
References


Ben Dor, A., S. Polbennikov, and J. Rosten, 2007b. "DTS\textsuperscript{SM} (Duration Times Spread) for CDS, A new measure of spread sensitivity", *The Journal of Fixed Income* 16 (Spring), 32-44.


