A Multivariable Super-Twisting Sliding Mode Approach

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Abstract

This communique proposes a multivariable super-twisting sliding mode structure which represents an extension of the well-known single input case. A Lyapunov approach is used to show finite time stability for the system in the presence of a class of uncertainty. This structure is used to create a sliding mode observer to detect and isolate faults for a satellite system.

Key words: sliding modes, fault estimation, super-twisting, fault detection and isolation

1 Introduction

Sliding mode control has been an active area of research for many decades due its (at least theoretical) invariance to a class of uncertainty known as matched uncertainty [2]. More recently these ideas have been exploited extensively for the development of robust observers and have found applications in the area of fault detection and fault tolerant control [15,1]. However one of the disadvantages of traditional sliding mode control (1st order sliding modes) is the 'chattering' due to the discontinuous control action [2]. Higher order sliding modes (HOSM) remove the chattering effect while retaining the robustness of first order sliding modes and improving on their accuracy [3,4]. A disadvantage of imposing an r-th order sliding mode is the necessity of having $s, \dot{s}..s^{r-1}$ available (where $s(t)$ is the switching surface). However in one special case of second order sliding modes, the derivative information is not required. This is the so-called 'super-twisting' approach [11]. Until very recently stability, robustness and convergence rates in higher order sliding mode methods have been analyzed in terms of homogeneity or geometric arguments [5]. However in a succession of papers [6,16,14], Lyapunov methods were employed successfully for the first time to analyze the properties of the super-twisting algorithm for uncertain systems. This has opened the door for the integration of these ideas with other nonlinear tools including gain adaptation [13,10,7]. However in all these developments a single input control structure has essentially been considered. In many situations it is possible by control input scaling to transform a multi-input control problem with m control inputs into a decoupled problem involving m single input control structures and so the approaches in [13,10,7] work satisfactorily. Instead, in this communique, a multi-variable super-twisting structure is proposed, which is then analyzed using an extension of the Lyapunov ideas from [14]. An example involving a fault detection problem in a satellite system is used to demonstrate a situation in which the proposed multi-input super-twisting structure is useful. The notation used in the paper is quite standard – in particular, throughout the paper, $\|\cdot\|$ is used to represent the Euclidean norm.

2 Problem Statement and System Description

In multivariable sliding mode control and observation, the objective is to force to zero in finite time a constraint (or switching) function given by $\sigma(x)$, where $x \in \mathbb{R}^n$ is the state of the dynamical system and $\sigma : \mathbb{R}^n \mapsto \mathbb{R}^m$ [17]. In calculating the total time derivative of σ, for the case of conventional (first order) sliding modes, an expression

$$\dot{\sigma}(t) = a(t,x) + b(t,x)v + \gamma(t,\sigma) \quad (1)$$

is established where v is the manipulated variable (the control signal or the output error injection in the case of observer problems), $a(t,x) \in \mathbb{R}^m$ and $b(t,x) \in \mathbb{R}^{m\times m}$ are assumed to be known, and $\gamma(\cdot)$ represents unknown (but usually bounded) uncertainty. If $\det(b(t,x)) \neq 0$ then using the expression $v = b(t,x)^{-1}(\bar{v} - a(t,x))$ where the components of $\bar{v}$ are

$$\bar{v}_i = -k_1 \text{sign}(\sigma_i)|\sigma_i|^{1/2} - k_2\sigma_i + z_i \quad (2)$$
$$\dot{z}_i = -k_3 \text{sign}(\sigma_i) - k_4\sigma_i \quad (3)$$

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and $k_1, \ldots, k_4$ are scalar gains, the system

$$\dot{\sigma}_i = -k_1 \text{sign}(\sigma_i)|\sigma_i|^{1/2} - k_2\sigma_i + z_i + \gamma_i(t,\sigma) \quad (4)$$
$$\dot{z}_i = -k_3 \text{sign}(\sigma_i) - k_4\sigma_i \quad (5)$$

for $i = 1 \ldots m$ is obtained. Suppose $|\gamma_i(t,\sigma)| \leq d_i|\sigma_i|$ for some scalars d_i, then if the gains $k_1 \ldots k_4$ are chosen properly, it can be proved that $\sigma_i = \dot{\sigma}_i = 0$ in finite time: see for example [14]. Alternatively if $|\dot{\gamma}_i(t,\sigma)| \leq \bar{d}_i$ for some finite gains $\bar{d}_i$, then for appropriate gains $k_1 \ldots k_4$, it can be proved that $\sigma_i = \dot{\sigma}_i = 0$ in finite time: see [3,14]. In the literature such a controller is usually known as a super-twisting controller [3,11,4].

Suppose instead of (2)-(3) a non-decoupled injection term

$$\bar{v} = -k_1 \frac{\sigma}{||\sigma||^{1/2}} + z - k_2\sigma \quad (6)$$
$$\dot{z} = -k_3 \frac{\sigma}{||\sigma||} - k_4\sigma \quad (7)$$

is used where $k_1, \ldots, k_4$ are scalars. Then the result is a set of coupled equations rather than the decoupled structure in (4)-(5), and the work in [14] cannot be employed directly. (Note however, if $m = 1$ then the scalar control structure in (6)-(7) reverts to (2)-(3). Also in this situation $k_2 = k_4 = 0$ is usually selected.) Substituting (6) into (1) yields a special case of the system

$$\dot{\sigma} = -k_1 \frac{\sigma}{||\sigma||^{1/2}} + z - k_2\sigma + \gamma(t,\sigma) \quad (8)$$
$$\dot{z} = -k_3 \frac{\sigma}{||\sigma||} - k_4\sigma + \phi(t) \quad (9)$$

when $\phi(t) \equiv 0$. The term $\phi(t)$ in (9) is included here to maintain compatibility with the more generic formulation in [14], and will be exploited in the example in Section 3. The terms $\gamma(t,\sigma)$ and $\phi(t)$ are assumed to satisfy

$$||\gamma(t,\sigma)|| \leq \delta_1||\sigma|| \quad (10)$$
$$||\phi(t)|| \leq \delta_2 \quad (11)$$

for known scalar bounds $\delta_1, \delta_2 > 0$.

Remark 1: Note that the uncertainty classes discussed earlier are a subset of the uncertainty in (10). Also note the matrix $b(t,x)$ must be known to achieve the structures in (8)-(9) (and also the decoupled one in (2)-(3)).

Remark 2: Also note that the differential equations in (4)-(5) and (8)-(9) have discontinuous right hand sides. The solutions to such equations must therefore be understood in the Filippov sense [8].

Remark 3: Equations such as (8)-(9) can also appear in the context of observer problems as will be demonstrated in Section 3.

Proposition 1 *For the system in (8)-(9), there exist a range of values for the gains $k_1 \ldots k_4$, such that the variables σ and $\dot{\sigma}$ are forced to zero in finite time and remain zero for all subsequent time.*

Proof: For the system (8)-(9), consider as a Lyapunov-function [1] candidate

$$V(\sigma,z) = 2k_3||\sigma|| + k_4\sigma^T\sigma + \frac{1}{2}z^Tz + \zeta^T\zeta \quad (12)$$

where $\zeta := k_1 \frac{\sigma}{||\sigma||^{1/2}} + k_2\sigma - z$. Define the subspace

$$\mathcal{S} = \{(\sigma,z) \in \mathbb{R}^{2m} \ : \ \sigma = 0\} \quad (13)$$

then $V(\sigma,z)$ in (12) is everywhere continuous, and differentiable everywhere except on the subspace $\mathcal{S}$. Furthermore it is easy to verify that $V(\cdot)$ is positive definite and radially unbounded.

Differentiating the expression in (12) yields

$$\begin{aligned}\dot{V}(\sigma,z) &= (2k_3 + \frac{k_1^2}{2})\frac{\sigma^T\dot{\sigma}}{||\sigma||} + 2(\frac{k_2^2}{2} + k_4)\sigma^T\dot{\sigma} + 2z^T\dot{z} \\ &+ \frac{3}{2}k_1k_2\frac{\sigma^T\dot{\sigma}}{||\sigma||^{1/2}} - k_2(\dot{\sigma}^Tz + \sigma^T\dot{z}) \\ &- k_1\left(-\frac{1}{2}\frac{(\sigma^T\dot{\sigma})(z^T\sigma)}{||\sigma||^{5/2}} + \frac{(\dot{z}^T\sigma + z^T\dot{\sigma})}{||\sigma||^{1/2}}\right) \quad (14)\end{aligned}$$

then substituting for (8)-(9) it follows from (14) using straightforward algebra that

$$\begin{aligned}\dot{V}(\sigma,z) &= -(k_1k_3 + \frac{k_1^3}{2})\frac{||\sigma||^2}{||\sigma||^{3/2}} + \frac{3}{2}k_1k_2\frac{\sigma^T\gamma}{||\sigma||^{1/2}} \\ &- (k_2k_4 + k_2^3)||\sigma||^2 - (k_4k_1 + \frac{5}{2}k_1k_2^2)\frac{||\sigma||^2}{||\sigma||^{1/2}} \\ &+ k_1^2\frac{\sigma^Tz}{||\sigma||} + 2k_2^2\sigma^Tz + 3k_1k_2\frac{\sigma^Tz}{||\sigma||^{1/2}} \\ &- k_2||z||^2 + \frac{k_1}{2}\frac{(\sigma^Tz)(z^T\sigma)}{||\sigma||^{5/2}} - k_1\frac{z^Tz}{||\sigma||^{1/2}} \\ &+ (2k_3 + \frac{k_1^2}{2})\frac{\sigma^T\gamma}{||\sigma||} + (2k_4 + k_2^2)\sigma^T\gamma \\ &- (k_3k_2 + 2k_1^2k_2)\frac{||\sigma||^2}{||\sigma||} \\ &- k_2\gamma^Tz + \frac{k_1}{2}\frac{\sigma^T\gamma z^T\sigma}{||\sigma||^{5/2}} - k_1\frac{z^T\gamma}{||\sigma||^{1/2}} \\ &+ 2z^T\phi - k_2\sigma^T\phi - k_1\frac{\phi^T\sigma}{||\sigma||^{1/2}} \quad (15)\end{aligned}$$

[1] Note that in the special case when $m = 1$, the Lyapunov function in (12) becomes the one originally proposed in [14].

for all $(\sigma, z) \notin \mathcal{S}$. Then from simple bounding arguments

$$
\begin{aligned}
\dot{V}(\sigma,z) \leq & -(k_1k_3 + \frac{k_1^3}{2})||\sigma||^{1/2} - (k_3k_2 + 2k_1^2k_2)||\sigma|| \\
& - (k_2k_4 + k_2^3)||\sigma||^2 - (k_4k_1 + \frac{5}{2}k_1k_2^2)||\sigma||^{3/2} \\
& + k_1^2\frac{|\sigma^Tz|}{||\sigma||} + 2k_2^2|\sigma^Tz| + 3k_1k_2\frac{|\sigma^Tz|}{||\sigma||^{1/2}} \\
& - k_2||z||^2 + \frac{k_1}{2}\frac{|\sigma^Tz|^2}{||\sigma||^{5/2}} + (2k_3 + \frac{k_1^2}{2})\frac{|\sigma^T\gamma|}{||\sigma||} \\
& + (2k_4 + k_2^2)|\sigma^T\gamma| + \frac{3}{2}k_1k_2\frac{|\sigma^T\gamma|}{||\sigma||^{1/2}} \\
& + k_2|\gamma^Tz| + \frac{k_1}{2}\frac{|\sigma^T\gamma||z^T\sigma|}{||\sigma||^{5/2}} + k_1\frac{|z^T\gamma|}{||\sigma||^{1/2}} \\
& + 2z^T\phi + k_2|\sigma^T\phi| + k_1\frac{|\phi^T\sigma|}{||\sigma||^{1/2}} \qquad (16)
\end{aligned}
$$

Using the Cauchy-Schwartz inequality on the inner product terms, together with the bounds on the terms $||\gamma||$ and $||\phi||$ from equation (10)-(11):

$$
\begin{aligned}
\dot{V}(\sigma,z) \leq & -(k_1k_3 + \frac{k_1^3}{2})||\sigma||^{1/2} - (k_2k_3 + 2k_1^2k_2)||\sigma|| \\
& - (k_1k_4 + \frac{5}{2}k_1k_2^2)||\sigma||^{3/2} + k_1^2||z|| - (k_2k_4 \\
& + k_2^3)||\sigma||^2 + 2k_2^2||\sigma||||z|| + 3k_1k_2||\sigma||^{1/2}||z|| \\
& - k_2||z||^2 + \frac{k_1}{2}\frac{||z||^2}{||\sigma||^{1/2}} + (2k_3 + \frac{k_1^2}{2})\delta_1||\sigma|| \\
& + (2k_4 + k_2^2)\delta_1||\sigma||^2 + \frac{3}{2}k_1k_2||\sigma||^{3/2}\delta_1 \\
& + k_2\delta_1||\sigma||||z|| + \frac{3}{2}k_1||\sigma||^{1/2}||z||\delta_1 \\
& + 2\delta_2||z|| + k_2\delta_2||\sigma|| + k_1\delta_2||\sigma||^{1/2} \qquad (17)
\end{aligned}
$$

Define $x = \text{col}(||\sigma||^{1/2}, ||\sigma||, ||z||)$ then from (17)

$$\dot{V} \leq -\frac{1}{||\sigma||^{1/2}}x^T\Omega x - x^T\Psi x \qquad (18)$$

where

$$\Omega = \begin{bmatrix} \Omega_{11} & 0 & \Omega_{13} \\ 0 & \Omega_{22} & \Omega_{23} \\ \Omega_{31} & \Omega_{32} & \Omega_{33} \end{bmatrix} \qquad (19)$$

with elements

$$
\begin{aligned}
\Omega_{11} &:= \tfrac{1}{2}k_1^3 + k_1k_3 - \delta_2k_1 \\
\Omega_{13} &:= -\tfrac{1}{2}k_1^2 - \delta_2 \\
\Omega_{22} &:= k_4k_1 + \tfrac{5}{2}k_2^2k_1 - \tfrac{3}{2}k_1k_2\delta_1 \\
\Omega_{23} &:= -\tfrac{3}{2}k_1k_2 \\
\Omega_{31} &:= \Omega_{13}, \Omega_{32} := \Omega_{23} \\
\Omega_{33} &:= \tfrac{1}{2}k_1
\end{aligned}
$$

and

$$\Psi = \begin{bmatrix} \Psi_{11} & 0 & \Psi_{13} \\ 0 & \Psi_{22} & \Psi_{23} \\ \Psi_{31} & \Psi_{32} & \Psi_{33} \end{bmatrix} \qquad (20)$$

with elements

$$
\begin{aligned}
\Psi_{11} &:= k_2k_3 + 2k_1^2k_2 - k_2\delta_2 - (2k_3 + \tfrac{1}{2}k_1^2)\delta_1 \\
\Psi_{13} &:= -\tfrac{3}{4}k_1\delta_1 \\
\Psi_{22} &:= k_4k_2 + k_2^3 - (k_2^2 + 2k_4)\delta_1 \\
\Psi_{23} &:= -k_2^2 - \tfrac{1}{2}k_2\delta_1 \\
\Psi_{31} &:= \Psi_{13}, \Psi_{32} := \Psi_{23} \\
\Psi_{33} &:= k_2
\end{aligned}
$$

It is easy to verify the symmetric matrix $\Omega > 0$ if the inequalities $k_1 > \sqrt{2\delta_2}$, $k_2 > 0$, $k_3 > k_3^\Omega$ and $k_4 > k_4^\Omega$ are satisfied where

$$k_3^\Omega := 3\delta_2 + \frac{2\delta_2^2}{k_1^2} \qquad (21)$$

$$k_4^\Omega := \frac{\beta_1}{\beta_2} + 2k_2^2 + \tfrac{3}{2}k_2\delta_1 \qquad (22)$$

with the positive scalar $\beta_1 = (\frac{3}{2}k_1^2k_2 + 3\delta_2k_2)^2$ and the scalar $\beta_2 = k_3k_1^2 - 2\delta_2^2 - 3\delta_2k_1^2$.

Likewise the remaining symmetric matrix $\Psi > 0$ if the inequalities $k_1 > 0$, $k_2 > 2\delta_1$, $k_3 > k_3^\Psi$ and $k_4 > k_4^\Psi$ are satisfied where

$$k_3^\Psi := \frac{\frac{9}{16}(k_1\delta_1)^2}{k_2(k_2 - 2\delta_1)} + \frac{\frac{1}{2}k_1^2\delta_1 - 2k_1^2k_2 + k_2\delta_2}{(k_2 - 2\delta_1)} \qquad (23)$$

$$k_4^\Psi := \frac{\alpha_1}{\alpha_2(k_2 - 2\delta_1)} + \frac{2k_2^2\delta_1 + \frac{1}{4}k_2\delta_1^2}{(k_2 - 2\delta_1)} \qquad (24)$$

in which the scalars $\alpha_1 := \frac{9}{16}(k_1\delta_1)^2(k_2 + \frac{1}{2}\delta_1)^2/k_2^2$ and $\alpha_2 := k_2(k_3 + 2k_1^2 - \delta_2) - (2k_3 + \frac{1}{2}k_1^2)\delta_1 - \frac{9}{16}(k_1\delta_1)^2/k_2$. In order to satisfy both $\Omega > 0$ and $\Psi > 0$, the k_i's are chosen as

$$
\left.
\begin{aligned}
k_1 &> \sqrt{2\delta_2} \\
k_2 &> 2\delta_1 \\
k_3 &> \max(k_3^\Omega, k_3^\Psi) \\
k_4 &> \max(k_4^\Omega, k_4^\Psi)
\end{aligned}
\right\} \qquad (25)
$$

and hence from (18)

$$\dot{V} \leq -\frac{1}{||\sigma||^{1/2}}x^T\Omega x \leq -\frac{1}{||\sigma||^{1/2}}\lambda_{min}(\Omega)||x||^2 \qquad (26)$$

using Rayleigh's inequality. Define $X := col(\frac{\sigma}{||\sigma||^{1/2}}, \sigma, z)$ and note that $||X|| = ||x||$ for all values of the states σ

and z. Therefore (26) can be written as

$$\dot{V} \leq -\frac{1}{||\sigma||^{1/2}}\lambda_{min}(\Omega)||X||^2 \tag{27}$$

Using similar arguments to [6], the Lyapunov function in (12) can be written as $V = X^T P X$ for an appropriate symmetric positive definite matrix $P \in \mathbb{R}^{3m\times 3m}$ and $V \leq \lambda_{max}(P)\|X\|^2$ from Rayleigh's inequality. Therefore from (27)

$$\dot{V} \leq -\frac{1}{||\sigma||^{1/2}}\frac{\lambda_{min}(\Omega)}{\lambda_{max}(P)}V \tag{28}$$

Because $V^{1/2} > \sqrt{\lambda_{min}(P)}||\sigma||^{1/2}$, it follows that

$$\dot{V} \leq -\alpha V^{1/2}, \qquad \text{where } \alpha = \frac{\lambda_{min}(\Omega)\sqrt{\lambda_{min}(P)}}{\lambda_{max}(P)} \tag{29}$$

for all $(\sigma(t), z(t)) \notin \mathcal{S}$. Note the absolutely continuous trajectories of the Filippov solution to (8)-(9) cannot stay on the set $\mathcal{S}\setminus\{0\}$ (i.e the set $\mathcal{S}$ from (13) excluding the origin when both $\sigma = z = 0$). This follows since if $(\sigma(t_0), z(t_0)) \in \mathcal{S}\setminus\{0\}$ at the time instant t_0, $\sigma(t_0) = 0$ and from equation (8), $\dot{\sigma}(t)|_{t=t_0} = z(t_0) \neq 0$ since $(\sigma(t_0), z(t_0)) \in \mathcal{S}\setminus\{0\}$. As a consequence, at least one component $\sigma_i(t)$ passes *monotonically* through zero during some (possibly small) time interval $T_0 \subset \mathbb{R}$ containing t_0 from the absolute continuity of $z_i(t)$ and the fact that $z_i(t_0) \neq 0$. Therefore *along the Filippov solution* to (8)-(9), inequality (29) holds almost everywhere, and thus $V(t)$ is a continuously decreasing function of time. Then using the 'Lyapunov Theorem' for differential inclusions in Proposition 14.1 [12], it can be concluded that the equilibrium point at the origin $(\sigma, z) = 0$ is reached in finite time [2]. Finally substituting for $\sigma = z = 0$ in the right hand side of (8) implies $\dot{\sigma} = 0$ (since $\gamma(0) = 0$) and therefore $\sigma = \dot{\sigma} = 0$ in finite time as claimed. ■

Remark 4: Note the proof given above is constructive and in particular if the gains are chosen to satisfy (25) where the scalars δ_1 and δ_2 are given (10)-(11) and the scalars $k_3^\Omega, k_3^\Psi, k_4^\Omega, k_4^\Psi$, which depend on δ_1 and δ_2, are given in (21)-(22) and (23)-(24), then from Proposition 1, the solution to (8)-(9) satisfies $\sigma = \dot{\sigma} = 0$ in finite time.

Remark 5: These conditions are not identical to the ones in [6], perhaps because of the different approximations used to obtain the expressions in (17).

3 Example

The nonlinear rigid body equations of motion of a satellite, with thrusters providing the required torque, can be represented in the following form [9]:

$$\dot{w} = J^{-1}(T - w^x J w) \tag{30}$$

where $T \in \mathbb{R}^3$ are the torques from the thrusters, $w \in \mathbb{R}^3$ denotes the inertial angular velocities, $J \in \mathbb{R}^{3\times 3}$ is a positive definite inertia matrix, and w^x denotes

$$w^x := \begin{bmatrix} 0 & -w_3 & w_2 \\ w_3 & 0 & -w_1 \\ -w_2 & w_1 & 0 \end{bmatrix} \tag{31}$$

where $w = col(w_1, w_2, w_3)$ are the rate components in the three axes. In the event of faults associated with the thrusters the system in (30) can be re-modelled as

$$\dot{w} = J^{-1}(T + f - w^x J w) \tag{32}$$

where $f \in \mathbb{R}^3$ represents the unknown torque arising from the fault. Assuming the inertia matrix J is known the objective is to create a fault detection scheme for such a system. One approach is to estimate f from knowledge of w and T only. For this purpose consider an observer of the form

$$\dot{\hat{w}} = J^{-1}(T - \hat{w}^x J\hat{w}) + \nu \tag{33}$$

where the output error injection signal

$$\nu = k_1\frac{\sigma}{||\sigma||^{1/2}} - \xi + k_2\sigma \tag{34}$$

$$\dot{\xi} = -k_3\frac{\sigma}{||\sigma||} - k_4\sigma \tag{35}$$

and $\sigma = w - \hat{w}$. Define $z = \xi + J^{-1}f$ then it follows the time varying vectors σ, z satisfy (8)-(9) where by definition

$$\gamma(\sigma) = J^{-1}(\hat{w}^x J\hat{w} - (\sigma + \hat{w})^x J(\sigma + \hat{w})) \tag{36}$$

and $\phi(t) = J^{-1}\dot{f}(t)$.

Remark 6: Because of the fact that discontinuities in the unit vector expression in (9) will only occur when all the components of $\sigma_i = 0$, the proposed structure is likely to have improved chattering reduction properties.

During the sliding motion $\sigma = \dot{\sigma} = 0$ and from (8) this implies $z = 0$ since from (36), $\gamma(0) = 0$. Consequently, since $z = 0$ during the sliding motion, by definition $z = \xi + J^{-1}f = 0$. If the fault estimate $\hat{f}$ is chosen as

$$\hat{f}(t) := -J\xi(t) \tag{37}$$

then during sliding $\hat{f} = f$. Note that $\xi(t)$ is available in realtime as the solution to (35) and so $\hat{f}(t)$ from (37) is

[2] The 'generalised' Lyapunov theorem in Proposition 14.1 [12] only requires continuity and not differentiability of $V(t)$ along the solution trajectories. This property is key to the proof above, which follows closely the arguments in [13].

a realtime estimate of thruster faults.
In the simulations, the initial conditions in the satellite model are $w(0) = [\,-0.0021\ -0.0067\ 0.0253\,]$ and

$$J = 1.0e^{003} \times \begin{bmatrix} 1.2757 & -0.0040 & -0.0230 \\ -0.0040 & 0.6597 & 0.0063 \\ -0.0230 & 0.0063 & 0.8750 \end{bmatrix}$$

The super-twisting observer gains are chosen as follows; $\delta_1 = 10, \delta_2 = 0.5, k_1 = 2, k_2 = 40, k_3 = 5.5625, k_4 = 60$ which satisfy the conditions of Proposition 1. Figure 1 shows that the state estimation error σ becomes zero in finite time as does the fault estimation error $e_f = \hat{f} - f$. Figure 2 shows that $\sigma = \dot{\sigma} = 0$ simultaneously at approximately 0.11 seconds. Figure 3 shows the fault estimates of two simultaneous unknown inputs comprising two different sinusoids in channels 1 and 3 beginning at $t = 0$. Visually perfect replication takes place.

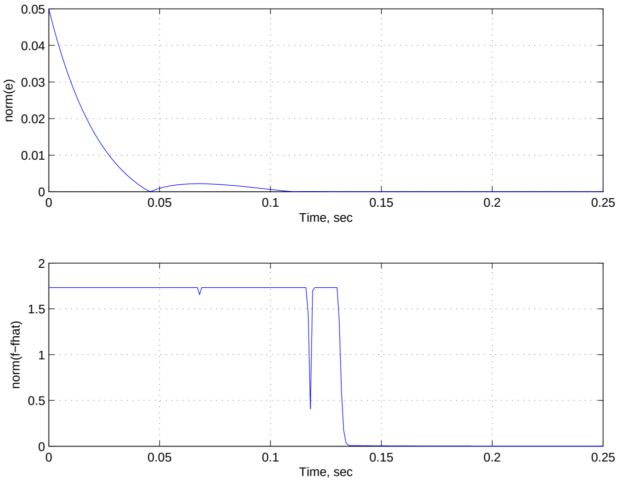

Fig. 1. States estimation and fault reconstruction errors

4 Conclusion

This communique has presented a novel Lyapunov based super twisting sliding mode structure for multivariable situations. This represents a generalization of the well-known single output case. A situation is presented in which this multivariable generalization provides a more elegant solution than trying to employ a decoupled collection of single variable structures.

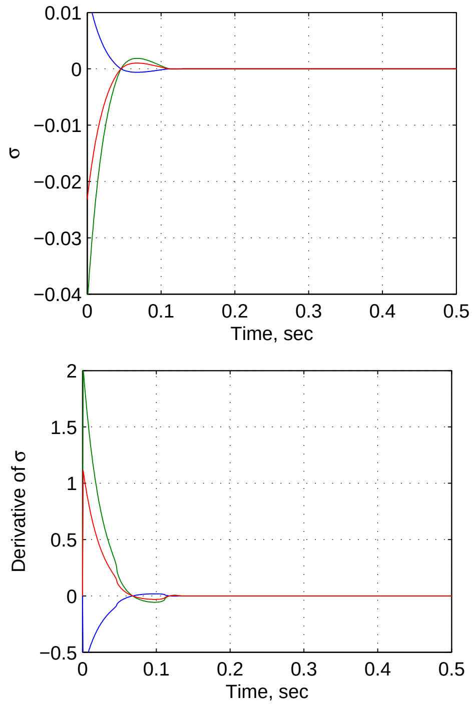

Fig. 2. Plots of σ and $\dot{\sigma}$

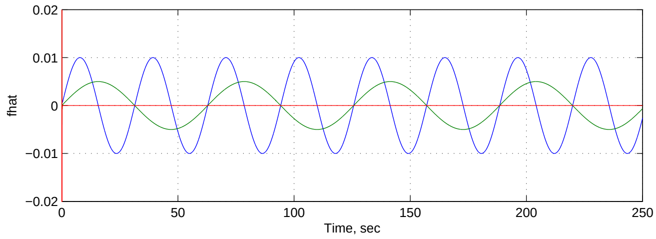

Fig. 3. Fault reconstruction errors

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