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FORTRAN PROGRAM **FOR** PREDICTING OFF-DESIGN PERFORMANCE OF CENTRIFUGAL COMPRESSORS

by Michael R. Galvas

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 $\label{eq:2.1} \mathcal{L}(\mathcal{L}^{\text{max}}_{\mathcal{L}}(\mathcal{L}^{\text{max}}_{\mathcal{L}})) \leq \mathcal{L}(\mathcal{L}^{\text{max}}_{\mathcal{L}}(\mathcal{L}^{\text{max}}_{\mathcal{L}}))$ $\mathcal{L}(\mathcal{L}^{\text{max}}_{\mathcal{L}})$. The set of $\mathcal{L}^{\text{max}}_{\mathcal{L}}$

 $\label{eq:2.1} \frac{1}{\sqrt{2}}\int_{\mathbb{R}^3}\frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{2}}\right)^2\frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{2}}\right)^2\frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{2}}\right)^2\frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{2}}\right)^2.$ $\mathcal{L}(\mathcal{A})$ and $\mathcal{L}(\mathcal{A})$

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FORTRAN PROGRAM FOR PREDICTING OFF-DESIGN PERFORMANCE

OF CENTRIFUGAL COMPRESSORS

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SUMMARY

A **FORTRAN** program for calculating the off-design performance **of** centrifugal compressors with channel diffusers is presented. Use of the program requires complete knowledge of the overall impeller and diffuser geometries. Individual losses are computed using analytical equations and empirical correlations which relate loss levels to velocity diagram characteristics and overall geometry. On a given speed line compressor performance is calculated for a range of inlet velocity levels. At flow rates between surge and choke, individual efficiency decrements, compressor overall efficiency, and compressor total pressure ratio are tabulated. An example case of performance comparison with a compressor built by a commercial engine manufacturer is presented to demonstrate the correlation with limited experimental data.

INTRODUCTION

Centrifugal **compressors are** frequently **used** in refrigeration **cycles, refining** systems, aircraft auxiliary systems, turboshaft engines, and other systems **where** light**weight** compact compression is required. The off-design performance characteristics of centrifugal compressors are of interest because of the large effects that compressor component performance has on overall cycle **performance** and because the compressor is required to operate at off-design conditions much of the time. For turboshaft engines assuming a drive turbine inlet temperature of 1422 K and efficiency of 90 percent, a 2.6-percent loss in compressor efficiencyresults in **a** 2-percent increase in engine specific fuel consumption (SFC) and a 2.8-percent loss in engine power at design speed. These losses become more pronounced at part-power settings. In addition to good performance at off-design flow rates it is important that the compressor operate stably over the **range** of flows **and** speeds required by the **engine** operating envelope. The usable range of the compressor pressure ratio-mass flow characteristic is bounded by the surge and choke mass flow rates. Operation at flows less than the surge point flow should be avoided because of potentially dangerous vibrations induced by the intermittent flow reversals and power loss. Operation with the compressor choked is generally avoided because of the poor compressor efficiency and pressure ratio at the choke point. The problem undertaken in this analysis is to determine the centrifugal compressor performance characteristics over a range of rotative speeds and flow rates and predict the usable range of flow rates at which the compressor can operate. The method of analysis uses the loss correlations and equations of reference 1 with surge and choke criteria added to predict compressor operating range.

A FORTRAN program has been developed at NASA Lewis Research Center which predicts centrifugal compressor performance through utilization of empirical correlations which are related to the compressor geometry and velocity diagram characteristics. A complete knowledge of the compressor overall geometry and working fluid inlet total conditions is required for its use. Working fluid state conditions and flow properties are calculated using a mean streamline one-dimensional analysis. The program is limited to centrifugal compressors with channel diffusers operating up to their choke point. A comparison of calculated and experimental performance is given to demonstrate the correlation with limited experimental data. The experimental data presented for comparison were obtained from a compressor developed by Solar Aircraft Company, Division of International Harvester, Inc. Shock losses in the rotor are neglected. Clearance losses are considered to be inherent in the impeller losses since good performance correlation is achieved on compressors operating with reasonable clearances.

METHOD OF ANALYSIS

Input values of (V/V_{cr}) are used to determine compressor mass flow rate. (Symbols are defined in appendix A and compressor stations are shown in fig. 1.) Individual losses are calculated using velocity diagram characteristics and empirical correlations determined by the input absolute velocity level and compressor geometry. For compressors having inlet guide vanes it is assumed that the vanes are placed in a constantarea annulus having no wall curvatures or slopes. Overall compressor efficiency, total pressure ratio, and mass flow rate are tabulated for each operating point inside the predicted range for each speed line that is input. A complete discussion of the loss calculations is discussed in appendix B.

Figure 1. - Meridional cross section of compressor.

Required **Information**

The compressor data **needed** to use the FORTRAN program are the following: inducer **inlet** tip diameter, inducer inlet hub-tip **diameter** ratio, inducer inlet blade angle at **the** root-mean-square **diameter,** inducer blade blockage at **the** rms **diameter, inducer** inlet hub and tip **wall** slopes, inducer inlet hub and **tip wall curvatures,** inlet solid-body swirl angle at **the** rms **diameter,** inducer-tip to impeller-exit **diameter** ratio, **impeller** exit blade angle, impeller blade exit height, **number** of impeller blades at **exit,** vaneless diffuser diameter ratio, vaned **diffuser** setting angle, total vaned **diffuser throat** area, vaned **diffuser** area ratio, *skin* friction **coefficient** of the **wetted** surfaces, **design** rotative speed, and **the** fraction of **design** rotative speed for **which** the **calculations** are **to** be performed.

The **working** fluid properties and state conditions required are: inlet total pressure, inlet total **temperature, inlet** dynamic viscosity based on **total** conditions, **specific** heat ratio, and gas **constant.**

Two control variables are used to select the options of solid-body prewhirl and **im**pellers **with** splitter blades. Inducer inlet hub and **tip** wall slopes and **curvatures** are all set equal to zero for compressors with inlet guide vanes.

Information which corresponds to the design speed line for the example compressor is as follows:

Calculated Losses

Inlet guide **vane** loss. - Equations **relating** working fluid kinetic energy level, boundary layer, and blade geometry were developed for axial turbine stators in reference 2. These equations were used to calculate state conditions downstream of the inlet guide vanes; thus the velocity diagram characteristics were found by satisfying onedimensional continuity.

An inlet guide vane loss coefficient is calculated using the same assumptions about blade geometry and boundary layer characteristics as used in reference 3. The inlet guide vane loss is then calculated as a fraction of the ideal kinetic energy at the rms diameter.

Inducer incidence loss. - Reference 4 developed equations for optimum inducer incidence angle of centrifugal compressors with axial inlets. The optimum incidence angle is calculated from the inlet velocity diagram characteristics and blade blockage at the rms diameter assuming an incompressible working fluid. The enthalpy loss due to incidence is calculated assuming that the relative velocity component normal to the optimum incidence angle is lost. Therefore, the incidence loss is simply

$$
\Delta h_{INC} = \frac{w_L^2}{2C_p}
$$

where

$$
W_{L} = W_{1MF} \sin \left| \beta_{opt} - \beta_{1MF} \right|
$$

Blade loading loss. - Boundary layer growth in the impeller is highly dependent on the diffusion of the working fluid internal to the impeller itself. Reference 5 proposed an equation for calculating the diffusion factor of the impeller based on a uniform velocity loading along the blade chord. This equation is used to calculate the impeller diffusion factor for impellers without splitters. A modified form, with reduced penalty due to aerodynamic work input, is used for impellers which have a set of splitter blades. With the diffusion factor calculated by these methods the blade **loading loss** was expressed as

$$
\Delta h_{BL} = 0.05 D_f^2 u_3^2
$$

Skin friction loss. - In addition to the losses resulting from the aerodynamic loading of the impeller blades, the impeller incurs losses due to skin friction of the impeller **and shroud wetted areas.** Reference 5 **developed an equation** for this loss **based on** fully **developed** turbulent pipe flow. In the **case** of impellers with splitter blade rows, the **empirical constant** appearing in the **equation** is modified to account for the higher mean **channel relative** velocity **caused** by the addition of splitters. The general **equation** used for skin friction loss is

$$
\Delta h_{SF} = K_{DF}C_f \frac{\frac{L}{D_3}}{\frac{D_{HYD}}{D_3}} \left(\frac{w}{u_3}\right)_{av}^2 u_3^2
$$

where $K_{\text{SF}} = 5.6$ for conventional impellers and $K_{\text{SF}} = 7.0$ for impellers with tandem blades.

Disk friction loss. - The specific loss due to windage on the compressor back face is **calculated** using the equation

$$
\Delta h_{\rm DF} = 0.01356 \frac{\rho_3}{w \rm Re} u_3^3 D_3^2
$$

This **is** a form of the **disk** friction power loss of reference 6.

Recirculation loss. - Losses resulting from **work done** on the working fluid **due to** backflow into the impeller are expressed as

$$
\Delta h_{\rm RC} = 0.02 \sqrt{\tan \alpha_3} D_{\rm f}^2 u_3^2
$$

This is **a** modification of **the equation proposed** in **reference** 5.

Vaneless diffuser loss. **-** The flow angle and Mach number variation with radius in the vaneless space are determined by numerical solution of the differential **equations** for vaneless space flow developed in reference **7.** The **equations** are simplified through the assumptions of adiabatic flow and constant geometric depth passage. The radial total **pressure** distribution in the vaneless space is calculated using the equation derived in reference 5. **When** the fluid **state** and flow properties are determined at the vaned diffuser leading **edge** radius by the methods described previously, the vaneless diffuser loss is calculated from the equation

$$
\Delta h_{\text{VLD}} = C_p T_3 \left[\left(\frac{p_4}{p_4^{'}} \right)^{\left(\gamma - 1\right) \big/ \gamma} \right. \left. - \left(\frac{p_4}{p_3^{'}} \right)^{\left(\gamma - 1\right) \big/ \gamma} \right]
$$

Vaned diffuser loss. - Curves of maximum static pressure recovery coefficient at a given area ratio were extrapolated from the test data for square throat diffusers reported in reference 8 for various combinations of vaned diffuser throat Mach number, aerodynamic blockage, and area ratio. These data were recorded for channel diffusers with symmetrical pressure Ioadings about the channel centerline. The vaned diffuser in a centrifugal compressor is loaded with a pressure gradient across the channel. In this analysis it was assumed that the difference in the loadings between the test diffusers of reference 8 and the compressor diffusers would have no effect on calculated diffuser recovery. A one-seventh power velocity distribution in the boundary layer along the vaneless space endwalls is used to calculate the displacement thickness representing vaned ** diffuser throat blockage. The value of static pressure recovery coefficient C_p corrected responding to the vaned diffuser geometric area ratio, inlet Mach number, and aerodynamic blockage is extrapolated from the test data.

The vaned diffuser exit critical velocity ratio is calculated using one-dimensional continuity. Vaned diffuser loss is then calculated using the equation

$$
\Delta h_{VD} = C_p T_3 \left[\left(\frac{p_6}{p_6'} \right)^{(\gamma - 1)/\gamma} - \left(\frac{p_6}{p_5'} \right)^{(\gamma - 1)/\gamma} \right]
$$

Calculation of Compressor **Choking** Flow

Two criteria are **used** in the prediction of compressor choking flow: (1) inducer choke and (2) vaned diffuser choke. Reference 9 presented data on inducer choking incidence levels as functions of relative Mach number and blade angle at the rms diameter for inducer pitch-chord ratios of 0.5 and thickness-chord ratios of 0.05. These data were used to predict compressor choking flow due to inducer choke.

Vaned diffuser choke is predicted from one-dimensional continuity using the computed values of weight flow, total temperature, total pressure, aerodynamic blockage, and the geometric throat area. The maximum value of the one-dimensional weight flow function is calculated from the equation

$$
\left(\frac{\mathbf{w}\,\sqrt{\mathbf{T}_3'}}{\mathbf{p}_5'A_5\mathbf{B}_5}\right)_{\text{max}} = \sqrt{\frac{\gamma}{\mathbf{R}}\left(\frac{2}{\gamma+1}\right)^{(\gamma+1)/(\gamma-1)}}
$$

At the first input **critical** velocity ratio where the left side of the **equation equals** or **ex**ceeds the right side the compressor is choked. One would expect **the** speed lines to have **infinite** slopes at **the** choke point. The vertical portions of the speed lines are approximated through the assumption of zero static pressure recovery in the vaned diffuser for the cases of vaned diffuser choke. The impeller characteristic should exhibit a vertical slope at the impeller choke point but this is not predicted by the **correlations** used.

Calculation of Compressor Surge Point

The vaned diffuser is assumed **to** be the component which governs the location of **the compressor** surge point. **Examination** of performance **characteristics** of **centrifugal** compressors covering a range of pressure ratios indicates **that** compressor flow range can be expressed as a function of the vaned diffuser leading edge Mach number. **With** a good impeller-diffuser **throat** area match the flow range **will** vary with **the** diffuser leading edge Mach number as shown in figure 2. This curve was deduced from unpublished operating range data for centrifugal compressors with design point pressure ratios ranging from 1.9 to 10.0 .

Figure 2. - Compressor flow range as a function of vaned diffuser leading edge Mach number.

This section compares the **results** obtained by the analytical **procedure** described herein to the experimental results obtained with a commercially manufactured centrifugal compressor.

Figure 3 shows the comparison between calculated and experimental pressure ratios as functions of corrected flow for 60, 80, 90, and 100 percent rotative speeds. *Choking* flow is caused by inducer choke at 100 percent speed and by diffuser choke at the other speeds. The assumption of zero static pressure recovery at the vaned diffuser choke point permits an estimate of the flow rate at which the compressor pressure ratio characteristic becomes vertical. Experimentally, the impeller characteristic exhibits behavior similar to that of the diffuser at the choked condition. However, the multidimensional effects that characterize this behavior apparently cannot be approximated accurately with strictly one-dimensional correlations. Modifying the correlations for evaluation at the hub, mean, and tip sections and averaging the results may help to predict the gradual pressure ratio reduction in moving toward choke. The potential refinements offered by this approach have not yet been considered herein. Maximum error in predicting the choking weight flow occurs at the 90 percent speed line and is 25 percent. Maximum error in predicting the compressor operating range $(w_{\text{surface}}/w_{\text{choice}})$ occurs at the 80 percent speed line and is 8.1 percent.

Figure 4 shows the comparison between calculated and experimental efficiencies as

Figure 4. - Comparison of calculated and experimental compressor total efficiencies as functions of equivalent weight flow and rotative speeds.

functions of corrected flow. The worst estimate of peak efficiency occurs at the 90 percent speed line and is in error by 1.2 points. Only partial data is shown for the efficiency contour at 60 percent speed. This results because the data is replotted from a performance map that had efficiency contours and more data could not be accurately interpolated. Predicted surge point efficiencies for all speed lines are considerably lower than those experimentally measured. This can be attributed to predicted diffuser pressure recovery being lower than that attainable experimentally. Aerodynamic blockage growth may not be as rapid as that predicted by the model in moving toward surge.

Compressor efficiency increases with increasing flow until the choke point is reached. At the choke point vaned diffuser static pressure recovery decreases, resulting in a decrease in overall compressor efficiency. At the impeller choke point one would expect the impeller losses to increase substantially, resulting in a decrease in overall compressor efficiency. However, this behavior is not predicted through the use of the empirical correlations.

CONCLUDING REMARKS

This section summarizes the **pertinent characteristics of** the **prediction** method, discusses the limitations of the method, and notes the effect of the limitations on the results.

For inducers with relative Mach numbers appreciably in excess of unity the predicted compressor total pressure ratio and efficiency are greater than that attainable experimentally. This is a result of neglecting shock losses in the inducer inlet.

Predicted static pressure recovery for vaned diffusers with throat Mach numbers greater than unity will be too large. Extrapolation of the channel diffuser data used in the program is valid only in the subsonic flow regime.

The prewhirl option available in the program is for solid-body vortex swirl. This type of swirl is used primarily because it minimizes inducer inlet tip relative Mach number.

Compressor total pressure ratio and efficiency are not adequately predicted with the one-dimensional correlations when the compressor mass flow rate is limited by the impeller rather than the vaned diffuser. The predicted total pressure ratio and efficiency levels are higher than those measured experimentally.

Predicted surge point efficiencies are considerably lower than those measured on the example compressor. Correlation with experimental data from other compressors indicates that the predicted surge point efficiencies are sometimes higher and sometimes lower than the corresponding experimental values. This may be due in part to the difficulty encountered in making precise measurements at this flow condition.

FORTRAN PROGRAM

Program Input

The information required for the program is input using NAMELIST format. A sample input sheet, with the data used for generating the 100 percent speed line for the example compressor, is shown in figure 5. The definitions of the FORTRAN input variables are:

inducer rms blade blockage function **BLOCK**

Sample Output

In the **normal** mode **of** operation the **program** performs two iterations over the range of inlet absolute critical velocity ratios which are specified. On the first iteration the compressor choking flow rate is determined. On the second iteration the printout of compressor performance at flow rates outside the usable range is deleted. **A** listing of the input information is tabulated for the compressor configuration studied. Then the compressor surge and choke flow rates are printed out. Finally, for each calculated point inside the operating range, compressor equivalent weight flow, total pressure ratio, total efficiency, and individual efficiency decrements are printed out. Sample output for the example compressor is as follows:

 $\overline{16}$

REC= 00000 FIL= 00002

Definitions of the abbreviated FORTRAN output variables are

Error Messages

If the compressor choking flow **rate** is not **reached with** the specified values of inlet absolute critical velocity ratios, the operating range cannot be determined with the methods used in this analysis. When this happens, the calculated performance output is deleted and the message COMPRESSOR CHOKING FLOW HAS NOT BEEN REACHED is printed.

If the first input value of (V/V_{cr}) results in calculated weight flow greater than the compressor choking weight flow, the message VOVCR ARRAY TOO LARGE is printed.

For input values of (V/V_{cr}) much too small for the compressor of interest a velocity diagram solution at the impeller exit cannot be obtained. When this occurs the message IRRATIONAL EXIT TRIANGLE is printed. All of these problems can be corrected by making the appropriate adjustments to the VOVCR array used.

Main Program FORTRAN Variables and *Corresponding* Engineering Symbols

 $A1$ a_1 $A2$ a₃ $ACOUSR$ a/a'

 $\mathcal{L}^{\text{max}}_{\text{max}}$, where $\mathcal{L}^{\text{max}}_{\text{max}}$

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Program Listing

```
DIMENSION VOVCR(15)
  DIPENSION P3P(20)+XMARR(20)
  DIMENSION F(20), S(20)
  DIMENSION AMT(4), BARR(6), PREC1(4, 6), PREC2(4, 6), PREC3(4, 6)
  DIMENSION PREC4(4,6), PREC5(4,6)
  DIMENSION MWRMS(8).CINC60(8).CINC50(8).CINC40(8)
  DIMENSION FLRNG(7), DIFLEM(7)
  INTEGER SPLT
  REAL LAMX+LCD+MUO+NONDES+MWRMS+INC+N
  DATA(AMT(I), [=1,4)/.2...4...6...8/DATA(BARR(I),[=1,6)/.02,.04,.06,.08,.10,.12/
  DATA((PREC1(I+J)+I=1+4)+J=1+6)/+234++244++257++269++215++224++233+
 1 - 243 - 207 - 215 - 223 - 232 - 193 - 199 - 206 - 212 - 183 - 190 - 196 - 202 - 1962169..176..182..188/
 DATA (IPREC2(I \cdot J) \cdot I=1,4) \cdot J=1,61/0644 \cdot 67) \cdot 696 \cdot 722 \cdot 620 \cdot 638 \cdot 6561 - 674 - 590 - 606 - 623 - 639 - 539 - 562 - 576 - 590 - 605 - 538 - 551 - 564 - 578 - 5642510..524..538..552/
 UATA((PREC3(I+J)+I=1+4)+J=1+6)/+782++789++796++802++750++756++762+
 1 - 768 - 708 - 716 - 724 - 732 - 672 - 680 - 687 - 695 - 652 - 648 - 654 - 6602604..612..619..626/
 DATA((PREC4(I+J)+I=1+4)+J=1+6)/+842++838++833++828++8++8++8++8++75
 12 \cdot 156 \cdot 160 \cdot 163 \cdot 110 \cdot 113 \cdot 116 \cdot 116 \cdot 19 \cdot 675 \cdot 678 \cdot 680 \cdot 683 \cdot 630 \cdot 6352.0640.06461DATA((PREC5(I.J), I=1.4).J=1.6)/.878..865..852..838..832..825..818.
 1 - 812 - 78 - 78 - 78 - 78 - 136 - 135 - 135 - 735 - 734 - 692 - 694 - 695 - 696 - 6442.647.2650.2652/DATA(CINC60(I), I=1,8)/-10,5,-5,,-2,,1,5,2,5,4,,5,,5,/
 DATA(CINC50(I),I=1.8)/-15.,-7.5,-3.,1.5,4.,6.5,9.,11./
 DATA(CINC40(I), [=1,8)/-20.,-12.,-5.,1.5,5.,8.,11.5,14.5/
 DATA(MWRMS(I).1=1,8)/.5..6..7..8..9.1..1.1.1.2/
 DATA(DIFLEM(I).I=1.7)/.4..5..6..7..8..9.1./
 DATA(FLRNG(I), [=1,7)/.48,.56,.64,.72,.79,.84,.89/
 NAMELIST /INPUT/ GAM.POP.TOP.N.ULT.MUO.CF.VOVCR.NVOVCR.RGAS.
 IDRAT.LAMX.B2X.Z.VLDRR.B2.BIMFB.AR.BLOCK.AL3.ADTH.NONDES.SPLT.
 2 ALIMF.CHIH.CHIT.CURVH.CURVT
1 REAC(5, INPUT)
```

```
WRITE(6.309)
 WRITE(6.315) POF.TOP.MUO.GAM.RGAS
 WRITE(6.310)
 WRITE(6,316) Z.CF.SPLT.N.ALIMF.BLOCK
 WRITE(6.311)
 WRITE(6,317) BIMFB.LAMX.DRAT
 WRITE(6,312)
 WRITE(6.318) B2X.82.VLDRR.AL3
 WRITE(6,313)
 WRITE(6.319) ADTH, AR
 WRITE(6,314)
 WRITE(6,320) (VOVCR(1),1=1,NVOVCR)
 PERDS=NONDES*100.
 G1 = GAM + 1.
 62 = 6AM-1.
 CP = GAMARGAS/G2B2X = -01745*B2XAL3=AL3*.01745
 ALIMF=.01745*ALIMF
 CHIH= - 01745*CHIH
 CHIT = .01745*CHITFLFUNC=SORT(GAM/RGAS*(2./G1)**(G1/G2))
  ROP=POP/RGAS/TOP
 VCR=SQRT(2.*GAM/G1*RGAS*TOP)
 Ull=3.14159*N*NONDES*D1T/60.
 NWR I E=1DO 18 1 = 1.2MIA = 12 DC 17 J=MIN.NVOVCR
 K = C81MF8=-01745*B1MF8
 VUIT=0.VU1MF=0.VU1H=0.VM1H=VOVCR(J)*VCR
 U2 = U1T/DRATD2 = D11/CRATUIH=UIT*LAMX
 DIH=DIT*LAMX
 DMF=SURT(DJT**2*(1.+LAMX**2)/2.)
 UINF=UIT*CMF/OIT
 CHIMF=(CHIT-CHIH)*(DMF-D1H)/(D1T-D1H)+CHIH
 CURVMF=(CURVT-CURVH)*(DMF-D1H)/(D1T-D1H)+CURVH
 HO = (OMF - D1H)/2.H = (CLI - DMF)/2.EINT=HO/2.*(CURVH+CURVMF)
 AMULT=EXPLEINT)
 VMINE=AMULT*VMIH
 GINT=(HO+H1) /6=*((2=-H1/HO)*CURVH+(HO+H1)**2/HO/H1*CURVMF+(2=-HO
L/HI) *CURVT)
 BMULT=EXP(GINT)
 VMIT=BMULT*VMIH
 VMIHN=VMIH*COS(CHIH)
 VMINEN=VMINE*COS(CHIME)
 VMITN=VMIT*COS(CHIT)
 RHOH=KOP*(1--VM1H**2/2-/CP/TOP)**(1./G2)
 RHEMF=ROP*(1.-VMIMF**2/2./CP/TOP)**(1./G2)
 RFCI=ROP*(1.-VM1T**2/2./CP/TOP)**(1./G2)
 FCN1=RFGH*VM1HN*D1H/2.
 FCN2=RHCMF*VM1MFN*DMF/2.
```

```
FCN3=RHOT*VM1TN*D1T/2.
   SW={HO+H1} /6.*({2.-H1/HO)*FCN1+{HO+H1}**2/HO/H1*FCN2+{2.-HO/H1}*
  1FCN3)*6.28318
   DFIO=0.VIME=SORT(VMIME=#2+VUIME=#2)
   TI = TOP - VIHFF * * 2/2 - ICPP1P = POPPOPP1=(1.-V1MF**2/2./CP/TOP)**(GAM/G2)
   PI = PIP*POPP1
   RIF=PIP/RGAS/TOP
  RI = RIP* (PI/PIP)**(1.7GAM)RE=U2*02/MU0*ROP
   ALSTAG=ALIMF/2.
  ES=0.0076/(COS(ALIME)-.025)*(1.+COS(ALSTAG)/0.7)
  VOVCR1=VOVCR(J)
   IFIALIMF.LT.0.001) GO TO 36
 S VOVCRI=VOVCRI+.001
   AKE=(VCVCR1*VCR)**2/2.
  POPP1=(1--AKE/CP/TOP)**(GAM/G2)
   AKEIC=AKE/(1.-ES)
  PIOPPO=(1.-AKEID/CP/TOP)**(GAM/G2)
  PIP=POP*PIOPPO/POPP1
  RIF=PIP/RGAS/TOP
  PI = PIP*POPP1
  R1 = R1P * (POPP1) * * (1*/GAM)01=3.14159*01T**2*(1.-LAMX**2)*VOVCR1*VCR*COS(AL1MF)/4.
  MTCHK = 01*R1
   IF(bICHK.LT.SW) GO TO 9
  VUIMF=VOVCRI#VCR*SIN(ALIMF)
   VMIME=VOVCRI#VCR#COS(ALIME)
  VINE=SORT(VUINE**2+VMINE**2)
   T1=T0P-V1MF#*2/2./CP
   XK=VUINF**2+2.*VMINF**2
  XC=VUlMF/DMF*2.
  VUII = KCHOII/2VUIH=XC*DIF/2.
  VMIT=SGRT(XK-2.*VUIT**2)
   VMIH=SORT(XK-2.*VUIH**2)
   OFIGV=ES*AKEID
36 VII=SURT(VMIT**2+VU1T**2)
   ALI=ARSIN(VUIT/VIT)
  WUIT=UIT-VUIT
  BI=ATAN(WUIT/VMIT)
  WIT=SORT(VMIT**2+WUIT**2)
  WUIH=UIH-VUIH
   BIH=ATAN(WUIH/VMIH)
  WIH=SORT(VMIH**2+WUIH**2)
  Al=SGRT(GAM*RGAS*T1)
  WUINF=UINF-VUINF
  WINF=SORT(VMINF**2+WUINF**2)
  RMRMS=WIMF/A1
  BIPF=ATAN(bUIMF/VMIMF)
   INC=(BIMF-BIMFB)*57.29577
  EPS=ATAN((1-BLOCK)*TAN(BIMF)/(1-+BLOCK*(TAN(BIMF))**2))
  BOPT=BIMF-EPS
   T1FP=T1+w1MF++2/2./CP
  WCR=SURT(2.*GAM/G1*RGAS*T1PP)
   TCT0=1.-G2/G1*(W1MF/WCR)**2
  WINFEF=WINF*COS(BOPT-BINFB)
```

```
TOT1=1.-G2/G1*(W1MFEF/WCR)**2
   TI = I1PP*IOI1WL=WIMF*SIN(ABS(BOPT-BIMF))
   DHINC=wL**2/2.
   PIFMF=PIP*EXP(-DHINC/TI/RGAS)
   DELTA=POP/101325.35
   THETA=10P/288.15
   EWF=SW*SORT(THETA)/DELTA
   BIMFB=BIMFB*57.29577
   IF(BIMFB.GE.40..AND.BIMFB.LE.50.) GO TO 71
   I = 281 IF(RMRMS.GE.MWRMS(I-1).AND.RMRMS.LE.MWRMS(I)) GO TO 80
   I = I + IGC TC 81
80 CINCL={RMRMS-MWRMS{I-1}}/(MWRMS{I)-MWRMS{I-1}}*{CINC60{I}-CINC60{I
  1 - 1) + C INC 60(1 - 1)
   CINC2=(RMRMS-MWRMS(I-1))/(MWRMS(I)-MWRMS(I-1))*(CINC50(I)-CINC50(I
  1 - 1) + CINC50(I-1)
   CINC=CINCI+(60.-BIMFB)/10.*(CINC2-CINC1)
   IF (INC.LE.CINC) K=1
   GO TO 74
71 I = 272 IFIRMANS.GE.MWRMS(I-1).AND.RMRMS.LE.MWRMS(I)) GO TO 73
   i = i + 1GC TC 72
73 CINC1=(RMRMS-MWRMS(I-1))/(MWRMS(I)-MWRMS(I-1))*(CINC50(I)-CINC50(I
  1 - 1 ) ) +C [NC50(1 - 1)
   CINC2=(RMRMS-MWRMS(I-1))/(MWRMS(I)-MWRMS(I-1))*(CINC40(I)-CINC40(I
  1 - 1) ) +C INC40(1 - 1)
   CINC=CINC1+(50--BIMFB)/10-*(CINC2-CINC1)
   IF(INC.LE.CINC) K=1
74 CCATINUE
   IF(K.EQ.1.AND.J.EQ.1) GO TO 202
   IF(K.EQ.1.AND.L.EQ.1) GO TO 18
   T2PP=T1PP+(U2**2-U1MF**2)/2./CP
   PHI=VMIMF/U2
   EPSLIM=1.7EXP(8.16*COS(82X)/2)VSL=SORT(COS(B2X))*U2/Z**.7
   IF((ONF/D2).GT.EPSLIM) VSL=U2*(SORT(COS(B2X))/Z**.7)*(1.-((ONF/D2-
  1EPSLIM)/(1.-EPSLIM))**3)+U2*((DMF/D2-EPSLIM)/(1.-EPSLIM))**3
   DHEST=02**2
   T2PEST=(DHEST/CP/TOP+1.)*TOP
   R2G=R1*(T2PEST/TOP)**(1./G2)
 4 RH62=R26VM2=Sw/(3.14159*RHO2*02*82)
   VU2=U2-VM2*TAN(B2X)-VSL
   MIN = JIF(VM2.LT.O..AND.L.EQ.2) GC TO 200
   IF(VM2.LT.0.) GO TO 17
   IF(VU2.LT.O..AND.L.EQ.2) GO TO 201
   IF(VU2.LT.0.) GO TO 17
   WUZ=02-VUZ
   W2=SORT(WU2**2+VM2**2)
   IF((VSL*COS(B2X)/W2).GT.1..AND.L.E0.2) GO TO 202
   IF((VSL*COS(B2X)/W2).GT.1.) GO TO 17
   T2=T2PP-W2**2/2./CP
   A2=SORT(GAM*RGAS*T2)
   H20h1T=h2/W1TWOU2={Phi++2+{DMF/D2}**2+W20W1T**2*(PHI++2+DRAT**2})/2.
```

```
AL2=ATAN(VU2/VM2)
  V2=SORT(VU2**2+VM2**2)
  T2P = T2+V2*+2/2./CP
  DHAERO=CP*TOP*(T2P/TOP-1.)
  DAFRO=CHAERO/U2**2
  DF= w11/U2*(2/3.14159*(1.-D1T/D2)+2.*D1T/D2)
  CONST1=0.75IF(SPLT.FC.1) CONST1=0.6
  DF=1 -- W20W1T+CONST1*0AERO/CF
  DHBL=0.05*DF**2*U2**2
  LOD=(1.-DMF/.3048)/COS(B2X)/2.
  DHDF=-01356*RH02*U2**3*D2**2/SW/RE**.2
  DHYD=Z/3.14159/COS(B2X)+D2/B2
  DFYD=1./DHYD+D1T/D2/(2./(1.-LAMX)+2.*2/3.14159/(1.+LAMX)
  1*SCRT(1.+(1.+LAMX**2)/2.*(TAN(B1))**2))
  DERC=0.02*SQRT(TAN(AL2))*DF**2*U2**2
  CCNSI2=5.6IF(SPLT.EC.1) CCNST2=7.0
  DHSF=CONST2*CF*LOD/DHYD*WOU2*U2**2
  DHACT=C+AERO+DHDF+DHRC
  HIS=DHAERO-DHBL-DHSF-DHIGV-DHINC
  ETAR=HIS/DHAERO
  TX=ETAR*DHAERO/CP/TOP+1.
51 P2P=TX**(GAM/G2)*PIPMF
  P2=P2P*(T2P/T2)**(-GAM/G2)
  R2G=P2/RGAS/T2
   IF(ABS((RHO2-R2G)/RHO2).GT.0.001) GO TO 4
  X M2 = V2/A2R2 = 02/2.
   AMU=1.4579E-6*T2**1.5/(T2+110.4)
   ANL=AMU/R2G
   80 = 1.XM = XM2ALPHA=AL2
  R = 1 - 0F(1)=XM2**3/(1.+G2/2.*XM2**2)**(G1/2./G2)
  P3P(1)=P2PXPARR(1)=XM2
  DELTAR=(VLDRR-1.)/10.
   ZETA=CF+R2/B2\triangleS=0.
  DC 1000 NO=2.10
   XMI = XMALPHI=ALPHA
   DS=CELTAR*R2/COS(ALPHA)
   A S = A S + D SDELTAS==037*AS**(-=2)*(V2/ANU)**(-=2)*DS
   8=80-2.*OELTAS/82
   DELTAB=80-B
  VARM=-2.*(1.+G2/2.*XM**2)/(XM**2-1./COS(ALPHA)**2)*((GAM*XM**2-TAN
  I(ALPHA) ** 2) * 7ETA/BO/COS(ALPHA) +1./BO*DELTAB/DELTAR-1./COS(ALPHA) **
  22/RJ*XM**2*DELTAR
  VARAL = 1.7COS(ALPHA)**2/(XM**2-(1.7COS(ALPHA)**2)*((1.6G2))*XM**2
  1)*ZETA/BO/COS(ALPHA)+1./BO*DELTAB/DELTAR-XM**2/R)*TAN(ALPHA)*DELTA
  2R
   VARMI=VARM
  VARALI=VARAL
   B1 = BB = B0
```

```
AS = AS - DSXM=XM1**2+VARM
    XM=SORT(XM)
    TALPH=TAN(ALPHI)+VARALI
    ALPHA=ATAN(TALPH)
    DS=DELTAR*R2/COS(ALPHA)
    R=R+DELTAR
    AS = AS + D S
    DELTAS=.037*AS**(-.2)*(V2/ANU)**(-.2)*DS
    B = 80 - 2.40ELTAS/B2
    DELTAR=BO-B
    VARM=-2.*(1.+G2/2.*XM**2)/(XM**2-1./COS(ALPHA)**2)*((GAM*XM**2-TAN
                                                                            A
   1(ALPHA) ** 2) * 7ETA/BO/COS(ALPHA) +1./BO*DELTAB/DELTAR-1./COS(ALPHA) **
   22/R)*XM**2*DELTAR
    VARAL=1./COS(ALPHA)**2/(XM**2-(1./COS(ALPHA))**2)*((1.+62)
                                                                   *XM**2
   1)*ZETA/BO/COS(ALPHA)+1./BO*DELTAB/DELTAR-XM**2/R)*TAN(ALPHA)*DELTA
   2R
    VARN={VARN]+VARM}/2.
  7 VARAL=(VARALI+VARAL)/2.
    B = (B + B))/2.
    XM=XM1**2+VARM
    XM=SORT(XM)
    TALPH=TAN(ALPHI)+VARAL
    ALPHA=ATAN(TALPH)
    BO = BACCUSR=SORT(1./(1.+G2/2.*XM**2))
    RHOR=1./(1.+G2/2.*XM**2)**(1./G2)
    FINGJ=XM**3*ACOUSR*RHOR*R
     IF(NO-EO-2) S(NO) = (F(NO)+F(NO-1)) * 0 - 5 * DELTARIF(NO-E0-2) GO TO 8
    CALL FNTGRL(NO.DELTAR.F.S)
   8 TPL=1./(1.+GAM*CF*R2*S(NO)/COS(AL2)/B2/XM2*(1.+G2/2.*XM2**2)**(G1/
    12.76211PP=TPL*P2P
     P3P(NO)=PPXMARR(NG)=XM
     XMACH=XMARR(NO)
1000 PTHP=P3P(NO)
     IF(L.EQ.1) GO TO 30
     M = 221 IF(XMACH.GE.DIFLEM(M-1).AND.XMACH.LE.DIFLEM(M)) GO TO 22
     M = M + 1GC TO 21
  22 RANGE=(XMACH-DIFLEM(M-1))/(DIFLEM(M)-DIFLEM(M-1))*(FLRNG(M)-FLRNG(
    1M-1)) +FLRNG(M-1)
     WSURGE=RANGE*WC+K
     IF(EWF.LT.WSURGE) GO TO 203
  30 CENTINUE
     PTH=PTHP/(1.+G2/2.*XMACH**2)**(GAM/G2)
     DHVLD=CP*T2P*((PTH/PTHP)**(G2/GAM)-(PTH/P2P)**(G2/GAM))
     A = 1 - AXMACH=XMACH*COS(ABS(ALPHA-AL3))
     PTHP=PTH*().+G2/2.*XMACH**2)**(GAM/G2)
     X1 = (ALPHA-AL3)*57*29577WEUNC=SW*SORT(T2P)/ADTh/PTHP/B
     IF(WFUNC.GE.FLFUNC) CPSTAR=0.
     IF(WFUNC.GE.FLFUNC) K=1
     IF(WFUNC+GE+FLFUNC) GO TO 14
     IF ((AR-1.2).GT.O..AND.(AR-2.).LT.O.) GO TO 10
     IF ((AR-2.).GT.O..AND.(AR-3.).LT.O.) GO TO 11
```

```
IF ((AR-3.).GT.O..AND.(AR-4.).LT.O.) GO TO 12
      IF ((AR-4.).GT.O..ANU.(AR-5.).LT.O.) GO TO 13
   10 CALL LININT(XMACH.BT, AMT.BARR.PREC1.4.6, FI)
      CALL LININT(XMACH, BT, AMT, BARR, PREC2, 4, 6, F2)
      CPSTAR = (AR-1.2)/.8*(F2-F1)*F1GC 10 14
   11 CALL LININT(XMACH,BT,AMT,BARR,PREC2,4,6,F2)
      CALL LININT(XMACH,BT,AMT,BARR,PREC3,4,6,F3)
      CPSTAR = (AR-2, )*(F3-F2)*F2GC TO 14
   12 CALL LININT(XMACH.BT.AMT.BARR.PREC3.4.6.F3)
      CALL LININT(XMACH.BT.AMT.BARR.PREC4.4.6.F4)
      CPSTAR=(AR-3.)*(F4-F3)+F3
      GG TO 14
   13 CALL LININT(XMACH, BT, AMT, BARR, PREC4, 4, 6, F4)
      CALL LININT(XMACH, BT, AMT, BARR, PREC5, 4, 6, F5)
      CPSTAR = (AR-4.1*(F5-F4)*F414 CONTINUE
      IF(K.EO.1.AND.J.EO.1) GO TO 202
      IF(K.EQ.1.AND.L.EQ.1) GO TO 18
      IF(K.NE.1.ANC.J.FO.NVOVCR) GD TO 204
      PEXIT=CPSTAR*(PTHP-PTH)+PTH
      RTHP=PTHP/RGAS/T2P
      VCRTH=SORT(2.*GAM/G1*RGAS*T2P)
      RVRVTH=SW/RTHP/VCRTH/ADTH/B
      V0VCR4 = -020100 P4PL=PEXIT/(1.-G2/GI*VOVCR4**2)**(GAM/G2)
      RVRVC4={1.-62/G1*VOVCR4**2}**(1./G2)*VOVCR4
      P4PG=RVRVTH*PTHP/AR/RVRVC4
      VEVCR4=VOVCR4+.001
      IFIPAPL.LT.P4PG) GO TO 100
      PPEXIT=P4PG
      DHCIF=T2P*CP*((PEXIT/PPEXIT)**(G2/GAM)-(PEXIT/PTHP)**(G2/GAM))
15<sub>1</sub>PR=PPFXIT/POP
      ETAC=DHAERO-DHSF-CHBL-DHVLC-DHDIF-DHIGV-DHINC
      ETAD=ETAD/DHACT
      DEIGV=DEIGV/DHACT
      DEBL=DFBL/DHACT
      DESF=DHSF/CHACT
      DEDF=DEDF/DHACT
      DERC=DHRC/DHACT
      DEVLD=DHVLD/DHACT
      DEVC=DHOIF/OHACT
      DEINC=DHINC/DHACT
      IF(L.FG.2) GO TO 19
17<sub>1</sub>CCNT INUE
      IFIL.EQ.1) GO TO 18
   19 IF(NWRITE.NE.1) GO TO 35
      WRITE(6,300)
      WRITE(6.301) PERDS
      WRITE(6,302)
      WRITE(6.303) WSURGE.WCHK
      NERITE=NWRITE+1
   35 MIN=MIN+1
      WRITE(6.304)
      WRITE(6.305) EWF.PR.ETAD
      WRITE(6,306)
      WRITE(6.307) DEIGV.DEINC.DEBL.DESF.DEDF.DERC.DEVLD.DEVD
      IF(K.NE. L.AND.MIN.LE.NVOVCR) GO TO 2
```

```
IF(K.EQ.1.AND.L.EQ.2) GO TO 1
200 WRITE(6,400)
    MIN=MIN+1
    IFIMIN.GT.NVOVCR) GO TO 1
    60-10-2201 WRITE(6,400)
    MIN=MIN+1
    IFIMIN.GT.NVOVCRJ GO TO 1
    GC 10 2
202 WRITE(6.402)
    GO TO 1
203 WRITE(6,403) VOVCR(J)
    MIN=MIN+1
    IFIMIN.GT.NVOVCR) GO TO 1
    GO TO 2
204 WRITE(6,308)
    GC TO 1
 18 WCHK=SW*SORT(THETA)/DELTA
    GO TO 1
300 FORMAT (59X, 13H PERCENT NDES)
301 FORMAT(63X.F5.1)
302 FORMAT145X.16H SURGE FLOW RATE.10X.15HCHOKE FLOW RATE)
303 FORMAT (51X.F5.3.20X.F5.3)
304 FORMAT(44X.4H WEQ.10X.14HPRESSURE RATIO,10X.4HETAT)
305 FORMAT(44X, F5.3, 13X, F5.3, 15X, F4.3)
306 FORMAT(11X,8H DETAIGV,8X,7HDETAINC,8X,6HDETABL,8X,6HDETASF,8X,6HDE
   ITADF, 8X, 6HDETARC, 8X, 7HDETAVLD, 8X, 6HDETAVD)
307 FORMAT(13X,F6.5,9X,F6.5,8X,F6.5,8X,F6.5,8X,F6.5,8X,F6.5,9X,F6.5,9X
   18X - 5 - 5 - 9X - 5 - 5308 FORMAT(45H COMPRESSOR CHOKING FLOW HAS NOT BEEN REACHED)
309 FORMAT(21H1INLET TOTAL PRESSURE,5X,23HINLET TOTAL TEMPERATURE,5X,2
   19HINLET TOTAL DYNAMIC VISCOSITY,5X,19HSPECIFIC HEAT RATIO,5X,12HGA
   25 CONSTANT)
310 FORMAT(17H NUMBER OF BLADES, 5X, 20HFRICTION CCEFFICIENT, 5X, 4HSPLT, 5
   1X.12HDESIGN SPEED.5X.23HINDUCER RMS SWIRL ANGLE.5X.27HINDUCER RMS
   2BLCCKAGE FACTOR)
311 FORMAT(11X,24H INDUCER RMS BLADE ANGLE, 5X, 30HINDUCER HUB-TIP DIAME
   ITER RATIO.5X.40HINDUCER TIP-IMPELLER EXIT DIAMETER RATIO)
312 FORMAT(30H IMPELLER BLADE EXIT BACKSWEEP,5X,23HIMPELLER BLADE EXIT
   1 HT.,5X,32HVANELESS DIFFUSER DIAMETER RATIO,5X,28HVANED DIFFUSER S
   2ETTING ANGLE)
313 FORMAT(33X,33H TOTAL VANED DIFFUSER THROAT AREA,5X,25HVANED DIFFUS
   JER AREA RATIO)
314 FCRMAT(47X+36H INLET CRITICAL VELOCITY RATIO ARRAY)
315 FORMAT(7x,F7.0,21x,F6.2,23x.F8.7,24x,F3.1,16X,F6.2)
316 FORMAT(7x,F3.0.20X,F4.3.15X,I1.9X,F6.0.17X,F4.2.26X,F4.2)
317 FORMAT(24X, F4.1, 27X, F6.4.34X, F6.4)
318 FORMAT(13X.F4.1.26X.F6.5.27X.F6.3.30X.F5.2)
319 FCRMAT(47X.F7.6.28X.F4.2)
320 FORMAT(8F10.2)
400 FORMAT(IX.24HIRRATIONAL EXIT TRIANGLE)
402 FORMAT(1X.21FVOVCR ARRAY TOO LARGE)
403 FORMAT(1X, 34HWT FLOW LESS THAN SURGE FOR V/VCR=, F4.2)
```
Subroutine $LINT(X1, Y1, X, Y, TN, MX,MY, F)$

This subroutine interpolates a value of maximum pressure recovery coefficient C_p^* ^{*} from a table of vaned diffuser throat Mach numbers and aerodynamic blockage given as **input.**

Xl input, **vaned diffuser** throat **Mach number**

- Y1 input, throat **aerodynamic blockage**
- **X** input array of throat **Mach** numbers
- **Y** input array of throat blockages
- **TN** input two-dimensional array of C_p^* corresponding to throat Mach numbers and blockages
- **MX** input, number of throat **Mach** numbers
- MY input, number of throat blockages
- **F** ** output, interpolated value of **Cp**

\$18FTC **LININT**

```
SUEROUTINE LININT (X1.YI.X.Y.TN.MX.MY.F)
      DiNF_SICN XlHX). Y(MYI. TNiMX,MY)
      I)E 1 J3=2.NX
      IF (X|.LE.X(J3)) GO TO 2
\mathbf{I}J3=NX
2
      DO 3 J4=2_NY
      IF (YI-LE.Y(J_)) GO TO 4
=I
      ,|4=NY
\ddot{\bullet}J/=J3-i
      d2=J4-|
      EPS l=( X l-X | J l ) )/| X( J3/-X ( Jll )
      FPS2=(YI-Y(J2) I/(Y(J4)-Y(J2))
      EPS3=I.-EPSI
      FFS4=I°-FPS2
      F=TN(J1.J2)*EPS3*EPS4+TN(J3,J2)*EPS1*EPS4+TN(J1,J4)*EPS2*EPS3+TN(J
     I_.JkI_EFSI_EPS2
      RFTURN
      ENC
```
Subroutine **FNTGRL(NO, DELTAR, F,** S)

This subprogram is used to integrate the function $M^3(\rho a/\rho' a')$ in the vaneless diffuser.

NO number of equally spaced radii DELTAR radius ratio between stations F function $M^3(\rho a/\rho' a')$ S integral of F

No FORTRAN listing of the subprogram is available but a brief description of the method of integration is as follows:

The subprogram uses Simpson's rule in the following manner: First interval:

$$
\int_{x_1}^{x_1} f(x) dx = 0
$$

Second interval:

$$
\int_{x_1}^{x_2} f(x) dx = \Delta x (5f_1 + 8f_2 - f_3)/12
$$

Remaining intervals:

$$
\int_{x_i}^{x_{i+1}} f(x) dx = \Delta x (5f_{i+1} + 8f_i - f_{i-1})/12
$$

Lewis Research Center,

National Aeronautics and Space Administration,

and

U.S. Army Air Mobility R&D Laboratory,

Cleveland, Ohio, August 2, 1973, 501-24.

APPENDIX A

SYMBOLS

" relative total condition

APPENDIX B

EQUATIONS

This section presents **a** more detailed discussion of the method **of calculating com**pressor performance than **was** presented in the section Method of Analysis.

The **equations** used in calculating the flow solution through the **compressor** are listed here in the approximate order of solution in the FORTRAN program. The known constants are the input fluid state and thermodynamic properties, compressor geometry, and velocity **diagram data.** The **working** fluid properties and state **conditions** are specific heat ratio γ , inlet total pressure p'_0 , inlet total temperature T'_0 , and inlet dynamic viscosity based on total conditions μ_0^* . The velocity diagram data are compressor design rotative speed N_{DES}, the fraction of design rotative speed for which performance (V / V_{cr}) for which performance is to data is desired N / N_{cr} or V / V_{cr}) for which performance is to ^V *)* **to** be used, **and** the **inducer solid-body** swirl **angle be calculated, the values of** */Vcr* **⁰** at the rms diameter α_{1MF} . The compressor data are inducer inlet hub and tip wall slopes x_{1H} and x_{1T} , inducer inlet hub and tip wall curvatures C_{1H} and C_{1T} , inducer inlet tip diameter D_{1T} , inducer inlet hub-tip diameter ratio λ , inducer-inlet-tip to impeller-exit diameter ratio D_{1T}/D_3 , number of blades at impeller exit (including splitters) Z_3 , impeller blade exit angle (positive opposite to direction of rotation) β_{3b} , im**peller exit blade height** b_3 , **inducer inlet blade angle** at the **rms diameter** β_{1MFD} , **inducer inlet blockage** factor **calculated at the rms diameter, vaneless diffuser diameter ratio** D_4/D_3 , vaned diffuser setting angle α_4 , total vaned diffuser throat area A_5 , **vaned** diffuser area ratio A_6/A_5 , and wetted surface friction coefficient C_f . With these known **values compressor performance can be evaluated by the** method **described.**

Calculation of Inlet Velocity Triangles **and** Compressor **Weight** Flow

Calculation of the inducer inlet velocity diagrams **and weight** flow **are handled** by two **methods depending on whether** the inlet **is swirl-free or subject to solid-body swirl. For the swirl cases it is assumed that the inlet guide vanes are located in a constant area annulus** with **zero wall slope** with **respect to the axial direction. The inner and outer diameters of the annulus are assumed to be equal to** the **inducer inlet hub and tip diameters. For the swirl-free cases it is assumed that the** flow **orthogonal can be approximated by a radial line. These assumptions** greatly **simplify calculation of the velocities specified by** the **general velocity** gradient **equation.**

Swirl-free inlets. - For compressor **configurations** with swirl-free inlets the input value of $\left(\frac{V}{V_{cr}}\right)_0$ is used as the hub streamline value of V/V_{cr} . The variation of inlet velocity with radius is calculated from the general velocity gradient equation simplified for swirl-free flow. This equation is (in simplified form from ref. 10)

$$
V_{m} = V_{m1H} e^{\int_{0}^{n} C dm}
$$
 (B1)

where V_{m1H} is the hub absolute velocity, C is the streamline curvature, and n is the distance along a radius. Streamline curvature and slope at the rms inlet diameter are approximated by linear interpolation of the input hub and tip values. The integral in the previous equation is approximated by the trapezoidal rule at the rms diameter and by Simpson's rule for unequal intervals at the tip. The numerical approximations are

╭

$$
V_{m1MF} = V_{m1H} e
$$

$$
\left[\frac{h_0}{2} (C_{1H} + C_{1MF})\right]
$$
(B2)

and

$$
V_{m1T} = V_{m1H} e^{\left\{ \frac{h_0 + h_1}{6} \right\} \left[\left(2 - \frac{h_1}{h_0} \right) C_{1H} + \frac{(h_0 + h_1)^2}{h_1 h_0} C_{1MF} + \left(2 - \frac{h_0}{h_1} \right) C_{1T} \right] \right\}
$$
(B3)

where

 $h_0 = r_{1MF} - r_{1H}$ (B4)

and

$$
h_1 = r_{1T} - r_{1MF} \tag{B5}
$$

Static densities **at the** hub, rms, and tip diameters are determined from the **inlet total** density and the calculated velocity at each station. Velocity components normal to the annular inlet plane are calculated using the velocities and streamline angles at the three stations:

$$
V_{\text{m1n}} = V_{\text{m1}} \cos \chi \tag{B6}
$$

where χ is the streamline angle. Inlet mass flow is then computed using Simpson's rule for unequal intervals

$$
w = 2\pi \left\{ \frac{h_0 + h_1}{6} \left[\left(2 - \frac{h_1}{h_0} \right) (\rho r)_{1H} V_{m1Hn} + \frac{(h_0 + h_1)^2}{h_0 h_1} (\rho r)_{1MF} V_{m1MFn} + \left(2 - \frac{h_0}{h_1} \right) (\rho r)_{1T} V_{m1Th} \right] \right\}
$$
(B7)

Inlets with **solid-body swirl. - For** impeller **inlets** with inlet guide **vanes producing** solid-body swirl the equations presented in reference 2 for kinetic energy loss were iteratively solved for state conditions and velocities downstream of the inlet guide vanes. Conditions at the rms diameter were assumed to be representative of the flow distribution at the impeller inlet. The inlet guide vane stagger angle was assumed to be onehalf of the turning angle at the rms diameter; that is,

$$
\alpha_{\text{ST}} = \frac{\alpha_{\text{1MF}}}{2} \tag{B8}
$$

The inlet guide vane loss coefficient was calculated from the equation

$$
e_S = {0.0076 \over \cos \alpha_{1MF} - 0.025} \left(1 + {\cos \alpha_{ST} \over 0.7} \right)
$$
 (B9)

 V / V_{cr} as a first approximation to the value of V / V_{cr} down- $\overline{}$ Using the input value of $\left(\frac{V}{V_{cr}}\right)_{0}$ as a first approximation to the value of V/V_{0} stream of the inlet guide vanes at the rms diameter, the following equations are solved iteratively with successive increments in V/V_{cr} until continuity checks the calculation of equation (BT):

$$
KE = \frac{\left(\frac{V}{V_{cr}}V_{cr}\right)^2}{2}
$$
 (B10)

$$
\left(\frac{\mathbf{p}}{\mathbf{p}'}\right)_{1\mathbf{M}\mathbf{F}} = \left(1 - \frac{\mathbf{KE}}{\mathbf{C}_1 \mathbf{T}_0'}\right)^{\gamma/(\gamma - 1)}\tag{B11}
$$

$$
KE_{id} = \frac{KE}{1 - e_S}
$$
 (B12)

$$
\frac{P_{1MF}}{P_0'} = \left(1 - \frac{KE_{id}}{C_p T_0'}\right)^{\gamma/(\gamma - 1)}
$$
(B13)

$$
p'_{1MF} = p'_0 \frac{p'_0}{\left(\frac{p}{p'}\right)_{1MF}}
$$
 (B14)

$$
\rho_{1MF}^{\prime} = \frac{p_{1MF}^{\prime}}{RT_0^{\prime}}
$$
 (B15)

$$
p_{1MF} = p'_{1MF} \left(\frac{p}{p'}\right)_{1MF}
$$
 (B16)

$$
\rho_{1MF} = \rho_{1MF} \left(\frac{p}{p'}\right)_{1MF}^{1/\gamma}
$$
 (B17)

$$
Q_1 = \frac{\pi}{4} D_{1T}^2 (1 - \lambda^2) \left(\frac{V}{V_{cr}}\right)_{1MF} V_{cr} \cos \alpha_{1MF}
$$
 (B18)

$$
w = Q_1 \rho_{1MF} \tag{B19}
$$

When continuity is satisfied downstream of the inlet guide vanes, the impeller inlet velocity triangles are calculated. *Combination* of the Euler equation with the solid-body vortex equation and simple radial equilibrium equation shows that the meridional velocity distribution can be expressed as

$$
V_{m1} = \sqrt{K_{Vm} - 2V_{u1}^2}
$$
 (B20)

Using the value of V/V_{cr} which satisfies continuity downstream of the inlet guide vanes and the input value of the rms turning angle allows solution of the constant in equation **(B20).** The tangential components of the inlet absolute velocity at the hub and tip are calculated from the vortex condition. Then the hub and tip meridional velocities can be determined as follows:

$$
V_{\mathbf{u}} = K_{\mathbf{V}\mathbf{u}} \mathbf{r}
$$
 (B21)

where

$$
K_{\text{Vu}} = \frac{V_{\text{u1MF}}}{r_{\text{1MF}}} \tag{B22}
$$

The following general **inducer inlet velocity** diagram characteristics are calculated after the particular swirl-free or solid-body swirl calculations are completed:

$$
V_{1T} = \sqrt{V_{m1T}^2 + V_{u1T}^2}
$$
 (B23)

$$
\alpha_{1T} = \sin^{-1}\left(\frac{V_{u1T}}{V_{1T}}\right) \tag{B24}
$$

$$
W_{u1T} = u_{1T} - V_{u1T}
$$
 (B25)

$$
\beta_{1T} = \tan^{-1} \left(\frac{W_{u1T}}{V_{m1T}} \right)
$$
 (B26)

$$
W_{1T} = \sqrt{V_{m1T}^2 + W_{u1T}^2}
$$
 (B27)

$$
W_{u1H} = u_{1H} - V_{u1H}
$$
 (B28)

$$
\beta_{1H} = \tan^{-1} \left(\frac{W_{u1H}}{V_{m1H}} \right)
$$
 (B29)

$$
W_{1H} = \sqrt{v_{m1H}^2 + w_{u1H}^2}
$$
 (B30)

$$
V_{1MF} = \sqrt{V_{u1MF}^2 + V_{m1MF}^2}
$$
 (B31)

$$
W_{u1MF} = u_{1MF} - V_{u1MF}
$$
 (B32)

$$
W_{1MF} = \sqrt{V_{m1MF}^2 + W_{u1MF}^2}
$$
 (B33)

$$
\beta_{1MF} = \tan^{-1} \left(\frac{W_{u1MF}}{V_{m1MF}} \right)
$$
 (B34)

The wheel speeds used in the previous equations are obtained from the following input constants:

$$
u_{1T} = \frac{\pi \left(\frac{N}{N_{DES}}\right) N_{DES} D_{1T}}{60}
$$
 (B35)

$$
u_{1H} = \lambda u_{1T} \tag{B36}
$$

$$
u_{1MF} = u_{1T} \frac{D_{1MF}}{D_{1T}}
$$
 (B37)

$$
D_{1MF} = \sqrt{\frac{1}{2} D_{1T}^2 (1 + \lambda^2)}
$$
 (B38)

Individual Losses

Inlet guide vane loss. **-** When the inducer inlet velocity diagram characteristics have been determined by the method described in the section Inlets With Solid-Body Swirl, the inlet guide vane loss is computed using the equation found in reference 1, sub**ject** to the assumptions made in reference 3.

$$
\Delta h_{IGV} = e_S KE_{id} \tag{B39}
$$

Inducer incidence loss. - The **equations** developed in **reference** 4 are **used** to determine the optimum inducer incidence angle for incompressible flow. *Compressibility* effects are ignored because the incidence loss is evaluated at the rms diameter and compressibility effects are insignificant except at the hub according to reference 4. Using the input blade blockage factor at the rms diameter the optimum incidence angle is found by

$$
\epsilon = \tan^{-1} \frac{(1 - B_1)\tan \beta_{1MF}}{1 + B_1 \tan^2 \beta_{1MF}}
$$
 (B40)

where

$$
B_1 = 1 - \frac{Z_1 t_u}{2\pi r_{1MF}}
$$
 (B41)

The optimum relative flow angle is then

$$
\beta_{\text{opt}} = \beta_{1\text{MF}} - \epsilon \tag{B42}
$$

and the component of the relative velocity lost is

$$
W_{L} = W_{1MF} \sin \left| \beta_{opt} - \beta_{1MF} \right| \tag{B43}
$$

The **enthalpy** loss due to incidence is **expressed** as

$$
\Delta h_{\text{INC}} = \frac{W_{\text{L}}^2}{2C_p} \tag{B44}
$$

and the total pressure just inside the blade row is

$$
p'_{2MF} = p'_{1MF} e^{-\Delta h_{INC}} \left(\frac{-\Delta h_{INC}}{T_2 R}\right)
$$
 (B45)

Impeller work and losses. - The **impeller** exit **density** is obtained through iteration of the impeller loss equations and calculated state properties. The enthalpy rise in the impeller is initially approximated as

$$
\Delta h_{\text{est}} = u_3^2 \tag{B46}
$$

and the corresponding impeller exit total temperature is then

$$
T'_{3est} = \left(\frac{\Delta h_{est}}{C_p T'_0} + 1\right) T'_0
$$
 (B47)

The impeller exit density is initially approximated to be

$$
\rho_3 = \rho_{1MF} \left(\frac{T_{3est}^{\prime}}{T_0^{\prime}} \right)^{1/(\gamma - 1)}
$$
(B48)

The meridional component **of exit** absolute velocity is calculated from continuity

$$
V_{m3} = \frac{w}{\pi \rho_3 D_3 b_3}
$$
 (B49)

The tangential component of exit absolute velocity is then

$$
V_{u3} = u_3 - V_{m3} \tan \beta_{3b} - V_{SL} \tag{B50}
$$

where the slip velocity V_{SL} is calculated by the method of reference 11.

$$
V_{SL} = \frac{u_3 \sqrt{\cos \beta_{3b}}}{Z_3^{0.7}}
$$
 (B51)

The **remaining** state **and** flow **properties** at the exit were calculated from the following relations:

$$
T_3' = T_{1MF}'' + \frac{u_3^2 - u_{1MF}^2}{2C_p}
$$
 (B52)

$$
W_{u3} = u_3 - V_{u3}
$$
 (B53)

$$
W_3 = \sqrt{V_{m3}^2 + W_{u3}^2}
$$
 (B54)

$$
T_3 = T_3' - \frac{w_3^2}{2C_p}
$$
 (B55)

$$
V_3 = \sqrt{v_{m3}^2 + v_{u3}^2}
$$
 (B56)

$$
T_3' = T_3 + \frac{V_3^2}{2C_p}
$$
 (B57)

The **work** loss due to **blade** loading is **calculated** from the equation

$$
\Delta h_{BL} = 0.05 \, D_f^2 u_3^2 \tag{B58}
$$

which is given by reference 5. The diffusion factor D_f is defined as

$$
D_{f} = 1 - \frac{W_{3}}{W_{1T}} + \frac{K_{BL}q_{aero}}{W_{1T}\left[\frac{Z_{3}}{\pi}\left(1 - \frac{D_{1T}}{D_{3}}\right) + 2\frac{D_{1T}}{D_{3}}\right]}
$$
(B59)

where a value of 0.75 is used for K_{BL} for conventional impellers and a value of 0.6 is used for impellers with splitters. A parametric study of calculated diffusion factors with a variation in the number of blades indicated that changing the constant to 0.6 would compensate for the changing solidity near the exit. The dimensionless actual head is obtained by dividing the enthalpy rise by the exit blade speed squared

$$
q_{\text{aero}} = \frac{\Delta h_{\text{aero}}}{u_3^2} \tag{B60}
$$

where

$$
\Delta h_{\text{aero}} = C_p T_0' \left(\frac{T_3'}{T_0'} - 1 \right) \tag{B61}
$$

Impeller disk friction loss is computed using the method of reference 6

$$
\Delta h_{\rm DF} = 0.01356 \frac{\rho_3}{\text{wRe}^{0.2}} u_3^3 D_3^2
$$
 (B62)

The Reynolds number in this **equation is based** on impeller exit dimensions **and** inlet total conditions

$$
Re = \frac{u_3 D_3 \rho'_1 MF}{\mu'_0}
$$
 (B63)

Skin friction loss is calculated from the **correlation** found **in reference** 5

$$
\Delta h_{\text{SF}} = \text{K}_{\text{SF}} \text{C}_{\text{f}} \frac{\frac{\text{L}}{\text{D}_3}}{\frac{\text{D}_{\text{HYD}}}{\text{D}_3}} \left(\frac{\text{w}}{\text{u}_3}\right)_{\text{av}}^2 \text{u}_3^2 \tag{B64}
$$

where the mean flowpath **blade** length-diameter **ratio** is expressed **as**

$$
\frac{L}{D_3} = \frac{1}{2} \frac{1 - \frac{D_{1MF}}{0.3048}}{\cos \beta_{3b}}
$$
 (B65)

The **ratio of mean hydraulic-exit** diameter ratio is **calculated** from the equation

$$
\frac{D_{\text{HTD}}}{D_3} = \frac{1}{\frac{Z_3}{\pi \cos \beta_{3b}} + \frac{D_3}{b_3}} + \frac{2Z_3}{1 - \lambda} + \frac{2Z_3}{\pi (1 + \lambda)} \sqrt{1 + \tan^2 \beta_{1T} \left(1 + \frac{\lambda^2}{2}\right)}
$$
(B66)

and the **ratio** of mean **relative velocity-impeller** exit **velocity** is **determined** by

$$
\left(\frac{w}{u_3}\right)_{av}^{2} = \frac{1}{2} \left\{ \left(\frac{V_{m1MF}}{u_3}\right)^{2} + \left(\frac{D_{1MF}}{D_3}\right)^{2} + \left(\frac{W_3}{W_{1T}}\right)^{2} \left[\left(\frac{V_{m1MF}}{u_3}\right)^{2} + \left(\frac{D_{1T}}{D_3}\right)^{2}\right] \right\}
$$
(B67)

Some studies of the effects of the addition of **splitter** blades on the impeller flow indicated that the mean channel relative velocity is increased in the splitter region. The ratio of mean flow path blade length and hydraulic diameter-exit diameter ratio remain virtually unchanged. *Computation* of the mean relative velocity ratios for an impeller both with and without splitters showed that, for splitters, the average relative velocity is about 14 percent higher than for conventional impellers. This phenomenon is accounted for by using the constant 7.0 for impellers with splitters and 5.6 for impellers without splitters.

At this point in the iteration procedure the impeller ideal enthalpy rise is calculated as follows:

$$
\Delta h_{id} = \Delta h_{aero} - \Delta h_{IGV} - \Delta h_{INC} - \Delta h_{SF} - \Delta h_{BL}
$$
 (B68)

Defining the impeller efficiency as

$$
\eta_{\mathbf{R}} = \frac{\Delta h_{\text{id}}}{\Delta h_{\text{aero}}} \tag{B69}
$$

the state conditions at the impeller exit are calculated from the relations

$$
p_3' = p_{2MF}' \left(\frac{\eta_R \Delta h_{aero}}{C_p T_0'} + 1 \right)^{\gamma / (\gamma - 1)}
$$
 (B70)

$$
p_3 = \frac{p_3'}{\left(\frac{T_3'}{T_3}\right)^{\gamma/\left(\gamma - 1\right)}}
$$
 (B71)

$$
\rho_3 = \frac{p_3}{RT_3} \tag{B72}
$$

The value of ρ_3 obtained from equation (B72) is compared to the value estimated in equation (B48).

If the values do not compare within 0.1 percent, the former value is used as a new approximation and equations (B49) to (B72) are iterated until two consecutive values of exit density agree within the tolerance specified.

Recirculation loss. - Recirculation of the working fluid from the vaneless space back into the impeller results in additional work which is lost to the system. Reference 5 expresses this loss as a function of the impeller exit absolute flow angle, impeller diffusion factor, and impeller exit tip speed. The following modified form of the equation of reference 5 is used to calculate the recirculation loss:

$$
\Delta h_{\rm RC} = 0.02 \sqrt{\tan \alpha_3} \ D_{\rm f}^2 u_3^2 \tag{B73}
$$

Vaneless diffuser loss. - Reference 7 developed differential equations **relating** Mach number, flow angle, and total temperature to vaneless diffuser radius ratio through the

fundamental relations of continuity, equilibrium, heat transfer, and fluid state. These equations are evaluated at ten equally spaced intervals in the vaneless space. Two iterations of the equations are performed at each station; the first iteration uses the calculated properties at the point of interest to compute the variation in Mach number and flow angle to the next point; the second iteration uses the estimated properties of the downstream point to compute the same variations. The arithmetic average of the two iterations is then used for the solution.

These equations, simplified for adiabatic flow in a geometrically constant depth radial passageare

$$
\frac{1}{M^2} \frac{dM^2}{dR} = \frac{-2\left(1 + \frac{\gamma - 1}{2}M^2\right)}{M^2 - \sec^2 \alpha} \left[(\gamma M^2 - \tan^2 \alpha) \frac{\zeta}{B \cos \alpha} + \frac{1}{B} \frac{dB}{dR} - \frac{\sec^2 \alpha}{\overline{R}} \right]
$$
(B74)

and

$$
\frac{1}{\tan \alpha} \frac{d \tan \alpha}{dR} = \frac{\sec^2 \alpha}{M^2 - \sec^2 \alpha} \left\{ \left[1 + (\gamma - 1)M^2 \right] \frac{\zeta}{B \cos \alpha} + \frac{1}{B} \frac{dB}{dR} - \frac{M^2}{R} \right\}
$$
(B75)

where

$$
\zeta = \frac{C_f r_3}{b_3} \tag{B76}
$$

$$
\overline{R} = \frac{r}{r_3}
$$
 (B77)

$$
B = \frac{b}{b_3}
$$
 (B78)

The **schedule of** effective **passage depth is determined by boundary** layer displacement **thickness** growth **on** the **end walls. The end walls in the vaneless diffuser are assumed to be** parallel **and have a spacing equal to the impeller exit blade height. The** flow pattern **between the end** walls is **approximated as a log spiral between adjacent calculation stations. With these assumptions the incremental** flow **path** length **is**

$$
\Delta S = \frac{\mathbf{r}_2 \Delta R}{\cos \alpha} \tag{B79}
$$

and the incremental boundary layer displacement thickness is

$$
\Delta \delta^* = 0.037 \, \text{S}^{-0.2} \left(\frac{V_2}{\nu}\right)^{-0.2} \, \Delta \text{S} \tag{B80}
$$

where

$$
S = \sum_{i=1}^{n} \Delta S_i
$$
 (B81)

The **effective** passage depth is

$$
B_{i+1} = B_i - \frac{2 \Delta \delta^*}{b_3} \tag{B82}
$$

and

$$
\Delta B = B_i - B_{i+1} \tag{B83}
$$

Loss the total pressure in the vaneless space is computed from the following equation derived in reference 4:

$$
\frac{1}{\left(\frac{p_4'}{p_3'}\right)} = 1 + \frac{\gamma C_f}{\cos \alpha_3} \frac{r_3}{b_3} \frac{\int_1^R M^3 \frac{a}{a} \frac{\rho}{\rho'} \overline{R} d\overline{R}}{M_3\left(\frac{a}{a'}\right)_3\left(\frac{\rho}{\rho'}\right)_3}
$$
(B84)

The integral in equation (B84) is determined by numerical methods. Vaned diffuser leading edge static pressure is calculated from the isentropic relation using the total pressure obtained from equation (B84) and the Mach number obtained from the distribution prescribed by equation (B74)

$$
p_{4} = \frac{p_{4}^{2}}{\left(1 + \frac{\gamma - 1}{2} M_{4}^{2}\right)^{\gamma / (\gamma - 1)}}
$$
(B85)

Vaneless **diffuser** loss **is** then **determined** by

$$
\Delta h_{\text{VLD}} = C_p T_3 \left[\left(\frac{p_4}{p_4'} \right)^{(\gamma - 1)/\gamma} - \left(\frac{p_4}{p_3'} \right)^{(\gamma - 1)/\gamma} \right]
$$
(B86)

Vaned **diffuser** loss. - Vaned **diffuser performance** is predicted by use of the test data reported in reference 8. Lines of maximum pressure recovery coefficient at a given area ratio **were** estimated from the performance maps reported for single plane divergence diffusers **with** square **throats.** The component of **the** vaned diffuser leading edge Mach number parallel **to the** vane setting angle is used as the **throat** Mach number. A loss **in** total pressure associated **with** this **incidence** is calculated holding static pressure constant between **the** leading edge and throat. The **throat** blockage is estimated from the displacement thickness growth on the vaneless **diffuser** end **walls.** Using **these** conditions of throat Mach number, total pressure, and blockage **the** pressure recovery of **the** vaned diffuser **is** extrapolated for **the** specified geometric area ratio. Vaned diffuser exit static pressure **is** then **calculated** from **the** relation

$$
p_6 = C_p^{**} (p_5 - p_5) + p_5
$$
 (B87)

The exit total pressure and velocity are determined by trial and error using estimated values of exit critical velocity ratio. The total pressure at the vaned diffuser throat is assumed to be constant across the free-stream area outside the boundary layer displacement thickness. The total pressure and velocity at the vaned diffuser exit are calculated assuming full flow across the geometric area. Starting with a low value of $V \cdot C \cdot C$ and fower estimate of exit total pressure is expressed as $\langle V_{\rm cr} \rangle$ the lower estimate of exit total press

$$
p_{6}^{t} = \frac{p_{6}}{\left[1 - \frac{\gamma - 1}{\gamma + 1} \left(\frac{V}{V_{cr}}\right)_{6}^{2}\right]^{\gamma / (\gamma - 1)}}
$$
(B88)

and the higher estimate is

$$
p_{6}^{\prime} = \frac{\left(\frac{\rho V}{\rho V_{cr}}\right) p_{5}^{\prime}}{AR\left(\frac{\rho V}{\rho V_{cr}}\right)}
$$
(B89)

where

$$
\left(\frac{\rho V}{\rho' V_{cr}}\right) = \frac{w}{\rho_5' (V_{cr})_5^A \rho_5^B}
$$
\n(B90)

Equations (B88) and (B89) are solved using increasing values of $(V/V_{cr})_6$ until convergence is achieved. The vaned diffuser loss is then calculated from the relation

$$
\Delta h_{VD} = C_p T_3 \left[\left(\frac{p_6}{p_6'} \right)^{(\gamma - 1)/\gamma} - \left(\frac{p_6}{p_5'} \right)^{(\gamma - 1)/\gamma} \right]
$$
(B91)

Efficiency decrements. - **Decrements** in efficiency caused by the individual losses are obtained from the following equations:

$$
\Delta \eta_{IGV} = \frac{\Delta h_{IGV}}{\Delta h_{act}}
$$
 (B92)

$$
\Delta \eta_{\text{INC}} = \frac{\Delta h_{\text{INC}}}{\Delta h_{\text{act}}} \tag{B93}
$$

$$
\Delta \eta_{BL} = \frac{\Delta h_{BL}}{\Delta h_{act}}
$$
 (B94)

$$
\Delta \eta_{\rm SF} = \frac{\Delta h_{\rm SF}}{\Delta h_{\rm act}} \tag{B95}
$$

$$
\Delta h_{DF} = \frac{\Delta h_{DF}}{\Delta h_{act}}
$$
 (B96)

$$
\Delta \eta_{\rm RC} = \frac{\Delta h_{\rm RC}}{\Delta h_{\rm act}} \tag{B97}
$$

$$
\Delta \eta_{\text{VLD}} = \frac{\Delta h_{\text{VLD}}}{\Delta h_{\text{act}}} \tag{B98}
$$

$$
\Delta \eta_{\text{VD}} = \frac{\Delta h_{\text{VD}}}{\Delta h_{\text{act}}} \tag{B99}
$$

where

$$
\Delta h_{\text{act}} = \Delta h_{\text{aero}} + \Delta h_{\text{RC}} + \Delta h_{\text{DF}} \tag{B100}
$$

Overall efficiency. - The overall **total efficiency** is **calculated as** follows:

$$
\eta_{AD} = \frac{\Delta h_{aero} - (\Delta h_{IGV} + \Delta h_{INC} + \Delta h_{BL} + \Delta h_{SF} + \Delta h_{VLD} + \Delta h_{VD})}{\Delta h_{aero} + \Delta h_{RC} + \Delta h_{DF}}
$$
(B101)

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