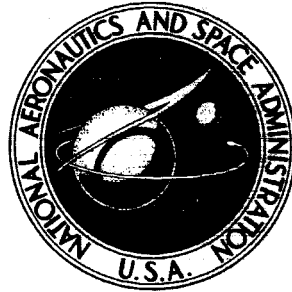


N74-10030

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NASA TN D-7387

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A STUDY OF METHODS WHICH PREDICT  
SUPERSONIC FLOW FIELDS FROM  
BODY GEOMETRY, DISTANCE, AND MACH NUMBER

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NATIONAL AERONAUTICS AND SPACE ADMINISTRATION • WASHINGTON, D. C. • NOVEMBER 1973

1. Report No. NASA TN D-7387	2. Government Accession No.	3. Recipient's Catalog No.	
4. Title and Subtitle A STUDY OF METHODS WHICH PREDICT SUPERSONIC FLOW FIELDS FROM BODY GEOMETRY, DISTANCE, AND MACH NUMBER		5. Report Date November 1973	
		6. Performing Organization Code	
7. Author(s) Robert J. Mack		8. Performing Organization Report No. L-8940	
		10. Work Unit No. 501-06-11-01	
9. Performing Organization Name and Address NASA Langley Research Center Hampton, Va. 23665		11. Contract or Grant No.	
		13. Type of Report and Period Covered Technical Note	
12. Sponsoring Agency Name and Address National Aeronautics and Space Administration Washington, D.C. 20546		14. Sponsoring Agency Code	
		15. Supplementary Notes	
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17. Key Words (Suggested by Author(s)) Supersonic flow Pressure fields - theoretical and experimental Bodies of revolution		18. Distribution Statement Unclassified - Unlimited	
19. Security Classif. (of this report) Unclassified	20. Security Classif. (of this page) Unclassified	21. No. of Pages 24	22. Price* Domestic, \$2.75 Foreign, \$5.25

# A STUDY OF METHODS WHICH PREDICT SUPERSONIC FLOW FIELDS FROM BODY GEOMETRY, DISTANCE, AND MACH NUMBER

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## SUMMARY

A study of seven methods for predicting flow-field pressure signatures from the parameters Mach number, body geometry, and field-path distance has been made. The methods included the method of characteristics, which served as a standard of comparison; a shock-capturing method; three Whitham theory methods; a modified characteristics method; and a bicharacteristics method. Results from each method were also compared with recently obtained wind-tunnel data for a cone-cylinder model at Mach numbers of 2.96 and 4.63 with ratios of radial distance to cone length of 2 and 5.

The comparisons at a Mach number of 2.96 showed that signatures from all the methods correlated well with wind-tunnel data and with the signatures predicted by the method of characteristics. At a Mach number of 4.63, however, the agreement between the signatures obtained in the wind tunnel and those predicted by theory varied from good to poor, as did the agreement between the signatures obtained by the method of characteristics and the other six methods. It should be noted that these results and comparisons indicate pressure prediction capabilities only for the near-field flow about bodies of revolution.

## INTRODUCTION

Early efforts to predict flow-field properties on and about slender bodies of revolution in supersonic flow were based on simple linearized theory. Although this theory was successful in providing estimates of drag, it could not be used for the accurate prediction of pressures at points in the flow field away from the body surface where pressure signatures may be required.

The introduction of the method of characteristics, which is outlined in reference 1, and the Taylor-Maccoll cone-theory solution (ref. 2) permitted accurate and complete flow-field predictions to be made. However, the usefulness of these methods was somewhat limited by the complicated analytical and computational procedures required, the

lack of cone-theory generality, and the difficulty in maintaining a sufficiently tight grid network in the flow-field representation as the solution progressed to large radial and longitudinal distances.

A significant breakthrough occurred when G. B. Whitham combined elements of characteristics theory and linearized theory. On the basis of the work in reference 3 by Lighthill, reference 4 by Friedrichs, and reference 5, Whitham introduced a method for axisymmetric bodies (ref. 6) which was relatively quick and easy to use and which described the essential first-order features of the far-field flow regime. Interest in the Whitham theory was mostly academic until the feasibility of a supersonic transport was demonstrated, and questions were raised about the sonic boom it would generate. Hayes (ref. 7) and Lomax (ref. 8) had shown that a complex aircraft configuration could be represented by an equivalent body of revolution insofar as its far-field influence was concerned. The combining of the equivalent-body concept and the Whitham theory formed the final step in devising a general method to predict far-field sonic-boom signatures.

Computer programs based on Whitham theory (refs. 9 and 10) were written and used in conjunction with wind-tunnel and flight-test data to evaluate the soundness and applicability of this new and ingenious concept. Results from these studies (refs. 9 and 11) showed that Whitham theory predicted pressure signatures reasonably well in the low and middle Mach number range (1.25 to 2.0) at large distances, but also indicated that it could be judiciously applied in the near field if the model or aircraft was smooth and slender in the linearized-theory sense.

Although used successfully in many applications, the Whitham theory was found to have limitations. Whitham theory signatures had been found to correlate poorly with wind-tunnel signatures obtained at Mach numbers above 3.0 (ref. 12) for bodies of varying fineness ratio. Another instance of poor agreement between experiment and theory at high supersonic Mach numbers was reported in reference 13. Consequently, other methods were developed to provide more accuracy in flow-field predictions.

These new near- and mid-field methods can be divided into three groups. The first group consists of methods that employ the Whitham approach, that is, obtain pressure signatures directly from a description of body geometry, field-path distance, and Mach number. Detailed presentations of these methods are found in references 14 to 17. In this report, predictions obtained by these methods will be examined and compared with experimental results from reference 12 and with results obtained from the method of characteristics. In addition, the results from the far-field Whitham theory will be shown and compared to give some idea of the progress that has been made. All these methods are of some use because they can be applied to a wide variety of problems to provide solutions without the need for costly wind-tunnel experimentation.

The second group consists of methods that predict mid- or far-field signatures by extrapolating measured near-field pressure distributions or calculated  $F$  functions (refs. 18 to 22). The third group contains solution-matching methods such as that of reference 23. Although interesting and useful, methods in the second and third groups are of little use in the near field, where all corroborative wind-tunnel data are obtained.

A simple cone-cylinder model was used to obtain pressure signatures from the methods contained in the first group. The signatures were cut off aft of the shoulder expansion, where a trend toward asymptotic recompression behavior was evident, and were compared with both signatures measured in the wind tunnel (ref. 12) and signatures predicted by the method of characteristics. Instead of showing all the experimental data, only the results at Mach numbers of 2.96 and 4.63 were used to keep the number of comparisons within reasonable bounds.

### SYMBOLS

$A$	normal areas along the longitudinal body axis
$F$	Whitham $F$ function
$\bar{F}$	integral of the $F$ function for a pointed body
$h$	Whitham influence function
$k = \frac{(\gamma + 1)M^4}{\sqrt{2\beta^3}}$	
$\bar{k} = \frac{M^2}{\sqrt{2\beta}}$	
$K = \frac{k}{\sqrt{2\beta}}$	
$l$	length of cone
$M$	Mach number
$p$	free-stream static pressure
$\Delta p$	incremental pressure due to flow field of model

r	radial distance
R	body radius
S	"Mach sliced" areas along the longitudinal body axis
t	dummy variable
x	distance along the longitudinal axis
y	value of the characteristic
$\beta = \sqrt{M^2 - 1}$	
$\gamma$	ratio of specific heats
$\theta$	semivertex cone angle

Primes indicate derivatives.

## SURVEY MODEL

A cone-cylinder with a semivertex angle of  $6.44^\circ$ , a forebody length of 5.08 cm (2.0 in.), an afterbody length of 20.32 cm (8.0 in.), and a maximum cross-sectional area of  $1.032 \text{ cm}^2$  ( $0.16 \text{ in}^2$ ) was used in the wind-tunnel tests and the mathematical analysis as the flow-field generating body. In figure 1, the model and some flow-field features are shown. The boundary-layer displacement thickness on the model was not shown or used in the analysis because an accurate representation could not be obtained.

## STUDY CONTENTS

Seven methods for predicting pressure signatures from body geometry, field-path distance, and Mach number conditions are discussed in this paper. The first two methods – the method of characteristics and the shock-capturing method – are "exact" methods since they include all flow effects except for viscosity and gravity. The next three are Whitham theory methods: the nonsmooth-body Whitham method, the smooth-body Whitham method, and the modified Whitham method. The last two methods are a near-field bicharacteristics method and a large-distance, modified characteristics method.

The methods are presented and described in the order and the grouping that has been outlined above. Comparisons of predicted signatures and wind-tunnel measured signatures are made for the conditions  $M = 2.96$ ,  $r/l = 5$ ;  $M = 4.63$ ,  $r/l = 2$ ; and  $M = 4.63$ ,  $r/l = 5$ . In addition, the method of characteristics is used as an analytical standard for comparison with the other six methods.

## WIND-TUNNEL COMPARISON DATA

The wind-tunnel results which were compared with the predicted signatures were obtained from reference 12. All the measured pressure signatures were rounded, to some extent, in locations where sharp discontinuities or shocks were expected. This rounding was caused by model and probe vibrations, by viscosity effects on the model and probe, and by random flow irregularities in the tunnel. More will be mentioned when the exact-method results are compared with experimental data.

## EXACT METHODS

### Method of Characteristics

Flow fields produced by bodies moving supersonically in inviscid fluids can be computed with reasonable accuracy by using numerical techniques based on the method of characteristics. In general, the characteristic lines form a nonlinear network which must be constructed, point by point, as the solution progresses. Although the network is non-uniform and requires complex computational procedures, the method has the advantage of correctly handling shock waves and expansions by properly accounting for the true region of influence of each field point. When used with care, the method can serve as a standard against which other methods for predicting inviscid, supersonic flow fields can be compared.

Complete equations, derivations, and explanation can be found in texts such as reference 1. It should be noted, however, that computational accuracy will depend on the tightness of the mesh describing the flow field. In most applications, the characteristics tend to diverge and the mesh points spread apart with increasing longitudinal and radial distance, making it increasingly difficult to maintain a high order of accuracy throughout the flow field. These difficulties were overcome by using several grid sizes to obtain solutions.

### Shock-Capturing Method

The shock-capturing method, like the method of characteristics, is based on the exact, inviscid-flow equations of continuity, momentum, energy, and state. Unlike the

method of characteristics, however, the shock-capturing method uses a finite-differencing scheme combined with a "predictor-corrector" technique to obtain a second-order solution. Instead of a characteristics mesh, a grid system of body-oriented coordinates is used to locate the calculated points. The complete set of equations in finite-difference form can be found in reference 17 along with an explanation and a number of sample solutions.

Free-stream flow conditions and an imposed tangent nose cone serve as the boundary and the starting points of a solution which is marched along the body and out into the flow field one grid station at a time. Shocks are simulated as rapidly increasing, but continuously varying, pressure changes.

### Comparisons

Pressure signatures obtained by the method of characteristics and the shock-capturing method are presented in figure 2 along with wind-tunnel data. The results from the method of characteristics were provided by Lillian R. Boney, of the Langley Research Center; the results from the shock-capturing method, by Paul Kutler, of the Ames Research Center.

Both these exact solutions are in good agreement with wind-tunnel data at  $M = 2.96$ . Peak positive and negative theoretical pressures appear to be overpredicted, but these differences between theory and experiment are probably due in part to errors caused by the effects of viscosity, model and probe vibration, and random flow irregularities. A pressure discontinuity across an oblique nose shock is spread over a finite distance as it passes through the boundary layer on the conical measuring probe and thus appears to be rounded instead of peaked. This rounding tendency is reinforced by contributions from model and probe vibration as well as random flow irregularities. At higher Mach numbers, the rounding becomes more pronounced, possibly because the characteristics are more closely aligned with the surface slope of the measuring probe and its displacement thickness.

In the trailing-shock region, the pressure signature will display rounding caused by model boundary-layer effects also. The recompression is less complete, the shock strength is less intense, and the signature is rounded by viscosity and vibration effects. A more complete discussion of signature rounding is found in appendix B of reference 9.

In view of these considerations, relatively good agreement between the method of characteristics, the shock-capturing method, and the wind-tunnel measured signatures is found at  $M = 4.63$  and  $r/l = 2$ . However, only fair agreement appears to be present at  $M = 4.63$  and  $r/l = 5$ . In contrast, the signatures predicted by the method of characteristics and the shock-capturing method correlate well; the excellent agreement indicates that the predicted signatures are theoretically correct.



The agreement between predicted and measured signatures in figure 2(c) would be good if the predicted signatures were shifted by about 0.33%. Such a shift would occur if the predicted pressures were calculated for  $M = 4.69$  instead of 4.63. In the wind-tunnel test section, the model and the probe were separated radially by 5 body lengths and longitudinally by about 22.6 body lengths. It is doubtful that conditions at  $M = 4.63$  were faithfully maintained throughout this flow region. Therefore, the disagreement between theory and experiment is probably due to nonuniform flow conditions, a consideration which must be kept in mind when the other pressure prediction methods are examined.

## WHITHAM THEORY METHODS

The classical method of predicting pressure signatures derived by G. B. Whitham is reported in reference 6. This theory is based on the assumptions that the body is slender; the flow is inviscid, adiabatic, and nearly isentropic; the shock waves are weak; and the pressure disturbances, apart from the shocks, are acoustical in magnitude and behavior far from the body axis. The resulting linearized equations are modified by the hypothesis that the predicted incremental pressures are nearly correct in magnitude but are longitudinally misplaced. Accordingly, the aft-running characteristics, which are the important ones in the Whitham theory, are corrected by terms representing the effects of the body area distribution and the field-path distance.

Whitham presented his theory in two forms: a nonsmooth-body solution and a smooth-body solution. Later, a third form, a modified Whitham method, was developed for use in the near field. In the following sections, each of these three methods will be examined.

### Nonsmooth-Body Method

The Whitham formulation of the far-field characteristic is given as

$$x - \beta r = y - kF(y)r^{1/2} \quad (1)$$

where  $F(y)$ , the  $F$  function, is given by

$$F(y) = \int_0^y \left[ \frac{2}{\beta R(t)} \right]^{1/2} h\left(\frac{y-t}{\beta R(t)}\right) \frac{dA'(t)}{2\pi} \quad (2)$$

and  $h$ , the influence function, is given in reference 6. This  $F$  function is applicable only to bodies of revolution.

Pressure signatures are found by distorting and normalizing the  $F$  function. For each  $y$ ,  $F(y)$  is relocated a distance  $x - \beta r$ , as given by the characteristic equation. Multiple values appearing in this distorted  $F$  function are removed by introducing jumps, that is shocks, by a scheme called "area balancing," which is outlined in reference 10. Thus, a normal appearance - the signature is single valued except for shocks - is restored.

#### Smooth-Body Method

The characteristic equation used with this method is the same as the one used with the nonsmooth-body method, except that the  $F$  function is

$$F(y) = \frac{1}{2\pi} \int_0^y \frac{S''(t)dt}{\sqrt{y-t}} \quad (3)$$

where  $S(t)$  may be considered the "Mach sliced" area distribution called for when applying the far-field analysis of Hayes (ref. 7) and Lomax (ref. 8). It should be noted that when the body is smooth and continuous, the same  $F$  function is calculated by both equations (2) and (3), as is shown in reference 5. However, only the smooth-body  $F$  function is applicable for the calculation of pressure signatures caused by a complex aircraft configuration which has both volume and lift contributions.

Shocks, or pressure jumps, appearing in the flow field are calculated in the same way as in the nonsmooth-body method, since the characteristic equation is the same for both.

#### Modified Whitham Method

In reference 12, pressure signatures predicted by the smooth-body Whitham method were compared with measured signatures. Reasonable correlations were found at  $M = 2.96$ , but agreement became poorer at  $M = 3.83$  and  $4.63$ . An attempt was made in reference 14 to improve the predicted signatures by using a more complete equation of the characteristic. By employing both analytical means and empirical simplifications, the equation of the characteristic for a pointed body was derived as

$$x - \beta r = y - kF(y)(r^{1/2} - R^{1/2}) - \bar{k}\bar{F}(y)(R^{-1/2} - r^{-1/2}) \quad (4)$$

where  $F(y)$  is the same as in the nonsmooth Whitham method (eq. (2)) and

$$\bar{F}(y) = \int_0^y F(t)dt \quad (5)$$

This characteristic starts on the body surface, rather than on the body axis, and makes the body size and shape a significant influencing factor in the near field. The equations of the nose and trailing-shock positions and strengths were found through the use of this near-field characteristic equation; therefore, they also contain body-geometry terms.

### Comparisons

Signatures predicted by the three Whitham methods are shown in figure 3 where they are compared with signatures predicted by the method of characteristics and those measured in the wind tunnel. At  $M = 2.96$ , two of the three Whitham signatures – the results from the nonsmooth-body and modified Whitham methods – appear to be in good agreement with the measured signature and in fair agreement with the exact, that is, method-of-characteristics signature. The signature obtained with the smooth-body Whitham method exhibits a conspicuously misplaced trailing shock, which is due to shoulder and thickness effects on the "Mach sliced" areas, and shows good agreement only in the forward half.

Similar behavior is seen at  $M = 4.63$ . The signatures from the Whitham theory methods tend to be in better agreement with the measured than with the exact signatures, but this could be fortuitous. Although the Whitham theory methods are approximate, and are probably outside their range of applicability at  $M = 4.63$ , they are useful in providing quick first-order estimates of peak pressures and signature shape.

## BICHARACTERISTICS AND MODIFIED CHARACTERISTICS METHODS

### Large-Distance, Modified Characteristics Method

In reference 15, Landahl, Rhyning, and Lofgren considered certain nonlinear effects in deriving a set of second-order, flow-prediction relations. These effects, which are important in the near field and at high Mach numbers, were obtained by working with both the aft-running and the forward-running characteristics. Landahl et al. believed that a simple set of approximate equations could be developed which would be good in the near as well as the far field.

By assuming that the field path was at a large, but not an infinite, distance from the body, an approximate equation of the aft-running characteristic was derived. For a sharp-nosed body of revolution, this equation has the form

$$x - \beta r = y - kF(y)r^{1/2} + \left(M^2 - \frac{K}{4}\right)(2\beta r)^{-1/2} \int_0^y F(t)dt \quad (6)$$

In applying this equation, the Stieltjes integral form of the  $F$  function was used because the body slopes were discontinuous. Shock positions were obtained from familiar area-balancing techniques and the characteristic equation, since no special shock relations were given. Equations for the longitudinal perturbation velocity were presented, however, and contained terms which made the velocity accurate to second order. Since these correction terms were small compared with the first-order term and tended to diminish rather than enhance pressures computed from the perturbation velocity, they were ignored.

A comparison of the characteristic of Landahl et al. (eq. (6)) with the Whitham far-field characteristic (eq. (1)) shows that the sole correction is the term containing the integral of the  $F$  function. It disappears at  $M = K^{1/2}/2$  and is at least one order of magnitude smaller than the  $F$  function term because of the influence of  $(2\beta r)^{-1/2}$ .

#### Near-Field Bicharacteristics Method

The lack of complete agreement between the predicted and the measured signatures in reference 12 prompted Sanford Davis, of the NASA Ames Research Center, to develop a near-field method for predicting pressure signatures (ref. 16). Using bicharacteristics to introduce both longitudinal and radial corrections, he derived a first-order set of characteristic and shock equations.

When recast in Whitham variables, the equation of the characteristic that Davis developed had the form

$$x - \beta r = y - \frac{kF(y)(r^{1/2} - R^{1/2})}{1 + \beta R'} \quad (7)$$

Similarities between this equation and the characteristic equation of the far-field Whitham method (eq. (1)), the modified Whitham method (eq. (4)), and the method of Landahl et al. (eq. (6)) are readily seen. In all cases, the  $kF(y)r^{1/2}$  term is the dominant factor. However, in the Davis method (ref. 16), both the body radius and the body surface slope are used to account for near-field effects.

Although derived and presented in a two-dimensional form for a cone-cylinder model, the approach could be used for three-dimensional-flow problems. The systems of equations are complex, but they can be readily programmed for the digital computer so that pressure signatures and flow-field data can be obtained.

#### Comparisons

Results from the large-distance, modified characteristics method and the near-field bicharacteristics method are shown in figure 4 where they are compared with results

obtained by the method of characteristics and in the wind tunnel. The comparisons show that a fairly good agreement is present at  $M = 2.96$  and  $r/l = 5$ . However, the situation changes to one of poor agreement at  $M = 4.63$  and  $r/l = 2$  and  $5$ . Relative to the exact solutions, the positive and negative peak pressures are underpredicted. Nose-shock positions are predicted fairly well by the modified characteristics method but poorly by the bicharacteristics method. When compared with wind-tunnel measured signatures, the correlation is somewhat better, but still is not satisfactory. The conclusion from these comparisons is that neither method seems to perform well at Mach numbers over  $3.0$ .

### CONCLUDING REMARKS

Several points stand out from this study of methods for predicting pressure signatures. The first is that the shock-capturing method and the method of characteristics give nearly the same solutions. Some rounding is present in the signatures predicted by the shock-capturing method, but this is due to the numerical scheme employed, as pointed out by Paul Kutler and Harvard Lomax in AIAA Paper No. 71-99, Jan. 1971.

The second point is that signatures predicted by most of the approximate methods agree reasonably well with signatures from the method of characteristics at a Mach number of  $2.96$  and a ratio of radial distance to cone length of  $5$ . A lone exception is the smooth-body Whitham method. Some difficulties could be expected since the model does have a discontinuity in surface slope at the shoulder.

A third point is that the good correlation between approximate and exact methods at a Mach number of  $2.96$  does not carry over to a Mach number of  $4.63$ . Positive pressure peaks are underpredicted and/or poorly located and trailing shocks, or recompressions, are not predicted or are underpredicted. An exception is the signature from the smooth-body Whitham method, where the trailing shock is of about the correct magnitude but is conspicuously misplaced. The conclusion that could reasonably be made is that nonlinear flow effects present at this Mach number and at these distances have rendered these approximate methods only nominally useful.

The fourth point is that the existing wind-tunnel measured signatures cannot be counted on to depict accurately all the significant features of the flow-field pressures at the high supersonic to low hypersonic Mach numbers used in the tests. These deficiencies are traceable to measuring-probe boundary-layer effects, probe and model vibration, random flow irregularities, and measuring-gage limitations.

Although the approximate methods mentioned in this study have shortcomings, they are likely to be the ones most used because of the ease and speed of their application. For distances in the mid-field and far field and for bodies that are complex, such as a supersonic cruise airplane, reasonable estimates of overpressure have been obtained and

can be expected from such schemes as the smooth-body Whitham method. In the near field, however, the shock-capturing method rather than the approximate methods seems to be most capable of accurately predicting flow-field pressures.

Langley Research Center,  
National Aeronautics and Space Administration,  
Hampton, Va., September 21, 1973.

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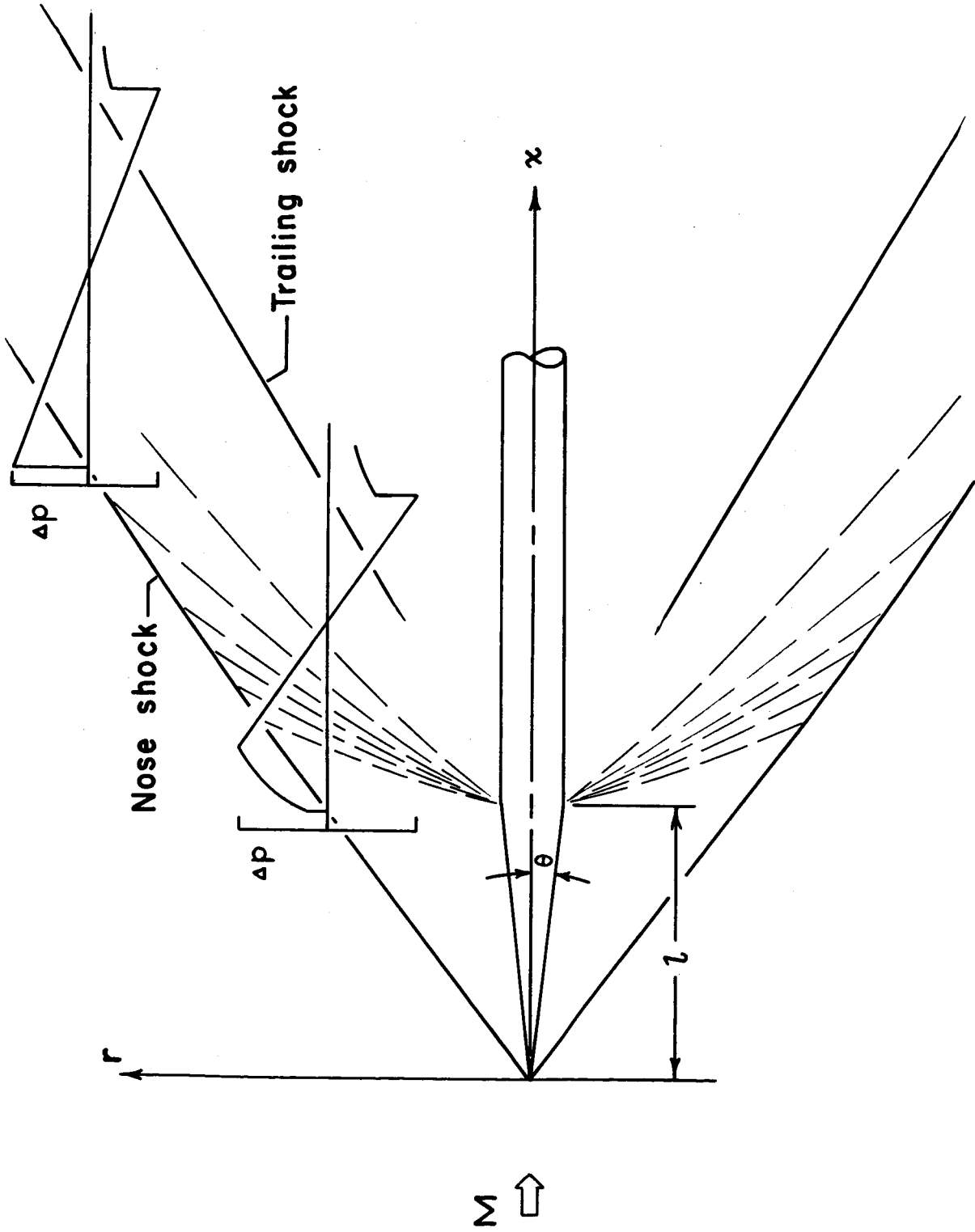
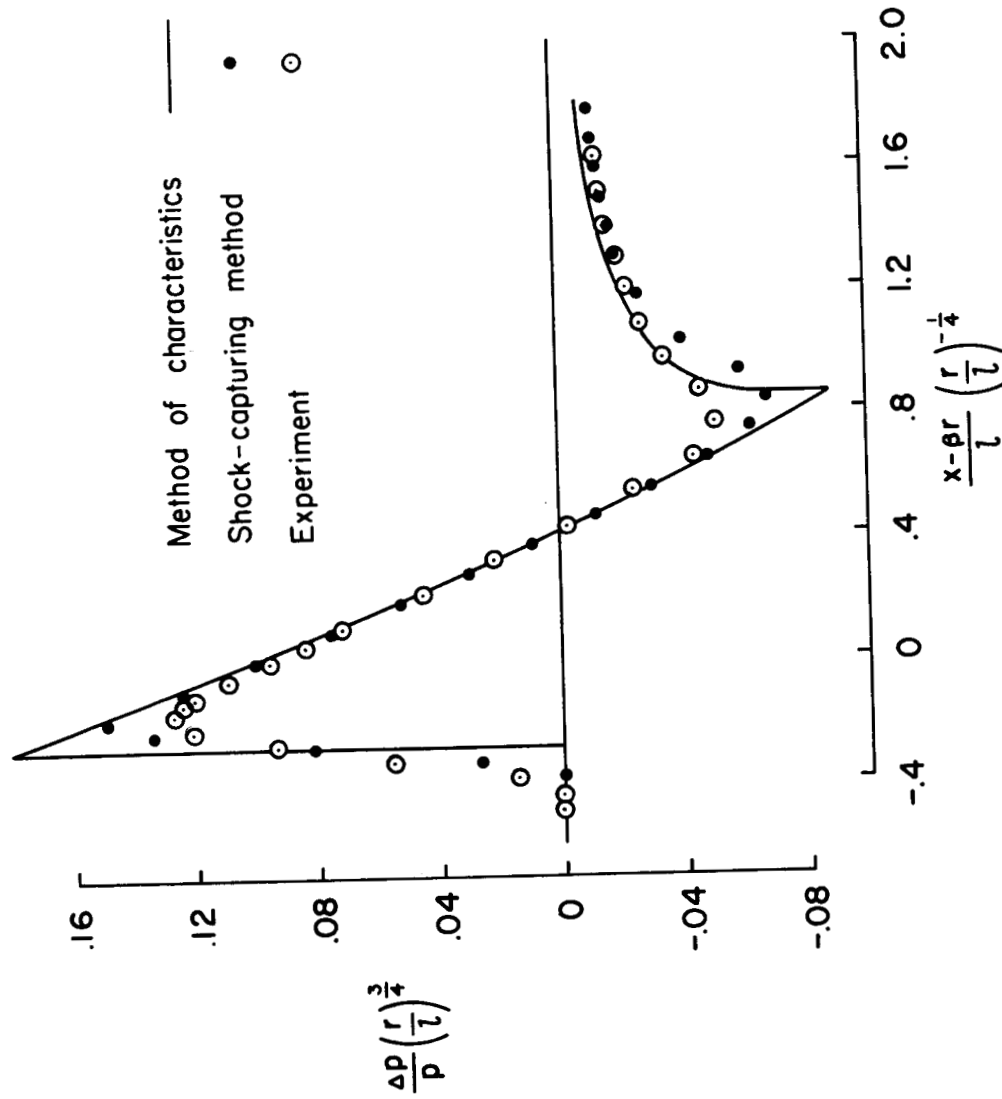
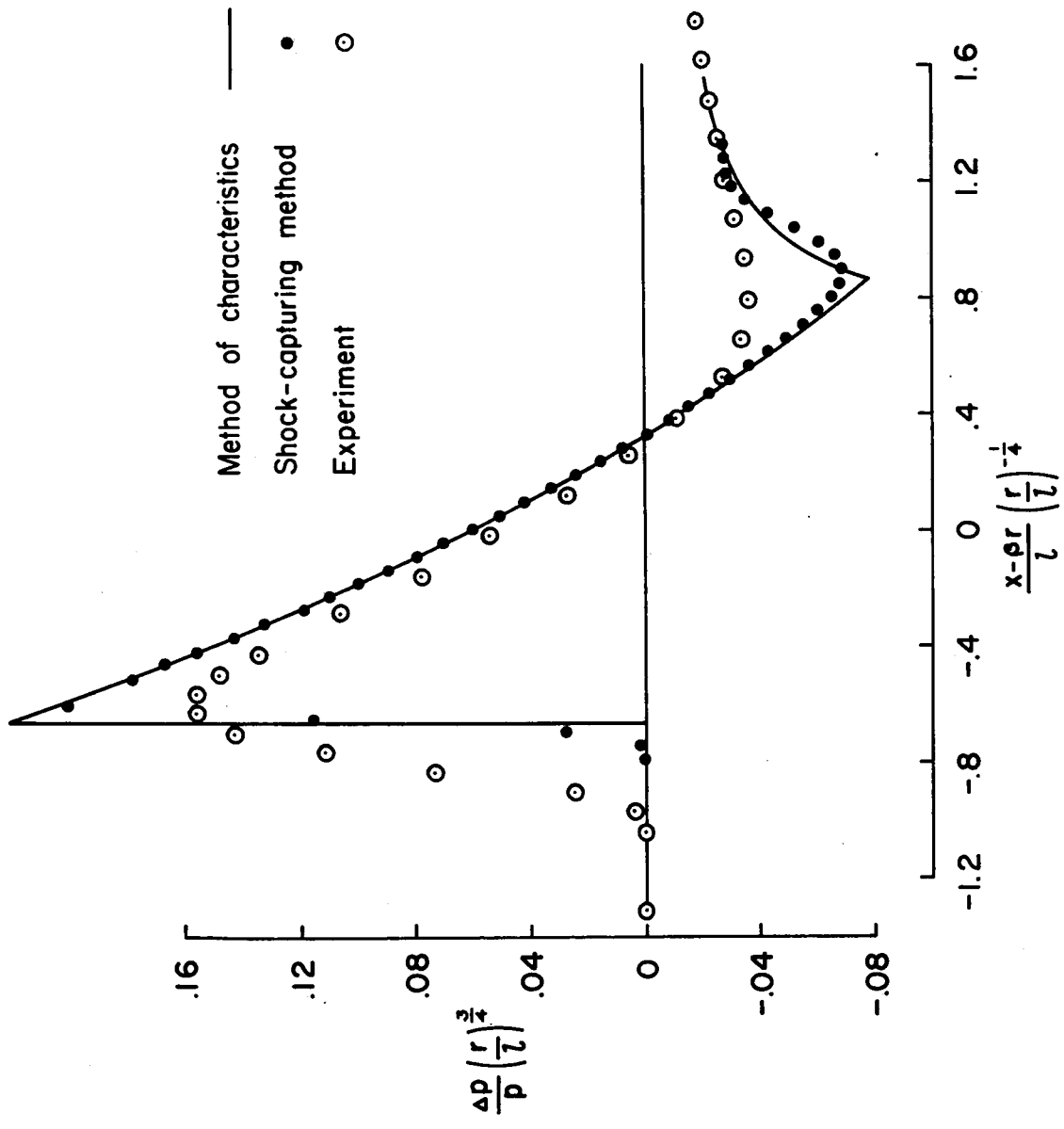


Figure 1.- Cone-cylinder model used in study.  $\theta = 6.438^\circ$ ;  $l = 5.08$  cm (2.0 in.).



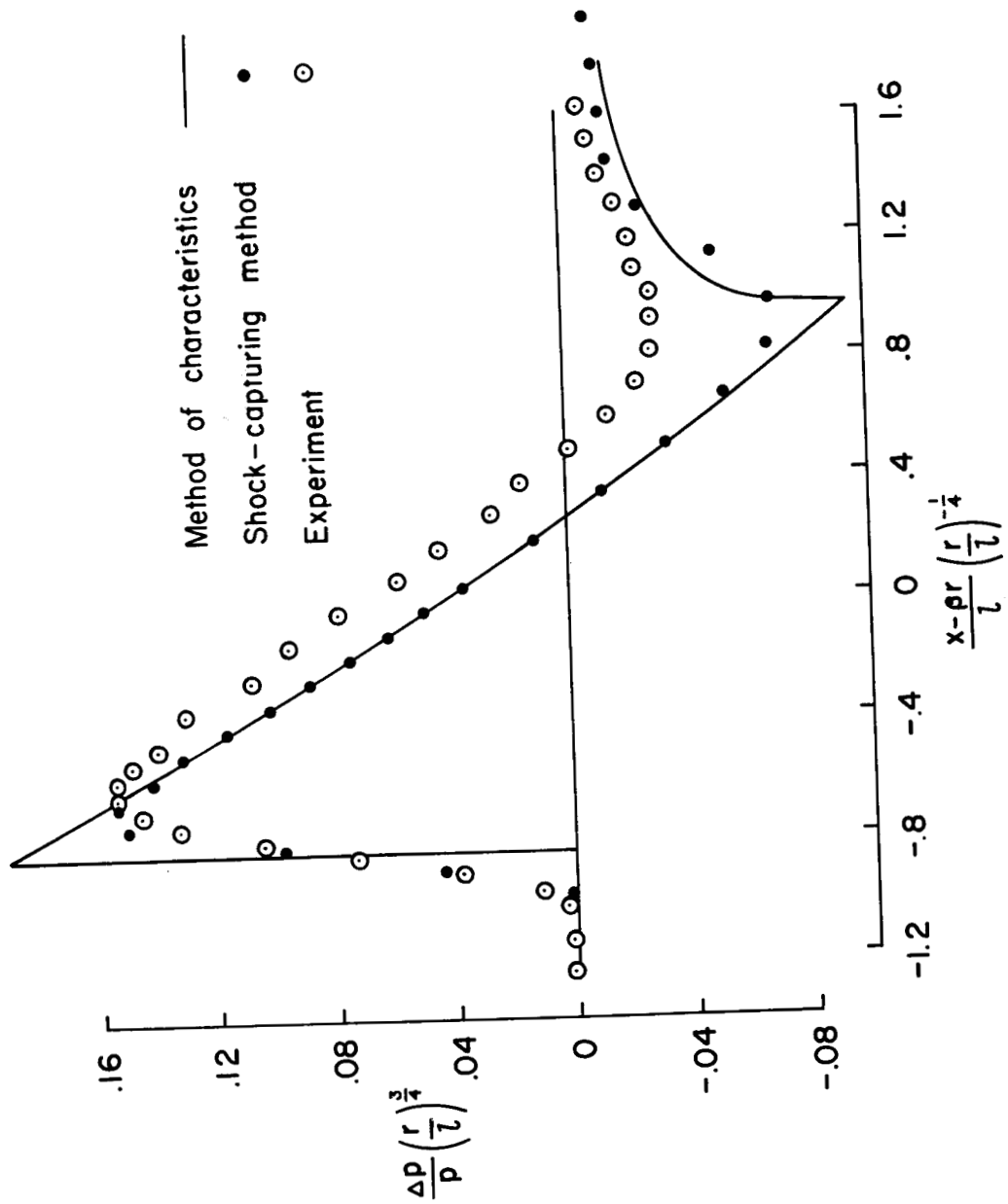
(a)  $M = 2.96$ ;  $r/l = 5$ .

Figure 2.- Signature comparisons - shock-capturing and characteristics methods.



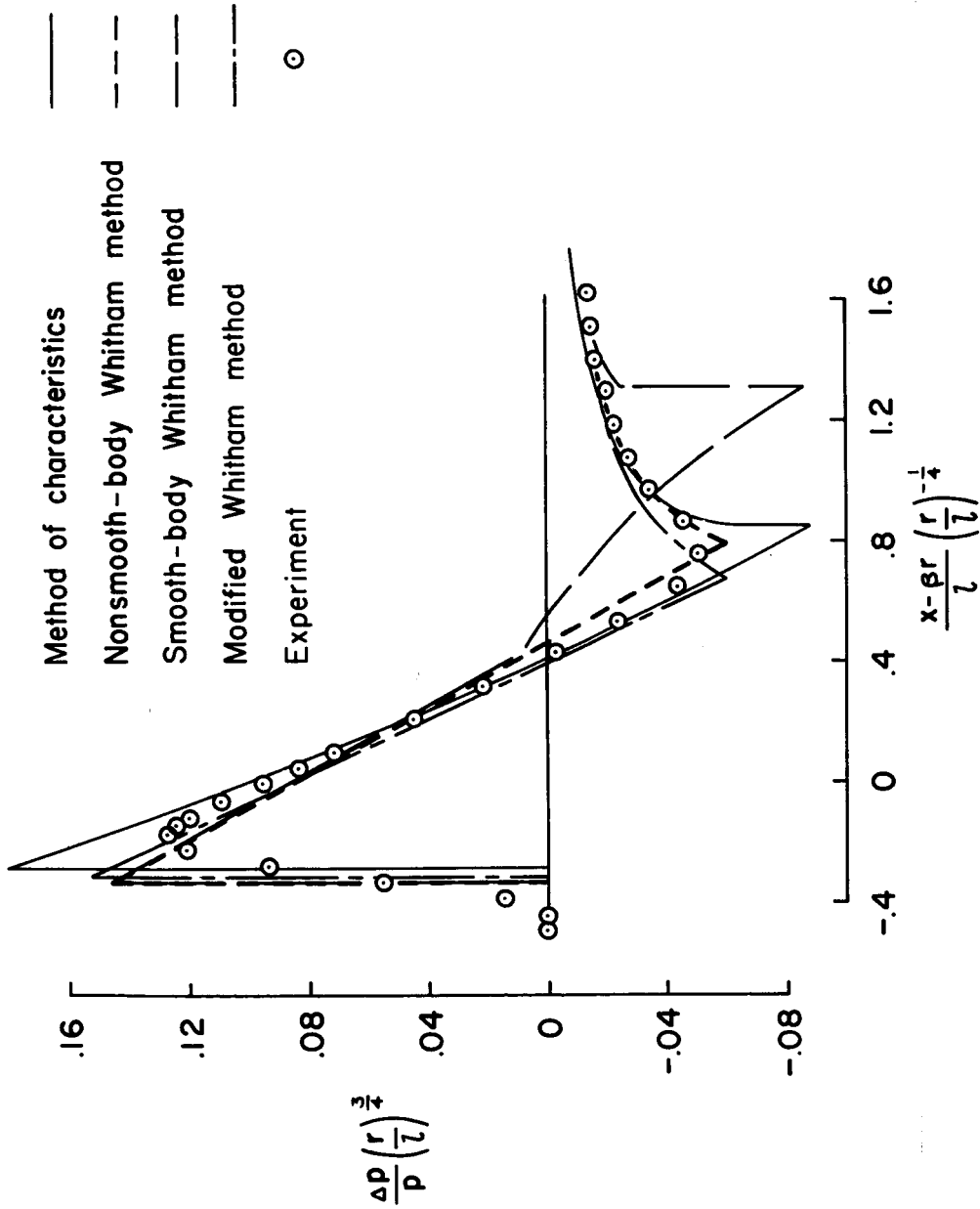
(b)  $M = 4.63$ ;  $r/l = 2$ .

Figure 2.- Continued.



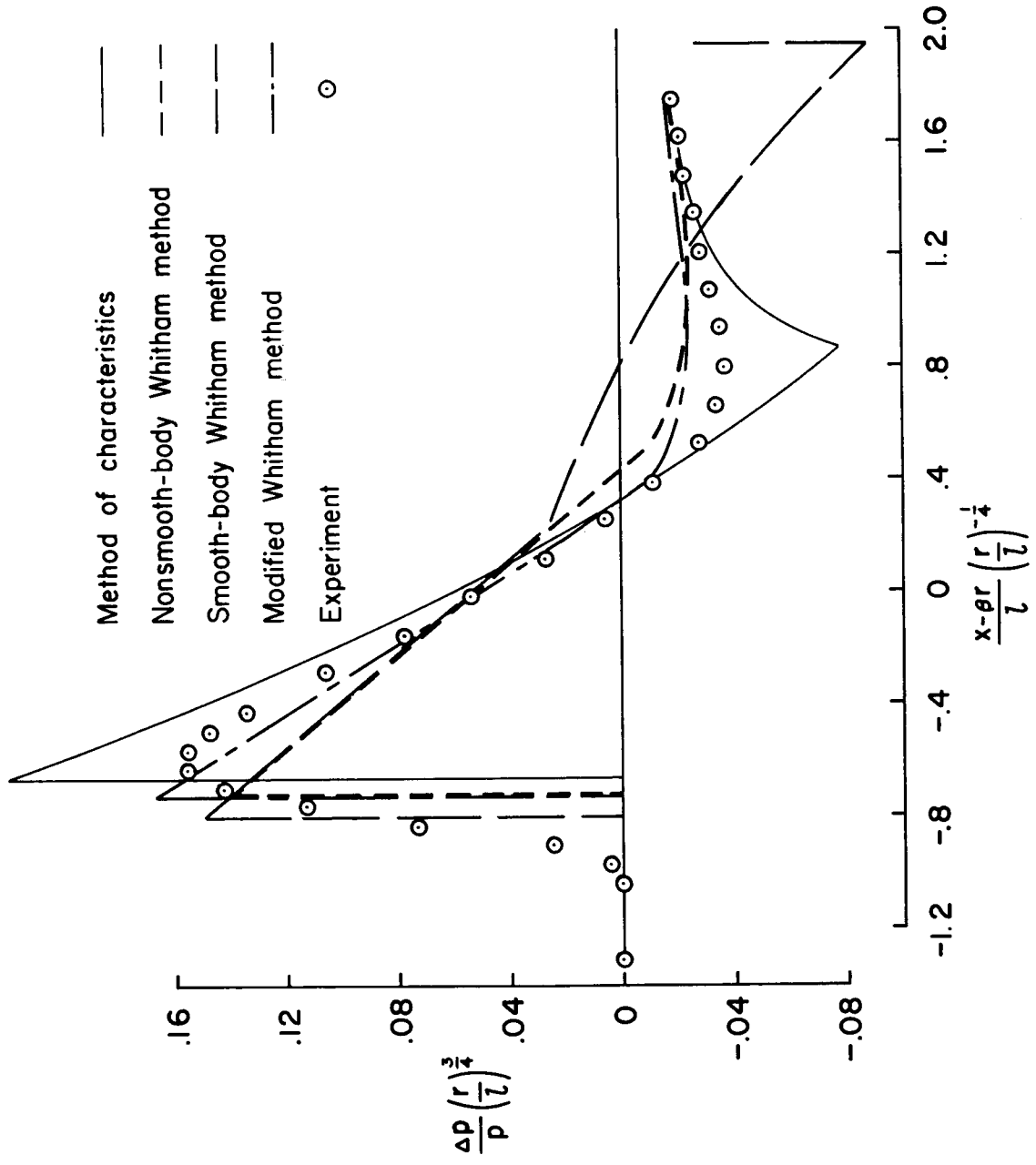
(c)  $M = 4.63$ ;  $r/l = 5$ .

Figure 2.- Concluded.



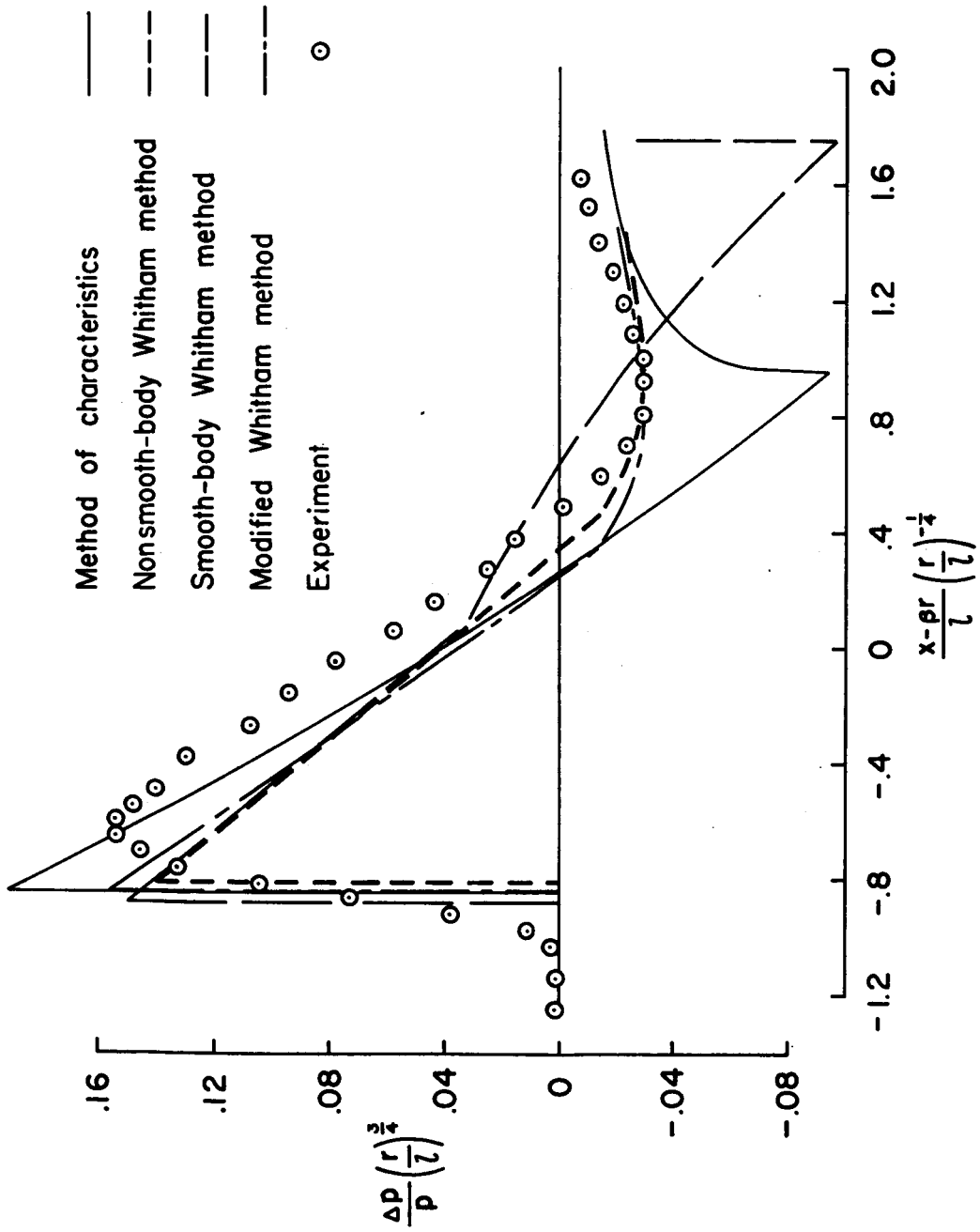
(a)  $M = 2.96$ ;  $r/l = 5$ .

Figure 3.- Signature comparisons - smooth-body, nonsmooth-body, and modified Whitham methods.



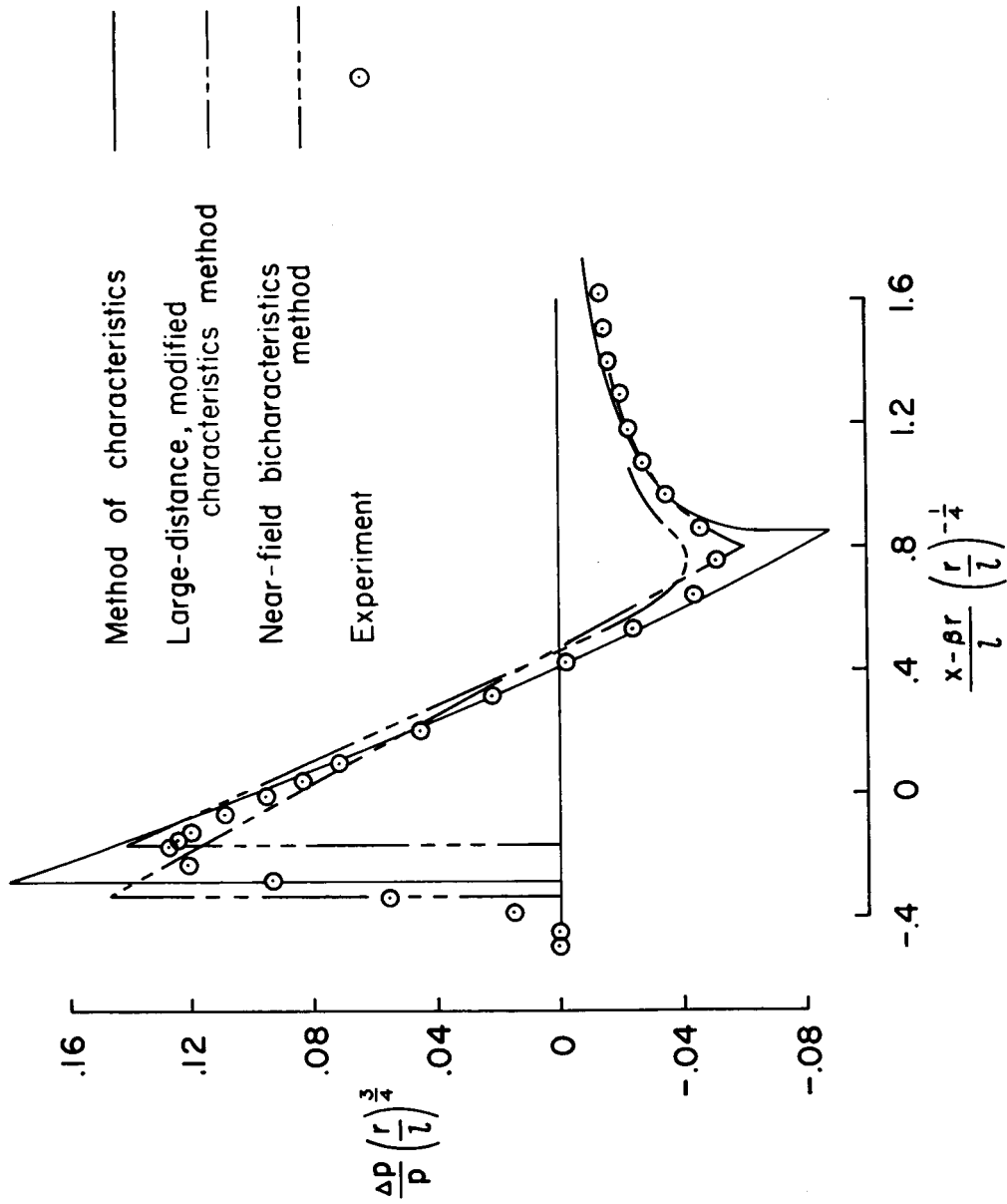
(b)  $M = 4.63$ ;  $r/l = 2$ .

Figure 3.- Continued.



(c)  $M = 4.63$ ;  $r/l = 5$ .

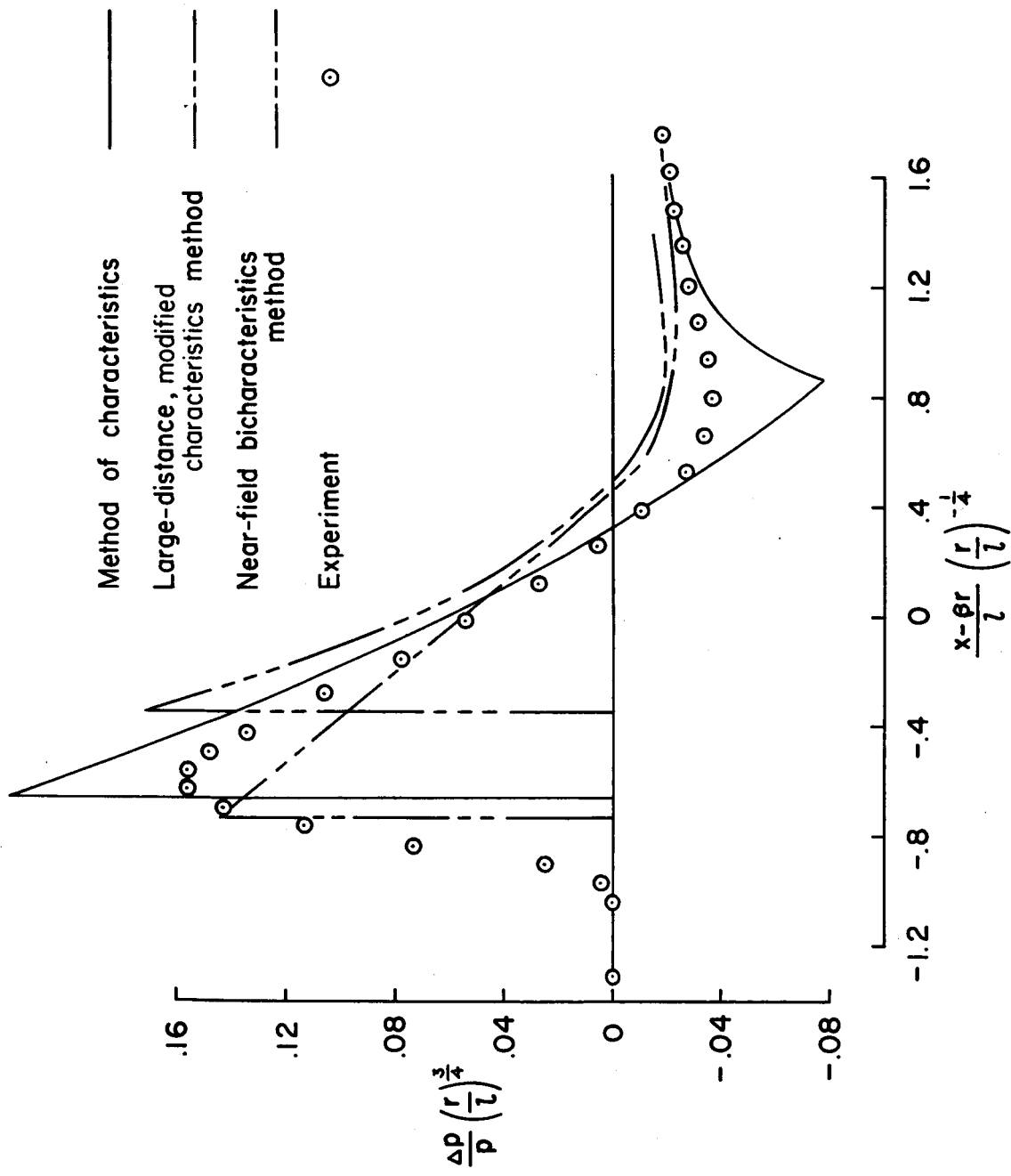
Figure 3.- Concluded.



(a)  $M = 2.96$ ;  $r/l = 5$ .

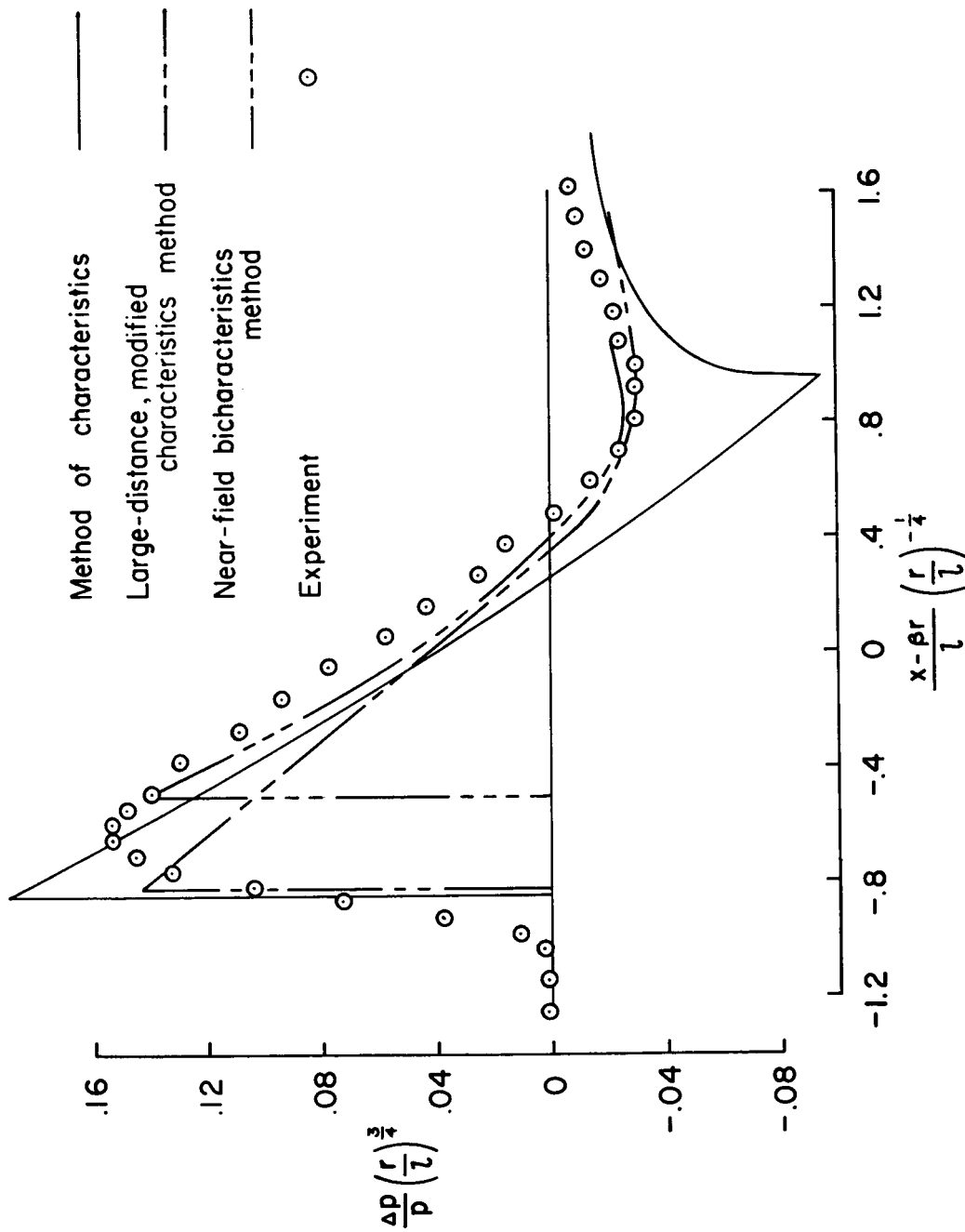
Figure 4.- Signature comparisons - near-field bicharacteristics and large-distance, modified characteristics methods.





(b)  $M = 4.63$ ;  $r/l = 2$ .

Figure 4.- Continued.



(c)  $M = 4.63$ ;  $r/l = 5$ .

Figure 4.- Concluded.