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GLOBAL DETAILED GRAVIMETRIC GEOID

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ABSTRACT

A global detailed gravimetric geoid has been computed by combining the Goddard Space Flight Center GEM-4 gravity model derived from satellite and surface gravity data and surface 1°-by-1° mean free-air gravity anomaly data. The accuracy of the geoid is ±2 meters on continents, 5 to 7 meters in areas where surface gravity data are sparse, and 10 to 15 meters in areas where no surface gravity data are available.

Comparisons have been made with the astrogeodetic data provided by Rice (United States), Bomford (Europe), and Mather (Australia). Comparisons have also been carried out with geoid heights derived from satellite solutions for geocentric station coordinates in North America, the Caribbean, Europe, and Australia.

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GLOBAL DETAILED GRAVIMETRIC GEOID

1. INTRODUCTION

This paper presents a global gravimetric geoid based upon a combination of a gravity model predominantly derived from satellite tracking data and surface 1°-by-1° gravity data. The early gravimetric geoid computations of Hirvonen (1934) and Tanni (1948, 1949) were based upon surface gravity data. The most ambitious of the pre-satellite gravimetric geoids was the Columbus geoid (Heiskanen, 1957). All of these pre-satellite geoids suffered from a lack of worldwide gravity coverage. With the advent of satellites it has been possible to derive the long wavelength components of the gravity field on a worldwide basis with considerable accuracy. The satellite-derived gravity data can be combined with the surface gravity data, in areas where surface gravity data are available, to provide accurate estimates of the details of the geoidal undulations.

The geoid is becoming increasingly important for the support of research in geodesy and geophysics. Geophysically, the independently derived gravimetric geoid (1) will provide a valuable complement to the GEOS-C and Skylab spacecraft radar altimeters, and (2) may be used for offshore mineral exploration. In geodesy the gravimetric geoid can be used to evaluate astrogeodetic geoids over the continents and to check the dynamically derived heights of tracking stations above mean sea level. The geoid can also be used as a constraint for geodetic solutions as was recently done by Mueller and Whiting, 1972.

In a previous publication (Vincent, et al., 1972) detailed geoid height maps were presented covering a substantial part of the northern hemisphere based on the

SAO 69 (Gaposchkin and Lambeck, 1970) gravity model and the surface gravity data available at that time. In the present computation a more extensive set of 1°-by-1° surface gravity data has been utilized. Also the Goddard Space Flight Center GEM-4 gravity model (Lerch, et al., 1972) derived from satellite and surface data has been used as a reference model.

The detailed gravimetric geoid presented here has an accuracy of ±2 meters rms on land and 5 to 7 meters where data were lacking. This accuracy was established by comparing the detailed gravimetric geoid with Rice's (1973) astrogeodetic geoid for the United States, Bomford's (1971) astrogeodetic geoid for Europe and astrogeodetic geoid of Mather et al., (1971) for Australia.

Comparisons have also been made between the detailed gravimetric geoid and satellite-derived tracking station positions of Goddard Space Flight Center (GSFC) (Marsh, et al., 1973 and Lerch, et al., 1972).

2. SURFACE GRAVITY DATA

The surface gravity data were collected from a number of sources. These sources included United States and foreign governmental agencies, research institutes, universities, and literature found in technical libraries and documentation centers (Casey, 1973).

2.1 SUMMARY OF DATA COLLECTED

A compilation of 23,947 records of 1°-by-1° mean free-air gravity anomaly values were obtained from the Aeronautical Chart and Information Center (ACIC), now the Defense Mapping Agency, Aerospace Center (DMA/AC). This gravity collection was augmented with data from NOAA (National Oceanic and Atmospheric Agency), Hawaii Institute of Geophysics worldwide 1°-by-1° collection, and many other sources. Some of the data were in the form of free-air anomalies at points, Bouguer anomalies, or free-air gravity contour maps. The free-air anomalies at points were compiled into average 1°-by-1° values. The Bouguer anomalies were first converted to free air anomalies before averaging.

2.2 DATA IDENTIFICATION

In general, the DMA/AC and Hawaii 1°-by-1° mean free air anomalies were used as a base in the detailed gravimetric-geoid computations. Whenever possible, local data, collected by local agencies were considered first in data-presentation. When these data were not sufficient, then DMA/AC or Hawaiian data were used, when available, to fill in the voids. With this in mind, the data used in major areas of computations are as in the following paragraphs.

2.2.1 Canada

The following sources of data were used:

1. Data were obtained from Dr. D. Nagy of the Gravity Division, Earth Physics Branch, Department of Energy, Ottawa, Ontario. The data were in the form 1°-by-2° means which were converted into 1°-by-1° means by assigning equal value to each of the two squares.

2. Canadian oceanographic data in the North Atlantic obtained from the Atlantic Oceanographic Laboratory, Bedford Institute.

3. Data from Dr. R. H. Rapp of Ohio State in the form 1°-by-1° mean anomalies, which were compiled from point gravity data.

2.2.2 North Atlantic, United States, and Northeast Pacific

The following sources were used:

1. Strange and Woollard (1964) 1°-by-1° data for the U.S.
2. Continental Shelf (East Coast) point station data obtained from NOAA. These data were reduced to 1°-by-1° values.
3. U.S. East Coast Continental Shelf point station data and U.S. Gulf Coast Continental Shelf point station data obtained from DMA/AC.
4. Bowin (1971), and Talwani (1971) point anomalies and 1°-by-1° data in the North Atlantic and Gulf Coast.
5. Strang Van Heese (1970) 1°-by-1° data in the North Atlantic.
6. Data in the North Atlantic provided by the Centre National Pour L'Exploitation De Oceans (CNEYO), Paris, France.
7. U.S. Pacific Ocean data offshore from Washington and Oregon obtained from NOAA.
8. A complete SEAMAP data series in the Northeast Pacific obtained from NOAA.
9. Hawaii Institute of Geophysics data in Hawaii.

2.2.3 Eurasia, Africa, and Australia

The following sources of data were used:

1. Kurt Arnold (1964) data of Eastern Europe in the form of 1°-by-1° means, 20'-by-12' means, 10'-by-6' means, and 30'-by-30' means.

2. Tengström (1965) 1°-by-1° mean gravity data collection for Europe.

3. Bowin (1971), Morelli (1970) point anomalies and contour maps in the Mediterranean.

4. ACIC, and Hawaii Institute of Geophysics 1°-by-1° data collection in Eurasia and Africa.

5. Point anomaly data in Kenya (1971), and Tanzania (1968) obtained from Department of Geophysics and Planetary Physics, University of Newcastle upon Tyne, England.

6. Professor Mather's 1°-by-1° mean values for Australia.

Several other sources of data were used for areas with sparse data. Some of these sources were:

1. Woollard (1968) 1°-by-1° mean values in Mexico and South America.

2. Japanese sea data in the areas of seamounts and trenches in the Pacific Ocean. The data were supplied by Prof. Tomoda, University of Tokyo.

3. Several contour maps in Venezuela were obtained from Dutch oil companies.

3. THEORY: GRAVIMETRIC GEOID COMPUTATION

The geoidal undulation at any point P on the earth can be computed using the well known Stokes' formula:

$$N(\psi, \lambda) = \frac{R}{4\pi\bar{\gamma}} \int_{\lambda'=0}^{2\pi} \int_{\psi'=-\frac{\pi}{2}}^{\pi/2} \Delta g_T(\psi', \lambda')\, S(\theta)\, \cos\psi'\, d\psi'\, d\lambda' \qquad (1)$$

where

ψ, λ = The geocentric latitude and longitude, respectively, of the computation point.

ψ', λ' = The geocentric latitude and longitude, respectively, of the variable integration point.

$N(\psi, \lambda)$ = Geoid undulation at ψ, λ.

R = Mean radius of the earth.

$\bar{\gamma}$ = Mean value of gravity over the earth.

$\Delta g_T(\psi', \lambda')$ = Free air gravity anomaly at the variable point ψ', λ'.

$$S(\theta) = \frac{1}{\sin(\theta/2)} - 6 \sin(\theta/2) + 1 + 5 \cos\theta$$

$$- 3 \cos\theta \ln \sin(\theta/2) + \sin^2(\theta/2)$$

where

$$\theta = \cos^{-1}\left[\sin\psi \sin\psi' + \cos\psi \cos\psi' \cos(\lambda - \lambda')\right] \quad (1.1)$$

In order to combine surface data and data derived from GEM-4 for computation of geoidal height at point P the earth is divided into two areas, a local area (A_1) surrounding the point P, and the remainder of the earth (A_2). Also each gravity anomaly in each area is partitioned into two parts represented by the symbols Δg_s and Δg_2. Δg_s is defined as that part of the gravity anomaly which can be represented by the coefficients in a spherical-harmonic expansion of the gravitational potential derived from satellite observations. The Δg_2 value is defined as the remainder of the gravity anomaly. Using this division of the earth's surface into two areas and of gravity anomalies into two components one can write Equation 1 in the form:

$$N(\psi, \lambda) = N_1 + N_2 + N_3 \tag{2}$$

where

$$N_1 = \frac{R}{4\pi\bar{\gamma}} \int_0^{2\pi} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \left[\Delta g_s(\psi', \lambda')\, S(\theta) \cos\psi' \, d\psi' \, d\lambda'\right]$$

$$N_2 = \frac{R}{4\pi\bar{\gamma}} \iint_{A_1} \left[\Delta g_2(\psi', \lambda')\, S(\theta) \cos\psi' \, d\psi' \, d\lambda'\right] \tag{3}$$

$$N_3 = \frac{R}{4\pi\bar{\gamma}} \iint_{A_2} \left[\Delta g_2(\psi', \lambda')\, S(\theta) \cos\psi' \, d\psi' \, d\lambda'\right]$$

The following paragraphs discuss how each of the three components presented in Equation 3 is handled in the computations.

Given a set of coefficients $\bar{C}_n^m$, $\bar{S}_n^m$, a number of methods exist for the computation of the N_1 component of the geoidal undulation.

The computation of N_1 was carried out using the procedure described by Bacon, et al., (1970). Briefly this procedure consists of fixing a value of the potential, W_0, and computing the component N_1 as

$$N_1 = r - r_E \tag{4}$$

where

r is the radial distance to the equipotential surface defined by W_0 and the potential coefficients of the GEM-4 gravitational potential model.

r_E is the radial distance to a selected reference-ellipsoid defined by a semi-major axis (a_e) and flattening (f).

The radial distance, r, to the equipotential surface W_0 at a particular latitude and longitude ψ, λ is determined by using the equation

$$W_0 = \Omega(r, \psi, \lambda) = \frac{GM}{r}\left[1 + \sum_{n=2}^{k} \sum_{m=0}^{n} \left(\frac{a_e}{r}\right)^n (\bar{C}_{nm} \cos m\lambda + \bar{S}_{nm} \sin m\lambda) \bar{P}_{nm}(\sin\psi)\right] + \frac{\omega^2 r^2}{2} \cos^2\psi \quad (5)$$

where

GM = the product of the gravitational constant and the mass of the earth

a_e = semimajor axis of the reference ellipsoid

r = geocentric radius

ω = earth's angular velocity

$\bar{C}_{nm}$ and $\bar{S}_{nm}$ = fully normalized spherical harmonic coefficients of the gravitational potential

$\bar{P}_{nm}(\sin\psi)$ = Normalized Associated Legendre Polynomial

The only unknown in this equation is r. Values of $r_1 = R + \epsilon$, $r_2 = R - \epsilon$, and $r_3 = (r_1 + r_2)/2$ are chosen for substitution into Equation 5 for evaluation of the functions

$$\Omega_1(r_1, \psi, \lambda),\ \Omega_2(r_2, \psi, \lambda),\ \text{and}\ \Omega_3(r_3, \psi, \lambda).$$

The r_i for which $|\Omega_i - W_0|$ is a maximum is identified and eliminated from consideration. The two remaining values of r_i are labeled r_1 and r_2 and are used for calculation of $r_3 = (r_1 + r_2)/2$. The potential functions are evaluated with these arguments and the worse-value elimination process is repeated. The process continues until an r is chosen such that $|\Omega(r, \psi, \lambda) - W_0| \leq 10^{-12}$. Using this value of r and the value of r_E computed using the input values of a_e and f of the reference ellipsoid, a geoid undulation component N_1 is computed.

For the computations described in this paper, the area A_1 for a point at which the geoid was being computed was defined to consist of a twenty degree-by-twenty degree area centered on the computation point. The computational formula used was:

$$N_2 = \frac{R}{4\pi\bar{\gamma}} \sum_{j=1}^{400} \overline{\Delta g_2}(\psi_j', \lambda_j')\ S(\theta_j)\ \cos\psi_j'\ \Delta\psi'\ \Delta\lambda' \tag{6}$$

where

$\overline{\Delta g_2}(\psi_j', \lambda_j')$ is the mean value of $\overline{\Delta g_2}$ within the j^{th} 1°-by-1° square.

$S(\theta_j)$ is the value of Stokes' function at the center of the j^{th} 1°-by-1° square.

$\Delta\psi' = \Delta\lambda' = 1°$.

The value of Δg_2 used for each 1°-by-1° square was computed using the formula

$$\overline{\Delta g_2} = \overline{\Delta g_e} - \overline{\Delta g_s}$$

The $\overline{\Delta g_e}$ values are mean 1°-by-1° free-air anomalies provided by surface gravity data. Values of Δg_e for each 1°-by-1° square were computed by carrying out the computation

$$\overline{\Delta g_e} = \overline{\Delta g_{IF}} + \gamma_{IF} + P.C. - \gamma_N$$

where

$\overline{\Delta g_{IF}}$ = Mean value of free air anomaly referred to the International Gravity Formula.

γ_{IF} = Value of surface gravity as defined by the International Gravity Formula.

P.C. = Potsdam correction with a value of -13.7 mgal.

γ_N = $978032.2\ (1 + .0053025 \sin^2 \psi - .00000585 \sin^2 2\psi)$ mgals.

In carrying out the computations γ_{IF} and γ_N were evaluated at the center of each 1°-by-1° square..

The $\overline{\Delta g_s}$ values are that part of the mean 1°-by-1° free-air anomalies represented by the GEM-4 harmonic coefficients used in computing N_1. The $\overline{\Delta g_s}$ values are obtained by evaluating the following equation at the center of each 1°-by-1° square.

$$\overline{\Delta g_s} = \bar{\gamma} \sum_{n=2}^{k} \sum_{m=0}^{n} (n - 1) [\bar{C}_{nm} \cos m\lambda' + \bar{S}_{nm} \sin m\lambda'] \bar{P}_{nm}(\sin \psi') \qquad (7)$$

where

$\bar{\gamma}$ = Mean value of gravity over the earth in milligals.

k = Upper limit on degree and order of the geopotential model.

n = Degree index of harmonic coefficients.

m = Order index of harmonic coefficients.

In Equation 7, the $\bar{C}_{20}$ and $\bar{C}_{40}$ terms do not represent the complete coefficients but rather the difference between the complete coefficients and the coefficients compatible with the ellipsoid used in computing N_1. The difference values used were $\overline{\Delta C}_{20} = .01954 \times 10^{-6}$ and $\overline{\Delta C}_{40} = -.2417 \times 10^{-6}$ (fully normalized). In order for the above described procedure to produce correct results, the quantities $\overline{\Delta g_e}$, $\overline{\Delta g_s}$, and the a and f which define the ellipsoid used to compute N_1 must all be compatible. Compatibility implies that the values of $\bar{C}_{20}$ and $\bar{C}_{40}$ used to compute the values of theoretical gravity needed to obtain $\overline{\Delta g_e}$ and $\overline{\Delta g_s}$ are the same as the values of $\bar{C}_{20}$ and $\bar{C}_{40}$ implied by the reference ellipsoid. Correct results in the absolute sense are also dependent upon the value of W_0 chosen to represent the true value of the potential of the geoid. The effects of not making $\overline{\Delta g_e}$, $\overline{\Delta g_s}$, a_e, and f compatible are twofold. First, all the computed geoid heights may be in error by a constant; in addition, there will be a systematic error as a function of latitude. The effect of selecting an incorrect value of W_0 would be to introduce a constant error in all geoid heights.

In the calculations described here, the term N_3 in Equation 2 is set equal to zero. This is equivalent to assuming that the GEM-4 derived approximation to the gravity field is adequate for the area A_2 at a distance of greater than ten degrees from the computation point.

The parameters used in this computation were:

$$W_0 = 6263687.5 \text{ kgal m}$$

$$\gamma_e = 978032.2 \text{ mgal}$$

$$a_e = 6378.142 \text{ km}$$

$$1/f = 298.255$$

$$GM = 3.986009 \times 10^5 \text{ km}^3/\text{sec}^2$$

4. DISCUSSION AND ANALYSIS OF RESULTS

4.1 THE REFERENCE GRAVITY MODEL

The reference gravity model provides information on the long wavelength (approximately 1000 km) contribution of the earth's gravity field. Previous detailed geoid computations were carried out using the SAO 69 Standard Earth Model as the reference field. This model has proven to be an invaluable tool for satellite-derived gravity anomaly analysis and comparison and evaluation of satellite derived positions of tracking stations. Recent GSFC computations have provided gravity fields complete to degree and order 16 based on combination of surface gravity data and satellite observations. When geoidal undulations computed using the SAO 69 model were compared with those derived from the GEM-4 model (Figure 1), variations as large as 15 to 20 meters were detected. The large magnitude of these differences prompted a series of tests on the two models. As a result of these tests the GEM-4 model was used in the computation of the global detailed gravimetric geoid (Figure 2). The GEM-4 coefficients are presented in the appendix. Some of these tests are discussed below.

Detailed gravimetric geoids computed using both the SAO 69 and GEM-4 models were compared with the astrogeodetic geoids of Bomford in Europe and Mather et al. in Australia. In both cases, the astrogeodetic geoids were transformed to a center of mass system before comparisons were made.

In Europe, a latitude profile at 48° north latitude recommended by Bomford as being the most representative was used for the comparison. Figure 4 presents a comparison of Bomford's transformed geoid with the detailed gravimetric geoids based upon the SAO 69 and GEM-4 models. The detailed gravimetric geoids were computed with the Stokes' function integrated 10° around the computation point. The detailed geoid based upon the SAO 69 model indicated a tilt of about 1.6 arc seconds with respect to the astrogeodetic geoid. However, when the detailed geoid based upon the GEM-4 model was considered, the differences became much less systematic and were on the order of ±2 meters.

In Figure 5 the detailed geoids computed with the two models were integrated for 20° around the computation point. This computation reduces the influence of long wavelength contribution from the gravity models. Comparisons of these detailed geoids indicated good agreement with the astrogeodetic geoid. The GEM-4 detailed geoid values did not change the computations based on the 10° integration interval, indicating a more accurate representation of the long wavelength features. A test was also performed with a profile at latitude 44°N. Similar conclusions were obtained.

Another test was conducted using the astrogeodetic geoid computed by Mather et al. for Australia. Figure 6 shows a profile at latitude 26° South. The detailed geoid, when based upon the SAO 69 model exhibited a tilt of 1 arc second with respect to Mather's geoid. However, the detailed geoid based upon the GEM-4 model showed only 0.5 arc seconds tilt; this matched the results Mather found in his studies on the Australian datum (Mather, 1970).

4.2 ANALYSIS OF RESULTS

To evaluate the accuracy of the detailed geoid for the areas computed, a number of comparisons were made. The first comparison was made with the astrogeodetic geoid data of Rice (1973) for the United States. Rice supplied 1100 points distributed over the United States, of which 200 well-distributed points were selected for comparison. Before any comparisons could be made, Rice's data were transformed from the North American Datum (NAD) to the geocentric coordinate system. Table 1 presents the differences between Rice's Astrogeodetic geoid and the gravimetric geoid. The rms difference is on the order of 2 meters or less.

As a means of evaluating the scale of the geoid, detailed geoidal heights and reference ellipsoid parameters were used together with mean sea level heights taken from the NASA Directory of Observation Station Locations (NASA, 1971) to compute geocentric radii for 32 satellite tracking stations. These geocentric radii were then compared with geocentric radii derived from satellite observations by GSFC investigators (Table 2). The dynamic radius vectors and those obtained using the gravimetrically derived parameters showed no systematic difference. This level of agreement is considered excellent taking into account the potential uncertainties in the various data used in deriving the computational parameters. Of the various potential sources of the differences, the most probable causes are:

1. Errors in values of γ_e, W_0, and a_e.
2. Errors in dynamic station coordinates.
3. Errors in mean sea level elevations for some tracking stations.
4. Errors in detailed gravimetric geoid heights at tracking stations due to the use of simple free-air anomalies rather than terrain-corrected free-air anomalies.

Theoretically, terrain-corrected free-air anomalies rather than simple free-air anomalies provide more accurate estimates of geoidal height. The effect of using simple free-air anomalies is to produce geoidal heights which are systematically too negative in the vicinity of land areas with rugged relief. Dimitrijevich (1972) has shown that the value of the difference in the United States ranges from in excess of +3.5 meters in the rugged mountains of the western United States to about +0.2 meters in the eastern part of the United States. Since most tracking stations used in the comparisons are on large land masses and several are in areas of rugged relief, one to two meters differences may arise from this source. It should be noted that differences due to this source are not the result of errors in basic parameters but the use of a slightly incorrect form of surface gravity anomalies in the computations.

Another scale evaluation was conducted by comparing Mather's gravimetric geoid (Mather 1970) with our gravimetric geoid. Mather's geoid was computed based on Rapp's model complete to (12, 12). The comparisons were made along two profiles, latitudes 24° and 26° (Table 3). In both instances the variation was less than 2 meters rms and no systematic scale differences were present.

4.3 COMPARISON OF GEM-4 GEOID WITH DETAILED GEOID

Figure 3 presents a contour map of the differences between the GEM-4 geoid and the detailed geoid. Several interesting features are apparent on the plot, These features are the representation of the surface gravity short-wavelength contribution to the geoid computation that are not provided from the GEM-4 geoid. For example, in Australia, prominent differences of 10 to 12 meters occur in the eastern and western parts of the country. These large differences are attributed to the dominance of mountain ranges that adjoin relatively flat plain and shallow continental slopes. A difference of -16 meters over the Puerto Rico Trench was not unexpected since the gravity gradient there is large over a small region. Other areas on the map when variations are on the order of 10 to 14 meters may indicate broad shallow features to which satellites are not sensitive. In general the differences between the gravimetric geoid and the GEM-4 geoid are on the order of 10 meters or less.

5. APPLICATIONS OF GRAVIMETRIC DETAILED GEOID

There are several important applications of gravimetric geoids in geodesy and geophysics. Some of these applications are described in the following paragraphs.

5.1 OCEANOGRAPHIC APPLICATIONS

Much attention has been focused on the departure of mean sea level relative to the equipotential surface. The amplitudes of these variations are as large as

3 to 4 meters in some places. A geoid more accurate than the amplitudes of these variations is essential to the determination of departures from mean sea level.

An accurate geoidal map is also valuable for satellite and inertial navigation systems which are being used for offshore mineral exploration.

5.2 GRAVIMETRIC APPLICATIONS

The long wavelength harmonics of the gravity field are well determined from satellite orbital analyses. The satellite data available at present are of limited usefulness for determining the shorter wavelength features of the earth's gravity field. New techniques, for example, satellite-to-satellite tracking and altimetry have the promise and the potential to determine these short wavelengths. The analysis of altimetry data will be greatly facilitated and simplified if an accurate reference geoid is available. The accuracy of a reference geoid must be of the order of 1 to 2 meters or better.

The SKYLAB and GEOS-C spacecraft are scheduled to carry radar altimeters for the purpose of measuring the geoidal undulations in oceanic areas. An independently derived geoid will provide a valuable complement to these experiments. For example, by studying this gravimetric geoid, optimum locations for experiments could be established.

5.3 GEODETIC APPLICATIONS

5.3.1 Astrogeodetic Surveys

The gravimetric geoid provides an independent means of comparison with astrogeodetic data over the continents. These comparisons provide information on the relative accuracy of the geoidal undulations and on datum orientations.

5.3.2 Station Coordinates

A number of experimenters have derived values for tracking station coordinates through dynamic and geometric analyses of satellite observations. Accurate geoidal undulations provide an independent check on the heights of the stations above mean sea level.

The detailed geoid can also be used as a constraint for geodetic solutions as was recently done by Mueller and Whiting (1972) who incorporated an earlier detailed gravimetric geoid map (Vincent, et al., 1972) into their global geometric solution.

5.3.3 Scale

Accurate determinations of the geoid provide one of the means of determining the scale of the mean Earth ellipsoid.

6. CONCLUSIONS

The gravimetric geoid presented here has an accuracy of ±2 meters over the continents and 5 to 7 meters where data are sparse.

The use of a consistent set of parameters references this geoid to an absolute datum. Comparisons of the detailed gravimetric geoid with astrogeodetic geoids and dynamic station positions show no systematic scale differences.

There seems to be no conclusive evidence of a rotation in the North American datum. However a slight rotation, which is prominent along the East-West profile, does exist in the European and Australian datums. This rotation could be attributed to long wavelength errors in the GEM-4 gravity model, a rotation of the astrogeodetic geoid, or a combination of both.

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Table 1

Comparison Between Detailed Gravimetric Geoid and Rice's Transformed Astrogeodetic Geoid for the U.S. (meters)

Latitude			Longitude			1	2	3
34°	59'	38".38	086°	59'	20".44	-29	-29	0
30	54	05.65	088	00	31.06	-31	-31	0
32	22	39.32	086	18	01.92	-31	-29	-2
32	45	32.93	086	57	21.46	-31	-29	-2
30	46	41.22	088	15	11.79	-31	-31	0
33	20	54.00	112	49	56.44	-31	-30	-1
34	55	25.52	110	08	44.60	-23	-23	0
34	32	01.83	112	40	59.96	-27	-27	0
32	19	27.77	110	55	37.31	-28	-28	0
32	54	46.40	110	25	38.44	-27	-27	0
31	27	57.27	110	34	38.77	-28	-27	-1
33	18	00.00	092	29	30.00	-28	-29	1
34	59	34.91	093	11	44.70	-31	-30	-1
33	03	19.89	093	00	55.81	-28	-29	1
34	58	40.73	091	52	36.23	-29	-29	0
34	58	16.07	090	54	31.48	-29	-29	0
34	43	40.75	115	15	29.49	-30	-31	1
33	27	39.84	117	33	03.29	-33	-37	4
34	54	16.13	117	00	37.60	-31	-33	2
36	13	00.16	121	45	28.21	-35	-39	4
40	53	49.61	122	14	40.82	-26	-27	1
36	42	29.69	118	07	47.15	-25	-30	5
39	08	29.64	121	35	17.36	-28	-29	1
38	40	09.97	122	37	56.60	-31	-34	3
38	49	43.54	104	49	35.06	-17	-19	2
38	02	20.58	103	14	55.25	-23	-23	0
39	39	50.99	104	29	35.10	-18	-19	1
40	21	23.29	106	49	41.02	-15	-14	1
40	10	36.12	102	49	02.37	-22	-22	0
39	52	14.51	104	58	23.01	-18	-17	1
38	31	27.30	106	54	23.01	-14	-16	2
41	40	40.60	073	13	23.73	-34	-31	-3
39	09	20.90	075	31	25.40	-37	-35	-2
29	16	53.45	082	00	05.16	-30	-32	2
26	13	43.59	080	17	55.34	-28	-33	5
30	25	54.12	085	54	06.94	-30	-30	0
25	45	41.99	080	20	25.05	-27	-31	4
27	53	35.16	082	43	33.27	-27	-30	3
32	09	42.27	081	53	21.49	-30	-29	-2
31	30	49.18	083	44	16.21	-31	-29	-1
33	31	58.51	084	18	10.83	-31	-31	-2
31	19	29.79	082	08	03.74	-18	-18	0
47	40	33.72	116	18	35.14	-14	-14	0

Table 1 (Continued)

Latitude			Longitude			1	2	3
43°	37'	07".54	113°	20'	40".27	-18	-18	0
45	57	41.34	116	17	51.86	-18	-17	-0
43	07	43.34	115	41	35.23	-15	-13	-1
45	06	45.04	113	45	44.30	-32	-30	-2
38	31	57.87	089	48	21.73	-30	-29	-2
36	59	47.92	089	09	30.28	-34	-32	-1
41	25	17.60	089	11	17.84	-34	-33	-2
41	15	36.64	090	01	53.24	-36	-33	-1
41	33	50.58	084	49	00.88	-36	-32	-3
40	18	31.54	085	26	55.33	-36	-32	-4
41	02	55.87	086	52	38.10	-36	-33	-4
40	12	16.24	085	06	54.27	-36	-33	-3
38	51	12.35	085	34	42.37	-33	-33	-3
41	42	08.06	092	00	15.15	-30	-31	0
42	59	22.00	093	10	04.00	-33	-32	1
41	01	47.05	093	33	40.64	-29	-30	-1
42	55	15.00	095	14	30.00	-29	-31	1
41	46	10.55	094	46	21.77	-29	-29	2
38	56	58.50	097	15	28.92	-30	-30	0
37	55	17.12	096	52	13.14	-28	-29	0
38	28	42.49	098	17	36.97	-26	-27	1
38	13	35.32	100	09	39.17	-27	-28	1
39	13	26.68	098	32	30.50	-26	-26	1
39	05	29.52	100	16	39.04	-35	-32	0
38	10	26.72	083	49	54.04	-32	-31	-3
36	39	12.86	085	14	04.46	-31	-30	-1
36	57	19.85	087	31	21.25	-28	-31	-1
29	54	28.87	090	05	02.50	-28	-30	3
30	31	02.74	091	31	50.18	-29	-31	2
31	28	01.70	093	12	00.04	-28	-30	2
46	04	49.04	070	02	56.76	-29	-27	-2
45	11	26.82	068	18	21.72	-27	-26	-1
46	13	09.34	067	52	42.72	-27	-26	-1
44	18	21.61	070	01	27.98	-29	-26	-3
39	08	52.64	077	04	02.73	-35	-34	-1
42	22	52.93	071	07	43.91	-31	-29	-2
42	02	22.55	070	03	39.84	-31	-28	-3
44	17	58.47	084	23	43.54	-38	-33	-5
44	01	45.01	085	22	45.44	-36	-33	-3
43	08	21.09	084	52	33.79	-36	-33	-3
47	45	04.43	095	37	17.77	-28	-29	1
44	18	04.37	093	14	36.88	-30	-31	1
46	50	55.20	094	54	37.94	-28	-29	1
48	27	22.88	096	51	20.71	-27	-27	0
47	31	53.30	092	32	56.07	-30	-32	2
44	47	22.94	095	11	15.83	-27	-29	2
30	53	39.80	088	50	07.26	-31	-31	0
32	29	15.99	089	15	19.80	-30	-30	0

Table 1 (Continued)

Latitude			Longitude			1	2	3
30°	59'	59".59	089°	20'	30".55	-30	-30	0
34	05	30.88	089	02	03.20	-30	-29	-1
31	45	24.23	089	56	31.42	-28	-30	2
39	47	31.50	092	05	41.43	-34	-33	-1
37	59	51.89	092	05	16.41	-32	-32	0
38	38	13.33	092	20	13.47	-33	-32	-1
40	20	25.32	094	38	27.19	-32	-32	0
37	07	19.81	093	04	46.65	-31	-31	0
38	07	30.00	094	09	00.00	-34	-33	-1
46	29	08.34	107	01	35.72	-16	-16	0
47	02	04.63	113	11	48.82	-15	-15	0
47	50	29.32	110	00	46.21	-16	-16	0
48	12	37.51	104	49	49.93	-18	-18	0
45	30	52.35	105	07	05.07	-16	-17	1
46	18	06.19	109	15	15.26	-13	-13	0
47	45	02.04	107	29	20.33	-17	-17	0
42	01	29.53	102	00	05.29	-21	-20	-1
42	54	44.73	102	57	24.18	-18	-18	0
40	11	51.47	100	09	56.48	-25	-25	0
42	25	25.39	098	25	59.51	-25	-25	0
40	10	33.73	098	30	19.15	-27	-27	0
38	42	28.96	115	30	43.27	-22	-22	0
37	00	11.19	114	56	27.04	-26	-26	0
40	53	41.10	115	26	53.08	-20	-19	-1
42	59	30.70	071	33	03.65	-30	-28	-2
38	55	59.57	074	57	38.07	-38	-35	-3
40	49	10.45	074	24	37.86	-36	-33	-3
32	34	40.60	106	19	39.84	-23	-22	-1
33	14	18.20	107	15	53.60	-22	-23	1
34	59	40.95	107	15	15.72	-21	-21	0
35	51	37.29	107	09	00.57	-20	-20	0
35	55	28.00	106	00	55.93	-18	-19	1
35	28	32.96	105	07	27.41	-20	-21	1
32	54	39.73	105	28	10.85	-21	-23	2
43	01	25.63	075	55	10.94	-36	-32	-4
42	11	54.49	075	02	27.49	-34	-31	-3
43	00	23.50	077	52	41.84	-38	-35	-3
40	58	23.30	072	42	12.72	-34	-31	-3
43	13	50.42	077	35	59.62	-39	-35	-4
35	24	37.19	081	07	27.60	-34	-33	-1
35	47	49.85	082	57	25.18	-31	-31	0
33	57	06.47	078	02	52.99	-40	-40	0
35	22	21.37	083	14	37.01	-30	-31	1
35	50	21.30	077	03	54.59	-39	-38	-1
48	04	38.80	099	53	10.19	-23	-23	0
46	45	40.17	097	55	24.40	-26	-25	-1
48	06	18.99	102	21	09.30	-19	-19	0
46	28	57.58	102	06	52.40	-20	-19	-1

Table 1 (Continued)

Latitude			Longitude			1	2	3
41°	31'	11".90	082°	50'	19".63	-37	-33	-4
40	07	07.04	082	02	09.44	-36	-32	-4
40	08	20.10	083	55	58.14	-36	-32	-4
41	22	55.87	084	44	53.32	-36	-33	-3
35	17	24.42	099	07	40.12	-28	-29	1
36	45	17.27	099	03	28.75	-29	-28	-1
34	55	39.67	096	07	24.65	-29	-31	2
36	30	02.00	096	49	23.00	-30	-28	-2
45	19	50.78	118	05	40.15	-20	-19	-1
42	59	01.78	121	56	21.21	-21	-23	2
44	26	41.54	118	42	09.43	-19	-19	0
45	14	20.76	121	48	47.58	-23	-21	-2
40	53	43.87	075	49	45.19	-36	-32	-4
41	52	18.52	079	06	43.35	-36	-35	-1
41	32	24.84	071	16	00.83	-33	-30	-3
34	06	38.64	082	07	36.67	-32	-32	0
34	11	21.09	079	03	38.08	-37	-38	1
35	00	22.53	080	56	51.44	-33	-33	0
32	13	11.89	081	04	27.76	-34	-32	-2
43	42	31.33	098	04	20.48	-26	-26	0
44	02	26.48	100	28	18.89	-25	-25	0
45	12	45.71	102	09	14.14	-20	-20	0
44	04	22.95	102	11	32.87	-20	-20	0
45	06	27.71	101	31	49.49	-21	-22	1
36	06	44.20	087	00	24.46	-31	-29	-2
35	57	24.58	083	55	33.51	-30	-30	0
35	01	24.47	085	01	25.89	-32	-31	-1
25	53	54.64	097	29	27.91	-26	-24	-2
30	55	14.79	103	11	36.72	-24	-24	0
33	15	08.66	095	54	20.75	-27	-29	2
32	57	15.47	101	08	48.86	-27	-27	0
30	32	05.42	095	23	56.09	-29	-31	2
35	00	08.94	101	12	61.35	-28	-28	0
29	42	52.84	098	09	52.10	-27	-26	-1
31	00	28.60	101	34	14.45	-25	-25	0
25	54	57.57	097	25	21.16	-25	-24	-1
33	02	51.64	098	08	03.57	-30	-29	-1
31	27	20.17	098	07	03.69	-28	-28	0
44	58	44.10	072	09	02.85	-27	-23	-4
42	58	28.15	072	36	10.57	-31	-29	-2
38	59	01.87	078	00	06.63	-36	-34	-2
37	02	20.21	077	01	13.29	-37	-37	0
38	08	46.19	079	04	19.29	-34	-34	0
45	59	27.70	121	04	57.58	-22	-21	-1
48	48	32.36	117	52	58.20	-19	-19	0
47	21	27.37	123	06	11.93	-21	-21	0
47	32	07.36	118	43	54.67	-20	-19	-1
38	21	02.04	081	37	59.43	-35	-32	-3

Table 1 (Continued)

Latitude			Longitude			1	2	3
39°	03'	31".08	079°	59'	58".40	-35	-32	-3
37	23	44.27	081	19	12.93	-33	-32	-1
38	30	53.39	079	16	48.88	-31	-33	2
45	28	33.75	091	06	43.53	-31	-33	2
45	38	52.82	089	24	36.57	-34	-33	-1
43	52	28.66	089	29	26.39	-37	-35	-2
44	21	24.40	105	59	45.76	-15	-16	1
42	23	35.19	108	02	05.06	-12	-14	2
41	10	56.57	105	35	37.35	-14	-16	2

1. Rice's Astrogeodetic geoid transformed to center of mass system.
2. Detailed gravimetric geoid.
3. Difference between Rice's transformed Astrogeodetic geoid and detailed gravimetric geoid.

Table 2

Comparison Between Dynamic Station Heights and Gravimetric Geoid (meters)

Station Name	Station No.	(1) GEM-4* Geoid Height†	(2) GSFC Long-Arc** Geoid Height†	(3) Gravimetric Geoid Height	(4) 1 - 3	(5) 2 - 3
United States						
St. Johns	1032		12	13		-1
Blossom Point	1021		-43	-34		-9
Ft. Myers	1022	-28	-29	-31	3	2
Goldstone	1030	-34	-30	-35	1	5
E. Grand Flks.	1034	-25	-27	-28	3	1
Rosman	1042	-30	-34	-32	2	-2
Edinburg	7036	-24	-27	-25	1	-2
Columbia	7037	-32	-35	-34	2	-1
Greenbelt	7050		-40	-34		-6
Denver	7045	-19	-18	-18	-1	0
Organ Pass	9001		-22	-23		1
Mt. Hopkins	9021		-30	-29		-1
Jupiter	7072	-32	-32	-36	4	4
Cold Lake	9424		-27	-29		2
Sudbury	7075	-34	-32	-37	-2	5
Caribbean						
Bermuda	7039	-36	-35	-39	3	4
San Juan	7040	-45	-46	-50	5	4
Europe						
Malvern	8011		45	47		-2
Winkfield	1035	49	47	48	1	-1
Delft	8009		45	43		2
Zimmerwald	8010		52	50		2
Haute Provence	8015		45	52		-7
Nice	8019		52	51		1
San Fernando	9004	43	43	50	-7	-7
Naini Tal	9006		-51	-60		9
Dionysos	9091	28	35	40	-12	-5
Oslo	9115		35	36		-5
Uzhgorod	9432		40	40		0
Helsinki	9435		15	13		2
Riga	9431		16	16		0
Australia						
Woomera	1024	12	6	0	12	6
Orroral	1038	25	23	20	5	3
Carnarvon	7054	-25	-20	-17	-8	-3
					rms = ±5.5 m.	rms = ±4.1 m.

*Lerch, et al., (1972)
**Marsh, Douglas, and Klosko (1973)
†Geoid Height = Height of tracking station above reference ellipsoid - height of tracking station above mean sea level.

Table 3

Comparison Between the Geoid of Mather and the Detailed Gravimetric Geoid for Australia (meters)

Latitude (-24°S)			
Longitude	Mather's Geoid	Detailed Geoid	Difference
114	-16	-15	-1
116	-11	-9	-2
118	-8	-6	-2
120	-4	-2	-2
122	-2	-1	-1
124	-1	-1	0
126	-0	1	-1
128	3	4	-1
130	4	6	-2
132	7	8	-1
134	12	14	-2
136	20	20	0
138	26	26	0
Latitude (-26°S)			
Longitude	Mather's Geoid	Detailed Geoid	Difference
114	-18	-17	-1
116	-14	-13	-1
118	-11	-10	-1
120	-9	-8	-1
122	-8	-8	0
124	-6	-7	1
126	-5	-5	0
128	0	-1	1
130	0	0	0
132	-1	0	-1
134	6	6	0
136	12	12	0
138	18	17	1
140	22	21	1

Absolute Mean = ±0.87 meters

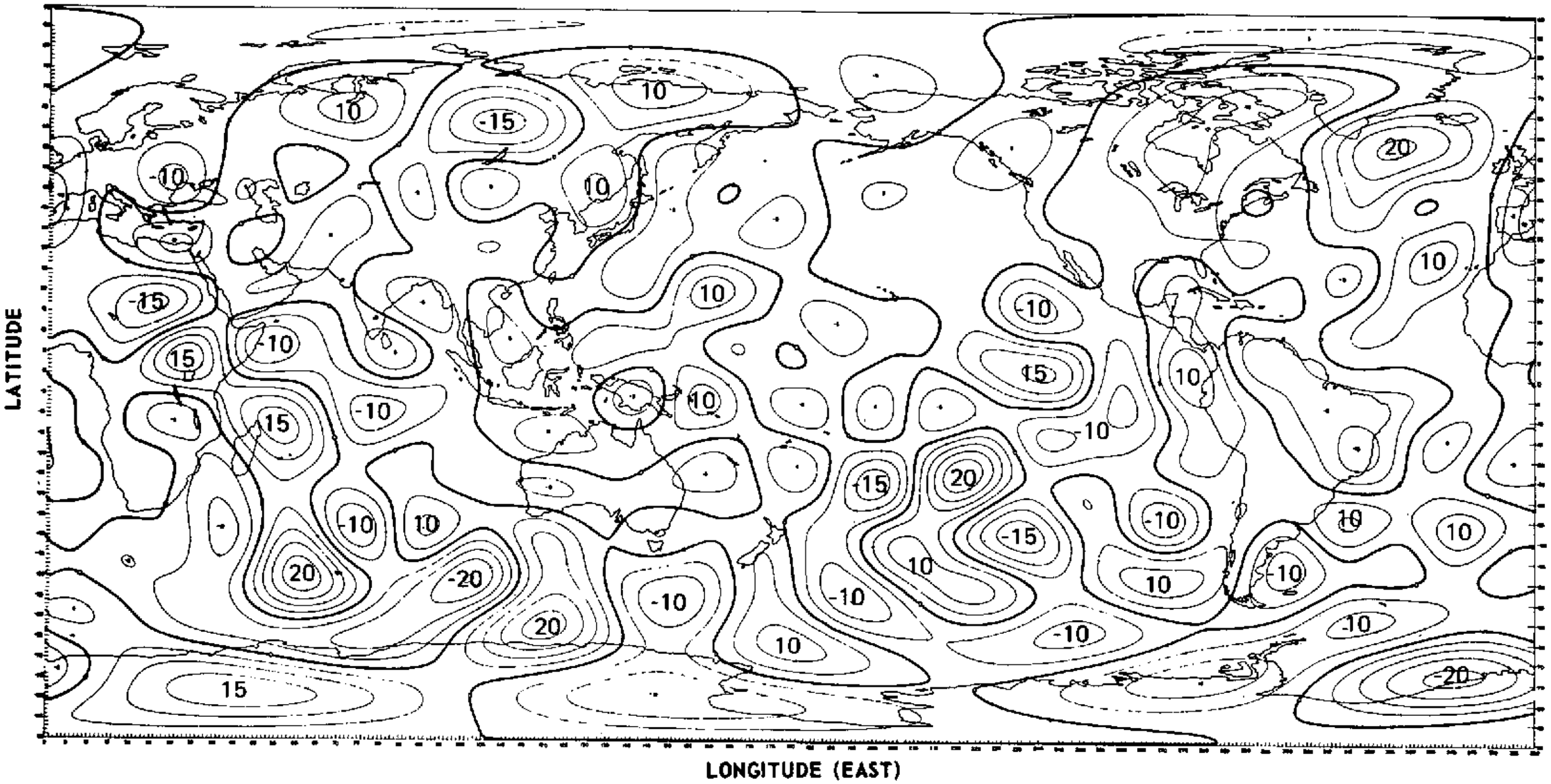

Figure 1. Difference Between GEM-4 and SAO 69 Geoid Heights (Contour Inverval = 5 meters)

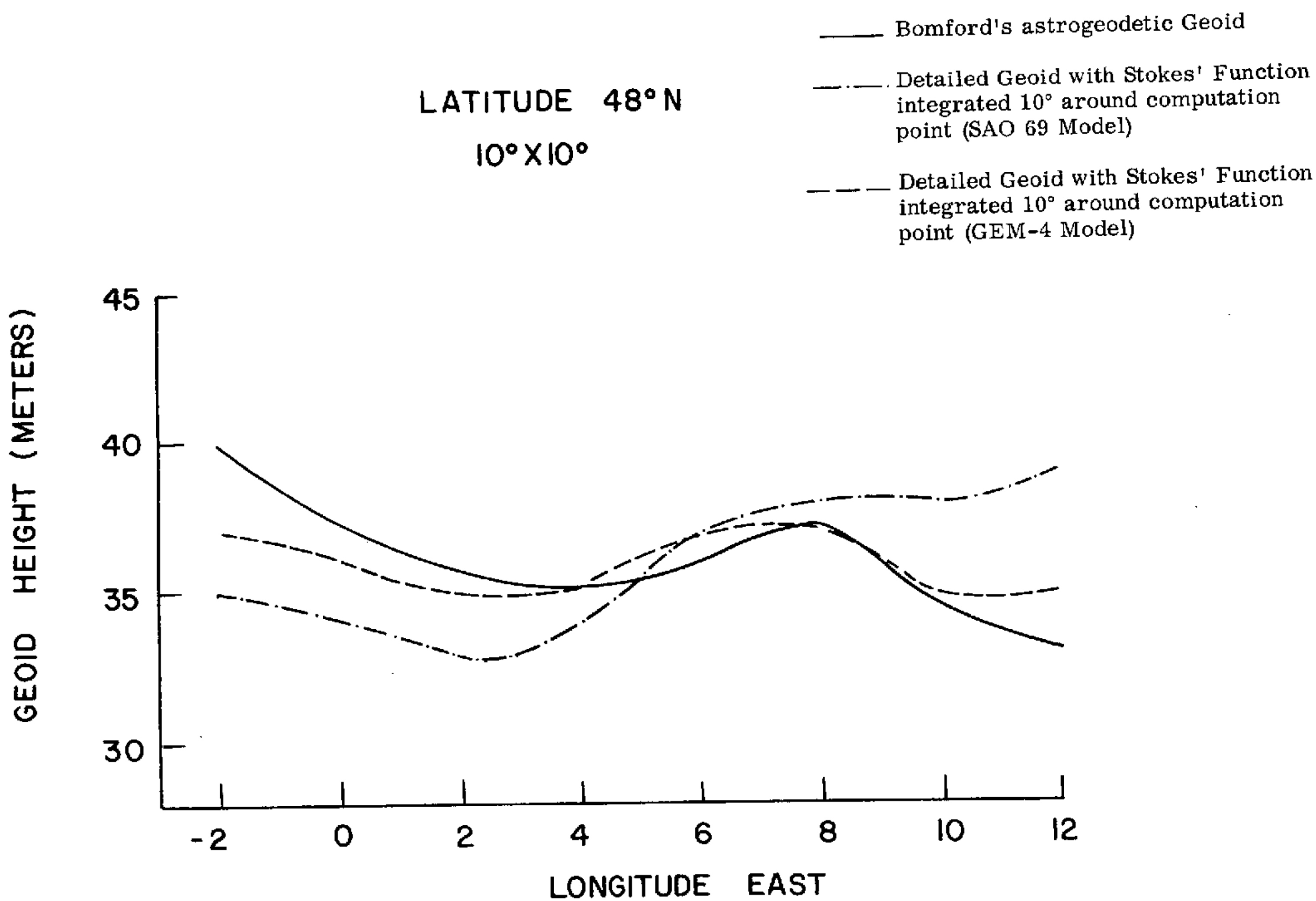

Figure 4. Comparison Between Bomford's Astrogeodetic Geoid and the Detailed Gravimetric Geoid (GEM-4 and SAO 69) Integrated 10° Around Computation Point for Europe

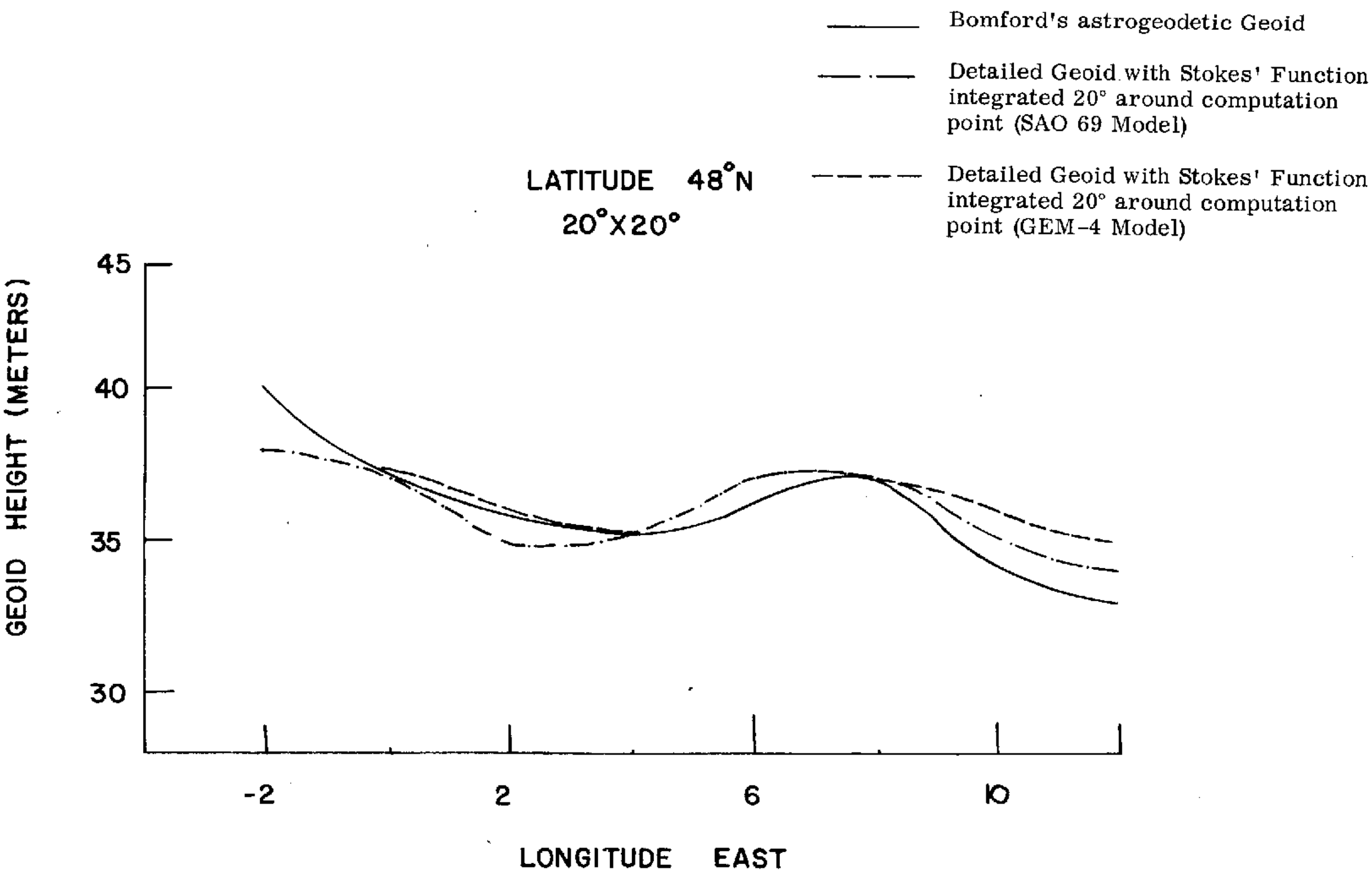

Figure 5. Comparison Between Bomford's Astrogeodetic Geoid and the Detailed Gravimetric Geoid (GEM-4 and SAO 69) Integrated 20° Around Computation Point for Europe

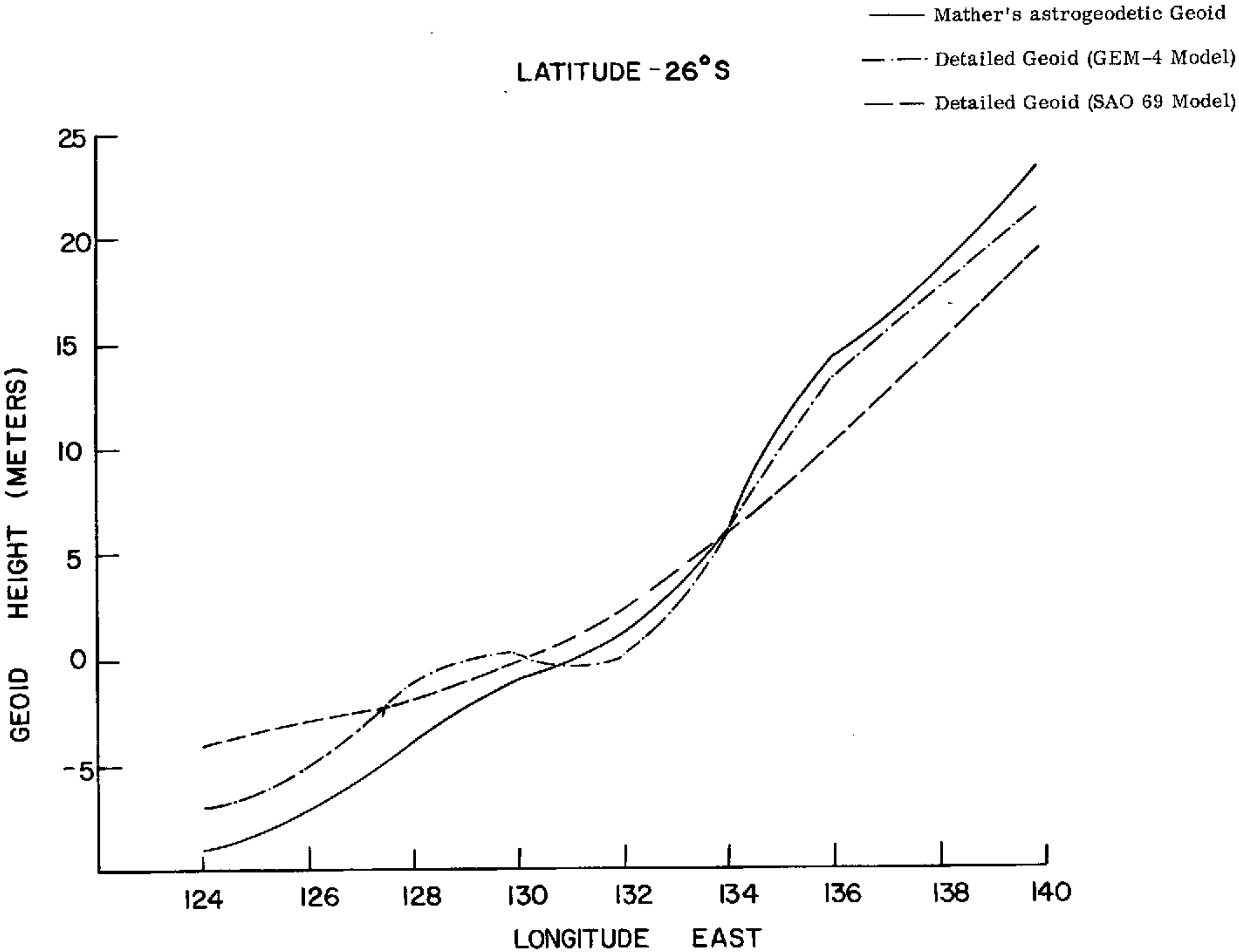

Figure 6. Comparison Between the Astrogeodetic Geoid for Australia by Mather et al. and the Detailed Gravimetric Geoid (GEM-4 and SAO 69) in Australia

APPENDIX

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GSFC Geopotential Solutions (Normalized Coefficients x 10^6)

	L	M	GEM 4
C	2	0	-484.1690
S	2	0	0.0
C	3	0	0.9570
S	3	0	0.0
C	4	0	0.5412
S	4	0	0.0
C	5	0	0.0692
S	5	0	0.0
C	6	0	-0.1528
S	6	0	0.0
C	7	0	0.0910
S	7	0	0.0
C	8	0	0.0515
S	8	0	0.0
C	9	0	0.0312
S	9	0	0.0
C	10	0	0.0562
S	10	0	0.0
C	11	0	-0.0561
S	11	0	0.0
C	12	0	0.0389
S	12	0	0.0
C	13	0	0.0477
S	13	0	0.0
C	14	0	-0.0266
S	14	0	0.0
C	15	0	-0.0050
S	15	0	0.0
C	16	0	-0.0093
S	16	0	0.0
C	17	0	0.0174
S	17	0	0.0
C	18	0	0.0113
S	18	0	0.0
C	19	0	0.0090
S	19	0	0.0
C	20	0	0.0090
S	20	0	0.0

	L	M	GEM 4
C	21	0	-0.0076
S	21	0	0.0
C	22	0	-0.0038
S	22	0	0.0
C	2	1	-0.0078
S	2	1	-0.0004
C	3	1	2.0164
S	3	1	0.2498
C	4	1	-0.5330
S	4	1	-0.4614
C	5	1	-0.0741
S	5	1	-0.0786
C	6	1	-0.0905
S	6	1	0.0084
C	7	1	0.2553
S	7	1	0.1334
C	8	1	0.0297
S	8	1	0.0578
C	9	1	0.1536
S	9	1	0.0088
C	10	1	0.0757
S	10	1	-0.1430
C	11	1	-0.0199
S	11	1	0.0371
C	12	1	-0.0592
S	12	1	-0.0466
C	13	1	0.0183
S	13	1	-0.0753
C	14	1	-0.0453
S	14	1	0.0371
C	15	1	0.1043
S	15	1	0.0419
C	16	1	-0.0314
S	16	1	0.0082
C	2	2	2.4237
S	2	2	-1.3895
C	3	2	0.9164
S	3	2	-0.6322

	L	M	GEM 4
C	4	2	0.3511
S	4	2	0.6652
C	5	2	0.6620
S	5	2	-0.3145
C	6	2	0.0679
S	6	2	-0.3795
C	7	2	0.3305
S	7	2	0.0748
C	8	2	0.0511
S	8	2	0.0739
C	9	2	0.0534
S	9	2	-0.0171
C	10	2	-0.0457
S	10	2	-0.0667
C	11	2	0.0158
S	11	2	-0.1250
C	12	2	-0.0449
S	12	2	0.0532
C	13	2	0.0194
S	13	2	-0.1477
C	14	2	-0.0376
S	14	2	0.1169
C	15	2	0.0008
S	15	2	-0.1010
C	16	2	0.0108
S	16	2	0.0217
C	3	3	0.7063
S	3	3	1.4231
C	4	3	0.9713
S	4	3	-0.2187
C	5	3	-0.4701
S	5	3	-0.2566
C	6	3	0.0169
S	6	3	-0.0127
C	7	3	0.2558
S	7	3	-0.2281
C	8	3	-0.0262
S	8	3	-0.0809

	L	M	GEM 4
C	9	3	-0.1700
S	9	3	-0.1049
C	10	3	-0.0483
S	10	3	-0.0976
C	11	3	-0.0205
S	11	3	-0.0887
C	12	3	0.1389
S	12	3	0.0429
C	13	3	-0.0335
S	13	3	0.0301
C	14	3	0.0386
S	14	3	-0.0157
C	15	3	0.0150
S	15	3	0.0552
C	16	3	0.0306
S	16	3	-0.0160
C	4	4	-0.1811
S	4	4	0.3153
C	5	4	-0.3107
S	5	4	0.0321
C	6	4	-0.1005
S	6	4	-0.4601
C	7	4	-0.2939
S	7	4	-0.1064
C	8	4	-0.2480
S	8	4	0.0466
C	9	4	0.0212
S	9	4	0.0139
C	10	4	-0.0934
S	10	4	-0.1177
C	11	4	0.0027
S	11	4	-0.0937
C	12	4	-0.0423
S	12	4	-0.0168
C	13	4	-0.0543
S	13	4	-0.0737
C	14	4	0.0346
S	14	4	0.0064

	L	M	GEM 4
C	15	4	0.0099
S	15	4	-0.0254
C	16	4	0.0308
S	16	4	0.0733
C	5	5	0.1700
S	5	5	-0.6845
C	6	5	-0.2964
S	6	5	-0.5115
C	7	5	0.0035
S	7	5	0.0321
C	8	5	-0.0884
S	8	5	0.0848
C	9	5	-0.0320
S	9	5	-0.0548
C	10	5	-0.0682
S	10	5	-0.0070
C	11	5	0.0736
S	11	5	0.0332
C	12	5	0.0399
S	12	5	-0.0048
C	13	5	0.0418
S	13	5	0.0548
C	14	5	0.0426
S	14	5	-0.0311
C	15	5	0.0237
S	15	5	-0.0175
C	16	5	0.0160
S	16	5	0.0334
C	6	6	0.0313
S	6	6	-0.2348
C	7	6	-0.3236
S	7	6	0.1664
C	8	6	-0.0476
S	8	6	0.2841
C	9	6	0.0651
S	9	6	0.2210
C	10	6	-0.0178
S	10	6	-0.1220

GSFC Geopotential Solutions (Normalized Coefficients x 10^6)

	L	M	GEM 4
C	11	6	-0.0211
S	11	6	0.0443
C	12	6	0.0634
S	12	6	-0.0252
C	13	6	-0.1284
S	13	6	0.0378
C	14	6	0.0534
S	14	6	-0.0323
C	15	6	-0.0174
S	15	6	-0.0481
C	16	6	-0.0407
S	16	6	-0.0189
C	7	7	0.0752
S	7	7	0.0130
C	8	7	0.0494
S	8	7	0.0679
C	9	7	-0.0685
S	9	7	-0.0212
C	10	7	0.0110
S	10	7	-0.0337
C	11	7	0.0223
S	11	7	-0.1104
C	12	7	-0.0335
S	12	7	0.0005
C	13	7	-0.0526
S	13	7	0.1473
C	14	7	0.1313
S	14	7	-0.0797
C	15	7	-0.0214
S	15	7	0.0968
C	16	7	0.0258
S	16	7	-0.0462
C	8	8	-0.1075
S	8	8	0.1158
C	9	8	0.2182
S	9	8	0.0052
C	10	8	0.0418
S	10	8	-0.1256

	L	M	GEM 4
C	11	8	0.0011
S	11	8	0.0639
C	12	8	-0.0317
S	12	8	0.0060
C	13	8	0.0412
S	13	8	-0.0192
C	14	8	0.0007
S	14	8	-0.0605
C	15	8	-0.1600
S	15	8	0.0290
C	16	8	0.0301
S	16	8	-0.0248
C	9	9	-0.0273
S	9	9	0.0801
C	10	9	0.1062
S	10	9	-0.0724
C	11	9	-0.0505
S	11	9	0.0857
C	12	9	0.0081
S	12	9	0.0208
C	13	9	0.0137
S	13	9	0.1196
C	14	9	0.0116
S	14	9	0.0460
C	15	9	0.0066
S	15	9	0.0769
C	16	9	0.0409
S	16	9	-0.0608
C	10	10	0.0786
S	10	10	-0.0232
C	11	10	-0.0727
S	11	10	-0.0063
C	12	10	-0.0057
S	12	10	0.0312
C	13	10	-0.0128
S	13	10	0.0171
C	14	10	0.0273
S	14	10	-0.1311

	L	M	GEM 4
C	15	10	0.0503
S	15	10	0.0345
C	16	10	-0.0602
S	16	10	-0.0093
C	11	11	0.0906
S	11	11	-0.0255
C	12	11	0.0052
S	12	11	0.0305
C	13	11	-0.0443
S	13	11	-0.0215
C	14	11	0.0980
S	14	11	-0.0331
C	15	11	-0.0567
S	15	11	0.0568
C	16	11	0.0046
S	16	11	-0.0064
C	12	12	-0.0117
S	12	12	0.0049
C	13	12	-0.0306
S	13	12	0.0994
C	14	12	0.0098
S	14	12	-0.0268
C	15	12	-0.0341
S	15	12	0.0153
C	16	12	0.0256
S	16	12	-0.0076
C	17	12	0.0261
S	17	12	-0.0011
C	18	12	-0.0568
S	18	12	-0.0229
C	19	12	-0.0256
S	19	12	-0.0203
C	20	12	0.0121
S	20	12	-0.0623
C	21	12	0.0072
S	21	12	-0.0347
C	22	12	-0.0537
S	22	12	-0.0333

	L	M	GEM 4
C	13	13	-0.0274
S	13	13	0.0930
C	14	13	0.0318
S	14	13	0.0087
C	15	13	-0.0023
S	15	13	0.0107
C	16	13	0.0064
S	16	13	-0.0213
C	17	13	0.0319
S	17	13	0.0423
C	18	13	-0.0027
S	18	13	-0.0834
C	19	13	-0.0068
S	19	13	-0.0012
C	20	13	0.0312
S	20	13	-0.0637
C	21	13	-0.0190
S	21	13	0.0257
C	22	13	-0.0137
S	22	13	-0.0348
C	14	14	-0.0521
S	14	14	-0.0074
C	15	14	0.0025
S	15	14	-0.0216
C	16	14	-0.0108
S	16	14	-0.0374
C	17	14	-0.0155
S	17	14	0.0060
C	18	14	-0.0234
S	18	14	-0.0043
C	19	14	0.0005
S	19	14	-0.0109
C	20	14	0.0117
S	20	14	-0.0035
C	21	14	0.0042
S	21	14	0.0134
C	22	14	0.0215
S	22	14	0.0071

	L	M	GEM 4
C	15	15	-0.0788
S	15	15	[illegible]
C	16	15	-0.0549
S	16	15	0.0090
C	16	16	-0.0048
S	16	16	[illegible]