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THE REMOTE SENSING OF SATURN'S RINGS

I: The Magnetic Alinement of the Ring Particles

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16. Abstract

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SYMBOLS

A,B,C,D	(in context) major elements of Saturn ring system
B	magnitude of $\vec{B}$
$\vec{B}$	magnetic field
$E[k]$	complete elliptical integral of the second kind
I_i	moment of inertia around i-axis ($i = \xi, \eta, \zeta$)
J	magnitude of $\vec{J}$
$\vec{J}$	angular momentum of ring particle
J_i	component of $\vec{J}$ in i-direction ($i = \xi, \eta, \zeta; x, y, z; X, Y, Z$)
M	magnetization
R	sun-planet distance
T_I	internal temperature of ring particles
T_d	kinetic temperature of the interplanetary dust and/or gas in the vicinity of the rings
a	radius of a spherical particle
a_s	symmetry semi-axis in a spheroid
a_t	transverse semi-axis in a spheroid
e	eccentricity of the ellipse formed by the intersection of the surface of a spheroid and a plane that includes the symmetry axis
f_i	stochastic phase-space distribution function
k	Boltzmann constant
m_d	average mass of the interplanetary dust and/or gas in the vicinity of the rings
m_r	average mass of a ring particle
q	ratio of transverse semi-axis to symmetry semi-axis in a spheroid
w	constant defined by equation (7)
θ	angle between local ring-plane normal and spheroid symmetry axis

β_J	angle between $\vec{J}$ and $\vec{B}$
θ_J	angle between $\vec{J}$ and $\hat{e}_\zeta$ (spheroid symmetry axis)
ν_d	number density of the interplanetary dust/gas particles
ξ,η,ζ	cartesian coordinate system centered on a spheroidal ring particle with ζ-axis along one of the symmetry semi-axes
ρ	radial distance from the center of Saturn
χ_M	magnetic susceptibility
χ',χ''	real and imaginary parts, respectively, of χ_M
ω	magnitude of $\vec{\omega}$
$\vec{\omega}$	angular velocity of a spinning ring particle

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SUMMARY

Because of the potential implications for the optical properties of Saturn's rings, the orientation of nonspherical ring particles at equilibrium is investigated with respect to four stochastic influences: interactions with the interplanetary medium, interactions with the expected magnetic field of Saturn, thermal fluctuations due to the internal temperature of the ring particles, and collisions between ring particles. The solution of the homogeneous Fokker-Planck equation for nearly spherical spheroids is presented and investigated in general. Values of the pertinent physical parameters in the vicinity of Saturn are estimated, and the implications for the alinement of the ring particles are investigated. It is concluded that for some alinement mechanisms, small ring particles (size $\lesssim$ 10 cm) can be expected to be almost completely alined ($\sigma \lesssim 1°$). This alinement results in each particle spinning around its shortest body axis with this axis parallel to the magnetic field direction (perpendicular to the ring plane).

INTRODUCTION

This is the first in a series of studies dealing with the potential scientific data obtainable by remote sensing of Saturn's rings from a spacecraft. This study deals with the configuration of nonspherical ring particles that would be expected in the presence of a Saturnian magnetic field. The second study (Part II)[1] is an analysis of the geometrical considerations of such remote sensing and the interaction of these considerations with such items as photometer thresholds, phase angle coverage, and areal and radial resolution. Subsequent studies (e.g., Part III)[1] will deal with the implications of the expected particle configuration for remote sensing data.

This series of studies has been prompted by the growing interest in flyby and orbiter missions to Saturn. Although the major thrust of the exploration of the outer planets is currently directed toward Jupiter, Saturn is the major secondary objective. One of the reasons for the sustained scientific interest in Saturn is undoubtedly the planet's rings. This interest is concentrated in two main areas: the features of the rings themselves, and the cosmological significance of their existence. The rings of Saturn are one of the few unique objects in the solar system; as such, the intrinsic interest in them is certainly not surprising. Their very existence raises questions concerning their structure, geometry, and dynamics.

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[1]Prospective NASA TN in preparation.

There are several other questions of intrinsic scientific interest. For instance, since Saturn is so similar to Jupiter in other respects, it may be expected to have a large magnetic field as well. This raises questions concerning the interaction between the rings and this possible magnetic field. Also, the surface roughness of the particles in the rings may give an indication of the average micrometeoroid flux in the vicinity of Saturn. Any inconsistency between this flux and the present one measured *in situ* on the same flight would suggest that the micrometeoroid flux has varied significantly in the past.

The properties of the individual ring particles are of cosmological interest. As several theoretical studies have pointed out (refs. 1-3), the rings occupy a region of space where, due to tidal forces, the presatellite material cloud would not be able to condense into a satellite. If this interpretation of the origin of the rings is correct, the rings may represent one of the very few regions of the solar system where the remains of the preplanetary cloud can be found and studied. Also, the dynamics of the rings' particles may be very similar to those of the preplanetary material cloud.

Because of the limitations of Earth-based observations, very little can be said concerning the properties of individual ring particles; in fact, only composition and size have been dealt with to date. Analysis of the spectrum of reflected infrared solar radiation allowed Pilcher *et al.* (ref. 4) to identify water ice as a major constituent of the ring particles. Using high-resolution spectrophotometry, Kuiper *et al.* (ref. 5) have identified six absorption bands of water ice in the light from the rings. Cook *et al.* (ref. 6) suggest that impurities, such as dust, are necessary in order to redden the visible spectrum. Lebofsky *et al.* (ref. 7) also contend that the ring particles are not pure water ice and suggest the possibility of frost-covered silicates.

As shown in table 1, recent estimates (refs. 8-12) of the size of the individual particles have varied over a range of *nine orders of magnitude* (0.1 μm to 100 m). The hypothesis of small particle sizes, however, raises questions of the stability of the rings. Radzievsky (ref. 13) has considered the effect of radiation braking on particles in circular orbits about a planet. For a sun-planet distance of R (AU), he found that the time required for the orbit of spherical particles of radius a (cm) and mass m_r (gm) to decay from radius r_o to radius r_f is given by

$$t = 2.3\times10^{6}(R/a)^{2}m_{r}\ln(r_{o}/r_{f}) \quad \text{years} \tag{1}$$

The lifetimes given in table 1 assume an average value of R for Saturn of 9.54 AU, a particle density of 0.9 gm/cm^3 (ice), $r_o = 1.37\times10^5$ km (outer radius of ring A), $r_f = 9\times10^4$ km (inner radius of ring B), and an additional factor of 3 to account for the effects of shadowing by the planet and other ring particles (ref. 11).

If the particles which comprise Saturn's rings are spherical, the alinement of their spin angular momenta relative to the ring plane is immaterial to a consideration of the optical properties of the rings. In the more likely case of nonspherical particles, however, the geometrical alinement of the particles must be considered. In this study, we consider the behavior of spinning nonspherical particles in the presence of a magnetic field. The previous treatments of this topic have been associated with attempts to explain starlight polarization observations in terms of the alinement of interstellar grains. The pioneering work in this field was done by Davis and Greenstein (ref. 14), (with an addendum by Davis (ref. 15)). At the same time, Spitzer and Tukey (ref. 16) proposed a different alinement mechanism, but it has largely been ignored since it is much less efficient in small magnetic fields than that proposed by Davis and Greenstein. The Davis and Greenstein analysis has been put on a firmer theoretical foundation and has been extended to other magnetic alinement mechanisms by Jones and Spitzer (ref. 17) and Henry (ref. 18; alinement mechanism only). Work in this area has been briefly reviewed by Dieter and Goss (ref. 19); recent work has included that by Greenburg (ref. 20), Purcell (ref. 21), Greenburg and Shah (ref. 22), Shah (ref. 23), and Goldstein (ref. 24).

These previous theoretical analyses have dealt almost exclusively with idealized models based on the assumptions of nearly spherical particles and a weak magnetic field. Although it is probably not unreasonable to assume that the ring particles are nearly spherical, the magnetic fields that could be expected in the rings are four or five orders of magnitude greater than those in interstellar space. Therefore, we will investigate the degree of alinement of nearly spherical particles at equilibrium in the presence of a strong magnetic field. Since the processes that influence the orientation of a real particle in a magnetic field are, in general, stochastic in nature, the analysis of this problem is centered around the Fokker-Planck equation. Contributions to the diffusion coefficients from each of the major factors influencing particle orientation are evaluated and approximate solutions of this equation at equilibrium are presented. The implications of these solutions for the Saturn ring system are then analyzed.

In order to simplify the discussion of the optical properties of the rings in a subsequent study in this series (Part III), we will confine our current analysis to the orientation of spheroidal ring particles.

GENERAL SPHEROID ALINEMENT

Consider a typical spheroidal ring particle, whose shape will be characterized by a symmetry semi-axis, a_s, and a transverse semi-axis, a_t. The particle will be assumed to be spinning with an angular velocity, $\vec{\omega}$ (angular momentum, $\vec{J}$); the local magnetic field is denoted by $\vec{B}$. Let (x, y, z) and (ξ, η, ζ) be orthogonal coordinate systems (fig. 1) defined so that $\hat{e}_z = \hat{e}_B$ and $\hat{e}_\zeta = \hat{e}_s$ ($\hat{e}_s$ = unit vector along the particle symmetry axis). If we let $f(J_i)dJ_i$ be the fraction of spheroids with angular momentum around the i-axis between J_i and $J_i + dJ_i$, we can express the Fokker-Planck equation as

$$\frac{\partial f}{\partial t} = -\sum_i \frac{\partial}{\partial J_i}\left[\langle \Delta J_i \rangle f\right] + \sum \frac{\partial^2}{\partial J_i^2}\left[\frac{1}{2}\langle (\Delta J)_i^2 \rangle f\right] \qquad i = \begin{matrix} x,\ y,\ z \\ \xi,\ \eta,\ \zeta \end{matrix} \tag{2}$$

where $\langle \Delta J_i \rangle$ represents the average change in J_i per unit time, and where $\langle (\Delta J)_i^2 \rangle$ represents the average value of $(\Delta J)_i^2$ per unit time; these quantities are referred to as "diffusion coefficients." If we denote by $\langle \Delta J_i \rangle_r$, $\langle \Delta J_i \rangle_d$, $\langle \Delta J_i \rangle_M$, and $\langle \Delta J_i \rangle_T$ the value of this diffusion coefficient resulting from collisions with other ring particles, from collisions with "dust" (or gas) particles, from magnetic torques, and from fluctuations due to the internal temperature of the ring particle, respectively, then from the linearity of equation (2), we have

$$\langle \Delta J_i \rangle = \langle \Delta J_i \rangle_r + \langle \Delta J_i \rangle_d + \langle \Delta J_i \rangle_M = \langle \Delta J_i \rangle_T \qquad i = \begin{matrix} x,\ y,\ z \\ \xi,\ \eta,\ \zeta \end{matrix} \tag{3}$$

A similar notation has been adopted for the other diffusion coefficient as well.

Gas/Dust Collisions

Expressions for $\langle \Delta J_i \rangle_d$ and $\langle (\Delta J)_i^2 \rangle_d$ can be derived in a straightforward manner (ref. 24). For simplicity, we will use the terms "dust" and "dust particles" to refer to interplanetary dust particles, gas atoms, and charged particles.

The average number of collisions between dust and ring particles that affect J_ξ is $\nu_d \pi a_\xi (a_\eta v_\eta + a_\zeta v_\zeta)$, where ν_d is the number density of dust particles; $a_\xi = a_\eta = a_t$; $a_\zeta = a_s$; and v_i is the average dust particle velocity along the i-axis. The average number of collisions affecting J_η and J_ζ can be expressed similarly. Since a ring particle can be assumed to be much more massive than a "dust particle," then in kinetic equilibrium its translational velocity will be much less, and is therefore ignored.

The quantity $\langle \Delta J_i \rangle d$ has two components: one due to the angular momentum of the dust at impact and the other due to the momentum imparted to the dust particles as they leave. The first component will be zero, since, on the average, as many dust particles will strike the spheroid on one side of each axis as on the other, even if the velocity field is anisotropic. One of the simplest assumptions for the other component is that, on the average, as the dust particles leave the spheroid, they have the same velocity as the surface element of the spheroid from which they left. Other assumptions are also plausible and might lead to slightly different numerical results. Under this assumption, however,

$$\langle \Delta J_i \rangle_{\text{one collision}} = \omega_i m_d a_i^{*2} \tag{4}$$

where m_d is the average mass of a dust particle and where a_i^* is the root mean square distance from the i-axis, which can be shown to be

$$a_\zeta^* = \frac{4a_t}{3\pi^2} \tag{5}$$

$$a_\xi^* = a_\eta^* = \frac{4wa_t}{3\pi^2} E(e) \tag{6}$$

where $E(k)$ is the complete elliptical integral of the second kind, e is the eccentricity of the ellipse formed by the intersection of the spheroid surface and a plane which includes the symmetry axis, and

$$w = \begin{cases} 1 & \text{prolate} \\ a_t/a_s & \text{oblate} \end{cases} \tag{7}$$

If we assume an isotropic Maxwellian distribution for the "dust particles" with an equivalent dust temperature T_d, we obtain

$$\langle \Delta J_\zeta \rangle_d = -\frac{16a_s a_t^3 \nu_d (2\pi k m_d T_d)^{1/2} J_\zeta}{3I_\zeta} \tag{8}$$

$$\langle \Delta J_\xi \rangle_d = \langle \Delta J_\eta \rangle_d = -\frac{64wa_s^2 a_t (a_s + a_t) E^2(e) \nu_d (2\pi k m_d T_d)^{1/2} J_{\xi,\eta}}{3\pi^2 I_\xi} \tag{9}$$

where k is the Boltzmann constant, and I_i is the moment of inertia around the i-axis.

The dominant contribution to $\langle (\Delta J)_i^2 \rangle_d$ is the angular momentum transferred to the spheroid by the dust particle in the collision. This leads to

$$\langle (\Delta J)_\zeta^2 \rangle_d = \frac{32a_t^5 \left(a_s^2 + 2a_t^2\right)\left(2a_s + a_t\right)}{a_t^4 + 2\left(a_s^2 + a_t^2\right)^2} \; \frac{\nu_d \left(2\pi m_d k^3 T_d^3\right)^{1/2}}{15} \tag{10}$$

$$\langle (\Delta J)_\xi^2 \rangle_d = \langle (\Delta J)_\eta^2 \rangle_d = \frac{32a_t \left(a_s^2 + a_t^2\right)^2 \left(a_s^2 + 2a_t^2\right)\left(2a_s + a_t\right)}{a_t^4 + 2\left(a_s^2 + a_t^2\right)^2} \; \frac{\nu_d \left(2\pi m_d k^3 T_d^3\right)^{1/2}}{15} \tag{11}$$

Magnetic Torques

Davis and Greenstein (ref. 14) dealt with the effects of magnetic torques on a spinning spheroid. They pointed out that if the spheroid is slightly paramagnetic, the component of the magnetization, $\vec{M}$, perpendicular to $\vec{B}$ will fluctuate as the spheroid spins. Since the particle is nonspherical, the symmetry axis will nutate around $\vec{J}$ as the particle spins; and, if the magnetization is averaged over one nutation of the particle, there is a nonzero resultant magnetization perpendicular to $\vec{B}$ and proportional to χ''/ω, where χ'' is the imaginary part of the magnetic susceptibility, χ_M:

$$\chi_M = \chi' + i\chi'' \tag{12}$$

This nonzero magnetization results in a dissipative torque, which tends to decrease the magnitude of the components of $\vec{J}$ perpendicular to $\vec{B}$. As a result, $\vec{J}$ tends to become alined in the same direction as $\vec{B}$. The results of the analysis of Davis and Greenstein (ref. 14), in the notation we are using here, can be expressed as

$$\langle(\Delta J_z)\rangle_M = 0 \tag{13}$$

$$\langle(\Delta J_x)\rangle_M = \langle(\Delta J_y)\rangle_M = \frac{-JV\chi'' B^2 \sin \beta_J}{2I_\xi a_t^2 \omega}\left[\left(a_s^2 + a_t^2\right)\cos\theta_J + 2a_t^2 \sin\theta_J\right] \tag{14}$$

$$\langle(\Delta J)^2\rangle_M = \frac{-2V\chi'' B^2 J^2 \sin^2 \beta_J}{2I_\xi a_t^2 \omega}\left[\left(a_s^2 + a_t^2\right)\cos\theta_J + 2a_t^2 \sin\theta_J\right] \tag{15}$$

where

$$\cos\beta_J = \hat{e}_J \cdot \hat{e}_B = J_z/J \tag{16}$$

$$\cos\theta_J = \hat{e}_J \cdot \hat{e}_s = J_\zeta/J \tag{17}$$

Internal Thermal Fluctuations

The fluctuations due to the internal temperature of the grains have recently been investigated by Goldstein (ref. 24). There are many mechanisms which may contribute to the internal temperature, T_I, of the grain, including: collisions with gas and dust particles, the dissipative torques involved in the paramagnetic relaxation process, bombardment by low-energy cosmic rays, by the remnants of the solar wind plasma, and by any trapped radiation, and energy absorption from the radiation field (sunshine, planet shine, starshine, and moonshine). Since the radiation field will be the dominant process for establishing T_I, we may treat T_I as an independent parameter.

If T_I is nonzero, the magnetization, $\vec{M}$, will fluctuate (ref. 17) in a random manner. The fluctuating magnetization will in turn produce a randomly varying torque perpendicular to $\vec{B}$ (torque $\propto \vec{M} \times \vec{B}$), which will in turn produce variations in $\vec{J}$. If we represent the random fluctuations in the magnetization by $\Delta\vec{M}$, then the symmetry of the problem implies

$$\langle(\Delta\vec{M})_i\rangle = 0 \qquad i = x,\ y,\ z \tag{18}$$

In addition, we can assume that the fluctuations of the orthogonal components are uncorrelated:

$$\langle(\Delta\vec{M})_i(\Delta\vec{M})_j\rangle = 0 \qquad i \neq j;\ i,j = x,\ y,\ z \tag{19}$$

Since changes in $\vec{J}$ are directly proportional to the applied torques, equation (18) implies

$$\langle(\Delta J_i)\rangle_T = 0 \qquad i = x,\ y,\ z \tag{20}$$

while the fact that the torques are perpendicular to $\vec{B}$ implies

$$\langle(\Delta J)_z^2\rangle_T = 0 \tag{21}$$

It can be shown (ref. 24) that the other two diffusion coefficients can be expressed as

$$\langle(\Delta J)_x^2\rangle_T = \langle(\Delta J)_y^2\rangle_T = 8\pi a_t^2 a_s kT_I B^2 \chi''/3\omega \tag{22}$$

Ring Particle Collisions

If we let $\Pi([\phi_1], [\phi_2], t)\ dt$ be the probability that two ring particles with distribution function parameter sets $[\phi_1]$ and $[\phi_2]$ collide in the time interval from t to $t + dt$, we can express the effects of such collisions on the angular momentum distribution in terms of Π. To a first approximation, however, we will follow the lead of most of the ring stability analyses (refs. 10, 12, 25-28) and assume that $\Pi([\phi_1],[\phi_2])$ is sufficiently small so that

and
$$\left.\begin{aligned} \langle(\Delta J_i)\rangle_r &\ll \langle(\Delta J_i)\rangle_d + \langle(\Delta J_i)\rangle_M + \langle(\Delta J_i)\rangle_T \\ \langle(\Delta J)_i^2\rangle_r &\ll \langle(\Delta J)_i^2\rangle_d + \langle(\Delta J)_i^2\rangle_M + \langle(\Delta J)_i^2\rangle_T \end{aligned}\right\} \quad i = \begin{matrix} x,\ y,\ z \\ \xi,\ \eta,\ \zeta \end{matrix} \tag{23}$$

Fokker-Planck Equation Solution

In principle, at least, we have now specified enough information to solve the Fokker-Planck equation, equation (2), for $f_j(J,\beta_J,\theta_J)$, j = obl, pro.

Since we are interested in the orientation of the ring particles in equilibrium, we need to find a solution to the homogeneous partial differential equation formed by setting the right-hand side of equation (2) equal to zero. Such a solution can be approximated in the case of nearly spherical spheroids (ref. 24). Using some of Goldstein's (ref. 24) notation and modifying his results to correspond to our current problem, we obtain the following solution for nearly spherical prolate particles:

$$f_{\text{pro}}(J,\beta_J,\theta_J) \approx \exp\left\{-\left[\cos^2 \beta_J + c_2^{-2} \sin^2 \beta_J + \frac{(1 - q^2)}{2q^2 c_1^2} \cos^2 \theta_J\right]\frac{J^2}{4kT_d I_\eta}\right\} \tag{24}$$

where c_1 and c_2 are constants defined by

$$c_1 \equiv \frac{1 + bT_I/T_d}{1 + b} \tag{25}$$

and

$$c_2 \equiv \frac{q^2(3 + 4bT_I/T_d) + 1}{q^2(3 + 4b) + 1} \tag{26}$$

and where b is a measure of the relative importance of magnetic torques and gas/dust collisions:

$$b = (\chi''/\omega)VB^2/2G\nu_d(2\pi m_d kT_d)^{1/2} \tag{27}$$

and $b >> 1$ indicates that magnetic alinement dominates over the effects of the interplanetary medium. In a similar manner, bT_I/T_d is a measure of the relative importance of magnetic torques and fluctuations due to the internal temperature of the grain. In equation (27), G is a constant related to the size and shape of the spheroid, and is defined as follows:

$$G_{\text{pro}} = a_s a_t^3[2qe^3 + qe + (4e^2 - 1)\sin^{-1}(e)]/2e^3 \tag{28}$$

$$G_{\text{obl}} = a_s^4[2q^4e^3 - q^2e + (4q^2e^2 + 1)\sin^{-1}(qe)]/2e^3 \tag{29}$$

where

$$q = a_t/a_s$$

It should be noted that $G_{\text{pro}}/a_s a_t^3$ and $G_{\text{obl}}/a_s^3 a_t$ are functions of q only; these two functions are illustrated in figure 2. The approximate solution for the case of nearly spherical oblate spheroids is

$$f_{\text{obl}}(J,\beta_J,\theta_J) \simeq \exp\left\{-\left[(1 + q^2) \cos^2 \beta_J + c_1{}^2 \sin^2 \beta_J + \frac{q^2 - 1}{c_1^2} \sin^2 \theta_J\right]\frac{J^2}{8q^2kT_g I_\eta}\right\} \tag{31}$$

The degree to which the particles are alined with the magnetic field can be expressed in terms of the average values of $(\hat{e}_s \cdot \hat{e}_B)^2$ and $(\hat{e}_j \cdot \hat{e}_B)^2$, which indicate the degree to which the symmetry axis and the angular momentum, respectively, are alined. Therefore, we are interested in finding

$$\langle \cos^2 \beta_J \rangle_i = \frac{1}{4}\int_{-1}^{+1} d(\cos \beta_J)\cos^2 \beta_J \int_{-1}^{+1} d(\cos \theta_J)\int_0^\infty dJ \frac{J}{8q^2 kT_g I_n} f_i(\beta_J,\theta_J) \qquad i = \text{obl, pro} \tag{32}$$

and

$$\langle \cos^2 \theta \rangle_i \equiv \frac{1}{8}\int_{-1}^{+1} d(\cos \beta_J)\int_{-1}^{+1} d(\cos \theta_J)\int_0^\infty dJ \frac{J}{8q^2 kT_g I_n}\Big(1 - \cos^2 \beta_J - \cos^2 \theta_J + 3\cos^2 \beta_J \cos^2 \theta_J\Big) f_i(\beta_J,\theta_J) \qquad i = \text{obl, pro} \tag{33}$$

with f_i normalized so that the integral of f_i over all of phase space is unity. Using equations (24) and (31) in equations (32) and (33), we can obtain for $T_I < T_d$ (see next section for justification):

$$\langle \cos^2 \beta_J \rangle_{\text{pro}} = \frac{1 - (1/N_1 c_4)\tan^{-1}(c_4/c_5)}{c_4^2} \tag{34}$$

$$\langle \cos^2 \theta \rangle_{\text{pro}} = \frac{q^2 c_1^2}{e^2 c_2^2 c_4^2 N_1}\left(c_6 - N_1 c_7^2\right) \tag{35}$$

$$\langle \cos^2 \beta_J \rangle_{\text{obl}} = \frac{q^2 e^2}{c_8^2 q_1^2}\left[1 - \frac{c_9}{N_2}\sin^{-1}(c_3)\right] \tag{36}$$

$$\langle \cos^2 \theta \rangle_{\text{obl}} = \frac{N_2 q_1^2\left(q^2 + q_1^2 c_1^2\right) - \sqrt{(8)}\,q_1 c_1 q^3}{N_2 q_1^2 e^2 q^2 c_3^2} \tag{37}$$

where

$$q_1^2 \equiv q^2 + 1 \tag{38}$$

and where the constants $c_3, \ldots, c_9$ have been introduced to simplify the above expressions:

$$c_3^2 \equiv 1 - c_1^2 \tag{39}$$

$$c_4^2 \equiv 1 - c_2^2 \tag{40}$$

$$c_5^2 \equiv \frac{c_2^2 c_4^2 e^2}{2c_1^2 q^2} \tag{41}$$

$$c_6^2 \equiv \frac{c_2^2}{2c_1^2 q^2}\left(e^2 + 2q^2 c_1^2\right) \tag{42}$$

$$c_7^2 \equiv c_6^2 - c_5^2 \tag{43}$$

$$c_8^2 \equiv \frac{e^2 c_3^2}{2} \tag{44}$$

$$c_9^2 \equiv \frac{q^2 e^2}{q_1^2 c_3^2} \tag{45}$$

N_1 and N_2 are the normalization constants for f_{pro} and f_{obl}, respectively:

$$N_1 \equiv \frac{c_1 q\sqrt{(2)}}{e c_2^2} \ln \frac{c_6 + c_5}{c_7} \tag{46}$$

$$N_2 \equiv \frac{\tan^{-1}(c_8/c_1)}{c_8 q_1} \tag{47}$$

We can, in a natural manner, define angles that are characteristic of the degree of alinement:

$$\beta_{J,i} \equiv \cos^{-1}[(\langle\cos^2 \beta_J\rangle_i)^{1/2}] \qquad i \equiv \text{pro, obl} \tag{48}$$

$$\Theta_{obl} \equiv \cos^{-1}[(\langle\cos^2\,\Theta\rangle_{obl})^{1/2}] \tag{49}$$

$$\bar{\Theta}_{pro} \equiv \pi/2 - \Theta_{pro} \equiv \sin^{-1}[(\langle\cos^2\,\Theta\rangle_{pro})^{1/2}] \tag{50}$$

Figure 3(a) shows contours of constant $\beta_{J,obl}$ in $(q,\ b/q)$-space for the range $q \in [1.0,\ 10.0]$. Similar results for Θ_{obl} are shown in figure 3(b), while contours of constant $\beta_{J,pro}$ and $\bar{\Theta}_{pro}$ in $(q,\ bq)$-space are shown for the range $q \in [0.1,\ 1.0]$ in figure 3(c) and (d).

It is clear that, for large b, $\beta_{J,obl}$, $\beta_{J,pro}$, and Θ_{obl} tend to zero while Θ_{pro} tends to $\pi/2$. This means that, if magnetic effects are dominant, oblate spheroids will tend to aline themselves with their spin axes and symmetry axes coincident and parallel to $\vec{B}$. Prolate spheroids, on the other hand, will tend to aline themselves with their spin axes parallel to $\vec{B}$ and their symmetry axes perpendicular to $\vec{B}$. Thus, both types of spheroids tend to rotate around the shortest body axis and to aline this axis with a strong magnetic field.

ALINEMENT IN SATURN'S RINGS

In order to apply these results to the particles in Saturn's rings, we must estimate the pertinent particle, planetary, and interplanetary parameters, namely, q, a_s, T_I, χ''/ω, $B_♄$, ν_d, m_d, and T_d.

Cis-Saturn Parameter Values

q.— In our discussion of ring particle alinement we will be principally concerned with nearly spherical particles ($q \approx 1$). As a consequence, although we will parameterize our results with respect to q, we will limit q to the range [0.1, 10.0].

a_s.— The range of particle sizes, characterized by a_s, is constrained by the current estimates of ring particle sizes (see table 1):

$$10^{-8}\ \text{m} \le a_s \le 10^{3}\ \text{m} \tag{51}$$

T_I.— The best current estimates of the temperature of the ring particles are based on radiometric and radio astronomical measurements. Two temperature levels appear to fit the observational data: $T_I \approx 30°$ K and $T_I \approx 65°$ K (ref. 11). Since the effectiveness of the magnetic alinement mechanisms is inversely proportional to the internal temperature of the particles, we will choose the latter level (65° K) as representative of "worst case" conditions.

χ''/ω.— The magnetic susceptibility of the particles depends on their internal composition, structure, and thermodynamic state. The value to be used for (χ''/ω) also depends on the magnetic relaxation mechanism being considered. Since it does not appear feasible to deal quantitatively with diamagnetic effects, as, for instance, in the case of graphite flakes imbedded in ice (ref. 29), we limit our considerations to paramagnetic and ferromagnetic effects. The derivations above are, in part, based on the assumption that the quantity χ''/ω is approximately constant. This assumption must be justified and the constant evaluated; this is done briefly for paramagnetic and ferromagnetic interactions. More detailed discussions are given by Davis and Greenstein (ref. 14) and Jones and Spitzer (ref. 17).

The dissipation of energy by paramagnetic forces is usually described in terms of a combination of spin-lattice and spin-spin interactions. However, if the paramagnetic ions are homogeneously distributed throughout the particle, there is no net magnetic moment in a coordinate system rotating with the particle; hence, spin-lattice interactions do not contribute to paramagnetic relaxation. The standard method of calculating the effects of spin-spin interactions is that of Van Vleck (ref. 30), which results in very tedious calculations. In order to avoid these calculations, Locher and Gorter (ref. 31) proposed an interpolating lineshape function which has been used successfully and predicts

$$\chi''(\omega) = \chi(o)\left(\frac{\pi}{2}\right)^{1/2}[\alpha\sqrt{\pi}\ \exp\ (\alpha^2)\ \mathrm{erfc}\ (\alpha)]^{-1}$$

$$\frac{\sqrt{2}a\tau\omega}{1 + \omega^2\tau^2}\exp(-\alpha^2\tau^2\omega^2) \tag{52}$$

where α is a function of $\langle\omega^4\rangle/\langle\omega^2\rangle^2$ and is a measure of the effectiveness of such other effects as hyperfine, exchange, and crystal field interactions, and τ is the time constant for spin-spin interactions. For any reasonable values of α ($>10^{-2}$) it is easy to show that the expression for $\chi''(\omega)$ reduces to a Gaussian (within a factor of 2):

$$\chi''(\omega) = \chi(o)\left(\frac{\pi}{2}\right)^{1/2}\frac{\omega}{\langle\omega^2\rangle^{1/2}}\exp(-\omega^2/2\langle\omega^2\rangle) \tag{53}$$

This, of course, leads to the conclusion that $\chi''(\omega)/\omega$ is approximately constant for small ω. Jones and Spitzer (ref. 17) have shown that $10^{-12}/T_I$ should represent a realistic lower limit for χ''/ω. They have also pointed out that if there are abundant concentrations of hydrogenic atoms, nuclear paramagnetic effects obviate the presence of paramagnetic ions, and the lower limit of χ''/ω is unchanged. Thus, we have

$$(\chi''/\omega)_{para} \gtrsim 10^{-12}/T_I \approx 1.5\times10^{-14}\ \mathrm{sec/rad} \tag{54}$$

nearly independent of the chemical composition of the ring particles.

Jones and Spitzer (ref. 17) have pointed out that if, instead of the uniform, diffuse distribution of paramagnetic impurities assumed above, one considers the equally likely possibility that these impurities may aggregate into particulates of, say Fe_3O_4 or γ-Fe_2O_3, then the value of $(\chi''/\omega)_{para}$ might be enhanced by six or seven orders of magnitude. Jones and Spitzer refer to this mechanism as "super-paramagnetism:"

$$(\chi''/\omega)_{s\text{-para}} \gtrsim 10^{-8}\ \text{sec/rad} \tag{55}$$

A consideration of ferromagnetic effects is complicated somewhat by the fact that there is a critical size (~0.01-1.0 µm) below which the ferromagnetic grains in the ring particles would consist of essentially a single domain, and above which they must be treated as being multidomained. Theoretical treatments of multidomain particles are rather sparse, but it is generally concluded that the low-frequency behavior of their permeability is due to reversible domain-wall motion (refs. 32-34). By appealing to the available experimental data, Jones and Spitzer (ref. 17) conclude that χ''/ω is constant in the limit of small frequencies. Excluding more exotic compounds such as $Ni_{0.4}\,Zn_{0.6}\,Mn_{0.02}\,Fe_{1.9}\,O_4$, the range of values of χ''/ω for multidomain ferromagnetic particles at room temperatures is approximately (ref. 35)

$$10^{-11} < (\chi''/\omega)_{ferro,md} < 10^{-7}\ \text{sec/rad} \tag{56}$$

Since the temperature dependence of this value is approximately like $\exp(\Delta E/kT_I)$ (0.1 eV < ΔE < 1.0 eV, ref. 36), we would expect χ''/ω to be much larger than indicated in equation (56). Experimental data at low temperatures is virtually nonexistent, however, and we therefore have used the vastly underestimated range indicated in equation (56), which results in an underestimation of the degree of particle alinement. The analysis of the permeability of single-domain ferromagnetic particles in the low-frequency limit is reviewed by Jones and Spitzer (ref. 17), who conclude that for interstellar grains

$$2\times10^{-13} < (\chi''/\omega)_{ferro,sd} < 4\times10^{-7}\ \text{sec/rad} \tag{57}$$

"with the lower limit extremely unlikely."

Clearly, the wide range of possible particle compositions and the several different possible relaxation mechanisms precludes a definitive estimate of χ''/ω; it is possible, however, to assert that 1.5×10^{-14} sec/rad is definitely a lower bound and that the actual value is probably several orders of mangitude larger.

$B_♄$.— Haffner (refs. 37, 38) has investigated the possibility of a Saturnian magnetic field. On the basis of the limited radioastronomical data available for Saturn and the assumption that the rings will effectively sweep trapped charged particles out of the inner magnetic field, he estimates the magnetic field strength at the equator to be 1 to 10 gauss. If we make the reasonable assumption that the magnetic field near the equatorial plane is essentially dipolar out to at least 2.5 $R_♄$, then the direction of the magnetic field in the rings will be normal to the ring plane and the magnitude of the field will vary as $\rho_♄^{-3}$, where $\rho_♄$ is the radial distance from the center of the planet. Table 2 shows the range of magnetic field strengths to be expected at the boundaries of the major ring elements.

ν_d, m_d, T_d.— The characteristics of the interplanetary dust/gas environment have been estimated by Cook and Franklin (ref. 27) in connection with a study to determine the effect of this environment on the stability of the rings. They estimate that the mass density of dust particles in cissaturnian interplanetary space is about 1.2×10^{-23} gm/cm^3. If we assume that the density of the individual dust particles is the same as that of meteoroids, 0.18 gm/cm^3 (ref. 39), then, assuming a dust particle radius of 1.0 μm, we have

$$\nu_d \approx 1.6\times10^{-12}\ \mathrm{cm}^{-3} \tag{58}$$

and

$$m_d \approx 7.5\times10^{-12}\ \mathrm{gm} \tag{59}$$

Cook and Franklin also estimate that the average dust particle velocity is about 34 km/sec, which leads to an effective kinetic temperature:

$$T_d \approx 2.5\times10^{17}\ {}^\circ\mathrm{K} \tag{60}$$

Degree of Ring Particle Alinement

With these estimates of the pertinent particle, planetary, and interplanetary parameters, we can now estimate the degree to which the particles in Saturn's rings could be expected to be alined. If we assume specific values for $B_{♄,\mathrm{equator}}$ and χ''/ω then we can express this degree of alinement in terms of the maximum size particle that would be alined to within a specific angle as a function of position in the rings for each of the four types of magnetic relaxation methods. Figure 4 (a)-(d) shows such a display for the two alinement angles for each of the two types of spheroids. Here we have chosen $B_{♄,\mathrm{equator}} = 3$ gauss as typical, and for each of the four relaxation methods we have used a value of χ''/ω that is two orders of magnitude above the absolute lower bound. All these choices appear to be conservative, and more realistic estimates of $a_{s,\mathrm{max}}$ are probably several orders of magnitude greater.

The data in figure 4(a), for instance, indicate that, with these parametric estimates, the spin axes of all prolate spheroids in the B ring smaller than 3×10^{-6} cm would be alined to within 1° of $\vec{B}$ even for the least efficient relaxation method (paramagnetic relaxation); if multidomain ferromagnetic particulates occur in the particles, however, their spin axes would be alined to within 1° of $\vec{B}$ for $a_s < 2.5$ cm. The effect of uncertainty in the correct value for χ''/ω, is indicated by the use of the upper bound for this parameter, which changes these two values to $a_s < 3\times10^{-2}$ cm and $a_s \lesssim 2.5$ m. Similar conclusions can be drawn from figure 4(b), (c), and (d) for $\bar{\Theta}_{\mathrm{pro}}$, $\beta_{J,\mathrm{obl}}$, and Θ_{obl}. Fully alined particles would appear as shown in figure 5.

CONCLUSIONS

If the particles in the rings of Saturn are spherical, the orientation of their spin angular momenta is immaterial to their optical properties. Perfectly spherical particles are, however, quite unlikely, and the orientation of nonspherical particles can be anticipated to have a significant effect on the optical properties of the rings. As a basis for subsequent investigation of these effects (Part III), we have determined the "preferred" orientation of spheroidal particles in a magnetic field and investigated the degree to which the ring particles will assume this orientation.

Since the alinement mechanisms, as well as the misalinement mechanisms, are stochastic in nature, we have used the Fokker-Planck formulism to determine the alinement distribution function. The four major factors contributing to the particle orientation — the interplanetary dust/gas, the Saturnian magnetic field, the internal ring particle temperature, and the collisions between ring particles — have been analyzed in turn, and the contribution of each to the Fokker-Planck diffusion coefficients has been determined. The solution of the resultant Fokker-Planck equation for nearly spherical particles has been presented, and the particle alinement implied by this solution (fig. 3) indicates that, at equilibrium, in the presence of a sufficiently large magnetic field any spheroidal particle will be spinning around its shortest body axis, with its spin axis oriented parallel to the magnetic field.

From estimates established for the pertinent physical parameters in the vicinity of the rings, implications of the previous analysis for the ring particles were investigated (fig. 4). The significance of the resulting data is somewhat obscured by the uncertainty in the value of the imaginary part of the magnetic susceptibility, χ''/ω. Nevertheless, it is clear that alinement of the ring particles is quite likely, especially for small (<10 cm) particles.

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TABLE 1. — ESTIMATED SIZE OF SATURN RING PARTICLES

Source	Characteristic radius, meter	Radiation braking lifetime for A & B rings (after ref. 13)	
		Years	Solar system lifetimes
Schoenberg (ref. 8)	$\gtrsim 3.6 \times 10^{-6}$	$\gtrsim 3.5 \times 10^{5}$	$\gtrsim 7.8 \times 10^{-5}$
Franklin and Cook (ref. 9) ("Model II")	3.1×10^{-4}	3×10^{7}	6.7×10^{-3}
Franklin and Columbo (ref. 10)	1×10^{2}	1×10^{13}	2.2×10^{3}
Bobrov (ref. 11)	1×10^{-6} - 7.5	$1 \times 10^{5} - 7.3 \times 10^{11}$	$2.2 \times 10^{-5} - 1.6 \times 10^{2}$
	$4.3 \times 10^{-2} - 2.8 \times 10^{2}$	$4.2 \times 10^{9} - 2.7 \times 10^{13}$	$9.3 \times 10^{-1} - 6 \times 10^{3}$
Price (ref. 12)	1×10^{-7}	1×10^{4}	2.2×10^{-6}
Pollack (unpublished estimate)	6×10^{-5} - 1.0	$6 \times 10^{6} - 1 \times 10^{11}$	1.3×10^{-3} - 22
Minimum radius for lifetime comparable to solar system lifetime	4.6×10^{-2}	4.5×10^{9}	1.0

TABLE 2. — DIMENSIONS OF SATURN'S RINGS (ref. 42)

	Ring boundary radius (10^3 km)	Ring boundary radius (R_p)	Magnetic field strength (est., ref. 38) (gauss)
	270? (ref. 41)	4.5?	0.010 - 0.10
Ring D			
	137	2.28	0.084 - 0.84
Ring A			
	120	2.00	0.125 - 1.25
Cassini division			
	116	1.93	0.139 - 1.39
Ring B			
	90	1.50	0.296 - 2.96
Gap			
	89	1.48	0.308 - 3.08
Ring C			
	72	1.20	0.579 - 5.79
Saturn			
	60	1.00	1.00 -10.0

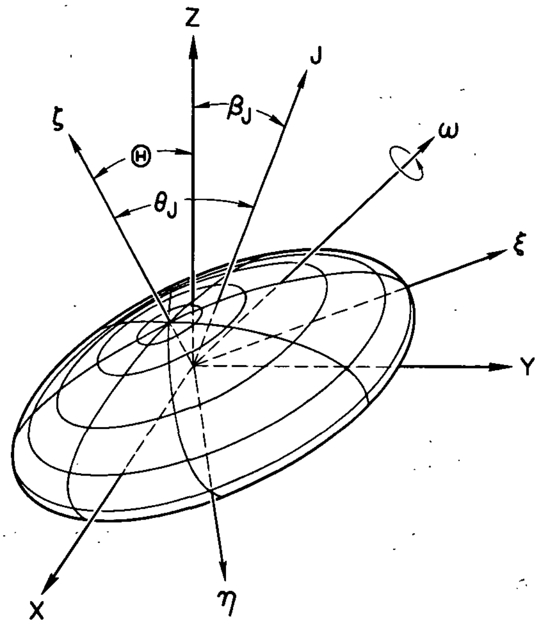
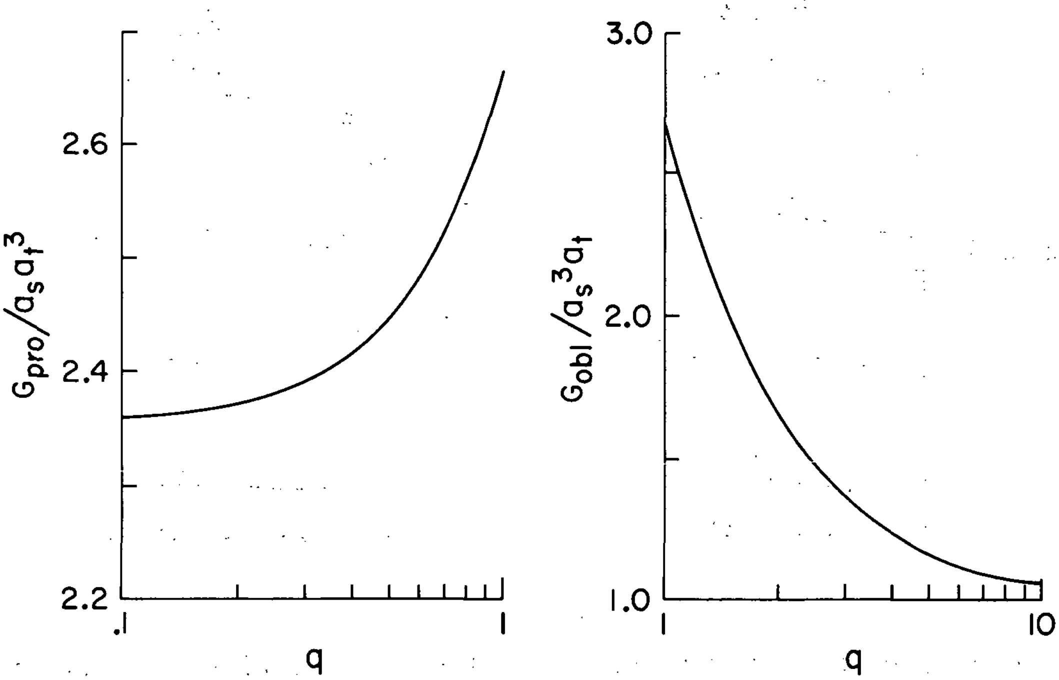


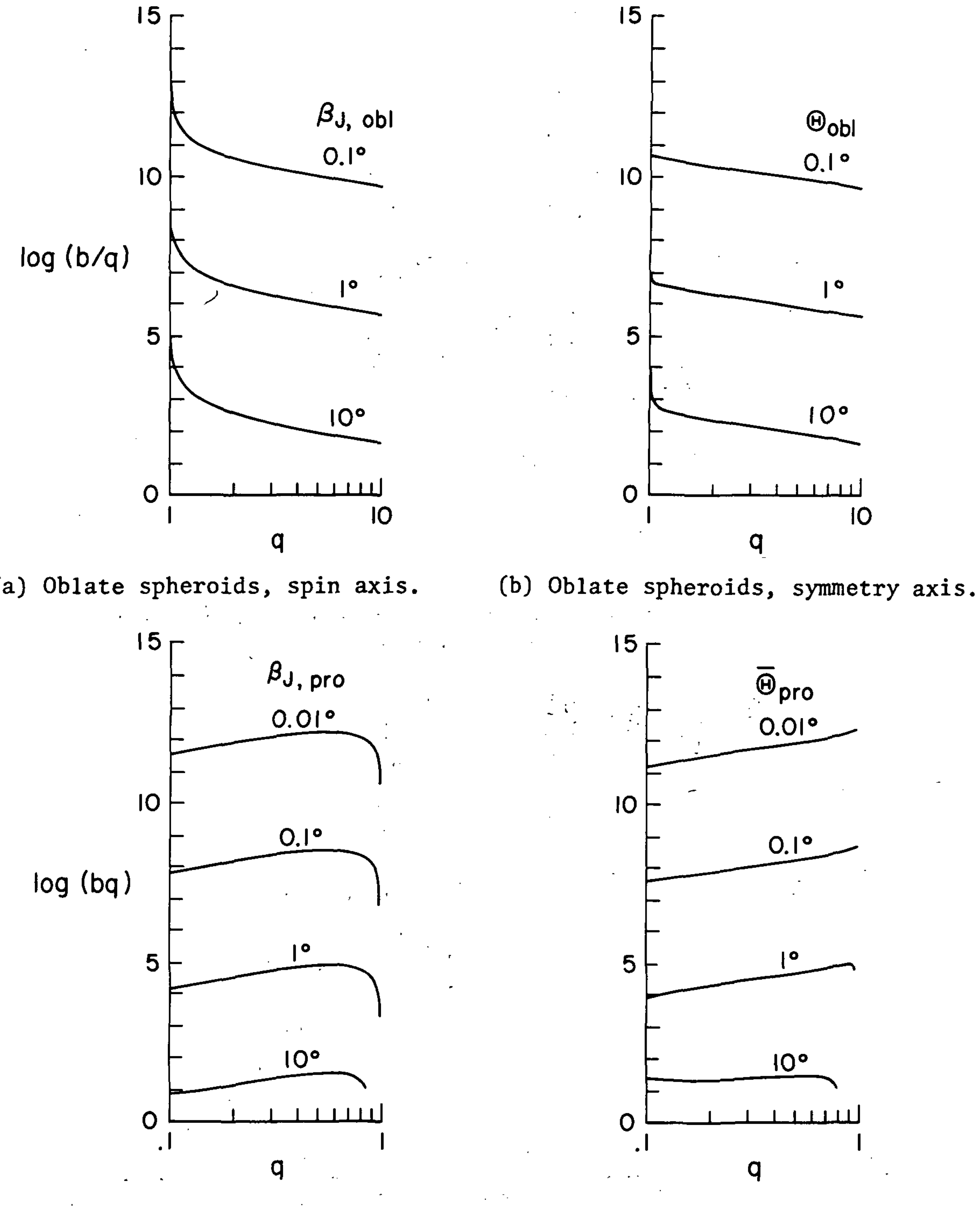
Figure 1.— Typical ring particle showing coordinate systems and angles used in the orientation analyses.



(a) Prolate spheroid.

(b) Oblate spheroid.

Figure 2.— Form factor G.



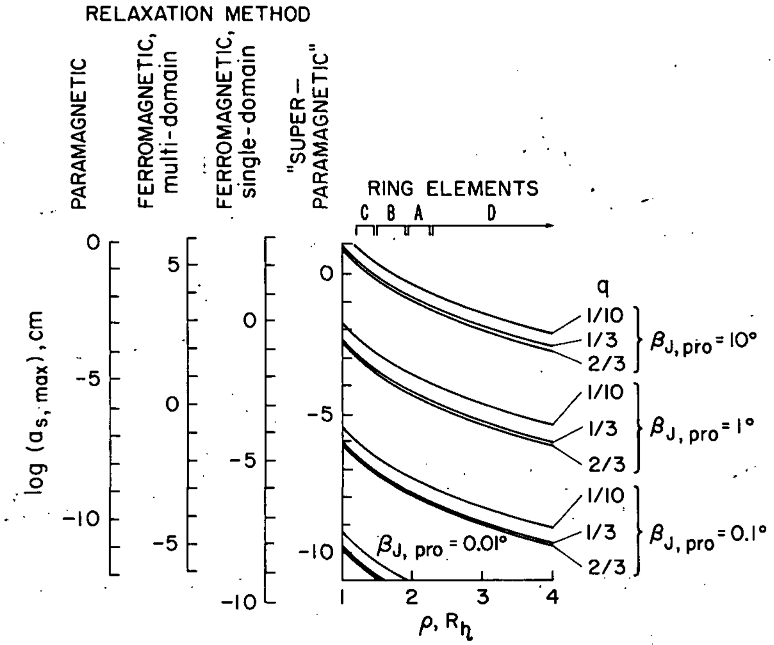
(a) Oblate spheroids, spin axis.

(b) Oblate spheroids, symmetry axis.

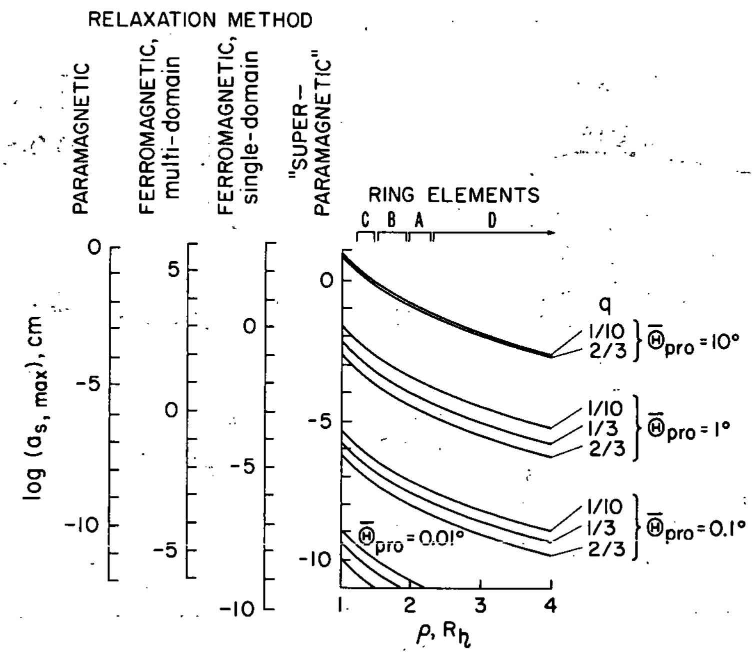
(c) Prolate spheroids, spin axis.

(d) Prolate spheroids, symmetry axis.

Figure 3.— Alinement of the spin and symmetry axes of spheroids with the magnetic field. The quantity, q, is the ratio of the transverse and symmetry semi-axes ($q = a_t/a_s$), and b is a measure of the relative effectiveness of magnetic and dust/gas interactions; ($b \gg 1$ implies that magnetic effects dominate).

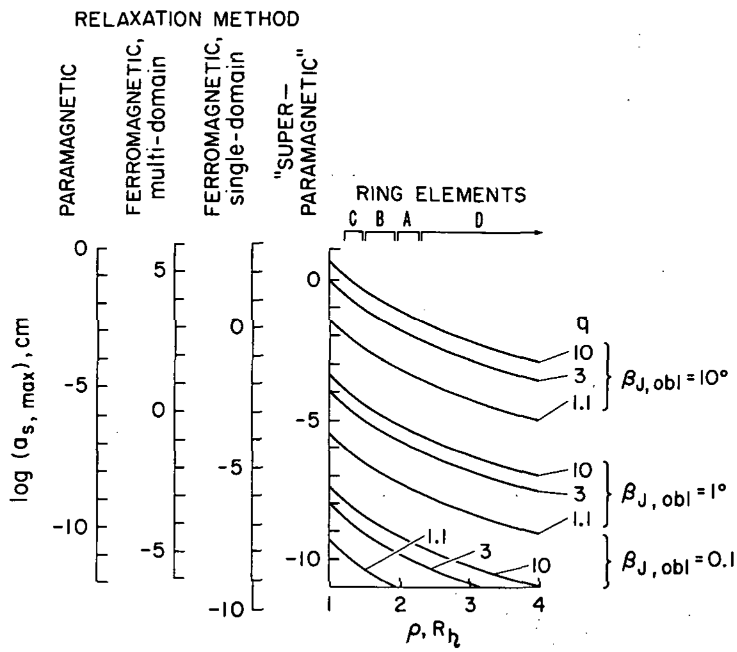


(a) Prolate spheroids, spin axis.

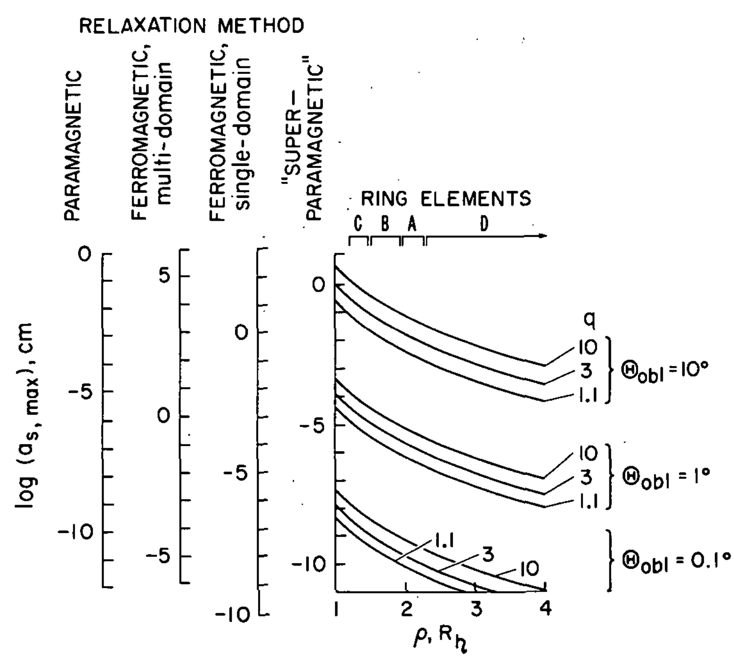


(b) Prolate spheroids, symmetry axis.

Figure 4.— Degree of alinement of the spin and symmetry axes of spheroids with the Saturnian magnetic field. The abscissa is the radial distance from the center of the planet in units of Saturn radii ($R_♄$). The ordinate scales represent the maximum size ring particles (in terms of a_s), which will give the indicated degree of alinement for each of the four magnetic relaxation mechanisms indicated.

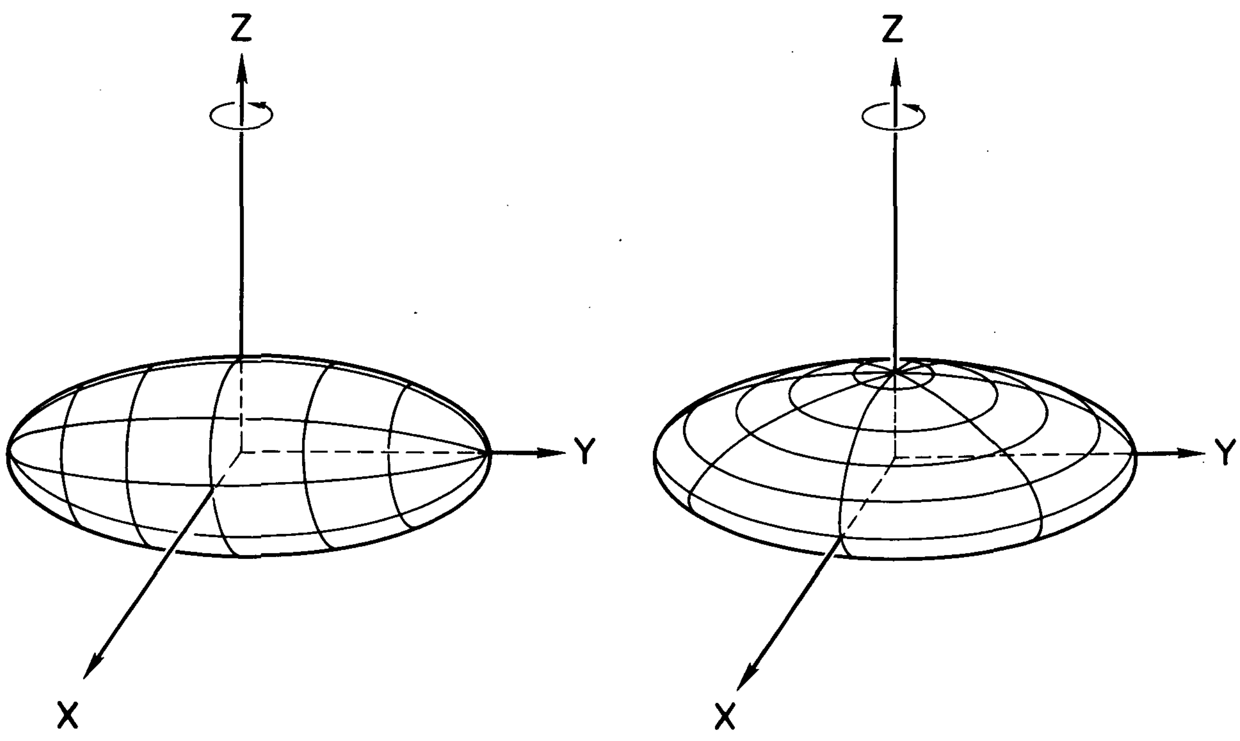


(c) Oblate spheroids, spin axis.



(d) Oblate spheroids, symmetry axis.

Figure 4.— Concluded.



(a) Prolate.

(b) Oblate.

Figure 5.– Fully alined spheroids.

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