NASA CR-121279

THE PREDICTION OF NONLINEAR THREE-DIMENSIONAL COMBUSTION INSTABILITY IN LIQUID ROCKETS WITH CONVENTIONAL NOZZLES

by

E. A. Powell and B. T. Zinn

GEORGIA INSTITUTE OF TECHNOLOGY

prepared for

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

NASA Lewis Research Center
Grant NGL 11-002-083
Richard J. Priem, Project Manager

Requests for copies of this report should be referred to

National Aeronautics and Space Administration
Scientific and Technical Information Facility
P. O. Box 33
College Park, Md. 20740

FOREWORD

The research described herein, which was conducted at Georgia Institute of Technology, was supported by NASA Grant No. NGL-11-002-083. The work was done under the management of the NASA Project Manager, Dr. Richard J. Priem, Chemical Rockets Division, NASA-Lewis Research Center.

ABSTRACT

An analytical technique is developed to solve nonlinear three-dimensional, transverse and axial combustion instability problems associated with liquid-propellant rocket motors. The Method of Weighted Residuals is used to determine the nonlinear stability characteristics of a cylindrical combustor with uniform injection of propellants at one end and a conventional DeLaval nozzle at the other end. Crocco's pressure sensitive time-lag model is used to describe the unsteady combustion process. The developed model predicts the transient behavior and nonlinear wave shapes as well as limit-cycle amplitudes and frequencies typical of unstable motor operation. The limit-cycle amplitude increases with increasing sensitivity of the combustion process to pressure oscillations. For transverse instabilities, calculated pressure waveforms exhibit sharp peaks and shallow minima, and the frequency of oscillation is within a few percent of the pure acoustic mode frequency. For axial instabilities, the theory predicts a steep-fronted wave moving back and forth along the combustor.

TABLE OF CONTENTS

TABLE OF CONTENTS (CONTINUED)

TABLE OF CONTENTS (CONTINUED)

LIST OF ILLUSTRATIONS

LIST OF TABLES

SUMMARY

An approximate analytical technique has been developed for the solution of nonlinear three-dimensional, transverse and axial combustion instability problems that are frequently observed in liquid-propellant rocket motors. This theory is an extension and generalization of previous analyses, which could analyze either transverse or axial instabilities in liquid combustors with quasi-steady nozzles, to the practical situations of three-dimensional instabilities in combustors with conventional DeLaval nozzles. Unlike the quasi-steady nozzle, the presence of a conventional nozzle imposes restrictions upon the behavior of both the amplitudes and phases of the oscillations at the nozzle entrance plane. The Method of Weighted Residuals is used to determine the nonlinear stability characteristics of a cylindrical combustor with uniform injection of propellants at one end and a conventional nozzle at the other end. Crocco's pressure sensitive time-lag model is used to describe the unsteady combustion process. The developed model can predict the transient behavior and nonlinear wave shapes as well as limit-cycle amplitudes and frequencies typical of unstable motor operation. These results establish the relationship that exists between the resulting instability (i.e., waveform, final amplitude and final frequency), the combustion parameters (i.e., interaction index, n, and time-lag, $\bar{\tau}$), and the chamber Mach number and length-to-diameter ratio. Results indicate that the limit-cycle amplitude increases with increasing sensitivity of the combustion process to pressure oscillations. For transverse instabilities, calculated pressure waveforms exhibit sharp peaks and shallow minima, and the frequency of oscillation is always within a few percent of the frequency of one of the chamber's acoustic modes. For axial instabilities, the theory predicts the presence of a steep-fronted wave moving back and forth along the combustor. In both cases calculations of pressure and velocity perturbations at the nozzle entrance plane show that the approximation to the nozzle boundary condition is very good. The theory described in this report represents the final stage in the development of a unified nonlinear theory for the solution of general three-dimensional, transverse and axial combustion instability problems.

INTRODUCTION

Observation of the behavior of unstable rocket motors indicates that combustion instability can be divided into two categories; that is, linear and nonlinear instabilities. Linear instabilities are spontaneous in nature, and they are usually an outgrowth of the random combustion and flow fluctuations present in the system. On the other hand, nonlinearly unstable motors require the introduction of a finite amplitude disturbance to produce (or trigger) combustion instability. In either case the instability, after a transient period, reaches a limiting maximum amplitude (i.e., limit-cycle amplitude) at which it oscillates with a given frequency that is usually close to the frequency of one of the chamber's acoustic modes. Pressure measurements taken during test firings of unstable motors indicate that the limit-cycle waveforms of transverse instabilities are non-sinusoidal; that is, they exhibit sharp peaks and flattened minima.[1] On the other hand, experimental observations of axial instabilities indicate the presence of shock-like steep-fronted waves in the chamber.[2] These results indicate that nonlinearities need to be considered in the theoretical treatment of combustion instability.

Any analytical treatment of combustion instability should be capable of solving nonlinear multi-dimensional combustion instability problems without exceeding memory core limitations of current computers and without requiring excessive computation time. To be of practical use, such a solution technique should be conceptually simple and easily adaptable for use by industry. This report describes the development and use of such a numerical solution technique.

Work on this problem has been in progress during the past several years, and due to its complexity, the problem had to be tackled in stages. In earlier investigations by these authors theories describing the nonlinear behavior of longitudinal[3,4] and transverse[5,6] instabilities in liquid combustors with quasi-steady nozzles were developed. These theories, which were based upon the application of the Method of Weighted Residuals (MWR), successfully predicted the transient behavior, nonlinear waveforms, and limit-cycle amplitudes of longitudinal and transverse instabilities in unstable liquid rockets. This report is concerned with the development of a generalized nonlinear theory that will be capable of analyzing three-dimensional, transverse and axial instabilities in the more practical situations where the combustors are attached to conventional nozzles. Obviously, this generalized theory will encompass the above-mentioned investigations as special cases. Contrary to the quasi-steady nozzle case, the presence of a conventional nozzle imposes both amplitude and phase boundary conditions that must be satisfied by the solutions of the problem at the nozzle entrance plane. The generalized theory presented herein also provides a better description of the unsteady flow field in the vicinity of the nozzle entrance plane.

The application of the theory presented herein will be demonstrated by considering the nonlinear stability of a liquid-propellant rocket combustor with uniform injection of propellants at one end and a conventional nozzle at the other end. Crocco's pressure sensitive time lag model[7] is used to describe the unsteady combustion process. In the sections to follow, the development of the wave equation for the analysis of nonlinear combustion instability in liquid rockets will be briefly described, the solution of this nonlinear wave equation will be outlined, and typical results will be presented and discussed. User's Manuals and program listings for the computer programs used to solve these problems are included as appendices to this report.

SYMBOLS

$A_{\ell mn}(t)$, $B_{\ell mn}(t)$	time-dependent amplitudes in series given by Eq. (6)
$A_p(t)$	time-dependent amplitudes in series given by Eq. (9)
$B(\Phi)$	boundary residual
$b_{\ell mn}$	complex axial acoustic eigenvalue
c^*	velocity of sound, ft/sec

C_0, C_1, C_2, C_3	coefficients of linear terms in Eqs.(12)
D_1, D_2, D_3, D_4	coefficients of nonlinear terms in Eqs. (12)
$E(\Phi)$	residual of Eq. (10)
i	imaginary unit, $\sqrt{-1}$
J_m	Bessel function of the first kind, order m
ℓ, m	axial and tangential mode numbers, respectively
n	pressure interaction index
p	dimensionless pressure, $\gamma p^*/\rho_o^* c_o^{*2}$
r	dimensionless radial coordinate, r^*/R_c^*
R_c^*	chamber radius, ft
$R_p(r)$	radial acoustic eigenfunction in Eq. (9)
S_{mn}	dimensionless transverse mode frequency
t	dimensionless time, $\frac{t^*}{(R_c^*/c_o^*)}$
u	dimensionless axial velocity, u^*/c_o^*
$\vec{V}$	dimensionless velocity vector, $\vec{V}^*/c_o^*$
W_m'	unsteady combustion mass source
Y	complex nozzle admittance

z	dimensionless axial coordinate, z^*/R_c^*
$Z_{\ell mn}(z)$, $Z_p(z)$	axial acoustic eigenfunctions
γ	ratio of specific heats
ϵ	ordering parameter
θ	azimuthal coordinate
$\Theta_p(\theta)$	tangential acoustic eigenfunction in Eq. (9)
ρ	dimensionless density, ρ^*/ρ_o^*
τ	dimensionless pressure sensitive time lag, $\frac{\tau^*}{(R_c^*/c_o^*)}$
Φ	velocity potential

Subscripts:

e	evaluated at the nozzle entrance
n	radial mode number
r, t, z, θ	partial differentiation with respect to r, t, z, or θ respectively
r, i	real and imaginary parts of a complex quantity, respectively
o	stagnation quantity

Superscripts:

′	perturbation quantity, differentiation with respect to argument

$\bar{\ }$	steady state quantity
$*$	dimensional quantity, complex conjugate
$\sim$	approximate solution

ANALYSIS

Development of the Wave Equation

To keep the problem as simple as possible, yet still physically meaningful, the following assumptions are made. The gas phase in the combustor is assumed to consist of a single constituent which is thermally and calorically perfect. Transport phenomena, such as diffusion, viscosity, and heat conduction are neglected. The momentum interchange between the liquid and gas phases is neglected (see Appendix A for a discussion of this assumption), and the specific stagnation enthalpy of the unburned propellant is assumed constant throughout the chamber. The presence of burning propellant drops is represented by a distribution of unsteady mass sources[7] and it is also assumed that the Mach number of the combustor's mean flow is small and that the waves have moderate amplitudes.

As a result of the last two assumptions, the governing conservation equations may be combined and the unsteady flow in the combustor can be described by a single nonlinear wave equation. The derivation of this equation appears in Refs. 8 and 9, where it was assumed that each perturbation quantity and the mean flow Mach number were of $O(\epsilon)$, where ϵ is an ordering parameter that is a measure of the wave amplitude. After neglecting all terms of $O(\epsilon^3)$ or higher and combining equations, one obtains the following nonlinear partial differential equation that describes the behavior of the velocity potential, Φ, of the combustor disturbance:

$$\nabla^2\Phi - \Phi_{tt} = 2\bar{\vec{V}}\cdot\nabla\Phi_t + \gamma(\nabla\cdot\bar{\vec{V}})\Phi_t + 2\nabla\Phi\cdot\nabla\Phi_t + (\gamma - 1)\Phi_t\nabla^2\Phi + W_m' \qquad (1)$$

Equation (1) is the desired wave equation, and it is similar to the inhomogeneous wave equation solved by Maslen and Moore[10] in a related study on nonlinear acoustics. This equation accounts for the following effects: (1) the effect of a steady state flow on the wave motion (viz., the first two terms on the right-hand side), (2) the coupling between the gas dynamical oscillations and the unsteady combustion process (viz., the last term on the right-hand side), and (3) the second order nonlinearities of the gas dynamical processes (viz., the third and fourth terms on the right-hand side).

In addition to satisfying Eq. (1), the desired solutions must satisfy rigid wall boundary conditions at the injector end of the chamber and at the chamber walls, while a nozzle admittance condition must be satisfied at the nozzle entrance. These boundary conditions are given (in a cylindrical coordinate system) by:

$$\Phi_r = 0 \text{ at } r = 1$$

$$\Phi_z = 0 \text{ at } z = 0$$

$$B(\Phi) = \Phi_z + \gamma Y \Phi_t = 0 \text{ at } z = z_e \tag{2}$$

The nozzle admittance, Y, is a complex number defined by

$$Y = Y_r + iY_i = (u'/p')_{z = z_e} \tag{3}$$

where u′ is the dimensionless axial velocity perturbation and p′ is the dimensionless pressure perturbation.

It should be pointed out that due to the absence of an appropriate nonlinear nozzle admittance boundary condition, the solutions of the problem are required to satisfy a linear nozzle admittance. Although inconsistent with the nonlinear wave equation, the linear nozzle admittance condition is used herein with the hope that the solution techniques developed herein will also be applicable when nonlinear nozzle admittance conditions become available. Also, the relative importance of nozzle nonlinearities is not known at the moment and it is quite possible that the linear nozzle boundary condition used herein adequately describes the flow conditions at the nozzle entrance.

The unsteady combustion process is represented by mass sources distributed throughout the volume of the chamber, and the response of the mass sources to pressure oscillations is assumed to be described by Crocco's pressure sensitive time-lag hypothesis.[7] The mass source perturbation, W'_m, is then given by:[5,8]

$$W'_m = -\gamma n \frac{d\bar{u}}{dz}\left[\Phi_t(r,\theta,z,t) - \Phi_t(r,\theta,z,t-\bar{\tau})\right] \tag{4}$$

where n is the pressure "interaction index" that describes the sensitivity of the combustion process to pressure oscillations, and $\bar{\tau}$, commonly referred to as the sensitive time-lag, is the part of the total combustion time-lag during which the combustion process is sensitive to pressure oscillations. The unsteady combustion response described by Eq. (4) is linear and the comments made above regarding the use of a linear nozzle admittance boundary condition are also applicable to this case.

Substituting Eq. (4) into Eq. (1) and expressing the resulting equation in a cylindrical coordinate system yields the following wave equation:

$$\begin{aligned}
&\Phi_{rr} + \frac{1}{r}\Phi_r + \frac{1}{r^2}\Phi_{\theta\theta} + \Phi_{zz} - \Phi_{tt} \\
&- 2\Phi_r\Phi_{rt} - \frac{2}{r^2}\Phi_\theta\Phi_{\theta t} - 2\Phi_z\Phi_{zt} \\
&- (\gamma - 1)\Phi_t\left(\Phi_{rr} + \frac{1}{r}\Phi_r + \frac{1}{r^2}\Phi_{\theta\theta} + \Phi_{zz}\right) \\
&- 2\bar{u}\Phi_{zt} - \gamma\Phi_t\frac{d\bar{u}}{dz} \\
&+ \gamma n\frac{d\bar{u}}{dz}\left[\Phi_t(r,\theta,z,t) - \Phi_t(r,\theta,z,t-\bar{\tau})\right] = 0
\end{aligned} \tag{5}$$

The combustor and nozzle geometries considered in this study, as well as the cylindrical coordinate system used in writing Eq. (5), are shown in Fig. 1.

Method of Solution

Since Eq. (5) has no known closed-form mathematical solution, it is necessary to resort to the use of either exact numerical solution techniques or approximate analytical techniques. For multi-dimensional problems, the exact numerical solution techniques generally exceed the computer storage capacities, therefore an approximate solution technique is used herein. The experience of previous investigators in the fields of structural stability and aeroelasticity indicates that an approximate solution technique known as the Method of Weighted Residuals[11, 12] may be effective in the solution of this nonlinear wave equation.

In order to employ the Method of Weighted Residuals in the solution on Eq. (5), it is first necessary to express the velocity potential, Φ, as an

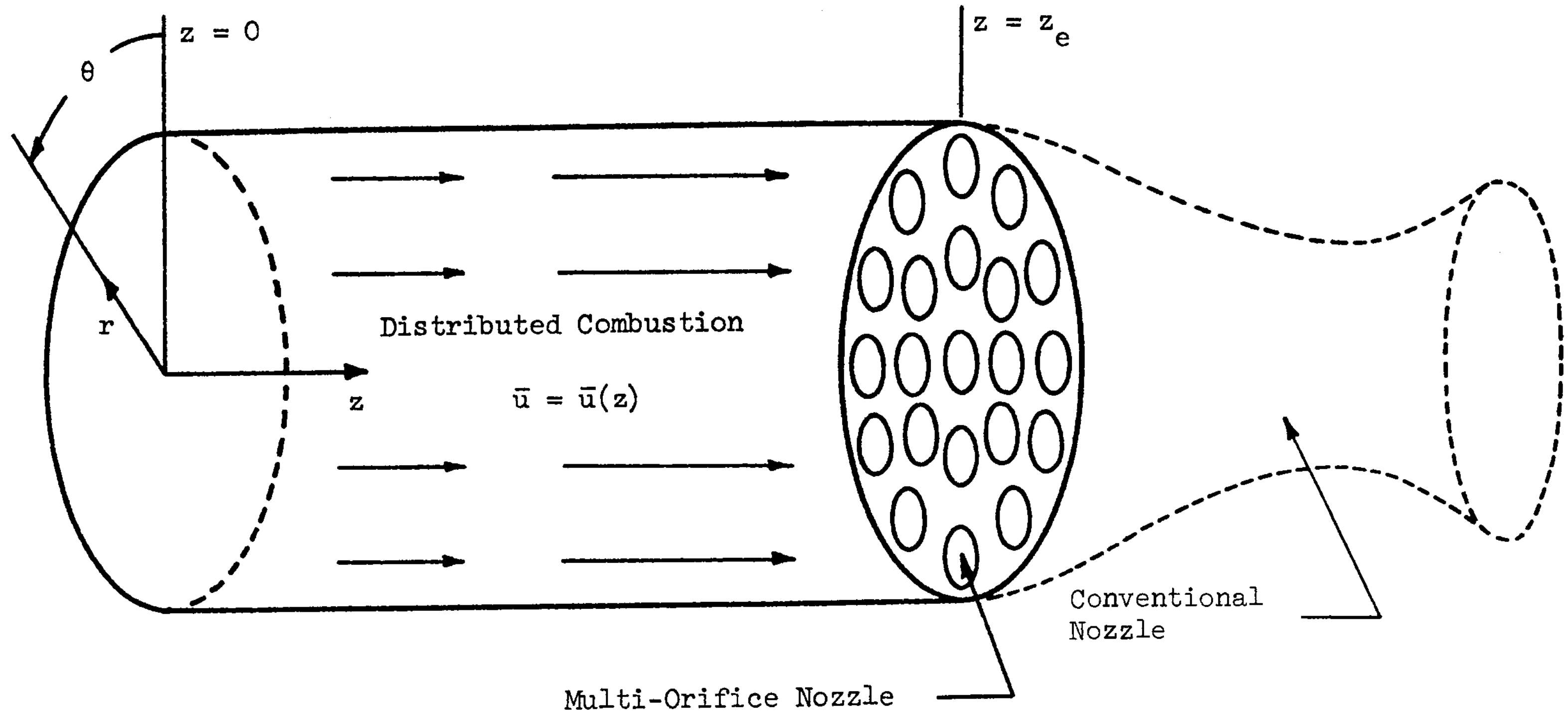


Figure 1. Combustor Configuration and Coordinate System.

approximating series expansion, $\tilde{\Phi}$. The question naturally arises as to what form of series expansion should be used. Inasmuch as the experimentally observed pressure oscillations during combustion instability usually resemble the natural acoustic modes of the chamber, the velocity potential, Φ, is expanded in terms of the natural acoustic modes of the chamber with unknown time-dependent amplitudes.

In previous analyses[3,6] of related problems the approximate solutions were expressed in terms of the acoustic modes for a cylindrical chamber with solid wall boundary conditions at both the injector and the nozzle ends. Consequently, the approximation of the flow conditions at the nozzle entrance was poor. In the present analysis a better approximation to the flow at the nozzle entrance is obtained by expanding the velocity potential in terms of the acoustic eigenfunctions for a chamber with a solid wall boundary condition at the injector end and a nozzle admittance condition at the other end. This removes both the two-dimensionality and the quasi-steady nozzle restrictions imposed upon the previous investigations.

The velocity potential, Φ, is therefore approximated by the following series expansion:

$$\tilde{\Phi} = \sum_{\ell}\sum_{m}\sum_{n}\left\{A_{\ell mn}(t)\ \sin m\theta + B_{\ell mn}(t)\ \cos m\theta\right\} Z_{\ell mn}(z)\ J_m(S_{mn}r) \tag{6}$$

where the A's and B's are unknown complex functions of time, and the Z's are the complex axial acoustic eigenfunctions. The complex form of the axial acoustic eigenfunctions is given by

$$Z_{\ell mn}(z) = \cosh(ib_{\ell mn}z) \tag{7}$$

where the $b_{\ell mn}$ are the axial acoustic eigenvalues which must satisfy the following transcendental equation:

$$b_{\ell mn}^2 \sin^2(b_{\ell mn}z_e) + \gamma^2 Y^2(S_{mn}^{\ 2} + b_{\ell mn}^2) \cos^2(b_{\ell mn}z_e) = 0 \tag{8}$$

Equations (7) and (8) are obtained by linearizing Eq. (5) and solving the resulting equation for the case of no mean flow or combustion (i.e., the acoustic case) subject to the boundary conditions specified in Eq. (2). Each term in the above expansion exactly satisfies the solid wall boundary conditions at the injector end (i.e., at z = 0) and at the chamber wall (i.e., at r = 1); however, due to the unknown time dependence of Eq. (6) the nozzle admittance condition imposed at $z = z_e$ is not exactly satisfied by the individual terms. Including both the sin mθ and cos mθ terms in the expansion for $\tilde{\Phi}$ allows for the possibility of either spinning or standing wave solutions.

In order to simplify the algebra involved in the application of the Method of Weighted Residuals, the development of the associated computer program, and the presentation of the results; the expansion of the velocity potential is written as a single summation as follows:

$$\tilde{\Phi} = \sum_{p=1}^{N} A_p(t) Z_p(z) \Theta_p(\theta) R_p(r) \tag{9}$$

where the A_p's are the unknown time-dependent amplitudes. In order to use Eq. (9) a correspondence must be established between the index, p, in Eq. (9) and the mode-numbers ℓ, m, and n in Eq. (6). Such a correspondence is given in Table 1 for a three mode series consisting of the spinning first tangential (1T) mode ($\ell = 0$, m = 1, n = 1), the spinning second tangential (2T) mode ($\ell = 0$, m = 2, n = 1), and the first radial (1R) mode ($\ell = 0$, m = 0, n = 1).

Table 1

Correspondence Between Eq. (6) and (9) for a Three-Mode Series

p	Mode	$\ell(p)$	m(p)	n(p)	A_p	Θ_p
1	1T	0	1	1	$A_{011}(t)$	sin θ
2	1T	0	1	1	$B_{011}(t)$	cos θ
3	2T	0	2	1	$A_{021}(t)$	sin 2θ
4	2T	0	2	1	$B_{021}(t)$	cos 2θ
5	1R	0	0	1	$B_{001}(t)$	1

Before proceeding with the analysis, the wave equation (i.e., Eq. (1)) must be modified for use with the assumed complex solution given by Equation (9). This modification is necessary because only the real part of the assumed solution is physically meaningful. It can easily be shown that if $\Phi = \varphi + i\Psi$ is a solution to Eq. (1), the real part, φ, is not a solution to Eq. (1).

This failure of φ to satisfy Eq. (1) is due to the presence of the nonlinear terms in this equation. It can also be shown, however, that a modified wave equation can be constructed for which the real part of its solution satisfies the original wave equation (i.e., Eq. (1)). This modified wave equation is given by:

$$
\begin{aligned}
E(\Phi) &= \nabla^2\Phi - \Phi_{tt} - 2\vec{\bar{V}} \cdot \nabla\Phi_t - \gamma(\nabla \cdot \vec{\bar{V}})\,\Phi_t - W_m' \\
&- \frac{1-i}{2}\left[\nabla\Phi\cdot\nabla\Phi_t + \nabla\Phi^*\nabla\Phi_t^*\right] - \frac{1+i}{2}\left[\nabla\Phi\cdot\nabla\Phi_t^* + \nabla\Phi^*\cdot\nabla\Phi_t\right] \\
&- \frac{\gamma-1}{4}\Big\{(1-i)\left[\Phi_t\nabla^2\Phi + \Phi_t^*\nabla^2\Phi^*\right] \\
&+ (1+i)\left[\Phi_t\nabla^2\Phi^* + \Phi_t^*\nabla^2\Phi\right]\Big\} = 0 \qquad (10)
\end{aligned}
$$

where Φ^* is the complex conjugate of Φ. The derivation of this equation is discussed in Appendix B. Thus, the Method of Weighted Residuals will be used to obtain approximate solutions to Eq. (10) (i.e., $\tilde{\Phi} = \tilde{\varphi} + i\tilde{\Psi}$) from which the real part, $\tilde{\varphi}$, will be taken as the approximate solution of Eq. (1).

In order to obtain a solution, the unknown time-dependent mode-amplitudes (i.e., $A_p(t)$) are determined by the following mathematical procedure. The assumed series expansion, $\tilde{\Phi}$, (i.e., Eq. (9)) is substituted into the wave equation (i.e., Eq. (10)) to form the equation residual, $E(\tilde{\Phi})$. Similarly, substituting the series expansion into the nozzle boundary condition (i.e., the last of Eq. (2)) yields the boundary residual, $B(\tilde{\Phi})$. In the event that these residuals are both identically zero, the solution is an exact solution. The residuals $E(\tilde{\Phi})$ and $B(\tilde{\Phi})$ represent the errors incurred by using the approximate solution, $\tilde{\Phi}$.

According to the modified version of the Method of Weighted Residuals, developed by the authors in Refs. 5 and 8, the residuals $E(\tilde{\Phi})$ and $B(\tilde{\Phi})$ must satisfy the following orthogonality conditions:

$$
\int_0^{z_e}\int_0^{2\pi}\int_0^1 E(\tilde{\Phi})Z_j^*(z)\Theta_j(\theta)R_j(r)\,r\,dr\,d\theta\,dz
$$

$$
-\int_0^{2\pi}\int_0^1 B(\Phi)Z_j^*(z_e)\Theta_j(\theta)R_j(r)\,r\,dr\,d\theta = 0 \qquad (11)
$$

$$j = 1, 2, \ldots N$$

where in the present study the complex conjugate of the axial eigenfunction, Z_j^*, is used in the weighting functions. The chosen weighting functions must correspond to the terms that appear in the assumed series solution; that is, Eq. (9).

Evaluating the spatial integrals in Eq. (11) yields the following system of N complex ordinary differential equations to be solved for the unknown complex amplitude functions, $A_p(t)$:

$$\sum_{p=1}^{N} \left\{ C_0(j,p) \frac{d^2 A_p}{dt^2} + C_1(j,p) A_p(t) + \left[C_2(j,p) - nC_3(j,p) \right] \frac{dA_p}{dt} \right.$$

$$\left. + nC_3(j,p) \frac{d[A_p(t - \bar{\tau})]}{dt} \right\} + \sum_{p=1}^{N} \sum_{q=1}^{N} \left\{ D_1(j,p,q)\, A_p \frac{dA_q}{dt} + D_2(j,p,q) A_p \frac{dA_q^*}{dt} \right.$$

$$\left. + D_3(j,p,q) A_p^* \frac{dA_q}{dt} + D_4(j,p,q) A_p^* \frac{dA_q^*}{dt} \right\} = 0 \qquad (12)$$

$$j = 1, 2, \ldots N$$

The coefficients appearing in the above equations are determined by evaluating the various integrals of hyperbolic, trigonometric, and Bessel functions that arise from the spatial integrations indicated in Eq. (11). A user's manual for the computer program COEFFS3D used to calculate these coefficients is given in Appendix C.

The time-dependent behavior of an engine following the introduction of a disturbance is determined by specifying the form of the initial disturbance and then following the subsequent behavior of the individual modes by numerically integrating Eqs. (12). Once the time-dependence of the individual modes is known, the velocity potential, $\tilde{\Phi}$, is calculated from Eq. (9). The pressure perturbation at any location within the chamber is related to the real part of $\tilde{\Phi}$ (i.e., $\tilde{\varphi}$) by the following second-order momentum equation (see Refs. 5 and 8):

$$\tilde{p}' = -\gamma[\tilde{\varphi}_t + \bar{u}(z)\tilde{\varphi}_z + \tfrac{1}{2}(\tilde{\varphi}_r^2 + \frac{1}{r^2}\tilde{\varphi}_\theta^2 + \tilde{\varphi}_z^2) - \tfrac{1}{2}\tilde{\varphi}_t^2] \qquad (13)$$

A user's manual for the computer program, LCYC3D, which obtains numerical solutions of Eqs. (12) and (13) is given in Appendix D.

In summary, the theory presented in this section represents a two-stage simplification of the original problem. In the first stage the problem has been reduced to the solution of a single nonlinear, partial differential equation (i.e., Eq. (1)). In the second stage the solution was expanded in a series of acoustic modes with time-dependent coefficients and the Method of Weighted Residuals was used to replace the solution of the nonlinear partial differential equation with the solution of a system of nonlinear, ordinary differential equations (i.e., Eq. (12)). Typical numerical solutions of these equations will be presented and discussed in the following section.

RESULTS AND DISCUSSION

The generalized three-dimensional theory introduced in the previous section has been used to obtain both linear and nonlinear data for pure transverse modes and pure longitudinal modes for rocket motors with conventional nozzles. Nonlinear data for the first tangential (1T) mode and the first longitudinal (1L) mode has also been obtained for combustors with quasi-steady nozzles for comparison with the results of the previous two-dimensional theories.

Linear Solutions

Before proceeding with the nonlinear analysis, it was desired to obtain numerical solutions of the linearized equations (i.e., Eqs. (12) with $D_1 = D_2 = D_3 = D_4 = 0$) in order to determine how closely the approximate solutions satisfied the nozzle boundary condition. The linear solution is also needed for comparison with the corresponding nonlinear results. The linear solutions were obtained by assuming a one-mode series expansion consisting only of the mode under consideration. Due to the presence of the retarded variables (i.e., $d[A_p(t - \bar{\tau})]/dt$) in Eqs. (12), it is necessary to specify the initial amplitudes over the interval $-\bar{\tau} \le t \le 0$. In this study the initial values were chosen such that the nozzle boundary condition was exactly satisfied during this initial time period. Solutions were obtained for values of n and $\bar{\tau}$ on the neutral stability limit (see Appendix E for the determination of neutral stability limits) for various conventional nozzle configurations. The nozzle admittance was expressed in the form, $Y = Ae^{i\varphi}$, where A is the amplitude factor and φ is the phase shift. The pressure perturbation, p', and the axial velocity perturbation, u', at the nozzle entrance were calculated numerically for several values of the nozzle phase shift, φ. These calculated values were then used to compute the ratios $(u'/p')_{z=z_e}$, which were then compared with the specified nozzle admittance values. These results are shown in Tables (2) and (3) where A_n and φ_n are the computed values of the amplitude factor and phase shift, respectively. These results show that the approximation to the nozzle boundary condition is very good for both the 1T and 1L modes; that is, the maximum error in the amplitude ratio is about 5% and the maximum error in phase is approximately 0.5 degree. These results are in contrast with previous theoretical investigations where the representation of the unsteady flow conditions in the vicinity of the nozzle entrance was very poor.

Table 2. 1T Mode Linear Solutions (Numerical).

A = 0.02			Error at Nozzle	
φ (Degrees)	$\bar{\tau}$	n	$\frac{A_n - A}{A}$	$\varphi_n - \varphi$ (Degrees)
0 0 0	1.2 1.7 2.2	0.66416 0.55001 0.64710	-.029 .003 .034	0.4 0.4 0.4
45 45 45	1.2 1.7 2.2	0.66137 0.54490 0.63665	-.031 .001 .032	0.5 0.5 0.5
90 90 90	1.2 1.7 2.2	0.62507 0.51252 0.59758	-.031 -.001 .028	0.3 0.3 0.3
135 135 135	1.2 1.7 2.2	0.57746 0.47274 0.55353	-.031 -.004 .023	-0.1 -0.1 -0.1
180 180 180	1.2 1.7 2.2	0.54825 0.45003 0.53121	-.030 -.004 .022	-0.4 -0.4 -0.4
225 225 225	1.2 1.7 2.2	0.55357 0.45677 0.54292	-.030 -.003 .024	-0.4 -0.4 -0.5

270 270 270	1.2 1.7 2.2	0.58854 0.48787 0.58090	-.029 .001 .028	-0.3 -0.3 -0.3
315 315 315	1.2 1.7 2.2	0.63362 0.52602 0.62368	-.029 .003 .032	0.1 0.1 0.1

Table 3. 1L Mode Linear Solutions (Numerical).

A = 0.02			Error at Nozzle	
φ (Degrees)	$\bar{\tau}$	n	$\frac{A_n - A}{A}$	$\varphi_n - \varphi$ (Degrees)
0 0 0	0.6 1.0 1.4	1.44680 1.01686 1.37491	-0.048 0.002 0.049	0.4 0.4 0.4
45 45 45	0.6 1.0 1.4	1.42414 0.99216 1.32746	-0.048 0.001 0.047	0.5 0.5 0.5
90 90 90	0.6 1.0 1.4	1.33681 0.92275 1.23131	-0.046 -0.001 0.042	0.3 0.3 0.3
135 135 135	0.6 1.0 1.4	1.23678 0.85007 1.14229	-0.044 -0.002 0.038	-0.1 -0.1 -0.1
180 180 180	0.6 1.0 1.4	1.18443 0.81682 1.11190	-0.043 -0.003 0.036	-0.4 -0.4 -0.4
225 225 225	0.6 1.0 1.4	1.20963 0.84176 1.15854	-0.043 -0.001 0.039	-0.5 -0.5 -0.5
270 270 270	0.6 1.0 1.4	1.29571 0.91003 1.25539	-0.044 0.001 0.044	-0.3 -0.3 -0.3
315 315 315	0.6 1.0 1.4	1.39320 0.98248 1.34523	-0.046 0.003 0.049	0.1 0.1 0.1

Nonlinear Solutions

Nonlinear solutions have been computed for both the 1T mode and the 1L mode. For the 1T mode calculations a three mode series expansion consisting of the 1T, 2T (second tangential), and 1R (first radial) modes was used. These are the same modes that were included in the series expansion used in the previous two-dimensional transverse instability studies.[5,6] In these studies it was shown that convergence was obtained with this three mode series; that is, the addition of higher transverse modes (i.e., 3T, 4T, etc.) to the basic series had little effect on the solution. The 1L mode computations were made using a series consisting of the first five longitudinal modes (i.e., 1L, 2L, 3L, 4L, and 5L). It has been shown by Lores and Zinn[3,4] that convergence is obtained with this five-mode series.

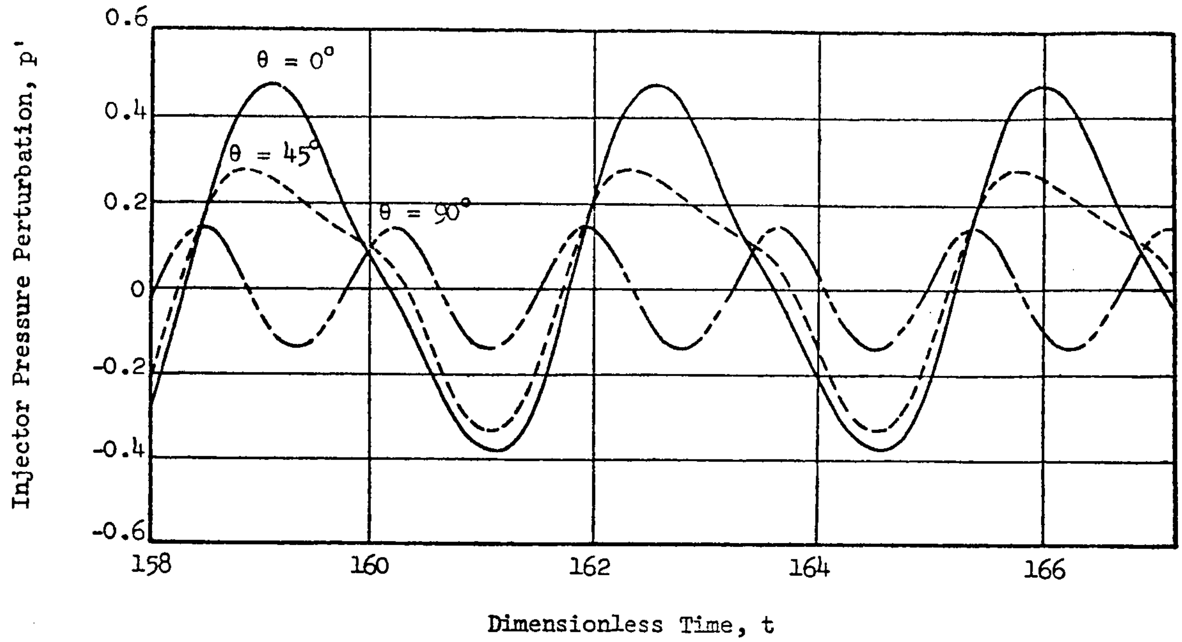
Transverse Mode Solutions. Nonlinear solutions have been computed for rocket combustors with quasi-steady nozzles (i.e., real admittances) and also for nozzles with complex admittances. The quasi-steady nozzle solutions were generated for comparison with the results of the previous two-dimensional theory.[5] For this case the nozzle admittance is given by:[13]

$$Y_r = \frac{\gamma - 1}{2\gamma}\bar{u}_e$$

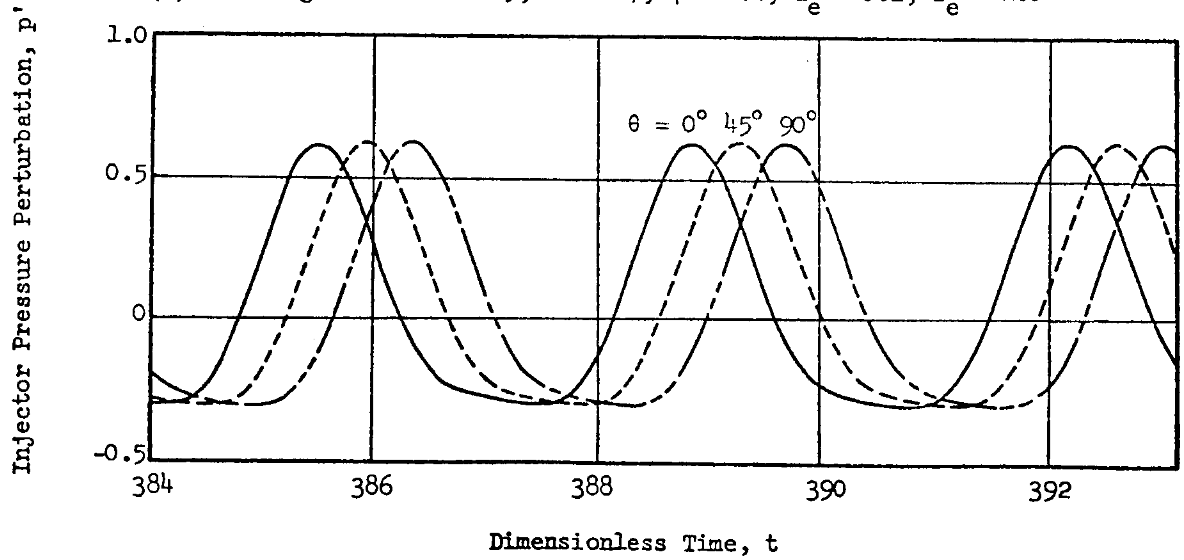
$$Y_i = 0 \qquad (14)$$

For nozzles with complex admittances the admittance was expressed in the form, $Y = Ae^{i\varphi}$. For both cases limit-cycle amplitudes and waveforms have been computed for both standing and spinning first tangential instability. This required three series terms to describe standing instability and five series terms to describe spinning instability. Typical computation times on a Univac 1108 computer to reach a limit-cycle were one minute for a standing wave and two minutes for a spinning wave.

Wall pressure waveforms ($r = 1$) were computed at the injector face ($z = 0$) and at the nozzle entrance ($z = z_e$) for three azimuthal locations, $\theta = 0^o$, $\theta = 45^o$, and $\theta = 90^o$. The initial conditions for standing waves were chosen such that a pressure anti-node occurred at $\theta = 0^o$. Injector pressure waveforms for both standing and spinning instability are shown in Fig. 2 for combustors with quasi-steady nozzles. These waveforms exhibit sharp peaks



(a) Standing Wave: $n = 0.65$, $\bar{\tau} = 1.7$, $\gamma = 1.2$, $\bar{u}_e = 0.2$, $z_e = 1.0$



(b) Spinning Wave: $n = 0.58$, $\bar{\tau} = 1.7$, $\gamma = 1.2$, $\bar{u}_e = 0.2$, $z_e = 1.0$

Figure 2. Nonlinear Pressure Waveforms for the 1T Mode.

and shallow minima; they are nearly identical in shape to those calculated using the previous two-dimensional theory.[5,6] Comparison of injector and nozzle pressure waveforms ($\theta = 0^{\circ}$) shows that there is very little variation in pressure with axial position. These waveforms are in qualitative agreement with the results of pressure measurements taken during test firings of unstable rocket motors.[1]

To check the accuracy of the approximation of the nozzle boundary condition, wall pressure and axial velocity waveforms were calculated at the nozzle entrance. The error at the nozzle boundary ($z = z_e$) is shown for nonlinear standing and spinning 1T mode instabilities in Fig. 3. Here the axial velocity perturbation, u', and the product of the quasi-steady nozzle admittance and the pressure perturbation, $Y_r p'$ are plotted as a function of time. The latter quantity is the axial velocity perturbation that would be obtained at the nozzle entrance if the nozzle boundary condition were exactly satisfied (i.e., the nozzle admittance condition requires that $u' = Y_r p'$ at $z = z_e$). Most of the discrepancy between the two curves is due to a slight phase shift between pressure and velocity and the second harmonic distortion of the pressure waveform resulting from the nonlinearities of the system. The nozzle boundary condition is satisfied in an average sense, however, for the ratio of the velocity amplitude (peak-to-peak) to pressure amplitude (peak-to-peak) is very close to the required value, Y_r.

In another study, limit-cycle amplitudes were calculated as a function of n and $\bar{\tau}$ for standing 1T mode instability. Values of n in the linearly unstable region were chosen for below resonant ($\bar{\tau} = 1.9$), resonant ($\bar{\tau} = 1.706$), and above resonant ($\bar{\tau} = 1.5$) conditions. The resulting amplitudes are compared with those obtained with the two-dimensional theory in Fig. 4. This figure shows that the three-dimensional theory predicts a slightly higher limit-cycle amplitude than the two-dimensional theory for chambers with quasi-steady nozzles.

Figure 4 also shows that the three-dimensional theory, like the previous two-dimensional one, cannot predict triggering of 1T mode instability by the introduction of finite amplitude disturbances. This result was expected since it was shown in Refs. 6 and 8 that the second order (i.e., $O(\epsilon^2)$) theory can predict triggering only for pure radial modes ($m = 0$, $n = 1, 2 \ldots$). Such triggering limits for the 1R mode are discussed in Ref. 9. It has also been

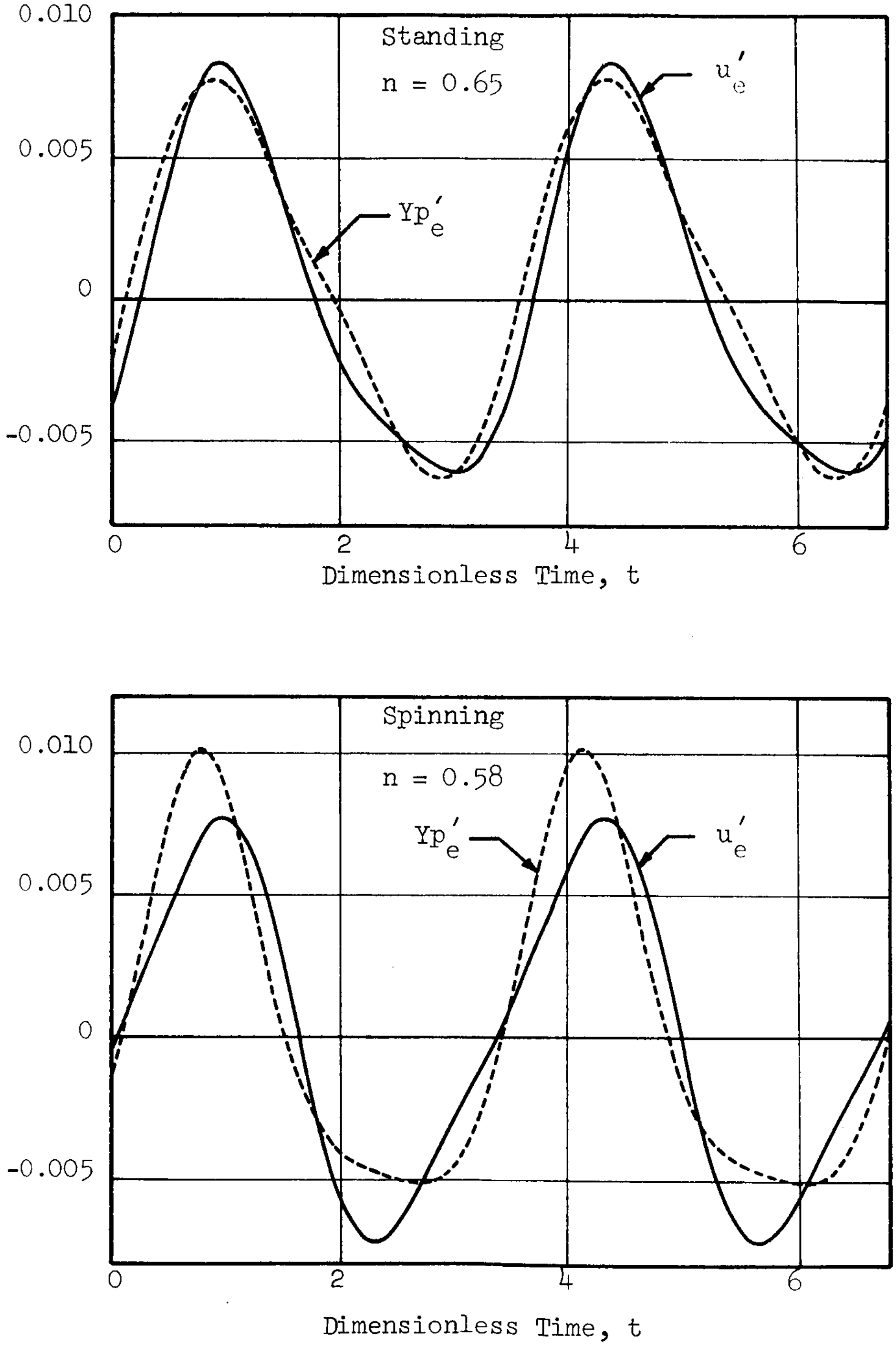


Figure 3. Nozzle Boundary Condition for Nonlinear 1T Mode Solutions for Quasi-Steady Nozzles.

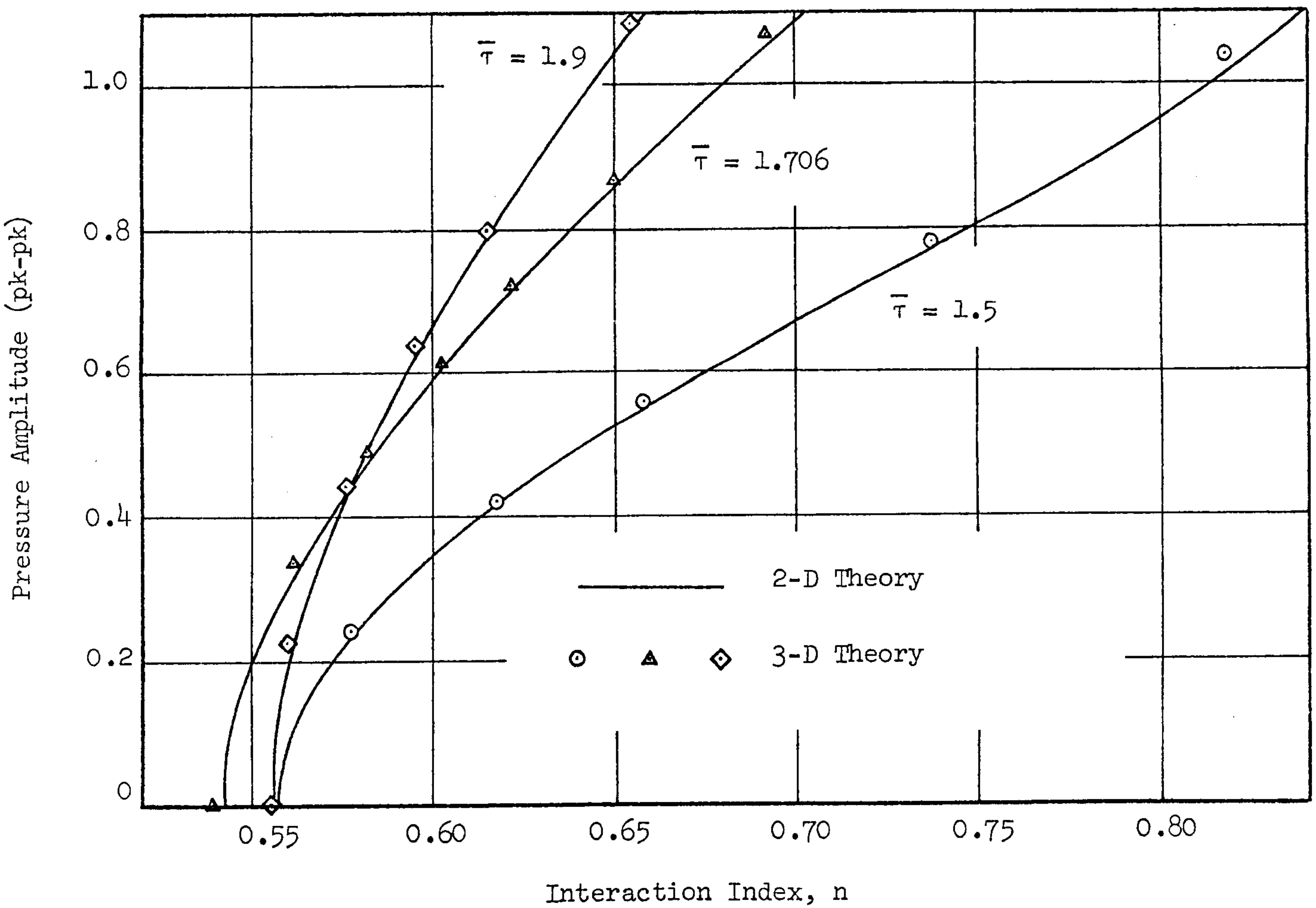


Figure 4. Limit-Cycle Amplitudes for the 1T Mode.

shown, however, that triggering of 1T mode instability can be described when the $O(\epsilon^3)$ terms are retained in the analysis.[8,14] The third order theory given in Refs. 8 and 14 is limited to a single mode in the approximating series expansions. A more general multi-mode third-order theory is now under development and the results will be presented in a future publication. It is also suspected that nonlinear unsteady combustion effects (not included in the present analysis) may play an important role in the triggering phenomenon.

For nozzles with complex admittances a study was conducted to determine the effect of the nozzle phase shift, φ, upon the limit-cycle amplitudes and waveforms for both standing and spinning 1T mode instability. The effect of nozzle phase shift on the nonlinear pressure and velocity waveforms at the nozzle entrance plane is shown in Fig. 5 for spinning waves. This figure shows that, while φ has little or no effect on the pressure waveforms, the phase and shape of the velocity waveforms is strongly dependent on φ. The effect of φ on the limit-cycle amplitude for standing 1T mode instability is shown in Fig. 6. For a given value of n and $\bar{\tau}$ (in the linearly unstable region for the 1T mode), Fig 6 shows a sinusoidal variation of limit-cycle amplitude with φ having a maximum amplitude at about $\varphi = 200^{\circ}$ and a minimum amplitude at about $\varphi = 20^{\circ}$. In this connection, it should be pointed out that according to linear results nozzle damping is a maximum at $\varphi = 0^{\circ}$ and a minimum at $\varphi = 180^{\circ}$; thus the observed shifts must be due to nonlinearities.

In order to determine how well the solutions approximate the nozzle boundary condition, the amplitude ratio and phase shift between pressure and velocity at the nozzle entrance have been calculated from the nonlinear solutions and have been compared with the specified nozzle admittance condition. Since the waveforms are non-sinusoidal, an approximate amplitude ratio, A_c, was calculated by taking the ratio of peak-to-peak velocity amplitude to peak-to-peak pressure amplitude. The approximate phase shift, φ_c was calculated from the following formula:

$$\varphi_c = \left[\frac{t_p - t_u}{T}\right] \times 360 \qquad (15)$$

where t_p is the average of an ascending zero-crossing and the following descending zero-crossing for the pressure perturbation, t_u is a similar average

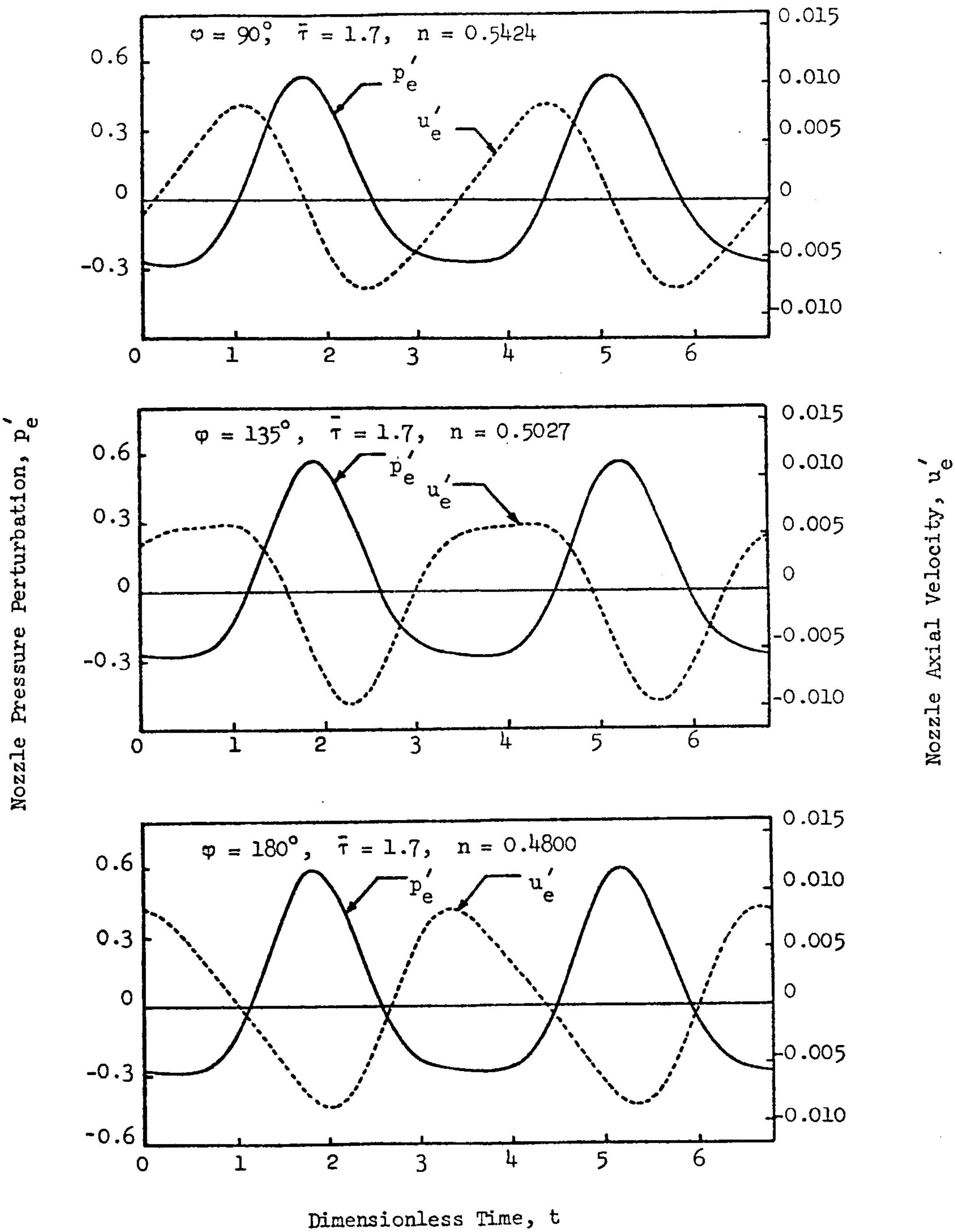


Figure 5. Effect of Nozzle Phase Shift, φ, on Nozzle Waveforms for Spinning 1T Modes.

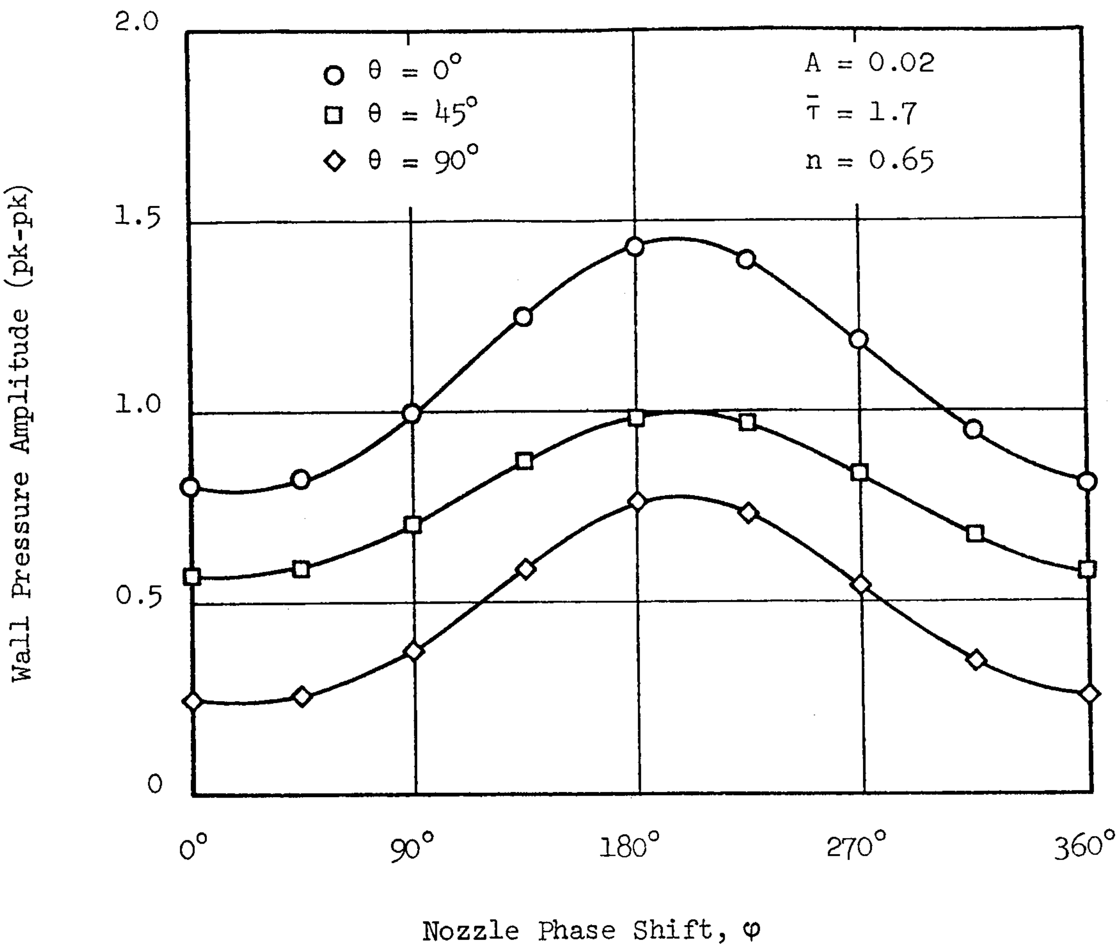


Figure 6. Effect of Nozzle Phase Shift on Limit-Cycle Amplitudes for the Standing 1T Mode.

for the velocity perturbation, and T is the period of oscillation. The results of this study are shown in Fig. 7 for both standing and spinning waves. For standing waves the calculated amplitude ratios are seen to be consistently higher than required by the nozzle admittance condition (dashed line), while for spinning waves the calculated amplitude ratios are lower than required. For both standing and spinning waves the calculated phase shifts are in excellent agreement with the imposed phase shifts. This study shows that the three-dimensional theory provides a good approximation to the nozzle boundary condition for the 1T mode, considering that the nonlinear solutions are being forced to satisfy a linear boundary condition.

Longitudinal Mode Solutions. Letting m and n equal zero in Eq. (6) and using a series consisting of the first five longitudinal modes (i.e., ℓ = 1, 2, ... 5), limit-cycle solutions were calculated for quasi-steady nozzles as well as for nozzles with complex admittances. The longitudinal mode solutions required somewhat longer computation times than the transverse mode solutions; the time required to reach a limit cycle was from three to four minutes on the Univac 1108 computer.

Longitudinal mode solutions for chambers with quasi-steady nozzles were compared with the solutions previously obtained by Lores and Zinn[3,4] using a one-dimensional theory. Pressure waveforms at the injector face are compared for both resonant and off-resonant conditions in Fig. 8 which shows excellent agreement between the two theories. Pressure and velocity waveforms at the nozzle entrance as well as injector face pressure waveforms are shown in Fig. 9 for quasi-steady nozzles, while Fig. 10 shows waveforms at the nozzle entrance for nozzles with complex admittance ($\varphi = 45^{\circ}$ and $\varphi = 90^{\circ}$). In each case the results indicate the presence of a steep-fronted pressure wave moving back and forth in the chamber. This behavior is in agreement with experimental observations of axial instabilities.[2] The relation between pressure and velocity waveforms at the nozzle entrance is a fairly good approximation to the nozzle admittance condition (see Figs. 9 and 10) in spite of the highly nonlinear waveforms. The results of this investigation indicate that the three-dimensional nonlinear theory is applicable to longitudinal instabilities as well as transverse instabilities. The theory can also be used to investigate the nonlinear behavior of combined

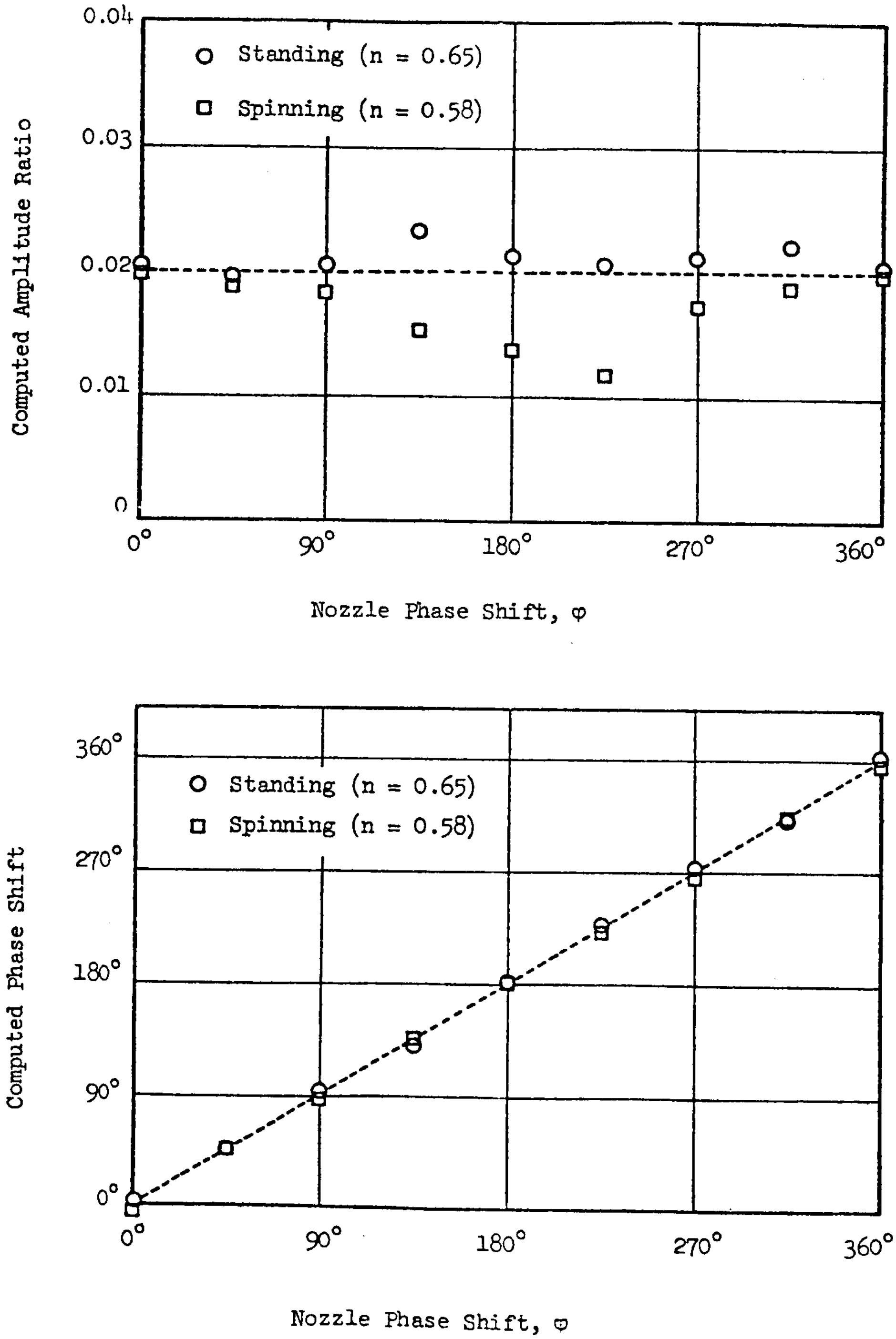


Figure 7. Nozzle Boundary Condition for Nonlinear 1T Mode Solutions.

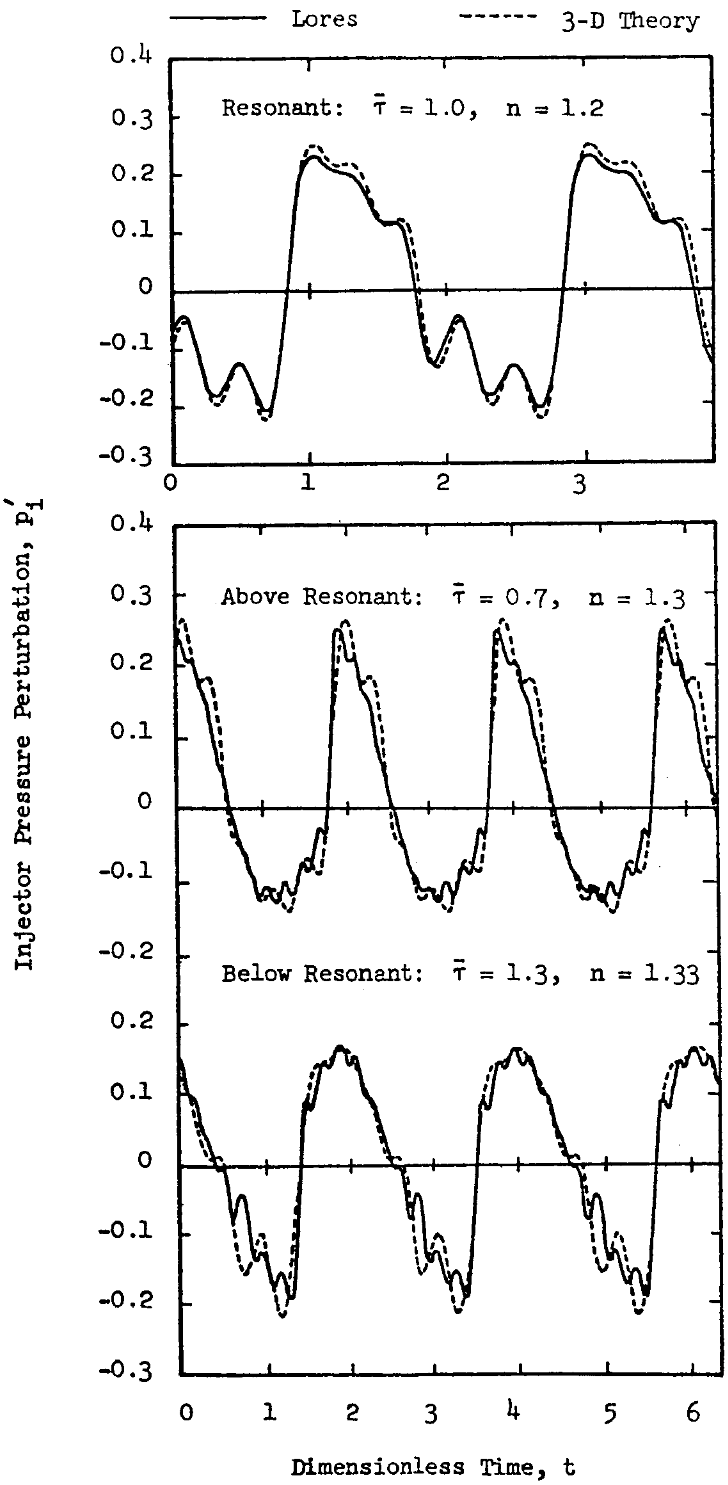


Figure 8. Comparison of Nonlinear 1L Mode Solutions for Quasi-Steady Nozzles.

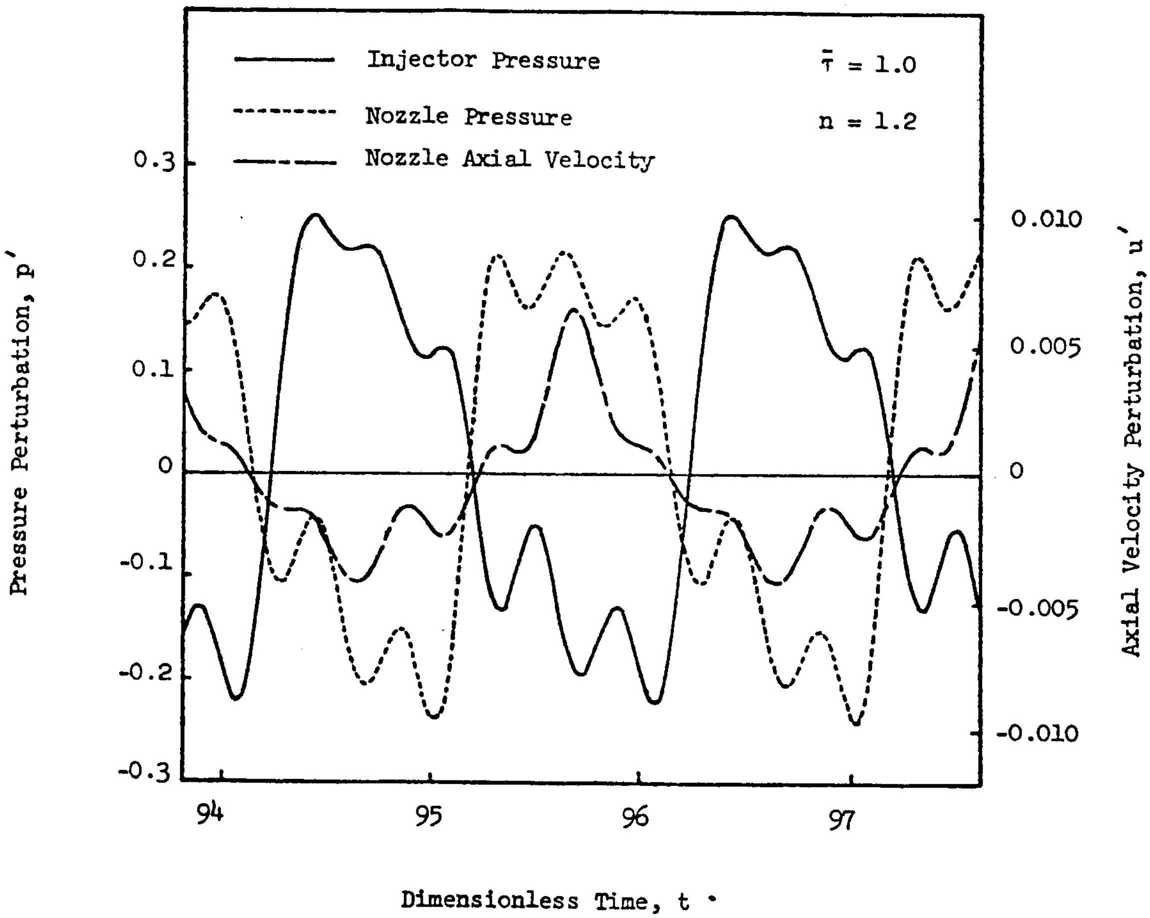


Figure 9. Longitudinal Mode Waveforms for Quasi-Steady Nozzles.

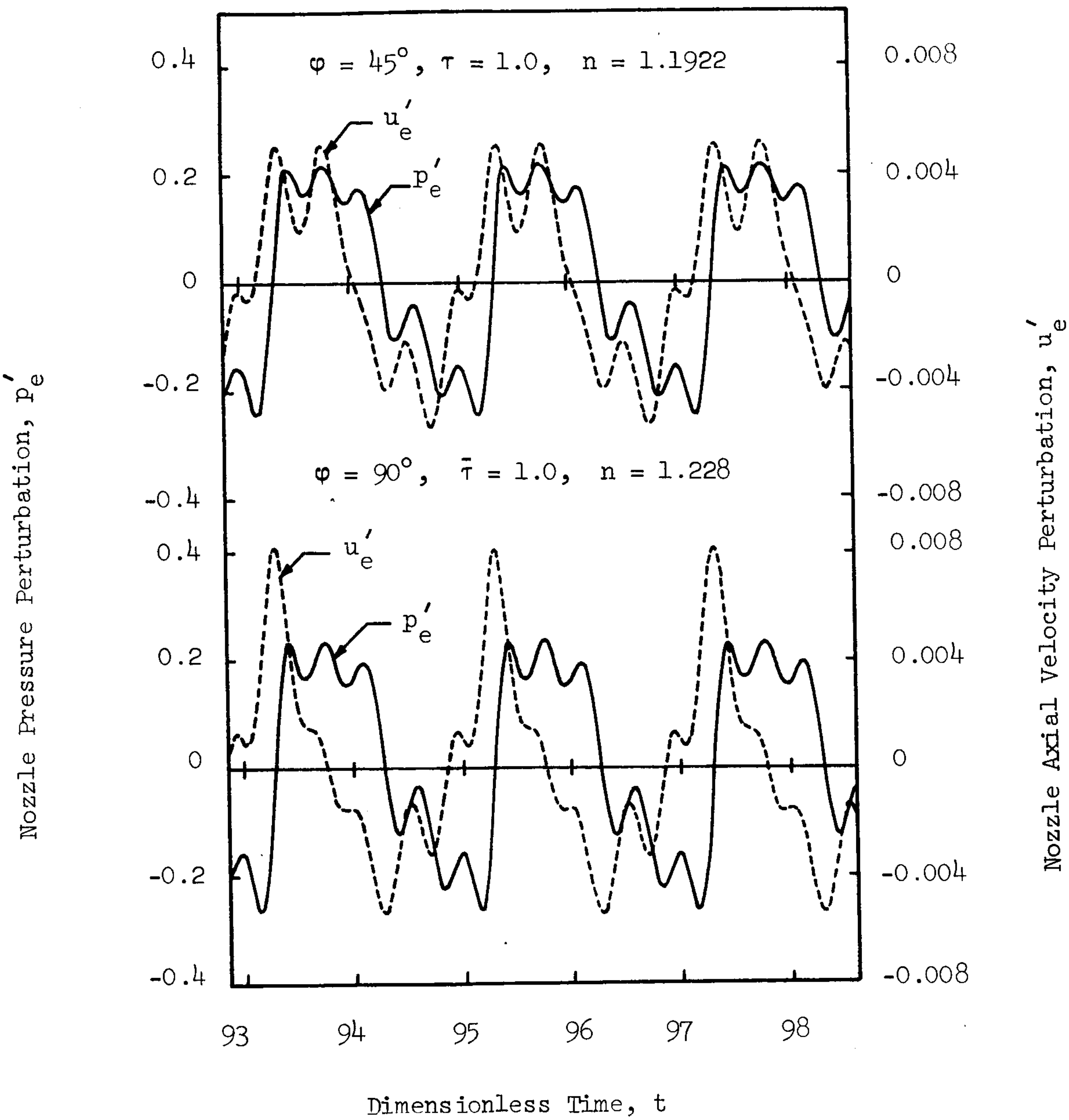


Figure 10. Longitudinal Mode Waveforms for Nozzles with Complex Admittances.

longitudinal-transverse instabilities, although no results for instabilities of this type are presented in this report.

CONCLUDING REMARKS

A general three-dimensional second-order nonlinear theory has been developed for predicting the linear and nonlinear behavior of combustion instability in liquid-propellant rocket combustors. This theory contains previous analyses of transverse and longitudinal instabilities as special cases. Furthermore it extends the previous analyses which were applicable only to combustors with quasi-steady nozzles, to the more practical cases of combustors with conventional DeLaval nozzles. The present theory can be used to predict the stability characteristics of longitudinal, transverse and combined longitudinal-transverse modes for various liquid-propellant rocket motor designs.

Results obtained for combustors with quasi-steady nozzles are in excellent agreement with the predictions of previous theories for both transverse and longitudinal instabilities. For combustors with conventional nozzles the limit-cycle amplitude varies sinusoidally with nozzle phase shift, φ, having a maximum value at $\varphi = 200^{\circ}$ and a minimum value at $\varphi = 20^{\circ}$. The nozzle phase shift has a strong effect on the axial velocity waveforms at the nozzle entrance while having only a minor influence on the nonlinear pressure waveforms. In both cases, the nonlinear theory developed in this paper provides a good approximation to the unsteady flow conditions at the nozzle entrance plane. This is in contrast to the previous theories which provided a relatively poor approximation to the nozzle boundary condition.

The results presented in this report establish the relationship that exists between the resulting instability (i.e., waveform, final amplitude, and final frequency), the combustion parameters (i.e., interaction index, n, and time-lag $\bar{\tau}$), and the chamber Mach number and length-to-diameter ratio. These results indicate that the limit-cycle amplitude increases with increasing sensitivity of the combustion process to pressure oscillations. For transverse instabilities, calculated pressure waveforms exhibit sharp peaks and shallow minima, and the frequency of oscillation is always within a few percent of the frequency of one of the chamber's acoustic modes. For axial instabilities, the theory predicts the presence of a steep-fronted wave moving back and forth along the combustor. In both cases the calculated pressure waveforms are in good qualitative agreement with available experimental data.

APPENDIX A

MOMENTUM INTERCHANGE BETWEEN LIQUID AND GAS PHASES

The results presented in this report were obtained under the assumption that the momentum interchange between the liquid droplets and the burned gases is negligible. This assumption will now be relaxed for the special case of uniformly distributed combustion, and it will be shown that this momentum interchange is an important stabilizing effect.

Analysis

The momentum equation for two-phase flow was derived in Ref. 8 and is given by:

$$\rho\left[\frac{\partial \vec{V}}{\partial t} + \vec{V}\cdot\nabla\vec{V}\right] + \frac{1}{\gamma}\nabla p = -(\vec{V} - \vec{V}_L)(C + W_m) \qquad \text{(A-1)}$$

where $\vec{V}$ and $\vec{V}_L$ are the gas and liquid velocity, respectively. The term on the right-hand-side of Eq. (A-1) represents a momentum source to the gas produced by the burning liquid drops. This momentum source consists of two parts: (1) the force necessary to accelerate the evolved gases from the droplet velocity to the gas velocity (i.e., the term $-W_m(\vec{V} - \vec{V}_L)$) and (2) the aerodynamic drag of the droplets (i.e., the term $-C(\vec{V} - \vec{V}_L)$).

In order to derive a wave equation for the velocity potential Φ it is necessary to make the following assumptions: (1) the drag term is negligible compared with the acceleration term, (2) liquid velocity fluctuations are negligible, and (3) the combustion is uniformly distributed throughout the chamber. Neglecting the drag term, perturbing, and neglecting third order quantities gives the following expression for the momentum source perturbation, $\vec{M}'$:

$$\vec{M}' = -(\vec{V}' - \vec{V}_L')\overline{W}_m \qquad \text{(A-2)}$$

This is simplified further by neglecting the liquid velocity perturbation, introducing the velocity potential, and using the steady-state relation, $\overline{W}_m = d\bar{u}/dz$, to obtain:

$$\underset{\rightarrow}{M}' = - \frac{d\bar{u}}{dz} \nabla\Phi \tag{A-3}$$

Finally, the assumption of uniformly distributed combustion gives $d\bar{u}/dz$ = constant which yields:

$$\underset{\rightarrow}{M}' = - \nabla\left[\frac{d\bar{u}}{dz} \Phi \right] \tag{A-4}$$

Perturbing the left-hand-side of Eq. (A-1), introducing the velocity potential, and combining with Eq. (A-4) gives:

$$\nabla\left[\frac{\partial\Phi}{\partial t} + \frac{1}{\gamma}p' + \bar{u}\Phi_z + \frac{d\bar{u}}{dz}\Phi + \frac{1}{2} \nabla\Phi\cdot\nabla\Phi - \frac{1}{2}\Phi_t^2 \right] = 0 \tag{A-5}$$

which can be integrated to obtain:

$$p' = -\gamma\left[\Phi_t + \bar{u}\Phi_z + \frac{d\bar{u}}{dz}\Phi + \frac{1}{2} \nabla\Phi\cdot\nabla\Phi - \frac{1}{2}\Phi_t^2 \right] \tag{A-6}$$

Equation (A-6) is similar to Eq. (13), where the additional term $(d\bar{u}/dz)\Phi$ arises from the droplet momentum source. Following the procedure outlined in Ref. 8, the momentum equation given by Eq. (A-6) is combined with the continuity and energy equations to obtain the desired wave equation:

$$\nabla^2\Phi - \Phi_{tt} = 2\bar{u}\Phi_{zt} + (\gamma + 1)\frac{d\bar{u}}{dz}\Phi_t + 2\nabla\Phi\cdot\nabla\Phi_t + (\gamma - 1)\Phi_t\nabla^2\Phi + W_m' \tag{A-7}$$

Comparing Eq. (A-7) with Eq. (1) shows that the droplet momentum source appears only in the second term on the right-hand-side of this equation, where the factor γ in Eq. (1) becomes $(\gamma + 1)$ in Eq. (A-7).

Applying the Method of Weighted Residuals to obtain approximate solutions to Eq. (A-7) yields a set of ordinary differential equations identical to Eqs. (12) where the coefficient $C_2(j,p)$ is now given by:

$$C_2(j,p) = \left\{2\int_0^{z_e} \bar{u}(z)Z_p'Z_j^* dz + (\gamma + 1)\int_0^{z_e} \frac{d\bar{u}}{dz}Z_pZ_j^* dz + \gamma Y Z_p(z_e)Z_j^*(z_e)\right\} \times$$

$$\times \int_0^{2\pi} \Theta_p\Theta_j d\theta \int_0^1 R_pR_j r dr \qquad \text{(A-8)}$$

Equation (A-8) is readily obtained from Eq. (C-3) by replacing γ in the second term by $\gamma + 1$.

Linear Stability Limits

Linear stability limits for the 1L mode were calculated by the method described in Appendix E for the following two cases: (1) the droplet momentum source was included in the analysis and (2) the droplet momentum source was neglected. The results were compared with the linear stability limit calculated by Mitchell[15] on a plot of interaction index, n, versus stretched time-lag, μ, where $\mu = \omega\bar{\tau}/\pi$ (see Fig. A-1). This figure shows excellent agreement between the results of Mitchell (solid curve) and the present theory (circle symbols) when the droplet momentum source is included. Neglecting the droplet momentum source shifts the stability curve to much lower values of n (dashed curve), which indicates that the droplet momentum source is an important stabilizing effect.

Nonlinear Solutions

In the second-order analysis presented in this report, the droplet momentum source affects the nonlinear solutions primarily by increasing the linear stability of the system. This is readily shown in Fig. (A-2) where the limit-cycle amplitude is plotted as a function of the displacement, δn, above the neutral stability limit. By plotting the limit-cycle amplitudes in this manner,

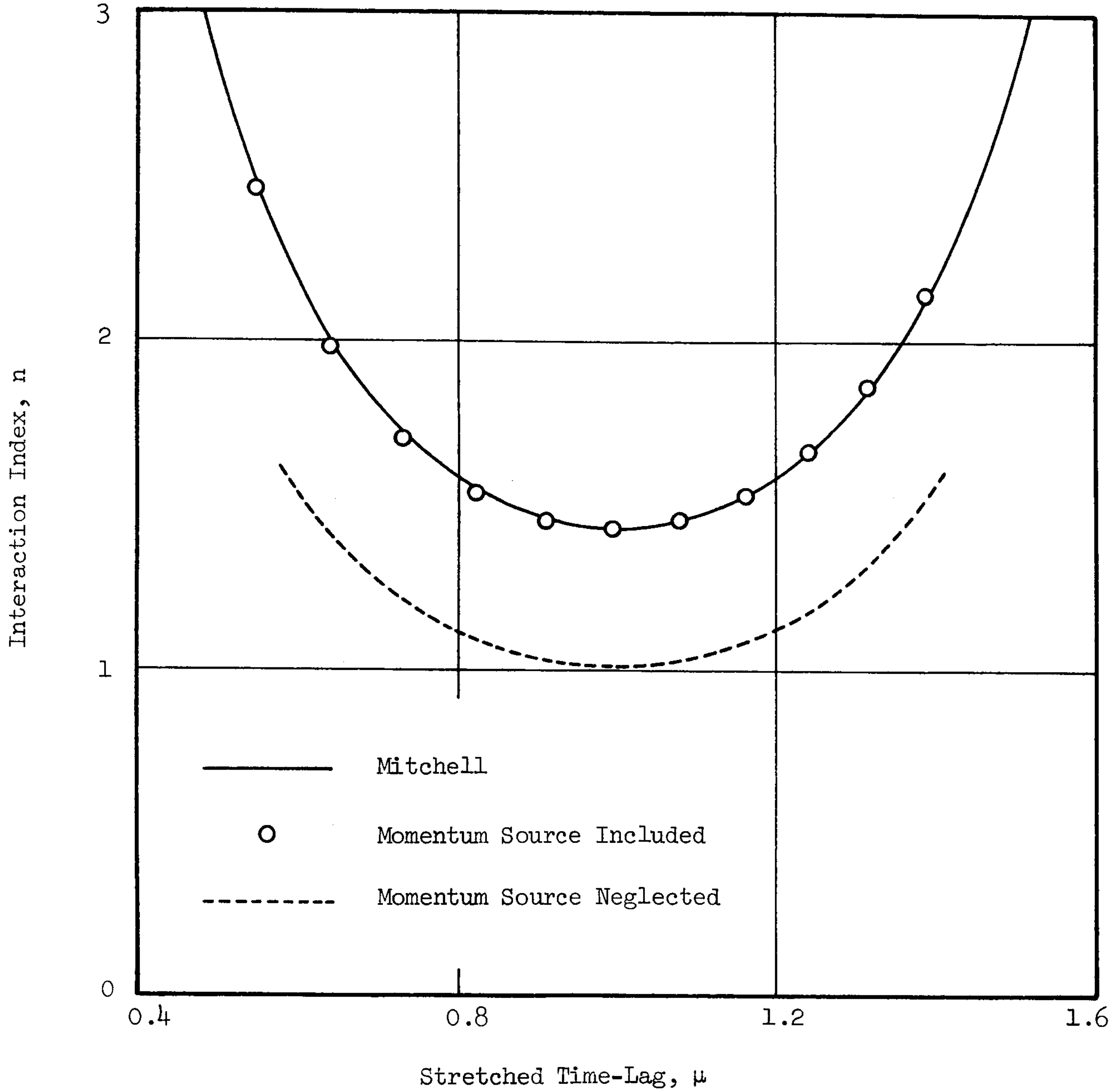


Figure A-1. Effect of Droplet Momentum Source on Linear Stability Limits for the 1L Mode.

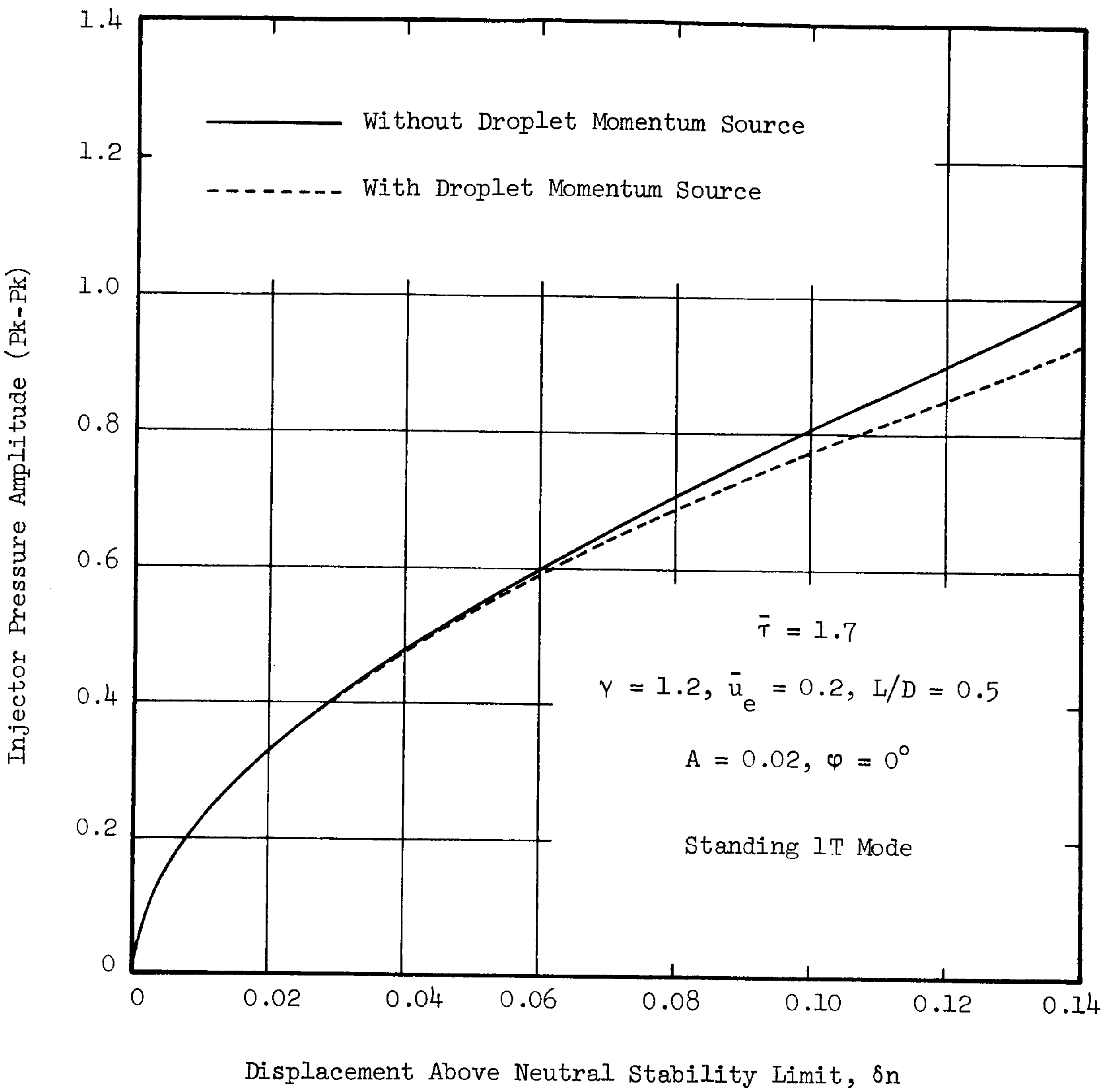


Figure A-2. Effect of Droplet Momentum Source on Limit-Cycle Amplitude.

the effect of the shift in the neutral stability curves is removed so that only the nonlinear effect of the momentum source is seen. Figure A-2 shows that, for equal displacements above the neutral stability limits, including the droplet momentum source results in a slightly smaller limit-cycle amplitude. This difference in limit-cycle amplitude is negligible for most practical purposes.

For combustors with uniformly distributed combustion it has been shown that the droplet momentum source is an important effect which is easily incorporated into the present analysis. Consequently the computer programs based on this theory include the droplet momentum source as an optional feature (see Appendices C, D, and E).

For chambers with non-uniform combustion distributions, Eqs. (A-6) and (A-7) are no longer applicable; however, the droplet momentum source can be taken into account in the following manner. Using the present theory with the droplet momentum source omitted, the neutral stability limit, $n_1(\bar{\tau})$, is calculated and the limit-cycle amplitudes are determined as a function of δn as in Fig. A-2. In addition, the linear stability limit, $n_2(\bar{\tau})$, is calculated using a linear theory which includes the droplet momentum source and is not restricted to uniformly distributed combustion (such as in Ref. (15)). Assuming that the nonlinear effect of the droplet momentum source is also small for non-uniformly distributed combustion and using the values of δn and $n_2(\bar{\tau})$ calculated above, the desired plot of limit-cycle amplitude as a function of n is readily obtained.

APPENDIX B

USE OF COMPLEX VARIABLES IN THE SOLUTION OF NONLINEAR DIFFERENTIAL EQUATIONS

It is often convenient to use complex variables in the solution of the linear equations which arise in acoustics, combustion instability and related fields. In this case the solution is expressed in complex form, and the real part represents the physically meaningful solution. However, care must be used when applying this technique in the solution of nonlinear equations. The difficulties that are encountered in applying the complex variable technique to nonlinear problems will be illustrated by analyzing the following simplified example. Consider the nonlinear wave equation given by:

$$\nabla^2 \Phi - \Phi_{tt} = \Phi \Phi_t \qquad \text{(B-1)}$$

A complex solution of Eq. (B-1) of the form $\Phi = \varphi + i\Psi$ would be useful only if its real part, φ, satisfies Eq. (B-1), which would be the case if the equation were linear. However, straightforward substitution of $\Phi = \varphi + i\Psi$ into Eq. (B-1) and separating its real and imaginary parts yields the following equation for φ :

$$\nabla^2 \varphi - \varphi_{tt} = \varphi \varphi_t - \Psi \Psi_t \qquad \text{(B-2)}$$

indicating that the real part, φ, does not satisfy Eq. (B-1) because of the extra term, $-\Psi\Psi_t$, appearing on the right hand side. In order to eliminate this extra term, the form of the original differential equation (i.e., Eq. (B-1)) must be modified.

Since Eq. (B-1) supposedly describes some physical phenomenon, and since only the real part of the complex solution is physically meaningful, then the nonlinear term $\Phi\Phi_t$ should really be expressed as the product Re(Φ) X Re(Φ_t) which is equivalent to $(\Phi\Phi_t + \Phi\Phi_t^* + \Phi^*\Phi_t + \Phi^*\Phi_t^*)/4$. Substituting this expression into Eq. (B-1) yields:

$$\nabla^2\Phi - \Phi_{tt} = \tfrac{1}{4}[\Phi\Phi_t + \Phi\Phi_t^* + \Phi^*\Phi_t + \Phi^*\Phi_t^*] \tag{B-3}$$

Substituting $\Phi = \varphi + i\Psi$ into Eq. (B-3) and separating its real and imaginary parts yield:

$$\begin{aligned} \nabla^2\varphi - \varphi_{tt} &= \varphi\varphi_t \\ \nabla^2\Psi - \Psi_{tt} &= 0 \end{aligned} \tag{B-4}$$

which shows that the real part of the solution of Eq. (B-3) satisfies the desired equation (i.e., Eq. (B-1)) and the imaginary part satisfies a homogeneous linear wave equation. This technique was applied to the solution of nonlinear combustion instability problems (i.e., to Eq. (1)), and the resulting modified wave equation was solved using the Method of Weighted Residuals. Due to the approximate nature of the Method of Weighted Residuals, however, the resulting solution contained an error term which grew without limit. Consequently, the above procedure had to be modified in order to obtain satisfactory solutions of Eq. (1) using the Method of Weighted Residuals.

An alternate technique is to modify Eq. (B-1) such that both the real and imaginary parts satisfy the original equation. This can be done by replacing terms of the form $\Phi\Phi_t$ with $\mathrm{Re}(\Phi)\mathrm{Re}(\Phi_t) + i\mathrm{Im}(\Phi)\mathrm{Im}(\Phi_t)$; using the relations:

$$\begin{aligned} \mathrm{Re}(\Phi)\mathrm{Re}(\Phi_t) &= \left(\frac{\Phi + \Phi^*}{2}\right)\left(\frac{\Phi_t + \Phi_t^*}{2}\right) = \tfrac{1}{4}\left[\Phi\Phi_t + \Phi\Phi_t^* + \Phi^*\Phi_t + \Phi^*\Phi_t^*\right] \\ i\mathrm{Im}(\Phi)\mathrm{Im}(\Phi_t) &= -i\left(\frac{\Phi - \Phi^*}{2}\right)\left(\frac{\Phi_t - \Phi_t^*}{2}\right) = -\tfrac{1}{4}\left[\Phi\Phi_t - \Phi\Phi_t^* - \Phi^*\Phi_t + \Phi^*\Phi_t^*\right] \end{aligned} \tag{B-5}$$

in Eq. (B-1) gives:

$$\nabla^2\Phi - \Phi_{tt} = \tfrac{1}{4}\left[(1 - i)(\Phi\Phi_t + \Phi^*\Phi_t^*) + (1 + i)(\Phi\Phi_t^* + \Phi^*\Phi_t)\right] \tag{B-6}$$

Substituting $\Phi = \varphi + i\Psi$ into Eq. (B-6) and separating into its real and imaginary parts gives:

$$\nabla^2\varphi - \varphi_{tt} = \varphi\varphi_t$$
$$\nabla^2\Psi - \Psi_{tt} = \Psi\Psi_t \qquad \text{(B-7)}$$

which shows that both φ and ψ satisfy Eq. (B-1). Applying this method to the solution of Eq. (1) yields the modified wave equation (i.e., Eq. (10)) used in the present investigation.

APPENDIX C

PROGRAM COEFFS3D: A USER'S MANUAL

Statement of the Problem

Program COEFFS3D calculates the coefficients of both the linear and non-linear terms which appear in Eqs. (12). These coefficients are required as input for Program LCYC3D (see Appendix D) which numerically integrates this system of equations. The coefficients that are required depend on the choice of terms to be included in the series solution for $\tilde{\Phi}$ (see Eq. (9)), therefore this information must be provided as input to Program COEFFS3D. The output of Program COEFFS3D is either punched onto cards or stored on drum (FASTRAND) for input to Program LCYC3D.

The coefficients to be calculated are functions of various integrals of hyperbolic, trigonometric, and Bessel functions and are given by the following expressions:

$$C_0(j,p) = \int_0^{z_e} Z_p Z_j^* \, dz \int_0^{2\pi} \Theta_p \Theta_j \, d\theta \int_0^1 R_p R_j r dr \tag{C-1}$$

$$C_1(j,p) = \left\{ s_{mn}^2(p) \int_0^{z_e} Z_p Z_j^* \, dz - \int_0^{z_e} Z_p'' Z_j^* \, dz + Z_p'(z_e) Z_j^*(z_e) \right\} \int_0^{2\pi} \Theta_p \Theta_j \, d\theta \int_0^1 R_p R_j r dr \tag{C-2}$$

$$C_2(j,p) = \left\{ 2 \int_0^{z_e} \bar{u}(z) Z_p' Z_j^* \, dz + \gamma \int_0^{z_e} \frac{d\bar{u}}{dz} Z_p Z_j^* \, dz + \gamma Y Z_p(z_e) Z_j^*(z_e) \right\} \tag{C-3}$$

$$\times \int_0^{2\pi} \Theta_p \Theta_j \, d\theta \int_0^1 R_p R_j r dr$$

$$C_3(j,p) = \left\{ \gamma \int_0^{z_e} \frac{d\bar{u}}{dz} Z_p Z_j^* \, dz \int_0^{2\pi} \Theta_p \Theta_j \, d\theta \int_0^1 R_p R_j r dr \tag{C-4}$$

$$D_1(j,p,q) = \tfrac{1}{2}(1 - i)\left\{T_1 \int_0^{z_e} Z_p Z_q Z_j^* \, dz + T_2 \left[\int_0^{z_e} Z_p' Z_q' Z_j^* \, dz + \right.\right.$$

$$\left.\left. + \frac{\gamma - 1}{2} \int_0^{z_e} Z_p'' Z_q Z_j^* \, dz\right]\right\} \qquad \text{(C-5)}$$

$$D_2(j,p,q) = \tfrac{1}{2}(1 + i)\left\{T_1 \int_0^{z_e} Z_p Z_q^* Z_j^* \, dz + T_2 \left[\int_0^{z_e} Z_p'(Z_q^*)' Z_j^* \, dz + \right.\right.$$

$$\left.\left. + \frac{\gamma - 1}{2} \int_0^{z_e} Z_p'' Z_q^* Z_j^* \, dz\right]\right\} \qquad \text{(C-6)}$$

$$D_3(j,p,q) = \tfrac{1}{2}(1 + i)\left\{T_1 \int_0^{z_e} Z_p^* Z_q Z_j^* \, dz + T_2 \left[\int_0^{z_e} (Z_p^*)' Z_q' Z_j^* \, dz + \right.\right.$$

$$\left.\left. + \frac{\gamma - 1}{2} \int_0^{z_e} (Z_p^*)'' Z_q Z_j^* \, dz\right]\right\} \qquad \text{(C-7)}$$

$$D_4(j,p,q) = \tfrac{1}{2}(1 - i)\left\{T_1 \int_0^{z_e} Z_p^* Z_q^* Z_j^* \, dz + T_2 \left[\int_0^{z_e} (Z_p^*)'(Z_q^*)' Z_j^* \, dz + \right.\right.$$

$$\left.\left. + \frac{\gamma - 1}{2} \int_0^{z_e} (Z_p^*)'' Z_q^* Z_j^* \, dz\right]\right\} \qquad \text{(C-8)}$$

where

$$T_1 = \int_0^{2\pi} \Theta_p \Theta_q \Theta_j \, d\theta \int_0^1 R_p' R_q' R_j r\,dr + \int_0^{2\pi} \Theta_p' \Theta_q' \Theta_j \, d\theta \int_0^1 R_p R_q R_j \frac{dr}{r} - \frac{\gamma - 1}{2} S_{mn}^2(p) \times$$

$$\times \int_0^{2\pi} \Theta_p \Theta_q \Theta_j \, d\theta \int_0^1 R_p R_q R_j r\,dr$$

$$T_2 = \int_0^{2\pi} \Theta_p \Theta_q \Theta_j \, d\theta \int_0^1 R_p R_q R_j r\,dr$$

In the equations on the prior page the notation of Eq. (9) is used; that is, a single index (i.e., j, p, or q) is used to identify a particular series term rather than the mode numbers used in Eq. (6). The index j identifies the equations in which a given coefficient appears which corresponds to the weighting function used in deriving that equation. For the coefficients of the linear terms (i.e., the C's) the index p identifies the amplitude function which the coefficient multiplies. For coefficients of the nonlinear terms, (i.e., the D's) p identifies the factor which is not differentiated with respect to time, (i.e., A_p or A_p^*), while q identifies the differentiated factor (i.e. dAp/dt or dAp*/dt. Due to the complex nature of the axial eigenfunctions, the above coefficients are complex numbers.

Structure of the Numerical Calculations

A flow chart for Program COEFFS3D is shown in Figure (C-1). The program can be divided into five major sections: (1) input, (2) calculation of the complex linear coefficients, (3) calculation of the complex nonlinear coefficients, (4) obtaining coefficients of the equivalent uncoupled real system, and (5) output.

The inputs to the program include the various parameters describing the chamber geometry, the nozzle boundary condition, the modes included in the approximating series expansion, and various control numbers, as well as the roots of the Bessel functions.

In the second section the axial acoustic eigenvalues are calculated by means of Subroutines EIGVAL and FCNS, and the integrals of the products of two axial eigenfunctions are computed by means of Subroutines AXIAL1 and UBAR. The integrals involving radial and tangential eigenfunctions are evaluated by using the orthogonality properties of these functions. The complex linear coefficients are then calculated according to Eqs. (C-1) through (C-4) and are normalized by dividing by $C_o(j, j)$.

In the third section the integrals of products of three Bessel functions are calculated using Subroutines RADIAL and JBES, while similar integrals involving azimuthal eigenfunctions and axial eigenfunctions are computed using Subroutines AZIMTL and AXIAL2 respectively. The normalized complex nonlinear coefficients are obtained from Eqs. (C-5) through (C-8) by dividing by $C_o(j, j)$.

In the fourth section the normalized complex coefficients are used to

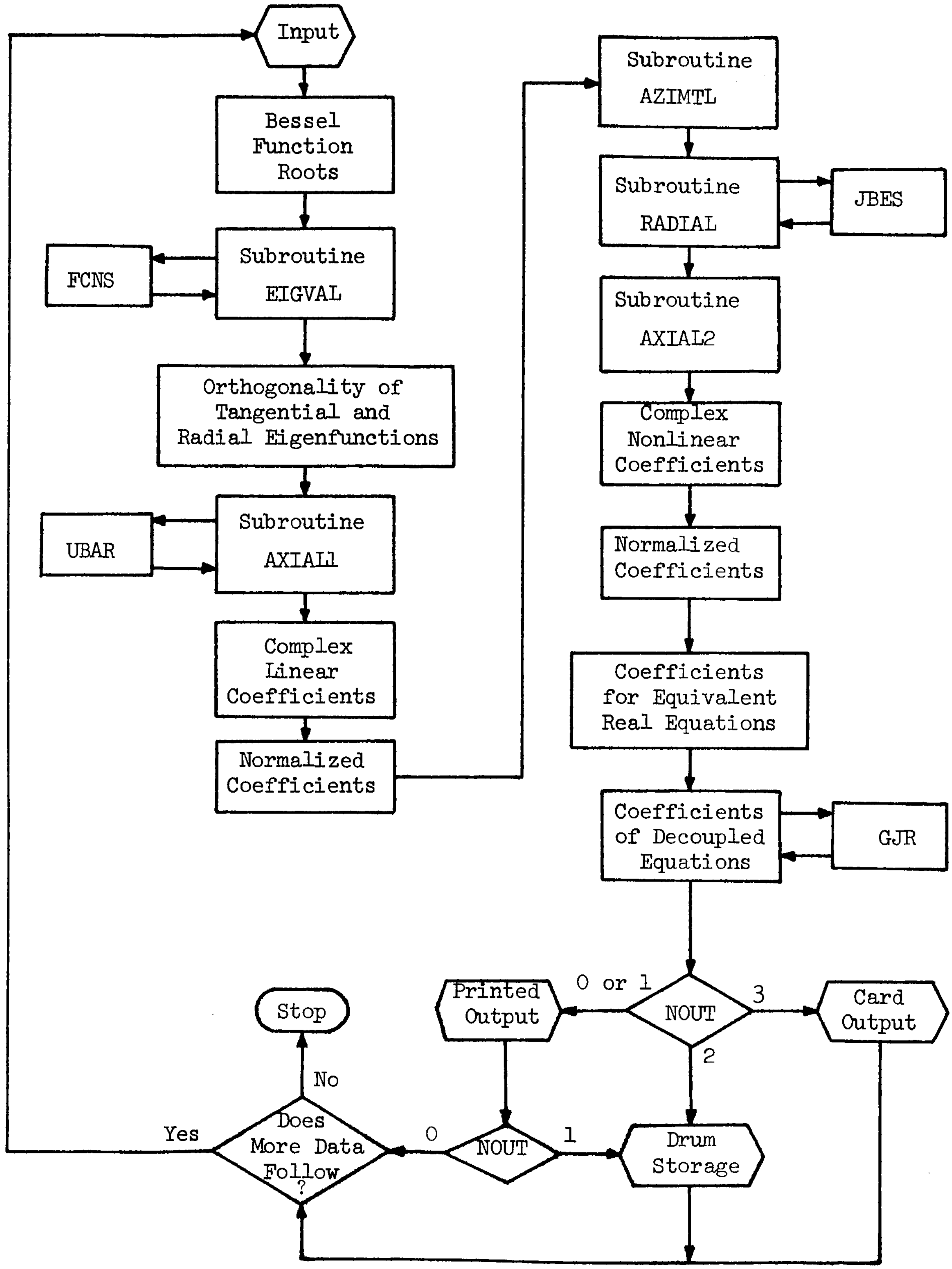


Figure C-1. Flow Chart for Program COEFFS3D obtain the coefficients for the equivalent system of real differential equations obtained by separating the real and imaginary parts of the complex equations. Since the axial eigenfunctions are non-orthogonal, the resulting system of equations may be coupled in the second derivative terms. Therefore, a matrix inversion procedure is used to obtain the coefficients of an equivalent system which is not coupled in the second derivatives.

In the last section the computed values of the coefficients are either printed out, punched onto cards, or stored on drum (FASTRAND file) as desired.

Input Data

The input data consists of the chamber parameters (i.e., ratio of specific heats, steady state Mach number, and length-to-diameter ratio), the nozzle admittance ratio, various control numbers, and information indicating which modes are included in the approximate series expansion. Regarding the latter information, each term in the series is identified by the integer variable J. The nature of each term is specified by the four integers L(J), M(J), N(J), and NS(J), and each term is given a four character name NAME(J). In this manner the coefficients are identified by the integers J associated with the modes involved rather than the corresponding axial, azimuthal, and radial mode numbers.

The following comments pertain to the detailed description of the input. The location number refers to columns of the card. Three formats are used for input: "A" indicates alphanumeric characters, "I" indicates integers, and "F" indicates real numbers with a decimal point. For the "I" and "F" formats the values are placed in fields of five and ten locations, respectively, and the numbers must be placed in the rightmost locations of the allocated field.

No. of Cards	Location	Type	Input Item	Comments
1	1-72	A	TITLE	Title of Case
1	1-10	F	GAMMA	Ratio of specific heats, γ.
	11-20	F	UE	Steady state Mach number at nozzle entrance, $\bar{u}_e$.
	21-30	F	RLD	Length-to-diameter ratio, $L/D = z_e/2$
	31-40	F	ZCOMB	Lenth of combustion zone, z_c/z_e.

No. of Cards	Location	Type	Input Item	Comments
	41-45	I	NDROPS	If 0: droplet momentum source neglected. If 1: droplet momentum source included.
	46-50	I	NOZZLE	If 0: quasi-steady nozzle. If 1: conventional nozzle.
1	1-5	I	NJMAX	Number of series terms (complex). (NJMAX ≤ 10)
	6-10	I	NONLIN	If 0: linear terms only. If 1: linear and nonlinear terms.
	11-15	I	NEGL	If 0: Nonzero coefficients calculated. If 1: Small coefficients neglected.
	16-20	I	NOUT	If 0: printed output only. If 1: printed and written into FASTRAND file. If 2: FASTRAND only. If 3: card output only.
If NEGL = 1:				
1	1-10	F	SM1	Linear coefficients with absolute value less than SM1 neglected.
	11-20	F	SM2	Nonlinear coefficients with absolute value less than SM2 neglected.
End of input for NEGL = 1.				
If NOZZLE = 1:				
NJMAX	1-5	I	J	Integer which identifies series term.
	6-15	F	AMPL(J)	Amplitude factor of nozzle admittance, A.
	16-25	F	PHASE(J)	Phase of nozzle admittance, φ.
End of Input for NOZZLE = 1.				
NJMAX	1-5	I	J	Integer which identifies series term.
	6-10	I	L(J)	Axial mode number, ℓ. ($0 \leq L(J) \leq 10$)

No. of Cards	Location	Type	Input Item	Comments
	11-15	I	M(J)	Tangential mode number, m. $(0 \leq L(J) \leq 8)$
	16-20	I	N(J)	Radial mode number, n. $(0 \leq N(J) \leq 5)$
	21-25	I	NS(J)	NS(J) = 1: $\Theta_j = \sin(m\theta)$ NS(J) = 2: $\Theta_j = \cos(m\theta)$
	26-30	A	NAME(J)	Four character name.

The first card gives a title (maximum 72 characters) used to identify the run. The second card gives the chamber parameters (i.e., γ, $\bar{u}_e$, L/D, z_c), determines whether the droplet momentum source is included in the analysis (see Appendix A), and specifies the type of nozzle (quasi-steady or conventional). If a quasi-steady nozzle is specified the nozzle admittance is calculated using Eqs. (14), and no further information concerning the nozzle is required. The contol numbers are given on the third card. Due to computer storage limitations the series expansion is limited to ten terms, thus NJMAX $\leq$ 10. The control number NEGL gives the option to neglect all coefficients with absolute value smaller than a given number, thus allowing a considerable saving in computation time when the equations are numerically integrated by Program LCYC3D. It has been found that neglecting coefficients with absolute value smaller than 0.1 (i.e., SM1 = SM2 = 0.1) reduces the computation time by half and has a negligible effect on the resulting solutions. For conventional nozzles a series of NJMAX cards is read which gives the nozzle admittance (amplitude and phase) for each term in the series. This is followed by another series of NJMAX cards giving the mode numbers for each series term.

The proper input for program COEFFS3D will be illustrated with the following example. Suppose the velocity potential Φ is expressed in terms of the first tangential (1T), the second tangential (2T), and the first radial (1R) modes. It is also desired to investigate instability of the spinning type, therefore both sin(mθ) and cos(mθ) terms are included in the series. However, for the 1R mode (m=0) there is no corresponding sin(mθ) term, therefore the resulting series will contain five terms. A nozzle admittance of $A = 0.02$ and $\varphi = 45^{\circ}$ will be assumed for each term in the series, and coefficients smaller than 0.1 as well as the droplet momentum source will be neglected.

The output data will be punched on cards. A sample input for this case is given in Table (C-1) below.

Table C-1. Sample Input.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51
1	T	,	2	T	,	1	R		S	P	I	N	N	I	N	G																																		
							1	.	2								0	.	2								0	.	5								1	.	0					0					1	
				5					1					1					3																															
							0	.	1								0	.	1																															
				1							0	.	0	2							4	5	.	0																										
				2							0	.	0	2							4	5	.	0																										
				3							0	.	0	2							4	5	.	0																										
				4							0	.	0	2							4	5	.	0																										
				5							0	.	0	2							4	5	.	0																										
				1					0					1					1					1		A	0	1	1																					
				2					0					1					1					2		B	0	1	1																					
				3					0					2					1					1		A	0	2	1																					
				4					0					2					1					2		B	0	2	1																					
				5					0					0					1					2		B	0	0	1																					

After the last card in the sequence described above is read, the program is executed and control returns to the input section. Thus, several cases can be executed on the same run. If no further cards are given the run is terminated.

In addition to the above card input, roots of the Bessel functions S_{mn} which give zero slope at r = 1 and the associated values $J_m(S_{mn})$ are needed for these calculations. These values were taken from Ref. (16) for m = 0, 1,...8 and n = 1,2...5; they are automatically put into the program by means of a DATA statement, which is an integral part of the program.

Complex Linear Coefficients

For NDROPS = 0 the complex linear coefficients are computed from Eqs. (C-1) through (C-4) and are stored in the complex array CC(KC,NJ,NP). For NDROPS = 1 the coefficients $C_2(j,p)$ are computed from Eq. (A-8).

In order to calculate these coefficients the following information is needed: (1) the axial acoustic eigenvalues, $b_{\ell mn}$, (2) the steady state Mach number distribution, $\bar{u}(z)$, (3) the orthogonality properties of the transverse eigenfunctions, and (4) the integrals of products of two axial eigenfunctions. The calculation of these quantities is described below.

Axial Acoustic Eigenvalues. The axial acoustic eigenvalues are determined by numerically solving the transcendental equation given by Eq. (8). This is done by first substituting $b_{\ell mn} = \epsilon_{\ell mn} + i\eta_{\ell mn}$ and $Y = Y_r + iY_i$ into Eq. (8) and separating real and imaginary parts. This yields a pair of simultaneous equations of the form:

$$\begin{aligned} f(\epsilon,\eta) &= 0 \\ g(\epsilon,\eta) &= 0 \end{aligned} \qquad \text{(C-9)}$$

where

$$\begin{aligned} f(\epsilon,\eta) = (\epsilon^2 - \eta^2)F(\epsilon,\eta) - 4\epsilon\eta H(\epsilon,\eta) \\ + \gamma^2\Big\{\Big[(Y_r^2 - Y_i^2)(S_{mn}^2 + \epsilon^2 - \eta^2) - 4Y_rY_i\epsilon\eta\Big]G(\epsilon,\eta) \\ + 4\Big[Y_rY_i(S_{mn}^2 + \epsilon^2 - \eta^2) + (Y_r^2 - Y_i^2)\epsilon\eta\Big]H(\epsilon,\eta)\Big\} \end{aligned} \qquad \text{(C-10)}$$

$$g(\epsilon,\eta) = (\epsilon^2 - \eta^2)H(\epsilon,\eta) + \epsilon\eta F(\epsilon,\eta)$$

$$+ \gamma^2\Big\{\Big[Y_r Y_i(S_{mn}^2 + \epsilon^2 - \eta^2) + (Y_r^2 - Y_i^2)\epsilon\eta\Big]G(\epsilon,\eta)$$

$$- \Big[(Y_r^2 - Y_i^2)(S_{mn}^2 + \epsilon^2 - \eta^2) + 4Y_r Y_i \epsilon\eta\Big]H(\epsilon,\eta)\Big\} \qquad \text{(C-11)}$$

and

$$F(\epsilon,\eta) = \sin^2(\epsilon z_e)\cosh^2(\eta z_e) - \cos^2(\epsilon z_e)\sinh^2(\eta z_e)$$

$$G(\epsilon,\eta) = \cos^2(\epsilon z_e)\cosh^2(\eta z_e) - \sin^2(\epsilon z_e)\sinh^2(\eta z_e) \qquad \text{(C-12)}$$

$$H(\epsilon,\eta) = \sin(\epsilon z_e)\cos(\epsilon z_e)\sinh(\eta z_e)\cosh(\eta z_e)$$

In the above equations the subscripts on ϵ and η have been omitted.

Equations (C-9) are solved by Subroutine EIGVAL using Newton's Method for two unknowns.[17] In this method successive approximations to the roots are generated by the recursion formulas:

$$\epsilon_{i+1} = \epsilon_i - \left[\frac{fg_\eta - gf_\eta}{J(f,g)}\right]_i$$

$$\eta_{i+1} = \eta_i - \left[\frac{gf_\epsilon - fg_\epsilon}{J(f,g)}\right]_i \qquad \text{(C-13)}$$

where the Jacobian $J(f,g)$ is given by:

$$J(f,g) = f_\epsilon g_\eta - g_\epsilon f_\eta \qquad \text{(C-14)}$$

and the subscripts indicate partial differentiation with respect to ϵ and η. The quantities f, g, f_ϵ, f_η, g_ϵ, g_η are calculated by the Subroutine FCNS. The iteration is started by assuming the following values for ϵ and η:

$$\epsilon_o = \epsilon_m + a\cos(\beta)$$
$$\eta_o = a\sin(\beta) \qquad \text{(C-15)}$$

where for $\ell = 0$:

$$\epsilon_m = 0$$
$$a = 10A/z_e$$
$$\beta = \varphi/2 + 45 \text{ (degrees)}$$

and for $\ell \neq 0$: $\qquad \text{(C-16)}$

$$\epsilon_m = \ell\pi/z_e$$
$$a = A/z_e$$
$$\beta = \varphi + 90 \text{ (degrees)}$$

The iteration is terminated when the errors $\Delta\epsilon$ and $\Delta\eta$ are smaller than 10^{-7}. If the iteration fails to converge after 40 iterations or the Jacobian vanishes a warning message is printed. FORTRAN listings of Subroutines EIGVAL and FCNS are given at the end of this appendix.

Steady State Mach Number Distribution. The steady state Mach number distribution is calculated by means of Subroutine UBAR which must be supplied by the user. This distribution must be of the form shown in Fig. (C-2) where the Mach number varies from zero at the injector face ($z = 0$) to its maximum value at the end of the combustion zone ($z = z_c$) and remains constant until the nozzle entrance ($z = z_e$) is reached. Thus the Mach number is given by

$$\bar{u}(z) = U(z)\bar{\mathbf{u}}_e \qquad (0 \leq z \leq z_c) \qquad \text{(C-17)}$$

$$\bar{u}(z) = \bar{u}_e \qquad (z_c \leq z \leq z_e)$$

where $U(0) = 0$ and $U(z_c) = 1$. Although the function $U(z)$ may be arbitrary, the results presented in this report were obtained using a linear Mach number distribution in the combustion zone (i.e., uniformly distributed combustion). Thus the function $U(z)$ in the listing of UBAR provided herein is given by:

$$U(z) = z/z_c \tag{C-18}$$

In addition to the Mach number distribution (NOPT = 1), the first (NOPT = 2) and second (NOPT = 3) derivatives are also calculated.

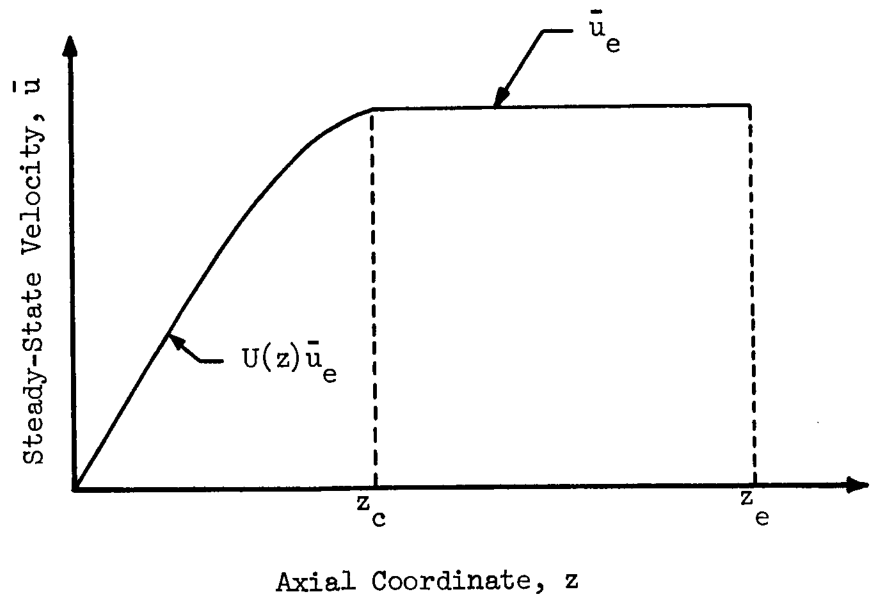


Figure C-2. Steady-State Mach Number Distribution.

Orthogonality of Transverse Eigenfunctions. The tangential eigenfunctions have the following orthogonality properties:

$$\int_0^{2\pi} \sin(m_p\theta)\sin(m_j\theta)d\theta = \int_0^{2\pi} \cos(m_p\theta)\cos(m_j\theta)d\theta = 0 \qquad m_p \neq m_j$$

$$= \pi \qquad m_p = m_j \neq 0$$

$$\int_0^{2\pi} \cos(m_p\theta)\cos(m_j\theta)d\theta = 2\pi \qquad m_p = m_j = 0 \tag{C-19}$$

$$\int_0^{2\pi} \sin(m_p\theta)\cos(m_j\theta)d\theta = 0 \qquad \text{for all } m_p \text{ and } m_j$$

For the special case of $m_p = m_j = 0$ the integral involving sines vanishes. The orthogonality property of the radial eigenfunctions is given by:

$$\int_0^1 R_pR_j rdr = 0 \qquad n_p \neq n_j \ (m_p = m_j)$$

$$\int_0^1 R_pR_j rdr = \frac{S_{mn}^2 - m^2}{2S_{mn}^2}\left[J_m(S_{mn})\right]^2 \qquad n_p = n_j \ (m_p = m_j) \tag{C-20}$$

Since the tangential integrals vanish when $m_p \neq m_j$ it is not necessary to calculate the radial integrals for $m_p \neq m_j$. These orthogonality properties are used to calculate the integrals, $\int_0^{2\pi} \Theta_p\Theta_j d\theta$ and $\int_0^1 R_pR_j rdr$, which appear in Eqs. (C-1) through (C-4). For a series containing pure transverse modes only ($\ell = 0$), it is easily seen that all of the linear coefficients vanish except those corresponding to p = j, yielding a system of equations which are not coupled in the linear terms.

Axial Integrals. The integrals of products of two axial eigenfunctions are calculated by Subroutine AXIAL1. According to the value of the input parameter NOPT these integrals are calculated as follows:

$$\text{NOPT} = 1: \quad \int_0^{z_e} Z_p Z_j^* dz = \frac{1}{2}\left\{ \frac{\sinh\left[i(b_p + b_j^*)z_e\right]}{i(b_p + b_j^*)} + \frac{\sinh\left[i(b_p - b_j^*)z_e\right]}{i(b_p - b_j^*)} \right\} \tag{C-21}$$

$$\text{NOPT} = 2: \quad \int_0^{z_e} Z_p'' Z_j^* dz = -b_p^2 \int_0^{z_e} Z_p Z_j^* dz \tag{C-22}$$

$$\text{NOPT} = 3: \quad \int_0^{z_e} \frac{d\bar{u}}{dz} Z_p Z_j^* dz \qquad \text{(evaluated numerically)}$$

$$\text{NOPT} = 4: \quad \int_0^{z_e} \bar{u}(z) Z_p' Z_j^* dz \qquad \text{(evaluated numerically)}$$

The last two integrals, which involve the mean flow Mach number, are evaluated by means of Simpson's Rule. A FORTRAN listing of AXIAL1 is provided at the end of this appendix.

Complex Nonlinear Coefficients.

The complex nonlinear coefficients are calculated from Eqs. (C-5) through (C-8) and are stored in the complex arrays, CD1(NJ,NP,NQ), CD2(NJ,NP,NQ), CD3(NJ,NP,NQ), and CD4(NJ,NP,NQ).

In order to calculate these coefficients, the various integrals of axial, azimuthal, and radial eigenfunctions must be evaluated. Since many of the azimuthal integrals are zero they are evaluated first, and the remaining integrals are computed only if the corresponding azimuthal integral is nonzero. The subroutines used to calculate these integrals are described in the following paragraphs.

Azimuthal Integrals. The azimuthal integrals are calculated by Subroutine AZIMTL according to the value of NOPT as follows:

$$\text{NOPT} = 1: \quad \int_0^{2\pi} \Theta_p \Theta_q \Theta_j d\theta$$

$$\text{NOPT} = 2 : \quad \int_0^{2\pi} \Theta_p \Theta'_q \Theta'_j \, d\theta$$

These integrals are easily evaluated analytically; for most values of p, q, and j they are zero. The nonzero integrals are readily expressed in terms of the following integrals:

$$\int_0^{2\pi} \cos(m_p\theta)\cos(m_q\theta)\cos(m_j\theta)d\theta = \pi/2 \quad \text{for} \quad m_j = m_p + m_q, \quad m_p = m_j + m_q, \text{ or} \quad m_q = m_j + m_p \tag{C-23}$$

$$\int_0^{2\pi} \cos(m_p\theta)\sin(m_q\theta)\sin(m_j\theta)d\theta = \pi/2 \quad \text{for} \quad m_q = m_p + m_j \text{ or} \quad m_j = m_p + m_q \tag{C-24}$$

$$\int_0^{2\pi} \cos(m_p\theta)\sin(m_q\theta)\sin(m_j\theta)d\theta = -\pi/2 \text{ for} \quad m_p = m_q + m_j \tag{C-25}$$

where m_p, m_q, and m_j are nonzero. If any one of the tangential mode numbers is zero (corresponding to a radial mode) the following values are obtained:

$$\int_0^{2\pi} \cos(m_p\theta)\cos(m_q\theta)\cos(m_j\theta)d\theta = 2\pi \quad m_p = m_q = m_j = 0$$

$$= \pi \quad m_p = 0,\ m_q = m_j; \quad m_q = 0,\ m_p = m_j; \quad m_j = 0,\ m_p = m_q \tag{C-26}$$

$$\int_0^{2\pi} \cos(m_p\theta)\sin(m_q\theta)\sin(m_j\theta)d\theta = \pi \qquad m_p = 0,\ m_q = m_j \qquad \text{(C-27)}$$

Subroutine AZIMTL consists of two sections. In the first section the azimuthal integral is expressed as the product of a constant factor and one of the basic forms given in Eqs. (C-23) and (C-24). The second section is essentially a series of logical tests to determine if the mode numbers, m_p, m_q, and m_j satisfy any of the conditions for Eqs. (C-23) through (C-27). If any of these conditions is satisfied the appropriate value is multiplied by the corresponding factor determined in the first section and the product is assigned to the output variable (i.e., RESULT), otherwise the value zero is assigned.

Radial Integrals. Subroutine RADIAL calculates the radial integrals which appear in Eqs. (C-5) through (C-8) according to NOPT as follows:

$$\text{NOPT} = 1 : \quad \int_0^1 R_p R_q R_j r dr$$

$$\text{NOPT} = 2 : \quad \int_0^1 R_p R_q R_j \frac{1}{r} dr$$

$$\text{NOPT} = 3 : \quad \int_0^1 R_p' R_q' R_j r dr$$

where the R's are the Bessel functions, $J_m(S_{mn}r)$. These integrals are computed numerically using Simpson's Rule with 100 subdivisions. In calculating the integrands the derivatives of the Bessel functions are given by:

$$J_m'\left(S_{mn}r\right) = \frac{1}{2}\left[J_{m-1}\left(S_{mn}r\right) - J_{m+1}\left(S_{mn}r\right)\right] \quad \text{for} \quad m = 1,2,3,\ldots$$

$$J_0'\left(S_{mn}r\right) = -J_1\left(S_{mn}r\right) \qquad \text{(C-28)}$$

The integrand of the second integral (NOPT = 2) is indeterminate at the lower limit of integration. However a limit exists, denoted by L, which vanishes with the following exceptions:

$$
\begin{aligned}
L &= S_{mn}(p)/2 \quad \text{for} \quad m_p = 1,\ m_q = m_j = 0 \\
L &= S_{mn}(q)/2 \quad \text{for} \quad m_q = 1,\ m_p = m_j = 0 \\
L &= S_{mn}(j)/2 \quad \text{for} \quad m_j = 1,\ m_p = m_q = 0
\end{aligned}
\qquad \text{(C-29)}
$$

All of the calculations in Subroutine RADIAL are carried out in double precision arithmetic. The results are given as a single precision number.

Subroutine JBES computes the double precision Bessel functions which are needed for the above calculations. A description of this subroutine and a program listing are given in Chapter 23 of Ref. (18).

Axial Integrals. The integrals of the products of three axial eigenfunctions (see Eqs. (C-5) through (C-8)) are computed by Subroutine AXIAL2 according to the input parameters NOPT and NCONJ. The three basic forms are specified by NOPT as follows:

$$\text{NOPT} = 1: \quad \int_o^{z_e} Z_p Z_q Z_j^* dz$$

$$\text{NOPT} = 2: \quad \int_o^{z_e} Z_p' Z_q' Z_j^* dz$$

$$\text{NOPT} = 3: \quad \int_o^{z_e} Z_p'' Z_q Z_j^* dz$$

When NCONJ = 1 these basic forms are calculated; these are the forms appearing in the expression for $D_1(j, p, q)$ (see Eq. (C-5)). For NCONJ = 2 the second function in the integrand is replaced by its complex conjugate to obtain the integrals appearing in the expression for $D_2(j,p,q)$. The integrals appearing in the expressions for $D_3(j,p,q)$ and $D_4(j,p,q)$ are obtained by setting NCONJ = 3 and NCONJ = 4 respectively.

The basic forms are calculated from the following analytical formulas:

$$\int_0^{z_e} Z_p Z_q Z_j^* dz = \frac{1}{4}\left\{ \frac{\sinh\left[i(b_p + b_q + b_j^*)z_e\right]}{i(b_p + b_q + b_j^*)} + \frac{\sinh\left[i(b_p + b_q - b_j^*)z_e\right]}{i(b_p + b_q - b_j^*)} + \frac{\sinh\left[i(b_p - b_q + b_j^*)z_e\right]}{i(b_p - b_q + b_j^*)} + \frac{\sinh\left[i(b_p - b_q - b_j^*)z_e\right]}{i(b_p - b_q - b_j^*)} \right\} \tag{C-30}$$

$$\int_0^{z_e} Z_p' Z_q' Z_j^* dz = -\frac{1}{4} b_p b_q \left\{ \frac{\sinh\left[i(b_p + b_q + b_j^*)z_e\right]}{i(b_p + b_q + b_j^*)} + \frac{\sinh\left[i(b_p + b_q - b_j^*)z_e\right]}{i(b_p + b_q - b_j^*)} - \frac{\sinh\left[i(b_p - b_q + b_j^*)z_e\right]}{i(b_p - b_q + b_j^*)} - \frac{\sinh\left[i(b_p - b_q - b_j^*)z_e\right]}{i(b_p - b_q - b_j^*)} \right\} \tag{C-31}$$

$$\int_0^{z_e} Z_p'' Z_q Z_j^* dz = -b_p^2 \int_0^{z_e} Z_p Z_q Z_j^* dz \tag{C-32}$$

The remaining forms are obtained from Eqs. (C-30) through (C-32) by replacing the appropriate eigenvalues with their complex conjugates; thus, for NOPT = 2 b_q is replaced by b_q^*, for NOPT = 3 b_p is replaced with b_p^*, and both b_p and b_q are replaced by their conjugates for NOPT = 4.

FORTRAN listings for Subroutines AZIMTL, RADIAL, and AXIAL2 are given at the end of this appendix.

Coefficients for Equivalent Real System.

Equations (12) are a system of complex differential equations to be solved for the unknown complex amplitude functions, $A_p(t)$. In order to solve these equations numerically they must first be separated into their real and imaginary parts. This is done by assuming that $A_p(t) = F_p(t) + iG_p(t)$, substituting into Eqs. (12), and separating real and imaginary parts to obtain an equivalent system of real differential equations that describe the behavior of the F_p's and G_p's. Since these equations contain twice as many unknown functions (i.e., $F_p(t)$ and $G_p(t)$) as Eqs. (12), it is convenient to re-index the unknown functions and their coefficients as follows:

$$
\begin{aligned}
F_p(t) &= B_{2p-1}(t) \\
G_p(t) &= B_{2p}(t)
\end{aligned}
\qquad \text{(C-33)}
$$

Thus the B's with odd indices correspond to the real parts, $F_p(t)$, and the B's with even indices correspond to the imaginary parts, $G_p(t)$. The corresponding set of differential equations is given by:

$$
\sum_{p=1}^{2N} \left\{ C_0'(j,p) \frac{d^2B_p}{dt^2} + C_1'(j,p)B_p(t) + \left[C_2'(j,p) - nC_3'(j,p)\right] \frac{dB_p}{dt} \right. +
$$

$$+ nC_3'(j,p) \frac{d\left[B_p(t-\bar{\tau})\right]}{dt} \Big\} + \sum_{p=1}^{2N} \sum_{q=1}^{2N} \left\{ D'(j,p,q) B_p \frac{dB_q}{dt} \right\} = 0$$

$$j = 1,2,3, \ldots 2N \quad \text{(C-34)}$$

The real coefficients in Eqs. (C-34) (i.e., C_0', C_1', C_2', C_3', and D') are related to the complex coefficients in Eqs. (12) (i.e., $C_0, \ldots C_3$, $D_1, \ldots D_4$) as follows:

$$\begin{aligned}
C_k'(2j-1,\ 2p-1) &= \mathrm{Re}\left[C_k(j,p)\right] \\
C_k'(2j-1,\ 2p) &= -\mathrm{Im}\left[C_k(j,p)\right] \\
C_k'(2j,\ 2p-1) &= \mathrm{Im}\left[C_k(j,p)\right] \\
C_k'(2j,\ 2p) &= \mathrm{Re}\left[C_k(j,p)\right]
\end{aligned} \quad \text{(C-35)}$$

for $k = 0,1,2,3$, $j = 1,2,\ldots N$, $p = 1,2,\ldots N$ and:

$$\begin{aligned}
D'(2j-1,2p-1,2q-1) &= \mathrm{Re}\left[D_1(j,p,q) + D_2(j,p,q) + D_3(j,p,q) + D_4(j,p,q)\right] \\
D'(2j-1,2p-1,2q) &= \mathrm{Im}\left[-D_1(j,p,q) + D_2(j,p,q) - D_3(j,p,q) + D_4(j,p,q)\right] \\
D'(2j-1,2p,2q-1) &= \mathrm{Im}\left[-D_1(j,p,q) - D_2(j,p,q) + D_3(j,p,q) + D_4(j,p,q)\right] \\
D'(2j-1,2p,2q) &= \mathrm{Re}\left[-D_1(j,p,q) + D_2(j,p,q) + D_3(j,p,q) - D_4(j,p,q)\right]
\end{aligned}$$

$$\text{(C-36)}$$

$$D'(2j,2p-1,2q-1) = \text{Im}\left[D_1(j,p,q) + D_2(j,p,q) + D_3(j,p,q) + D_4(j,p,q)\right]$$

$$D'(2j,2p-1,2q) = \text{Re}\left[D_1(j,p,q) - D_2(j,p,q) + D_3(j,p,q) - D_4(j,p,q)\right]$$

$$D'(2j,2p,2q-1) = \text{Re}\left[D_1(j,p,q) + D_2(j,p,q) - D_3(j,p,q) - D_4(j,p,q)\right]$$

$$D'(2j,2p,2q) = \text{Im}\left[-D_1(j,p,q) + D_2(j,p,q) + D_3(j,p,q) - D_4(j,p,q)\right]$$

for $j = 1,2,\ldots N$, $p = 1,2,\ldots N$, $q = 1,2,\ldots N$. The linear coefficients are stored in the arrays C1(NJ,NP) for $k = 0$ and C(KC,NJ,NP) for $k = 1,2,3$. The nonlinear coefficients are stored in the array D(NJ,NP,NQ).

In general Eqs. (C-34) are coupled in the second derivatives; that is, they are of the form:

$$\sum_{p=1}^{2N} \left\{ C_0'(j,p) \frac{d^2B_p}{dt^2} \right\} = g_j(B_1,B_2,\ldots B_{2N}) \qquad \text{(C-37)}$$

where there are two or more C_0' terms in each equation. This coupling results from the non-orthogonality of the axial eigenfunctions. In order to numerically integrate Eqs. (C-34), they must be decoupled by transforming to the form:

$$\frac{d^2B_j}{dt^2} = f_j(B_1,B_2,\ldots B_{2N}) \qquad \text{(C-38)}$$

in which only one second derivative appears in each equation. Using Eq. (C-38), it is seen that Eq. (C-37) can be expressed as

$$C_0'f = g \qquad \text{(C-39)}$$

where C_0' is the 2N X 2N matrix of coefficients of the coupled system, f is the column matrix corresponding to the right-hand-side of the decoupled system, and g is the column matrix corresponding to the right-hand-side of the coupled system. To decouple Eqs. (C-37), therefore, Eq. (C-39) is solved for f, thus:

$$f = C_0^{-1} g \tag{C-40}$$

where C_0^{-1} is the inverse of the matrix C_0'. Performing these operations and equating the coefficients of like terms in f and $C_0^{-1}g$ gives the following relations:

$$\begin{aligned} \tilde{C}_i(j,p) &= \sum_{k=1}^{2N} C_0^{-1}(j,k)C_i'(k,p) \qquad i = 1,2,3 \\ \tilde{D}(j,p,q) &= \sum_{k=1}^{2N} C_0^{-1}(j,k)D'(k,p,q) \end{aligned} \tag{C-41}$$

where $\tilde{C}_i$ and $\tilde{D}$ are the corresponding coefficients of the decoupled system. The matrix inverse, C_0^{-1}, is computed by the subroutine GJR, which is a standard Univac 1108 library program, and is stored in the array C1(NJ,NP). A listing of GJR and instructions for its use are given in Ref. (19).

The calculation of $\tilde{C}_i(j,p)$ and $\tilde{D}(j,p,q)$, which are the coefficients for the equivalent set of real, decoupled equations, is the final step in the computations performed by COEFFS3D. The coefficients are stored in the arrays C(KC,NJ,NP) and D(NJ,NP,NQ), replacing those computed from Eqs. (C-35) and (C-36). The output of these coefficients is described below.

Output

According to the value of the control number NOUT, the coefficients calculated by Program COEFFS3D are printed, punched onto cards, or stored on drum (FASTRAND). These three output modes will now be discussed individually.

Printed Output. Since the printed output cannot be used as input to

Program LCYC3D, the option "printed output only" (NOUT = 0) is only used for checkout purposes. Printed output can also be obtained in conjunction with the drum storage mode (NOUT = 1). Since the printed output format can only accommodate five series terms (complex), it should only be used for NJMAX ≤ 5.

The first page of printed output gives a restatement of the input parameters. This page is headed by the title of the case (TITLE) which is followed by the ratio of specific heats (GAMMA), the steady state Mach number at the nozzle entrance (UE), the length-to-diameter ratio (L/D), and the length of the combustion zone as a fraction of the chamber length (ZCOMB). After statements concerning the presence or absence of the liquid droplet momentum source and the type of nozzle considered, a restatement of the input parameters J, L(J), M(J), N(J), NS(J), and NAME(J) which describe the terms in the series expansion of Φ is given. This tabulation also includes additional parameters needed by Program LCYC3D: S_{mn}, the dimensionless frequency of the mode (SMN); $J_m(S_{mn})$, the associated value of the Bessel function (JM(SMN)); the real part (EPS) and the imaginary part (ETA) of the axial acoustic eigenvalue; and the real part (YR) and imaginary part (YI) of the nozzle admittance.

The next three pages give the decoupled linear coefficients, $\tilde{C}_1(j,p)$, $\tilde{C}_2(j,p)$, and $\tilde{C}_3(j,p)$. These coefficients are presented in the matrix format with the rows corresponding to the index j and the columns corresponding to the index p. The remaining pages give the decoupled nonlinear coefficients $\tilde{D}(j,p,q)$ for each value of j. Here the rows correspond to the index p and the columns correspond to the index q.

A sample printed output for the five term series used in the sample input is given in Tables (C-2) through (C-4).

Drum Storage. When available drum storage, such as the FASTRAND system used with the Univac 1108, is the most convenient means of storing the output of Program COEFFS3D. In the absence of such a system, the program can be easily modified to store the coefficients on magnetic tape. In either case magnetic tape can be used as a back-up file or for permanent storage of the data. The control statements needed to execute these procedures depend upon the computer facilities being used and cannot be described in

Table C-2. Sample Printed Output, Page 1.

```
1T,2T,1R SPINNING.

GAMMA =  1.20     UE =   .20     L/D =    .50000     ZCOMB =  1.00
DROPLET MOMENTUM SOURCE NEGLECTED
```

NAME	J	L	M	N	NS	SMN	JM(SMN)	EPS	ETA	YR	YI
A011	1	0	1	1	1	1.84118	.58187	.08122	.19451	.01414	.01414
B011	2	0	1	1	2	1.84118	.58187	.08122	.19451	.01414	.01414
A021	3	0	2	1	1	3.05424	.48650	.10617	.25115	.01414	.01414
B021	4	0	2	1	2	3.05424	.48650	.10617	.25115	.01414	.01414
B001	5	0	0	1	2	3.83171	-.40276	.11993	.28170	.01414	.01414

Table C-3. Sample Printed Output, Page 2.

DECOUPLED COEFFICIENT OF B(P): C(1,J,P)

J \ P	1	2	3	4	5	6	7	8	9	10
1	3.390599	.000000	.000000	.000000	.000000	.000000	.000000	.000000	.000000	.000000
2	-.000000	3.390599	.000000	.000000	.000000	.000000	.000000	.000000	.000000	.000000
3	.000000	.000000	3.390599	.000000	.000000	.000000	.000000	.000000	.000000	.000000
4	.000000	.000000	-.000000	3.390599	.000000	.000000	.000000	.000000	.000000	.000000
5	.000000	.000000	.000000	.000000	9.330212	.000000	.000000	.000000	.000000	.000000
6	.000000	.000000	.000000	.000000	.000000	9.330212	.000000	.000000	.000000	.000000
7	.000000	.000000	.000000	.000000	.000000	.000000	9.330212	.000000	.000000	.000000
8	.000000	.000000	.000000	.000000	.000000	.000000	.000000	9.330212	.000000	.000000
9	.000000	.000000	.000000	.000000	.000000	.000000	.000000	.000000	14.684905	.000000
10	.000000	.000000	.000000	.000000	.000000	.000000	.000000	.000000	-.000000	14.684905

Table C-4. Sample Printed Output, Page 5.

DECOUPLED COEFFICIENT OF B(P) * DB(Q)/DT IN EQUATION FOR B(1)

P \ Q	1	2	3	4	5	6	7	8	9	10
1	.000000	.000000	.000000	.000000	.000000	.000000	-1.735038	-.015812	-2.338657	-.026798
2	.000000	.000000	.000000	.000000	.000000	.000000	-.010850	.008834	-.013979	.011946
3	.000000	.000000	.000000	.000000	1.735038	.015812	.000000	.000000	.000000	.000000
4	.000000	.000000	.000000	.000000	.010850	-.008834	.000000	.000000	.000000	.000000
5	.000000	.000000	1.497831	.008314	.000000	.000000	.000000	.000000	.000000	.000000
6	.000000	.000000	.015813	-.007594	.000000	.000000	.000000	.000000	.000000	.000000
7	-1.497831	-.008314	.000000	.000000	.000000	.000000	.000000	.000000	.000000	.000000
8	-.015813	.007594	.000000	.000000	.000000	.000000	.000000	.000000	.000000	.000000
9	-1.962811	-.010896	.000000	.000000	.000000	.000000	.000000	.000000	.000000	.000000
10	-.024780	.009986	.000000	.000000	.000000	.000000	.000000	.000000	.000000	.000000

this manual.

Card Output. When a drum or magnetic tape storage is not available, punched card output can be used (NOUT = 3). This method becomes unwieldy, however, when a large number of coefficients is involved since only one coefficient can be punched on a card. The format for both drum and card output is the same and is given below:

Number of Cards	Location	Type	Output Item	Comments
1	1-10	F	GAMMA	Same as for input.
	11-20	F	UE	Same as for input.
	21-30	F	ZE	Dimensionless chamber length, (2L/D).
	31-40	F	ZCOMB	Same as for input.
	41-45	I	NDROPS	Same as for input.
	46-50	I	NJMAX	Number of unknown functions, $B_p(t)$ (see Eq. (C-34)).
NJMAX/2	1-5	I	J	Same as input.
	6-10	I	L(J)	"
	11-15	I	M(J)	"
	16-20	I	N(J)	"
	21-25	I	NS(J)	"
	26-35	F	S(J)	Root of Bessel function, S_{mn}.
	36-45	F	SJ(J)	Associated value of Bessel function, $J_m(S_{mn})$.
	46-50	A	NAME(J)	Same as input.
NJMAX/2	1-5	I	J	Same as input.
	6-15	F	YR	Real part of nozzle admittance, Y_r.
	16-25	F	YI	Imaginary part of nozzle admittance, Y_i.
	26-35	F	EPS	Real part of axial eigenvalue, ϵ.
	36-45	F	ETA	Imaginary part of axial eigenvalue, η.

Number of Cards	Location	Type	Output Item	Comments
1	1-5	I	KMAX(1)	Number of nonzero linear coefficients of type $\tilde{C}_1(j,p)$.
KMAX(1)	1-5	I	NJ	Index, j.
	6-10	I	NP	Index, p.
	11-25	F	C(1,NJ,NP)	Linear coefficient, $\tilde{C}_1(j,p)$.
1	1-5	I	KMAX(2)	Number of nonzero linear coefficients of type $\tilde{C}_2(j,p)$.
KMAX(2)	1-5	I	NJ	Index, j.
	6-10	I	NP	Index, p.
	11-25	F	C(2,NJ,NP)	Linear coefficient, $\tilde{C}_2(j,p)$.
1	1-5	I	KMAX(3)	Number of nonzero linear coefficients of type $\tilde{C}_3(j,p)$.
KMAX(3)	1-5	I	NJ	Index, j.
	6-10	I	NP	Index, p.
	11-25	F	C(3,NJ,NP)	Linear coefficient, $\tilde{C}_3(j,p)$.
1	1-5	I	KMAX(4)	Number of nonzero nonlinear coefficients.
KMAX(4)	1-5	I	NJ	Index, j.
	6-10	I	NP	Index, p.
	11-15	I	NQ	Index, q.
	16-30	F	D(NJ,NP,NQ)	Nonlinear coefficient, $\tilde{D}(j,p,q)$.

The first card of output gives the chamber parameters γ, $\bar{u}_e$, L/D, and z_c/z_e; the droplet momentum source control number, NDROPS; and the number of unknown real functions (i.e., $B_p(t)$), NJMAX. This is followed by NJMAX/2 cards (the number of unknown complex functions, $A_p(t)$) describing the terms included in the series expansion of Φ. The next NJMAX/2 cards gives the complex nozzle admittance (Y_r and Y_i) and the corresponding complex axial eigenvalue (ϵ and η) for each complex series term. The linear coefficients are given in three sets of cards. The first card in the set gives the number of coefficients of the given type, while the remaining cards give the indices j and p and the coefficient $\tilde{C}_i(j,p)$. The next card gives the number of nonlinear coefficients and is followed by cards giving the indices j, p, q and the corresponding coefficient D(j,p,q). Both linear and nonlinear coefficients are given in a field of 15 spaces with six decimal places. For NEGL = 0 only the nonzero coefficients (absolute value greater than 10^{-5}) are given, while for NEGL = 1 only linear coefficients with absolute value greater than SM1 and nonlinear coefficients with absolute value greater than SM2 are given.

A sample card output produced by the sample input of Table (C-1) is given in Table (C-5) below.

Table C-5. Sample Card Output.

```
   1.20000    .20000   1.00000   1.00000    0   10
    1    0    1    1    1   1.84118    .58187 A011
    2    0    1    1    2   1.84118    .58187 B011
    3    0    2    1    1   3.05424    .48650 A021
    4    0    2    1    2   3.05424    .48650 B021
    5    0    0    1    2   3.83171   -.40276 B001
    1    .01414    .01414    .08122    .19451
    2    .01414    .01414    .08122    .19451
    3    .01414    .01414    .10617    .25115
    4    .01414    .01414    .10617    .25115
    5    .01414    .01414    .11993    .28170
   10
    1    1       3.390599
    ⋮    ⋮           ⋮
   10   10      14.684905
   10
    1    1        .261527
    ⋮    ⋮           ⋮
   10   10        .266538
```

```
   10
    1    1        .240000
    :    :           :
   10   10        .240000
   50
    1    1    7      -1.735038
    1    1    9      -2.338657
    :    :    :          :
    :    :    :          :
   10    8    8       -.210907
   10   10   10       4.187931
```

FORTRAN Listing.

```
C
C
C     *********************PROGRAM COEFFS3D ***************************
C
C          THIS PROGRAM COMPUTES THE COEFFICIENTS WHICH APPEAR
C     IN THE DIFFERENTIAL EQUATIONS WHICH GOVERN THE MODE-AMPLITUDE
C     FUNCTIONS.   THESE COEFFICIENTS ARE STORED ON DRUM OR
C     PUNCHED ONTO CARDS FOR INPUT INTO PROGRAM LCYC3D.
C
C     THE FOLLOWING INPUTS ARE REQUIRED:
C     THE TITLE OF THE CASE.
C     GAMMA IS THE SPECIFIC HEAT RATIO.
C     UE IS THE STEADY STATE MACH NUMBER AT THE NOZZLE ENTRANCE.
C     RLD IS THE LENGTH-TO-DIAMETER RATIO.
C     ZCOMB IS THE LENGTH OF THE REGION OF UNIFORMLY DISTRIBUTED
C     COMBUSTION, EXPRESSED AS A FRACTION OF THE CHAMBER LENGTH.
C     NDROPS DETERMINES THE PRESENCE OF DROPLET MOMENTUM SOURCES:
C        NDROPS = 0  DROPLET MOMENTUM SOURCE NEGLECTED.
C        NDROPS = 1  DROPLET MOMENTUM SOURCE INCLUDED.
C     NOZZLE SPECIFIES THE TYPE OF NOZZLE USED:
C        NOZZLE = 0      QUASI-STEADY.
C        NOZZLE = 1      CONVENTIONAL NOZZLE.
C     FOR CONVENTIONAL NOZZLE:
C        AMPL IS THE NOZZLE AMPLITUDE RATIO.
C        PHASE IS THE NOZZLE PHASE SHIFT.
C     NJMAX IS THE NUMBER OF MODE-AMPLITUDE FUNCTIONS IN THE ASSUMED
C     SERIES SOLUTION.  NJMAX MUST NOT EXCEED 10.
C     THE COEFFICIENTS COMPUTED ARE DETERMINED BY NONLIN AS FOLLOWS:
C        NONLIN = 0   LINEAR COEFFICIENTS ONLY.
C        NONLIN = 1   BOTH LINEAR AND NONLINEAR COEFFICIENTS.
C     COEFFICIENTS TO BE NEGLECTED ARE DETERMINED BY NEGL
C     AS FOLLOWS:
C        NEGL = 0   TERMS SMALLER THAN 0.00001 ARE NEGLECTED.
C        NEGL = 1   LINEAR TERMS SMALLER THAN SM1 AND NONLINEAR
C                   TERMS SMALLER THAN SM2 ARE NEGLECTED.
C     THE OUTPUT IS DETERMINED BY NOUT AS FOLLOWS:
C        NOUT = 0  PRINTED OUTPUT ONLY.
C        NOUT = 1  PRINTED AND STORED ON DRUM (FASTRAND FILE).
C        NOUT = 2  FASTRAND FILE ONLY.
C        NOUT = 3  CARD OUTPUT ONLY.
C     EACH MODE-AMPLITUDE IS ASSIGNED AN INTEGER J.
C     THE MODE IS SPECIFIED BY THE INDICES L(J), M(J), AND N(J).
C     L(J) IS THE AXIAL MODE NUMBER AND MUST NOT EXCEED 10.
C     M(J) IS THE AZIMUTHAL MODE NUMBER AND MUST NOT EXCEED 8.
C     N(J) IS THE RADIAL MODE NUMBER AND MUST NOT EXCEED 5.
C     THE INTEGER NS(J) IS ASSIGNED AS FOLLOWS:
C        NS = 1    A-FUNCTION    SIN(M*THETA) * COSH(I*B*Z)
C        NS = 2    B-FUNCTION    COS(M*THETA) * COSH(I*B*Z)
C     NAME(J) IS A FOUR-CHARACTER NAME.
C
```

```
C     ******************************************************************
C
      DIMENSION    L(10), N(10), NAME(10), S(10), SJ(10), TITLE(80),
     1             RJROOT(10,5), RJVAL(10,5), C1(20,20), C(3,20,20),
     2             D(20,20,20), AMPL(10), PHASE(10), AZI(2),
     3             BES1(9,9,9), BES2(9,9,9), BES3(9,9,9),
     4             V(2), JC(20), TS(3,20), TSQ(20), KMAX(4)
      COMPLEX      CRSLT, CI, ZEJ, ZEP1, ZEP2, CZE, CAZ, CRAD,
     1             G1, DCOEF, CGAM, CAX, B(10), BC(10), YNOZ(10),
     2             CNORM(10), CSSQ(10), TANINT(2), RADINT(3),
     3             AXINT(4,3), CC(4,10,10), CD1(10,10,10),
     4             CD2(10,10,10), AX(4), T1, T2, D1, D2, D3, D4,
     5             CD3(10,10,10), CD4(10,10,10)
      COMMON       B    /BLK2/  M(10), NS(10)
C
C     DATA INPUT.
C
      PI = 3.1415927
      SM1 = 0.00001
      SM2 = 0.00001
      CI = (0.0,1.0)
C
C     INPUT ROOTS AND VALUES OF BESSEL FUNCTIONS.
      DATA ((RJROOT(I,J), J = 1,5), I = 1,9)/
     1   3.83171,  7.01559, 10.17347, 13.32369, 16.47063,
     2   1.84118,  5.33144,  8.53632, 11.70600, 14.86359,
     3   3.05424,  6.70613,  9.96947, 13.17037, 16.34752,
     4   4.20119,  8.01524, 11.34592, 14.58585, 17.78875,
     5   5.31755,  9.28240, 12.68191, 15.96411, 19.19603,
     6   6.41562, 10.51986, 13.98719, 17.31284, 20.57551,
     7   7.50127, 11.73494, 15.26818, 18.63744, 21.93172,
     8   8.57784, 12.93239, 16.52937, 19.94185, 23.26805,
     9   9.64742, 14.11552, 17.77401, 21.22906, 24.58720/
      DATA ((RJVAL(I,J), J = 1,5), I = 1,9)/
     1  -0.40276,  0.30012, -0.24970,  0.21836, -0.19647,
     2   0.58187, -0.34613,  0.27330, -0.23330,  0.20701,
     3   0.48650, -0.31353,  0.25474, -0.22088,  0.19794,
     4   0.43439, -0.29116,  0.24074, -0.21097,  0.19042,
     5   0.39965, -0.27438,  0.22959, -0.20276,  0.18403,
     6   0.37409, -0.26109,  0.22039, -0.19580,  0.17849,
     7   0.35414, -0.25017,  0.21261, -0.18978,  0.17363,
     8   0.33793, -0.24096,  0.20588, -0.18449,  0.16929,
     9   0.32438, -0.23303,  0.19998, -0.17979,  0.16539/
C
C     INPUT PARAMETERS.
    4 READ (5,5000, END = 600)  (TITLE(I), I = 1, 72)
      READ (5,5001)  GAMMA, UE, RLD, ZCOMB, NDROPS, NOZZLE
      IF (GAMMA) 600, 600, 8
    8 READ (5,5004) NJMAX, NONLIN, NEGL, NOUT
      IF (NEGL .EQ. 1) READ (5,5005) SM1, SM2
```

```
      IF (NOZZLE .EQ. 1)  GO TO 5
C     COMPUTE ADMITTANCE FOR QUASI-STEADY NOZZLE.
      Y = (GAMMA - 1.0) * UE/(2.0 * GAMMA)
      DO 3 J = 1, NJMAX
      AMPL(J) = Y
      PHASE(J) = 0.0
    3 CONTINUE
      GO TO 7
    5 DO 6 I = 1, NJMAX
      READ (5,5003) J, AMPL(J), PHASE(J)
    6 CONTINUE
    7 DO 10 I = 1, NJMAX
      READ (5,5002) J, L(J), M(J), N(J), NS(J), NAME(J)
   10 CONTINUE
C
      DO 12 J = 1, NJMAX
      THETA = PHASE(J) * PI/180.0
      YR = AMPL(J) * COS(THETA)
      YI = AMPL(J) * SIN(THETA)
      YNOZ(J) = CMPLX(YR,YI)
   12 CONTINUE
C
      ZE = 2.0 * RLD
      CZE = CMPLX(ZE,0.0)
      CGAM = CMPLX(GAMMA,0.0)
      CAX = CGAM
      IF (NDROPS .EQ. 1) CAX = CGAM + (1.0,0.0)
C
C     ******************************************************************
C
C     ASSIGN ARRAYS FOR ROOTS OF BESSEL FUNCTIONS.
      DO 20 J = 1, NJMAX
      IF ((M(J) .EQ. 0) .AND. (N(J) .EQ. 0))  GO TO 15
      MM = M(J) + 1
      NN = N(J)
      S(J) = RJROOT(MM,NN)
      SJ(J) = RJVAL(MM,NN)
      GO TO 25
   15 S(J) = 0.0
      SJ(J) = 1.0
   25 SSQ = S(J) * S(J)
      CSSQ(J)  = CMPLX(SSQ,0.0)
   20 CONTINUE
C
C     ******************************************************************
C
C     CALCULATE AXIAL ACOUSTIC EIGENVALUES.
C
C     FIND MAXIMUM VALUES OF L(J), M(J), AND N(J).
      KN = 0
```

```
      LMAX = 0
      MMAX = 0
      NMAX = 0
      DO 30 J = 1, NJMAX
      IF (L(J) .GT. LMAX)  LMAX = L(J)
      IF (M(J) .GT. MMAX)  MMAX = M(J)
      IF (N(J) .GT. NMAX)  NMAX = N(J)
      IF (N(J) .NE. N(1))  KN = 1
   30 CONTINUE
      LMAX = LMAX + 1
      MMAX = MMAX + 1
C
C     COMPUTE EIGENVALUES.
      DO 40 J = 1, NJMAX
      LL = L(J)
      SMN = S(J)
      YAMPL = AMPL(J)
      YPHASE = PHASE(J)
      CALL EIGVAL(LL,SMN,GAMMA,ZE,YAMPL,YPHASE,CRSLT)
      B(J) = CRSLT
      BC(J) = CONJG(CRSLT)
   40 CONTINUE
C
C     *****************************************************************
C
C     CALCULATE LINEAR COEFFICIENTS.
C
      DO 100 NJ = 1, NJMAX
      DO 100 NP = 1, NJMAX
C
C     ZERO COEFFICIENT ARRAYS.
      DO 105 KC = 1, 4
      CC(KC,NJ,NP) = (0.0,0.0)
  105 CONTINUE
C
C     ORTHOGONALITY PROPERTY OF TANGENTIAL EIGENFUNCTIONS.
      IF ( NS(NP) .NE. NS(NJ) )  GO TO 100
      IF (M(NP) .NE. M(NJ))  GO TO 100
      IF (M(NJ) .EQ. 0)  GO TO 112
      AZ = PI
      GO TO 120
  112 IF ( NS(NJ) .EQ. 1)  GO TO 100
      AZ = 2.0 * PI
C
C     ORTHOGONALITY PROPERTY OF RADIAL EIGENFUNCTIONS.
  120 IF (N(NP) .NE. N(NJ))  GO TO 100
      IF (S(NP))  125, 122, 125
  125 SQM = M(NJ) * M(NJ)
      SSQ = S(NP) * S(NP)
      SJSQ = SJ(NJ) * SJ(NJ)
```

```
      RAD = (SSQ - SQM) * SJSQ/(2.0 * SSQ)
      GO TO 127
  122 RAD = 0.5
C
C     CALCULATE AXIAL INTEGRALS.
  127 DO 130 NOPT = 1, 4
      CALL AXIAL1(NOPT,NP,NJ,UE,ZE,ZCOMB,CRSLT)
      AX(NOPT) = CRSLT
  130 CONTINUE
C
C     EVALUATE FUNCTIONS AT NOZZLE END.
      ZEJ = CCOSH(CI*BC(NJ)*CZE)
      ZEP1 = CCOSH(CI*B(NP)*CZE)
      ZEP2 = CI * B(NP) * CSINH(CI*B(NP)*CZE)
C
      CAZ = CMPLX(AZ,0.0)
      CRAD = CMPLX(RAD,0.0)
C
C     COEFFICIENT OF THE SECOND DERIVATIVE OF A(P).
      CC(1,NJ,NP) = AX(1) * CAZ * CRAD
C
C     COEFFICIENT OF A(P).
      CC(2,NJ,NP) = (CSSQ(NP)*AX(1) - AX(2) + ZEP2*ZEJ) * CAZ * CRAD
C
C     COEFFICIENT OF THE FIRST DERIVATIVE OF A(P).
      CC(3,NJ,NP) = (CAX*AX(3) + (2.0,0.0)*AX(4)
     1             + CGAM*YNOZ(NP)*ZEP1*ZEJ) * CAZ * CRAD
C
C     COEFFICIENT OF THE RETARDED DERIVATIVE OF A(P).
      CC(4,NJ,NP) = CGAM * AX(3) * CAZ * CRAD
C
  100 CONTINUE
C
C     NORMALIZE LINEAR COEFFICIENTS.
      DO 140 NJ = 1, NJMAX
      CNORM(NJ) = CC(1,NJ,NJ)
      DO 140 NP = 1, NJMAX
      DO 140 KC = 1, 4
      CC(KC,NJ,NP) = CC(KC,NJ,NP)/CNORM(NJ)
  140 CONTINUE
C
C     ******************************************************************
C
C     COMPUTE NONLINEAR COEFFICIENTS.
C
      IF (NONLIN .EQ. 0)  GO TO 402
      G1 = (CGAM - (1.0,0.0)) * (0.5,0.0)
C
C     COMPUTATIONS OF BESSEL INTEGRALS WHEN ALL SERIES TERMS HAVE THE
C     SAME RADIAL MODE NUMBER N(J).
```

```
      IF (KN .EQ. 1)  GO TO 170
      DO 150 MP = 1, MMAX
      DO 150 MQ = 1, MMAX
      DO 150 MJ = 1, MMAX
      BES1(MP,MQ,MJ) = 0.0
      BES2(MP,MQ,MJ) = 0.0
      BES3(MP,MQ,MJ) = 0.0
      L1 = MP - 1
      L2 = MQ - 1
      L3 = MJ - 1
      LM = L1 + L2
      LN = L1 + L3
      MN = L2 + L3
      IF ((L3.EQ.LM) .OR. (L2.EQ.LN) .OR. (L1.EQ.MN))  GO TO 160
      GO TO 150
  160 IF (NMAX .EQ. 0)  GO TO 165
      A1 = RJROOT(MP,NMAX)
      A2 = RJROOT(MQ,NMAX)
      A3 = RJROOT(MJ,NMAX)
      GO TO 167
  165 A1 = 0.0
      A2 = 0.0
      A3 = 0.0
  167 CALL RADIAL(1,L1,L2,L3,A1,A2,A3,RESULT)
      BES1(MP,MQ,MJ) = RESULT
      CALL RADIAL(2,L1,L2,L3,A1,A2,A3,RESULT)
      BES2(MP,MQ,MJ) = RESULT
      CALL RADIAL(3,L1,L2,L3,A1,A2,A3,RESULT)
      BES3(MP,MQ,MJ) = RESULT
  150 CONTINUE
C
  170 DO 200 NJ = 1, NJMAX
      DO 200 NP = 1, NJMAX
      DO 200 NQ = 1, NJMAX
C
      CD1(NJ,NP,NQ) = (0.0,0.0)
      CD2(NJ,NP,NQ) = (0.0,0.0)
C
      DO 210 J = 1, 2
      CALL AZIMTL(J,NP,NQ,NJ,RESULT)
      AZI(J) = RESULT
      TANINT(J) = CMPLX(RESULT,0.0)
  210 CONTINUE
C
      IF (AZI(1)) 220, 225, 220
  225 IF (AZI(2)) 220, 200, 220
C
  220 IF (KN .EQ. 0)  GO TO 222
      L1 = M(NP)
      L2 = M(NQ)
```

```
      L3 = M(NJ)
      A1 = S(NP)
      A2 = S(NQ)
      A3 = S(NJ)
      GO TO 244
C
  222 MP = M(NP) + 1
      MQ = M(NQ) + 1
      MJ = M(NJ) + 1
      RADINT(1) = CMPLX(BES1(MP,MQ,MJ),0.0)
      RADINT(2) = CMPLX(BES2(MP,MQ,MJ),0.0)
      RADINT(3) = CMPLX(BES3(MP,MQ,MJ),0.0)
C
  244 DO 240 J = 1, 3
      IF (KN .EQ. 0)  GO TO 242
      CALL RADIAL (J,L1,L2,L3,A1,A2,A3,RESULT)
      RADINT(J) = CMPLX(RESULT,0.0)
  242 DO 240 NC = 1,4
      CALL AXIAL2(J,NC,NP,NQ,NJ,ZE,CRSLT)
      AXINT(NC,J) = CRSLT
  240 CONTINUE
C
C
      DO 250 J = 1,4
      T1 = G1 * CSSQ(NP) * AXINT(J,1)
      T2 = G1 * AXINT(J,3)
      D1 = AXINT(J,1) * TANINT(1) * RADINT(3)
      D2 = AXINT(J,1) * TANINT(2) * RADINT(2)
      D3 = AXINT(J,2) * TANINT(1) * RADINT(1)
      D4 = (T2 - T1) * TANINT(1) * RADINT(1)
      DCOEF = (0.5,0.0) *  (D1 + D2 + D3 + D4)/CNORM(NJ)
      IF (J .EQ. 1)  CD1(NJ,NP,NQ) = (1.0,-1.0) * DCOEF
      IF (J .EQ. 2)  CD2(NJ,NP,NQ) = (1.0,1.0) * DCOEF
      IF (J .EQ. 3)  CD3(NJ,NP,NQ) = (1.0,1.0) * DCOEF
      IF (J .EQ. 4)  CD4(NJ,NP,NQ) = (1.0,-1.0) * DCOEF
  250 CONTINUE
  200 CONTINUE
C
C     ******************************************************************
C
C     CALCULATE COEFFICIENTS FOR EQUIVALENT REAL SYSTEM.
C
  402 DO 350 NJ = 1, NJMAX
      NEWJ = (2 * NJ) - 1
      NEWJ1 = NEWJ + 1
      DO 350 NP = 1, NJMAX
      NEWP = (2 * NP) - 1
      NEWP1 = NEWP + 1
C
C     COEFFICIENTS OF LINEAR TERMS.
```

```
      CCR = REAL(CC(1,NJ,NP))
      CCI = AIMAG(CC(1,NJ,NP))
      C1(NEWJ,NEWP) = CCR
      C1(NEWJ,NEWP1) = -CCI
      C1(NEWJ1,NEWP) = CCI
      C1(NEWJ1,NEWP1) = CCR
      DO 360 KC = 1,3
      CCR = REAL(CC(KC+1,NJ,NP))
      CCI = AIMAG(CC(KC+1,NJ,NP))
      C(KC,NEWJ,NEWP) = CCR
      C(KC,NEWJ,NEWP1) = -CCI
      C(KC,NEWJ1,NEWP) = CCI
      C(KC,NEWJ1,NEWP1) = CCR
  360 CONTINUE
C
C     COEFFICIENTS OF NONLINEAR TERMS.
      IF (NONLIN .EQ. 0) GO TO 350
      DO 370 NQ = 1, NJMAX
      NEWQ = (2 * NQ) - 1
      NEWQ1 = NEWQ + 1
      CD1R = REAL(CD1(NJ,NP,NQ))
      CD1I = AIMAG(CD1(NJ,NP,NQ))
      CD2R = REAL(CD2(NJ,NP,NQ))
      CD2I = AIMAG(CD2(NJ,NP,NQ))
      CD3R = REAL(CD3(NJ,NP,NQ))
      CD3I = AIMAG(CD3(NJ,NP,NQ))
      CD4R = REAL(CD4(NJ,NP,NQ))
      CD4I = AIMAG(CD4(NJ,NP,NQ))
      D(NEWJ,NEWP,NEWQ) = CD1R + CD2R + CD3R + CD4R
      D(NEWJ,NEWP,NEWQ1) = -CD1I + CD2I - CD3I + CD4I
      D(NEWJ,NEWP1,NEWQ) = -CD1I - CD2I + CD3I + CD4I
      D(NEWJ,NEWP1,NEWQ1) = -CD1R + CD2R + CD3R - CD4R
      D(NEWJ1,NEWP,NEWQ) = CD1I + CD2I + CD3I + CD4I
      D(NEWJ1,NEWP,NEWQ1) = CD1R - CD2R + CD3R - CD4R
      D(NEWJ1,NEWP1,NEWQ) = CD1R + CD2R - CD3R - CD4R
      D(NEWJ1,NEWP1,NEWQ1) = -CD1I + CD2I + CD3I - CD4I
  370 CONTINUE
  350 CONTINUE
C
C     ********************************************************************
C
C     COMPUTE COEFFICIENTS FOR THE EQUATIONS WHICH ARE DECOUPLED
C     IN THE SECOND DERIVATIVES.
C
      DO 405 KC = 1, 4
      KMAX(KC) = 0
  405 CONTINUE
C
C     CALCULATE INVERSE OF THE MATRIX C1(I,J).
      JMAX = NJMAX
```

```
      NJMAX = 2 * NJMAX
C
C
      V(1) = 1
      CALL GJR(C1,20,20,NJMAX,0,$500,JC,V)
C
C     USE INVERSE TO CALCULATE DECOUPLED COEFFICIENTS.
C
      DO 410 NP = 1, NJMAX
C
C     LINEAR COEFFICIENTS.
      DO 420 NJ = 1, NJMAX
      DO 420 KC = 1, 3
      TS(KC,NJ) = 0.0
      DO 420 K = 1, NJMAX
      TS(KC,NJ) = TS(KC,NJ) + C1(NJ,K) * C(KC,K,NP)
  420 CONTINUE
      DO 430 NJ = 1, NJMAX
      DO 430 KC = 1, 3
      C(KC,NJ,NP) = TS(KC,NJ)
      ABSVAL = ABS(C(KC,NJ,NP))
      IF (ABSVAL .GE. SM1)  KMAX(KC) = KMAX(KC) + 1
  430 CONTINUE
C
C     NONLINEAR COEFFICIENTS.
      IF (NONLIN .EQ. 0)  GO TO 410
      DO 415 NQ = 1, NJMAX
      DO 440 NJ = 1, NJMAX
      TSQ(NJ) = 0.0
      DO 440 K = 1, NJMAX
      TSQ(NJ) = TSQ(NJ) + C1(NJ,K) * D(K,NP,NQ)
  440 CONTINUE
      DO 445 NJ = 1, NJMAX
      D(NJ,NP,NQ) = TSQ(NJ)
      ABSVAL = ABS(D(NJ,NP,NQ))
      IF (ABSVAL .GE. SM2)  KMAX(4) = KMAX(4) + 1
  445 CONTINUE
  415 CONTINUE
C
  410 CONTINUE
C
C     *****************************************************************
C     OUTPUT.
C
      IF (NOUT .GE. 2) GO TO 455
C
C     PRINTED OUTPUT.
      WRITE (6,6001)  (TITLE(I), I = 1, 72)
      WRITE (6,6002)  GAMMA, UE, RLD, ZCOMB
      IF (NDROPS .EQ. 0)  WRITE (6,6020)
```

```
      IF (NDROPS .EQ. 1)  WRITE (6,6021)
      IF (NOZZLE .EQ. 0)  WRITE (6,6012)
      WRITE (6,6004)
      DO 310 J = 1, JMAX
      WRITE (6,6003)  NAME(J), J, L(J), M(J), N(J), NS(J),
     1                S(J), SJ(J), B(J), YNOZ(J)
  310 CONTINUE
      IF (NONLIN .EQ. 0)  WRITE (6,6013)
C
C     OUTPUT OF LINEAR COEFFICIENTS.
      DO 320 KC = 1, 3
      IF (KC .EQ. 1)  WRITE (6,6005)
      IF (KC .EQ. 2)  WRITE (6,6006)
      IF (KC .EQ. 3)  WRITE (6,6007)
      WRITE (6,6008)  (J, J = 1, NJMAX)
      WRITE (6,6014)
      DO 320 NJ = 1, NJMAX
      WRITE (6,6009)  NJ, (C(KC,NJ,NP), NP = 1, NJMAX)
  320 CONTINUE
C
C     OUTPUT OF NONLINEAR COEFFICIENTS.
      IF (NONLIN .EQ. 0)  GO TO 452
      DO 400 NJ = 1, NJMAX
      WRITE (6,6010)  NJ
      WRITE (6,6011)  (J, J = 1, NJMAX)
      WRITE (6,6015)
      DO 400 NP = 1, NJMAX
      WRITE (6,6009) NP, (D(NJ,NP,NQ),  NQ = 1, NJMAX)
  400 CONTINUE
  452 IF (NOUT .EQ. 0)  GO TO 4
C
  455 IF (NOUT .EQ. 3)  GO TO 480
C
C    WRITE COEFFICIENTS ON FASTRAND FILE.
C
      WRITE (9,7001)  GAMMA, UE, ZE, ZCOMB, NDROPS, NJMAX
C
      DO 450 J = 1, JMAX
      WRITE (9,7002)  J, L(J), M(J), N(J), NS(J), S(J), SJ(J),
     1           NAME(J)
  450 CONTINUE
C
      DO 457 J = 1, JMAX
      WRITE (9,7006) J, YNOZ(J), B(J)
  457 CONTINUE
C
      DO 460 KC = 1, 3
      WRITE (9,7003) KMAX(KC)
      DO 460 NJ = 1, NJMAX
      DO 460 NP = 1, NJMAX
```

```
      ABSVAL = ABS(C(KC,NJ,NP))
      IF (ABSVAL .GE. SM1)  WRITE (9,7004)  NJ, NP, C(KC,NJ,NP)
  460 CONTINUE
C
      WRITE (9,7003) KMAX(4)
      IF (NONLIN .EQ. 0)  GO TO 4
      DO 470 NJ = 1, NJMAX
      DO 470 NP = 1, NJMAX
      DO 470 NQ = 1, NJMAX
      ABSVAL = ABS(D(NJ,NP,NQ))
      IF (ABSVAL .GE. SM2)  WRITE (9,7005) NJ, NP, NQ, D(NJ,NP,NQ)
  470 CONTINUE
      GO TO 4
C
C     PUNCHED CARD OUTPUT.
C
  480 PUNCH 7001  GAMMA, UE, ZE, ZCOMB, NDROPS, NJMAX
C
      DO 482 J = 1, JMAX
      PUNCH 7002  J, L(J), M(J), N(J), NS(J), S(J), SJ(J),
     1            NAME(J)
  482 CONTINUE
C
      DO 484 J = 1, JMAX
      PUNCH 7006 J, YNOZ(J), B(J)
  484 CONTINUE
C
      DO 486 KC = 1, 3
      PUNCH 7003 KMAX(KC)
      DO 486 NJ = 1, NJMAX
      DO 486 NP = 1, NJMAX
      ABSVAL = ABS(C(KC,NJ,NP))
      IF (ABSVAL .GE. SM1)  PUNCH 7004  NJ, NP, C(KC,NJ,NP)
  486 CONTINUE
C
      PUNCH 7003 KMAX(4)
      IF (NONLIN .EQ. 0)  GO TO 4
      DO 488 NJ = 1, NJMAX
      DO 488 NP = 1, NJMAX
      DO 488 NQ = 1, NJMAX
      ABSVAL = ABS(D(NJ,NP,NQ))
      IF (ABSVAL .GE. SM2)  PUNCH 7005 NJ, NP, NQ, D(NJ,NP,NQ)
  488 CONTINUE
      GO TO 4
C
C     ERROR EXIT
  500 IF (JC(1))  510, 510, 520
  510 JC(1) = ABS(JC(1))
      WRITE (6,6017) JC(1)
      GO TO 4
```

```
  520 WRITE (6,6018) JC(1)
      GO TO 4
  600 CONTINUE
C
C     ****************************************************************
C
C     FORMAT SPECIFICATIONS.
 5000 FORMAT (72A1)
 5001 FORMAT (4F10.0,2I5)
 5002 FORMAT (5I5,1X,A4)
 5003 FORMAT (I5,2F10.0)
 5004 FORMAT (4I5)
 5005 FORMAT (2F10.0)
 6001 FORMAT (1H1,1X,72A1//)
 6002 FORMAT (2X,8HGAMMA = ,F5.2,5X,5HUE = ,F5.2,5X,6HL/D = ,F8.5,
    1          5X,8HZCOMB = ,F5.2/)
 6003 FORMAT (2X,A4,5I5,6F10.5/)
 6004 FORMAT (2X////2X,29HNAME    J    L    M    N   NS,7X,3HSMN,3X,
    1          7HJM(SMN),7X,3HEPS,7X,3HETA,8X,2HYR,8X,2HYI//)
 6005 FORMAT (1H1,45H  DECOUPLED COEFFICIENT OF B(P):     C(1,J,P)///)
 6006 FORMAT (1H1,44H  DECOUPLED COEFFICIENT OF THE DERIVATIVE OF,
    1          6H B(P):,5X,8HC(2,J,P)///)
 6007 FORMAT (1H1,39H  DECOUPLED COEFFICIENT OF THE RETARDED,
    1          20H DERIVATIVE OF B(P):,5X,8HC(3,J,P)///)
 6008 FORMAT (7X,1HP,I8,9I12)
 6009 FORMAT (2X//2X,I3,3X,10F12.6)
 6010 FORMAT (1H1,42H  DECOUPLED COEFFICIENT OF B(P) * DB(Q)/DT,
    1          19H IN EQUATION FOR B(,I2,1H)///)
 6011 FORMAT (7X,1HQ,I8,9I12)
 6012 FORMAT (2X,19HQUASI-STEADY NOZZLE/)
 6013 FORMAT (2X//2X,24HLINEAR COEFFICIENTS ONLY)
 6014 FORMAT (4X,1HJ)
 6015 FORMAT (4X,1HP)
 6017 FORMAT (1H1,31H OVERFLOW DETECTED, LAST ROW = ,I5)
 6018 FORMAT (1H1,34H SINGULARITY DETECTED, LAST ROW = ,I5)
 6020 FORMAT (2X,'DROPLET MOMENTUM SOURCE NEGLECTED'/)
 6021 FORMAT (2X,'DROPLET MOMENTUM SOURCE INCLUDED'/)
 7001 FORMAT (4F10.5,2I5)
 7002 FORMAT (5I5,2F10.5,1X,A4)
 7003 FORMAT (I5)
 7004 FORMAT (2I5,F15.6)
 7005 FORMAT (3I5,F15.6)
 7006 FORMAT (I5,4F10.5)
      END
```

```
      SUBROUTINE EIGVAL(L,SMN,GAMMA,ZE,YAMPL,YPHASE,RESULT)
C
      COMPLEX   RESULT
      COMMON  /BLK1/ GSQ, ABSQ, ALBET, SMNSQ
C
C     ******************************************************************
C
C     THIS SUBROUTINE COMPUTES THE COMPLEX AXIAL ACOUSTIC EIGENVALUES
C     FOR A CYLINDRICAL CHAMBER WITH A NOZZLE AND STORES THEM IN
C     RESULT.
C     THE EIGENVALUES ARE COMPUTED BY MEANS OF NEWTONS METHOD.
C
C     THE INPUT PARAMETERS ARE AS FOLLOWS:
C     L IS THE AXIAL MODE NUMBER.
C     SMN IS THE DIMENSIONLESS ACOUSTIC FREQUENCY.
C     GAMMA IS THE SPECIFIC HEAT RATIO.
C     ZE IS THE LENGTH-TO-RADIUS RATIO.
C     YAMPL IS THE NOZZLE AMPLITUDE FACTOR.
C     YPHASE IS THE NOZZLE PHASE SHIFT IN DEGREES.
C
C     ******************************************************************
C
      PI = 3.1415927
      ERR = 0.0000001
C
      IF (YAMPL) 5, 60, 5
C     CALCULATE CONSTANTS.
    5 PHASE = YPHASE * PI/180.0
      ALPHA = YAMPL * COS(PHASE)
      BETA  = YAMPL * SIN(PHASE)
      GSQ = GAMMA * GAMMA
      ABSQ = (ALPHA * ALPHA) - (BETA * BETA)
      ALBET = ALPHA * BETA
      SMNSQ = SMN * SMN
C
C     ASSIGN INITIAL GUESS FOR EIGENVALUE.
      IF (L .EQ. 0)  GO TO 45
      RL = L
      PHI = PI/2.0 + PHASE
      XM = RL * PI/ZE
      A = YAMPL/ZE
      XO = XM + A*COS(PHI)
      YO = A*SIN(PHI)
      GO TO 47
   45 PHI = PI/4.0 + 0.5*PHASE
      A = YAMPL * 10.0/ZE
      XO = A * COS(PHI)
      YO = A * SIN(PHI)
C
C     ITERATION USING NEWTONS METHOD FOR A SYSTEM OF TWO EQUATIONS
```

```
C       IN TWO UNKNOWNS.
   47 L1 = 0
      X = X0
      Y = Y0
   40 CALL FCNS(X,Y,ZE,F,G,FX,FY,GX,GY)
      IF (L1 .EQ. 40)  GO TO 50
      RJFG = (FX * GY) - (GX * FY)
      IF (RJFG) 20, 30, 20
   20 DELTAX = (-F * GY   +   G * FY)/RJFG
      DELTAY = (-G * FX   +   F * GX)/RJFG
      L1 = L1 + 1
      X = X + DELTAX
      Y = Y + DELTAY
C
C       TEST FOR CONVERGENCE.
      IF (ABS(DELTAX) .GE. ERR .OR. ABS(DELTAY) .GE. ERR)  GO TO 40
      GO TO 10
C
C       WARNING MESSAGES
   30 WRITE (6,6005)
      GO TO 10
   50 WRITE (6,6006)
      GO TO 10
C
C       CASE OF HARD WALL (YAMPL = 0).
   60 RL = L
      X = RL * PI/ZE
      Y = 0.0
C
   10 RESULT = CMPLX(X,Y)
C
C       FORMAT SPECIFICATIONS.
 6005 FORMAT (2X//2X,16HJACOBIAN IS ZERO//)
 6006 FORMAT (2X//2X,35HFAILED TO CONVERGE IN 40 ITERATIONS//)
      RETURN
      END
```

```
      SUBROUTINE FCNS(X,Y,ZE,F,G,FX,FY,GX,GY)
C
C     THIS SUBROUTINE COMPUTES THE FUNCTIONS F(X,Y) AND G(X,Y)
C     AND THEIR PARTIAL DERIVATIVES WITH RESPECT TO X AND Y.
C
      COMMON  /BLK1/ GSQ, ABSQ, ALBET, SMNSQ
C
C     COMPUTE THE TRIGONOMETRIC FUNCTIONS, THE HYPERBOLIC FUNCTIONS
C     AND THEIR SQUARES.
C
      I = 1
      ARGX = ZE * X
      ARGY = ZE * Y
   10 SX = SIN(ARGX)
      CX = COS(ARGX)
      SHY = SINH(ARGY)
      CHY = COSH(ARGY)
      IF (I .EQ. 2)  GO TO 20
      SXSQ = SX * SX
      CXSQ = CX * CX
      SHYSQ = SHY * SHY
      CHYSQ = CHY * CHY
      ARGX = 2.0 * ARGX
      ARGY = 2.0 * ARGY
      I = 2
      GO TO 10
C
C     COMPUTE TRANSCENDENTAL FUNCTIONS AND THEIR DERIVATIVES
C
   20 FF = (SXSQ * CHYSQ) - (CXSQ * SHYSQ)
      GG = (CXSQ * CHYSQ) - (SXSQ * SHYSQ)
      HH = 0.25 * SX * SHY
      FFX = ZE * SX * CHY
      GGY = ZE * CX * SHY
      FFY = -GGY
      GGX = -FFX
      HHX = 0.5 * GGY
      HHY = 0.5 * FFX
C
C     COMPUTE FACTORS
      XYSQ = (X * X) - (Y * Y)
      XY = X * Y
      SMNXY = SMNSQ + XYSQ
      F1 = (ABSQ * SMNXY) - (4.0 * ALBET * XY)
      F2 = (ALBET * SMNXY) + (ABSQ * XY)
      G1 = (ABSQ * SMNXY) + (4.0 * ALBET * XY)
      FX1 = (2.0 * X * ABSQ) - (4.0 * ALBET * Y)
      FX2 = (2.0 * X * ALBET) + (ABSQ * Y)
      FY1 = (-2.0 * Y * ABSQ) - (4.0 * ALBET * X)
      FY2 = (-2.0 * Y * ALBET) + (ABSQ * X)
```

```
      GX1 = (2.0 * X * ABSQ) + (4.0 * ALBET * Y)
      GY1 = (-2.0 * Y * ABSQ) + (4.0 * ALBET * X)
C
C     COMPUTE F(X,Y) AND G(X,Y)
C
      F = (XYSQ * FF) - (4.0 * XY * HH)
     1    + GSQ * ((F1 * GG) + (4.0 * F2 * HH))
      G = (XYSQ * HH) + (XY * FF)
     1    + GSQ * ((F2 * GG) - (G1 * HH))
C
C     COMPUTE THE PARTIAL DERIVATIVES OF F AND G
C
      FX = (2.0 * X * FF) + (XYSQ * FFX)
     1    -4.0 * ((Y * HH) + (XY * HHX))
     2    + GSQ * ((FX1 * GG) + (F1 * GGX)
     3    + (4.0 * FX2 * HH) + (4.0 * F2 * HHX))
      FY = (-2.0 * Y * FF) + (XYSQ * FFY)
     1    -4.0 * ((X * HH) + (XY * HHY))
     2    + GSQ * ((FY1 * GG) + (F1 * GGY)
     3    + (4.0 * FY2 * HH) + (4.0 * F2 * HHY))
      GX = (2.0 * X * HH) + (XYSQ * HHX)
     1    + (Y * FF) + (XY * FFX)
     2    + GSQ * ((FX2 * GG) + (F2 * GGX)
     3    -(GX1 * HH) - (G1 * HHX))
      GY = (-2.0 * Y * HH) + (XYSQ * HHY)
     1    + (X * FF) + (XY * FFY)
     2    + GSQ * ((FY2 * GG) + (F2 * GGY)
     3    -(GY1 * HH) - (G1 * HHY))
      RETURN
      END
```

```
      SUBROUTINE AXIAL1(NOPT,NP,NJ,UE,ZE,ZCOMB,RESULT)
C
C
C     THIS SUBROUTINE CALCULATES THE INTEGRAL OVER THE INTERVAL
C     (0,ZE) OF THE FOLLOWING FUNCTIONS ACCORDING TO THE VALUE
C     OF NOPT:
C
C     NOPT = 1      Z(NP) * ZC(NJ)
C     NOPT = 2      ZPP(NP) * ZC(NJ)
C     NOPT = 3      UP * Z(NP) * ZC(NJ)
C     NOPT = 4      U * ZP(NP) * ZC(NJ)
C
C     IN THE ABOVE EQUATIONS:
C     Z(NP) IS THE AXIAL ACOUSTIC EIGENFUNCTION OF INDEX NP.
C     Z(NJ) IS THE AXIAL ACOUSTIC EIGENFUNCTION OF INDEX NJ.
C     ZC IS THE COMPLEX CONJUGATE OF THE AXIAL EIGENFUNCTION.
C     ZP AND ZPP ARE THE FIRST AND SECOND DERIVATIVES OF THE
C     AXIAL EIGENFUNCTIONS RESPECTIVELY.
C     U IS THE STEADY STATE VELOCITY DISTRIBUTION AND UP IS ITS
C     AXIAL DERIVATIVE.
C     THE VELOCITY DISTRIBUTION IS COMPUTED BY THE SUBROUTINE UBAR.
C
C
      REAL      MAG
      COMPLEX   CI, CZE, BP, BJ, T1, T2, CH, F1, F2, F3, CZ, ARG,
     1          S1, S2, S3, RESULT, FUNCT(500), B(10)
      COMMON    B
C
      CI = (0.0,1.0)
      CZE = CMPLX(ZE,0.0)
      BP = B(NP)
      BJ = CONJG(B(NJ))
C
      IF (NOPT .GT. 2)  GO TO 50
C     CALCULATE INTEGRALS BY MEANS OF ANALYTICAL EXPRESSIONS FOR
C     NOPT = 1 AND NOPT = 2.
      ARG = (BP + BJ) * CI
      MAG = CABS(ARG)
      IF (MAG) 20, 25, 20
   20 T1 = CSINH(ARG*CZE)/ARG
      GO TO 30
   25 T1 = CZE
   30 ARG = (BP - BJ) * CI
      MAG = CABS(ARG)
      IF (MAG) 35, 40, 35
   35 T2 = CSINH(ARG*CZE)/ARG
      GO TO 45
   40 T2 = CZE
   45 RESULT = (T1 + T2) * (0.5,0.0)
      IF (NOPT .EQ. 2)  RESULT = -B(NP) * B(NP) * RESULT
      GO TO 100
C
```

```
C     NUMERICAL EVALUATION OF INTEGRALS FOR NOPT = 3 AND NOPT = 4.
C
C     COMPUTE STEP SIZE FOR SIMPSON INTEGRATION.
   50 N = 50
      RN = N
      RESULT = (0.0,0.0)
      IC = ZCOMB
      IC = 2 - IC
C
      DO 90 J = 1, IC
      IF (J .EQ. 1)  H = ZCOMB * ZE/RN
      IF (J .EQ. 2)  H = (1.0 - ZCOMB) * ZE/RN
      IF (J .EQ. 1)  ZO = 0.0
      IF (J .EQ. 2)  ZO = ZCOMB * ZE
      NP1 = N + 1
      CH = CMPLX(H,0.0)
C
C     COMPUTE INTEGRANDS.
      DO 60 I = 1, NP1
      STEP = I - 1
      Z = (STEP * H) + ZO
      IF ((I.EQ.1) .AND. (J.EQ.2)) Z = Z + H/100.0
      IF (NOPT .EQ. 3)  CALL UBAR(2,UE,ZE,ZCOMB,Z,F)
      IF (NOPT .EQ. 4)  CALL UBAR(1,UE,ZE,ZCOMB,Z,F)
      F1 = CMPLX(F,0.0)
      CZ = CMPLX(Z,0.0)
      ARG = CI * BP
      IF (NOPT .EQ. 3)  F2 = CCOSH(ARG*CZ)
      IF (NOPT .EQ. 4)  F2 = ARG * CSINH(ARG*CZ)
      ARG = CI * BJ
      F3 = CCOSH(ARG*CZ)
      FUNCT(I) = F1 * F2 * F3
   60 CONTINUE
C
C     PERFORM SIMPSON INTEGRATION.
      NM1 = N - 1
      S1 = FUNCT(1) + FUNCT(NP1)
      S2 = (0.0,0.0)
      S3 = (0.0,0.0)
      DO 70 I = 2, N, 2
      S2 = S2 + FUNCT(I)
   70 CONTINUE
      DO 80 I = 3, NM1, 2
      S3 = S3 + FUNCT(I)
   80 CONTINUE
      RESULT = RESULT +
     1         CH * (S1 + (4.0,0.0)*S2 + (2.0,0.0)*S3)/(3.0,0.0)
   90 CONTINUE
C
  100 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE UBAR(NOPT,UE,ZE,ZCOMB,Z,RESULT)
C
C     THIS SUBROUTINE CALCULATES THE STEADY STATE VELOCITY
C     DISTRIBUTION FOR UNIFORMLY DISTRIBUTED COMBUSTION COMPLETED AT
C     Z = ZCOMB * ZE WHERE:
C     UE IS THE EXIT MACH NUMBER.
C     ZE IS THE DIMENSIONLESS LENGTH.
C     Z IS THE AXIAL COORDINATE.
C
C     IF NOPT = 1 THE DISTRIBUTION IS CALCULATED.
C     IF NOPT = 2 THE DERIVATIVE IS CALCULATED.
C     IF NOPT = 3 THE SECOND DERIVATIVE IS CALCULATED.
C
C
      ECZ = ZCOMB * ZE
      GO TO (10,20,30), NOPT
   10 IF (Z .LE. ECZ) RESULT = UE * Z/ECZ
      IF (Z .GT. ECZ) RESULT = UE
      GO TO 40
   20 IF (Z .LE. ECZ) RESULT = UE/ECZ
      IF (Z .GT. ECZ) RESULT = 0.0
      GO TO 40
   30 RESULT = 0.0
   40 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE AZIMTL(NOPT,NP,NQ,NJ,RESULT)
C
      DIMENSION    NFCN(3), SG(2)
      COMMON  /BLK2/    M(10), NS(10)
C
C     ******************************************************************
C
C     THIS SUBROUTINE CALCULATES THE INTEGRAL OVER THE INTERVAL
C     (O, 2*PI)  OF THE FOLLOWING FUNCTIONS ACCORDING TO THE VALUE
C     OF NOPT:
C
C     NOPT = 1      TH(NP) * TH(NQ) * TH(NJ)
C
C     NOPT = 2      THP(NP) * THP(NQ) * TH(NJ)
C
C     IN THE ABOVE EQUATIONS:
C     TH(NP), TH(NQ), AND TH(NJ) ARE THE TANGENTIAL EIGENFUNCTIONS
C     AND NP, NQ, AND NJ ARE THEIR INDICES.
C     THP IS THE DERIVATIVE OF THE TANGENTIAL EIGENFUNCTIONS.
C
C     IF NS = 1  TH = SIN(M*THETA)
C     IF NS = 2  TH = COS(M*THETA)
C
C     ******************************************************************
C
      RESULT = 0.0
      FACTOR = 1.0
      PI = 3.1415927
C
C     DISTINGUISH BETWEEN SINES AND COSINES.
      DO 10 K1 = 1, 3
      NFCN(K1) = 1
   10 CONTINUE
      IF  (NS(NJ).EQ.2) NFCN(3) = 2
      IF (NOPT .EQ. 2)  GO TO 20
      IF  (NS(NP).EQ.2) NFCN(1) = 2
      IF  (NS(NQ).EQ.2) NFCN(2) = 2
      GO TO 30
   20 IF  (NS(NP).EQ.1) NFCN(1) = 2
      IF  (NS(NQ).EQ.1) NFCN(2) = 2
      DO 40 K1 = 1,2
      SG(K1) = 1.0
      IF (NFCN(K1) .EQ. 1)  SG(K1) = -1.0
   40 CONTINUE
      FACTOR = SG(1) * SG(2) * M(NP) * M(NQ)
C
   30 NSUM = 0
      DO 50 K1 = 1, 3
      NSUM = NSUM + NFCN(K1)
   50 CONTINUE
C
```

```
      IF ((NSUM .EQ. 3) .OR. (NSUM .EQ. 5))  GO TO 60
      IF (NSUM .EQ. 4)  GO TO 70
      IF (NSUM .EQ. 6)  GO TO 80
C
   70 KOPT = 2
      IF (NFCN(1) .EQ. 2)  GO TO 72
      GO TO 74
   72 LL = M(NP)
      MM = M(NQ)
      NN = M(NJ)
      GO TO 90
   74 IF (NFCN(2) .EQ. 2)  GO TO 76
      GO TO 78
   76 LL = M(NQ)
      MM = M(NP)
      NN = M(NJ)
      GO TO 90
   78 LL = M(NJ)
      MM = M(NP)
      NN = M(NQ)
      GO TO 90
C
   80 KOPT = 1
      LL = M(NP)
      MM = M(NQ)
      NN = M(NJ)
C
C     COMPUTE VALUES OF THE INTEGRALS.
C
   90 IF ((LL.NE.0) .AND. (MM.NE.0) .AND. (NN.NE.0))  GO TO 101
      GO TO 103
  101 LM = LL + MM
      LN = LL + NN
      MN = MM + NN
      IF ((NN.EQ.LM) .OR. (MM.EQ.LN))    RESULT = PI/2.0
      IF (LL .EQ. MN)  GO TO 102
      GO TO 104
  102 IF (KOPT .EQ. 1)  RESULT = PI/2.0
      IF (KOPT .EQ. 2)  RESULT = -PI/2.0
      GO TO 104
  103 IF ((LL.EQ.0) .AND. (MM.EQ.0) .AND. (NN.EQ.0))  GO TO 105
      IF ((KOPT.EQ.1) .AND. (NN.EQ.0) .AND. (LL.EQ.MM)) RESULT = PI
      IF ((KOPT.EQ.1) .AND. (MM.EQ.0) .AND. (LL.EQ.NN)) RESULT = PI
      IF ((LL .EQ. 0) .AND. (MM .EQ. NN))  RESULT = PI
      GO TO 104
  105 IF (KOPT .EQ. 1)  RESULT = 2.0 * PI
  104 CONTINUE
      RESULT = FACTOR * RESULT
   60 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE RADIAL(NOPT,L,M,N,A,B,C,RESULT)
C
C     THIS SUBROUTINE CALCULATES THE INTEGRAL OVER THE INTERVAL
C     (0,1) OF THE FOLLOWING PRODUCTS OF THREE BESSEL FUNCTIONS:
C
C     NOPT = 1   JL(A*R) * JM(B*R) * JN(C*R) * R
C
C     NOPT = 2   JL(A*R) * JM(B*R) * JN(C*R)/R
C
C     NOPT = 3   JPL(A*R) * JPM(B*R) * JN(C*R) * R
C
C     JL  IS THE BESSEL FUNCTION OF FIRST KIND OF ORDER L
C     JPL IS THE DERIVATIVE OF JL WITH RESPECT TO R
C     L, M, N  ARE NON-NEGATIVE INTEGERS
C     A, B, C  ARE REAL NUMBERS
C
      DIMENSION  FUNCT(200)
      DOUBLE PRECISION  DN, DH, DSTEP, DR, ARG1, ARG2, ARG3,
     1                  BES1, BES2, BES3, BESH, BESL, PROD,
     2                  FUNCT, BESLIM, S1, S2, S3
C
      NN = 100
      DN = NN
      DH = 1.0/DN
      NP1 = NN + 1
C
      DO 10 I = 1, NP1
      DSTEP = I - 1
      DR = DH * DSTEP
      ARG1 = A * DR
      ARG2 = B * DR
      ARG3 = C * DR
C
      CALL JBES(N,ARG3,BES3,$500)
      IF (NOPT .EQ. 3)  GO TO 101
      CALL JBES(L,ARG1,BES1,$500)
      CALL JBES(M,ARG2,BES2,$500)
      GO TO 102
  101 IF (L .EQ. 0)  GO TO 103
      CALL JBES(L+1,ARG1,BESH,$500)
      CALL JBES(L-1,ARG1,BESL,$500)
      BES1 = A * (BESL - BESH)/2.0
      GO TO 104
  103 CALL JBES(1,ARG1,BES1,$500)
      BES1 = -BES1 * A
  104 IF (M .EQ. 0)  GO TO 105
      CALL JBES(M+1,ARG2,BESH,$500)
      CALL JBES(M-1,ARG2,BESL,$500)
      BES2 = B * (BESL - BESH)/2.0
      GO TO 102
```

```
  105 CALL JBES(1,ARG2,BES2,$500)
      BES2 = -BES2 * B
  102 PROD = BES1 * BES2 * BES3
C
      IF (NOPT .EQ. 2)  GO TO 110
      FUNCT(I) = PROD * DR
      GO TO 10
  110 IF (I .EQ. 1)  GO TO 111
      FUNCT(I) = PROD/DR
      GO TO 10
  111 BESLIM = 0.0
      IF ((L.EQ.1) .AND. (M.EQ.0) .AND. (N.EQ.0))  BESLIM = A/2.0
      IF ((L.EQ.0) .AND. (M.EQ.1) .AND. (N.EQ.0))  BESLIM = B/2.0
      IF ((L.EQ.0) .AND. (M.EQ.0) .AND. (N.EQ.1))  BESLIM = C/2.0
      FUNCT(I) = BESLIM
   10 CONTINUE
C
      NM1 = NN - 1
      S1 = FUNCT(1) + FUNCT(NP1)
      S2 = 0.0
      S3 = 0.0
      DO 20 I = 2, NN, 2
      S2 = S2 + FUNCT(I)
   20 CONTINUE
      DO 30 I = 3, NM1, 2
      S3 = S3 + FUNCT(I)
   30 CONTINUE
      RESULT = DH * (S1 + 4.0*S2 + 2.0*S3)/3.0
      GO TO 501
  500 WRITE (6, 6000)
 6000 FORMAT (1H1,10HERROR JBES)
  501 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE AXIAL2(NOPT,NCONJ,NP,NQ,NJ,ZE,RESULT)
C
C
C     THIS SUBROUTINE CALCULATES THE INTEGRAL OVER THE INTERVAL
C     (0,ZE) OF THE FOLLOWING FUNCTIONS ACCORDING TO THE VALUES
C     OF NOPT AND NCONJ:
C
C     FOR NCONJ = 1 AND:
C     NOPT = 1      Z(NP) * Z(NQ) * ZC(NJ)
C     NOPT = 2      ZP(NP) * ZP(NQ) * ZC(NJ)
C     NOPT = 3      ZPP(NP) * Z(NQ) * ZC(NJ)
C
C     FOR NCONJ = 2 AND:
C     NOPT = 1      Z(NP) * ZC(NQ) * ZC(NJ)
C     NOPT = 2      ZP(NP) * ZPC(NQ) * ZC(NJ)
C     NOPT = 3      ZPP(NP) * ZC(NQ) * ZC(NJ)
C
C     FOR NCONJ = 3 AND:
C     NOPT = 1      ZC(NP) * Z(NQ) * ZC(NJ)
C     NOPT = 2      ZPC(NP) * ZP(NQ) * ZC(NJ)
C     NOPT = 3      ZPPC(NP) * Z(NQ) * ZC(NJ)
C
C     FOR NCONJ = 4 AND:
C     NOPT = 1      ZC(NP) * ZC(NQ) * ZC(NJ)
C     NOPT = 2      ZPC(NP) * ZPC(NQ) * ZC(NJ)
C     NOPT = 3      ZPPC(NP) * ZC(NQ) * ZC(NJ)
C
C     IN THE ABOVE EQUATIONS:
C     Z(NP), Z(NQ), AND Z(NJ) ARE THE AXIAL ACOUSTIC EIGENFUNCTIONS
C     AND NP, NQ, AND NJ ARE THEIR INDICES.
C     ZP IS THE FIRST DERIVATIVE OF THE AXIAL EIGENFUNCTIONS.
C     ZPP IS THE SECOND DERIVATIVE OF THE AXIAL EIGENFUNCTIONS.
C     ZC AND ZPC ARE COMPLEX CONJUGATES OF Z AND ZP RESPECTIVELY.
C
C
      REAL       MAG
      COMPLEX    CI, CF, CZE, BP, BQ, BJ, SUM, RESULT,
     1           ARG(4), FUNCT(4), B(10)
      COMMON     B
C
C     CALCULATE INTEGRALS BY MEANS OF ANALYTICAL EXPRESSIONS.
      CI = (0.0,1.0)
      CF = (0.25,0.0)
      CZE = CMPLX(ZE,0.0)
      BP = B(NP)
      BQ = B(NQ)
      BJ = CONJG(B(NJ))
      IF ((NCONJ .EQ. 2) .OR. (NCONJ .EQ. 4))  BQ = CONJG(BQ)
      IF (NCONJ .GT. 2)  BP = CONJG(BP)
      ARG(1) = (BP + BQ + BJ) * CI
```

```
      ARG(2) = (BP + BQ - BJ) * CI
      ARG(3) = (BP - BQ + BJ) * CI
      ARG(4) = (BP - BQ - BJ) * CI
      DO 10 J = 1,4
      MAG = CABS(ARG(J))
      IF (MAG) 12, 15, 12
   12 FUNCT(J) = CSINH(ARG(J)*CZE)/ARG(J)
      GO TO 10
   15 FUNCT(J) = CZE
   10 CONTINUE
      IF (NOPT .EQ. 2)  GO TO 30
      SUM = FUNCT(1) + FUNCT(2) + FUNCT(3) + FUNCT(4)
      RESULT = CF * SUM
      IF (NOPT .EQ. 3)  RESULT = -BP * BP * RESULT
      GO TO 50
   30 SUM = FUNCT(1) + FUNCT(2) - FUNCT(3) - FUNCT(4)
      RESULT = -CF * BP * BQ * SUM
   50 CONTINUE
      RETURN
      END
```

APPENDIX D

PROGRAM LCYC3D: A USER'S MANUAL

General Description

Using the three-dimensional second-order theory described in this report Program LCYC3D calculates the nonlinear stability characteristics of a cylindrical combustion chamber with distributed combustion and a conventional nozzle. The response of the burning rate to pressure oscillations is described by Crocco's time-lag model. For given values of the operating parameters (i.e., n, $\bar{\tau}$, γ, $\bar{u}_e$, and L/D), a given series expansion, and a given initial disturbance Program LCYC3D integrates Eqs. (C-38) to obtain the time behavior of the unknown mode-amplitude functions (i.e., $B_j(t)$). From this information a time history of the pressure oscillation is determined. The program determines the final amplitude of the pressure oscillation attained in a linearly unstable engine (i.e., limit-cycle amplitude). Since the second-order analysis does not predict "triggering", however, the threshold amplitude above which a finite amplitude disturbance can trigger instability in a linearly stable engine (i.e., triggering limit) is not calculated by Program LCYC3D. For either transient or limit-cycle conditions, the program prints out time histories of both pressure and axial velocity perturbations from which the amplitude, frequency, and wave shapes can be determined. The option to produce plotted output using a CALCOMP plotter is also provided.

Program Structure

A flow chart for Program LCYC3D is given in Fig. (D-1). This program performs the following operations: (1) reads the input data, (2) calculates the initial conditions, (3) numerically integrates the differential equations, (4) tests for limit cycles (optional), and (5) prints and plots the resulting solutions.

The inputs to the program include the data generated by Program COEFFS3D, the combustion parameters n and $\bar{\tau}$, various control numbers, and a description of the initial disturbance. The data from COEFFS3D is read first and then printed out. Next the space dependent coefficients appearing in the series

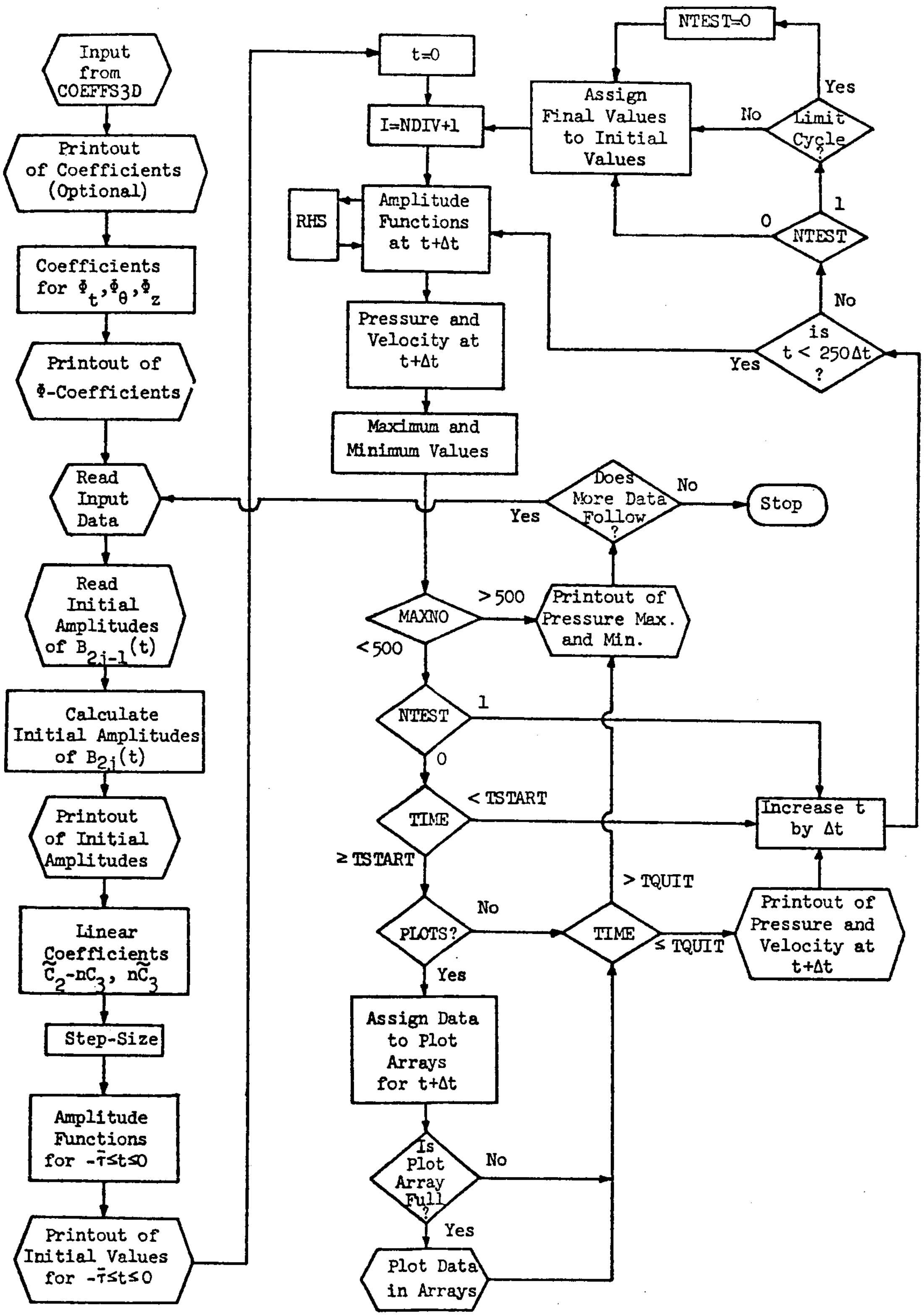


Figure D-1. Flow Chart for Program LCYC3D.

expansions for Φ_t, Φ_θ, and Φ_z are computed and printed out. These coefficients are calculated by Subroutine PHICFS for use in the computation of the pressure and axial velocity perturbations. The remaining input data is then read, and following program execution, control is returned to this point (see Fig. D-1) so that several cases (i.e., different values of n and $\bar{\tau}$) may be run for a given set of coefficients generated by COEFFS3D.

After input of the initial amplitudes of the real parts (i.e., $B_{2j-1}(t)$) of the complex amplitude functions, the initial amplitudes of the imaginary parts (i.e., $B_{2j}(t)$) are calculated such that the nozzle admittance condition is satisfied for $-\bar{\tau} \le t \le 0$. These amplitudes are then printed out. Next the integration step-size, Δt, is calculated such that the interval $-\bar{\tau} \le t \le 0$ is divided into NDIV equal increments. Assuming a sinusoidal initial disturbance, the initial amplitudes of $B_{2j-1}(t)$ and $B_{2j}(t)$ are used to calculate these functions and their derivatives at each of the NDIV + 1 discrete points in $-\bar{\tau} \le t \le 0$. These values are needed in order to start the numerical solution of the differential equations (i.e., Eqs. (C-38)). The initial values of the amplitude functions are stored in the array U(I,J) where the index I varies from 1 ($t = -\bar{\tau}$) to NDIV + 1 ($t = 0$) and the index J identifies the function. The corresponding initial values of the pressure and velocity perturbations are then printed out. This section also calculates the coefficients $\tilde{C}_2(j,p) - n\tilde{C}_3(j,p)$ and $n\tilde{C}_3(j,p)$ which are the coefficients of dB_p/dt and $d\left[B_p(t - \bar{\tau})\right]/dt$ in Eqs. (C-38).

After the starting values are calculated, Eqs. (C-38) are solved using a modified form of the fourth order Runge Kutta method. Starting at $t = 0$ (I = NDIV+1), the amplitude functions at $t + \Delta t$ are calculated, using the Subroutine RHS to evaluate the functions $f_j(B_1, B_2, \ldots B_{2N})$ on the right hand sides of Eqs. (C-38). The amplitude functions and the coefficients from PHICFS are then used to compute the pressure and axial velocity perturbations by Subroutine PRSVEL. The values of the amplitude functions at $t + \Delta t$ are stored in U(I + 1,J), while the pressure and axial velocity perturbations are stored in the arrays PRESS(NPRES) and AXVEL(NPRES) where NPRES specifies the locations in the chamber where the data is calculated. Pressure data at one location (specified by NLOC) is also stored in the array PRS(I + 1). After checking for maximum and minimum values of U(I,J) and PRS(I), the data may be printed out (if NTEST = 0 and TSTART $\leq$ t $\leq$ TQUIT) or stored in plot arrays as desired. The time is then increased by Δt (i.e., I is increased by 1) and the calculations are repeated. This process continues until 250 integration steps have been computed (t = 250Δt), after which transfer is made to the limit-cycle section.

In the limit-cycle section a test for a limit-cycle is made if NTEST = 1. If the test is satisfied, NTEST is set to zero so that no further tests will be made and the results can be printed or plotted. In either case the final values (for 250-NDIV $\leq$ I $\leq$ 250) replace the initial values (for 1 $\leq$ I $\leq$ NDIV+1) in the arrays U(I,J) and PRS(I), I is again assigned the value NDIV+1, and another 250 integration steps are calculated. This process continues until one of the following conditions is satisfied: (1) NTEST = 0 and t $>$ TQUIT, (2) a limit-cycle is reached and t > TQUIT, and (3) more than 250 cycles of the pressure oscillation have been computed (MAXNO > 500). At this point the numerical calculations are terminated and the time history of the pressure amplitude (maxima and minima) are printed out and/or plotted as desired.

As can be seen from Fig. D-1 the output is not confined to a single section of the program but is produced in several different sections. Thus data is printed out or plotted shortly after it is calculated, which greatly reduces the amount of core storage required. All plots are generated by Subroutine GRAPHS which uses standard Univac 1108 plot routines.[20]

FORTRAN listings of Program LCYC3D and Subroutines PHICFS, PRSVEL, RHS, and GRAPHS are provided at the end of this appendix.

Input Data

A precise definition of the input data required to run the computer program is given below. This input data consists of three parts: (1) the control number NOUTCF, (2) the parameters and coefficients generated by Program COEFFS3D and (3) the data describing the cases to be run (see Fig. D-1). For each input case the following information must be provided: (1) the combustion parameters n and $\bar{\tau}$; (2) a series of control numbers; and (3) information describing the initial disturbance.

The control number NOUTCF determines whether the coefficients from COEFFS3D will be printed, and it appears on the first card of input. This card is followed by the coefficient deck generated by COEFFS3D and the data describing the cases to be run. Since the coefficient data has already been described in Appendix C, it will be omitted from the following detailed description of the input. As in Appendix C the location number refers to the columns of the card. Again three formats are used for input: "A" indicates alphanumeric characters, "I" indicates integers, and "F" indicates real numbers with a decimal point. For the "I" formats the values are placed in fields of five locations, while a field of ten locations is used with the "F" formats. In either case the numbers must be placed in the rightmost locations of the allocated field.

No. of Cards	Location	Type	Input Item	Comments
1	1-5	I	NOUTCF	If 0: coefficients are not printed out. If 1: linear coefficients only are printed out. If 2: all coefficients are printed out.
1	1-72	A	TITLE	Title used to label plots.
1	1-10	F	EN	Interaction index, n.
	11-20	F	TAU	Time-lag, $\bar{\tau}$.
	21-30	F	H	Time-increment for numerical integration, Δt. *
	31-40	F	TSTART	Time at which output of solutions begins.
	41-50	F	TQUIT	Time at which output of solutions ends.
1	1-5	I	NTEST	If 0: compute transient behavior. If 1: compute limit-cycle behavior.
	6-10	I	JMODE	Identifies the amplitude function used to test for limit-cycles.

* This value is adjusted slightly by the program to divide the interval $-\bar{\tau} \leq t \leq 0$ into NDIV equal parts.

No. of Cards	Location	Type	Input Item	Comments
	11-15	I	NLOC	Determines location for wall pressure maxima and minima. If 1: $z = 0$, $\theta = 0°$ If 2: $z = 0$, $\theta = 45°$ If 3: $z = 0$, $\theta = 90°$
	16-20	I	NTERMS	Number of amplitude functions given initial values.
	21-25	I	NPZ	Determines how secondary instability zones are handled. If 0: all instability zones retained. If 1: secondary zones eliminated.
	26-30	I	NOUT	Determines output. If 0: printed output only. If $1 \leq$ NOUT ≤ 6: both printed and plotted output, NOUT gives number of last plot produced.

If $1 \leq$ NOUT ≤ 6 the following two cards are read:

No. of Cards	Location	Type	Input Item	Comments
1	1-10	F	YHI(1)	Maximum ordinate for pressure plots.
	11-20	F	YHI(5)	Maximum ordinate for velocity plots.
	21-30	F	YLAB(1)	Interval for ordinate labeling of pressure plots.
	31-40	F	YLAB(5)	Interval for ordinate labeling of velocity plots.
1	1-5	I	ITICY(1)	Number of ordinate tic marks for pressure plots.
	6-10	I	ITICY(5)	Number of ordinate tic marks for velocity plots.
	11-15	I	NFIRST	Gives the number of the first plot produced.
	16-20	I	NOMIT	If 0: amplitude plot produced. If 1: amplitude plot omitted.

No. of Cards	Location	Type	Input Item	Comments
End of input for 1 ≤ NOUT ≤ 6.				
NTERMS	1-5	I	J	Identifies complex amplitude function.
	6-15	I	AST	Amplitude of sin(ωt) term in initial conditions.
	16-25	I	ACT	Amplitude of cos(ωt) term in initial conditions.

The input data describing the cases to be run is given on a series of three or more cards. These cards are preceded by a title card which gives a title (TITLE) to be used to identify any plots produced by the run. This title appears before the first plot generated and does not appear on the printed output. The title card is included only for the first case of the run; on all subsequent cases it is omitted.

The first card of the series gives the interaction index, n, and the time-lag, $\bar{\tau}$, for the motor under consideration (EN and TAU); the time-increment, Δt, used in the numerical integrations (H); and the times (TSTART and TQUIT) at which output begins and ends. For all cases considered in this report a time-increment (dimensionless) of H = 0.050 was used, which gives about 70 steps per cycle for the 1T mode. For $\bar{\tau} = 1.7$ this input value was adjusted by the program to obtain H = 0.04857 which divides $-\bar{\tau} \leq t \leq 0$ into 35 equal parts. For transient cases (NTEST = 0) printed output is given for TSTART ≤ t ≤ TQUIT. When the limit-cycle behavior is calculated (NTEST = 1), TSTART and TQUIT are measured from the time at which the limit-cycle is reached, t_{LC}. Thus the limit-cycle solutions are printed out for $(t_{LC} + \text{TSTART}) \leq t \leq (t_{LC} + \text{TQUIT})$. Two or three cycles of limit-cycle data for the 1T mode are obtained with TSTART = 0 and TQUIT = 10. For plotted output, the time axis is always 10 units long, therefore (TQUIT - TSTART) > 10 to obtain plots.

The second card of the series gives the control numbers, NTEST, JMODE, NLOC, NTERMS, NPZ, and NOUT. The task to be performed by Program LCYC3D is specified by NTEST. If NTEST = 0 the transient behavior (growth or decay) of the pressure oscillation is determined, while for NTEST = 1 the program searches for a limit-cycle amplitude. JMODE identifies the "principal" series term, the amplitude function used in the limit-cycle test. This is usually the lowest frequency mode (i.e., 1T or 1L) in the approximating series expansion. NLOC gives the location at which the amplitude-time history (maxima and minima) of the wall pressure perturbation is calculated. The number of complex series terms $A_j(t)$ receiving initial values is specified by NTERMS, while all other series terms are initially zero. The parameter NPZ determines how the secondary instability zones (phantom zones) are handled by Program LCYC3D. For NPZ = 1 the phantom zones are eliminated by dropping the combustion terms for a given mode when $\bar{\tau} > \bar{\tau}_{cut}$ where:

$$\bar{\tau}_{cut} = \frac{2\pi}{\omega} = 2\pi \left[S_{mn}^2 + \frac{\ell^2 \pi^2}{z_e^2} \right]^{-\frac{1}{2}} \tag{D-1}$$

A similar procedure was used in the axial instability studies by Lores and Zinn.[3] The transverse instability data presented herein was obtained with NPZ = 0, while NPZ = 1 was used in the axial instability studies to facilitate comparison with the results of Ref. (3). The last control number NOUT determines which plots, if any, are produced. For NOUT = 0 no plots are produced. For 1 ≤ NOUT ≤ 6, NOUT gives the number of the last plot produced, where the plots are numbered as given in Table D-1 below:

Table D-1. Numbering of Plots.

No. of Plot (NPLOT)	Quantity Plotted	Axial Location	Azimuthal Coordinate
1	Pressure	Injector	0°
2	"	"	45°
3	"	"	90°
4	"	Nozzle	0°
5	Axial Velocity	"	0°
6	Nozzle Boundary Term	"	0°

The nozzle boundary term given on the last plot is discussed later in this appendix.

If plots are produced, two additional cards are needed to give the maximum and minimum values of the variables to be plotted, YHI(NPLOT) and YLO(NPLOT); the intervals for ordinate labeling (YLAB(NPLOT)); and the number of ordinate tic marks, ITICY(NPLOT). All of the plots are symmetric about the time-axis so that YLO(NPLOT) = -YHI(NPLOT), and ITICY(NPLOT) must be negative to obtain the centerline. Since the ordinate scales and labeling are the same for all pressure plots (NPLOT = 1,2,3,4) this data is read for NPLOT = 1 only; likewise the data for the last two plots is read for NPLOT = 5 only. In addition NFIRST gives the number of the first plot produced, giving additional control over the number of plots produced. NOMIT determines whether a plot of pressure amplitude versus time (location specified by NLOC) is produced.

The remaining cards give the initial amplitudes of the complex series terms, $A_j(t)$, needed to start the numerical integration. Only the amplitudes of the real parts, $B_{2j-1}(t)$, are given on these cards, while the amplitudes of the imaginary parts, $B_{2j}(t)$, are determined from the nozzle admittance condition. For each value of J the amplitudes AST and ACT are assigned to the arrays AS(NP) and AC(NP) where NP = 2J - 1. The computation of the amplitudes of the imaginary parts, AS(NP + 1) and AC(NP + 1), is discussed later. The initial values of the series terms are then calculated from the formula:

$$B_p(t) = AS(NP)\sin(\omega_p t) + AC(NP)\cos(\omega_p t) \qquad (-\bar{\tau} \le t \le 0) \qquad \text{(D-2)}$$

where ω_p is the acoustic frequency. The derivatives, dB_p/dt, are also required for starting the numerical integration; they are obtained simply by differentiating Eq. (D-2).

The proper input for pure standing and pure spinning single-mode initial disturbances is given as follows. For a standing mode, only the $\cos(m\theta)$ terms are retained in the series and NTERMS = 1. A single card is read giving the amplitude of the initial disturbance. For a spinning mode, both $\sin(m\theta)$ and cos(mθ) terms are included in the series expansion. It is convenient to pair these terms such that the index J corresponds to a sin(mθ) term and J + 1 corresponds to a cos(mθ) term. For an initial disturbance of amplitude A spinning in the counterclockwise direction (θ increasing), NTERMS = 2 and two cards are read giving the following data:

$$\begin{aligned} J &: \quad AST = A \text{ and } ACT = 0 \\ J + 1 &: \quad AST = 0 \text{ and } ACT = A \end{aligned} \tag{D-3}$$

In both cases above initial amplitudes are required only for the mode initially present, and the initial amplitudes of all other modes included in the series expansion are zero.

The proper input for Program LCYC3D will be illustrated with the following example. Assuming that the velocity potential Φ is expressed in terms of the 1R, 1T, and 2T modes*, it is desired to determine the limit-cycle behavior of a linearly unstable engine ($n = 0.57486$, $\bar{\tau} = 1.7$, $\bar{u}_e = 0.2$, $L/D = 0.5$) with a nozzle admittance of $A = 0.02$ and $\varphi = 45°$. Sample input is given for the case of a spinning 1T mode disturbance of amplitude 0.3. The principal series term is the cos(mθ) term for the 1T mode (i.e., $B_{011}(t)$), thus JMODE = 2. Plots are desired for the pressure, axial velocity, and nozzle boundary condition at the nozzle entrance, thus NOUT = 6 and NFIRST = 4.

To run the case described above the data deck must be assembled as follows. The card specifying NOUTCF is followed by the coefficient deck produced by Program COEFFS3D; in this example it contains the information given in the sample output for COEFFS3D shown in Appendix C. The coefficient deck is followed by the data for the case to be run as shown in the sample input below:

---

* This is the same case used to illustrate Program COEFFS3D.

Table D-2. Sample Input.

```
    2
1T,2T,1R SPINNING, A=0.02 PHI=45 DEGREES.
   0.57486       1.7      0.05       0.0      11.0
    1    2    1    2    0    6
       1.5      0.03       0.5      0.01
  -61  -61    4    0
    1       0.3       0.0
    2       0.0       0.3
```

Coefficients in Series for Φ_t, Φ_θ, and Φ_z.

As seen from Eq. (13) the real parts of the time and space derivatives of the velocity potential (i.e., φ_t, φ_r, φ_θ, φ_z) are needed in order to compute the pressure perturbation. Differentiating the complex series expansion given by Eq. (9) and evaluating at the chamber wall (r = 1) gives the following expansions:

$$\Phi_t = \sum_{p=1}^{N} \frac{dA_p}{dt} Z_p(z)\Theta_p(\theta)R_p(1) = \sum_{p=1}^{N} C_t(p,z,\theta) \frac{dA_p}{dt} \qquad \text{(D-4)}$$

$$\Phi_\theta = \sum_{p=1}^{N} A_p(t)Z_p(z)\Theta_p'(\theta)R_p(1) = \sum_{p=1}^{N} C_\theta(p,z,\theta)A_p(t) \qquad \text{(D-5)}$$

$$\Phi_z = \sum_{p=1}^{N} A_p(t) Z_p'(z) \Theta_p(\theta) R_p(1) = \sum_{p=1}^{N} C_z(p,z,\theta) A_p(t) \qquad \text{(D-6)}$$

where the complex coefficients C_t, C_θ, and C_z are functions of z and θ. The quantity, Φ_r, is not needed since $\Phi_r = 0$ at the chamber wall. The complex coefficients C_t, C_θ, and C_z are calculated by Subroutine PHICFS and are assigned to the variables, C1, C2, and C3 respectively. The coefficients in the series expansions for the corresponding real parts (i.e., φ_t, φ_θ, φ_z) are related to the complex coefficients by:

$$\begin{aligned} C_t'(2p-1, z, \theta) &= \mathrm{Re}\left[C_t(p, z, \theta)\right] \\ C_t'(2p, z, \theta) &= -\mathrm{Im}\left[C_t(p, z, \theta)\right] \end{aligned} \qquad \text{(D-7)}$$

where similar relations hold for C_θ' and C_z'. The real coefficients are stored in the arrays CFT(NPRES, NP), CFTH(NPRES, NP), and CFZ(NPRES, NP) where NPRES determines the location in the chamber as given in Table D-3 below:

Table D-3. Chamber Locations for Pressure Calculations.

NPRES	Axial Location (z)	Azimuthal Location (θ)
1	0	0°
2	0	45°
3	0	90°
4	z_e	0°
5	z_e	45°
6	z_e	90°

Initial Amplitudes

The initial amplitudes of the real parts of the complex series terms (i.e., $B_{2j-1}(t)$) are specified in the input to the program. The initial amplitudes of the imaginary parts (i.e., $B_{2j}(t)$), however, are calculated such that the nozzle admittance condition is satisfied for $-\bar{\tau} \leq t \leq 0$. This is done by introducing the linear expressions for u′ and p′ into the nozzle admittance relation and assuming periodic solutions. This yields a set of linear algebraic equations relating the amplitudes of the real and imaginary parts of the complex series terms. For given values of the amplitudes of the real parts, AS(NP) and AC(NP), these equations are solved to obtain the amplitudes of the imaginary parts, AS(NP + 1) and AC(NP + 1). The following formulas are used in this calculation.

$$\begin{aligned} AS(NJ + 1) &= -(r_2 a_1 - r_1 a_2) / (a_1^2 + a_2^2) \\ AC(NJ + 1) &= (r_1 a_1 + r_2 a_2) / (a_1^2 + a_2^2) \end{aligned} \qquad \text{(D-8)}$$

where

$$\begin{aligned} r_1 &= a_3 \left[AC(NJ)\right] - a_4 \left[AS(NJ)\right] \\ r_2 &= -a_4 \left[AC(NJ)\right] - a_3 \left[AS(NJ)\right] \end{aligned} \qquad \text{(D-9)}$$

and

$$\begin{aligned} a_1 &= (1 + \gamma Y_r \bar{u}_e) CFZ(NPRES,\ NJ{+}1) - \gamma Y_i \omega_j CFT(NPRES,\ NJ{+}1) \\ a_2 &= \gamma Y_r \omega_j CFT(NPRES,\ NJ{+}1) + \gamma Y_i \bar{u}_e CFZ(NPRES,\ NJ{+}1) \\ a_3 &= -(1 + \gamma Y_r \bar{u}_e) CFZ(NPRES,\ NJ) + \gamma Y_i \omega_j CFT(NPRES,\ NJ) \\ a_4 &= \gamma Y_r \omega_j CFT(NPRES,\ NJ) + \gamma Y_i \bar{u}_e CFZ(NPRES,\ NJ) \end{aligned} \qquad \text{(D-10)}$$

In Eqs. (D-8) through (D-10) ω_j is the acoustic frequency and CFT and CFZ are the coefficients in the series for φ_t and φ_z computed previously. The above conditions are applied at a pressure anti-node for each series term, therefore NPRES = 4 ($z = z_e$, $\theta = 0°$) for a cos(mθ) term and NPRES = 6 ($z = z_e$, $0 = 90°$) for a sin(mθ) term.

For nozzles with phase shifts of $\varphi = 90°$ and $\varphi = 270°$ the quantity $a_1^2 + a_2^2$ vanishes and Eqs. (D-8) become indeterminate. In these cases the amplitudes of the imaginary parts are given by:

$$AS(NJ + 1) = AC(NJ)$$
$$AC(NJ + 1) = AS(NJ) \qquad \text{(D-11)}$$

which provides a good approximation to the nozzle admittance condition.

Integration of the Differential Equations

For purposes of numerical integration Eqs. (C-38) are written as an equivalent system of first order differential equations as follows:

$$\frac{dB_j}{dt} = B_j' \qquad \text{(D-12)}$$

$$\frac{dB_j'}{dt} = f_j(B_p, B_p') \qquad \text{(D-13)}$$

where the dependent variables are now B_j and B_j'. These equations are solved numerically using the fourth order Runge-Kutta method. Due to the presence of retarded variables in Eqs. (D-12) and (D-13) the formulas (see Ref. 21) used in the Runge-Kutta method must be slightly modified.

The appropriate formulas for applying the Runge-Kutta method to problems involving a time-delay are readily obtained by considering a single equation of the following form:

$$\frac{dx}{dt} = f(x,t) + g[x(t - \bar{\tau})] \qquad \text{(D-14)}$$

Noting that at any step of the integration the value of $x(t - \bar{\tau})$ has already been determined from previous steps, the function g can be considered to be a known function of time g(t).

Since x(t) is computed only at discrete points $x_n(t_n)$ it is desired that the retarded variable $x(t_n - \bar{\tau})$ will coincide with such previously computed points. This can be accomplished by choosing the step-size Δt such that it divides the time-lag $\bar{\tau}$ into k equal increments. Thus $\bar{\tau} = k\Delta t$ and the Runge-Kutta formulas which apply to Eq. (D-14) can now be written as:

$$x_{n+1} = x_n + \frac{1}{6}\left(k_1 + 2k_2 + 2k_3 + k_4\right)$$

$$k_1 = \left\{f(x_n, t_n) + g(x_{n-k})\right\}\Delta t$$

$$k_2 = \left\{f(x_n + k_1/2,\ t_n + \Delta t/2) + g(x_{n-k+\frac{1}{2}})\right\}\Delta t \quad \text{(D-15)}$$

$$k_3 = \left\{f(x_n + k_2/2,\ t_n + \Delta t/2) + g(x_{n-k+\frac{1}{2}})\right\}\Delta t$$

$$k_4 = \left\{f(x_n + k_3,\ t_n + \Delta t) + g(x_{n-k+1})\right\}\Delta t$$

Equations (D-15) are readily extended to handle the system of equations given by Eqs. (D-12) and (D-13). It is seen from Eqs. (D-15) that k values of the dependent variables prior to the initial values are needed to start the integration.

Although the initial wave shape can be an arbitrary function of time, it is assumed that initially the mode-amplitudes are sinusoidal functions of time oscillating with the natural frequency ω_j. Thus each mode-amplitude function is expressed in the following form:

$$B_j(t) = AS(J)\sin(\omega_j t) + AC(J)\cos(\omega_j t) \quad \text{(D-16)}$$

$$B_j'(t) = \omega_j\left[AS(J)\cos(\omega_j t) - AC(J)\sin(\omega_j t)\right]$$

where $-\bar{\tau} \le t \le 0$.

In Program LCYC3D both the functions $B_j(t)$ and the derivatives $B_j'(t)$ are stored in the same array U(I,J). The $B_j(t)$ (N functions) are stored in the first half of the array ($1 \le J \le N$), while the remaining space ($N + 1 \le J \le 2N$) is used to store the values of $B_j'(t)$. Thus for a given value of j ($1 \le j \le N$), $B_j(t)$ is stored in U(I,J) and $B_j'(t)$ is stored in U(I,J + N). In addition the retarded variables $B_j'(t - \bar{\tau})$ are stored in the array RV(J,K) as follows:

$$RV(J,1) = B_j'(t - \bar{\tau})$$

$$RV(J,2) = RV(J,3) = B_j'(t - \bar{\tau} + \Delta t/2) \qquad \text{(D-17)}$$

$$RV(J,4) = B_j'(t - \bar{\tau} + \Delta t)$$

The values of $B_j'(t - \bar{\tau} + \Delta t/2)$ are computed from $B_j'(t - \bar{\tau})$, $B_j'(t - \bar{\tau} + \Delta t)$, and $B_j'(t - \bar{\tau} + 2\Delta t)$ using a three-point interpolation.

Pressure and Axial Velocity Perturbations

From the calculated time dependence of the series terms Program LCYC3D computes the dimensionless pressure perturbation, p', with the aid of Eqs. (D-4) through (D-6) and either Eq. (13) for NDROPS = 0 or Eq. (A-6) for NDROPS = 1. The pressure is calculated at the injector face ($z = 0$) and the nozzle entrance plane ($z = z_e$) for three angular positions along the periphery of the chamber (i.e., $r = 1$; $\theta = 0°, 45°, 90°$). The results are stored in the array PRESS(NPRES) where NPRES gives the location according to Table D-3. The axial velocity perturbation at the nozzle entrance, u_e', is calculated for $\theta = 0°, 45°, 90°$ using the relation $u' = \varphi_z$ and Eq. (D-6), and the results are stored in AXVEL(K), where K = NPRES-3. In addition the quantity, $Re\left[-\gamma Y \Phi_t\right]$, is calculated at the nozzle entrance for $\theta = 0°$ and assigned to the variable YPHI. From Eq. (2) it is seen that YPHI is the axial velocity at the nozzle entrance (i.e., u'_e) if the nozzle admittance condition is exactly satisfied. Since the solutions generated by Program LCYC3D are approximate, the difference between u'_e and YPHI is a measure of the accuracy of this approximation at the nozzle boundary.

Maximum and Minimum Values

In order to determine the transient behavior and limit-cycle amplitudes it is necessary to follow the growth or decay of the amplitudes of the series terms and the pressure perturbation. The maxima and minima of the principal series term (specified by JMODE) are assigned to the array UMAX(MAXNO) where MAXNO is a counter variable. For the pressure perturbation, maximum and minimum values at the location specified by NLOC are stored in PMAX(MAXP), and the corresponding times of maximum and minimum are stored in TIMAX(MAXP). Since the solutions are calculated only at discrete points, the maximum and minimum values are computed using a three-point interpolation scheme.

Calculation of Limit-Cycle Amplitude

A limit-cycle amplitude is calculated by specifying an initial disturbance and continuing the step-by-step integration of Eqs. (D-12) and (D-13) until a periodic solution is obtained; that is, the amplitude of the oscillation remains essentially constant. The test for convergence to a limit cycle is performed upon a single series term, usually the most important term in the series, in the following manner. After the first 500 integration steps, usually about 10 cycles for the 1T mode, the amplitude of the principal series term A_1 is compared with its amplitude after 250 integration steps A_0. If the change in amplitude $|A_1 - A_0|$ is greater than the maximum permissible change ϵ, the calculations are continued and the change in amplitude during the next 250 integration steps is calculated. The process is repeated until $|A_k - A_{k-1}| < \epsilon$ at which point the computation is terminated. The amplitudes used in the above calculations are determined by averaging the absolute values of UMAX(MAXNO) over the last two complete cycles for each 250 integration steps. A value of $\epsilon = 0.001$ is used in Program LCYC3D which gives sufficient accuracy for most cases.

Output

Printed Output. The printed output produced by Program LCYC3D consists of the five sections discussed below.

Section 1 is a restatement of the input from Program COEFFS3D. It includes the following information: (a) the ratio of specific heats (GAMMA), the mean flow Mach number at the nozzle entrance (UE), the dimensionless chamber length (ZE), the length of the combustion zone as a fraction of the chamber length (ZCOMB), and the number of series terms (real) NJMAX; (b) a statement regarding the presence or absence of the droplet momentum source; (c) the parameters which describe and identify each term in the series expansion; (d) the nozzle admittance (YR and YI) and the axial acoustic eigenvalue (EPS and ETA) for each series term; (e) the nonzero linear coefficients, C(KC, NJ, NP); and (f) the nonzero nonlinear coefficients, D(NJ, NP, NQ). The nonlinear coefficients are omitted from the output for NOUTCF = 1, and no coefficients are printed out for NOUTCF = 0.

Section 2 gives the coefficients needed for computation of the wall pressure waveforms; that is, the coefficients in the series for φ_t, φ_θ, and φ_z. These are given for each of the NJMAX series terms at each of the six locations specified by NPRES (see Table D-3).

Section 3 gives the initial amplitudes (AS(J) and AC(J)) of all series terms included in the assumed initial disturbance. This section also states whether the limit-cycle behavior is calculated and whether plots are produced.

Section 4 gives the time-dependent solutions for the following quantities: (a) the injector pressure perturbation at $\theta = 0°$, $45°$, $90°$; (b) the nozzle pressure perturbation at $\theta = 0°$, $45°$, $90°$; (c) the nozzle axial velocity perturbation at $\theta = 0°$, $45°$, $90°$; and (d) the nozzle boundary term, $\mathrm{Re}\left[-\gamma Y \Phi_t\right]$, at $\theta = 0°$. This output is given in two parts: (1) the initial values for $-\bar{\tau} \le t \le 0$ and (2) the solutions for $t_i \le t \le t_f$, where t_i and t_f are determined by the input parameters TSTART and TQUIT (see discussion on Input). On the first page of each part a heading gives the interaction index, n, and the time-lag, $\bar{\tau}$, and the chamber parameters, γ, $\bar{u}_e$, and L/D.

Section 5 gives the time history of the pressure amplitude (maximum and minimum values) for the chamber location specified by NLOC. This information is printed as an array of number pairs giving the value of the pressure maximum or minimum (upper number) and the corresponding time of maximum or minimum (lower number). This information is useful in determining the growth (or decay) rate of the transient solutions, and it provides a check on the convergence of the solution to a limit-cycle.

Plotted Output. According to the values of NOUT and NFIRST the pressure and axial velocity waveforms given in Section 4 of the printed output may be plotted using a Calcomp plotter. The data over the dimensionless time interval for printed output, $t_i \leq t \leq t_f$, is plotted in sections of 10 units in length beginning at $t = t_i$. Thus for each quantity plotted, N plots are produced where N is the largest multiple of 10 contained in the interval $t_i \leq t \leq t_f$. The data left over (i.e., for $t_i + 10N \leq t \leq t_f$) is not plotted. All quantities to be plotted for a given time interval are plotted before proceeding to the next time interval.

The data given in Section 5 of the printed output (pressure maxima only) is also plotted if NOUT > 0 and NOMIT = 0. The abscissa and ordinate ranges for this plot are not specified in the input, but are calculated such that all of the data falls within these ranges. This plot is always the last plot produced.

All of the above plots are scaled to fit on standard $8\frac{1}{2}$" x 11" paper and scissor-lines are plotted for trimming plots to this size. The data is plotted as individual points using a small circle symbol, and all of the values computed during the given time interval are plotted. Before the first plot is produced the identifying title (see Input) is printed.

Sample Output. The following sample output illustrates the printed and plotted output produced by Program LCYC3D for the sample input given in Table D-2.

Table D-4. Sample Output, Section 1.

GAMMA = 1.200 UE = .200 ZE = 1.00000 ZCOMB = 1.00 NJMAX = 10

DROPLET MOMENTUM SOURCE IS NEGLECTED

NAME	J	L	M	N	NS	SMN	JM(SMN)
A011	1	0	1	1	1	1.84118	.58187
B011	2	0	1	1	2	1.84118	.58187
A021	3	0	2	1	1	3.05424	.48650
B021	4	0	2	1	2	3.05424	.48650
B001	5	0	0	1	2	3.83171	-.40276

J	YR	YI	EPS	ETA
1	.01414	.01414	.08122	.19451
2	.01414	.01414	.08122	.19451
3	.01414	.01414	.10617	.25115
4	.01414	.01414	.10617	.25115
5	.01414	.01414	.11993	.28170

NUMBER OF COEFFICIENTS C(1,NJ,NP) IS 10

```
C(1, 1, 1) =    3.39060
C(1, 2, 2) =    3.39060
C(1, 3, 3) =    3.39060
C(1, 4, 4) =    3.39060
C(1, 5, 5) =    9.33021
C(1, 6, 6) =    9.33021
C(1, 7, 7) =    9.33021
C(1, 8, 8) =    9.33021
C(1, 9, 9) =   14.68491
C(1,10,10) =   14.68491
```

NUMBER OF COEFFICIENTS C(2,NJ,NP) IS 10

```
C(2, 1, 1) =     .26153
C(2, 2, 2) =     .26153
C(2, 3, 3) =     .26153
C(2, 4, 4) =     .26153
C(2, 5, 5) =     .26457
C(2, 6, 6) =     .26457
C(2, 7, 7) =     .26457
C(2, 8, 8) =     .26457
C(2, 9, 9) =     .26654
C(2,10,10) =     .26654
```

NUMBER OF COEFFICIENTS C(3,NJ,NP) IS 10

```
C(3, 1, 1) =     .24000
C(3, 2, 2) =     .24000
C(3, 3, 3) =     .24000
C(3, 4, 4) =     .24000
C(3, 5, 5) =     .24000
C(3, 6, 6) =     .24000
C(3, 7, 7) =     .24000
```

Table D-4. (Continued)

```
C(3, 8, 8) =      .24000
C(3, 9, 9) =      .24000
C(3,10,10) =      .24000

NUMBER OF COEFFICIENTS D(NJ,NP,NQ) IS    50

D( 1, 1, 7) =   -1.73504
D( 1, 1, 9) =   -2.33866
D( 1, 3, 5) =    1.73504
D( 1, 5, 3) =    1.49783
D( 1, 7, 1) =   -1.49783
D( 1, 9, 1) =   -1.96281
D( 2, 2, 8) =   -1.73505
D( 2, 2,10) =   -2.33867
D( 2, 4, 6) =    1.73505
D( 2, 6, 4) =    1.49784
D( 2, 8, 2) =   -1.49784
D( 2,10, 2) =   -1.96282
D( 3, 1, 5) =    1.73504
D( 3, 3, 7) =    1.73504
D( 3, 3, 9) =   -2.33866
D( 3, 5, 1) =    1.49783
D( 3, 7, 3) =    1.49783
D( 3, 9, 3) =   -1.96281
D( 4, 2, 6) =    1.73505
D( 4, 4, 8) =    1.73505
D( 4, 4,10) =   -2.33867
D( 4, 6, 2) =    1.49784
D( 4, 8, 4) =    1.49784
D( 4,10, 4) =   -1.96282
D( 5, 1, 3) =   -1.13133
D( 5, 3, 1) =   -1.13133
D( 5, 5, 9) =   -3.07465
D( 5, 9, 5) =   -2.81865
D( 6, 2, 4) =   -1.13132
D( 6, 4, 2) =   -1.13132
D( 6, 6,10) =   -3.07469
D( 6,10, 6) =   -2.81868
D( 7, 1, 1) =    1.13133
D( 7, 3, 3) =   -1.13133
D( 7, 7, 9) =   -3.07465
D( 7, 9, 7) =   -2.81865
D( 8, 2, 2) =    1.13132
D( 8, 4, 4) =   -1.13132
D( 8, 8,10) =   -3.07469
D( 8,10, 8) =   -2.81868
D( 9, 1, 1) =    1.04087
D( 9, 3, 3) =    1.04087
D( 9, 5, 5) =    -.21090
D( 9, 7, 7) =    -.21090
D( 9, 9, 9) =    4.18784
D(10, 2, 2) =    1.04087
D(10, 4, 4) =    1.04087
D(10, 6, 6) =    -.21091
D(10, 8, 8) =    -.21091
D(10,10,10) =    4.18793
```

Table D-5. Sample Output, Section 2.

COEFFICIENTS FOR COMPUTATION OF WALL PRESSURE WAVEFORMS

J	Z	THETA (DEGREES)	COEFFICIENTS IN SERIES FOR: TIME DERIVATIVE	THETA DERIVATIVE	AXIAL DERIVATIVE
1	.000	.0	.0000000	.5818700	.0000000
2	.000	.0	.0000000	.0000000	.0000000
3	.000	.0	.5818700	.0000000	.0000000
4	.000	.0	.0000000	.0000000	.0000000
5	.000	.0	.0000000	.9730000	.0000000
6	.000	.0	.0000000	.0000000	.0000000
7	.000	.0	.4865000	.0000000	.0000000
8	.000	.0	.0000000	.0000000	.0000000
9	.000	.0	-.4027600	.0000000	.0000000
10	.000	.0	.0000000	.0000000	.0000000
1	.000	45.0	.4114442	.4114442	.0000000
2	.000	45.0	.0000000	.0000000	.0000000
3	.000	45.0	.4114442	-.4114442	.0000000
4	.000	45.0	.0000000	.0000000	.0000000
5	.000	45.0	.4865000	-.0000000	.0000000
6	.000	45.0	.0000000	.0000000	.0000000
7	.000	45.0	-.0000000	-.9730000	.0000000
8	.000	45.0	.0000000	.0000000	.0000000
9	.000	45.0	-.4027600	.0000000	.0000000
10	.000	45.0	.0000000	.0000000	.0000000
1	.000	90.0	.5818700	-.0000000	.0000000
2	.000	90.0	.0000000	.0000000	.0000000
3	.000	90.0	-.0000000	-.5818700	.0000000
4	.000	90.0	.0000000	.0000000	.0000000
5	.000	90.0	-.0000000	-.9730000	.0000000
6	.000	90.0	.0000000	.0000000	.0000000
7	.000	90.0	-.4865000	.0000001	.0000000
8	.000	90.0	.0000000	.0000000	.0000000
9	.000	90.0	-.4027600	.0000000	.0000000
10	.000	90.0	.0000000	.0000000	.0000000
1	1.000	.0	.0000000	.5909575	.0000000
2	1.000	.0	.0000000	.0092403	.0000000
3	1.000	.0	.5909575	.0000000	.0181736
4	1.000	.0	.0092403	.0000000	.0185766
5	1.000	.0	.0000000	.9981959	.0000000
6	1.000	.0	.0000000	.0261690	.0000000
7	1.000	.0	.4990979	.0000000	.0251885
8	1.000	.0	.0130845	.0000000	.0263938
9	1.000	.0	-.4158379	.0000000	-.0261428
10	1.000	.0	-.0137546	.0000000	-.0278051
1	1.000	45.0	.4178700	.4178700	.0128507
2	1.000	45.0	.0065339	.0065339	.0131356
3	1.000	45.0	.4178700	-.4178700	.0128507

Table D-5. (Continued)

4	1.000	45.0	.0065339	-.0065339	.0131356
5	1.000	45.0	.4990979	-.0000000	.0251885
6	1.000	45.0	.0130845	-.0000000	.0263938
7	1.000	45.0	-.0000000	-.9981959	-.0000000
8	1.000	45.0	-.0000000	-.0261690	-.0000000
9	1.000	45.0	-.4158379	.0000000	-.0261428
10	1.000	45.0	-.0137546	.0000000	-.0278051
1	1.000	90.0	.5909575	-.0000000	.0181736
2	1.000	90.0	.0092403	-.0000000	.0185766
3	1.000	90.0	-.0000000	-.5909575	-.0000000
4	1.000	90.0	-.0000000	-.0092403	-.0000000
5	1.000	90.0	-.0000000	-.9981959	-.0000000
6	1.000	90.0	-.0000000	-.0261690	-.0000000
7	1.000	90.0	-.4990979	.0000001	-.0251885
8	1.000	90.0	-.0130845	.0000000	-.0263938
9	1.000	90.0	-.4158379	.0000000	-.0261428
10	1.000	90.0	-.0137546	.0000000	-.0278051

Table D-6. Sample Output, Section 3.

INITIAL CONDITIONS ARE OF THE FORM:

U(I,J) = AC(J)*COS(FREQ*T) + AS(J)*SIN(FREQ*T)), * EXP(DAMP*T)

J	DAMPING	FREQUENCY	AC(J)	AS(J)
1	.00000000	1.84118000	.00000000	.30000000
2	.00000000	1.84118000	-.30278619	-.00209447
3	.00000000	1.84118000	.30000000	.00000000
4	.00000000	1.84118000	-.00209447	.30278619

THE LIMIT-CYCLE BEHAVIOR IS CALCULATED.

THIS RUN PRODUCES PLOTTED OUTPUT.

Table D-7. Sample Output, Section 4.

COMBUSTION PARAMETERS: INTERACTION INDEX = .57486 TIME-LAG = 1.70000
MOTOR PARAMETERS: GAMMA = 1.20000 EXIT MACH NUMBER = .20000 LENGTH/DIAMETER = .50000

INITIAL CONDITIONS

		INJECTOR PRESSURE			NOZZLE PRESSURE			NOZZLE AXIAL VELOCITY			
STEP	TIME	0. DEG.	45. DEG.	90. DEG.	0. DEG.	45. DEG.	90. DEG.	0. DEG.	45. DEG.	90. DEG.	YPHI
-35	-1.70000	-.00446	.29088	.42934	.00294	.30085	.43542	-.00548	.00006	.00556	-.00557
-34	-1.65143	-.03845	.26129	.42696	-.03161	.27126	.43380	-.00595	-.00064	.00505	-.00605
-33	-1.60286	-.07144	.22989	.42083	-.06520	.23977	.42837	-.00638	-.00133	.00450	-.00648
-32	-1.55429	-.10320	.19702	.41103	-.09758	.20673	.41918	-.00676	-.00201	.00391	-.00685
-31	-1.50571	-.13352	.16303	.39768	-.12854	.17250	.40636	-.00708	-.00268	.00329	-.00718
-30	-1.45714	-.16221	.12826	.38093	-.15788	.13741	.39006	-.00734	-.00332	.00265	-.00744
-29	-1.40857	-.18912	.09306	.36100	-.18543	.10182	.37048	-.00755	-.00394	.00198	-.00765
-28	-1.36000	-.21410	.05775	.33812	-.21107	.06606	.34786	-.00770	-.00453	.00130	-.00779
-27	-1.31143	-.23704	.02265	.31256	-.23466	.03047	.32246	-.00778	-.00508	.00060	-.00788
-26	-1.26286	-.25785	-.01193	.28461	-.25611	-.00465	.29459	-.00781	-.00559	-.00009	-.00790
-25	-1.21429	-.27645	-.04572	.25460	-.27536	-.03901	.26456	-.00777	-.00605	-.00079	-.00785
-24	-1.16571	-.29280	-.07847	.22285	-.29234	-.07236	.23270	-.00766	-.00647	-.00148	-.00775
-23	-1.11714	-.30685	-.10993	.18970	-.30700	-.10445	.19937	-.00750	-.00683	-.00216	-.00758
-22	-1.06857	-.31856	-.13991	.15552	-.31932	-.13507	.16492	-.00728	-.00714	-.00282	-.00735
-21	-1.02000	-.32792	-.16823	.12062	-.32927	-.16403	.12969	-.00700	-.00739	-.00346	-.00706
-20	-.97143	-.33491	-.19472	.08537	-.33684	-.19118	.09404	-.00666	-.00759	-.00407	-.00672
-19	-.92286	-.33953	-.21926	.05008	-.34202	-.21638	.05829	-.00627	-.00772	-.00465	-.00632
-18	-.87429	-.34176	-.24175	.01507	-.34480	-.23951	.02278	-.00583	-.00779	-.00519	-.00587
-17	-.82571	-.34160	-.26209	-.01936	-.34519	-.26049	-.01220	-.00534	-.00780	-.00569	-.00538
-16	-.77714	-.33906	-.28021	-.05294	-.34317	-.27925	-.04636	-.00481	-.00775	-.00614	-.00484
-15	-.72857	-.33414	-.29606	-.08543	-.33875	-.29572	-.07946	-.00425	-.00763	-.00655	-.00426
-14	-.68000	-.32684	-.30960	-.11659	-.33194	-.30988	-.11124	-.00364	-.00746	-.00690	-.00365
-13	-.63143	-.31718	-.32080	-.14622	-.32273	-.32169	-.14152	-.00301	-.00722	-.00720	-.00301
-12	-.58286	-.30516	-.32964	-.17415	-.31115	-.33112	-.17010	-.00236	-.00693	-.00744	-.00235
-11	-.53429	-.29081	-.33612	-.20023	-.29721	-.33817	-.19683	-.00168	-.00658	-.00762	-.00167
-10	-.48571	-.27417	-.34021	-.22434	-.28094	-.34283	-.22159	-.00100	-.00618	-.00774	-.00097
-9	-.43714	-.25527	-.34193	-.24636	-.26237	-.34509	-.24426	-.00030	-.00573	-.00780	-.00027
-8	-.38857	-.23418	-.34125	-.26622	-.24157	-.34495	-.26477	.00040	-.00523	-.00780	.00044
-7	-.34000	-.21097	-.33819	-.28385	-.21860	-.34241	-.28303	.00109	-.00469	-.00773	.00114
-6	-.29143	-.18574	-.33275	-.29920	-.19355	-.33747	-.29900	.00178	-.00412	-.00760	.00183
-5	-.24286	-.15859	-.32494	-.31223	-.16653	-.33014	-.31265	.00245	-.00351	-.00741	.00251
-4	-.19429	-.12968	-.31476	-.32293	-.13767	-.32041	-.32394	.00310	-.00287	-.00716	.00317
-3	-.14571	-.09916	-.30223	-.33126	-.10714	-.30832	-.33286	.00373	-.00221	-.00686	.00380
-2	-.09714	-.06723	-.28739	-.33721	-.07512	-.29386	-.33939	.00433	-.00154	-.00650	.00441
-1	-.04857	-.03410	-.27025	-.34079	-.04183	-.27709	-.34353	.00489	-.00085	-.00609	.00497
0	.00000	.00000	-.25087	-.34198	-.00749	-.25803	-.34527	.00541	-.00015	-.00562	.00550

Table D-7. (Continued)

COMBUSTION PARAMETERS: INTERACTION INDEX = .57486 TIME-LAG = 1.70000
MOTOR PARAMETERS: GAMMA = 1.20000 EXIT MACH NUMBER = .20000 LENGTH/DIAMETER = .50000

		INJECTOR PRESSURE			NOZZLE PRESSURE			NOZZLE AXIAL VELOCITY			
STEP	TIME	0. DEG.	45. DEG.	90. DEG.	0. DEG.	45. DEG.	90. DEG.	0. DEG.	45. DEG.	90. DEG.	YPHI
3006	146.00571	-.12500	.17758	.48179	-.11918	.16709	.48937	-.00618	-.00133	.00429	-.00790
3007	146.05428	-.14707	.13466	.46189	-.14184	.14395	.47008	-.00654	-.00200	.00372	-.00800
3008	146.10286	-.16640	.09290	.43622	-.16178	.10189	.44492	-.00684	-.00265	.00312	-.00800
3009	146.15143	-.18316	.05286	.40547	-.17917	.06150	.41458	-.00709	-.00329	.00248	-.00790
3010	146.20000	-.19756	.01498	.37044	-.19421	.02321	.37985	-.00727	-.00389	.00183	-.00770
3011	146.24857	-.20984	-.02038	.33200	-.20714	-.01261	.34161	-.00739	-.00446	.00116	-.00743
3012	146.29714	-.22024	-.05300	.29104	-.21820	-.04573	.30075	-.00745	-.00499	.00048	-.00710
3013	146.34571	-.22903	-.08273	.24846	-.22764	-.07599	.25816	-.00745	-.00548	-.00021	-.00671
3014	146.39428	-.23645	-.10952	.20511	-.23571	-.10334	.21472	-.00739	-.00593	-.00090	-.00629
3015	146.44286	-.24273	-.13339	.16179	-.24265	-.12779	.17123	-.00726	-.00632	-.00157	-.00583
3016	146.49143	-.24810	-.15444	.11923	-.24866	-.14944	.12842	-.00708	-.00666	-.00224	-.00537
3017	146.54000	-.25273	-.17281	.07804	-.25393	-.16842	.08691	-.00683	-.00694	-.00289	-.00490
3018	146.58857	-.25678	-.18868	.03874	-.25860	-.18492	.04724	-.00653	-.00716	-.00351	-.00443
3019	146.63714	-.26036	-.20227	.00175	-.26277	-.19916	.00982	-.00618	-.00732	-.00410	-.00399
3020	146.68571	-.26352	-.21384	-.03263	-.26650	-.21138	-.02504	-.00577	-.00742	-.00466	-.00356
3021	146.73428	-.26627	-.22362	-.06420	-.26980	-.22181	-.05712	-.00532	-.00746	-.00518	-.00316
3022	146.78286	-.26859	-.23188	-.09286	-.27263	-.23073	-.08631	-.00483	-.00744	-.00565	-.00279
3023	146.83143	-.27038	-.23885	-.11857	-.27490	-.23836	-.11260	-.00430	-.00735	-.00608	-.00244
3024	146.88000	-.27152	-.24477	-.14140	-.27649	-.24493	-.13601	-.00374	-.00720	-.00645	-.00213
3025	146.92857	-.27183	-.24985	-.16144	-.27721	-.25065	-.15666	-.00314	-.00699	-.00677	-.00183
3026	146.97714	-.27111	-.25426	-.17887	-.27686	-.25568	-.17472	-.00252	-.00673	-.00703	-.00156
3027	147.02571	-.26914	-.25813	-.19388	-.27522	-.26016	-.19036	-.00189	-.00641	-.00723	-.00130
3028	147.07428	-.26565	-.26154	-.20671	-.27203	-.26417	-.20383	-.00124	-.00603	-.00737	-.00104
3029	147.12286	-.26039	-.26456	-.21759	-.26702	-.26774	-.21537	-.00058	-.00561	-.00744	-.00079
3030	147.17143	-.25309	-.26716	-.22679	-.25993	-.27088	-.22522	.00009	-.00515	-.00746	-.00052
3031	147.22000	-.24347	-.26929	-.23455	-.25048	-.27352	-.23364	.00075	-.00464	-.00741	-.00023
3032	147.26857	-.23129	-.27086	-.24111	-.23843	-.27555	-.24086	.00141	-.00410	-.00730	.00008
3033	147.31714	-.21639	-.27172	-.24671	-.22354	-.27684	-.24710	.00205	-.00352	-.00713	.00042
3034	147.36571	-.19830	-.27168	-.25152	-.20559	-.27720	-.25255	.00268	-.00292	-.00690	.00080
3035	147.41428	-.17713	-.27054	-.25572	-.18441	-.27641	-.25737	.00328	-.00229	-.00662	.00121
3036	147.46285	-.15265	-.26804	-.25941	-.15987	-.27423	-.26167	.00386	-.00165	-.00628	.00167
3037	147.51143	-.12481	-.26394	-.26268	-.13192	-.27041	-.26552	.00441	-.00100	-.00589	.00217
3038	147.56000	-.09362	-.25796	-.26555	-.10055	-.26467	-.26893	.00492	-.00033	-.00545	.00270
3039	147.60857	-.05917	-.24984	-.26799	-.06585	-.25675	-.27190	.00540	.00033	-.00497	.00327
3040	147.65714	-.02164	-.23932	-.26994	-.02801	-.24638	-.27433	.00583	.00099	-.00445	.00386
3041	147.70571	.01867	-.22614	-.27126	.01268	-.23332	-.27612	.00621	.00164	-.00389	.00447
3042	147.75423	.06137	-.21007	-.27182	.05584	-.21733	-.27709	.00655	.00228	-.00330	.00509
3043	147.80285	.10594	-.19094	-.27140	.10095	-.19822	-.27705	.00683	.00290	-.00269	.00570
3044	147.85143	.15176	-.16857	-.26979	.14738	-.17583	-.27579	.00706	.00350	-.00206	.00629
3045	147.90000	.19811	-.14287	-.26674	.19442	-.15005	-.27304	.00723	.00407	-.00141	.00685
3046	147.94857	.24418	-.11380	-.26198	.24125	-.12084	-.26855	.00734	.00460	-.00075	.00736
3047	147.99714	.28908	-.08140	-.25526	.28698	-.08825	-.26204	.00739	.00510	-.00009	.00780
3048	148.04571	.33190	-.04580	-.24629	.33068	-.05238	-.25326	.00739	.00556	.00057	.00818
3049	148.09428	.37171	-.00721	-.23482	.37141	-.01346	-.24194	.00732	.00597	.00123	.00846

Table D-8. Sample Output, Section 5.

PRESSURE MAXIMA AND MINIMA AT: Z = .00 THETA = .0
VALUES COMPUTED: 94

.456418 .771543	-.308270 2.863856	.580628 4.172075	-.272996 5.603093	.478533 7.544092	-.308486 9.446675	.542703 10.853099	-.252604 12.786936
.510174 14.257257	-.294000 16.063277	.515242 17.564400	-.268418 19.537089	.523881 20.950593	-.279236 22.744107	.506445 24.284235	-.277079 26.215260
.522925 27.645648	-.271794 29.485428	.507847 30.999033	-.278825 32.887355	.517217 34.347683	-.271253 36.223408	.511007 37.707680	-.277434 39.573612
.512651 41.054779	-.273096 42.940318	.512507 44.413538	-.275527 46.276354	.510428 47.763727	-.274546 49.647243	.512173 51.119364	-.274260 52.988021
.509667 54.472655	-.274897 56.351470	.510885 57.826298	-.273707 59.701087	.509343 61.181129	-.274477 63.056576	.509485 64.534305	-.273529 66.411903
.508919 67.889316	-.273837 69.763511	.508374 71.242904	-.273437 73.120358	.508277 74.597562	-.273341 76.471918	.507590 77.951710	-.273339 79.827781
.507531 81.306071	-.273099 83.180643	.507004 84.660571	-.273171 86.534866	.506817 88.014882	-.272970 89.888968	.506484 91.369514	-.272988 93.242868
.506195 94.723948	-.272868 96.597583	.505966 98.078606	-.272806 99.951679	.505655 101.433208	-.272734 103.306509	.505445 104.787896	-.272628 106.660972
.505162 108.142635	-.272564 110.015746	.504934 111.497398	-.272457 113.370504	.504692 114.852245	-.272387 116.725281	.504466 118.207081	-.272298 120.080224
.504254 121.561975	-.272226 123.435121	.504037 124.916878	-.272157 126.790210	.503844 128.271814	-.272090 130.145315	.503647 131.626776	-.272062 133.500187
.503467 134.981756	-.272003 136.854771	.503289 138.336773	-.271953 140.209436	.503120 141.691811	-.271899 143.564207	.502957 145.046881	-.271849 146.919048
.502798 148.401972	-.271799 150.273991	.502646 151.757095	-.271749 153.629004	.502497 155.112246	-.271700 156.984108		

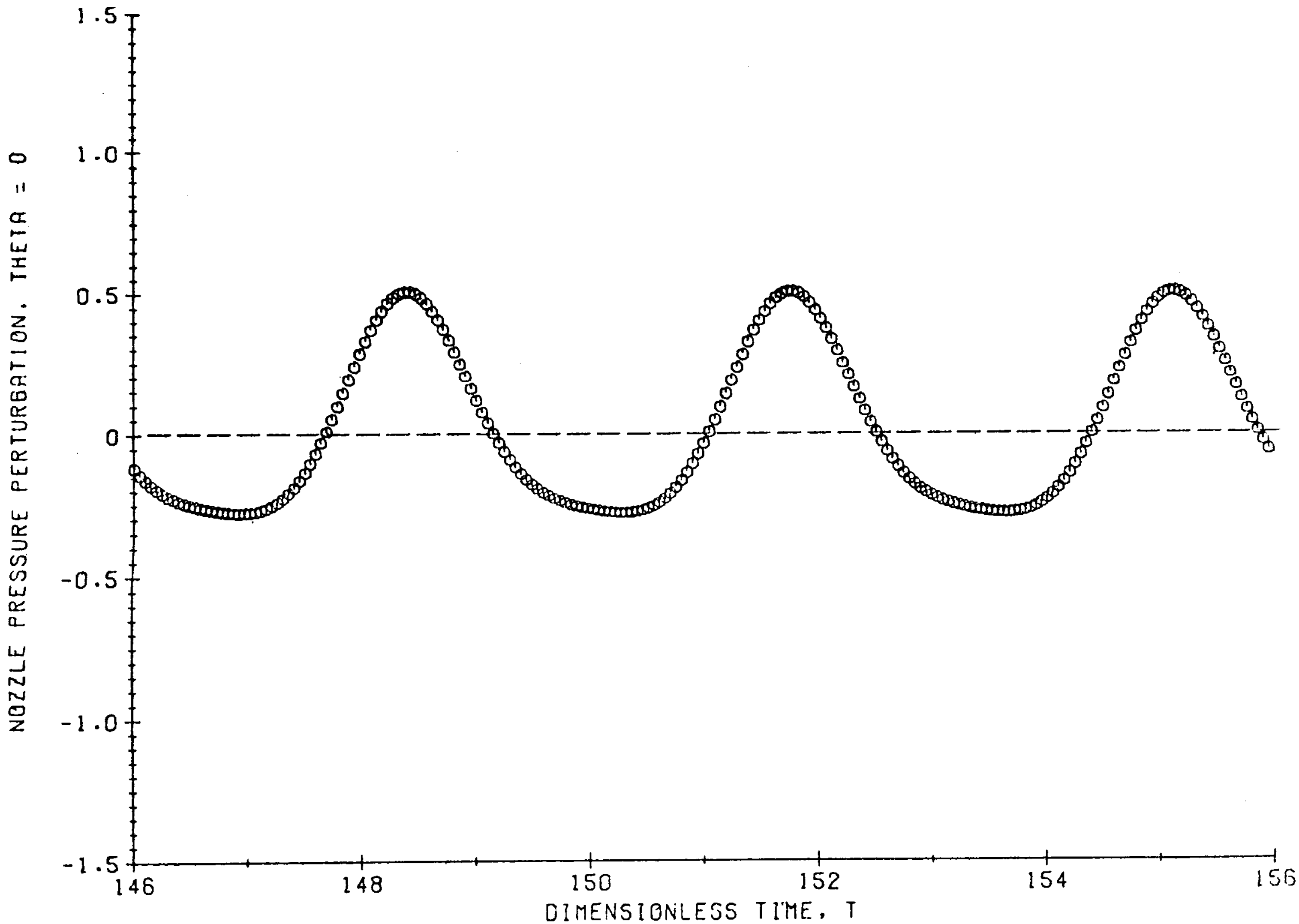


Figure D-2. Sample Pressure Plot.

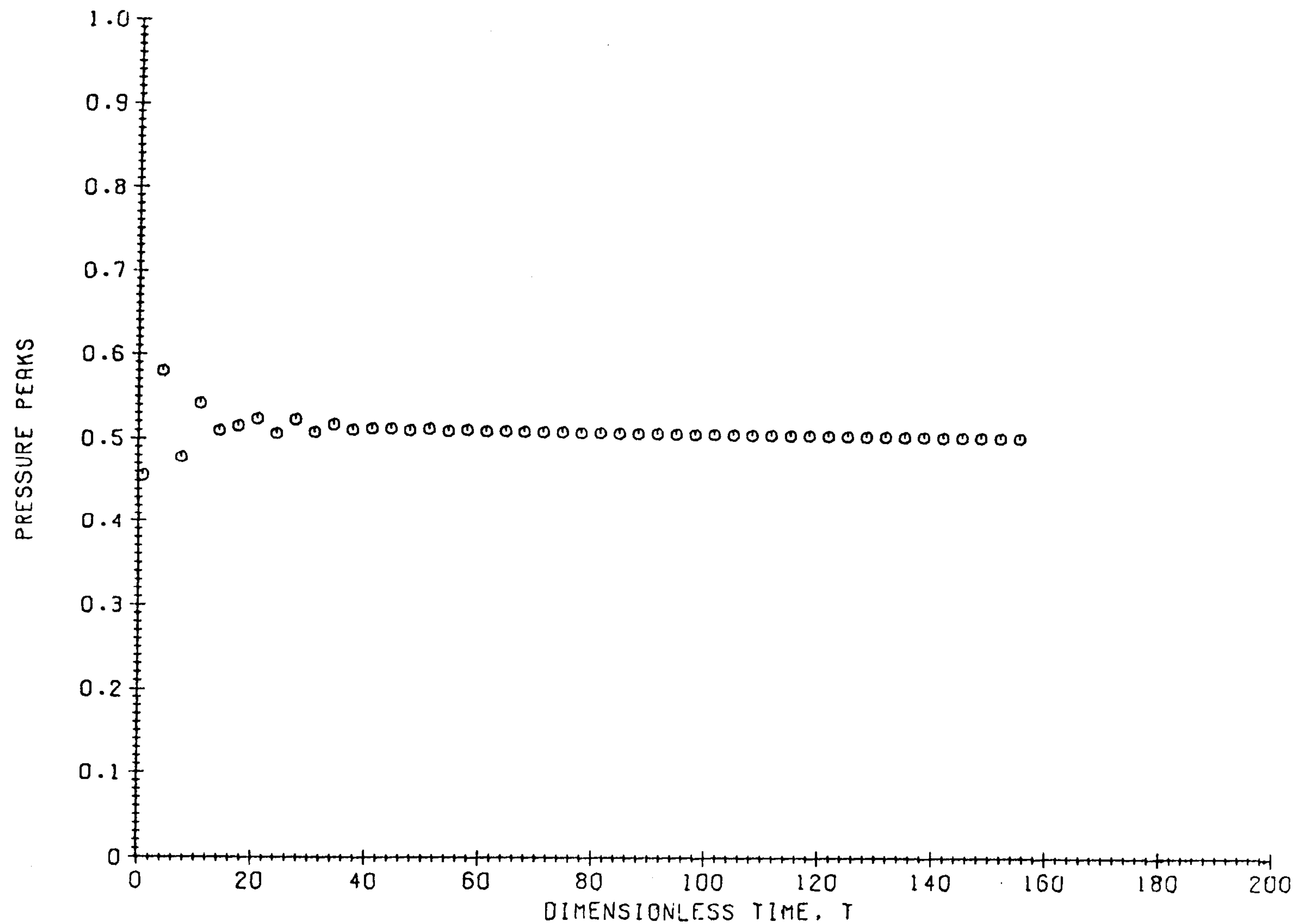


Figure D-3. Sample Amplitude Plot.

```
C      *************** PROGRAM LCYC3D ********************************
C
C         THIS PROGRAM CALCULATES THE NONLINEAR BEHAVIOR OF
C      TRANSVERSE, AXIAL, OR COMBINED LONGITUDINAL-TRANSVERSE
C      INSTABILITIES IN A CYLINDRICAL COMBUSTION CHAMBER WITH
C      UNIFORM PROPELLANT INJECTION, DISTRIBUTED COMBUSTION
C      PROCESS, AND A CONVENTIONAL NOZZLE.  THE COMBUSTION PROCESS
C      IS DESCRIBED BY CROCCO'S TIME-LAG MODEL.  BOTH TRANSIENT
C      AND LIMIT-CYCLE SOLUTIONS ARE CALCULATED.
C
C      THE FOLLOWING INPUTS ARE REQUIRED:
C
C      (1)  THE CONTROL NUMBER, NOUTCF.
C      (2)  THE COEFFICIENTS FROM PROGRAM COEFFS3D.
C      (3)  THE DATA DECK.
C
C      NOUTCF DETERMINES PRINTOUT OF COEFFICIENTS.
C         IF NOUTCF = 0  COEFFICIENTS ARE NOT PRINTED OUT.
C         IF NOUTCF = 1  LINEAR COEFFICIENTS ONLY ARE PRINTED OUT.
C         IF NOUTCF = 2  ALL COEFFICIENTS ARE PRINTED OUT.
C
C      THE DATA DECK CONSISTS OF THE FOLLOWING CARDS:
C
C      FIRST CARD:
C
C      EN IS THE INTERACTION INDEX.
C      TAU IS THE TIME LAG.
C      H IS THE INTEGRATION STEP SIZE.
C      TSTART IS THE TIME AT WHICH OUTPUT STARTS.
C      TQUIT IS THE TIME AT WHICH COMPUTATIONS ARE TERMINATED.
C
C      SECOND CARD:
C
C      NTEST IS TASK CONTROL NUMBER:
C         IF NTEST = 0   COMPUTE TRANSIENT BEHAVIOR.
C         IF NTEST = 1   COMPUTE THE LIMIT-CYCLE BEHAVIOR.
C      JMODE IS THE MODE-AMPLITUDE USED TO TEST FOR LIMIT-CYCLES.
C      NLOC DETERMINES THE LOCATION OF THE WALL PRESSURE MAXIMA
C      AND MINIMA:
C         IF NLOC = 1    LOCATION IS Z = 0, THETA = 0 DEGREES.
C         IF NLOC = 2    LOCATION IS Z = 0, THETA = 45 DEGREES.
C         IF NLOC = 3    LOCATION IS Z = 0, THETA = 90 DEGREES.
C      NTERMS IS THE NUMBER OF TERMS GIVEN INITIAL VALUES.
C      NPZ DETERMINES HOW SECONDARY STABILITY ZONES (PHANTOM
C      ZONES) ARE HANDLED.
C         IF NPZ = 0 PHANTOM ZONES ARE RETAINED.
C         IF NPZ = 1 PHANTOM ZONES ARE ELIMINATED.
C      NOUT IS THE OUTPUT CONTROL NUMBER.
C         IF NOUT = 0    PRINTED OUTPUT ONLY.
C         IF NOUT > 0    BOTH PRINTED AND PLOTTED OUTPUT, NOUT
```

```
C                          DETERMINES THE NUMBER OF THE LAST PLOT
C                          PRODUCED.
C
C      DATA FOR SETTING UP PLOTS (THIRD AND FOURTH CARDS):
C
C      YHI(1) IS THE MAXIMUM ORDINATE FOR PRESSURE PLOTS.
C      YHI(5) IS THE MAXIMUM ORDINATE FOR VELOCITY PLOTS.
C      NOTE: THE ORDINATE SCALES FOR PRESSURE AND VELOCITY PLOTS
C            ARE SYMMETRIC ABOUT ZERO.
C      YLAB IS THE INTERVAL FOR ORDINATE LABELING FOR ABOVE PLOTS.
C      ITICY IS THE NUMBER OF ORDINATE TIC MARKS FOR ABOVE PLOTS.
C      NOTE: ITICY SHOULD BE NEGATIVE FOR PRESSURE AND VELOCITY PLOTS
C            TO OBTAIN CENTERLINE.
C      NFIRST IS THE NUMBER OF THE FIRST PLOT PRODUCED.
C      NOMIT DETERMINES WHETHER AMPLITUDE PLOT IS PRODUCED:
C         IF NOMIT = 0 AMPLITUDE PLOT IS PRODUCED.
C         IF NOMIT = 1 AMPLITUDE PLOT IS OMITTED.
C
C      INITIAL AMPLITUDES OF F-FUNCTIONS (REMAINING CARDS):
C
C      AS(J) IS THE AMPLITUDE OF THE SINE TERM.
C      AC(J) IS THE AMPLITUDE OF THE COSINE TERM.
C
C
C
       COMPLEX      YNOZ(10), B(10), C1, C2, C3, CPHIT(10), CSUM, A
       DIMENSION    L(10), N(10), S(10), NAME(10), AS(20), AC(20),
     1              U(250,40), AA(4), Y(40), FZ(4,40), YP(40), UZ(40),
     2              CP(3,20,20), FRQ1(20), DMP1(20), UMAX(500),UAVG(100),
     3              Z(6), ANGLE(6), THETA(6), CFT(6,20), YI(20),
     4              CFTH(6,20), CFZ(6,20), PRESS(6), AXVEL(3), YR(20),
     5              TPLOT(500), YPLOT(6,500), DUMMYT(500), DUMMYY(500),
     6              IBUF(3000), ITT(4), ITY1(7), ITY2(7), ITY3(7),
     7              ITY4(7), ITY5(6), TAUCUT(20), ITY6(8),
     8              ITP(3),  TITLE(12), PRS(500), TI(500), PMAX(500),
     9              TIMAX(500), YLO(6), YHI(6), YLAB(6), ITICY(6)
C
       COMMON       RV(20,4), C(3,20,20), D(20,400),
     1              KPMAX(3,20), IC(3,20,20), KPQMAX(20),
     2              IDP(20,400), IDQ(20,400)
       COMMON       /BLK2/       M(10), NS(10), SJ(10), B
       COMMON       /BLK3/    NJMAX, NLMAX, GAMMA, COEF(3,20)
C
       DATA     ITT/'DIMENSIONLESS TIME, T'/,
     1          ITY1/'INJECTOR PRESSURE PERTURBATION, THETA = 0'/,
     2          ITY2/'INJECTOR PRESSURE PERTURBATION, THETA = 45'/,
     3          ITY3/'INJECTOR PRESSURE PERTURBATION, THETA = 90'/,
     4          ITY4/'NOZZLE PRESSURE PERTURBATION, THETA = 0'/,
     5          ITY5/'NOZZLE AXIAL VELOCITY, THETA = 0'/,
     6          ITY6/'NOZZLE B.C. (RE(-GAMMA*Y*PHIT)) AT THETA = 0'/,
```

```
     7          ITP/'PRESSURE PEAKS'/
C
      LAST = 250
      ERR = 0.001
      TDEL = 10.0
      NPT = 0
      AA(1) = 0.0
      AA(2) = 0.5
      AA(3) = 0.5
      AA(4) = 1.0
      PI = 3.1415927
      READ (5,5003) NOUTCF
C
C     ************* COEFFICIENT INPUT SECTION **************************
C
C     THIS VERSION OF LCYC3D READS THE COEFFICIENT DATA FROM
C     A FASTRAND FILE GENERATED BY PROGRAM COEFFS3D.  TO READ
C     THIS DATA FROM CARDS, USE READ (5,XXXX) INSTEAD OF
C     READ (9,XXXX) IN THIS SECTION.
C
C     INPUT OF MOTOR PARAMETERS AND NUMBER OF TERMS.
      READ (9,5001)  GAMMA, UE, ZE, ZCOMB, NDROPS, NJMAX
      WRITE (6,6001)  GAMMA, UE, ZE, ZCOMB, NJMAX
      IF (NDROPS .EQ. 0)  WRITE (6,6030)
      IF (NDROPS .EQ. 1)  WRITE (6,6031)
      NU = 2 * NJMAX
      JMX = NJMAX/2
      RLD = 0.5 * ZE
C
      WRITE (6,6002)
C
C     INPUT OF DESCRIPTION OF SERIES EXPANSION.
      DO 10 K = 1, JMX
      READ (9,5002) NJ, L(NJ), M(NJ), N(NJ), NS(NJ), S(NJ), SJ(NJ),
     1              NAME(NJ)
      WRITE (6,6003)  NAME(NJ), NJ, L(NJ), M(NJ), N(NJ), NS(NJ),
     1                S(NJ), SJ(NJ)
   10 CONTINUE
C
      WRITE (6,6010)
      DO 15 K = 1, JMX
      READ (9,5010) J, YNOZ(J), B(J)
      WRITE (6,6015) J, YNOZ(J), B(J)
      NJ = (2 * J) - 1
      YR(NJ) = REAL(YNOZ(J))
      YI(NJ) = AIMAG(YNOZ(J))
      YR(NJ+1) = YR(NJ)
      YI(NJ+1) = YI(NJ)
   15 CONTINUE
C
```

```
C     ZERO LINEAR COEFFICIENT ARRAYS.
      DO 20 KC = 1, 3
      DO 20 NJ = 1, 20
      DO 20 NP = 1, 20
      C(KC,NJ,NP) = 0.0
      CP(KC,NJ,NP) = 0.0
   20 CONTINUE
C
C     ZERO NONLINEAR COEFFICIENT ARRAY.
      DO 30 NJ = 1, 20
      DO 30 NPQ = 1, 400
      D(NJ,NPQ) = 0.0
   30 CONTINUE
C
C     INPUT OF LINEAR COEFFICIENTS.
      DO 40 KC = 1, 3
      READ (9,5003) KMAX
      IF (NOUTCF .GT. 0) WRITE (6,6004) KC, KMAX
      IF (KMAX .EQ. 0)  GO TO 40
      DO 45 K = 1, KMAX
      READ (9,5004)  NJ, NP, CP(KC,NJ,NP)
      IF (NOUTCF .GT. 0)  WRITE (6,6005) KC, NJ, NP, CP(KC,NJ,NP)
   45 CONTINUE
   40 CONTINUE
C
C
C     INPUT OF NONLINEAR COEFFICIENTS.
      READ (9,5003) NLMAX
      IF (NOUTCF .EQ. 2)  WRITE (6,6006) NLMAX
      IF (NLMAX .EQ. 0)  GO TO 50
      DO 52 NJ = 1, 20
      KPQMAX(NJ) = 0
   52 CONTINUE
      DO 55 K = 1, NLMAX
      READ (9,5005)  NJ, NP, NQ, DT
      IF (NOUTCF .EQ. 2)  WRITE (6,6007)  NJ, NP, NQ, DT
      KPQMAX(NJ) = KPQMAX(NJ) + 1
      KPQ = KPQMAX(NJ)
      IDP(NJ,KPQ) = NP
      IDQ(NJ,KPQ) = NQ
      D(NJ,KPQ) = DT
   55 CONTINUE
   50 CONTINUE
C
C     ************* PRESSURE COEFFICIENT SECTION ***********************
C
C     CALCULATE SPATIAL COORDINATES FOR PRESSURE COMPUTATION.
      DO 51 NPRES = 1, 3
      Z(NPRES) = 0.0
      RTHETA = NPRES - 1
```

```
      ANGLE(NPRES) = RTHETA * 45.0
      THETA(NPRES) = RTHETA * PI/4.0
      Z(NPRES + 3) = ZE
      ANGLE(NPRES + 3) = ANGLE(NPRES)
      THETA(NPRES + 3) = THETA(NPRES)
   51 CONTINUE
C
C     CALCULATE COEFFICIENTS FOR PRESSURE TIME HISTORIES.
      DO 53 NPRES = 1, 6
      DO 53 J = 1, JMX
      NP = (2 * J) - 1
      Z1 = Z(NPRES)
      ANG = THETA(NPRES)
      CALL PHICFS(J,Z1,ANG,C1,C2,C3)
      IF (NPRES .EQ. 4)  CPHIT(J) = C1
      CFT(NPRES,NP) = REAL(C1)
      CFT(NPRES,NP+1) = -AIMAG(C1)
      CFTH(NPRES,NP) = REAL(C2)
      CFTH(NPRES,NP+1) = -AIMAG(C2)
      CFZ(NPRES,NP) = REAL(C3)
      CFZ(NPRES,NP+1) = -AIMAG(C3)
   53 CONTINUE
C
C     OUTPUT OF COEFFICIENTS FOR PRESSURE TIME HISTORIES.
      WRITE (6,6020)
      DO 56 NPRES = 1, 6
      WRITE (6,6014)
      DO 56 J = 1, NJMAX
      WRITE (6,6021)            J, Z(NPRES), ANGLE(NPRES),
     1                  CFT(NPRES,J), CFTH(NPRES,J), CFZ(NPRES,J)
   56 CONTINUE
C
C     ************* DATA INPUT SECTION **********************************
C
      READ (5,5000) TITLE
C
C     ZERO INITIAL VALUE AND FREQUENCY ARRAYS.
    5 DO 57 K = 1, NJMAX
      AS(K) = 0.0
      AC(K) = 0.0
      FRQ1(K) = 0.0
   57 CONTINUE
C
C
C     READ COMBUSTION AND CONTROL PARAMETERS.
      READ (5,5006, END = 300) EN, TAU, H, TSTART, TQUIT
C
C     READ CONTROL NUMBERS.
      READ (5,5008)  NTEST, JMODE, NLOC, NTERMS, NPZ, NOUT
      JMODE = (2 * JMODE) - 1
```

```
      JPMODE = JMODE + NJMAX
      IF (NOUT .GT. 0)  NPT = 1
C
C
      IF (NOUT .EQ. 0)  GO TO 9
C     READ DATA FOR SETTING UP PLOTS.
      READ (5,5009) YHI(1), YHI(5), YLAB(1), YLAB(5)
      READ (5,5008) ITICY(1), ITICY(5), NFIRST, NOMIT
C
C     ************ INITIAL AMPLITUDES SECTION **************************
C
    9 DO 58 K = 1, NTERMS
C
C     INPUT INITIAL AMPLITUDES FOR F-FUNCTIONS.
      READ (5,5007)  J, AST, ACT
      NJ = (2 * J) - 1
      AS(NJ) = AST
      AC(NJ) = ACT
C
C     CALCULATE FREQUENCY AND DAMPING.
      RL = L(J)
      AX = RL * PI/ZE
      AXSQ = AX * AX
      SSQ = S(J) * S(J)
      FRQ1(NJ) = SQRT(SSQ + AXSQ)
      DMP1(NJ) = 0.0
      FRQ1(NJ+1) = FRQ1(NJ)
      DMP1(NJ+1) = DMP1(NJ)
C
C     CALCULATE INITIAL AMPLITUDES FOR G-FUNCTIONS.
C
      IF (FRQ1(NJ)) 58, 58, 581
  581 GYRU = GAMMA*YR(NJ)*UE
      GYIF = GAMMA*YI(NJ)*FRQ1(NJ)
      GYRF = GAMMA*YR(NJ)*FRQ1(NJ)
      GYIU = GAMMA*YI(NJ)*UE
C
      NPRES = 4
      IF (NS(J) .EQ. 1)   NPRES = 6
C
      A1 = (1.0 + GYRU)*CFZ(NPRES,NJ+1)
     1       - GYIF*CFT(NPRES,NJ+1)
      A2 = GYRF*CFT(NPRES,NJ+1) + GYIU*CFZ(NPRES,NJ+1)
      A3 = -(1.0 + GYRU)*CFZ(NPRES,NJ) + GYIF*CFT(NPRES,NJ)
      A4 = GYRF*CFT(NPRES,NJ) + GYIU*CFZ(NPRES,NJ)
C
      DET = A1*A1 + A2*A2
      IF (DET .LT. 0.0000001) GO TO 583
      R1 = A3*AC(NJ) - A4*AS(NJ)
      R2 = -A4*AC(NJ) - A3*AS(NJ)
```

```
C
      AC(NJ+1) = (R1*A1 + R2*A2)/DET
      AS(NJ+1) = -(R2*A1 - R1*A2)/DET
      GO TO 58
  583 AC(NJ+1) = -AS(NJ)
      AS(NJ+1) = AC(NJ)
C
   58 CONTINUE
C
C
C     OUTPUT OF INITIAL AMPLITUDES.
      WRITE (6,6016)
      DO 590 J = 1, NJMAX
      IF (AS(J))  591, 592, 591
  592 IF (AC(J))  591, 590, 591
  591 WRITE (6,6017) J, DMP1(J), FRQ1(J), AC(J), AS(J)
  590 CONTINUE
      IF (NTEST .EQ. 0)  WRITE (6,6025)
      IF (NTEST .EQ. 1)  WRITE (6,6026)
      IF (NPZ .EQ. 1) WRITE (6,6028)
      IF (NOUT .GE. 1)  WRITE (6,6027)
C
C     ************* LINEAR COEFFICIENTS SECTION ***********************
C
      DO 59 KC = 1, 3
      DO 59 NJ = 1, 10
      KPMAX(KC,NJ) = 0
   59 CONTINUE
C
      IF (NPZ .EQ. 0) GO TO 605
      DO 602 J = 1, JMX
      NJ = (2 * J) - 1
      RL = L(J)
      AX = RL * PI/ZE
      AXSQ = AX * AX
      SSQ = S(J) * S(J)
      OMEGA = SQRT(SSQ + AXSQ)
      TAUCUT(NJ) = 2.0 * PI/OMEGA
      TAUCUT(NJ+1) = TAUCUT(NJ)
  602 CONTINUE
C
      DO 604 NJ = 1, NJMAX
      DO 604 NP = 1, NJMAX
      IF (TAU .GT. TAUCUT(NP))  CP(3,NJ,NP) = 0.0
  604 CONTINUE
C
C     COMPUTE LINEAR COEFFICIENTS FOR GIVEN VALUES OF EN AND TAU.
  605 DO 60 NJ = 1, NJMAX
      DO 60 NP = 1, NJMAX
      CT = CP(1,NJ,NP)
```

```
      IF (CT) 61, 62, 61
   61 KPMAX(1,NJ) = KPMAX(1,NJ) + 1
      KP = KPMAX(1,NJ)
      IC(1,NJ,KP) = NP
      C(1,NJ,KP) = CT
   62 CT = CP(2,NJ,NP) - EN*CP(3,NJ,NP)
      IF (CT) 63, 64, 63
   63 KPMAX(2,NJ) = KPMAX(2,NJ) + 1
      KP = KPMAX(2,NJ)
      IC(2,NJ,KP) = NP
      C(2,NJ,KP) = CT
   64 CT = EN * CP(3,NJ,NP)
      IF (CT) 65, 60, 65
   65 KPMAX(3,NJ) = KPMAX(3,NJ) + 1
      KP = KPMAX(3,NJ)
      IC(3,NJ,KP) = NP
      C(3,NJ,KP) = CT
   60 CONTINUE
C
C     ************* STEP-SIZE COMPUTATION ******************************
C
      NDIV = 1.0 + TAU/H
      RN = NDIV
      H = TAU/RN
      H6 = H/6.0
C
C     ************* INITIAL VALUES SECTION *****************************
C
      WRITE (6,6008)  EN, TAU, GAMMA, UE, RLD
      WRITE (6,6009)
      WRITE (6,6022) (ANGLE(J), J = 1,6), (ANGLE(J), J = 1,3)
      WRITE (6,6012)
      NP1 = NDIV + 1
      DO 70 I = 1, NP1
      NSTEP = I - NP1
      RSTEP = NSTEP
      TIME = RSTEP * H
      TI(I) = TIME
      DO 75 J = 1, NJMAX
      JP = J + NJMAX
      IF (AC(J)) 751, 753, 751
  753 IF (AS(J)) 751, 752, 751
  752 U(I,J) = 0.0
      U(I,JP) = 0.0
      GO TO 75
  751 ARG = FRQ1(J) * TIME
      FSIN = SIN(ARG)
      FCOS = COS(ARG)
      FEXP = EXP(DMP1(J)*TIME)
      U(I,J) = (AS(J)*FSIN + AC(J)*FCOS) * FEXP
```

```
      U(I,JP) = ((AS(J) * FCOS) - (AC(J) * FSIN)) * FRQ1(J) * FEXP
     1             + DMP1(J) * U(I,J)
   75 CONTINUE
C     CALCULATE INITIAL VALUES OF PRESSURE AND VELOCITY.
      DO 704 NPRES = 1, 6
      DO 702 J = 1, NJMAX
      COEF(1,J) = CFT(NPRES,J)
      COEF(2,J) = CFTH(NPRES,J)
      COEF(3,J) = CFZ(NPRES,J)
  702 CONTINUE
      DO 703 J = 1, NU
      Y(J) = U(I,J)
  703 CONTINUE
      UBAR = 0.0
      IF (NPRES .GT. 3) UBAR = UE
      UMS = 0.0
      IF ((NDROPS.EQ.1) .AND. (NPRES.LT.4)) UMS = UE/(ZE*ZCOMB)
      CALL PRSVEL(UBAR,UMS,Y,P,VTH,VZ)
      PRESS(NPRES) = P
      IF (NPRES .GT. 3)  AXVEL(NPRES - 3) = VZ
  704 CONTINUE
      PRS(I) = PRESS(NLOC)
C
C     CALCULATE INITIAL VALUES OF NOZZLE B.C.
      CSUM = (0.0,0.0)
      DO 710 J = 1, JMX
      JP = NJMAX + (2 * J) - 1
      FT = Y(JP)
      GT = Y(JP+1)
      A = CMPLX(FT,GT)
      CSUM = CSUM + YNOZ(J) * CPHIT(J) * A
  710 CONTINUE
      SUM = REAL(CSUM)
      YPHI = -GAMMA * SUM
      WRITE (6,6011)  NSTEP, TIME, (PRESS(J), J = 1,6),
     1                (AXVEL(J), J = 1,3), YPHI
   70 CONTINUE
C
      WRITE (6,6008)  EN, TAU, GAMMA, UE, RLD
      WRITE (6,6022) (ANGLE(J), J = 1,6), (ANGLE(J), J = 1,3)
C
C     ************* INITIALIZE CONTROL NUMBERS *************************
C
      LINE = 8
      K = 0
      MAXNO = 0
      MAXP = 0
      IF (NOUT .EQ. 0)  GO TO 100
      JPLOT = 0
      TMIN = TSTART
```

```
      TMAX = TSTART + TDEL
      YLO(1) = -YHI(1)
      DO 90 J = 2,4
      YHI(J) = YHI(1)
      YLO(J) = YLO(1)
      YLAB(J) = YLAB(1)
      ITICY(J) = ITICY(1)
   90 CONTINUE
      YLO(5) = -YHI(5)
      YHI(6) = YHI(5)
      YLO(6) = YLO(5)
      YLAB(6) = YLAB(5)
      ITICY(6) = ITICY(5)
C
C     ************* NUMERICAL CALCULATIONS SECTION *********************
C
  100 I = NP1
C
C     RUNGE-KUTTA INTEGRATION SCHEME.
  105 NSTEP = (I - NP1 + (LAST - NP1) * K)
      RSTEP = NSTEP
      TIME = RSTEP * H
      TI(I) = TIME
      DO 110 J = 1, NJMAX
      JP = J + NJMAX
      RV(J,1) = U(I-NDIV,JP)
      RV(J,4) = U(I-NDIV+1,JP)
      RV(J,2) = 0.375*RV(J,1) + 0.75*RV(J,4) - 0.125*U(I-NDIV+2,JP)
      RV(J,3) = RV(J,2)
  110 CONTINUE
      DO 120 J = 1, NU
      Y(J) = U(I,J)
  120 CONTINUE
      CALL RHS(NU,1,Y,YP)
      DO 130 J = 1, NU
      FZ(1,J) = YP(J)
  130 CONTINUE
      DO 140 II = 2,4
      DO 144 J = 1, NU
      UZ(J) = Y(J) + AA(II) * H * FZ(II-1,J)
  144 CONTINUE
      CALL RHS(NU,II,UZ,YP)
      DO 148 J = 1, NU
      FZ(II,J) = YP(J)
  148 CONTINUE
  140 CONTINUE
      DO 150 J = 1, NU
      U(I+1,J) = Y(J) + (FZ(1,J)+2.0*(FZ(2,J)+FZ(3,J)) + FZ(4,J)) * H6
  150 CONTINUE
C
```

```
C     CALCULATE PRESSURE TIME HISTORIES.
      DO 154 NPRES = 1, 6
      DO 152 J = 1, NJMAX
      COEF(1,J) = CFT(NPRES,J)
      COEF(2,J) = CFTH(NPRES,J)
      COEF(3,J) = CFZ(NPRES,J)
  152 CONTINUE
      UBAR = 0.0
      IF (NPRES .GT. 3) UBAR = UE
      UMS = 0.0
      IF ((NDROPS.EQ.1) .AND. (NPRES.LT.4)) UMS = UE/(ZL*ZCOMB)
      CALL PRSVEL(UBAR,UMS,Y,P,VTH,VZ)
      PRESS(NPRES) = P
      IF (NPRES .GT. 3)  AXVEL(NPRES - 3) = VZ
  154 CONTINUE
      PRS(I) = PRESS(NLOC)
C
C     CALCULATE VALUES OF NOZZLE B.C.
      CSUM = (0.0,0.0)
      DO 650 J = 1, JMX
      JP = NJMAX + (2 * J) - 1
      FT = Y(JP)
      GT = Y(JP+1)
      A = CMPLX(FT,GT)
      CSUM = CSUM + YNOZ(J) * CPHIT(J) * A
  650 CONTINUE
      SUM = REAL(CSUM)
      YPHI = -GAMMA * SUM
C
C
C     DETERMINE MAXIMA AND MINIMA OF PRINCIPAL MODE-AMPLITUDE
C     FUNCTION FOR USE IN DETERMINING LIMIT-CYCLE BEHAVIOR.
      IF (U(I,JPMODE) * U(I+1,JPMODE)) 170, 170, 160
  170 PDEN = U(I,JPMODE) - U(I+1,JPMODE)
      IF (PDEN) 171, 160, 171
  171 PP = U(I,JPMODE)/PDEN
      PA = (PP - 1.0) * PP * 0.5
      PB = 1.0 - (PP * PP)
      PC = (PP + 1.0) * PP * 0.5
      MAXNO = MAXNO + 1
      UMAX(MAXNO) = PA*U(I-1,JMODE) + PB*U(I,JMODE) + PC*U(I+1,JMODE)
      IF (MAXNO .GE. 500)  GO TO 250
  160 CONTINUE
C
C     DETERMINE MAXIMUM AND MINIMUM PRESSURE AT LOCATION SPECIFIED
C     BY NLOC.
      DPL = PRS(I) - PRS(I-1)
      DPS = PRS(I-1) - PRS(I-2)
      IF (DPL*DPS) 173, 173, 175
  173 PNUM = PRS(I-2) - PRS(I)
```

```
      PDEN = 2.0 * (PRS(I-2) + PRS(I) - 2.0*PRS(I-1))
      IF (PDEN) 174, 175, 174
  174 PP = PNUM/PDEN
      PA = (PP - 1.0) * PP * 0.5
      PB = 1.0 - (PP * PP)
      PC = (PP + 1.0) * PP * 0.5
      MAXP = MAXP + 1
      PMAX(MAXP) = PA*PRS(I-2) + PB*PRS(I-1) + PC*PRS(I)
      TIMAX(MAXP) = TI(I-1) + PP*H
      IF (MAXP .GE. 500)  GO TO 250
  175 CONTINUE
C
      IF (NTEST .EQ. 1)  GO TO 155
      IF (TIME .LT. TSTART)  GO TO 155
      IF ((NOUT .EQ. 0) .OR. (NOUT .GT. 6))  GO TO 156
C
C     ************* TIME HISTORY PLOTTING SECTION **********************
C
      IF (TMAX .GT. TQUIT)  GO TO 156
      IF ((TIME .GT. TMAX) .OR. (JPLOT .GE. 500))  GO TO 1000
C
      JPLOT = JPLOT + 1
C
C     FILL TIME ARRAY FOR PLOTTING.
      TPLOT(JPLOT) = TIME
C
C     FILL INJECTOR PRESSURE ARRAYS FOR PLOTTING (THETA = 0, 45, 90)
      DO 1001 J = 1,3
      YPLOT(J,JPLOT) = PRESS(J)
 1001 CONTINUE
C
C     FILL NOZZLE PRESSURE ARRAY FOR PLOTTING (THETA = 0)
      YPLOT(4,JPLOT) = PRESS(4)
C
C     FILL NOZZLE AXIAL VELOCITY ARRAY FOR PLOTTING (THETA = 0)
      YPLOT(5,JPLOT) = AXVEL(1)
C
C     FILL NOZZLE B.C. ARRAY FOR PLOTTING (THETA = 0).
      YPLOT(6,JPLOT) = YPHI
C
      GO TO 156
C
 1000 NUM = JPLOT
C
C     PLOT TIME HISTORIES.
C
      DO 1020 NPLOT = NFIRST, NOUT
C
      JPLOT = 0
C
```

```
C     ASSIGN PLOTTING PARAMETERS.
      YMIN = YLO(NPLOT)
      YMAX = YHI(NPLOT)
      NTICY = ITICY(NPLOT)
      DELY = YLAB(NPLOT)
C
C     ELIMINATE POINTS THAT ARE OUT OF THE ORDINATE RANGE.
      DO 1010 J = 1, NUM
      IF ((YPLOT(NPLOT,J) .LT. YMIN) .OR. (YPLOT(NPLOT,J) .GT. YMAX))
     1   GO TO 1010
      JPLOT = JPLOT + 1
      DUMMYT(JPLOT) = TPLOT(J)
      DUMMYY(JPLOT) = YPLOT(NPLOT,J)
 1010 CONTINUE
C
      IF (JPLOT .EQ. 0)  GO TO 1020
      GO TO (1011,1012,1013,1014,1015,1016), NPLOT
C
C     PLOT INJECTOR PRESSURE AT THETA = 0 DEGREES.
 1011 CALL GRAPHS(IBUF,3000,4,JPLOT,11,NTICY,TMAX,YMAX,TMIN,YMIN,
     1               ITT,ITY1,21,41,DUMMYT,DUMMYY,2.0,DELY,TITLE)
      GO TO 1020
C
C     PLOT INJECTOR PRESSURE AT THETA = 45 DEGREES.
 1012 IF (M(JMODE) .EQ. 0)  GO TO 1020
      CALL GRAPHS(IBUF,3000,4,JPLOT,11,NTICY,TMAX,YMAX,TMIN,YMIN,
     1               ITT,ITY2,21,42,DUMMYT,DUMMYY,2.0,DELY,TITLE)
      GO TO 1020
C
C     PLOT INJECTOR PRESSURE AT THETA = 90 DEGREES.
 1013 IF (M(JMODE) .EQ. 0)  GO TO 1020
      CALL GRAPHS(IBUF,3000,4,JPLOT,11,NTICY,TMAX,YMAX,TMIN,YMIN,
     1               ITT,ITY3,21,42,DUMMYT,DUMMYY,2.0,DELY,TITLE)
      GO TO 1020
C
C     PLOT NOZZLE PRESSURE AT THETA = 0 DEGREES.
 1014 CALL GRAPHS(IBUF,3000,4,JPLOT,11,NTICY,TMAX,YMAX,TMIN,YMIN,
     1               ITT,ITY4,21,39,DUMMYT,DUMMYY,2.0,DELY,TITLE)
      GO TO 1020
C
C     PLOT NOZZLE AXIAL VELOCITY AT THETA = 0 DEGREES.
 1015 CALL GRAPHS(IBUF,3000,4,JPLOT,11,NTICY,TMAX,YMAX,TMIN,YMIN,
     1               ITT,ITY5,21,32,DUMMYT,DUMMYY,2.0,DELY,TITLE)
      GO TO 1020
C
C     PLOT NOZZLE B.C. AT THETA = 0 DEGREES.
 1016 CALL GRAPHS(IBUF,3000,4,JPLOT,11,NTICY,TMAX,YMAX,TMIN,YMIN,
     1               ITT,ITY6,21,44,DUMMYT,DUMMYY,2.0,DELY,TITLE)
C
 1020 CONTINUE
```

```
C
C     REASSIGN PLOTTING PARAMETERS FOR NEXT SET OF PLOTS.
      JPLOT = 0
      TMIN = TMAX
      TMAX = TMAX + TDEL
C
C     ************* TIME HISTORY PRINTED OUTPUT SECTION ****************
C
  156 WRITE (6,6011)  NSTEP, TIME, (PRESS(J), J = 1,6),
     1                (AXVEL(J), J = 1,3), YPHI
      LINE = LINE + 1
  157 IF (TIME .GT. TQUIT)  GO TO 250
      IF (LINE .LT. 52)  GO TO 155
      WRITE (6,6013)
      WRITE (6,6022) (ANGLE(J), J = 1,6), (ANGLE(J), J = 1,3)
      LINE = 4
C
  155 I = I + 1
      IF (I .LT. LAST)  GO TO 105
C
C     ************* LIMIT-CYCLE SECTION *********************************
C
C     TEST FOR LIMIT CYCLE.
      K = K + 1
      IF ((NTEST .EQ. 0) .OR. (MAXNO .LT. 80))  GO TO 190
      UTOT = 0.0
      DO 180 J = 0, 3
      JMAX = MAXNO - J
      UTOT = UTOT + ABS(UMAX(JMAX))
  180 CONTINUE
      UAVG(K) = UTOT/4.0
      IF (K .EQ. 1)  GO TO 190
      CHANGE = UAVG(K) - UAVG(K-1)
      ABSCHG = ABS(CHANGE/UAVG(K))
      IF (ABSCHG .GT. ERR)  GO TO 190
      TM = TIME/2.0
      ITM = TM
      ITM = 2*ITM + 2
      TM = ITM
      TSTART = TM + TSTART
      TQUIT = TM + TQUIT
      TMIN = TSTART
      TMAX = TSTART + TDEL
      NTEST = 0
C
C     RE-ASSIGN ARRAYS.
  190 DO 200 I = 1, NP1
      ILAST = LAST - NP1 + I
      PRS(I) = PRS(ILAST)
      TI(I) = TI(ILAST)
```

```
      DO 200 J = 1, NU
      U(I,J) = U(ILAST,J)
  200 CONTINUE
      GO TO 100
C
C
C     ************* PRESSURE MAXIMA AND MINIMA PRINTOUT ****************
C
  250 WRITE (6,6023) Z(NLOC), ANGLE(NLOC), MAXP
      LINE = 4
      DO 255 JST = 1, MAXP, 8
      JSTART = JST
      JSTOP = JST + 7
      IF (JSTOP .GT. MAXP) JSTOP = MAXP
      WRITE (6,6024) (PMAX(J), J = JSTART, JSTOP)
      WRITE (6,6024) (TIMAX(J), J = JSTART, JSTOP)
      WRITE (6,6014)
      LINE = LINE + 3
      IF (LINE .LT. 52)  GO TO 255
      LINE = 0
      WRITE (6,6013)
  255 CONTINUE
      IF ((NOUT .EQ. 0) .OR. (NOMIT .EQ. 1))  GO TO 5
C
C     ************* PRESSURE MAXIMA PLOTTING SECTION ******************
C
C     DETERMINE LARGEST VALUE OF PMAX.
      AMPMAX = 0.0
      DO 260 J = 1, MAXP
      IF (PMAX(J) .LT. AMPMAX)  GO TO 260
      AMPMAX = PMAX(J)
  260 CONTINUE
C
C     RANGE OF PLOT AND COORDINATE LABELING.
      ITM = AMPMAX + 1.0
      AMPMAX = ITM
      ITM = 1.0 + TIMAX(MAXP)/50.0
      TMAX = ITM * 50
      DELX = TMAX/10.0
      DELY = AMPMAX/10.0
C
C     ELIMINATE NEGATIVE VALUES.
      JPLOT = 0
      DO 262 J = 1, MAXP
      IF (PMAX(J)) 262, 264, 264
  264 JPLOT = JPLOT + 1
      DUMMYT(JPLOT) = TIMAX(J)
      DUMMYY(JPLOT) = PMAX(J)
  262 CONTINUE
C
```

```
C     PLOT VALUES.
      CALL GRAPHS(IBUF,3000,4,JPLOT,101,101,TMAX,AMPMAX,0.0,0.0,
     1            ITT,ITP,21,14,DUMMYT,DUMMYY,DELX,DELY,TITLE)
C
      GO TO 5
C
C     TURN OFF PLOTTING ROUTINE.
  300 IF (NPT .EQ. 1)  CALL SHPARG
C
C     ************* READ FORMAT SPECIFICATIONS ************************
C
 5000 FORMAT (12A6)
 5001 FORMAT (4F10.0,2I5)
 5002 FORMAT (5I5,2F10.5,1X,A4)
 5003 FORMAT (I5)
 5004 FORMAT (2I5,F15.6)
 5005 FORMAT (3I5,F15.6)
 5006 FORMAT (5F10.0)
 5007 FORMAT (I5,2F10.0)
 5008 FORMAT (7I5)
 5009 FORMAT (7F10.0)
 5010 FORMAT (I5,4F10.5)
C
C     ************* WRITE FORMAT SPECIFICATIONS ***********************
C
 6001 FORMAT (1H1,9H GAMMA = ,F5.3,5X,5HUE = ,F5.3,
     1        5X,5HZE = ,F8.5,5X,8HZCOMB = ,F5.2,
     2        5X,8HNJMAX = ,I2//)
 6002 FORMAT (2X,29HNAME    J    L    M    N    NS,7X,3HSMN,3X,
     1        7HJM(SMN)/)
 6003 FORMAT (2X,A4,5I5,2F10.5)
 6004 FORMAT (1H0,26H NUMBER OF COEFFICIENTS C(,I1,10H,NJ,NP) IS,I5/)
 6005 FORMAT (2X,2HC(,I1,1H,,I2,1H,,I2,4H) = ,F10.5)
 6006 FORMAT (1H0,38H NUMBER OF COEFFICIENTS D(NJ,NP,NQ) IS,I5/)
 6007 FORMAT (2X,2HD(,I2,1H,,I2,1H,,I2,4H) = ,F10.5)
 6008 FORMAT(1H1,45H COMBUSTION PARAMETERS:  INTERACTION INDEX = ,F7.5,
     1        12X,11HTIME-LAG = ,F7.5/2X,17HMOTOR PARAMETERS:,19X,
     2        8HGAMMA = ,F7.5,23H     EXIT MACH NUMBER = ,F7.5,
     3        22H     LENGTH/DIAMETER = ,F7.5//)
 6009 FORMAT (2X,18HINITIAL CONDITIONS//)
 6010 FORMAT (1H0,5X,1HJ,8X,2HYR,8X,2HYI,7X,3HEPS,7X,3HETA//)
 6011 FORMAT (2X,I5,F12.5,10F10.5)
 6012 FORMAT (1H0)
 6013 FORMAT (1H1)
 6014 FORMAT (1H )
 6015 FORMAT (2X,I5,4F10.5)
 6016 FORMAT (1H1,36H INITIAL CONDITIONS ARE OF THE FORM://
     1        2X,49HU(I,J) =  AC(J)*COS(FREQ*T) + AS(J)*SIN(FREQ*T)),
     2        14H * EXP(DAMP*T)///6X,1HJ,8X,7HDAMPING,
     3        6X,9HFREQUENCY,10X,5HAC(J),10X,5HAS(J)//)
```

```
6017 FORMAT (2X,I5,4F15.8/)
6020 FORMAT (1H1,46H COEFFICIENTS FOR COMPUTATION OF WALL PRESSURE,
    1          10H WAVEFORMS///43X,27HCOEFFICIENTS IN SERIES FOR://
    2          22X,5HTHETA,10X,4HTIME,10X,5HTHETA,10X,5HAXIAL/
    3                      6X,1HJ,9X,1HZ,3X,9H(DEGREES),5X,10HDERIVATIVE,
    4          5X,10HDERIVATIVE,5X,10HDERIVATIVE//)
6021 FORMAT (2X,I5,F10.3,F12.1,3F15.7)
6022 FORMAT (26X,17HINJECTOR PRESSURE,14X,15HNOZZLE PRESSURE,
    1          12X,21HNOZZLE AXIAL VELOCITY/3X,4HSTEP,8X,4HTIME,
    2          F5.0,5H DEG.,F5.0,5H DEG.,F5.0,5H DEG.,
    3          F5.0,5H DEG.,F5.0,5H DEG.,F5.0,5H DEG.,
    4          F5.0,5H DEG.,F5.0,5H DEG.,F5.0,5H DEG.,6X,4HYPHI//)
6023 FORMAT (1H1,38H PRESSURE MAXIMA AND MINIMA AT:    Z = ,F5.2,
    1          11H   THETA = ,F4.1/19H VALUES COMPUTED:   ,I3//)
6024 FORMAT (1H ,7X,8F13.6)
6025 FORMAT (2X//2X,37HTHE TRANSIENT BEHAVIOR IS CALCULATED.)
6026 FORMAT (2X//2X,39HTHE LIMIT-CYCLE BEHAVIOR IS CALCULATED.)
6027 FORMAT (2X//2X,33HTHIS RUN PRODUCES PLOTTED OUTPUT.)
6028 FORMAT (2X//2X,'THE PHANTOM ZONES ARE ELIMINATED.')
6030 FORMAT (2X,'DROPLET MOMENTUM SOURCE IS NEGLECTED'/)
6031 FORMAT (2X,'DROPLET MOMENTUM SOURCE IS INCLUDED'/)
     END
```

```
      SUBROUTINE PHICFS(NP,Z,THETA,CT,CTH,CZ)
C
C     THIS SUBROUTINE COMPUTES THE COEFFICIENTS NEEDED TO
C     CALCULATE THE WALL PRESSURE PERTURBATION.
C
C     NP IS THE INDEX OF THE COMPLEX SERIES TERM.
C     Z IS THE AXIAL LOCATION.
C     THETA IS THE AZIMUTHAL LOCATION.
C     CT IS THE COEFFICIENT IN THE SERIES FOR THE TIME DERIVATIVE OF
C     THE VELOCITY POTENTIAL.
C     CTH IS THE COEFFICIENT IN THE SERIES FOR THE THETA DERIVATIVE
C     OF THE VELOCITY POTENTIAL.
C     CZ IS THE COEFFICIENT IN THE SERIES FOR THE AXIAL DERIVATIVE
C     OF THE VELOCITY POTENTIAL.
C
      COMPLEX     CI, CZ, CAXI, CAXIZ, CRAD, CAZI, CAZITH,
     1            B(10), CT, CTH, CZ
      COMMON        /BLK2/  M(10), NS(10), SJ(10), B
C
      CI = (0.0,1.0)
      CZ = CMPLX(Z,0.0)
      CAXI = CCOSH(CI * B(NP) * CZ)
      CAXIZ = CI * B(NP) * CSINH(CI * B(NP) * CZ)
      CRAD = CMPLX(SJ(NP),0.0)
      EM = M(NP)
      ARG = EM * THETA
      FSIN = SIN(ARG)
      FCOS = COS(ARG)
      AZI = FCOS
      IF (NS(NP) .EQ. 1)   AZI = FSIN
      AZITH = EM * FCOS
      IF (NS(NP) .EQ. 2)   AZITH = -EM * FSIN
      CAZI = CMPLX(AZI,0.0)
      CAZITH = CMPLX(AZITH,0.0)
C
      CT = CAZI * CAXI * CRAD
      CTH = CAZITH * CAXI * CRAD
      CZ = CAZI * CAXIZ * CRAD
C
      RETURN
      END
```

```
      SUBROUTINE PRSVEL(UBAR,UMS,Y,P,VTH,VZ)
C
C     THIS SUBROUTINE COMPUTES THE WALL PRESSURE AND VELOCITY.
C
C     UBAR IS THE LOCAL AXIAL STEADY STATE MACH NUMBER.
C     UMS IS THE DERIVATIVE OF THE MACH NUMBER FOR THE CASE
C      WHEN DROPLET MOMENTUM SOURCES ARE INCLUDED.
C     Y IS THE ARRAY CONTAINING VALUES OF THE MODE-AMPLITUDE
C     FUNCTIONS AND THEIR DERIVATIVES.
C     P IS THE VALUE OF THE WALL PRESSURE PERTURBATION.
C     VTH IS THE TANGENTIAL COMPONENT OF VELOCITY AT THE WALL.
C     VZ IS THE AXIAL COMPONENT OF VELOCITY AT THE WALL.
C
      DIMENSION    Y(40), SUM(4), SUMSQ(3)
      COMMON       /BLK3/    NJMAX, NLMAX, GAMMA, COEF(3,20)
C
      DO 10 I = 1, 4
      SUM(I) = 0.0
   10 CONTINUE
C
      DO 20 I = 1, 4
      DO 20 J = 1, NJMAX
      JY = J
      IF (I .EQ. 1)  JY = J + NJMAX
      II = I
      IF (I .EQ. 4) II = 1
      SUM(I) = SUM(I) + Y(JY) * COEF(II,J)
   20 CONTINUE
C
      PLIN = SUM(1) + UBAR*SUM(3) + UMS*SUM(4)
      PNL = 0.0
      IF (NLMAX .EQ. 0)  GO TO 40
      DO 30 I = 1, 3
      SUMSQ(I) = SUM(I) * SUM(I)
   30 CONTINUE
      PNL = 0.5 * (SUMSQ(2) + SUMSQ(3) - SUMSQ(1))
C
   40 P = -GAMMA * (PLIN + PNL)
      VTH = SUM(2)
      VZ = SUM(3)
C
      RETURN
      END
```

```
      SUBROUTINE RHS(NU,II,U,UP)
C
      DIMENSION     U(NU), UP(NU)
      COMMON        RV(20,4), C(3,20,20), D(20,400),
     1              KPMAX(3,20), IC(3,20,20), KPQMAX(20),
     2              IDP(20,400), IDQ(20,400)
      COMMON        /BLK3/    NJMAX, NLMAX, GAMMA, COEF(3,20)
C
      DO 10 NJ = 1, NJMAX
      NJP = NJ + NJMAX
      UP(NJ) = U(NJP)
      SL1 = 0.0
      SL2 = 0.0
      SL3 = 0.0
      SNL = 0.0
      MAX = KPMAX(1,NJ)
      IF (MAX .EQ. 0) GO TO 25
      DO 20 KP = 1, MAX
      NP = IC(1,NJ,KP)
      SL1 = SL1 + (C(1,NJ,KP) * U(NP))
   20 CONTINUE
   25 MAX = KPMAX(2,NJ)
      IF (MAX .EQ. 0) GO TO 35
      DO 30 KP = 1, MAX
      NPP = IC(2,NJ,KP) + NJMAX
      SL2 = SL2 + (C(2,NJ,KP) * U(NPP))
   30 CONTINUE
   35 MAX = KPMAX(3,NJ)
      IF (MAX .EQ. 0) GO TO 45
      DO 40 KP = 1, MAX
      NP = IC(3,NJ,KP)
      SL3 = SL3 + (C(3,NJ,KP) * RV(NP,II))
   40 CONTINUE
   45 IF (NLMAX .EQ. 0)  GO TO 55
      MAX = KPQMAX(NJ)
      IF (MAX .EQ. 0) GO TO 55
      DO 50 KPQ = 1, MAX
      NP = IDP(NJ,KPQ)
      NQP = IDQ(NJ,KPQ) + NJMAX
      SNL = SNL + (D(NJ,KPQ) * U(NP) * U(NQP))
   50 CONTINUE
   55 UP(NJP) = -(SL1 + SL2 + SL3 + SNL)
   10 CONTINUE
      RETURN
      END
```

```
      COMPILER (FLD=ABS)
      SUBROUTINE GRAPHS(IBUF,NLOC,LDEV,NTOT,NTICX,NTICY,
     1 XMAX,YMAX,XMIN,YMIN,ITITLX,ITITLY,LTITLX,LTITLY,XARRAY,
     2 YARRAY,DELX,DELY,TITLE)
C-----------------------------------------------------------------------
C
C IDENTIFIER               MEANING                                          TYPE
C
C IBUF:    ADDRESS OF BUFFER AREA FOR PLOT OUTPUT              INTEGER
C NLOC:    NUMBER OF LOCATIONS IN BUFFER AREA (>=2000)         INTEGER
C LDEV:    LOGICAL DEVICE NUMBER FOR PLOT                      INTEGER
C NTOT:    NUMBER OF POINTS TO BE PLOTTED                      INTEGER
C NTICX:   NUMBER OF TIC MARKS ON ABSCISSA (>=2)               INTEGER
C NTICY:   NUMBER OF TIC MARKS ON ORDINATE (>=2)               INTEGER
C XMAX:    UPPER LIMIT OF ABSCISSA DOMAIN                      REAL
C YMAX:    UPPER LIMIT OF ORDINATE RANGE                       REAL
C XMIN:    LOWER LIMIT OF ABSCISSA DOMAIN                      REAL
C YMIN:    LOWER LIMIT OF ORDINATE RANGE                       REAL
C ITITLX:  ABSCISSA LABEL                                      FIELDATA ARRAY
C ITITLY:  ORDINATE LABEL                                      FIELDATA ARRAY

C LTITLX:  NUMBER OF CHARACTERS IN ITITLX                      INTEGER
C LTITLY:  NUMBER OF CHARACTERS IN ITITLY                      INTEGER
C XARRAY:  ABSCISSA POINTS IN TERMS OF XMIN-XMAX COORD'S       REAL ARRAY
C YARRAY:  ORDINATE POINTS IN TERMS OF YMIN-YMAX COORD'S       REAL ARRAY
C DELX:    INTERVALS OF ABSCISSA TIC MARK LABELING
C          IN TERMS OF XMIN-XMAX COORDINATES                   REAL
C DELY:    INTERVALS OF ORDINATE TIC MARK LABELING
C          IN TERMS OF YMIN-YMAX COORDINATES                   REAL
C TITLE:   LABEL FOR THE WHOLE RUN                             FIELDATA ARRAY
C
C-----------------------------------------------------------------------
      DIMENSION IBUF(NLOC),XARRAY(NTOT),YARRAY(NTOT),ITITLX(1),
     1 ITITLY(1),YLIT(100)
      DIMENSION TITLE(1)
C-----------------------------------------------------------------------
C
C     FIXED BASIC PARAMETERS
C
C-----------------------------------------------------------------------
      LOGICAL ZERO
      DEFINEZERO=NDEC.LT.0.AND.ABS(FPN).LT..5
     1 .OR.NDEC.GT.0.AND.ABS(FPN).LT.5.*10.**(-NDEC-1)
      DEFINE DNDEC=NDEC-FLD(0,36,ZERO)*NDEC-FLD(0,36,ZERO)
      DEFINE IFIX(FARG)=INT(FARG+.5)
      DATA J/1/
      DATA HEIGHT/.105/
      DATA INTEG/1/
      DATA ABSCIS/8./
      DATA ORDINA/6./
      DATA ICODE/-1/
```

```
      DATA TOPMAR/1./
      DATA BOTMAR/1.5/
      REAL LEFMAR
      DATA LEFMAR/1.9/
      DATA RYTMAR/1.1/
      DATA FACT/1./
      DATA MAXIS/1/
      DATA MLINE/1/
      DATA HTLAB/.105/
C ----------------------------------------------------------------------
C
C     19 INITIAL COMPUTATION OF DERIVED PARAMETERS
C        AND INITIAL PLOTS CALL
C     20 SKIPS PRELIMINARIES FOR 2ND AND SUBSEQUENT CALLS
C
C ----------------------------------------------------------------------
      GO TO (19,20),J
19    YDIT(1) = 3./19.
      TICKLE = HEIGHT/2.
      ROTFAC =  - 3./14. * HEIGHT - 4./7. * HEIGHT
      STARTL = 6 * HEIGHT + ROTFAC + TICKLE
      SEPLAB = STARTL + 1.5 * HEIGHT
      SYMBLH = 0.070
      REAL LABSEP
      LABSEP = 4. * HEIGHT
      ASTART = 2. * HEIGHT
      DO 1 I = 2,100
1     YDIT(I) = YDIT(I - 1) + (2 * MOD(I,2) + 1)/19.
      YDIT(100) = YDIT(100) + .5
      CALL PLOTS(IBUF,NLOC,LDEV)
      CALL FACTOR(1.)
      J = 2
      CALL SYMBOL (HEIGHT,36 * HEIGHT + 5.5,HEIGHT,TITLE,270.,72)
      CALL PLOT(1., - .5, - 3)
3     DO 2 I = 1,100
2     CALL PLOT(0.,YDIT(I),3 - MOD(I,2))
      DO 33 I = 1,100
33    YDIT(I) = YDIT(I) - ABSCIS - RYTMAR
C ----------------------------------------------------------------------
C
C     RESET ORIGIN
C
C ----------------------------------------------------------------------
      XPAGE = BOTMAR + ORDINA
      GO TO 2019
20    XPAGE = BOTMAR + ORDINA + TOPMAR
2019  CALL WHERE(RXPAGE,RYPAGE,FACT)
      YPAGE = RYPAGE - LEFMAR
      CALL PLOT(XPAGE,YPAGE, - 3)
      CALL FACTOR(FACT)
```

```
C -----------------------------------------------------------------
C
C     DRAW AXES AND LABELING MAXIS TIMES
C
C -----------------------------------------------------------------
      DO 100 I = 1,MAXIS
100   CALL MYAXIS
C -----------------------------------------------------------------
C
C     DRAW POINTS, OPTIONAL CENTERLINE,AND PAGE SCISSORLINE
C     MLINE TIMES
C
C -----------------------------------------------------------------
      DO 200 I = 1,MLINE
200   CALL MYLINE
      RETURN
C -----------------------------------------------------------------
C
C     ENTRY POINT SHPARG
C     TERMINATE PLOTTING SEQUENCE
C
C -----------------------------------------------------------------
      ENTRY SHPARG
      CALL WHERE(RXPAGE,RYPAGE,I)
      CALL PLOT(RXPAGE,RYPAGE,999)
      RETURN
C -----------------------------------------------------------------
C
C     SUBROUTINE MYAXIS (INTERNAL)
C
C -----------------------------------------------------------------
      SUBROUTINE MYAXIS
      STARTL = 6 * HEIGHT + ROTFAC + TICKLE
      IMAX = IFIX((YMAX - YMIN)/DELY)
      TICSEP = ORDINA/(ABS(NTICY) - 1)
      CALL DENDEC(YMAX,DELY,NDEC)
      K = 1
      N = (ABS(NTICY)/IMAX) - 1 + MOD(ABS(NTICY),2)
      DO 9 I = 0,IMAX
      GO TO (11,12),K
11    IF(2 * I.LT.IMAX)GO TO 12
      CALL AXLAB(0.,ITITLY,LTITLY,HTLAB)
      K = 2
12    FPN = YMAX - I * DELY
      IF(ZERO)FPN = 0.
      TMID = 1.
      XPAGE =  - I * ORDINA/IMAX - .5 * HEIGHT
      IF(FPN)113,122,118
113   IF(NDEC - 2)115,114,112
114   YPAGE = STARTL
```

```
      GO TO 112
115   IF(NDEC - 1)117,116,112
116   YPAGE = STARTL - HEIGHT
      GO TO 112
117   IF(ABS(FPN) - 100.)119,116,116
119   IF(ABS(FPN) - 10.)120,121,121
120   YPAGE = STARTL - 3 * HEIGHT
      GO TO 112
121   YPAGE = STARTL - 2 * HEIGHT
      GO TO 112
122   YPAGE = STARTL - 4 * HEIGHT
      GO TO 112
118   IF(NDEC - 2)123,116,112
123   IF(NDEC - 1)125,124,112
124   IF(FPN - 10.)121,116,116
125   IF(FPN - 10.)122,120,126
126   IF(FPN - 100.)120,121,127
127   IF(FPN - 1000.)121,116,128
128   IF(FPN - 10000.)116,114,114
112   NNDEC = DNDEC
      CALL NUMBER(XPAGE,YPAGE,HEIGHT,FPN,270.,NNDEC)
      XPAGE =  - I * (ORDINA/IMAX)
      DO 10 JJ = 1,N
      YPAGE = TICKLE * TMID
      CALL PLOT(XPAGE,YPAGE,3)
      YPAGE = YPAGE * ( - 1 + I/IMAX * .5)
      CALL PLOT(XPAGE,YPAGE,2)
      IF(I/IMAX)110,110,9
110   YPAGE = 0
      CALL PLOT(XPAGE,YPAGE,3)
      XPAGE = XPAGE - TICSEP
      CALL PLOT(XPAGE, YPAGE,2)
      TMID = .5
10    CONTINUE
9     CONTINUE
      K = 1
      IMAX = IFIX((XMAX - XMIN)/DELX)
      TICSEP = ABSCIS/(NTICX - 1)
      XPAGE =  - ASTART - ORDINA
      CALL DENDEC(XMAX,DELX,NDEC)
      DO 28 I = 0,IMAX
      STARTL =  - I * ABSCIS/IMAX
      GO TO (24,25),K
24    IF(2 * I.LT.IMAX)GO TO 25
      CALL AXLAB(270.,ITITLX,LTITLX,HTLAB)
      K = 2
      XPAGE =  - ASTART - ORDINA
25    FPN = XMIN + I * DELX
      IF(ZERO)FPN = 0.
      IF(FPN)813,822,818
813   IF(NDEC - 2)815,814,23
814   YPAGE = STARTL + 16./7. * HEIGHT
      GO TO 23
815   IF(NDEC - 1)817,816,23
```

```
816   YPAGE = STARTL + 25./14. * HEIGHT
      GO TO 23
817   IF(ABS(FPN) - 100.)819,816,816
819   IF(ABS(FPN) - 10.)820,821,821
820   YPAGE = STARTL + 11./14. * HEIGHT
      GO TO 23
821   YPAGE = STARTL + 9./7. * HEIGHT
      GO TO 23
822   YPAGE = STARTL + 2./7. * HEIGHT
      GO TO 23
818   IF(NDEC - 2)823,816,23
823   IF(NDEC - 1)825,824,23
824   IF(FPN - 10.)821,816,816
825   IF(FPN - 10.)822,820,826
826   IF(FPN - 100.)820,821,827
827   IF(FPN - 1000.)821,816,828
828   IF(FPN - 10000.)816,814,814
23    NNDEC = DNDEC
28    CALL NUMBER(XPAGE,YPAGE,HEIGHT,FPN,270.,NNDEC)
      N = (NTICX/IMAX) - 1 + MOD(NTICX,2)
      DO 26 I = IMAX,0, - 1
      TMID = 1.
      YPAGE =  - I * ABSCIS/IMAX
      DO 27 JJ = 1,N
      XPAGE =  - ORDINA - TICKLE * TMID
      CALL PLOT(XPAGE,YPAGE,3)
      XPAGE = XPAGE + (TICKLE + FLD(0,36,I.NE.0) * TICKLE) * TMID
      CALL PLOT(XPAGE,YPAGE,2)
      IF(I)111,26,111
111   XPAGE =  - ORDINA
      CALL PLOT(XPAGE,YPAGE,3)
      YPAGE = YPAGE + TICSEP
      CALL PLOT(XPAGE,YPAGE,2)
      TMID = .5
27    CONTINUE
26    CONTINUE
      RETURN
C ------------------------------------------------------------------
C
C     SUBROUTINE MYLINE (INTERNAL)
C
C ------------------------------------------------------------------
      SUBROUTINE MYLINE
      ITOP = IFIX((ABSCIS + RYTMAR + .5)/11. * 99.)
      IBOT = IFIX(RYTMAR/11. * 99.)
      DO 17 I = 1,NTOT
      XPAGE = (YARRAY(I) - YMAX)/(YMAX - YMIN) * ORDINA
      YPAGE = (XMIN - XARRAY(I))/(XMAX - XMIN) * ABSCIS
17    CALL SYMBOL(XPAGE,YPAGE,SYMBLH,INTEQ,270.,ICODE)
      IF(NTICY.GE.0)GO TO 22
      XPAGE =  - ORDINA/2.
      YPAGE =  - ABSCIS
      CALL PLOT(XPAGE,YPAGE,3)
      DO 18 I = IBOT,ITOP
```

```
18     CALL PLOT(XPAGE,YDIT(I),3 - MOD(I,2))
22     XPAGE = TOPMAR
       YPAGE =  - ABSCIS - RYTMAR - .5
       CALL PLOT(XPAGE,YPAGE,3)
       DO 21 I = 1,100
21     CALL PLOT(XPAGE,YDIT(I),3 - MOD(I,2))
       RETURN
C -------------------------------------------------------------------
C
C      SUBROUTINE AXLAB (INTERNAL)
C
C -------------------------------------------------------------------
       SUBROUTINE AXLAB(ANGLE,IBCD,NCHARX,HEIGHT)
       DIMENSION IBCD(7)
       LOGICAL S
       INTEGER QSQ/' S'/
       K = 2
       NCHAR = NCHARX
       S = .FALSE.
       IF(ABS(ANGLE).GT..1)GO TO 30
       XPAGE =  - ORDINA/2. - NCHAR * HEIGHT/2
       YPAGE = SEPLAB
       GO TO 31
30     XPAGE =  - ORDINA - LABSEP
       YPAGE =  - ABSCIS/2. + NCHAR * HEIGHT/2
31     LSTART = 6 * MOD(NCHAR,6) - 12
       IF(LSTART.EQ. - 12)LSTART = 24
       LOOK = NCHAR/6 + 1.1
       IF(LSTART.EQ. - 6)GO TO 13
       IF(FLD(0,12,',S').EQ.FLD(LSTART,12,IBCD(LOOK)))GO TO 15
       GO TO 14
13     IF(FLD(0,6,',').NE.FLD(30,6,IBCD(LOOK - 1)))GO TO 14
       IF(FLD(0,6,'S').NE.FLD(0,6,IBCD(LOOK)))GO TO 14
15     NCHAR = NCHAR - 1
       S = .TRUE.
14     CALL SYMBOL(XPAGE,YPAGE,HEIGHT,IBCD,ANGLE,NCHAR)
       IF(S)CALL SYMBOL(999.,999.,2 * HEIGHT/3,QSQ,ANGLE,2)
       RETURN
C -------------------------------------------------------------------
C
C      SUBROUTINE DENDEC (INTERNAL)
C
C -------------------------------------------------------------------
       SUBROUTINE DENDEC(QMAX,DELQ,NDEC)
       IF(INT(ABS(QMAX)).GE.10)GO TO 5
       IF(AMOD(ABS(QMAX - DELQ),.1).GE..01)GO TO 7
       NDEC = 1
       RETURN
5      NDEC =  - 1
       RETURN
7      NDEC = 2
       RETURN
       END
```

APPENDIX E

USER'S MANUAL FOR THE LINEAR STABILITY PROGRAMS: LINSOL AND LSTB3D

General Description

Two auxiliary programs, LINSOL and LSTB3D, calculate the linear stability characteristics of a cylindrical combustion chamber with distributed combustion and a conventional nozzle. For given values of the operating parameters (i.e., n, $\bar{\tau}$, γ, $\bar{u}_e$, and L/D) and a given nozzle admittance (i.e., A and φ), Program LINSOL calculates the growth rate, Λ, and the frequency, ω, of a given acoustic mode. For given values of $\bar{\tau}$ Program LSTB3D calculates the corresponding values of n and ω for neutral stability ($\Lambda = 0$). These programs are based on an analytical solution of the linearized version of Eqs. (12). After a discussion of the linear analysis, Programs LINSOL and LSTB3D will be described.

Linear Analysis

For a single acoustic mode, dropping the nonlinear terms in Eqs. (12) yields the following linear equation:

$$\frac{d^2A}{dt^2} + C_1A + (C_2 - nC_3)\frac{dA}{dt} + nC_3\frac{d\left[A(t - \bar{\tau})\right]}{dt} = 0 \qquad \text{(E-1)}$$

where A(t) is the unknown complex amplitude function for the mode under consideration and the coefficients are obtained from Eqs. (C-1) through (C-4) by dividing by C_0. Thus the coefficients are complex numbers given by:

$$C_1 = S_{mn}^2 + \frac{Z'(z_e)Z^*(z_e) - \int_o^{z_e} Z''Z^*dz}{\int_o^{z_e} ZZ^*dz} \qquad \text{(E-2)}$$

$$C_2 = \frac{2\int_0^{z_e} \bar{u}(z) Z' Z^* dz + \gamma \int_0^{z_e} \frac{d\bar{u}}{dz} Z Z^* dz + \gamma Y Z(z_e) Z^*(z_e)}{\int_0^{z_e} Z Z^* dz} \tag{E-3}$$

$$C_3 = \frac{\gamma \int_0^{z_e} \frac{d\bar{u}}{dz} Z Z^* dz}{\int_0^{z_e} Z Z^* dz} \tag{E-4}$$

where the droplet momentum source has been neglected. When the droplet momentum source is included, the γ in the second term of Eq. (E-3) is replaced by $\gamma + 1$ (see Appendix A).

The linear solutions are determined by substituting a solution of the form:

$$A(t) = a e^{(\Lambda + i\omega)t} \tag{E-5}$$

into Eq. (E-1) and separating real and imaginary parts to obtain:

$$\omega^2 = C_{1r} + \Lambda^2 + (C_{2r} - nC_3)\Lambda - C_{2i}\omega + C_3 n e^{-\Lambda\bar{\tau}}(\Lambda \cos\omega\bar{\tau} + \omega \sin\omega\bar{\tau}) \tag{E-6}$$

$$\Lambda = -\left\{ \frac{C_{1i} + (C_{2r} - nC_3)\omega + nC_3 e^{-\Lambda\bar{\tau}} \omega \cos\omega\bar{\tau}}{2\omega + C_{2i} - nC_3 e^{-\Lambda\bar{\tau}} \sin\omega\bar{\tau}} \right\} \tag{E-7}$$

where $C_1 = C_{1r} + iC_{1i}$, $C_2 = C_{2r} + iC_{2i}$, and C_3 is always real. The above equations are solved numerically by Program LINSOL to obtain the growth rate, Λ, and the frequency, ω, for given values of n and $\bar{\tau}$.

The equations describing the neutral stability limits are obtained by substituting $\Lambda = 0$ into Eqs. (E-6) and (E-7). Solving the resulting equations for n and ω^2 gives:

$$n = \frac{C_{2r} + C_{1i}/\omega}{C_3(1 - \cos\omega\bar{\tau})} \quad \text{(E-8)}$$

$$\omega^2 = C_{1r} + \omega(nC_3 \sin\omega\bar{\tau} - C_{2i}) \quad \text{(E-9)}$$

which are solved numerically by Program LSTB3D.

Program LINSOL

Program Structure. A flow chart for Program LINSOL is given in Fig. (E-1). This program consists of the following major sections: (1) input, (2) calculation of the coefficients C_1, C_2, and C_3, (3) iterative solution for Λ and ω, and (4) output.

Input. The input data required by Program LINSOL includes: (1) a title for the run, (2) the chamber parameters γ, u_e, L/D, and z_c/z_e, (3) several control numbers, (4) the nozzle admittance, (5) the mode under consideration, and (6) the values of n and $\bar{\tau}$ for the cases to be run. This data is described in the following table where the location number refers to the columns of the card and the following three formats are used: alphanumeric characters (A), integers (I), and numbers with a decimal point (F). For the "I" formats the values are placed in fields of five locations, while a field of ten locations is used with the "F" formats. In either case the numbers must be placed in the rightmost locations of the allocated field.

No. of Cards	Location	Type	Input Item	Comments
1	1-72	A	TITLE	Title of run.
1	1-10	F	GAMMA	Specific heat ratio, γ.
	11-20	F	UE	Steady state Mach number at nozzle entrance, $\bar{u}_e$.
	21-30	F	RLD	Length-to-diameter ratio, $L/D = z_e/2$.

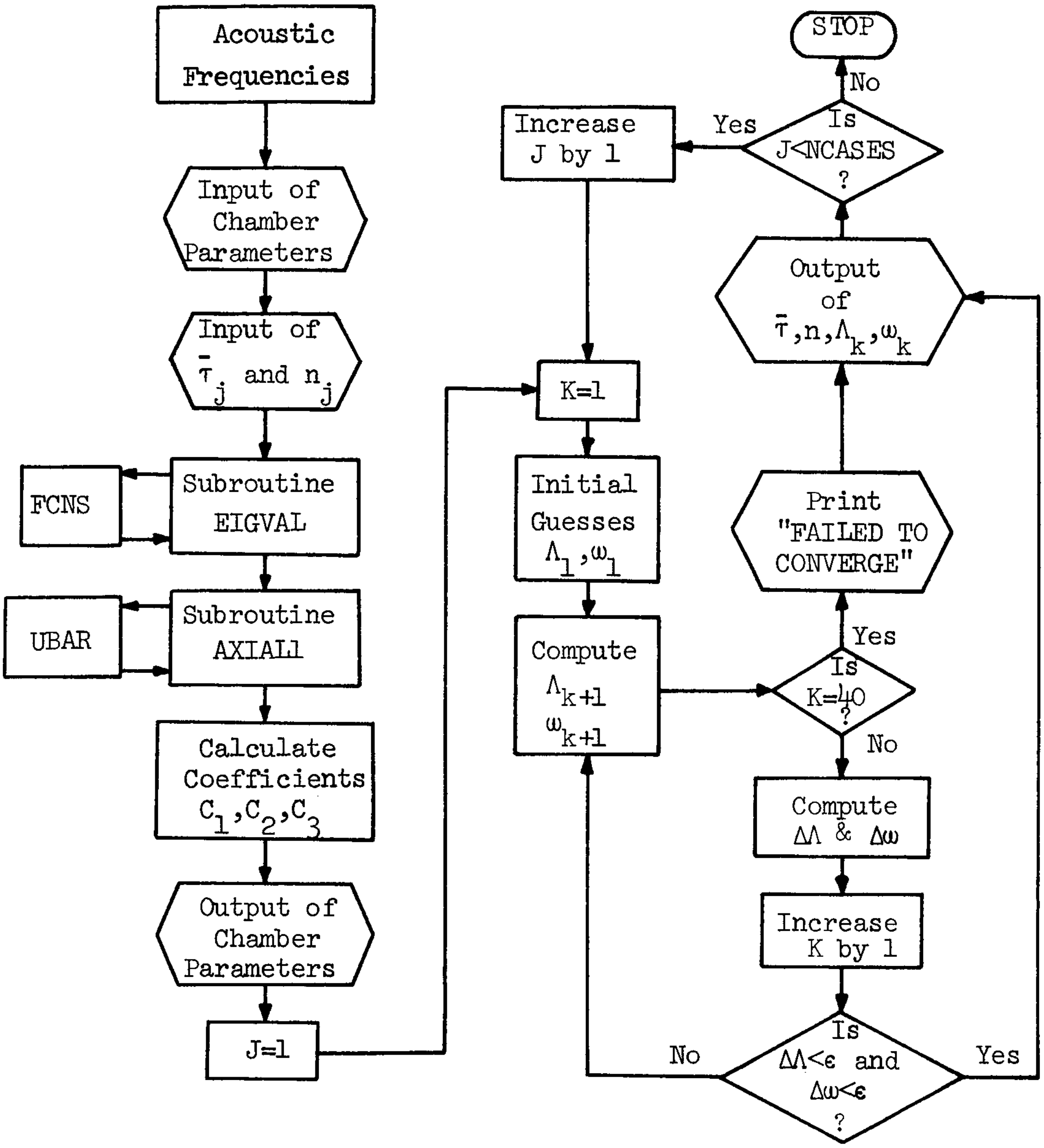


Figure E-1. Flow Chart for Program LINSOL.

No. of Cards	Location	Type	Input Item	Comments
	31-40	F	ZCOMB	Length of combustion zone, z_c/z_e.
	41-45	I	NDROPS	If 0: droplet momentum source neglected. If 1: droplet momentum source included.
	46-50	I	NOZZLE	If 0: quasi-steady nozzle. If 1: conventional nozzle.
	51-55	I	NOPT	If 1: all coefficients included. If 2: imaginary parts neglected.
If NOZZLE = 1 :				
1	1-10	F	YAMPL	Amplitude factor of nozzle admittance, **A**.
	11-20	F	YPHASE	Phase of nozzle admittance, φ.
End of input for NOZZLE = 1.				
1	1-5	I	L	Axial mode number, ℓ $(0 \leq L \leq 10)$.
	6-10	I	M	Tangential mode number, m $(0 \leq M \leq 8)$.
	11-15	I	N	Radial mode number, n $(0 \leq N \leq 5)$.
	16-20	I	NCASES	Number of cases to be run (NCASES $\leq$ 100).
NCASES	1-10	F	TAU	Time-lag, $\bar{\tau}$.
	11-20	F	EN	Interaction Index, n.

The title on the first card should identify the mode under consideration. On the second card of input all quantities are the same as those given in the input to COEFFS3D (see Appendix C) except NOPT. NOPT gives the option to neglect the imaginary parts of the coefficients C_1 and C_2 which are an order of magnitude smaller than the corresponding real parts. Neglecting these imaginary parts (NOPT = 2) yields linear solutions consistent with the non-linear solutions obtained when the small coefficients are neglected (NEGL = 1 in input to COEFFS3D). The values of n and $\bar{\tau}$ for the cases to be run are given on a series of NCASES cards. These cards are all read and the values of $\bar{\tau}$ and n are stored in the arrays TAU(J) and EN(J) before any computations are made.

In addition to the above card input, the acoustic frequencies S_{mn} are also needed for these calculations. As in Program COEFFS3D these values are given in a DATA statement, which is an integral part of the program.

Calculation of C_1, C_2, and C_3. In this section the coefficients C_1, C_2, and C_3 appearing in Eqs. (E-6) and (E-7) are calculated using Eqs. (E-2) through (E-4). As in Program COEFFS3D the axial acoustic eigenvalues necessary for these computations are calculated by Subroutines EIGVAL and FCNS, and the integrals of the products of two axial eigenfunctions appearing in Eqs. (E-2) through (E-4) are computed by Subroutines AXIAL1 and UBAR. Listings of these subroutines are given in Appendix C.

Iterative Solution for Λ and ω. Equations (E-6) and (E-7) are of the form:

$$\begin{aligned} \omega^2 &= C_{1r} + f(\Lambda,\omega) \\ \Lambda &= g(\Lambda,\omega) \end{aligned} \tag{E-10}$$

where the quantity $f(\Lambda,\omega)$ is small compared to C_{1r} and Λ is small in most cases. Starting with an initial guess of

$$\begin{aligned} \omega_1 &= \sqrt{S_{mn}^2 + \frac{\ell^2\pi^2}{z_e^2}} \\ \Lambda_1 &= 0 \end{aligned} \tag{E-11}$$

Eqs. (E-10) are solved iteratively using the following recursion formulas:

$$\omega_{k+1}^{2} = C_{1r} + f(\Lambda_k, \omega_k)$$

$$\Lambda_{k+1} = g(\Lambda_k, \omega_k) \tag{E-12}$$

At each step of the iteration the quantities $\Delta\Lambda$ and $\Delta\omega$ are calculated, where

$$\Delta\Lambda = | \Lambda_{k+1} - \Lambda_k |$$

$$\Delta\omega = | \omega_{k+1} - \omega_k | \tag{E-13}$$

and the computations are terminated when k = 40 or when $\Delta\Lambda$ and $\Delta\omega$ are less than $\epsilon = 10^{-6}$. The process usually converges in less than 15 iterations.

Output. The output generated by Program LINSOL consists of a restatement of the input data followed by the calculated results in tabular form. For each case the tabulated results give the values of $\bar{\tau}$ and n (TAU and EN), the corresponding values of the growth rate Λ and the frequency ω (LAMBDA and OMEGA), and the number of iterations (ITER). When ITER is 40 the last values of Λ and ω are given followed by the warning message "FAILED TO CONVERGE."

Sample Input and Output. A sample input for the 1T mode is given in Table E-1 followed by the resulting output in Table E-2.

Program LSTB3D

Program Structure. A flow chart for Program LSTB3D is given in Figure (E-2). This program consists of the following major sections: (1) input, (2) calculation of the coefficients C_1, C_2, and C_3, (3) iterative solution for n and ω for neutral stability, and (4) output.

Table E-1. Sample Input for LINSOL.

```
1T MODE.
       1.2       0.2       0.5       1.0    0    1    1
      0.02      45.0
    0    1   .1    9
       1.4       0.5
       1.4   0.58396
         :        :
       2.0   0.57562
       2.0       0.6
```

Table E-2. Sample Output for LINSOL.

```
1T MODE.
DROPLET MOMENTUM SOURCE NEGLECTED
GAMMA =  1.20    UE =  .20    L/D =  .50000    ZCOMB =  1.00
AMPL =  .02000    PHASE =  45.0
```

TAU	EN	LAMBDA	OMEGA	ITER
1.400	.50000	-.01789	1.86593	7
1.400	.58396	.00000	1.87005	7
1.400	.60000	.00339	1.87078	7
1.700	.50000	-.00975	1.83602	7
1.700	.54490	-.00000	1.83612	6
1.700	.60000	.01176	1.83618	7
2.000	.50000	-.01537	1.80691	8
2.000	.57562	.00000	1.80410	8
2.000	.60000	.00487	1.80322	8

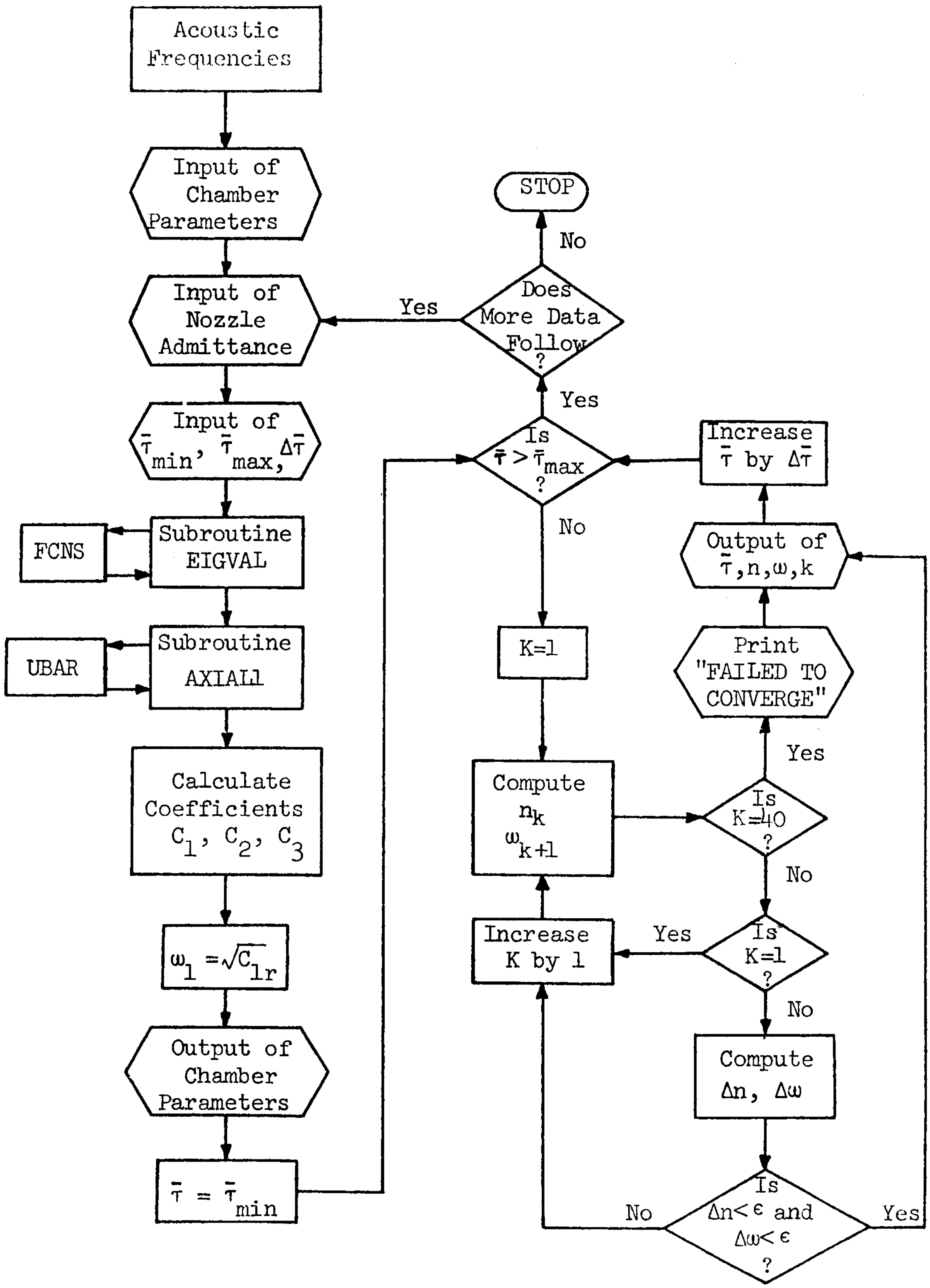


Figure E-2. Flow Chart for Program LSTB3D.

Input. The input data required by Program LSTB3D is basically the same as required by Program LINSOL. The first two cards, which give the title of the case, the chamber parameters, and the control numbers, are identical in content and format to those required by LINSOL. The third card gives the mode numbers ℓ, m, and n and is followed by a card giving the nozzle admittance if a conventional nozzle is specified. The last card gives the values of $\bar{\tau}$ for the cases to be run. A detailed description of this input is given below.

No. of Cards	Location	Type	Input Item	Comments
1	1-72	A	TITLE	See input for LINSOL.
1	1-40	F	GAMMA, UE, RLD, ZCOMB	See input for LINSOL.
	41-55	I	NDROPS, NOZZLE, NOPT	See input for LINSOL.
1	1-15	I	L, M, N	See input for LINSOL
If NOZZLE = 1 :				
1	1-20	F	YAMPL, YPHASE	See input for LINSOL.
End of input for NOZZLE = 1.				
1	1-10	F	TAUMIN	Smallest value of $\bar{\tau}$.
	11-20	F	TAUMAX	Largest value of $\bar{\tau}$.
	21-30	F	DELTAU	Increment in $\bar{\tau}$.

The last card gives the values of $\bar{\tau}$ which are used in the computation of the neutral stability limit. Thus computations are begun for $\bar{\tau}$ = TAUMIN, $\bar{\tau}$ is increased by increments of DELTAU, and computations are terminated when $\bar{\tau} \geq$ TAUMAX.

After completion of the computations program control returns to the read statement for the nozzle admittance, thus neutral stability curves can be calculated for several different nozzles for the same set of chamber and mode parameters.

Calculation of C_1, C_2, and C_3. The calculation of the coefficients C_1, C_2, and C_3 appearing in Eqs. (E-8) and (E-9) is performed in the same manner as given in program LINSOL.

Iterative Solution for n and ω. The values of n and ω for neutral stability are calculated for each value of $\bar{\tau}$ by solving Eqs. (E-8) and (E-9) using the following iteration scheme:

$$n_k = \frac{C_{2r} + C_{1i}/\omega_k}{C_3(1 - \cos\omega_k\bar{\tau})}$$

$$\omega_{k+1}^2 = C_{1r} + \omega_k(n_k C_3 \sin\omega_k\bar{\tau} - C_{2i}) \tag{E-14}$$

The iteration is started by using $\omega_1 = \sqrt{C_{1r}}$ and is stopped when k = 40 or Δn and $\Delta\omega$ are less than $\epsilon = 10^{-6}$. Convergence is usually obtained in less than 20 iterations.

Output. The output generated by Program LSTB3D consists of a restatement of the input data followed by the calculated results in tabular form. For each value of $\bar{\tau}$ in the range TAUMIN $\leq \bar{\tau} \leq$ TAUMAX, the tabulated results give the value of $\bar{\tau}$ (TAU), the corresponding values of n and ω for neutral stability (EN and OMEGA), and the number of iterations (ITER). If ITER is 40 the last values of n and ω computed are given followed by the warning message "FAILED TO CONVERGE."

Sample Input and Output. A sample input for the 1T mode is given in Table E-3 and is followed by the resulting output in Table E-4.

Table E-3. Sample Input for LSTB3D.

```
1T MODE.
       1.2       0.2       0.5       1.0    0    1    1
    0    1    1
      0.02      45.0
       0.6       2.8       0.1
```

Table E-4. Sample Output for LSTB3D.

1T MODE.

DROPLET MOMENTUM SOURCE NEGLECTED

GAMMA = 1.20 UE = .20 RLD = .50000 ZCOMB = 1.00

AMPL = .02000 PHASE = 45.00

TAU	EN	OMEGA	ITER
.60000	1.66353	2.03102	6
.70000	1.31671	1.99646	6
.80000	1.08482	1.96911	6
.90000	.92333	1.94663	6
1.00000	.80765	1.92753	6
1.10000	.72330	1.91089	6
1.20000	.66137	1.89605	6
1.30000	.61616	1.88255	6
1.40000	.58396	1.87005	6
1.50000	.56230	1.85827	6
1.60000	.54961	1.84702	5
1.70000	.54490	1.83612	5
1.80000	.54769	1.82542	6
1.90000	.55785	1.81479	7
2.00000	.57562	1.80410	8
2.10000	.60157	1.79325	8
2.20000	.63666	1.78210	9
2.30000	.68221	1.77055	10
2.40000	.74006	1.75847	11
2.50000	.81258	1.74575	13
2.60000	.90278	1.73224	14
2.70000	1.01446	1.71783	17
2.80000	1.15226	1.70240	21

```
C
C     ****************** PROGRAM LINSOL ********************************
C
C     THIS PROGRAM COMPUTES THE DAMPING (LAMBDA) AND FREQUENCY
C     (OMEGA) FOR GIVEN VALUES OF THE INTERACTION INDEX (EN) AND
C     THE TIME-LAG (TAU).  THIS PROGRAM IS BASED ON AN ANALYTICAL
C     SOLUTION OF THE COMPLEX DIFFERENTIAL EQUATION.
C
C     THE FOLLOWING INPUTS ARE REQUIRED:
C
C     FIRST CARD:
C     THE TITLE OF THE CASE.
C
C     SECOND CARD:
C     GAMMA IS THE SPECIFIC HEAT RATIO.
C     UE IS THE STEADY STATE MACH NUMBER AT THE NOZZLE ENTRANCE.
C     RLD IS THE LENGTH-TO-DIAMETER RATIO.
C     ZCOMB IS THE LENGTH OF THE COMBUSTION ZONE, EXPRESSED
C     AS A FRACTION OF THE CHAMBER LENGTH.
C     NDROPS DETERMINES THE PRESENCE OF DROPLET MOMENTUM SOURCES:
C        NDROPS = 0  DROPLET MOMENTUM SOURCE NEGLECTED.
C        NDROPS = 1  DROPLET MOMENTUM SOURCE INCLUDED.
C     NOZZLE SPECIFIES THE TYPE OF NOZZLE USED:
C        NOZZLE = 0     QUASI-STEADY
C        NOZZLE = 1     CONVENTIONAL NOZZLE
C     NOPT SPECIFIES THE SOLUTIONS DESIRED.
C        NOPT = 1   COUPLING COEFFICIENTS INCLUDED.
C        NOPT = 2   COUPLING COEFFICIENTS NEGLECTED.
C
C     THIRD CARD (FOR CONVENTIONAL NOZZLE ONLY):
C        YAMPL IS THE AMPLITUDE OF THE NOZZLE ADMITTANCE.
C        YPHASE IS THE PHASE OF THE NOZZLE ADMITTANCE.
C
C     FOURTH CARD:
C     THE MODE IS SPECIFIED BY THE INDICES L, M, AND N.
C     L IS THE AXIAL MODE NUMBER AND MUST NOT EXCEED 10.
C     M IS THE AZIMUTHAL MODE NUMBER AND MUST NOT EXCEED 8.
C     N IS THE RADIAL MODE NUMBER AND MUST NOT EXCEED 5.
C     NCASES IS THE NUMBER OF CASES TO BE RUN.
C
C     REMAINING CARDS:
C     TAU IS THE TIME LAG.
C     EN IS THE INTERACTION INDEX.
C
C     ******************************************************************
C
      COMPLEX       YNOZ, RESULT, B(10), BC, AX(4), CI, CZE,
     1              CGAM, ZEJ, ZEP1, ZEP2, CC, CD, CE, CSSQ, CAX
      DIMENSION     TITLE(72),
     1              RJROOT(10,5),
     2              D(5), OMEGA(100),
     3              EN(100), TAU(100)
      REAL           LAMBDA(100)
      COMMON    B
```

```
C
C      ************* DATA INPUT SECTION **********************************
C
       ERR = 0.000001
       PI = 3.1415927
       CI = (0.0,1.0)
C
C      INPUT ROOTS AND VALUES OF BESSEL FUNCTIONS.
       DATA ((RJROOT(I,J), J = 1,5), I = 1,9)/
      1    3.83171,  7.01559, 10.17347, 13.32369, 16.47063,
      2    1.84118,  5.33144,  8.53632, 11.70600, 14.86359,
      3    3.05424,  6.70613,  9.96947, 13.17037, 16.34752,
      4    4.20119,  8.01524, 11.34592, 14.58585, 17.78875,
      5    5.31755,  9.28240, 12.68191, 15.96411, 19.19603,
      6    6.41562, 10.51986, 13.98719, 17.31284, 20.57551,
      7    7.50127, 11.73494, 15.26818, 18.63744, 21.93172,
      8    8.57784, 12.93239, 16.52937, 19.94185, 23.26805,
      9    9.64742, 14.11552, 17.77401, 21.22906, 24.58720/
C
C      INPUT PARAMETERS.
       READ (5,5000)  (TITLE(I), I = 1, 72)
       READ (5,5001)  GAMMA, UE, RLD, ZCOMB, NDROPS, NOZZLE, NOPT
       IF (NOZZLE .EQ. 1)  GO TO 5
C      COMPUTE ADMITTANCE FOR QUASI-STEADY NOZZLE.
       YAMPL = (GAMMA - 1.0) * UE/(2.0 * GAMMA)
       YPHASE = 0.0
       GO TO 7
     5 READ (5,5002) YAMPL, YPHASE
     7 READ (5,5003) L, M, N, NCASES
C
       THETA = YPHASE * PI/180.0
       YR = YAMPL * COS(THETA)
       YI = YAMPL * SIN(THETA)
       YNOZ = CMPLX(YR,YI)
C
       ZE = 2.0 * RLD
       CZE = CMPLX(ZE,0.0)
       CGAM = CMPLX(GAMMA,0.0)
       CAX = CGAM
       IF (NDROPS .EQ. 1)  CAX = CGAM + (1.0,0.0)
C
       DO 10 J = 1, NCASES
       READ (5,5002) TAU(J), EN(J)
    10 CONTINUE
C
C      ************* PRELIMINARY CALCULATIONS ****************************
C
C
C      ASSIGN ARRAYS FOR ROOTS OF BESSEL FUNCTIONS.
       IF ((M .EQ. 0) .AND. (N .EQ. 0))  GO TO 15
       MM = M + 1
       NN = N
       SMN = RJROOT(MM,NN)
```

```
      GO TO 20
   15 SMN = 0.0
C
   20 SSQ = SMN * SMN
      CSSQ = CMPLX(SSQ,0.0)
C
C     CALCULATE AXIAL ACOUSTIC EIGENVALUES.
      CALL EIGVAL(L,SMN,GAMMA,ZE,YAMPL,YPHASE,RESULT)
      B(1) = RESULT
      BC = CONJG(RESULT)
C
C     ************* CALCULATE AXIAL INTEGRALS ***************************
C
      DO 100 NT = 1, 4
      CALL AXIAL1(NT,1,1,UE,ZE,ZCOMB,RESULT)
      AX(NT) = RESULT
  100 CONTINUE
C
C     ************* CALCULATE VALUES AT NOZZLE ENTRANCE ****************
C
      ZEJ = CCOSH(CI*BC*CZE)
      ZEP1 = CCOSH(CI*B(1)*CZE)
      ZEP2 = CI * B(1) * CSINH(CI*B(1)*CZE)
C
C     ************* CALCULATE COEFFICIENTS *****************************
C
      CC = (CSSQ*AX(1) - AX(2) + ZEP2*ZEJ)/AX(1)
      CD = (CAX*AX(3) + (2.0,0.0)*AX(4)
     1     + CGAM*YNOZ*ZEP1*ZEJ)/AX(1)
      CE = CGAM*AX(3)/AX(1)
C
      D(1) = REAL(CC)
      D(3) = REAL(CD)
      D(5) = REAL(CE)
      IF (NOPT .EQ. 2) GO TO 50
      D(2) = AIMAG(CC)
      D(4) = AIMAG(CD)
      GO TO 55
   50 D(2) = 0.0
      D(4) = 0.0
C
C     ****** CALCULATION OF DAMPING AND FREQUENCY **********************
C
   55 WRITE (6,6001)  (TITLE(I), I = 1, 72)
      IF (NDROPS .EQ. 0)  WRITE (6,6020)
      IF (NDROPS .EQ. 1)  WRITE (6,6021)
      IF (NOPT .EQ. 2) WRITE (6,6015)
      WRITE (6,6002)  GAMMA, UE, RLD, ZCOMB
      IF (NOZZLE .EQ. 0)  WRITE (6,6012)
      WRITE (6,6005) YAMPL, YPHASE
      WRITE (6,6011)
      LINE = 14
```

```
C
C     CALCULATE INITIAL GUESSES FOR FREQUENCY.
      RL = L
      AXI = RL * PI/ZE
      AXSQ = AXI * AXI
      SSQ = SMN * SMN
      FRQ = SQRT(SSQ + AXSQ)
C
C
      DO 200 J = 1, NCASES
C
      C2R = D(3) - EN(J) * D(5)
      C3 = EN(J) * D(5)
C
      LAMBDA(1) = 0.0
      OMEGA(1) = FRQ
C
      K = 1
  210 X = LAMBDA(K)
      Y = OMEGA(K)
      XT = X * TAU(J)
      YT = Y * TAU(J)
      EX = EXP(-XT)
      SN = SIN(YT)
      CS = COS(YT)
      XSQ = X * X
      WSQ = D(1) + XSQ + C2R*X - D(4)*Y
     1      + C3*EX*(X*CS + Y*SN)
      A = D(2) + C2R*Y + C3*EX*Y*CS
      BB = 2.0*Y + D(4) - C3*EX*SN
C
      OMEGA(K+1) = SQRT(WSQ)
      LAMBDA(K+1) = -A/BB
C
      IF (K .EQ. 40) GO TO 216
      DX = ABS(LAMBDA(K+1) - LAMBDA(K))
      DY = ABS(OMEGA(K+1) - OMEGA(K))
      K = K + 1
      IF ((DX .LT. ERR) .AND. (DY .LT. ERR)) GO TO 217
      GO TO 210
C
  216 WRITE (6,6009) TAU(J), EN(J), LAMBDA(K), OMEGA(K), K
      GO TO 220
C
  217 WRITE (6,6008) TAU(J), EN(J), LAMBDA(K), OMEGA(K), K
C
  220 LINE = LINE + 2
      IF (LINE .LT. 54)  GO TO 200
      WRITE (6,6007)
      WRITE (6,6011)
      LINE = 4
C
  200 CONTINUE
```

```
C
C
C      ************* FORMAT SPECIFICATIONS ******************************
C
C      READ FORMATS
 5000 FORMAT (72A1)
 5001 FORMAT (4F10.0,3I5)
 5002 FORMAT (2F10.0)
 5003 FORMAT (4I5)
C
C      WRITE FORMATS
 6001 FORMAT (1H1,1X,72A1/)
 6002 FORMAT (2X,8HGAMMA = ,F5.2,5X,5HUE = ,F5.2,5X,6HL/D = ,F8.5,
     1          5X,8HZCOMB = ,F5.2/)
 6005 FORMAT (2X,7HAMPL = ,F8.5,5X,8HPHASE = ,F6.1/)
 6007 FORMAT (1H )
 6008 FORMAT (2X,F5.3,F8.5,2F10.5,I6/)
 6009 FORMAT (2X,F5.3,F8.5,2F10.5,I6,5X,18HFAILED TO CONVERGE/)
 6011 FORMAT (2X///4X,3HTAU,6X,2HEN,4X,6HLAMBDA,5X,5HOMEGA,
     1        2X,4HITER/)
 6012 FORMAT (2X,19HQUASI-STEADY NOZZLE/)
 6015 FORMAT (2X,24HCOUPLING TERMS NEGLECTED/)
 6020 FORMAT (2X,'DROPLET MOMENTUM SOURCE NEGLECTED'/)
 6021 FORMAT (2X,'DROPLET MOMENTUM SOURCE INCLUDED'/)
      END
```

```
C
C     ******************* PROGRAM LSTB3D ******************************
C
C     THIS PROGRAM COMPUTES THE LINEAR STABILITY LIMITS CONSISTENT
C     WITH THE THREE-DIMENSIONAL SECOND-ORDER THEORY.
C
C     THE FOLLOWING INPUTS ARE REQUIRED:
C
C     FIRST CARD:
C     THE TITLE OF THE CASE.
C
C     SECOND CARD:
C     GAMMA IS THE SPECIFIC HEAT RATIO.
C     UE IS THE STEADY STATE MACH NUMBER AT THE NOZZLE ENTRANCE.
C     RLD IS THE LENGTH-TO-DIAMETER RATIO.
C     ZCOMB IS THE LENGTH OF THE COMBUSTION ZONE, EXPRESSED
C     AS A FRACTION OF THE CHAMBER LENGTH.
C     NDROPS DETERMINES THE PRESENCE OF DROPLET MOMENTUM SOURCES:
C        NDROPS = 0  DROPLET MOMENTUM SOURCE NEGLECTED.
C        NDROPS = 1  DROPLET MOMENTUM SOURCE INCLUDED.
C     NOZZLE SPECIFIES THE TYPE OF NOZZLE USED:
C        NOZZLE = 0     QUASI-STEADY
C        NOZZLE = 1     CONVENTIONAL NOZZLE
C     NOPT SPECIFIES WHICH SOLUTION WILL BE COMPUTED.
C        NOPT = 1    COUPLING COEFFICIENTS INCLUDED.
C        NOPT = 2    COUPLING COEFFICIENTS NEGLECTED.
C
C     THIRD CARD:
C     THE MODE IS SPECIFIED BY THE INDICES L, M, AND N.
C     L IS THE AXIAL MODE NUMBER AND MUST NOT EXCEED 10.
C     M IS THE AZIMUTHAL MODE NUMBER AND MUST NOT EXCEED 8.
C     N IS THE RADIAL MODE NUMBER AND MUST NOT EXCEED 5.
C
C     FOURTH CARD (IF CONVENTIONAL NOZZLE):
C        YAMPL IS THE AMPLITUDE OF THE NOZZLE ADMITTANCE.
C        YPHASE IS THE PHASE OF THE NOZZLE ADMITTANCE.
C
C     REMAINING CARDS:
C        TAUMIN IS THE MINIMUM VALUE OF THE TIME-LAG.
C        TAUMAX IS THE MAXIMUM VALUE OF THE TIME-LAG.
C        DELTAU IS THE INCREMENT IN TIME-LAG.
C
C     ******************************************************************
C
      COMPLEX       YNOZ, RESULT, B(10), BC, AX(4), CI, CZE,
     1              CGAM, ZEJ, ZEP1, ZEP2, CC, CD, CE, CSSQ, CAX
      DIMENSION     TITLE(72),
     1              RJROOT(10,5),
     2              OMEGA(100), EN(100)
      COMMON        B
C
```

```
C     ************* DATA INPUT SECTION *********************************
C
      ERR = 0.000001
      PI = 3.1415927
      CI = (0.0,1.0)
C
C     INPUT ROOTS AND VALUES OF BESSEL FUNCTIONS.
      DATA ((RJROOT(I,J), J = 1,5), I = 1,9)/
     1   3.83171,  7.01559, 10.17347, 13.32369, 16.47063,
     2   1.84118,  5.33144,  8.53632, 11.70600, 14.86359,
     3   3.05424,  6.70613,  9.96947, 13.17037, 16.34752,
     4   4.20119,  8.01524, 11.34592, 14.58585, 17.78875,
     5   5.31755,  9.28240, 12.68191, 15.96411, 19.19603,
     6   6.41562, 10.51986, 13.98719, 17.31284, 20.57551,
     7   7.50127, 11.73494, 15.26818, 18.63744, 21.93172,
     8   8.57784, 12.93239, 16.52937, 19.94185, 23.26805,
     9   9.64742, 14.11552, 17.77401, 21.22906, 24.58720/
C
C     INPUT PARAMETERS.
      READ (5,5000)  (TITLE(I), I = 1, 72)
      READ (5,5001)  GAMMA, UE, RLD, ZCOMB, NDROPS, NOZZLE, NOPT
      READ (5,5002) L, M, N
    8 IF (NOZZLE .EQ. 1)  GO TO 5
C     COMPUTE ADMITTANCE FOR QUASI-STEADY NOZZLE.
      YAMPL = (GAMMA - 1.0) * UE/(2.0 * GAMMA)
      YPHASE = 0.0
      GO TO 7
    5 READ (5,5003,END = 300) YAMPL, YPHASE
    7 READ (5,5003, END = 300) TAUMIN, TAUMAX, DELTAU
C
      THETA = YPHASE * PI/180.0
      YR = YAMPL * COS(THETA)
      YI = YAMPL * SIN(THETA)
      YNOZ = CMPLX(YR,YI)
C
      ZE = 2.0 * RLD
      CZE = CMPLX(ZE,0.0)
      CGAM = CMPLX(GAMMA,0.0)
      CAX = CGAM
      IF (NDROPS .EQ. 1)  CAX = CGAM + (1.0,0.0)
C
C     ************* PRELIMINARY CALCULATIONS ***************************
C
C     ASSIGN ARRAYS FOR ROOTS OF BESSEL FUNCTIONS.
      IF ((M .EQ. 0) .AND. (N .EQ. 0))  GO TO 15
      MM = M + 1
      NN = N
      SMN = RJROOT(MM,NN)
      GO TO 20
   15 SMN = 0.0
   20 SSQ = SMN * SMN
      CSSQ = CMPLX(SSQ,0.0)
C
```

```
C     CALCULATE AXIAL ACOUSTIC EIGENVALUES.
      CALL EIGVAL(L,SMN,GAMMA,ZE,YAMPL,YPHASE,RESULT)
      B(1) = RESULT
      BC = CONJG(RESULT)
C
C     ************* CALCULATE AXIAL INTEGRALS **************************
C
      DO 100 NT = 1, 4
      CALL AXIAL1(NT,1,1,UE,ZE,ZCOMB,RESULT)
      AX(NT) = RESULT
  100 CONTINUE
C
C     ************* CALCULATE VALUES AT NOZZLE ENTRANCE ****************
C
      ZEJ = CCOSH(CI*BC*CZE)
      ZEP1 = CCOSH(CI*B(1)*CZE)
      ZEP2 = CI * B(1) * CSINH(CI*B(1)*CZE)
C
C     ************* CALCULATE COEFFICIENTS *****************************
C
      CC = (CSSQ*AX(1) - AX(2) + ZEP2*ZEJ)/AX(1)
      CD = (CAX*AX(3) + (2.0,0.0)*AX(4)
     1       + CGAM*YNOZ*ZEP1*ZEJ)/AX(1)
      CE = CGAM*AX(3)/AX(1)
C
      C1 = REAL(CC)
      D1 = REAL(CD)
      E = REAL(CE)
      IF (NOPT .EQ. 2) GO TO 50
      C2 = AIMAG(CC)
      D2 = AIMAG(CD)
      GO TO 55
   50 C2 = 0.0
      D2 = 0.0
C
C     *********** CALCULATION OF LINEAR STABILITY LIMIT ******
C
   55 OMEGA(1) = SQRT(C1)
C
      WRITE (6,6001) (TITLE(J), J = 1,72)
      IF (NDROPS .EQ. 0)  WRITE (6,6025)
      IF (NDROPS .EQ. 1)  WRITE (6,6026)
      IF (NOPT .EQ. 2) WRITE (6,6022)
      WRITE (6,6002)  GAMMA, UE, RLD, ZCOMB
      IF (NOZZLE .EQ. 0)  WRITE (6,6012)
      WRITE (6,6005) YAMPL, YPHASE
      WRITE (6,6010)
      LINE = 12
C
      TAU = TAUMIN
  370 IF (TAU .GT. TAUMAX)  GO TO 8
```

```
C
      K = 1
  310 WT = OMEGA(K) * TAU
      BB = (D1 + C2/OMEGA(K))/E
      EN(K) = BB/(1.0 - COS(WT))
      G = (E*EN(K)*SIN(WT) - D2) * OMEGA(K)
      OMEGA(K+1) = SQRT(C1 + G)
      IF (K .EQ. 40)  GO TO 316
      IF (K .EQ. 1) GO TO 311
      DN = ABS(EN(K) - EN(K-1))
      DW = ABS(OMEGA(K+1) - OMEGA(K))
      IF ((DN .LT. ERR) .AND. (DW .LT. ERR)) GO TO 317
  311 K = K + 1
      GO TO 310
C
  316 WRITE (6,6013) TAU, EN(K), OMEGA(K), K
      GO TO 318
  317 WRITE (6,6014) TAU, EN(K), OMEGA(K), K
C
  318 LINE = LINE + 2
      TAU = TAU + DELTAU
      IF ((LINE .LT. 60) .OR. (TAU .GT. TAUMAX)) GO TO 370
      WRITE (6,6015)
      WRITE (6,6010)
      LINE = 6
      GO TO 370
C
  300 CONTINUE
C
C     ************* FORMAT SPECIFICATIONS *******************************
C
C     READ FORMATS
 5000 FORMAT (72A1)
 5001 FORMAT (4F10.0,3I5)
 5002 FORMAT (3I5)
 5003 FORMAT (3F10.0)
C
C     WRITE FORMATS
 6001 FORMAT (1H1,1X,72A1/)
 6002 FORMAT (2X,8HGAMMA = ,F5.2,5X,5HUE = ,F5.2,5X,6HRLD = ,F8.5,
     1           5X,8HZCOMB = ,F5.2/)
 6003 FORMAT (2X,A4,5I5,4F10.5/)
 6005 FORMAT (2X,7HAMPL = ,F8.5,5X,8HPHASE = ,F7.2/)
 6007 FORMAT (1H )
 6008 FORMAT (1H0)
 6010 FORMAT (2X//8X,3HTAU,8X,2HEN,5X,5HOMEGA,6X,4HITER/)
 6012 FORMAT (2X,19HQUASI-STEADY NOZZLE/)
 6013 FORMAT (2X,3F10.5,I10,5X,19H FAILED TO CONVERGE/)
 6014 FORMAT (2X,3F10.5,I10/)
 6015 FORMAT (1H1)
 6022 FORMAT (2X,24HCOUPLING TERMS NEGLECTED/)
 6025 FORMAT (2X,'DROPLET MOMENTUM SOURCE NEGLECTED'/)
 6026 FORMAT (2X,'DROPLET MOMENTUM SOURCE INCLUDED'/)
      END
```

REFERENCES

1. Hefner, R. J., "Diagnosis of High Frequency Combustion Stability Characteristics from Pressure Measurements," Third ICRPG Combustion Conference, CPIA Publication No. 138, Vol. I (October 1966).

2. Price, E. W., "Combustion Instability in Rocket Motors with Aluminized Solid Propellants," Naval Weapons Center Report TP 5505 (June 1973).

3. Lores, M. E. and Zinn, B. T., "The Prediction of Nonlinear Longitudinal Combustion Instability in Liquid Propellant Rockets," NASA CR-120904 (April 1972).

4. Lores, M. E. and Zinn, B. T., "Nonlinear Longitudinal Combustion Instability in Rocket Motors," AIAA Paper 73-217, presented at the AIAA 11th Aerospace Sciences Meeting (January 1973) and accepted for publication in Combustion Science and Technology.

5. Zinn, B. T. and Powell, E. A., "Nonlinear Combustion Instability in Liquid-Propellant Rocket Engines," Proceedings of the 13th Symposium (International) on Combustion, The Combustion Institute, 1971, pp. 491-503.

6. Powell, E. A. and Zinn, B. T., "The Prediction of the Nonlinear Behavior of Unstable Liquid Rockets," NASA CR-72902 (July 1971).

7. Crocco, L. and Cheng, S. E., Theory of Combustion Instability in Liquid Propellant Rocket Motors, AGARD Monograph No. 8, 1-24. Butterworths Scientific Pub., Ltd., London (1956).

8. Powell, E. A., "Nonlinear Combustion Instability in Liquid Propellant Rocket Engines," Ph.D. Thesis, Georgia Institute of Technology, GITAER 70-6 (September 1970).

9. Powell, E. A. and Zinn, B. T., "Stable Limit Cycles and Triggering Limits of the First Radial Mode in Unstable Liquid Rockets," Israel Journal of

Technology, March 1971, pp. 177-187.

10. Maslen, S. H. and Moore, F. K., "On Strong Transverse Waves Without Shocks in a Circular Cylinder," Journal of Aeronautical Sciences 23, No. 6, pp. 583-593 (1956).

11. Finlayson, B. A. and Scriven, L. E., "The Method of Weighted Residuals -- a Review," Applied Mechanics Reviews 19, No. 9, pp. 735-744 (September 1966).

12. Ames, W. F., Nonlinear Partial Differential Equations in Engineering, pp. 243-262. Academic Press, New York (1965).

13. Crocco, L. and Sirignano, W. A., "Effect of Transverse Component on the Nonlinear Behavior of Short Nozzles," AIAA J. 4, No. 8, pp. 697-706 (August 1966).

14. Powell, E. A. and Zinn, B. T., "A Single Mode Approximation in the Solution of Nonlinear Combustion Instability Problems," Combustion Science and Technology 3, pp. 121-132 (1971).

15. Mitchell, C. E., "Axial Mode Shock Wave Combustion Instability in Liquid Propellant Rocket Engines," Princeton University AMS Technical Report No. 798, (NASA CR 72259), July 1967, (Ph.D. Thesis).

16. Abramowitz, M. and Stegun, I. A., Handbook of Mathematical Functions, Dover Publications, Inc., New York, 1965, p. 411.

17. Conte, S. D., Elementary Numerical Analysis, McGraw-Hill Book Company, New York, 1965, pp. 45-46.

18. McCracken, D. D., FORTRAN with Engineering Applications, John Wiley and Sons, Inc., New York, 1967, pp. 146-153.

19. Univac 1108 Math-Pack Programmers Reference, UP-7542, Sperry Rand Corporation (Univac Division).

20. Univac 1108 Calcomp Plotter Manual, Georgia Institute of Technology, Office of Computing Services, October 1972.

21. Conte, S. D., Elementary Numerical Analysis, McGraw-Hill Book Company, New York, 1965, pp. 223-224.

REPORT DISTRIBUTION LIST

Dr. R. J. Priem MS 500-209
NASA Lewis Research Center
21000 Brookpark Road
Cleveland, Ohio 44135

NASA-Lewis Research Center
Attention: N. T. Musial (M.S. 500-311)
21000 Brookpark Road
Cleveland, Ohio 44135

NASA-Lewis Research Center
Attention: Library (M.S. 60-3)
21000 Brookpark Road
Cleveland, Ohio 44135

NASA-Lewis Research Center
Attention: Report Control Office(M.S. 5-5)
21000 Brookpark Road
Cleveland, Ohio 44135

Brooklyn Polytechnic Institute
Attn: V. D. Agosta
Long Island Graduate Center
Route 110
Farmingdale, New York 11735

Chemical Propulsion Information Agency
Johns Hopkins University/APL
Attention: T. W. Christian
8621 Georgia Avenue
Silver Spring, Maryland 20910

NASA-Lewis Research Center
Attention: E. W. Conrad (M.S. 500-204)
21000 Brookpark Road
Cleveland, Ohio 44135

North American Rockwell Corporation
Rocketdyne Division
Attention: L. P. Combs, D/991-350
Zone 11
6633 Canoga Avenue
Canoga Park, California 91304

NASA Scientific and Technical
Information Facility
Attention: Acquisitions Branch
P. O. Box 33
College Park, Maryland 20740

Aerospace Corporation
Attn: O. W. Dykema
Post Office Box 95085
Los Angeles, California 90045

Ohio State University
Department of Aeronautical and
Astronautical Engineering
Attn: R. Edse
Columbus, Ohio 43210

TRW Systems
Attn: G. W. Elverum
One Space Park
Redondo Beach, California 90278

Bell Aerospace Company
Attn: T. F. Ferger
Post Office Box 1
Mail Zone J-81
Buffalo, New York 14205

Pratt & Whitney Aircraft
Florida Research & Development
Center
Attn: G. D. Garrison
Post Office Box 710
West Palm Beach, Florida 33402

NASA-Lewis Research Center
Attn: L. Gordon, MS 500-209
21000 Brookpark Road
Cleveland, Ohio 44135

Purdue University
School of Mechanical Engineering
Attn: R. Goulard
Lafayette, Indiana 47907

Air Force Office of Scientific
Research
Chief Propulsion Division
Attn: Lt. Col. R. W. Haffner (NAE)
1400 Wilson Boulevard
Arlington, Virginia 22209

Pennsylvania State University
Mechanical Engineering Department
Attn: G. M. Faeth
207 Mechanical Engineering Bldg.
University Park, Pennsylvania 16802

University of Illinois
Aeronautics/Astronautic Engineering
Department
Attn: R. A. Strehlow
Transportation Building, Room 101
Urbana, Illinois 61801

NASA
Manned Spacecraft Center
Attn: J. C. Thibadaux
Houston, Texas 77058

Massachusetts Institute of Technology
Department of Mechanical Engineering
Attn: T. Y. Toong
77 Massachusetts Avenue
Cambridge, Massachusetts 02139

Illinois Institute of Technology
Attn: T. P. Torda
Room 200 M. H.
3300 S. Federal Street
Chicago, Illinois 60616

U. S. Army Missile Command
AMSMI-RKL, Attn: W. W. Wharton
Redstone Arsenal, Alabama 35808

University of California
Aerospace Engineering Department
Attn: F. A. Williams
Post Office Box 109
LaJolla, California 92037

Georgia Institute of Technology
School of Aerospace Engineering
Attn: B. T. Zinn
Atlanta, Georgia 30332

Marshall Industries
Dynamic Science Division
2400 Michelson Drive
Irvine, California 92664

Mr. Donald H. Dahlene
U. S. Army Missile Command
Research, Development, Engineering
and Missile Systems Laboratory
Attn: AMSMI-RK
Redstone Arsenal, Alabama 35809

TISIA
Defense Documentation Center
Cameron Station
Building 5
5010 Duke Street
Alexandria, Virginia 22314

Office of Assistant Director
(Chemical Technician)
Office of the Director of Defense
Research and Engineering
Washington, D. C. 20301

D. E. Mock
Advanced Research Projects Agency
Washington, D. C. 20525

Dr. H. K. Doetsch
Arnold Engineering Development Center
Air Force Systems Command
Tullahoma, Tennessee 37389

Library
Air Force Rocket Propulsion
Laboratory (RPR)
Edwards, California 93523

Library
Bureau of Naval Weapons
Department of the Navy
Washington, D. C.

Library
Director (Code 6180)
U. S. Naval Research Laboratory
Washington, D. C. 20390

APRP (Library)
Air Force Aero Propulsion Laboratory
Research and Technology Division
Air Force Systems Command
United States Air Force
Wright-Patterson AFB, Ohio 45433

Technical Information Department
Aeronutronic Division of Philco
Ford Corporation
Ford Road
Newport Beach, California 92663

Library-Documents
Aerospace Corporation
2400 E. El Segundo Boulevard
Los Angeles, California 90045

AFRPL
Attn: R. R. Weiss
Edwards, California 93523

Princeton University
James Forrestal Campus Library
Attn: D. Harrje
Post Office Box 710
Princeton, New Jersey 08540

U. S. Naval Weapons Center
Attn: T. Inouye, Code 4581
China Lake, California 93555

Office of Naval Research
Navy Department
Attn: R. D. Jackel, 473
Washington, D. C. 20360

Air Force Aero Propulsion Laboratory
Attn: APTC Lt. M. Johnson
Wright Patterson AFB, Ohio 45433

Naval Underwater Systems Center
Energy Conversion Department
Attn: Dr. R. S. Lazar, Code TB 131
Newport, Rhode Island 02840

NASA
Langley Research Center
Attn: R. S. Levine, MS 213
Hampton, Virginia 23365

Aerojet General Corporation
Attn: David A. Fairchild, Mech. Design
Post Office Box 15847 (Sect. 9732)
Sacremento, California 95809

Colorado State University
Mechanical Engineering Department
Attn: C. E. Mitchell
Fort Collins, Colorado 80521

ARL (ARC)
Attn: K. Scheller
Wright Patterson AFB, Ohio 45433

University of Wisconsin
Mechanical Engineering Department
Attn: P. S. Myers
1513 University Avenue
Madison, Wisconsin 53706

North American Rockwell Corporation
Rocketdyne Division
Attn: J. A. Nestlerode,
AC46 D/596-121
6633 Canoga Avenue
Canoga Park, California 91304

University of Michigan
Aerospace Engineering
Attn: J. A. Nicholls
Ann Arbor, Michigan 41804

Tulane University
Attn: J. C. O'Hara
6823 St. Charles Avenue
New Orleans, Louisiana 70118

University of California
Department of Chemical Engineering
Attn: A. K. Oppenheim
6161 Etcheverry Hall
Berkeley, California 94720

Sacramento State College
School of Engineering
Attn: F. H. Reardon
6000 J. Street
Sacramento, California 95819

Purdue University
School of Mechanical Engineering
Attn: B. A. Reese
Lafayette, Indiana 47907

NASA
George C. Marshall Space Flight Center
Attn: R. J. Richmond, SNE-ASTN-PP
Huntsville, Alabama 35812

Jet Propulsion Laboratory
California Institute of Technology
Attn: J. H. Rupe
4800 Oak Grove Drive
Pasadena, California 91103

University of California
Mechanical Engineering Thermal Systems
Attn: Prof. R. Sawyer
Berkeley, California 94720

Library
Bell Aerosystems, Inc.
Box 1
Buffalo, New York 14205

Report Library, Room 6A
Battelle Memorial Institute
505 King Avenue
Columbus, Ohio 43201

D. Suichu
General Electric Company
Flight Propulsion Laboratory Department
Cincinnati, Ohio 45215

Library
Ling-Temco-Vought Corporation
Post Office Box 5907
Dallas, Texas 75222

Marquardt Corporation
16555 Saticoy Street
Box 2013 - South Annex
Van Nuys, California 91409

P. F. Winternitz
New York University
University Heights
New York, New York

R. Stiff
Propulsion Division
Aerojet-General Corporation
Post Office Box 15847
Sacramento, California 95803

Library, Department 596-306
Rocketdyne Division of Rockwell
North American Rockwell Inc.
6633 Canoga Avenue
Canoga Park, California 91304

Library
Stanford Research Institute
333 Ravenswood Avenue
Menlo Park, California 94025

Library
Susquehanna Corporation
Atlantic Research Division
Shirley Highway and Edsall Road
Alexandria, Virginia 22314

STL Tech. Lib. Doc. Acquisitions
TRW System Group
1 Space Park
Redondo Beach, California 90278

Dr. David Altman
United Aircraft Corporation
United Technology Center
Post Office Box 358
Sunnyvale, California 94088

Library
United Aircraft Corporation
Pratt and Whitney Division
Florida Research and Development
Center
Post Office Box 2691
West Palm Beach, Florida 33402

Library
Air Force Rocket Propulsion
Laboratory (RPM)
Edwards, California 93523

Professor Kenneth R. Purdy
Post Office Box 5014
Tennessee Technological University
Cookeville, Tennessee 38501

NASA-Lewis Research Center
Attn: E. O. Bourke MS 500-209
21000 Brookpark Road
Cleveland, Ohio 44135

NASA-Lewis Research Center
MS 500-313
Rockets & Spacecraft Procurement
Section
21000 Brookpark Road
Cleveland, Ohio 44135

1. Report No.	2. Government Accession No.	3. Recipient's Catalog No.
NASA CR-121279		

4. Title and Subtitle	5. Report Date
THE PREDICTION OF NONLINEAR THREE-DIMENSIONAL COMBUSTION INSTABILITY IN LIQUID ROCKETS WITH CONVENTIONAL NOZZLES	October 1973
	6. Performing Organization Code
7. Author(s)	8. Performing Organization Report No.
Eugene A. Powell and Ben T. Zinn	
	10. Work Unit No.
9. Performing Organization Name and Address	11. Contract or Grant No.
Georgia Institute of Technology Atlanta, Georgia 30332	NGL 11-002-083
	13. Type of Report and Period Covered
12. Sponsoring Agency Name and Address	Contractor Report
National Aeronautics and Space Administration Washington, D. C. 20546	14. Sponsoring Agency Code

15. Supplementary Notes

Project Manager, Richard J. Priem, Chemical Propulsion Division, NASA Lewis Research Center, Cleveland, Ohio

16. Abstract

An analytical technique is developed to solve nonlinear three-dimensional, transverse and axial combustion instability problems associated with liquid-propellant rocket motors. The Method of Weighted Residuals is used to determine the nonlinear stability characteristics of a cylindrical combustor with uniform injection of propellants at one end and a conventional DeLaval nozzle at the other end. Crocco's pressure sensitive time-lag model is used to describe the unsteady combustion process. The developed model predicts the transient behavior and nonlinear wave shapes as well as limit-cycle amplitudes and frequencies typical of unstable motor operation. The limit-cycle amplitude increases with increasing sensitivity of the combustion process to pressure oscillations. For transverse instabilities, calculated pressure wave-forms exhibit sharp peaks and shallow minima, and the frequency of oscillation is within a few percent of the pure acoustic mode frequency. For axial instabilities, the theory predicts a steep-fronted wave moving back and forth along the combustor.

17. Key Words (Suggested by Author(s))	18. Distribution Statement
Combustion instability Liquid rockets Method of Weighted Residuals	Unclassified - unlimited

19. Security Classif. (of this report)	20. Security Classif. (of this page)	21. No. of Pages	22. Price*
Unclassified	Unclassified	173	$3.00

* For sale by the National Technical Information Service, Springfield, Virginia 22151