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INTRODUCTION

ANALYSIS AND DESIGN OF THREE
DIMENSIONAL SUPERSONIC NOZZLES

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SIMILARITY LAWS FOR NOZZLE FLOWS

By

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I. INTRODUCTION

The development of nozzles for hypersonic aircraft requires the ability to analyze the behavior of high temperature gases, often chemically reactive, flowing through ducts of complicated geometry. For this purpose, analyses such as those presented in the preceding volumes of this report have been and will continue to be developed. However, the extremely complicated nature of these flow fields necessitates the adoption of certain simplifying assumptions in the construction of the analytical models. It is therefore both necessary and desirable to carry out the parallel development of an experimental program aimed at assessing the accuracy and areas of applicability of the analytical results as well as at the acquisition of data for those physically interesting configurations for which suitable analytical tools may not yet be available.

Since the exact duplication of all gas conditions at the nozzle entrance station, especially the extremely high temperature levels, makes experiments both difficult and expensive, it is logical to investigate the possibility of simulating the actual nozzle flows with low temperature nonreactive gases. To pursue this goal, it is necessary to develop the similarity laws for nozzle flows and determine the parameters which must be duplicated. This effort has been carried out and the results of the

analysis are presented here. In addition to the development of these similarity rules, a number of cool gas flows have been considered to determine their suitability as substitutes for actual high temperature gases in an experimental program.

II. SIMILARITY REQUIREMENTS

The proper simulation of a flow field requires that at any geometrically similar point the Mach number and flow direction be the same as those which exist at the corresponding point in the alternate flow and that the ratio of static pressure from one point to another in each of the flows be identical. The necessity to match the variation of p with θ requires that the value of Γ used in the characteristics equations and defined for equilibrium chemistry as

$$\Gamma = \frac{\rho}{p} \left(\frac{\partial p}{\partial \rho} \right)_s$$

be duplicated at corresponding points. The requirement that the Mach number and flow deflection be identical at corresponding points requires that the initial conditions be similar in p and identical in M and θ . In addition, the variation of V/V_i and a/a_i with p must be the same, where V_i and a_i are the gas velocity and speed of sound at the reference initial conditions.

It should be noted that although the ratio V_i/a_i must be the same for the nozzle and the model, the actual values V_i and a_i can be different since only the local Mach number must be the same.

The assumption is made that the chemistry is in equilibrium. Therefore, the conditions of similarity require that the variation of pressure ratio (p/p_i), Γ and c_p with temperature ratio (T/T_i) be the same at geometrically similar points. This will assure that the variation of velocity ratio, Mach number and pressure ratio, as functions of temperature ratio are also the same. These conditions imply that the variation of three thermodynamic parameters with temperature must be closely simulated in the model, within the range of temperature ratio existing from the throat to the nozzle exit.

III. THERMODYNAMIC RELATIONS

Consider a volume containing a unit weight of gas in thermodynamic equilibrium consisting of n moles and with a static enthalpy H per mole. In this case, the following thermodynamic relations are valid if the gas is a mixture of species i and n_i is the number of moles of each specie:

$$\sum n_i = n \quad (1)$$

$$\left(\frac{\partial H}{\partial T}\right)_p = n c_p = T \left(\frac{\partial S}{\partial T}\right)_p \quad (2)$$

where c_p is the specific heat per mole at constant pressure. The flow is isentropic between discontinuities and is in chemical equilibrium. Therefore,

$$\left(\frac{\partial S}{\partial T}\right)_p dT + \left(\frac{\partial S}{\partial p}\right)_T dp = 0 \text{ (isentropic flow)} \quad (3)$$

$$\left(\frac{\partial n}{\partial T}\right)_p dT + \left(\frac{\partial n}{\partial p}\right)_T dp = 0 \text{ (equilibrium chemistry)} \quad (4)$$

The Gibbs free energy is given by

$$G = H - TS \quad (4)$$

and

$$dH = T dS - V dp \quad (5)$$

Then

$$\left(\frac{\partial V}{\partial T}\right)_p = \left(\frac{\partial S}{\partial p}\right)_T \quad (6)$$

because

$$\left(\frac{\partial n}{\partial p}\right)_T = V \left(\frac{\partial n}{\partial T}\right)_p = -S \quad (7)$$

but Equation (3) gives

$$\left(\frac{\partial S}{\partial p}\right)_T = - \left(\frac{\partial S}{\partial T}\right)_p \left(\frac{\partial T}{\partial p}\right)_S \quad (8)$$

Therefore,

$$\left(\frac{\partial T}{\partial p}\right)_S = \frac{T \left(\frac{\partial V}{\partial T}\right)_p}{n c_p} \quad (9)$$

The equation of state gives

$$\left(\frac{\partial V}{\partial T}\right)_p = \frac{R_0 T}{p} \left(\frac{\partial n}{\partial T}\right)_p + \frac{n R_0}{p} \quad (10)$$

Therefore

thus

$$\left(\frac{\partial \ln T}{\partial \ln p}\right)_S = \frac{R_0}{c_p} \left[\left(\frac{\partial \ln n}{\partial \ln T}\right)_p + 1 \right] \quad (11)$$

If we define
 where

$$D_T = \sum n_i \left(\frac{\partial \ln n_i}{\partial \ln T} \right) p = n \alpha_T \quad (12)$$

and
 The relation between α_T and α_p is given by

$$\alpha_T = \frac{\left(\frac{\partial \ln p}{\partial \ln T} \right) \left(\frac{\partial p}{\partial T} \right)_{c_p}}{\left(\frac{\partial p}{\partial T} \right)_{c_p}} \left[\frac{1}{1 + D_T} \right] \quad (13)$$

α_T must be the same at corresponding points or at the same value of temperature ratio $\left(\frac{T}{T_i} \right)$. In addition, since

$$p = \frac{pW}{R_0 T} \quad (20)$$

$$nH = \sum n_i H_i \quad (14)$$

where W is the molecular weight and $nW = 1$,
 then

In addition,

$$n c_p = \left(\frac{\partial nH}{\partial T} \right) p = \sum_i n_i c_{pi} + \sum_i \frac{n_i H_i}{T} D_{Ti} \quad (15)$$

where D_{Ti} is defined as

and

$$D_{Ti} = \frac{\left(\frac{\partial \ln n_i}{\partial \ln T} \right) p \left(\frac{\partial p}{\partial T} \right)_{c_p}}{\left(\frac{\partial p}{\partial T} \right)_{c_p}} \quad (16)$$

Thus

then, if we define

$$c_p = \sum \frac{n_i c_{pi}}{n} + \sum \frac{n_i H_i D_{Ti}}{nT} \quad (17)$$

$$\alpha_p = \left(\frac{\partial \ln p}{\partial \ln T} \right)_{c_p} \quad (23)$$

$$\alpha_T = \frac{c_p - p}{(1 + D_T) p} \quad (24)$$

where

$$\sum n_i D_{T_i} = n D_T \quad (18)$$

The relation between p and ρ is given by

$$\left(\frac{\partial p}{\partial \rho}\right)_S = a^2 = \frac{\left(\frac{\partial p}{\partial T}\right)_S}{\left(\frac{\partial \rho}{\partial T}\right)_S} \quad (19)$$

But

$$\rho = \frac{pW}{R_0 T} \quad (20)$$

where W is the molecular weight and $nW = 1$.

In addition,

$$\left(\frac{\partial p}{\partial T}\right)_S = \frac{W}{R_0 T} \left(\frac{\partial p}{\partial T}\right)_S + \frac{p}{R_0 T} \left(\frac{\partial W}{\partial T}\right)_S - \frac{pW}{R_0 T^2} \quad (21)$$

and

$$\left(\frac{\partial W}{\partial T}\right)_S = \left(\frac{\partial W}{\partial T}\right)_p + \left(\frac{\partial W}{\partial p}\right)_T \left(\frac{\partial p}{\partial T}\right)_S \quad (22)$$

where $\left(\frac{\partial W}{\partial T}\right)_p$ is the partial derivative of W with respect to T and $\left(\frac{\partial W}{\partial p}\right)_T$ is the coefficient defined in Equation (16).

Then, if we define

$$D_p = \left(\frac{\partial \ln \frac{n}{p}}{\partial \ln p}\right)_T \quad (23)$$

$$a^2 = \frac{c_p \frac{p}{\rho}}{(1-D_p)c_p - (1+D_T)^2 R_0} \quad (24)$$

Then if Γ is defined as

$$\Gamma = \frac{c_p R_0}{(1-D_p)c_p - (1+D_T)^2 R_0} \quad (25)$$

Unfortunately, the gases entering the nozzle are in actual

$$a^2 = \Gamma \left(\frac{p}{\rho} \right) \quad (26)$$

If we transform c_p into specific heat per unit mass c_{pm}

then

$$\Gamma = \frac{c_{pm} R_0}{c_{pm}(1-D_p) - (1+D_T)^2 \frac{R_0}{W}} \quad (27)$$

and

$$c_{pm} = \sum \alpha_i c_{pmi} + \alpha_i h_i D_{Ti} \quad (28)$$

$$\alpha_T = \frac{c_{pm}}{R_0 W} \left(\frac{1}{1+D_T} \right) \quad (29)$$

where α_i is the mass fraction of species i and D_{Ti} is the coefficient defined in Equation (16).

From the preceding we conclude that the three thermodynamic parameters which must be duplicated are Γ , α_T and c_p or equivalently D_p , D_T and c_p .

Unfortunately, the gases entering the nozzle of an actual vehicle undergo a substantial change in chemical composition as they flow through the duct and this change has a pronounced effect on the values of D_T and D_p . Since any cold gas simulation will, of necessity, preclude these chemical reactions, it does not appear possible to obtain perfect simulation over the entire range of temperatures existing in the nozzle. However, gases are available which permit close simulation of Γ , c_p and α_T for some conditions of interest. Some of these gases are investigated and defined in the next section where their variation of critical thermodynamic properties with temperature ratio is presented within the temperature range of interest.

IV. COLD FLOW SIMULATION

In order to assess the relative merits of various gases with regard to their ability to simulate the hot exhaust flow through a hypersonic nozzle, it is first necessary to define the properties of a typical hot flow. To do this, we select the case of a scramjet propelled vehicle flying at a Mach number of 8 at an altitude of 100,000 feet. The scramjet engine operation is characterized by an inlet static pressure ratio of 140 with a total pressure recovery of 60% and constant pressure combustion. For hydrogen-air combustion at equivalence ratios of 0.6 and 1.0 these operational assumptions result in the following set of gas stream characteristics at the nozzle entrance.

ϕ	$p(\text{pa})$	$h(\text{j/kg})$	$\rho(\text{kg/m}^3)$	$T(^{\circ}\text{K})$
0.6	1.56×10^5	6×10^6	0.162	2821
1.0	1.56×10^5	4.2×10^6	0.204	2381

Assuming equilibrium chemistry, the temperature-pressure history through an isentropic expansion from the nozzle entrance conditions to a static pressure 200 times lower than the initial value is obtained from the tables of Reference (1) and is shown in Figure (1). It should be noted that the actual degree of expansion would be expected to be significantly less than a factor

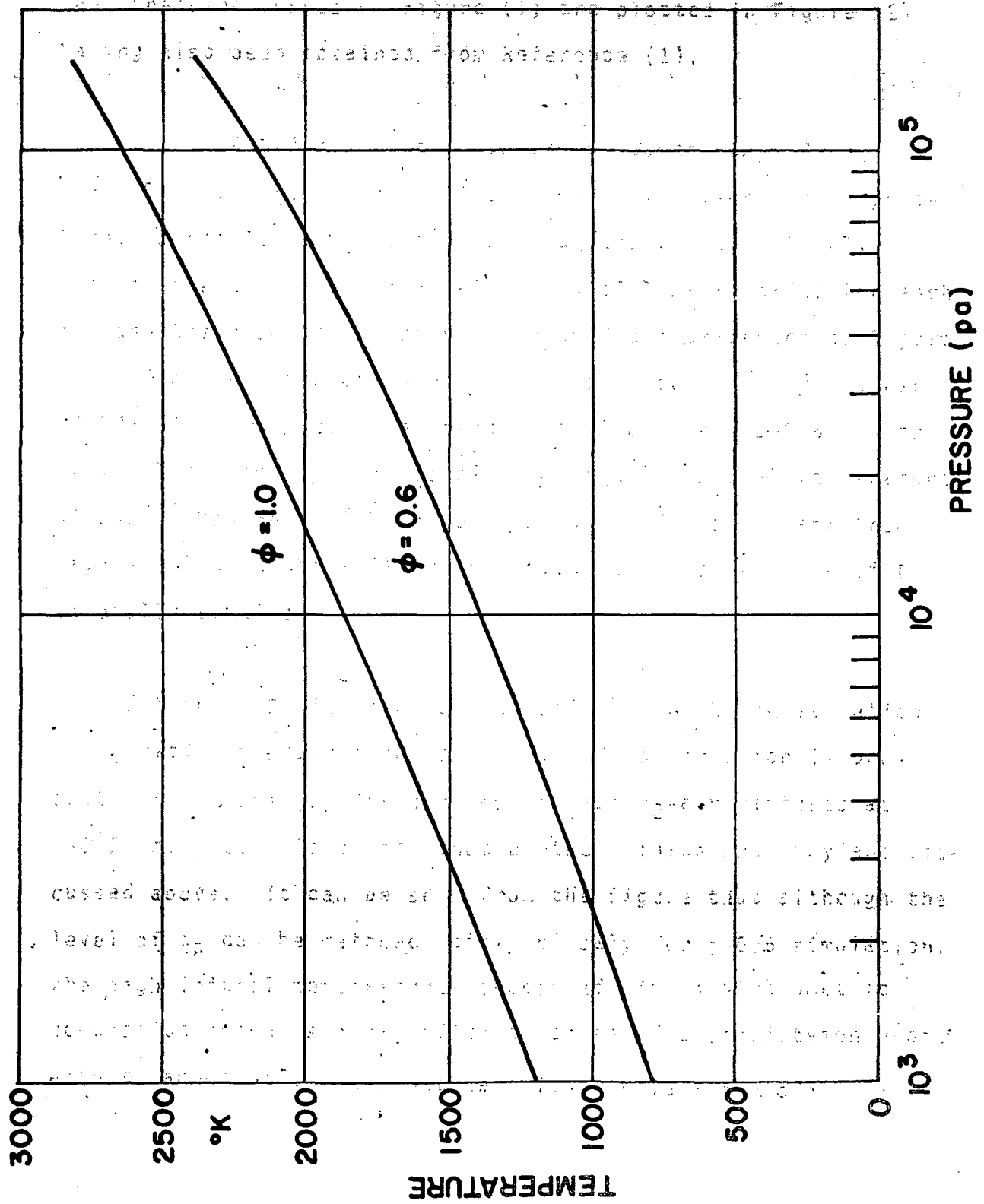


FIGURE 1. PRESSURE-TEMPERATURE PATH THROUGH EQUILIBRIUM ISENTROPIC EXPANSION.

of 200 but this wider range provides an adequate margin for the purposes of comparison. The values of Γ corresponding to the expansion curves of Figure (1) are plotted in Figure (2), having also been obtained from Reference (1).

The values of Γ for several hydrocarbon gases (CH_4 , C_2H_4 , C_2H_6 , C_4H_8-2) are shown in Figure (3) as functions of temperature. Since it is necessary to match the function $\Gamma(T/T_i)$ through the expansion, several values of T_i were tried for each gas and the resulting Γ compared with the requirement of Figure (2). The best simulation was found to be obtained by the use of CH_4 with an initial temperature of 550°K when $\phi=0.6$ was to be simulated and by the use of C_2H_4 with an initial temperature of 500°K when $\phi=1.0$ was to be simulated. It is evident from Figure (4) that the ability to provide the proper value of Γ is easily attained.

In addition to Γ , it is also desirable to match the variation of c_p with temperature ratio (T/T_i). This variation is presented in Figure (5) for the equilibrium H_2 -air mixtures at $\phi=0.6$ and $\phi=1.0$ along with that of the methane and ethylene discussed above. It can be seen from the figure that although the level of c_p can be matched fairly closely for $\phi=0.6$ simulation, the high initial temperature associated with stoichiometric combustion produces a significant degree of dissociation whose effect cannot be closely matched with a relatively cool gas.

FIGURE 2. VARIATION OF EQUILIBRIUM Γ THROUGH NOZZLE EXPANSION.

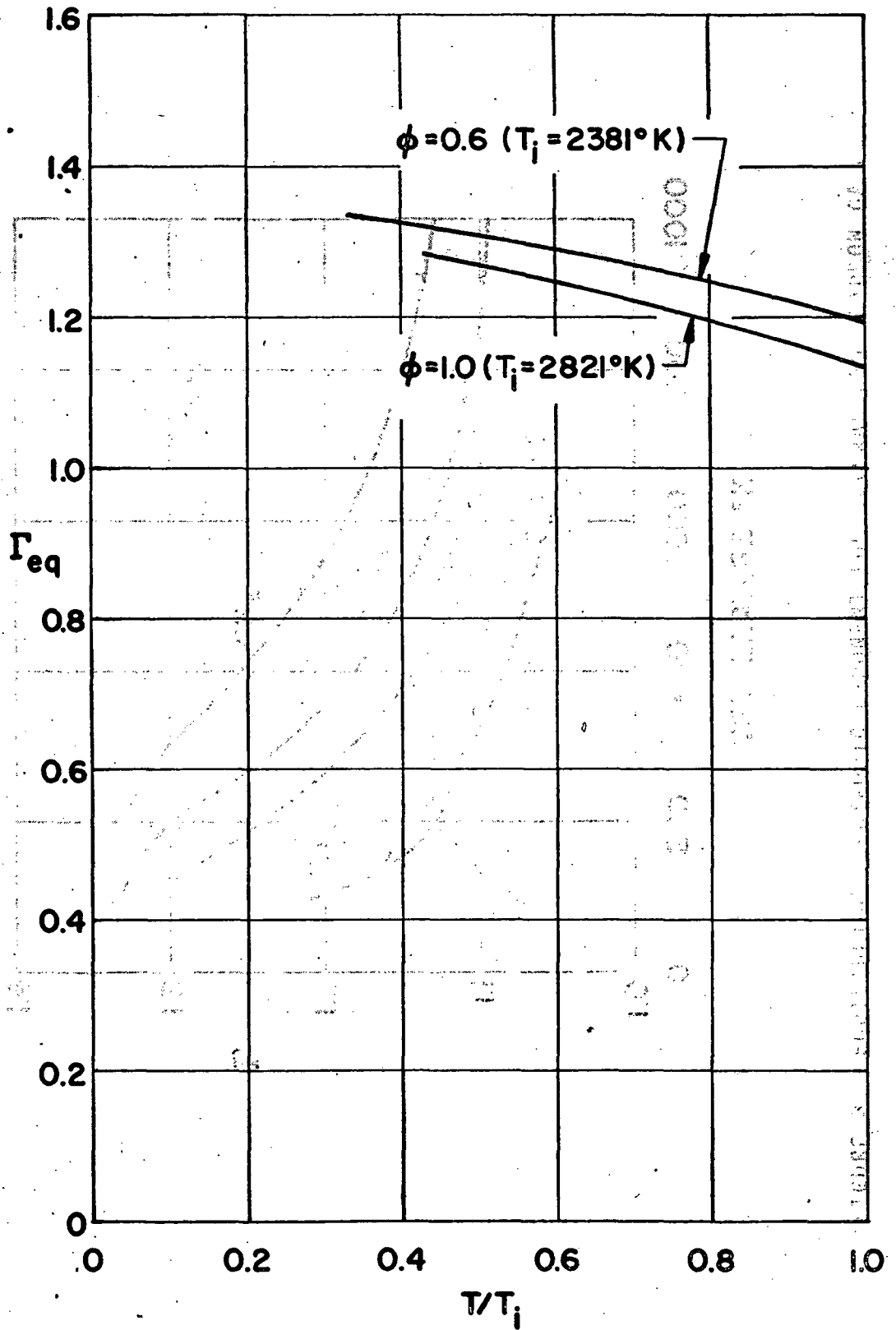


FIGURE 2. VARIATION OF EQUILIBRIUM Γ THROUGH NOZZLE EXPANSION.

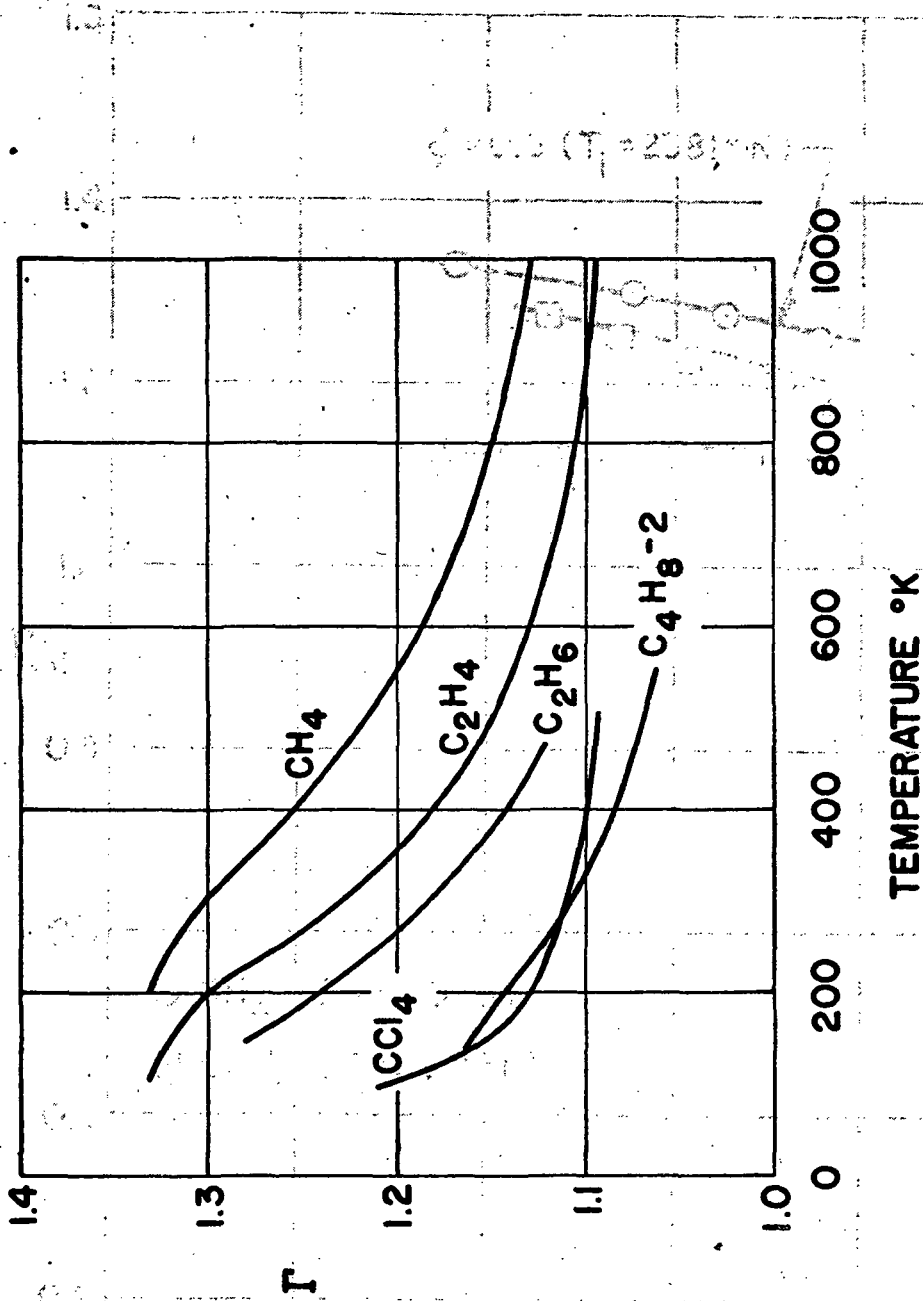


FIGURE 3. EQUILIBRIUM-ISENTROPIC-EXPONENT FOR SEVERAL HYDROCARBON GASES.

(REFERENCE 2)

FIGURE 4. EQUILIBRIUM-ISENTROPIC-EXPONENT FOR SEVERAL HYDROCARBON GASES.

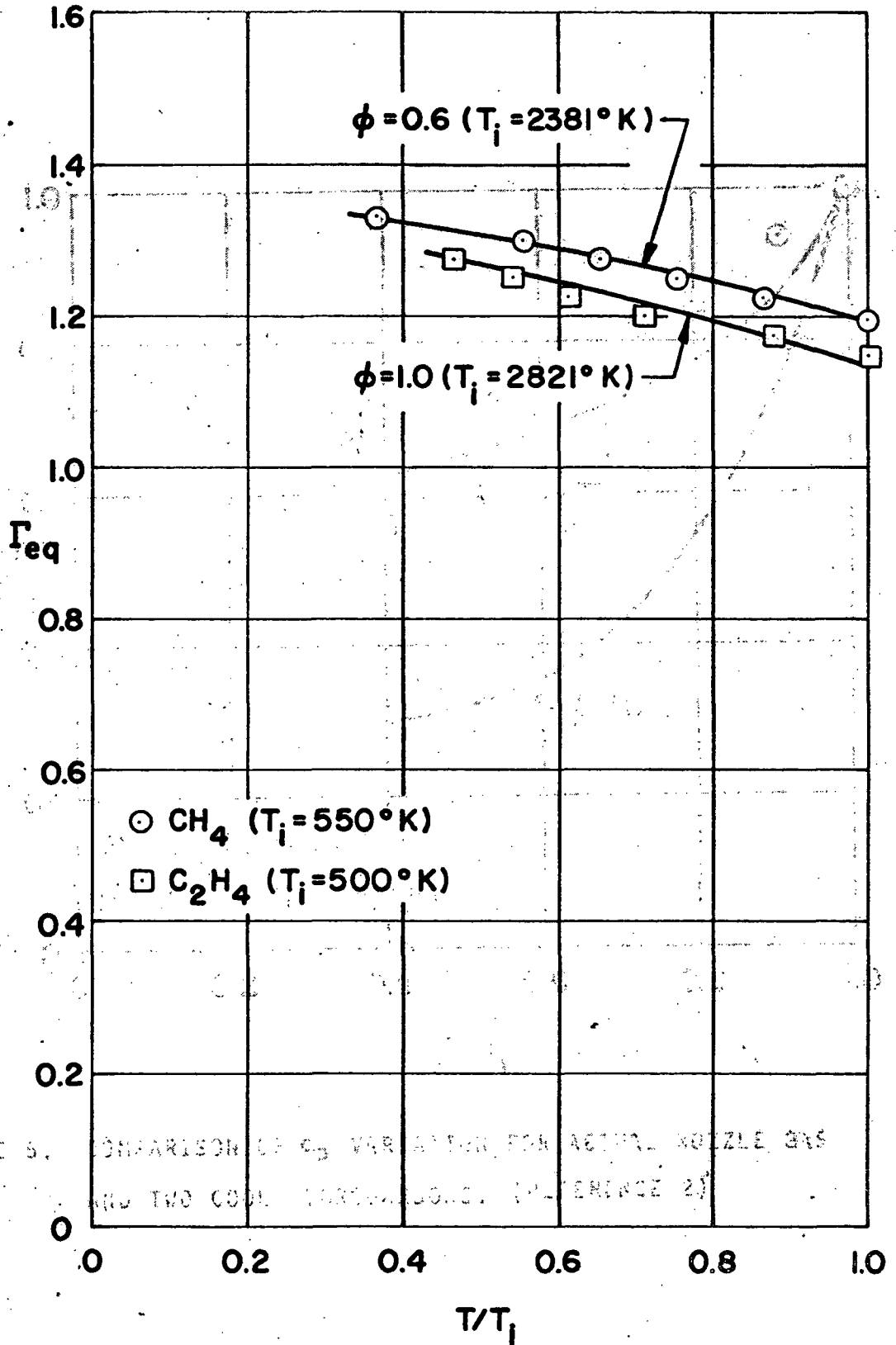


FIGURE 5. COMPARISON OF Γ_{eq} VARIATION FOR ACTUAL NOZZLE GAS AND TWO COOL HYDROCARBONS. (REFERENCE 2)

FIGURE 4. COMPARISON OF Γ VARIATION FOR ACTUAL NOZZLE GAS AND TWO COOL HYDROCARBONS.

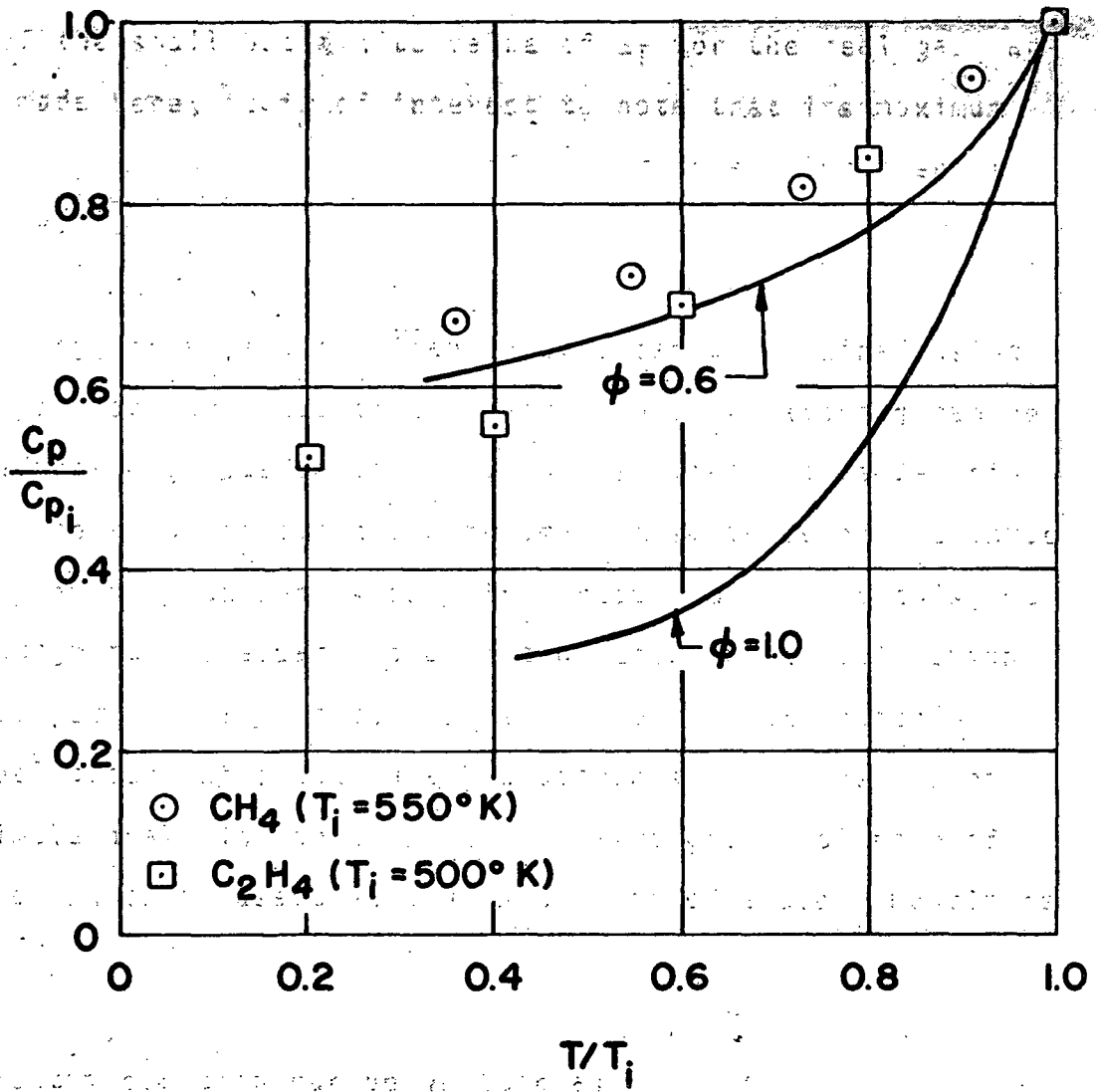


FIGURE 5. COMPARISON OF c_p VARIATION FOR ACTUAL NOZZLE GAS AND TWO COOL HYDROCARBONS. (REFERENCE 2)

Finally, the value of α_T is zero for the nonreactive substitute gases while it varies from zero to a small negative number for the high temperature gases. Although a quantitative evaluation of the small but finite value of α_T for the real gas cannot be made here, it is of interest to note that its maximum magnitude for $\phi=0.6$ is an order of magnitude smaller than it is for $\phi=1.0$.

Summarizing, excellent duplication of Γ can be obtained using a cool gas for any value of ϕ . Good c_p and α_T matching can be obtained for moderate values of ϕ , but a quantitative definition of the error associated with small mismatches of the three thermodynamic functions is certainly required. To do this, we will integrate the equations of motion used in an equilibrium characteristics analysis to analyze a one wave expansion of both real gases and substitute hydrocarbons. The results of this simple analysis can then be used as a good indicator of the error which is associated with the use of a cool substitute gas.

The momentum equation can be written as

$$\frac{dp}{\rho} + \frac{1}{2} dV^2 = 0 \tag{30}$$

and the isentropic law in the form

$$\int \frac{dp}{p} = \Gamma \int \frac{dp}{p} \quad (31)$$

For a one wave flow, a convenient coordinate system can be defined by the local Mach line and a normal to it. Defining the velocity component along the characteristic as V_t , we have for one wave flow

$$\frac{d V_t}{d \delta} = a \quad (32)$$

where δ is the angle defining the wave orientation from the y-axis and a is the local sonic velocity, given by Equation (26). From the definition of our coordinate system, we have

$$v^2 = v_t^2 + a^2 \quad (33)$$

Integrating (30) and (31) over a small step in Δp through which

Γ may be assumed constant, we obtain

$$\frac{\Gamma+1}{\Gamma-1} a^2 + v_t^2 = c^2 \quad (34)$$

Substituting into (32), we have

$$\frac{dV_t}{d\delta} = \sqrt{\frac{\Gamma-1}{\Gamma+1}} \sqrt{c^2 - v_t^2} \quad (35)$$

or

$$\int_{v_{t1}}^{v_{t2}} \frac{dv_t}{\sqrt{c^2 - v_t^2}} = \int_{\delta_1}^{\delta_2} \sqrt{\frac{\Gamma-1}{\Gamma+1}} d\delta \quad (36)$$

which yields

$$\beta_2 = \beta_1 - \sqrt{\frac{\Gamma-1}{\Gamma+1}} (k_2 - k_1) \quad (37)$$

where

$$k \equiv \tan^{-1} \frac{v_t/c}{\sqrt{1 - (v_t/c)^2}} \quad (38)$$

and

$$\beta = \theta + \mu = \pi/2 - \delta \quad (39)$$

Equations (33), (34), (37) and (38) can be used to evaluate the expansion of a gas through a series of steps in Δp , the value of Γ being a constant in each step. This analysis was programmed for numerical solution using the curvefits for $h(T,p,\phi)$ and $\Gamma(T,p,\phi)$ described in Volume I of this report. The program listing is presented in the Appendix. The program was used to compute the variation of pressure and impulse

function* with flow turning angle through a one wave expansion for the $\phi=0.6$ and 1.0 nozzle entrance conditions described earlier. The program was also used with the curvefits of Reference (1) for the functions $h(T)$ and $[c_p/R](T)$ for the gases CH_4 and C_2H_4 . Since the temperature range for the hydrocarbons was fairly low, the equilibrium isentropic exponent was defined by the expression

$$\Gamma = \frac{c_p/R}{(c_p/R) - 1} \quad (40)$$

The results of these calculations are presented in Figures (6) and (7) where it is clear that fairly good representation of pressure distribution and impulse can be achieved with the use of cool substitute gases.

*Impulse function is defined here as

$$F/F^* = \frac{1 + \Gamma m^2}{m \sqrt{2(\Gamma+1) \left[1 + \left(\frac{\Gamma-1}{2} \right) m^2 \right]}}$$

following the notation of Reference (3).

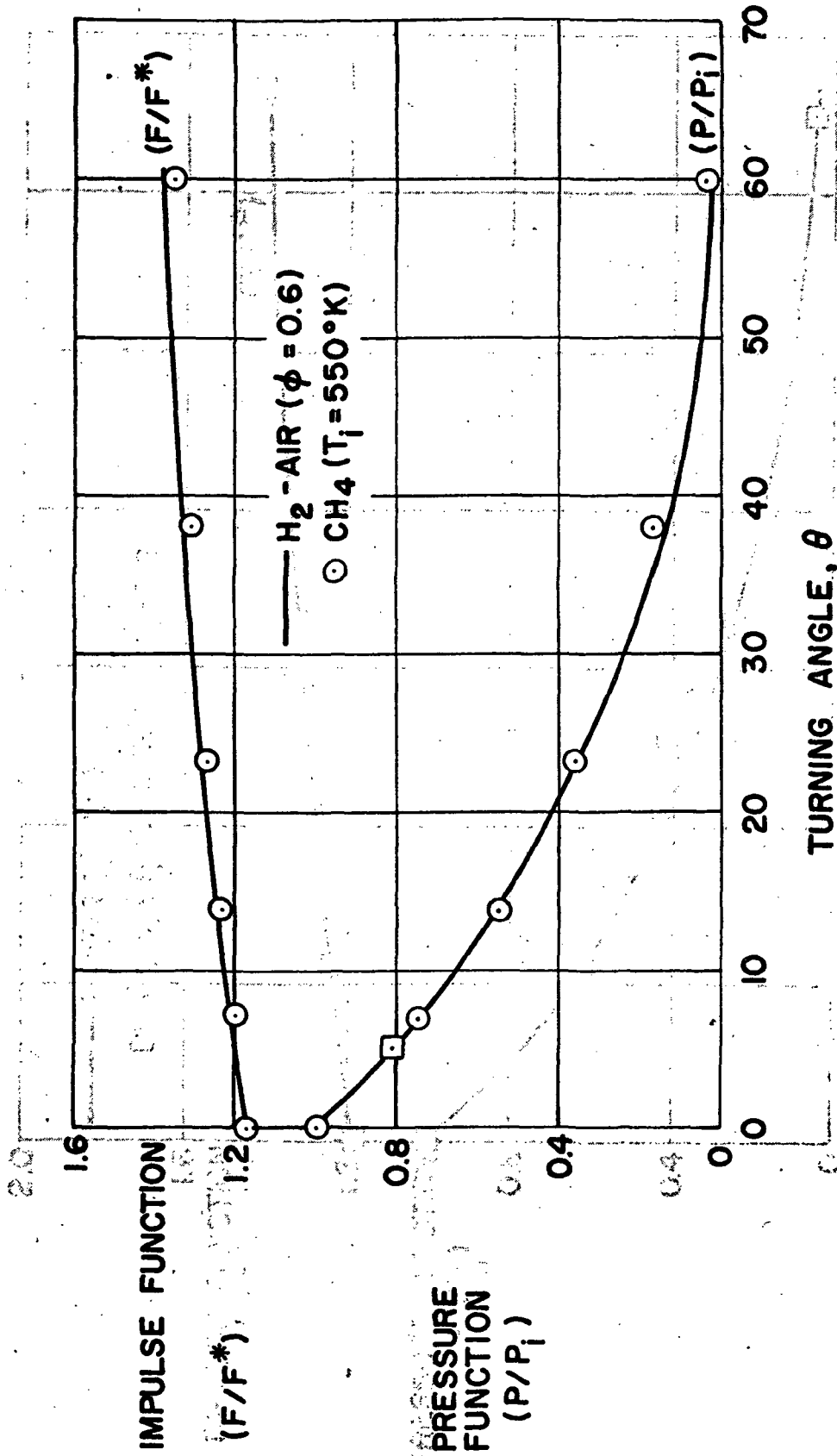


FIGURE 6. COMPARISON OF STATIC PRESSURE DISTRIBUTION AND IMPULSE FUNCTION FOR EQUILIBRIUM H₂-AIR ($\phi = 0.6$) AND CH₄ (T₁ = 550°K).

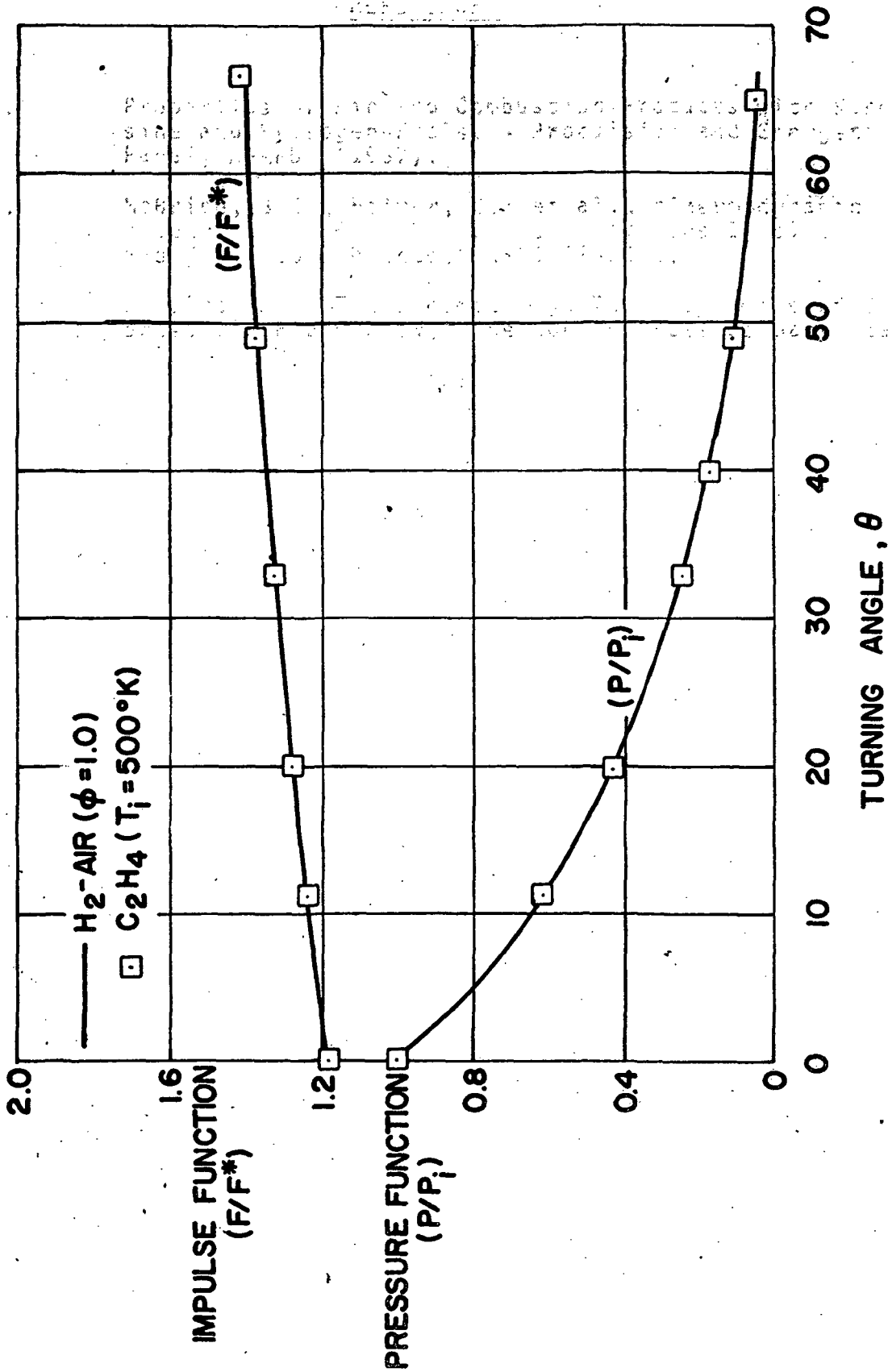


FIGURE 7. COMPARISON OF STATIC PRESSURE DISTRIBUTION AND IMPULSE FUNCTION FOR EQUILIBRIUM STOICHIOMETRIC H_2 -AIR AND C_2H_4 ($T_i = 500^\circ K$).

REFERENCES

1. Properties of Air and Combustion Products with Kerosine and Hydrogen Fuels. - Propulsion and Energetics Panel, AGARD (1967).
2. McBride, B.J., Heimer, S.; et al., "Thermodynamic Properties to 6000°K for 210 Substances Involving the First 18 Elements," NSAS SP-3001.
3. Shapiro, A., "The Dynamics and Thermodynamics of Compressible Fluid Flow," The Ronald Press Company (1954).

```

10  LET A=1
20  LET B=2
30  LET C=3
40  LET D=4
50  LET E=5
60  LET F=6
70  LET G=7
80  LET H=8
90  LET I=9
100 LET J=10
110 LET K=11
120 LET L=12
130 LET M=13
140 LET N=14
150 LET O=15
160 LET P=16
170 LET Q=17
180 LET R=18
190 LET S=19
200 LET T=20
210 LET U=21
220 LET V=22
230 LET W=23
240 LET X=24
250 LET Y=25
260 LET Z=26
270 LET AA=27
280 LET AB=28
290 LET AC=29
300 LET AD=30
310 LET AE=31
320 LET AF=32
330 LET AG=33
340 LET AH=34
350 LET AI=35
360 LET AJ=36
370 LET AK=37
380 LET AL=38
390 LET AM=39
400 LET AN=40
410 LET AO=41
420 LET AP=42
430 LET AQ=43
440 LET AR=44
450 LET AS=45
460 LET AT=46
470 LET AU=47
480 LET AV=48
490 LET AW=49
500 LET AX=50
510 LET AY=51
520 LET AZ=52
530 LET BA=53
540 LET BB=54
550 LET BC=55
560 LET BD=56
570 LET BE=57
580 LET BF=58
590 LET BG=59
600 LET BH=60
610 LET BI=61
620 LET BJ=62
630 LET BK=63
640 LET BL=64
650 LET BM=65
660 LET BN=66
670 LET BO=67
680 LET BP=68
690 LET BQ=69
700 LET BR=70
710 LET BS=71
720 LET BT=72
730 LET BU=73
740 LET BV=74
750 LET BW=75
760 LET BX=76
770 LET BY=77
780 LET BZ=78
790 LET CA=79
800 LET CB=80
810 LET CC=81
820 LET CD=82
830 LET CE=83
840 LET CF=84
850 LET CG=85
860 LET CH=86
870 LET CI=87
880 LET CJ=88
890 LET CK=89
900 LET CL=90
910 LET CM=91
920 LET CN=92
930 LET CO=93
940 LET CP=94
950 LET CQ=95
960 LET CR=96
970 LET CS=97
980 LET CT=98
990 LET CU=99
1000 LET CV=100

```

APPENDIX

PROGRAM LISTING IN BASIC LANGUAGE

```

1000 LET DA=100
1010 LET DB=101
1020 LET DC=102
1030 LET DD=103
1040 LET DE=104
1050 LET DF=105
1060 LET DG=106
1070 LET DH=107
1080 LET DI=108
1090 LET DJ=109
1100 LET DK=110
1110 LET DL=111
1120 LET DM=112
1130 LET DN=113
1140 LET DO=114
1150 LET DP=115
1160 LET DQ=116
1170 LET DR=117
1180 LET DS=118
1190 LET DT=119
1200 LET DU=120
1210 LET DV=121
1220 LET DW=122
1230 LET DX=123
1240 LET DY=124
1250 LET DZ=125
1260 LET EA=126
1270 LET EB=127
1280 LET EC=128
1290 LET ED=129
1300 LET EE=130
1310 LET EF=131
1320 LET EG=132
1330 LET EH=133
1340 LET EI=134
1350 LET EJ=135
1360 LET EK=136
1370 LET EL=137
1380 LET EM=138
1390 LET EN=139
1400 LET EO=140
1410 LET EP=141
1420 LET EQ=142
1430 LET ER=143
1440 LET ES=144
1450 LET ET=145
1460 LET EU=146
1470 LET EV=147
1480 LET EW=148
1490 LET EX=149
1500 LET EY=150
1510 LET EZ=151
1520 LET FA=152
1530 LET FB=153
1540 LET FC=154
1550 LET FD=155
1560 LET FE=156
1570 LET FF=157
1580 LET FG=158
1590 LET FH=159
1600 LET FI=160
1610 LET FJ=161
1620 LET FK=162
1630 LET FL=163
1640 LET FM=164
1650 LET FN=165
1660 LET FO=166
1670 LET FP=167
1680 LET FQ=168
1690 LET FR=169
1700 LET FS=170
1710 LET FT=171
1720 LET FU=172
1730 LET FV=173
1740 LET FW=174
1750 LET FX=175
1760 LET FY=176
1770 LET FZ=177
1780 LET GA=178
1790 LET GB=179
1800 LET GC=180
1810 LET GD=181
1820 LET GE=182
1830 LET GF=183
1840 LET GG=184
1850 LET GH=185
1860 LET GI=186
1870 LET GJ=187
1880 LET GK=188
1890 LET GL=189
1900 LET GM=190
1910 LET GN=191
1920 LET GO=192
1930 LET GP=193
1940 LET GQ=194
1950 LET GR=195
1960 LET GS=196
1970 LET GT=197
1980 LET GU=198
1990 LET GV=199
2000 LET GW=200

```

```

5 PRINT "I", "P", "T", "RHO", "H"
6 PRINT "GAMMA", "MACH", "THETA"
7 PRINT
10 DIM J[1000], K[1000]
15 DIM B[1000], Z[1000]
20 GOTO 512
30 LET T=TO=1500
40 GOSUB 180
50 LET EO=(H-H1)/H
60 LET HO=H1
70 LET T1=TO*.1
80 LET T=T1
90 GOSUB 180
100 LET E1=(H-H1)/H
110 IF ABS(E1)<.0001 THEN 390
120 LET T9=TO-EO*(T1-TO)/(E1-EO)
130 LET EO=E1
140 LET TO=T1
150 LET HO=H1
160 LET T1=T9
170 GOTO 80
180 LET A=1.E-07*(-.1042*F+2+.8242*F+.987)
190 LET B=.001*(.01167*F+2+.1503*F+.938)
200 LET C=-.0284*F+2+.6731*F+.4293
210 IF F <= 1 THEN 250
220 LET A=1.E-07*(1.787*F+2-5.48*F+5.4)
230 LET B=.001*(-.1867*F+2+1.11*F+.176)
240 LET C=-.0933*F+2+3.975*F-2.808
250 IF T <= 2000 THEN 330
260 LET A=.000001*(1.792*F+2+.3983*F+.31)
270 LET B=.001*(-9.05*F+2-.07917*F+.245)
280 LET C=10.86*F+2-.1183*F+.97
290 IF F <= 1 THEN 330
300 LET A=.000001*(4.81*F+2-13.9*F+11.59)
310 LET B=.001*(-23.08*F+2+66.82*F-52.61)
320 LET C=27.05*F+2-73.73*F+58.39
330 LET H1=A*T+2+B*T+C
340 IF T <= 2000 THEN 370
350 LET Z9=.125*(LOG(P)/2.3-5)+2-.275*(LOG(P)/2.3-5)
360 LET H1=H1*(1+(1+F)*(T/2000-1)*Z9)
370 LET H1=H1*1.E+06
380 RETURN
390 LET M=0
400 IF T <= 1000 THEN 420
410 LET M=2.15E-08*T+2+.000091*T-.0695
420 LET N=4.E-09*T+2-.00002*T-.019
430 IF F <= 1 THEN 450
440 LET N=.0339*SQR(T)-.000391*T-.681
450 LET C=-1.833E-07*T+2+.000075*T+1.367
460 IF T<500 THEN 500
470 LET C=2.E-08*T+2-.000138*T+1.423
480 IF T<2000 THEN 500
490 LET C=7.267E-08*T+2-.000457*T+1.85
500 LET C=C+M*(LOG(P)/2.3-5)+N*(F-1)
510 RETURN
512 READ P, P9, R1, D1, H, V, F
514 DATA 1.56E+06, 15600, 1.62, 5000, 6.E+06, 2075, 1
516 GOTO 600
520 PRINT "INITIAL PRESSURE";
530 INPUT P
540 PRINT "FINAL PRESSURE";
550 INPUT P9

```

```
560 PRINT "RHO";
570 INPUT R1
580 PRINT "DELTA PRESSURE";
590 INPUT D1
600 LET I1=1000
610 LET J[1]=P
620 LET K[1]=R1
622 GOTO 690
630 PRINT "ENTHALPY";
640 INPUT H
650 PRINT "VELOCITY";
660 INPUT V
670 PRINT "FUEL TO AIR RATIO";
680 INPUT F
690 LET H2=H+.5*V*V
700 GOSUB 30
710 LET C8=P/R1*G
720 LET R=R1
722 GOSUB 2000
724 LET Z[1]=ATN(V1/C1/SQR(1-(V1/C1)^2))
726 LET B[1]=ATN(1/SQR((V/A1)^2-1))
728 LET T1=0
730 FOR I=2 TO I1
740 LET C8=P/R*G
744 IF I<2 THEN 750
746 : BETA(###)=#####. MU=##### THETA=#####
750 IF INT(I/20)*20 <> I THEN 770
760 PRINT I,P,T,R,H
765 PRINT G,V/A1,57.3*T1
766 PRINT
770 LET J[I]=J[I-1]-D1
780 LET K[I]=(J[I]/C8)*(1/G)
790 LET V=SQR(V*V+2*G/(G-1)*(J[I-1]/K[I-1]-J[I]/K[I]))
800 LET H=H2-.5*V*V
810 LET P=J[I]
820 LET R=K[I]
830 GOSUB 30
835 GOSUB 950
840 LET I2=I
850 IF J[I] <= P9 THEN 890
860 IF J[I]-D1>P9 THEN 880
870 LET D1=J[I]-P9
880 NEXT I
890 LET I=K[I2]*V*V+J[I2]
900 LET R=K[I2]
910 LET P=J[I2]
920 GOSUB 30
925 GOSUB 950
930 PRINT I2,J[I2],T,K[I2],H,G,V,I
940 STOP
950 GOSUB 2000
990 LET Z[I]=ATN(V1/C1/SQR(1-(V1/C1)^2))
1000 LET B[I]=B[I-1]-SQR(G1)*(Z[I]-Z[I-1])
1010 LET U=ATN(1/SQR((V/A1)^2-1))
1020 LET T1=B[I]-U
1024 GOTO 1030
1025 PRINT USING 746,I,B[I],57.3*U,57.3*T1
1030 RETURN
1990 STOP
2000 LET A1=SQR(G*P/R)
2010 LET G1=(G+1)/(G-1)
2020 LET C1=SQR(G1*A1^2+V1^2-A1^2)
2030 LET V1=SQR(V1^2-A1^2)
2040 RETURN
3000 END
```