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Appendix D

NUMERICAL SIMULATION OF WEATHER AND CLIMATE

132873

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A SIMPLIFIED SCHEME FOR  
 COMPUTING RADIATION TRANSFER IN THE TROPOSPHERE

by  
 A. Katayama

*DRA*

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## ABSTRACT

A simplified scheme is presented, for the heating of clear and cloudy air by solar and infrared radiation transfer, designed for use in tropospheric general circulation models with coarse vertical resolution.

A bulk transmission function is defined for the infrared transfer. The interpolation factors, required for computing the bulk transmission function, are parameterized as functions of such physical parameters as the thickness of the layer, the pressure, and the mixing ratio at a reference level.

The computation procedure for solar radiation is significantly simplified by the introduction of two basic concepts. The first is that the solar radiation spectrum can be divided into a scattered part, for which Rayleigh scattering is significant but absorption by water vapor is negligible, and an absorbed part for which absorption by water vapor is significant but Rayleigh scattering is negligible. The second concept is that of an equivalent cloud water vapor amount which absorbs the same amount of radiation as the cloud.

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Insert 1, p. II-2

$u_{H_2O}^*$  (3), is given by

$$u_{H_2O}^* = \frac{1}{g} \int_{p(z)}^{p_g} q_{H_2O} \left( \frac{p}{p_{oo}} \right)^{\alpha_{H_2O}} dp \quad (II.3)$$

where  $q_{H_2O}$  is the mixing ratio of water vapor. A similar expression for the effective carbon dioxide amount is given by equation (IV.17).

Insert 2, p. II-2

Following Yamamoto (1952), the total transmission function of a mixture of water vapor and carbon dioxide is assumed to be represented by the product of their respective transmission functions, that is

$$\tau(u^*, T) = \tau_{H_2O}(u_{H_2O}^*, T) \tau_{CO_2}(u_{CO_2}^*, T)$$

and

$$\tilde{\tau}(u^*, T) = \tilde{\tau}_{H_2O}(u_{H_2O}^*, T) \tau_{CO_2}(u_{CO_2}^*, T)$$

As discussed in section 3(ii),  $\tau_{CO_2}(u_{CO_2}^*, T)$  is assumed to be constant between any two isobaric levels. Thus  $\tau$  and  $\tilde{\tau}$  vary only with  $\tau_{H_2O}$  and  $\tilde{\tau}_{H_2O}$ , respectively. Consequently, unless required for clarity, the subscript  $H_2O$  will henceforth be dropped from  $\tau_{H_2O}$  and  $\tilde{\tau}_{H_2O}$ .

Insert 3, p. II-9

Experiments have been conducted with the GCM to determine, once and for all, the functional dependence expressed by equation (II.22). In these experiments, the integrals appearing in (II.16) and (II.17) were evaluated numerically by dividing the GCM layer under consideration into thin sublayers of 10 mb thickness. In this way,...

Insert 4, p. II-15

Following Yamamoto (1952), the total transmission function of a mixture of water vapor and carbon dioxide is assumed to be represented by the product of their respective transmission functions, that is

$$\tau(u_{\text{H}_2\text{O}}^*, u_{\text{CO}_2}^*, T) = \tau_{\text{H}_2\text{O}}(u_{\text{H}_2\text{O}}^*, T) \tau_{\text{CO}_2}(u_{\text{CO}_2}^*, T)$$

and

$$\tilde{\tau}(u_{\text{H}_2\text{O}}^*, u_{\text{CO}_2}^*, T) = \tilde{\tau}_{\text{H}_2\text{O}}(u_{\text{H}_2\text{O}}^*, T) \tau_{\text{CO}_2}(u_{\text{CO}_2}^*, T)$$

Yamamoto presents values of  $\tau_{\text{H}_2\text{O}}$ ,  $\tilde{\tau}_{\text{H}_2\text{O}}$  and  $\tau_{\text{CO}_2}$  (denoted in Tables II-2 to

II-5 by Obs.) that are based upon experimental laboratory measurements of a generalized absorption coefficient in certain discrete spectral ranges. Using this experimental data, we deduce the following empirical equations for the transmission functions of water vapor and carbon dioxide.

Insert 5, p. IV-1

The effective water vapor amount in an air column of height  $z$ ,  $u^*(z)$ , can be obtained from equation (II.3) if the continuous vertical distribution of water vapor mixing ratio  $q$  is known. Since the UCLA 3-layer GCM predicts  $q$  from the moisture conservation equation only at discrete levels, some approximation technique must be employed for equation (II.3). Smith (1966) has shown that the seasonal and latitudinal northern hemisphere mean value of the mixing ratio in the lower troposphere can be expressed by

$$q = q_0 \left( \frac{p}{p_0} \right)^k, \quad (\text{IV.1})$$

where  $q_0$  is a known value of the mixing ratio at some pressure  $p_0$ . Utilizing different forms of this relation, in conjunction with the values of  $q$  predicted by the GCM, two methods for the evaluation of  $u^*$  will be developed subsequently.

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*Variation of the zonal*

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## I. - INTRODUCTION

To accurately compute the fluxes of long wave radiation and solar radiation insolation in the atmosphere, it is necessary to discretize the spectrum and apply the equation of radiative transfer to each narrow wavelength interval. Furthermore, the atmosphere must be divided into a large number of thin layers for <sup>an accurate</sup> ~~the~~ radiation computation, even when the general circulation model (GCM) has only a few layers. The amount of computer time required to calculate the radiative fluxes throughout the ~~entire~~ <sup>combined</sup> global atmosphere by this method is enormous and would exceed the <sup>the</sup> time required for all other computations of the GCM. ~~combined~~ Consequently, it is desirable to have a simplified radiation computation scheme for use with a GCM having <sup>with</sup> coarse vertical resolution.

~~C-~~ <sup>clouds</sup> The ~~cloud system~~ of the atmosphere strongly influences the radiational heating field. When the sky is covered by a dense cloud layer, the net flux of long wave radiation in the cloud layer is zero <sup>and the</sup> ~~the~~ top and bottom of the cloud layer are ~~treated as~~ black body radiators at their respective temperatures. The cooling due to long wave radiation is therefore significantly modified throughout the atmospheric column containing the cloud layer. Furthermore, <sup>because</sup> ~~since~~ a substantial fraction of the incident solar radiation is reflected or absorbed by a cloud layer, the vertical distribution of heating <sup>the</sup> ~~due to~~ <sup>by</sup> solar radiation is <sup>considerably</sup> ~~largely~~ affected by the cloud layer.

The radiative heating of clear and cloudy air is very complicated and at present we do not have a simple and accurate calculation scheme for a GCM. This notwithstanding, the scheme for calculating radiative heating in the UCLA multi-level GCM is presented in this study. A brief presentation for the UCLA two-layer GCM, has been given earlier by Arakawa, Katayama and Mintz (1969).

## II. LONG WAVE RADIATION

### 1. Basic equations

The atmospheric long wave (thermal) radiation with wavelengths between  $2.5 \mu$  and  $40 \mu$  is mainly affected by the absorption by water vapor, carbon dioxide and ozone. <sup>of long wave radiation</sup> The scattering by air molecules is negligible, but ~~however~~ the scattering by large aerosol particles in the air and water droplets in a cloud layer may be significant. <sup>Because</sup> Since the present UCLA GCM ~~only~~ covers

~~only~~ the troposphere, the absorption by ozone <sup>will</sup> can be neglected. Furthermore, for simplicity, any scattering of long wave radiation is assumed to be negligible. *However, for simplicity, the long wave scattering will be neglected.*

By solving the equation of radiative transfer, subject to the boundary conditions that ~~the~~ the downward long wave radiation flux ~~is zero~~ at the top of the atmosphere <sup>is zero</sup> and ~~the~~ the upward long wave radiation flux at the earth's surface is ~~that given by~~ <sup>the</sup> black body radiation at the surface temperature, the downward and upward radiation fluxes  $R_z^\downarrow$  and  $R_z^\uparrow$  at a reference level  $z$ , can be expressed by:

$$R_z^\downarrow = \int_0^\infty \pi B_\nu(T_z) d\nu - \int_0^\infty \pi B_\nu(T_\infty) \tau_f \{l_\nu(u_\infty - u_z)\} d\nu + \int_0^\infty \int_{T_z}^{T_\infty} \pi \frac{dB_\nu(T)}{dT} \tau_f \{l_\nu(u - u_z)\} dT d\nu, \quad (II.1)$$

$$R_z^\uparrow = \int_0^\infty \pi B_\nu(T_z) d\nu + \int_0^\infty \int_{T_z}^{T_s} \pi \frac{dB_\nu(T)}{dT} \tau_f \{l_\nu(u_z - u)\} dT d\nu, \quad (II.2)$$

where  $u(z)$  is the ~~water vapor~~ amount in the vertical air column from the earth's surface to the level  $z$ ,  $T(z)$  is the temperature,  $\pi B_\nu(T)$  is the flux of black body radiation of frequency  $\nu$  at temperature  $T$ ,  $l_\nu$  is the generalized absorption coefficient defined by Elsasser (1942) and  $\tau_f \{l_\nu u\}$  is the transmission

*of absorbing medium (water vapor and carbon dioxide)*

function of a slab at frequency  $\nu$ . The subscripts  $g, z$  and  $\infty$  denote the earth's surface, the reference level and the top of the atmosphere, respectively.

Since the width of an absorption line increases by collision damping, the absorption coefficient  $k_\nu$  is a function of pressure  $p$ . This effect can be approximately expressed by  $(p/p_{oo})^\alpha$  where  $p_{oo}$  is a standard pressure and  $\alpha$  is a pressure scaling factor. Since the absorption coefficient is always multiplied by the amount of the absorbing medium  $u$ , we can conveniently use the absorption coefficient at the standard pressure and apply the pressure correction to  $u$ . The water vapor amount modified in this manner, the effective water vapor amount

Insert 1

$u^*(z)$ , is given by

$$u^* = \frac{1}{g} \int_{p(z)}^{p_{oo}} q \left( \frac{p}{p_{oo}} \right)^\alpha dp \quad (11.3)$$

where  $q$  is the mixing ratio of water vapor. In the following discussion,  $u^*$  is used in lieu of  $u$ .

We define two types of weighted mean transmission function as follows:

$$\tau(u^*, T) = \tau(u_{H_2O}^*, u_{CO_2}^*, T) \quad \tau(u^*, T) = \frac{1}{\pi \frac{dB(T)}{dT}} \int_0^\infty \pi \frac{dB_\nu(T)}{dT} \tau_f(l_\nu u) d\nu \quad (11.4)$$

$$\tilde{\tau}(u^*, T) = \tilde{\tau}(u_{H_2O}^*, u_{CO_2}^*, T) \quad \tilde{\tau}(u^*, T) = \frac{1}{\pi B(T)} \int_0^\infty \pi B_\nu(T) \tau_f(l_\nu u) d\nu \quad (11.5)$$

where

$$\pi B(T) = \int_0^\infty \pi B_\nu(T) d\nu = \sigma T^4 \quad (11.6)$$

Relation (11.6) is the Stefan-Boltzmann law and  $\sigma$ , the Stefan-Boltzmann constant, is  $0.817 \times 10^{-10} \text{ ly min}^{-1} \text{ }^\circ\text{K}^{-4}$  ( $1.17 \times 10^{-7} \text{ ly day}^{-1} \text{ }^\circ\text{K}^{-4}$ ).  $\tau$  and  $\tilde{\tau}$  are functions of temperature because the spectrum distribution of black body radiation gradually changes with temperature. Yamamoto (1952) calculated the mean transmission function  $\tau(u^*, T)$  for water vapor and found that: 1) the dependence of  $\tau$

Insert 2

on temperature is weak in the range 210°K to 320°K and 2) below 210°K,  $\tau$  decreases very rapidly with decreasing temperature. Therefore, following Manabe and Möller (1961), if we select some critical temperature  $T_c$  near 220°K,  $\tau(u^*, T)$  may be approximated for  $T > T_c$  either by  $\tau(u^*, \bar{T})$  with, say,  $\bar{T} = 260^\circ\text{K}$ , or by a mean transmission function averaged over the range 210°K to 320°K. Also,  $\tilde{\tau}(u^*, T)$  may be approximated by  $\tilde{\tau}(u^*, T_c)$ .

The calculation procedure for long wave radiation is considerably simplified by use of the mean transmission functions. The second term of the right hand side of (II.1) is rewritten as follows:

$$\begin{aligned} & \int_0^\infty \pi B_\nu(T_\infty) \tau_f \{l_\nu(u_\infty - u_z)\} d\nu \\ &= \int_0^\infty \pi B_\nu(T_c) \tau_f \{l_\nu(u_\infty - u_z)\} d\nu \\ &+ \int_0^\infty \int_{T_c}^\infty \pi \frac{dB_\nu}{dT} \tau_f \{l_\nu(u_\infty - u_z)\} dT d\nu, \\ &= \pi B(T_c) \tilde{\tau}(u_\infty^* - u_z^*, T_c) + \int_{T_c}^\infty \tau(u_\infty^* - u_z^*, \bar{T}) \pi \frac{dB}{dT} dT, \\ &= \sigma T_c^4 \tilde{\tau}(u_\infty^* - u_z^*, T_c) + (\sigma T_\infty^4 - \sigma T_c^4) \tau(u_\infty^* - u_z^*, \bar{T}). \end{aligned}$$

Substituting this relation into (II.1) we obtain

$$\begin{aligned} R_Z^L &= \pi B_Z - \pi B_c \tilde{\tau}(u_\infty^* - u_z^*, T_c) - (\pi B_\infty - \pi B_c) \tau(u_\infty^* - u_z^*, \bar{T}) \\ &+ \int_0^\infty \tau(u_\infty^* - u_z^*, \bar{T}) d(\pi B), \end{aligned} \tag{II.7}$$

where  $\pi B_i \equiv \sigma T_i^4$ .

Similarly, the upward flux at level  $z$  is

$$R_z^\uparrow = \pi B_z + \int_{\pi B_z}^{\pi B_g} \tau(u_z^* - u^*, \bar{T}) d(\pi B) \quad (11.8)$$

The net upward flux at level  $z$  becomes

$$R_z \equiv R_z^\uparrow - R_z^\downarrow = A + B + C \quad (11.9)$$

where

$$A = \pi B_c \tau(u_\infty^* - u_z^*, T_c)$$

$$B = (\pi B_\infty - \pi B_c) \tau(u_\infty^* - u_z^*, \bar{T})$$

$$C = \int_{\pi B_\infty}^{\pi B_z} \tau(u^* - u_z^*, \bar{T}) d(\pi B) + \int_{\pi B_z}^{\pi B_g} \tau(u_z^* - u^*, \bar{T}) d(\pi B)$$

$$= \int_{-\pi B_\infty}^{\pi B_g} \tau(|u^* - u_z^*|, \bar{T}) d(\pi B) \quad (11.10)$$

The relations above are schematically illustrated by Yamamoto's Radiation Chart shown in Figure (11.1) →

The heating rate due to long wave radiation is given by

$$\frac{\partial T}{\partial t} = \frac{g}{c_p} \frac{\partial R_z}{\partial p} \quad (11.11)$$

where  $g$  is the acceleration of gravity and  $c_p$  is the specific heat of air at constant pressure.

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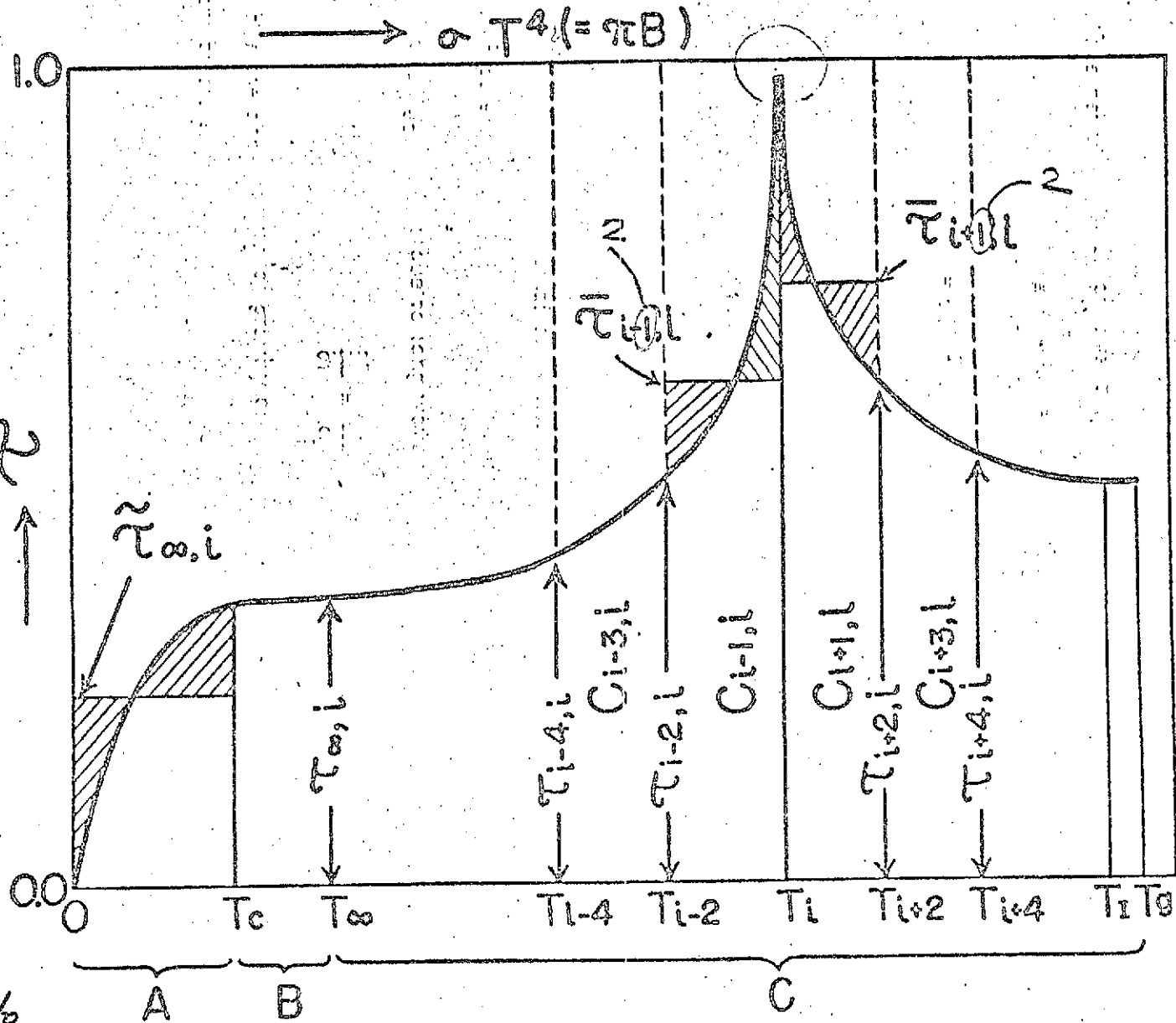


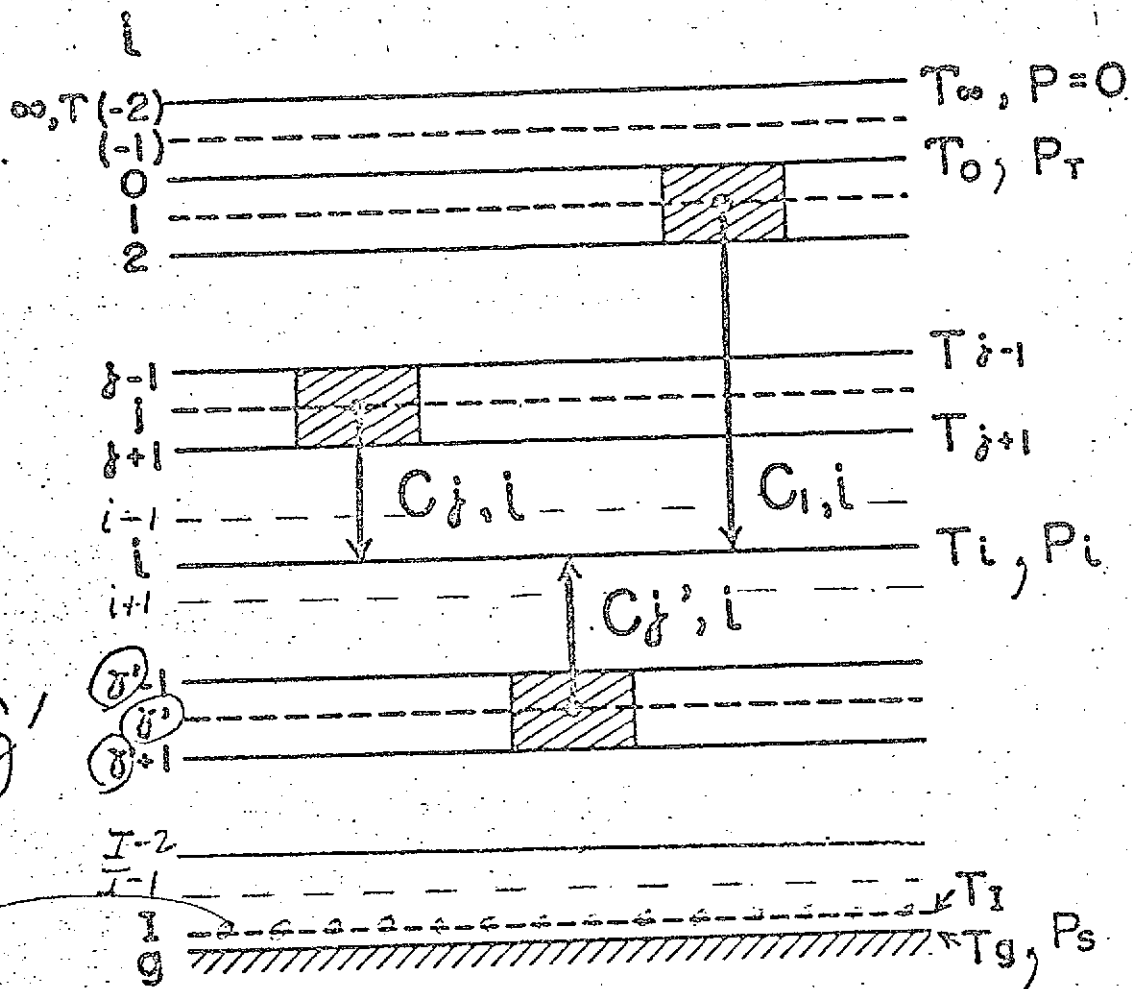
Figure 1

11-5

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Fig. 11-2. Indexing scheme for <sup>the</sup> levels of the model.

Figure 2

However,  $\tau$  exponentially decreases as  $|u^* - u_i^*|$  increases and the change of  $|u^* - u_i^*|$  with  $\pi B$  is also roughly exponential. As shown in Figure 11-1,  $\tau$  in the neighborhood of level  $i$  usually decreases more rapidly with  $\pi B$  than a linear relationship. Therefore a careful formulation is required to compute the contribution to reference level  $i$  from its adjacent layers,  $C_{i-1,i}$  and  $C_{i+1,i}$ .

To parameterize this contribution, a bulk transmission function  $\bar{\tau}$  is defined as follows:

$$C_{i-1,i} = \int_{-\pi B_{i-2}}^{\pi B_i} \tau(u^* - u_i^*, \bar{T}) d(\pi B) \equiv (\pi B_i - \pi B_{i-2}) \bar{\tau}_{i-2,i} \quad (II.16)$$

$$C_{i+1,i} = \int_{-\pi B_i}^{\pi B_{i+2}} \tau(u_i^* - u^*, \bar{T}) d(\pi B) \equiv (\pi B_{i+2} - \pi B_i) \bar{\tau}_{i+2,i} \quad (II.17)$$

Since  $\tau = 1$  for  $|u^* - u_i^*| = 0$ ,  $\bar{\tau}_{i-2,i}$  (or  $\bar{\tau}_{i+2,i}$ ) must have some value between 1 and  $\tau_{i-2,i}$  (or  $\tau_{i+2,i}$ ). Thus we define a linear interpolation factor  $m$  by the following relations:

$$\bar{\tau}_{i-2,i} = (1 + m_{i-2,i} \tau_{i-2,i}) / (1 + m_{i-2,i}) \quad (II.18)$$

$$\bar{\tau}_{i+2,i} = (1 + m_{i+2,i} \tau_{i+2,i}) / (1 + m_{i+2,i}) \quad (II.19)$$

The values of  $m$  must be determined by the physical parameters of the adjacent layer under consideration. We assume that the vertical distributions of the water vapor mixing ratio  $q$  and the temperature  $T$  can be represented by

$$q = q_i (p/p_i)^{k_i} \quad (II.20)$$

$$T = T_i + \Gamma_i (p_i - p) \quad (II.21)$$

Thus,  $m$  may be a function of pressure, mixing ratio and temperature at the reference level  $i$ , the lapse rate parameters  $k_i$  and  $\Gamma_i$  and the pressure thickness  $\Delta p$  of the layer (see Figure II-3). Thus,

$$m_i^\pm = f^\pm(p_i, q_i, T_i, k_i, \Gamma_i, \Delta p) \quad (II.22)$$

where  $m_i^+ \equiv m_{i+2,i}$  and  $m_i^- \equiv m_{i-2,i}$ .

a linear relationship... the contribution to...

Replace by  
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*model*

For the GCM atmosphere, in which the above parameters <sup>may</sup> vary over a wider range than that observed in the real atmosphere, the integrals in (II.16) and (II.17) are computed numerically by dividing the GCM layer under consideration into thin sublayers of 10 mb thickness. In this way, the values

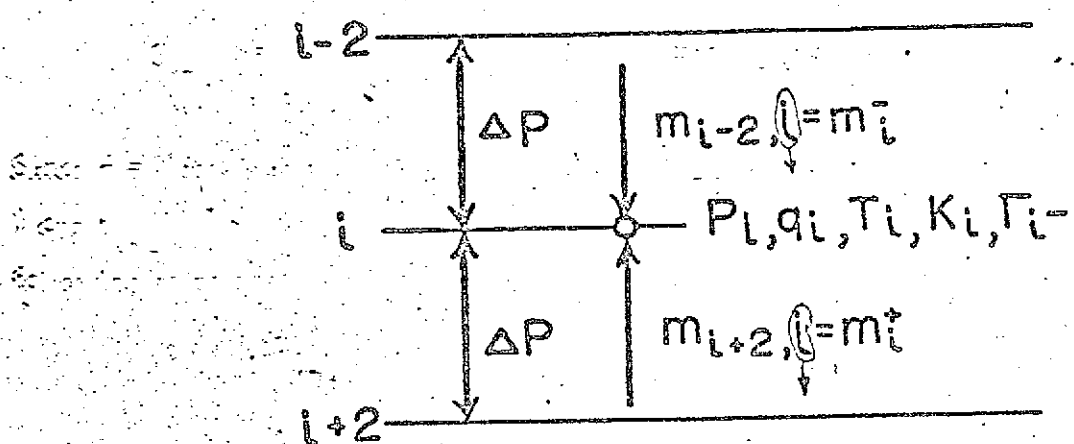


Fig. 11-3. Schematic chart for the parameterization of the interpolation factor  $m_i$ .

of  $\bar{T}_i^\pm$  <sup>here</sup> are estimated from (II.16) and (II.17). Then, from (II.18) and (II.19), the values of  $m_i^\pm$  <sup>here</sup> can be obtained as a function of  $p_i, q_i, T_i, k_i, \Gamma_i$  and  $\Delta p$ . The functional form of  $m_i^\pm$  <sup>was</sup> then ~~is~~ found by a curve fitting technique.

<sup>These</sup> Preliminary calculations have shown that the dependence of  $m_i^\pm$  on  $\bar{T}_i^\pm$  is negligibly small; it is therefore ignored. Figure 11-4 shows that  $m_i^\pm$  can be expressed approximately as a linear function of  $\Delta p$  when the values of the remaining parameters are fixed. Thus,

$$m_i^\pm = a_i^\pm + b_i^\pm \Delta p / 100 \quad (II.23)$$

where  $a_i^\pm$  and  $b_i^\pm$  may be functions of  $p_i, q_i, k_i$  and  $\Gamma_i$ . Since the dependence of  $a_i^\pm$  on  $k_i$  and  $\Gamma_i$  is weak, it is neglected and the following empirical relations are obtained:

Fig 11-4. Interpolation factor, respectively.

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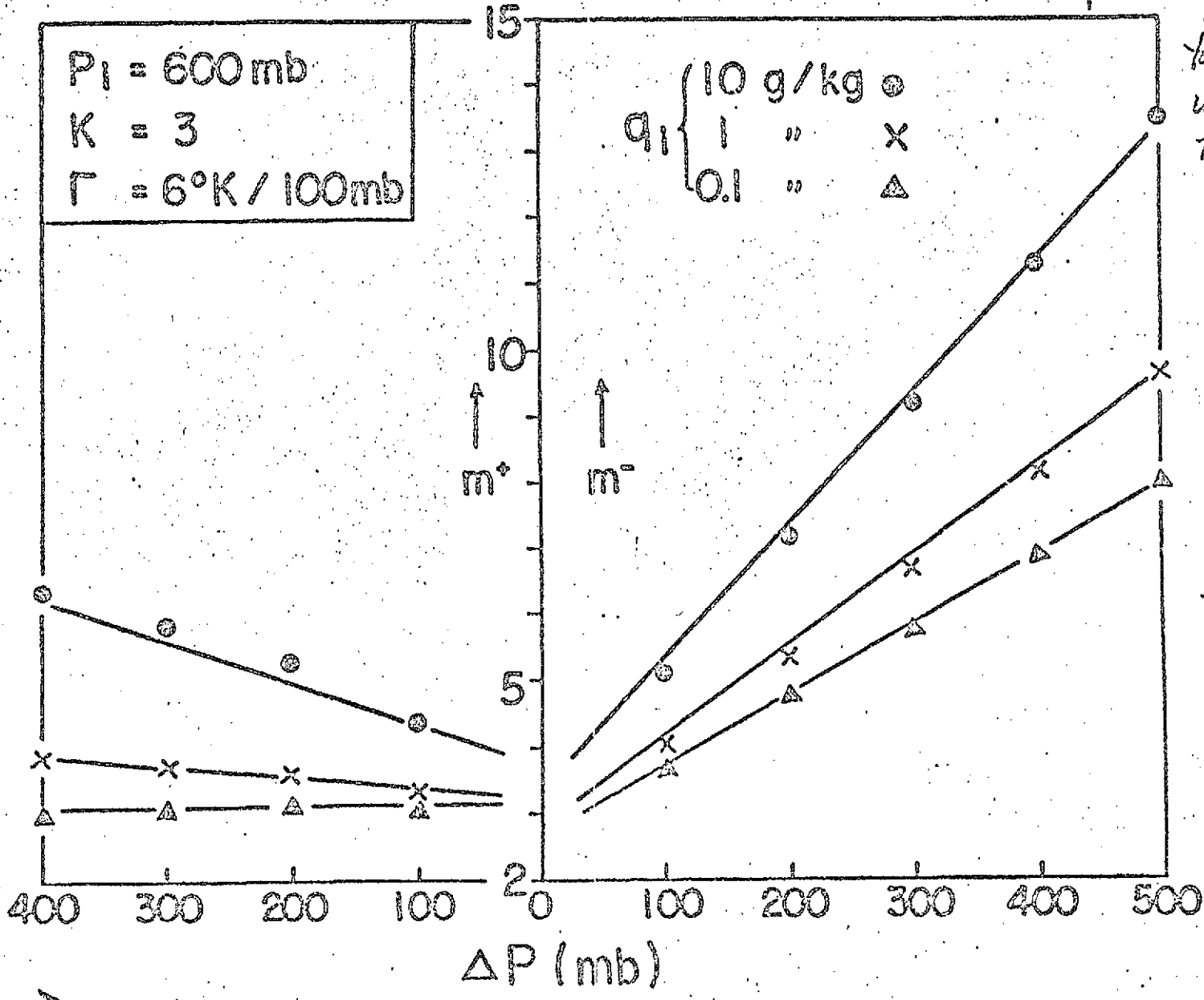


Fig 11-4

11-10

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$$a_i^+ = L_a^+(P_i) + F_a^+(Z_i), \quad (11.24)$$

$$b_i^+ = L_b^+(P_i) + F_b^+(Z_i) + \left(\frac{\partial b}{\partial k}\right)^+ \Delta k_i + \left(\frac{\partial b}{\partial \Gamma}\right)^+ \Delta \Gamma_i. \quad (11.25)$$

$$L_a^+(P_i) = -1.66 + 1.76 \log_{10} P_i,$$

$$L_b^+(P_i) = -0.197 + 0.0002 P_i,$$

$$F_a^+(Z_i) = 0.30 Z_i + 0.28 Z_i^{\uparrow 2} + 0.04 Z_i^{\uparrow 3},$$

$$F_b^+(Z_i) = 0.0812 Z_i - 0.045 Z_i^{\uparrow 2} + 0.02334 Z_i^{\uparrow 3},$$

$$\left(\frac{\partial b}{\partial \Gamma}\right)^+ = \text{Min}(-0.041 + 0.021 Z_i, -0.06) < 0,$$

$$\left(\frac{\partial b}{\partial \Gamma}\right)^+ = \text{Max}(0.01225 + 0.007 Z_i, 0.0093) > 0.$$

(11.26)

$$a_i^- = -0.09 L_a^-(P_i) + F_a^-(Z_i - 0.105 L_a^-(P_i)),$$

(11.26)

$$b_i^- = -0.09 L_b^-(P_i) + F_b^-(Z_i - 0.105 L_b^-(P_i)) + \left(\frac{\partial b}{\partial k}\right)^- \Delta k_i + \left(\frac{\partial b}{\partial \Gamma}\right)^- \Delta \Gamma_i. \quad (11.27)$$

$$L_a^-(P_i) = \text{Max}(61.86 - 22.92 \log_{10} P_i, 76.63 - 28.39 \log_{10} P_i),$$

$$L_b^-(P_i) = \text{Min}(-42.59 + 15.78 \log_{10} P_i, -60.81 + 22.53 \log_{10} P_i),$$

$$F_a^-(X) = 2.57 + 0.233 X + 0.18 X^{\uparrow 2} + 0.027 X^{\uparrow 3},$$

$$F_b^-(X) = 1.42 + 0.48 X + 0.16 X^{\uparrow 2} + 0.011 X^{\uparrow 3},$$

$$\left(\frac{\partial b}{\partial k}\right)^- = 0.08 + (0.371 - 0.102 \log_{10} P_i) (Z_i + 2.1) > 0,$$

$$\left(\frac{\partial b}{\partial \Gamma}\right)^- = \text{Min}(-0.0325 - 0.005 Z_i, -0.0275) < 0,$$

where  $Z_i = \log_{10} q_i$ ,  $Z_i = |Z_i + 2.5|$ ,  $\Delta k_i = k_i - 3$ ,  $\Delta \Gamma_i = \Gamma_i - 10$  ( $^{\circ}\text{K}/100 \text{ mb}$ ) and  $X$  is a dummy variable.

Figure II-4 shows the relation between  $m_i^{\pm}$  and  $\Delta p$  for the cases of  $q_i = 10, 1, 0.1 \text{ g/Kg}$ ,  $p_i = 600 \text{ mb}$ ,  $k_i = 3$  and  $\Gamma_i = 6^{\circ}\text{K}/100 \text{ mb}$ . The plotted points give the values of  $m_i^{\pm}$  obtained from equations (II-16) - (II-19), while the straight lines represent  $m_i^{\pm}$  determined from (II.23) - (II.27). The agreement appears to be satisfactory.

The values of  $a_i^{\pm}$  and  $b_i^{\pm}$  for various values of  $p_i$  and  $q_i$  are presented in Table II-1 for the case of  $k_i = 3$ ,  $\Gamma_i = 6^{\circ}\text{K}/100 \text{ mb}$ .  $a_i^{\pm}$  and  $b_i^{\pm}$  increase while  $b_i^{-}$  decreases as  $p_i$  increases. Their dependence on  $q_i$  is not monotonic.  $a_i^{\pm}$  gradually decrease with decreasing mixing ratio and reach minimum values near  $q_i = 0.1 \text{ g/Kg}$ .  $b_i^{\pm}$  exhibit a similar behavior with minimum values near  $q_i = 0.01 \text{ g/Kg}$ . Since  $b_i^{+}$  is considerably smaller than  $b_i^{-}$ , while the difference between  $a_i^{+}$  and  $a_i^{-}$  is usually less than 1,  $m_i^{-}$  is larger than  $m_i^{+}$  in most cases.

As discussed in Appendix C, if the permissible error in  $\frac{\partial T}{\partial t}$  and  $\frac{\partial T}{\partial z}$  is  $0.2^{\circ}\text{C}/\text{day}$ , the accuracy requirement for  $m_i^{\pm}$  is not too severe and usually more than 20% error in  $m_i^{\pm}$  can be tolerated.

If the vertical coordinate of a GCM is pressure or geopotential height,  $p_i$  and  $\Delta p$  are exactly or nearly constant in space and time. For such a GCM, only  $q_i$ ,  $k_i$  and  $\Gamma_i$  are variable parameters in the expression for  $m_i^{\pm}$ . Considering the range of these parameters in the actual atmosphere, appropriate mean values for  $k_i$  and  $\Gamma_i$ , such as their globally averaged climatological values, may be used in the empirical expressions for  $m_i^{\pm}$ . For example,  $k \approx 3$  and  $\Gamma \approx 6^{\circ}\text{K}/100 \text{ mb}$  may be adopted.

TABLE II-1. The values of  $a_i^\pm$  and  $b_i^\pm$  for various  $p_i$  and  $q_i$ . The values of  $b_i^\pm$  are shown for the case of  $k_L = 3$ ,  $\Gamma_L = 6^\circ\text{K}/100\text{ mb}$ .

-	$P_i(\text{mb})$	$q_i(\text{g/Kg})$					
		30	10	1	0.1	0.01	0.001
$a_i^+$	100	3.05	2.48	1.86	1.80	2.06	2.40
	200	3.58	3.01	2.39	2.33	2.59	2.94
	400	4.11	3.54	2.92	2.86	3.12	3.46
	600	4.42	3.85	3.23	3.17	3.43	3.77
	800	4.64	4.07	3.45	3.39	3.65	3.99
	1000	4.81	4.24	3.62	3.56	3.82	4.16
$b_i^+$	100	0.81	0.48	0.06	-0.12	-0.18	-0.18
	200	0.83	0.50	0.08	-0.10	-0.16	-0.16
	400	0.87	0.54	0.12	-0.06	-0.12	-0.12
	600	0.91	0.58	0.16	-0.02	-0.08	-0.08
	800	0.95	0.62	0.20	0.03	-0.04	-0.04
	1000	0.99	0.66	0.24	0.07	-0.00	-0.00
$a_i^-$	200	1.64	1.52	1.49	1.62	1.77	1.75
	400	2.90	2.59	2.27	2.26	2.41	2.54
	600	3.75	3.31	2.79	2.65	2.74	2.90
	800	4.36	3.83	3.15	2.92	2.96	3.11
	1000	4.87	4.27	3.47	3.14	3.13	3.28
$b_i^-$	200	4.64	3.99	2.96	2.31	2.00	1.93
	400	3.12	2.62	1.87	1.45	1.32	1.39
	600	2.43	2.00	1.38	1.06	1.02	1.15
	800	2.06	1.67	1.12	0.86	0.85	1.01
	1000	1.79	1.43	0.92	0.71	0.73	0.91

As a rough approximation, the effect of the variation of  $q_i$  may also be ignored. In that case, the hemispheric mean climatological value given by (IV.16) is recommended as an appropriate constant value of  $q$ . However, the mixing ratio at a reference level in the lower troposphere varies by more than two orders of magnitude over the global domain. Therefore, it is desirable to retain the spatial and temporal variation of  $q$  for the estimation of  $m_i^\pm$ .

When the  $\sigma$ -coordinate system is employed, as in the UCLA GCM,  $p_i$  on a  $\sigma$ -level and  $\Delta p$  between  $\sigma$ -levels may vary in space and time. For example, over high mountain areas such as the Himalayas and the Antarctic continent,  $p_i$  and  $\Delta p$  are significantly reduced, which leads to a decrease in  $m_i^\pm$  and an increase in  $\bar{T}_i^\pm$ . If  $p_i$  is assumed constant on a reference  $\sigma$ -level, the contribution to the flux of long wave radiation at the reference level from its adjacent layers is underestimated over high mountain areas.

*Change section 3 to section 4 and move accordingly*

3. The effect of clouds

When the sky is covered by a thick cloud, the top and bottom of the cloud <sup>may be</sup> treated as perfect black body radiators at their respective temperatures (the net flux in the cloud is assumed to be zero). This assumption can safely be <sup>made</sup> adopted for a dense cloud, but <sup>it</sup> must be checked for cirriform clouds which contain small amounts of water vapor and solid water (ice crystal concentration <sup>S</sup> less than  $0.1 \text{ gm}^{-3}$ ).

Fritz and Winston (1962) analyzed the radiation measurements of TIROS II and found that the cirrostratus layer did not strongly absorb <sup>in</sup> the window region of ~~the~~ long wave radiation,  $8 \mu - 12 \mu$ . A detailed discussion of the greyiness of cirriform clouds has been presented by Katayama (1966). ~~It is unlikely that~~ *Because*



do not  
 cirriform clouds completely absorb long wave radiation, <sup>we assume, for</sup> ~~for simplicity,~~  
~~assume that, under mean conditions, cirriform clouds~~ <sup>that they</sup> behave as grey body  
 radiators with greyness of 0.5. In the computation of the radiation flux, the  
 above assumption <sup>is</sup> may be introduced roughly by modifying the amount of cirriform  
 clouds by the weighting factor of 0.5.

It has been confirmed by laboratory experiments that there is a critical  
 temperature, about  $-40^{\circ}\text{C}$ , at which homogeneous nucleation transforms all  
 liquid water to ice crystals. Clodman (1957) found from aircraft observations  
 that the base of cirrus clouds is situated at the level where the air temperature  
 is about  $-40^{\circ}\text{C}$ . We therefore assume that any non-convective cloud formed in  
 a layer with temperature less than  $-40^{\circ}\text{C}$  is cirriform and that the greyness of  
 the cloud is 0.5

*change section 4 to section 3 and move accordingly*

4. Empirical transmission function equations

*Following*  
 According to Yamamoto (1952), the total transmission function of a  
 mixture of water vapor and carbon dioxide is assumed to be represented by the  
 product of their respective transmission functions, that is

$$\tau(u_{\text{H}_2\text{O}}^*, u_{\text{CO}_2}^*, T) = \tau_{\text{H}_2\text{O}}(u_{\text{H}_2\text{O}}^*, T) \tau_{\text{CO}_2}(u_{\text{CO}_2}^*, T) \quad (II.28)$$

Using the table presented by Yamamoto (1952), we deduce the  
 following empirical equations for the transmission functions of water vapor  
 and carbon dioxide.

(i) Water vapor

Two empirical equations for the transmission function of water vapor at  
 a temperature of  $260^{\circ}\text{K}$  are presented below; the first has a form similar to  
 Callendar's expression (1941) for carbon dioxide while the second is expressed

*Replace with insert 4*

*and*

$$\tau(u_{\text{H}_2\text{O}}^*, u_{\text{CO}_2}^*, T) = \tau_{\text{H}_2\text{O}}(u_{\text{H}_2\text{O}}^*, T) \tau_{\text{CO}_2}(u_{\text{CO}_2}^*, T)$$

by a quadratic polynomial

$$(A) \tau_{H_2O}(u^*, 260^\circ K) = \frac{1}{1 + 1.750 u^{*0.416}} \quad (II.29)$$

$$(B) \tau_{H_2O}(u^*, 260^\circ K) = 0.384 - 0.2785 Z + 0.0345 Z^2, \quad (u^* \geq 1),$$

$$= 0.384 - 0.259 Z - 0.028 Z^2, \quad (10^{-4} \leq u^* < 1), \quad (II.30)$$

$$= 1/(1 + 288 u^*), \quad (u^* < 10^{-4}),$$

where  $Z = \log_{10} u^*$  and  $u^* = u_{H_2O}^*$ .

Table II-2 shows the accuracy of the two expressions above. In the real atmosphere,  $u^*$  is usually less than  $6 \text{ g cm}^{-2}$ . Therefore the maximum error of expression A is about 0.025 while expression B has a negligibly small error for  $u^*$  between  $10 \text{ g cm}^{-2}$  and  $0.0001 \text{ g cm}^{-2}$ .

TABLE II-2. Accuracy of the empirical equations for the transmission function of water vapor at a temperature of  $260^\circ K$ . The errors are shown in units of  $10^{-3}$ .

$u_{H_2O}^*$ ( $\text{g cm}^{-2}$ )	$\tau_{H_2O}(u_{H_2O}^*, 260^\circ K)$				
	Obs.	A	O-A	B	O-B
10	0.140	0.180	-40	0.140	0
6	0.188	0.212	-24	0.188	0
3	0.259	0.266	-7	0.259	0
1	0.384	0.364	20	0.384	0
0.3	0.510	0.485	25	0.512	-2
0.1	0.612	0.598	14	0.615	-3
0.03	0.713	0.711	2	0.714	-1
0.01	0.790	0.795	-5	0.790	0
0.001	0.910	0.910	0	0.909	1
0.0001	0.972	0.963	9	0.972	0

Define "Obs" 17

Laboratory  
 What is Obs  
 15/1/24  
 measurements

11-3.  
 TABLE II-3: Accuracy of the empirical equation<sup>S</sup> for the transmission function of water vapor averaged over the temperature range between 220°K and 300°K. The errors are shown in units of 10<sup>-3</sup>.

$u_{H_2O}^*$ (g cm <sup>-2</sup> )	$\tau_{H_2O}^A(u_{H_2O}^*)$						
	Obs.	C	O-C	D	O-D	E	O-E
10	0.134	0.172	-28	0.134	0	0.134 ✓	0
6	0.181	0.204	-23	0.185	-4	0.181	0
3	0.250	0.268	-18	0.248	2	0.250	0
1	0.373	0.364	-9	0.373	0	0.373	0
0.3	0.499	0.489	10	0.498	1	0.501	-2
0.1	0.603	0.597	6	0.601	2	0.605 ✓	-2
0.03	0.704	0.715	-11	0.704	0	0.705	-1
0.01	0.783	0.800	-17	0.784	-1	0.782	1
0.001	0.905	0.914	-9	0.905	0	0.904	1
0.0001	0.971	0.966	5	0.969	2	0.971 ✓	0

As previously mentioned, the transmission function of a slab of water vapor is nearly constant with respect to temperature; it varies by about 0.02 from 220°K to 300°K. It may therefore be more suitable to use an averaged value of the transmission function over this temperature range rather than the value at 260°K.

We define  $\tau_{H_2O}^A$  as

$$\tau_{H_2O}^A(u^*) = \frac{1}{3} [\tau_{H_2O}^A(u^*, 300^\circ K) + \tau_{H_2O}^A(u^*, 260^\circ K) + \tau_{H_2O}^A(u^*, 220^\circ K)] \quad (II.31)$$

The observed values of  $\tau_{H_2O}^A$  are shown in the second column of Table II-3. From the observed values, three different empirical expressions are deduced as follows:

$$(C) \tau_{H_2O}^A(u^*) = \frac{1}{1+1.746 u^{*0.423}} \quad (II.32)$$

$$(D) \tau_{H_2O}^A(u^*) = F(1.681, 0.535), \quad (u^* \geq 0.724), \\ = F(1.600, 0.3815), \quad (0.00186 \leq u^* < 0.724), \\ = F(3.853, 0.522), \quad (u^* < 0.00186), \quad (II.33)$$

where  $F(a,b) = 1/(1+au^{*b})$ .

$$(E) \tau_{H_2O}^A(u^*) = 0.373 - 0.274 Z + 0.035 Z^2, \quad (u^* \geq 1), \\ = 0.373 - 0.2595 Z - 0.0275 Z^2, \quad (10^{-4} \leq u^* < 1), \\ = 1/(1+298.7 u^*), \quad (u^* < 10^{-4}), \quad (II.34)$$

Expressions (C) and (E) are similar to (A) and (B), respectively; expression (D) is an extension of (C). The accuracy of these three expressions is shown in Table II-3. Expression (C) exhibits the largest error, but its maximum error is less than 0.025 for  $u^* \leq 6 \text{ cm}^{-2}$ .

It is also necessary to evaluate  $\bar{\tau}(u^*, T)$ , the mean transmission function defined by (II.5). As suggested in section II.1, 220°K is adopted as the critical temperature  $T_c$ . The empirical expressions for  $\bar{\tau}$  are similar to those for  $\tau$ , namely

$$(A') \bar{\tau}_{H_2O}(u^*, 220^\circ K) = \frac{1}{1+3.0 u^{*0.408}}, \quad u^* \geq 0.1 \quad (II.35)$$

$$(B') \bar{\tau}_{H_2O}(u^*, 220^\circ K) = 0.254 - 0.1985 Z + 0.0205 Z^2, \quad (0.1 \leq u^* < 10), \\ = 0.216 - 0.2827 Z - 0.0258 Z^2, \quad (10^{-4} \leq u^* < 0.1), \\ = 1/(1+2.56 u^{*0.39}), \quad (u^* < 10^{-4}). \quad (II.36)$$

The observed values of  $\bar{\tau}$  are shown in the second column of Table II-3.

From the observed values,  $\bar{\tau} = 1/(1+2.56 u^{*0.39})$ ,  $(u^* < 10^{-4})$ .

$$(C') \bar{\tau}_{H_2O}(u^*, 220^\circ K) = 0.254 - 0.2224 Z + 0.0444 Z^2, \quad (1 \leq u^* < 10), \\ = 0.254 - 0.226 Z - 0.007 Z^2, \quad (0.03 \leq u^* < 1), \\ = 0.194 - 0.297 Z - 0.028 Z^2, \quad (10^{-4} \leq u^* < 0.03), \quad (II.37)$$

The accuracy of these empirical expressions is shown in Table II-4.

Expressions (A) and (A') were used for  $\tau_{H_2O}$  and  $\tilde{\tau}_{H_2O}$ , respectively, in the 2-layer moist and early-stage 3-layer UCLA general circulation models. In the current 3-layer <sup>UCLA</sup>GCM, expressions (E) and (B') have been adopted for  $\tau_{H_2O}$  and  $\tilde{\tau}_{H_2O}$ , respectively.

TABLE II-4. Accuracy of the empirical equations for  $\tilde{\tau}_{H_2O}$  at the temperature of 220°K. The errors are shown in unit of  $10^{-3}$ .

$u_{H_2O}^*$ (g cm <sup>-2</sup> )	$\tilde{\tau}_{H_2O}(u_{H_2O}^*, 220^\circ K)$						
	Obs.	A'	O-A'	C'	O-C'	B'	O-B'
10	0.076	0.115	-39	0.076	0	0.076	0
3	0.158	0.176	-18	0.158	0	0.164	-6
1	0.254	0.250	4	0.254	0	0.254	0
0.3	0.368	0.353	15	0.370	-2	0.366	2
0.1	0.473	0.460	13	0.473	0	0.473	0
0.03	0.584	0.582	2	0.582	2	0.585	-1
0.01	0.676	0.685	-9	0.676	0	0.678	-2
0.001	0.833	0.848	-15	0.833	0	0.832	1
0.0001	0.934	0.935	1	0.934	0	0.934	0
0.00001	0.972	0.973	1	0.979	-7	0.985	-13

(ii) Carbon dioxide

The transmission function of carbon dioxide at normal temperature and pressure (NTP) is shown in Table II-5. These observations can be expressed by

$$\tau_{CO_2}(u_{CO_2}^*) = 0.930 - 0.066 \log_{10} u_{CO_2}^* \quad \text{for } u_{CO_2}^* \geq 0.3 \text{ cm} - \text{NTP} \quad (II.38)$$

where  $u_{CO_2}^*$  is the effective amount of carbon dioxide at NTP within a layer of thickness  $|p_i - p_j|$  (see section IV.2, equation (IV.19)). The agreement between (II.38) and the observations is good, as shown in Table II-5. Introducing (IV.19) into (II.38) yields

$$\tau_{CO_2}(i,j) = 0.791 - 0.066 \log_{10} \left[ \frac{|p_i^2 - p_j^2|}{p_{\infty}^2} \right] \quad (II.39)$$

The temperature dependence of the transmission function of carbon dioxide is neglected in the present formulation.

11-5,  
TABLE II-5. The transmission function of carbon dioxide. Calc. shows the value obtained from equation (II.38).  
*obtained from equation (II.38)*

$u_{CO_2}^*$ (cm/NTP)		1	3	10	30	100	150
$\tau_{CO_2}$	Obs.	0.930	0.900	0.865	0.834	0.796	0.784
	Calc.	0.930	0.899	0.864	0.833	0.798	0.786

If pressure is the vertical coordinate of a GCM,  $\tau_{CO_2}(i,j)$  does not change in space or time. In the  $\sigma$ -coordinate system, however, ~~since the~~ pressure at a constant  $\sigma$ -level varies with the surface pressure,  $p_s$ ,  $\tau_{CO_2}(i,j)$  also varies with  $p_s$ . Thus, with the exception of the layer above the  $\sigma \approx 0$  level,  $\tau_{CO_2}(i,j)$  ~~should~~ <sup>must</sup> be calculated at each grid point and ~~at~~ <sup>of</sup> each time step. From an examination of the dependence of  $\tau_{CO_2}(i,j)$  on  $p_s$ , the following simplified empirical relation is adopted:

$$\tau_{CO_2}(i,j;p_s) = \beta \tau_{CO_2}(i,j;p_{\infty}) \quad (II.40)$$

where

$$\beta = 1 + 0.09(1 - p_s/p_{\infty}) \quad (II.41)$$

and  $\tau_{\text{CO}_2}(i, j; p_s)$  is  $\tau_{\text{CO}_2}(i, j)$  when the surface pressure is  $p_s$  and  $p_{\infty} = 1000$  mb. Figure II-5 shows the values of  $\tau_{\text{CO}_2}(i, j; p_{\infty})$  for various layers of the 3-layer GCM.

Finally, the total transmission function for the layer between levels  $i$  and  $j$  is given by

$$\begin{aligned} \tau_{ij} &\equiv \tau(u_{\text{H}_2\text{O}}^*(i, j), u_{\text{CO}_2}^*(i, j), T) \\ &= \beta(p_s) \tau_{\text{CO}_2}(i, j; p_{\infty}) \tau_{\text{H}_2\text{O}}(i, j) \end{aligned} \quad (\text{II.42})$$

where  $\tau_{\text{H}_2\text{O}}(i, j) = \tau_{\text{H}_2\text{O}}(|u_i^* - u_j^*|, \bar{T})$

### 5. Finite difference schemes

In this section, the finite difference scheme for the long wave radiation calculation is summarized. The indexing scheme is shown in Fig. II-2. The following assumptions are made regarding cloud layers:

- (i) a cloud layer is located about a level with an odd index
- (ii) the cloud thickness is equal to the thickness of the layer in which it is situated.
- (iii) the distribution of cloud elements is horizontally quasi-homogeneous
- (iv) there are no clouds in the layer above level 0
- (v) the cloudiness and greyness of a cloud in layer  $j$  (between levels  $j-1$  and  $j+1$ ) are  $CL_j$  and  $\alpha_j^!$ , respectively; the modified cloudiness  $CL_j^!$  is defined by  $CL_j^! = \alpha_j^! CL_j$ .

Combining (II.9) and (II.13), the net flux of long wave radiation at reference level  $i$  in a clear sky is

$$R_i = \pi B_c \bar{\tau}_{\infty i} + (\pi B_{\infty} - \pi B_c) \tau_{\infty i}$$

$$+ \sum_{j=-1}^{I-1} C_{j,i} + (\pi B_g - \pi B_I) \tau_{(I)}$$

(11.43)

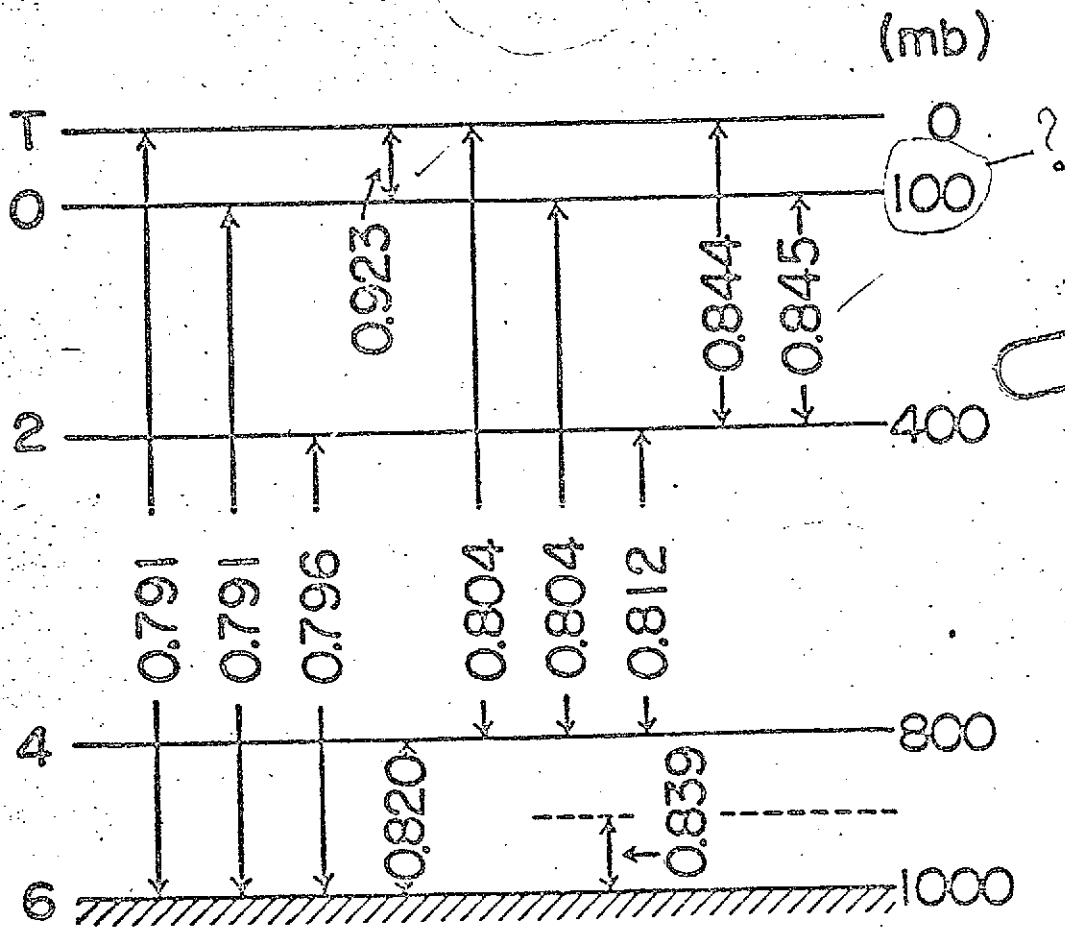


Fig. 11-5. Transmission function of carbon dioxide for various layers.

~~Figure 5~~



In a cloudy sky, (II.43) is modified as

$$\begin{aligned}
 R_i = & [\pi B_c \tilde{\tau}_{\infty i} + (\pi B_{\infty} - \pi B_c) \tau_{\infty i} + C_{-1,i} + C_{1,i}] \prod_{j=1}^{i-1} (1-CL_j^!) + C_{3,i} \prod_{j=3}^{i-1} (1-CL_j^!) + \dots \\
 & + C_{i-3,i} (1-CL_{i-3}^!) (1-CL_{i-1}^!) + C_{i-1,i} (1-CL_{i-1}^!) \\
 & + C_{i+1,i} (1-CL_{i+1}^!) + C_{i+3,i} (1-CL_{i+1}^!) (1-CL_{i+3}^!) \\
 & + \dots + C_{I-3,i} \prod_{j=i+1}^{I-3} (1-CL_j^!) \\
 & + [C_{I-1,i} + (\pi B_g - \pi B_I) \tau_{Ii}] \prod_{j=i+1}^{I-1} (1-CL_j^!) \quad (II.44)
 \end{aligned}$$

where  $\prod_j^!$  denotes multiplication over  $j$  odd.  $\tau_{ij}$  is expressed by (II.42).

Substituting (II.14) and (II.16) - (II.19) into the above relation yields

$$\begin{aligned}
 R_i = & \left( \sigma T_c^4 \tilde{\tau}_{\infty i} + \Delta B_{c,\infty} \tau_{\infty i} + \Delta B_{\infty,o} \frac{\tau_{\infty i} + \tau_{oi}}{2} \right. \\
 & + \Delta B_{o,2} \left. \frac{\tau_{oi} + \tau_{2i}}{2} \right) \prod_{j=1}^{i-1} (1-CL_j^!) \\
 & + \Delta B_{2,4} \frac{\tau_{2i} + \tau_{4i}}{2} \prod_{j=3}^{i-1} (1-CL_j^!) + \dots \\
 & + \Delta B_{i-4,i-2} \frac{\tau_{i-4,i} + \tau_{i-2,i}}{2} (1-CL_{i-3}^!) (1-CL_{i-1}^!) \\
 & + \Delta B_{i-2,i} \frac{1+m_i^- \tau_{i-2,i}}{1+m_i^-} (1-CL_{i-1}^!) \\
 & + \Delta B_{i,i+2} \frac{1+m_i^+ \tau_{i+2,i}}{1+m_i^+} (1-CL_{i+1}^!) \\
 & + \Delta B_{i+2,i+4} \frac{\tau_{i+2,i} + \tau_{i+4,i}}{2} (1-CL_{i+3}^!) (1-CL_{i+1}^!) \\
 & + \dots
 \end{aligned}$$

$$\begin{aligned}
& + \Delta B_{I^{-4}, I^{-2}} \frac{\tau_{I^{-4}, i} + \tau_{I^{-2}, i}}{2} \frac{I^{-3}}{i=i+1} (1-CL)_i \\
& + \left( \Delta B_{I^{-2}, I} \frac{\tau_{I^{-2}, i} + \tau_{I, i}}{2} + \Delta B_{I, g} \tau_{I, i} \right) \frac{I^{-1}}{i=i+1} (1-CL)_i
\end{aligned} \quad (11.45)$$

where  $\Delta B_{l, m} = \sigma T_m^4 - \sigma T_l^4$ .

As an example, the finite difference scheme for the current UCLA 3-layer GCM is shown in Appendix A.

### III. SOLAR RADIATION

There are three principal factors that contribute to the depletion of solar radiation in the cloudless troposphere:

- (i) absorption by water vapor
- (ii) Rayleigh scattering by air molecules
- (iii) absorption and Mie scattering by aerosol particles

According to Katayama's preliminary result (1966), absorption by aerosol particles is comparable to absorption by a cloud layer in the normal mean cloud state and is of the same order as the absorption by water vapor in high latitudes at large sun zenith angles. Thus the depletion by aerosol absorption should be included in the atmospheric radiation process. At present, however, <sup>we do not know very much</sup> ~~very little is known~~ about the optical properties of aerosol particles <sup>and therefore, we</sup> ~~nor~~ <sup>are neglecting the absorption and Mie scattering</sup> ~~about the spatial and temporal variation of the aerosol distribution,~~ ~~sequently, absorption of solar radiation by aerosol particles is neglected in~~ ~~the report.~~

#### 1. Basic quantities

The effective absorption bands of water vapor for the solar spectrum are in the wavelength range  $\lambda > 0.9 \mu$ . There are two absorption bands in the region  $\lambda < 0.9 \mu$ , namely  $\lambda = 0.75 \mu$  and  $0.85 \mu$ , however their contribution to the total water vapor absorption is negligibly small.

On the other hand, since the amount of Rayleigh scattering is inversely proportional to the fourth power of the wavelength  $\lambda$ , the scattering by air molecules rapidly decreases with increasing wavelength. With an error of a few per cent, scattering in the wavelength range  $\lambda > 0.9 \mu$  can be neglected.

Based upon these considerations, Joseph <sup>(1966, 1970)</sup> ~~(1956)~~ divided the solar radiation into two <sup>components:</sup> ~~parts~~ the scattered and the absorbed parts. This idea

was also presented by Feigel'son (1964), using a critical wavelength of  $0.75 \mu$  instead of  $0.9 \mu$ . Thus, for  $\lambda < 0.9 \mu$ , solar radiation is assumed to be Rayleigh scattered but not absorbed by water vapor, while for  $\lambda > 0.9 \mu$ , the converse is assumed. The solar radiation incident at the top of the atmosphere is then separated as follows:

$$\text{The scattered part: } S_o^s = 0.651 S_o \cos \zeta \quad (\text{III.1})$$

$$\text{The absorbed part: } S_o^a = 0.349 S_o \cos \zeta \quad (\text{III.2})$$

where  $S_o$  is the solar constant and  $\zeta$  is the zenith angle of the sun.

(i) Absorptivity of water vapor

Mügge and Möller (1932) found the following empirical relation for the absorption by water vapor of the mean solar constant:

$$a = 0.172 (u^* \sec \zeta)^{0.303} \text{ ly min}^{-1}$$

where  $u^*$  is the effective water vapor amount.

Yamamoto and Onishi (1948) and MacDonald (1960) also constructed absorption curves. Their values are smaller than those of Mügge and Möller by about 15 per cent. Manabe and Möller (1961) recalculated the absorption and obtained a 10 per cent larger value than Mügge and Möller.

Since absorption by aerosol particles is neglected in the present treatment, Manabe and Möller's larger value of the absorption by water vapor,

$$a' = 0.189 (u^* \sec \zeta)^{0.303} \text{ ly min}^{-1}, \quad (\text{III.3})$$

is employed. Since this absorption refers to the mean solar constant  $\bar{S}_o$ , the absorptivity for the absorbed part is

$$\frac{a'}{0.349 \bar{S}_o} = 0.271 (u^* \sec \zeta)^{0.303}, \quad (\text{III.4})$$

where  $\bar{S}_o = 2 \text{ ly min}^{-1}$ .

*absorptivity*

Defining the function  $A(x)$  by

$$A(x) = 0.271 (x)^{0.303} \quad (III.5)$$

the absorption by water vapor of the direct solar radiation is

$$\begin{aligned} \alpha' &= S_0^a A(u^* \sec \zeta) \\ &= 0.349 S_0 \cos \zeta A(u^* \sec \zeta) \end{aligned} \quad (III.6)$$

where  $S_0 = \bar{S}_0 \left(\frac{\bar{r}_E}{r_E}\right)^2$  *Constant* (III.7)

and  $r_E$  is the sun's distance from the earth and  $\bar{r}_E$  is the mean value of  $r_E$ , that is one astronomical unit. The zenith angle  $\zeta$  is given by

$$\cos \zeta = \sin \phi \sin \delta + \cos \phi \cos \delta \cos h \quad (III.8)$$

where  $\phi$  is the latitude,  $\delta$  is the solar declination and  $h$  is the hour angle of the sun.

In a numerical simulation of the seasonal variation of the general circulation, the seasonal variation of  $\delta$  and  $r_E$  must be ~~predicted or tabulated~~ *known*. As shown in Appendix D,  $\delta$  and  $r_E$  can be estimated by a perturbation solution of Kepler's second law.

(ii) The albedo of the *cloudless* atmosphere  
 The albedo of the *cloudless atmosphere* clear sky due to Rayleigh scattering, was estimated by Coulson (1959) and is presented in Table II-3 of the paper by Joseph (1966). Joseph *fitted* to express the albedo by a least-squares polynomial in  $\cos \zeta$ , of the form

$$\alpha_0(\zeta) = \sum_i C_i \cos^i \zeta$$

In the following, the empirical expression

$$\alpha_0(\zeta, p_s) = 0.085 - 0.245 \log_{10} \left( \frac{p_s}{p_{\infty}} \cos \zeta \right) \quad (III.9)$$

will be employed, where  $p_{\infty}$  is 1000 mb and  $p_s$  is the surface pressure.

Table III-1 gives a comparison of (III.9) for  $p_s = p_{\infty}$ , with the values of Coulson. The error of (III.9) is less than 10 per cent.

cloudless

TABLE III-1. The albedo of the clear atmosphere.

→ *Coulson*

$\zeta$		$0^\circ$	$36.9^\circ$	$56.4^\circ$	$78.5^\circ$	$84.3^\circ$	$88.8^\circ$
$\alpha_0$	Coulson	.085	.097	.158	.234	.333	.519
	(III.9)	.085	.108	.148	.257	.331	.470

(iii) The albedo and absorptivity of clouds

The albedo and absorptivity of a cloud are functions of the cloud thickness, cloud height, liquid and/or solid water content, water vapor content and zenith angle of the sun. Since observations are sparse and theoretical studies are insufficient, it is not possible to introduce the above effects explicitly into the model. Therefore, following Rogers (1967), the albedo and absorptivity of clouds are prescribed as shown in Table III-2 (here subscript  $i$  denotes cloud type). The definitions of  $u_{ct}^*$  and  $u_{ci}^*$  will be given later.

TABLE III-2. The albedo and absorptivity of clouds. The units of  $u_{ct}^*$  and  $u_{ci}^*$  are  $\text{g cm}^{-2}$ .

Cloud Type $i$	Scattered part	Absorbed part			
	Albedo ( $R_i$ )	Albedo ( $R_i$ )	Absorptivity ( $A_i$ )	$u_{ct}^*$	$u_{ci}^*$
High cloud	0.21	0.19	0.04	0.01 ~0.0002	0.04 ~ 0.01 (0.03)
Middle cloud	0.54	0.46	0.20	0.1 ~ 0.01	3.2 ~ 2.4 (3.0)
Low cloud	0.66	0.50	0.30	1 ~ 0.1	14 ~ 11 (12)
Cb	0.76	0.60	0.30	0.1 ~ 0.01	22 ~ 20

(0.01)  
unit  
page

2. Interaction between cloud layers and solar radiation

For the scattered part <sup>of the solar radiation,</sup> ~~visible radiation,~~ molecular scattering in clouds is negligible compared to scattering by water droplets. Absorption by water droplets and water vapor in clouds can be neglected.

For the absorbed part <sup>of the solar radiation,</sup> ~~(near-infrared radiation),~~ scattering by water droplets and absorption by water vapor and droplets in clouds must both be considered. Thus the behavior of the absorbed part in a cloud layer is very complicated. As yet, an analytical solution suitable for the present purpose has not been obtained. Therefore, the absorption and scattering of the absorbed part in a cloud layer are treated separately under the assumptions presented in subsection (iii).

(i) The albedo of a cloudy atmosphere <sup>for the</sup> scattered part <sup>of the solar radiation</sup>

Consider a single cloud layer with albedo  $R'$  for the scattered part. Since absorption by the cloud is assumed negligible for the scattered part, the transmissivity of the cloud is  $(1-R')$ . The transmissivity of a cloudless atmospheric column due to Rayleigh scattering is  $(1-\alpha_0)$ . Thus the transmissivity for the scattered part of the atmosphere with a single cloud layer can be roughly estimated by  $(1-R')(1-\alpha_0)$ . Then the albedo for the scattered part of the atmosphere with a single cloud layer is

$$\begin{aligned} \alpha_c &= 1 - (1-R')(1-\alpha_0), \\ &= R' + \alpha_0 - R'\alpha_0. \end{aligned} \tag{III.10}$$

For a partly cloudy sky with cloudiness  $CL$ ,  $\alpha_c$  is modified as follows:

$$\begin{aligned} \alpha_c &= (1-CL)\alpha_0 + CL \cdot (R' + \alpha_0 - R'\alpha_0), \\ &= \alpha_0 + CR'(1-\alpha_0), \end{aligned} \tag{III.11}$$

where  $CR' = CL \cdot R'$ . (III.11) is identical to (III.10) with  $R'$  replaced by  $CR'$ .

(ii) The albedo of multi-layer clouds ~~(containing water)~~

(a) 2 cloud layers

Let the albedos of the two cloud layers be  $R_1$  and  $R_2$ . Multiple reflection between the two cloud layers must be considered. As shown schematically in Fig. III-1, the total transmissivity  $T_{12}$  is the sum of

$T_{12}^I, T_{12}^{II}, T_{12}^{III}, \dots$ , where

$$T_{12}^I = (1-R_1)(1-R_2),$$

$$T_{12}^{II} = (1-R_1)(1-R_2)R_1R_2,$$

$$T_{12}^{III} = (1-R_1)(1-R_2)R_1^2R_2^2.$$

⊙ ⊙ ⊙ ⊙ ⊙ ⊙

^  
^  
^

do not add  $R$  on  $R$

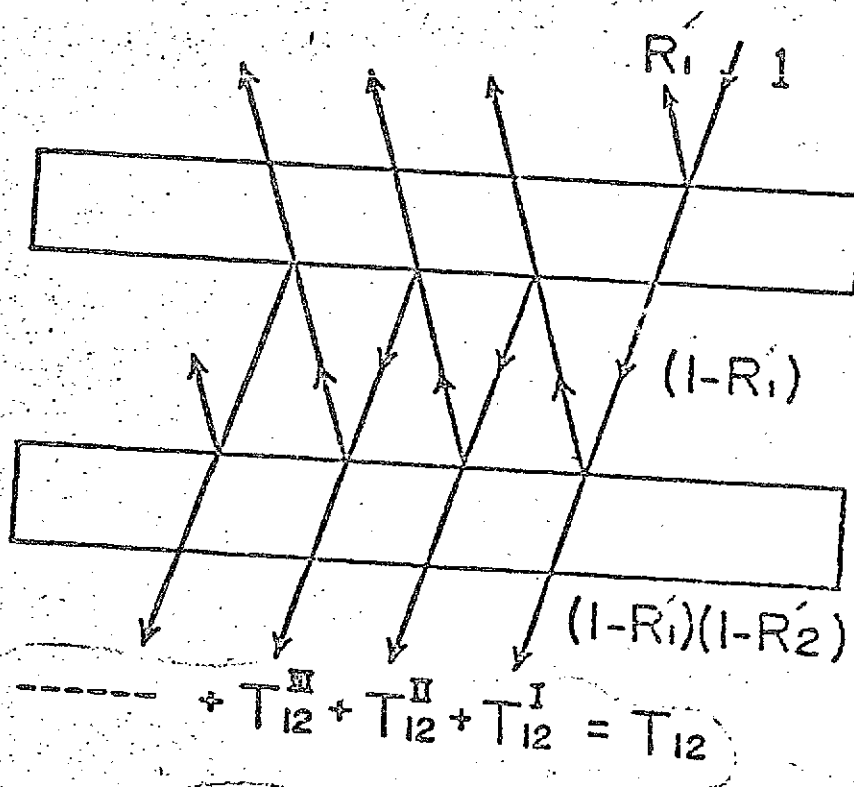


Figure 6

Fig. III-1. Schematic chart for the total albedo of two cloud layers.



Then  $T_{12} = T_{12}^I + T_{12}^{II} + T_{12}^{III} + \dots$   
 $= (1-R_1')(1-R_2')[1 + R_1'R_2 + (R_1'R_2)^2 + \dots]$   
 $= (1-R_1')(1-R_2')/(1-R_1'R_2)$  (III.12)

and  $R'_{12} = 1 - T_{12} = 1 - \frac{(1-R_1')(1-R_2')}{1-R_1'R_2}$  (III.13)

(b) 3 cloud layers

Let the albedos of the three cloud layers be  $R_1'$ ,  $R_2'$  and  $R_3'$ . The total albedo of the 3 cloud layers can be found by considering two cloud layers with albedos  $R'_{12}$  and  $R_3'$ . Then from (III.13),

$R'_{123} = 1 - T_{123}$   
 $= 1 - \frac{(1-R'_{12})(1-R_3')}{1-R'_{12}R_3'}$   
 $= 1 - \frac{(1-R_1')(1-R_2')(1-R_3')}{1-(R_1'R_2+R_2'R_3+R_3'R_1) + 2R_1'R_2R_3}$  (III.14)

(c) Multiple cloud layers

Let the albedos of the n cloud layers be  $R_1', R_2', \dots, R_n'$ . Repeating the procedure by which (III.14) was obtained, the following general expression for the total albedo of the n cloud layers is obtained:

$R'_{12\dots n} = 1 - E(R_1', \dots, R_n')/D(R_1', \dots, R_n')$  (III.15)

where  $E(R_1', \dots, R_n') = \prod_{i=1}^n (1-R_i')$

$D(R_1', \dots, R_n') = 1 - \sum_{i < j} R_i'R_j' + 2 \sum_{i < j < k} R_i'R_j'R_k' - 3 \sum_{i < j < k < l} R_i'R_j'R_k'R_l' + \dots + (-1)^{n-1} (n-1)R_1'R_2' \dots R_n'$

do not add primes on R

and, for example,  $\sum_{1 \leq i < j \leq n} R_i R_j$  means the summation of all  $R_i R_j$  for which  $i < j$

and  $i < j$ ; the number of terms is equal to  $n \cdot C_2 \rightarrow n C_2$ .

(iii)

(ii) The equivalent cloud water vapor amount and assumptions for the absorbed part of the solar radiation

The following assumptions are made for the absorbed part of the solar radiation (subscript  $i$  denotes cloud type):

- (a) The albedo  $R_i$  and absorptivity  $A_i$  are prescribed as shown in Table III-2.
- (b) The albedo of a cloud for the absorbed part results from back scattering by cloud water droplets. This reflected insolation may, before it exits the cloud top, be subject to considerable absorption; however this absorption is neglected.
- (c) The absorption of the reflected radiation after it leaves the cloud is neglected.
- (d) Solar radiation in a cloud and radiation transmitted by a cloud are diffuse radiation. The effective optical thickness for diffuse radiation is assumed to be 1.66 times the length of a vertical column, that is  $\sec \zeta = 1.66$  ( $\zeta = 53^\circ$ ).
- (e) The cloud layer is regarded as a fictitious water vapor layer which absorbs the same amount of insolation as that absorbed by the cloud. The water vapor amount of such a fictitious layer is denoted by  $u_{ci}^*$ , the equivalent cloud water vapor amount.

Let the effective water vapor amount between the top of the atmosphere and the cloud top be  $u_{ct}^*$ . Since the absorbed part of the solar radiation incident on the cloud top is  $S_0^a [1 - A(u_{ct}^*, \sec \zeta)]$ , the total absorption by the cloud is  $A_i S_0^a [1 - A(u_{ct}^*, \sec \zeta)]$ . By assumptions (b) and (e), this total absorption is

by  
 equal to the absorption of a fictitious layer of water vapor having an equivalent cloud water vapor amount  $u_{ci}^*$ . By assumption (d), the following relation ~~is~~ ~~section (17)~~ is obtained:

$$A_i S_o^a [1 - A(u_{ct}^* \sec \zeta)] = (1 - R_i) S_o^a [A(u_{ct}^* \sec \zeta + 1.66 u_{ci}^*) - A(u_{ct}^* \sec \zeta)] \quad (III.16)$$

By (III.5), the above relation yields the solution for  $u_{ci}^*$ :

$$u_{ci}^* = \left[ \left( \frac{A_i [1 - A(x)] + (1 - R_i) A(x)}{0.271 (1 - R_i)} \right)^{3.3} - x \right] / 1.66 \quad (III.17)$$

where  $x = u_{ct}^* \sec \zeta$ .

Fig. III.2 shows the relation between  $u_{ci}^*$  and  $x$  for the four combinations of  $R_i$  and  $A_i$  in Table III-2. The values of  $u_{ci}^*$  shown in Table III-2 correspond to the possible range of  $u_{ct}^*$  with  $\sec \zeta = 1$ . The values of  $u_{ci}^*$  within the parentheses in Table III-2 represent the mean values adopted in the current model.

*delete*

The above procedure for the estimation of  $u_{ci}^*$  is somewhat sophisticated. The values of  $u_{ci}^*$  in Table III-2 show only the normal conditions for the four cloud types. If the value of  $u_{ct}^*$  is fixed for each cloud type, the absorptivity  $A_i$  of a cloud type will change with  $u_{ct}^*$ ; an increase of  $u_{ct}^*$  corresponds to a decrease of  $A_i$ .

In the current model, a cloud with a temperature less than  $-40^\circ\text{C}$  is assumed to be a cirrus type cloud. The normal value of  $u_{ci}^*$  for cirrus is very small, say  $0.03 \text{ g cm}^{-2}$ . Thus, when a middle or low cloud is assumed to be cirrus,  $u_{ci}^*$  may be smaller than the water vapor amount of the environmental layer. To avoid such an unlikely condition, the fixed value of  $u_{ci}^*$  for cirrus

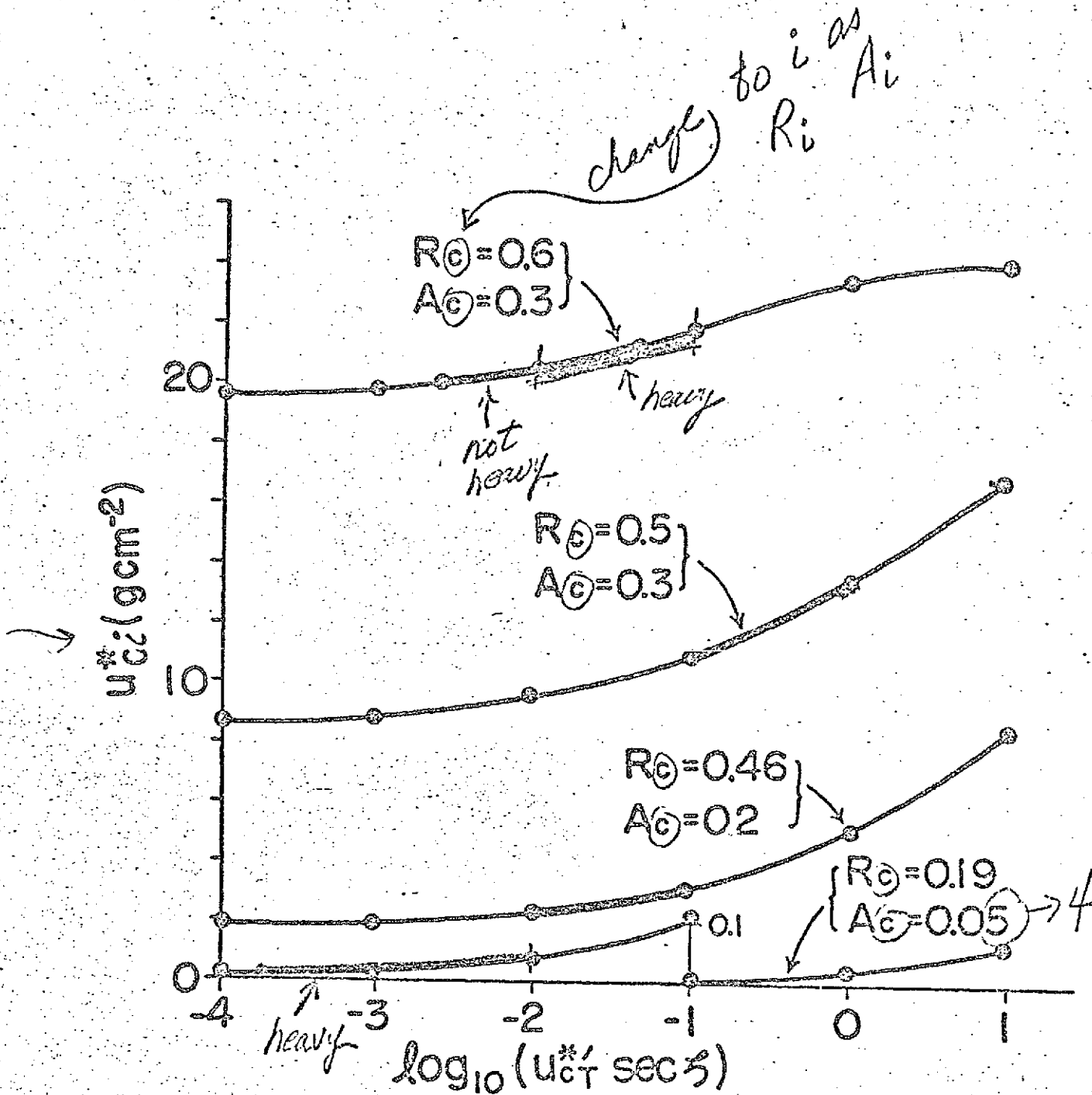


Fig. III-2. The relation . . .  
 ~ figure 7 ~ atmosphere.  
 (see p. iv)

is modified by adding to it the water vapor amount of the clear area in the corresponding layer.

### 3. The finite difference scheme for a clear sky

The indexing scheme is shown in Fig. II-2. In the following, superscript  $\circ$  denotes conditions for a clear sky.

The downward flux at level  $i$  of the absorbed part of the solar radiation is

$$S_{ai}^{\circ} = S_{\circ}^{\circ} [1 - A((u_T^* - u_i^*) \sec \zeta)] , \quad (III.18)$$

where index  $T$  denotes the top of the atmosphere. The absorption of solar radiation in the layer between  $j-1$  and  $j+1$ ,  $AS_j^{\circ}$ , is therefore

$$AS_j^{\circ} = S_{aj-1}^{\circ} - S_{aj+1}^{\circ} , \quad (III.19)$$

The solar radiation reaching the earth's surface is:

$$\text{The absorbed part: } S_{aI}^{\circ} = S_{\circ}^{\circ} [1 - A(u_T^* \sec \zeta)] , \quad (III.20)$$

$$\text{The scattered part: } S_{sI}^{\circ} = S_{\circ}^{\circ} (1 - \alpha_{\circ}) / (1 - \alpha_{\circ} \alpha_s) , \quad (III.21)$$

where  $\alpha_s$  is the surface albedo and the denominator for the scattered part represents the correction factor due to multiple reflection between the atmosphere and the earth's surface. The total solar radiation absorbed by the earth's surface is then

$$S_I^{\circ} = (1 - \alpha_s) [S_{aI}^{\circ} + S_{sI}^{\circ}] , \quad (III.22)$$

4. The finite difference scheme for a cloudy sky

(i) One cloud layer

When the sky is covered by only a single cloud layer, the complex behavior of the solar radiation can be calculated by use of the concept of the equivalent cloud water vapor amount. In the following, superscript c denotes conditions for a cloudy sky.

From the assumptions discussed in sections 2(i) and (iii), the downward flux of the absorbed part of the solar radiation is (see Fig. III-3):

(a) at a level above the cloud

$$S_{QU}^c = S_0^a [1 - A((u_r^* - u_u^*) \sec \xi)] \quad (III.23)$$

(b) at a level within the cloud

$$S_{QM}^c = S_0^a (1 - R_c) \left[ 1 - A \left( X + 1.66 \frac{\Delta p_H}{\Delta p_c} u_c^* \right) \right] \quad (III.24)$$

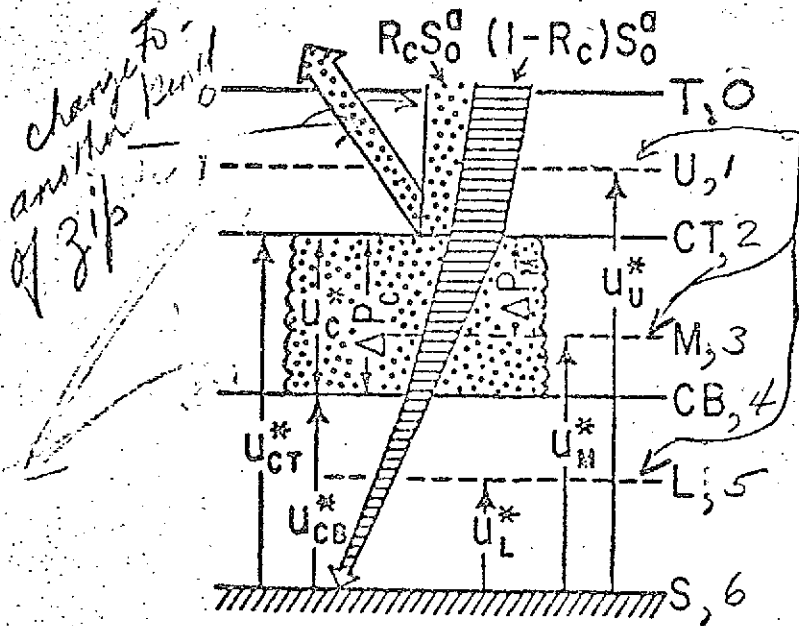


Fig. III-3. Schematic . . . . . layer.

Figure 8

*cloud,  $\Delta p_M$  is that of the*

(c) at a level beneath the cloud

$$S_{aL}^c = S_o^a (1 - R_c) [1 - A(X + 1.66(u_c^* + u_{cB}^* - u_L^*))], \quad (III.25)$$

(d) at the cloud top

$$S_{act}^c = S_o^a [1 - A(X)], \quad (III.26)$$

where  $X \equiv (u_T^* - u_{CT}^*) \sec \zeta$ . Subscripts  $U_{\Lambda}^M$  and  $L$  denote the levels at the top of, above, within, and beneath the cloud, respectively.  $\Delta p_c$  is the pressure thickness of the layer between CT and M.  $R_c$  and  $u_c^*$  are used in lieu of  $R_i$  and  $u_{ci}^*$ , respectively.

The absorption of solar radiation in the layer between  $j-1$  and  $j+1$ ,

$AS_j^c$  is then:

(a) if both levels  $j-1$  and  $j+1$  are above or beneath the cloud top,

$$AS_j^c = S_{aj-1}^c - S_{aj+1}^c, \quad (III.27)$$

(b) if level  $j-1$  is above, and level  $j+1$  beneath, the cloud top,

$$AS_j^c = S_{aj-1}^c - (S_{aj+1}^c + R_c S_{act}^c), \quad (III.28)$$

The last term on the right hand side represents the radiation of the absorbed part reflected at the cloud top.

The solar radiation reaching the earth's surface is divided into the absorbed part  $S_{aI}^c$  and the scattered part  $S_{sI}^c$ . The absorbed part is given by (III.25) with  $u_c^* = 0$ . The scattered part is given by (III.21) with  $\alpha_o$  replaced by  $\alpha_c$ , where by (III.10),

$$\alpha_c = 1 - (1 - R_c)(1 - \alpha_o),$$

The total solar radiation absorbed by the earth's surface is then

$$S_I^c = (1 - \alpha_s) [S_{aI}^c + S_{sI}^c], \quad (III.29)$$

where

$$S_{aI}^c = S_o^a (1-R_c) [1-A(X+1.66(u_c^* + u_{cb}^*))] / (1-R_c \alpha_s) \quad (III.30)$$

$$S_{sI}^c = S_o^s (1-\alpha_c) / (1-\alpha_c \alpha_s) \quad (III.31)$$

The denominator of (III.30) is the correction factor due to the multiple reflection between the cloud and the earth's surface.

For a partly cloudy condition with cloudiness CL, the absorption in a reference layer  $AS_i$  and at the earth's surface  $S_I$  are approximated by

$$AS_i = (1-CL)AS_i^o + CL \cdot AS_i^c \quad (III.32)$$

$$S_I = (1-CL)S_I^o + CL \cdot S_I^c \quad (III.33)$$

The above single cloud layer model <sup>was used</sup> has been employed in the UCLA moist 2-layer general circulation model. <sup>The</sup> detailed description of the programming <sup>was</sup> presented by Gates, et al (1971).

## (ii) Multiple cloud layers

The assumptions presented in section II.5 regarding the distribution of cloud layers are again made. Whereas the procedure developed in the preceding subsection (i) applied to quasi-vertical clouds such as cumulonimbus, the following treatment applies to multiple layers of quasi-horizontal clouds.

The indexing scheme is shown in Fig. II-2. Let  $i$  and  $j$  be even and odd integers, respectively. Layer  $j$  means the layer between levels  $j-1$  and  $j+1$ .

Define a cloud state index,  $\ell_j$ , which can have only the values 0 and 1. If  $\ell_j = 1$  (or 0), the incident solar radiation passes through a cloudy area (or a clear area) in layer  $j$ . In an  $n$ -layer model, the state index is an  $n$ -dimensional vector,



where  $N = 2n - 1$

$(l_1, \dots, l_i, \dots, l_N)$  If the cloud layers are scattered, there are  $2^n$  different possible cloud states. Fig. III-4 shows the four possible cloud states  $\mathcal{C}$  for the lower layer of a 2-layer model.

that may contain clouds

Let the optical path length of the layer between the top of the atmosphere, level T, and a reference level i be denoted by  $D_{Ti}$ . Since there are  $i/2$  layers between levels T and i, there are  $2^{i/2}$  different possible cloud states for the solar radiation transferred from the top of the atmosphere to level i. Each cloud state corresponds to a different value of  $D_{Ti}$ . Let  $D_{Ti}(l_1, l_2, \dots, l_{i-1})$  denote  $D_{Ti}$  for the cloud state  $(l_1, l_2, \dots, l_{i-1})$ .

The optical path length in the layer between levels i-2 and i,  $\Delta D_{i-2,i}$ , may have three different expressions corresponding to the different cloud states above level i as follows:

- (a) when solar radiation passes through a cloud area in the i-1 layer ( $l_{i-1} = 1$ ),

$$\Delta D_{i-2,i} = 1.66 u_{c_{i-1}}^* \quad (III.34)$$

- (b) when solar radiation does not pass through any cloud area in the layers above level i ( $l_j = 0, 1 \leq j \leq i-1$ ),

$$\Delta D_{i-2,i} = (u_{i-2}^* - u_i^*) \sec \zeta \quad (III.35)$$

- (c) when solar radiation passes through a cloud area in any layer above level i-2 and passes through a clear area in layer i-1 (one or more of the  $l_j$  for  $1 \leq j \leq i-3$  has a value of 1 and  $l_{i-1} = 0$ ),

$$\Delta D_{i-2,i} = 1.66 (u_{i-2}^* - u_i^*) \quad (III.36)$$

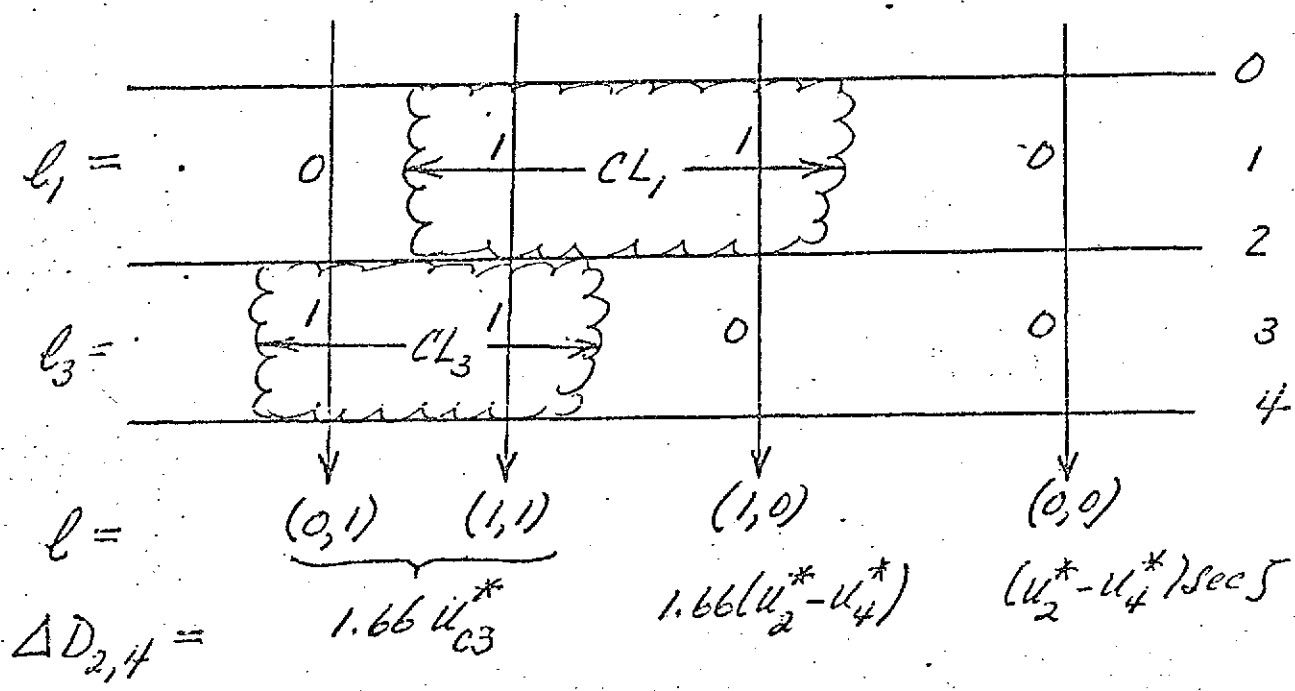


Fig. III-4. Cloud state indices  $l_1$  and  $l_3$ , the four possible cloud states  $l$  for the lower layer and ~~the~~ corresponding ~~four three~~ three possible optical path lengths ~~for the~~  $\Delta D_{2,4}$  for the lower layer of a 2-layer model.

Defining

$$M_i = \begin{cases} 1 & \text{for } i < 4 \\ \prod_{j=1}^{i-3} (1 - l_j) & \text{for } i \geq 4 \end{cases} \quad (III.37)$$

the cloud state for the three cases above can be classified as

- (a)  $l_{i-1} = 1$
- (b)  $l_{i-1} = 0, M_i = 1$
- (c)  $l_{i-1} = 0, M_i = 0$

The general expression for  $\Delta D_{i-2,i}$  is then

$$\Delta D_{i-2,i} = 1.66 u_{c,i-1}^* l_{i-1} + (u_{i-2}^* - u_i^*) [M_i \sec \zeta + 1.66 (1 - M_i)] (1 - l_{i-1}) \quad (III.38)$$

$D_{Ti}$  for a cloud state  $(l_1, \dots, l_{i-1})$  is

$$D_{Ti}(l_1, \dots, l_{i-3}, l_{i-1}) = D_{T,i-2}(l_1, \dots, l_{i-3}) + \Delta D_{i-2,i} \quad (III.39)$$

For example,  $D_{Ti}$  at levels 0, 2 and 4 *are of a 2-layer model are (see Fig. III-4)*

$$\begin{aligned} D_{T0} &= (u_T^* - u_0^*) \sec \zeta, \\ D_{T2}(0) &= D_{T0} + (u_0^* - u_2^*) \sec \zeta, \\ D_{T2}(1) &= D_{T0} + 1.66 u_{c1}^* \end{aligned} \quad (III.40)$$

$$\begin{aligned} D_{T4}(0,0) &= D_{T2}(0) + (u_2^* - u_4^*) \sec \zeta, \\ D_{T4}(0,1) &= D_{T2}(0) + 1.66 u_{c3}^*, \\ D_{T4}(1,0) &= D_{T2}(1) + 1.66 (u_2^* - u_4^*), \\ D_{T4}(1,1) &= D_{T2}(1) + 1.66 u_{c3}^* \end{aligned} \quad (III.41)$$

Let the fractional area occupied by a cloud state  $(l_1, \dots, l_n)$  be denoted by  $W(l_1, \dots, l_n)$ . Assuming that all cloud layers are horizontally quasi-homogeneous,

$$W(l_1, \dots, l_n) = \prod_{i=1}^n w(l_i) \quad (III.42)$$

where

$$w(l_i) = (1-l_i)(1-CL_i) + l_i CL_i \quad (III.43)$$

and  $CL_i$  is the cloudiness of layer  $i$ .  $W$  has the property that  $\sum W(l_1, \dots, l_n) = 1$ , where the summation is taken over all possible cloud states. For example, for a 2-layer model,

$$\begin{aligned} W(0,0) &= (1-CL_1)(1-CL_2) , \\ W(0,1) &= (1-CL_1)CL_2 , \\ W(1,0) &= CL_1(1-CL_2) , \\ W(1,1) &= CL_1 \cdot CL_2 , \end{aligned}$$

and  $W(0,0) + W(0,1) + W(1,0) + W(1,1) = 1$ .

If the insolation incident at the top of layer  $i$  meets a cloud area, a fraction  $R_i$  is reflected back. It is therefore useful to define another weighting function  $W'(l_1, \dots, l_n)$  such that reflection is taken into account, namely

$$W'(l_1, \dots, l_n) = \prod_{i=1}^n w'(l_i) \quad (III.44)$$

where

$$\begin{aligned} w'(l_i) &= w(l_i)[(1-l_i) + l_i(1-R_i)] , \\ &= (1-l_i)(1-CL_i) + l_i CL_i(1-R_i) . \end{aligned} \quad (III.45)$$

The last expression was obtained via the relation  $l_i^2 = l_i$ ,  $(1-l_i)^2 = (1-l_i)$ , and  $l_i(1-l_i) = 0$ .  $W'(l_1, \dots, l_n)$  represents the transmissivity-weighted fractional area occupied by the cloud state  $(l_1, \dots, l_n)$ .

Let the total albedo for both the scattered part of the radiation and the fraction of the absorbed part that is reflected by clouds be, for a cloud

*change n to N*

state  $(l_1, \dots, l_n)$ ,  $R(l_1, \dots, l_n)$ . By (III-15), with  $R_i$  replaced by  $l_i R_i$ ,

$$R(l_1, \dots, l_n) = 1 - [E(l_1 R_1, \dots, l_n R_n) / D(l_1 R_1, \dots, l_n R_n)] \quad (III.46)$$

Thus, for example,  $R(1,1) = R_{13}$ ,  $R(0,1,0) = R_3$ ,  $R(1,1,1) = R_{135}$  and  $R(0,0,0) = 0$ .

The radiation budgets for an atmospheric layer and at the earth's surface can now be formulated on the basis of the above results.

Absorption in an atmospheric layer

The downward flux of the absorbed part of the solar radiation at levels  $i$  and  $i+2$  can be expressed as

$$S_{ai} = S_o^a \sum^{m_i} W'(l_1, \dots, l_{i-1}) [1 - A(D_{Ti}(l_1, \dots, l_{i-1}))], \quad (III.47)$$

$$S_{ai+2} = S_o^a \sum^{m_{i+2}} W'(l_1, \dots, l_{i-1}, l_{i+1}) [1 - A(D_{Ti+2}(l_1, \dots, l_{i-1}, l_{i+1}))], \quad (III.48)$$

where the summation is taken over all the  $m_i = 2^{i/2}$  different possible cloud states.

Since the net downward flux through level  $i$  of the absorbed part of the radiation

is  $(1 - R_{i+1}) S_{ai}$ , the absorption in layer  $i+1$  is

$$AS_{i+1} = (1 - R_{i+1}) S_{ai} - S_{ai+2} \quad (III.49)$$

Absorption by the earth's surface

The absorbed part of the solar radiation that reaches the earth's surface is estimated by (III.47) as

$$S'_{aI} = S_o^a \sum^m W'(l_1, \dots, l_{I-1}) \left[ \frac{1 - A(D_{TI}(l_1, \dots, l_{I-1}))}{1 - \alpha_s R(l_1, \dots, l_{I-1})} \right], \quad (III.50)$$

where the denominator is the correction factor due to multiple reflection between the clouds and the earth's surface. However, in addition to  $S'_{aI}$ , some fraction of the absorbed part of the solar radiation that is reflected at the tops of clouds

can reach the earth's surface after multiple reflections between cloud layers.

Let the contribution to the downward insolation at the surface from radiation reflected at level  $i$  (and subsequently multiply-reflected between cloud layers) be  $S_{aI}^i$ . Let  $R_U$  and  $R_L$  be the total albedos of the cloud groups above and below level  $i$  and let the total albedo of all the cloud groups in the entire atmosphere be  $R_{U+L}$ . The absorbed part of the solar radiation reflected at level  $i$  is  $S_{aI}^i R_{i+1}$ . After multiple reflections between the upper and lower cloud groups, the fraction  $R_U / (1 - R_U R_L)$  has been scattered back to level  $i$  as a downward flux. Of this, the fraction  $(1 - R_L)$  is transmitted through the lower cloud group to reach the earth's surface. Multiple reflections between the earth's surface and all the cloud groups can be introduced via the factor  $1 / (1 - R_{U+L} \alpha_s)$ .

$CL_{i+1}$

If the  $i+1$  layer is devoid of clouds, reflection at level  $i$  does not occur. Considering this fact,  $S_{aI}^i$  for a cloud state  $(l_1, \dots, l_{I-1})$  is given by

$$S_{aI}^i(l_1, \dots, l_{I-1}) = S_{aI}^{i+1} R_{i+1} R_U (1 - R_U) / [(1 - R_U R_L)(1 - R_{U+L} \alpha_s)] \quad (III.51)$$

where

$CL_{i+1}$

$$R_U = R(l_1, \dots, l_{i-1})$$

$$R_L = R(l_{i+1}, \dots, l_{I-1})$$

$$R_{U+L} = R(l_1, \dots, l_{I-1})$$

and

$$S_{aI}^i = \sum_{I=1}^m W(l_1, \dots, l_{I-1}) S_{aI}^i(l_1, \dots, l_{I-1}) \quad (III.52)$$

Finally, the absorbed part of the solar radiation that reaches the earth's surface is

$$S_{aI}^i + \sum_{i=2}^{I-1} S_{aI}^i \quad (III.53)$$

Since different values are adopted for the albedos of the scattered and absorbed parts of the solar radiation, let  $R_1^s$  and  $R^s(l_1, \dots, l_n)$  denote the former and  $R_1$  and  $R(l_1, \dots, l_n)$  denote the latter.

In analogy to (III.20) and (III.21), the scattered part of the solar radiation that reaches the earth's surface is

$$S_{sI} = S_o^s \sum_{I=1}^m \cancel{w(l_1, \dots, l_{I-1})} \left[ \frac{1 - \alpha_c(l_1, \dots, l_{I-1})}{1 - \alpha_s \alpha_c(l_1, \dots, l_{I-1})} \right] \quad (III.54)$$

where

$$\alpha_c(l_1, \dots, l_{I-1}) = 1 - (1 - \alpha_o) [1 - R^s(l_1, \dots, l_{I-1})] \quad (III.55)$$

Finally, the total absorption <sup>by</sup> of the earth's surface of both the scattered and absorbed part <sup>of</sup> the solar radiation is

$$S_I = (1 - \alpha_s) \left( S_{aI}^s + \sum_{i=2}^{I-2} S_{aI}^i + S_{sI} \right) \quad (III.56)$$

The symbolic expressions developed in this section for multiple cloud layers appear to be relatively simple. However, the complexity and computer time requirement of the computation increase very rapidly with increasing vertical resolution. Therefore, it may be advantageous to introduce the following simplifications into the model.

(a) No overlapping cloud layers.

Overlapping of cloud layers is prohibited; at most, only one of the  $l_j$ 's is equal to one. Thus, for an  $n$ -layer model, there are only  $n+1$  different possible cloud states  $(l_1, \dots, l_n)$ . This simplification reduces the computation to a superposition of single cloud layers.

(b) No partly cloudy condition.

The cloudiness is restricted to either 0 or 1;  $l_i$  in any layer can have only a single value. Thus, at any time and position, there is only a single cloud state  $(l_1, \dots, l_n)$ .

The radiation scheme for the current UCLA 3-layer GCM has been formulated with ~~the above simplifications~~. As shown in Appendix B, the scheme is relatively straightforward. *simplification (b)*

## 5. Surface albedo

The earth's surface is divided into three categories: sea surface, snow-free land, and snow and ice surface.

### i) Sea surface

The albedo of the sea surface is a function of the overlying cloud condition, the solar zenith angle, and the wave state of the sea surface.

Under a clear sky, a majority of the solar insolation reaches the earth's surface as direct radiation. The albedo of a smooth sea surface for direct radiation increases from a few per cent to about 100 per cent as the solar zenith angle increases. Since the mean solar zenith angle increases with latitude, the albedo of the sea surface is usually somewhat larger in high latitudes than in low latitudes.

Under an overcast sky, almost all of the solar insolation reaches the surface as diffuse radiation. The albedo of the sea surface for diffuse radiation is nearly a constant value of 0.07.

In the present 3-layer GCM, a constant value of 0.07 is adopted as the sea surface albedo.



## (ii) Snow-free land

The albedo of snow-free land depends upon the soil condition (especially color) and the extent and color of natural vegetation. Coniferous forests with dark leaves have an albedo of about 0.14 while deserts covered by light colored sand have an albedo larger than 0.3. Seasonal variations in albedo caused by leaves withering or turning yellow are not uncommon.

It appears that the spatial and seasonal changes in albedo are not essential to the simulation of the general circulation. Therefore, following Posey and Clapp (1964), a constant albedo of 0.14 is adopted for snow-free land.

## (iii) Snow and ice

The albedo of snow and ice varies considerably with their crystalline structure. The latter is a function of temperature and, in the case of snow, age. Thus, the albedo of snow depends upon whether the snow is wet or dry, fresh or old, and whether or not it is in the melting stage. The albedo of ice depends upon whether <sup>it</sup> ~~the ice~~ is sea pack <sup>ice</sup> or permanent land ice, and whether or not it is in the melting stage.

In "Climates of the Polar Regions" (edited by Orvig, 1970), Putniss, Vowinckel and Orvig, and Schwerdtfeger discuss <sup>e</sup> the albedo of the snow and ice surfaces of Greenland, the North Polar Basin, and Antarctica, respectively. Their key results are:

- (a) the albedo of snow varies from 55% to 100% ,
- (b) the albedo of sea ice varies from 20% to 60% ,
- (c) the albedo of permanent snow and ice fields over the interior of land has a lower limit of about 80%.

In addition, the albedo of snow-covered land may vary considerably with the type of underlying surface. For example, the albedo of a snow-covered forest area depends upon the type of forest, the density of trees and the depth of the snow cover.

In midsummer, the ice and snow surfaces of the polar region receive more than  $700 \text{ ly day}^{-1}$  of solar radiation. If the surface albedo changes by 0.1, the solar radiation absorbed by a surface changes by more than  $70 \text{ ly day}^{-1}$ ; this amount is of the same order as the net outgoing long wave radiation from the earth's surface. Therefore, in midsummer, the radiation budget at the polar regions is very sensitive to the surface albedo. Since the albedos of snow and ice surfaces may vary from 0.2 to 1.0, special attention must be paid to the prescription of the surface albedo for the summer polar region.

In the current 3-layer model, albedos of 0.4 for sea ice and 0.7 for snow have been adopted. For permanent ice and snow fields over land interiors,  $\text{Min}(0.85, 0.7 + 0.00015 h)$  has been employed, where  $h$  is the land surface elevation in meters.

*in the current UCLA 3-layer GME,*  
 In summary, the assumed values of surface albedo  $\alpha_s$  are:

<u>Surface</u>	<u>Albedo, <math>\alpha_s</math></u>
snow-free land	0.14
sea	0.07
sea ice	0.4
temporary snow	0.7
permanent land ice and snow	$\text{Min}(0.85, 0.7 + 0.00015 h)$

IV. VERTICAL DISTRIBUTION OF WATER VAPOR AND CARBON DIOXIDE

1. Water vapor

The effective water vapor amount in an air column of height  $z$ ,  $u_z^*$ , can be obtained from equation (II.3) if the vertical distribution of water vapor mixing ratio  $q$  is known. In principle, since the UCLA 3-layer GCM predicts  $q$  for each layer from the moisture conservation equation,  $u_z^*$  for each level in the troposphere could be estimated straightforwardly from (II.3). However, for simplicity, an assumed vertical distribution of  $q$  will be employed rather than that predicted by the GCM. ~~Smith (1966) has shown that the climatic value of the mixing ratio in the lower troposphere can be expressed by~~

~~$q = \frac{p_0}{p} \exp(-\frac{z}{H})$~~   $q = \frac{p_0}{p} \exp(-\frac{z}{H})$   $q = \frac{p_0}{p} \exp(-\frac{z}{H})$

Two methods, based upon different forms of this relation for the entire troposphere, will be developed subsequently.

Although the present UCLA 3-layer GCM does not predict the water vapor content in the stratosphere, this quantity is <sup>necessary</sup> requisite for the radiation calculation. Aircraft observations by the British group (for example, Murgatroyd et al (1955)) have shown that the frost-point temperature decreases rapidly near the tropopause and tends to asymptotically approach a constant value of about 190°K in the lower stratosphere. Manabe and Möller (1961) summarized the results obtained by the British group and showed that the frost-point temperature in the lower stratosphere could be assumed to be 190°K, irrespective of latitude and season. This assumption was employed in the UCLA 2-layer moist model.

More recently, Williamson and Houghton (1965) estimated from radiometer measurements that the stratospheric mixing ratio is in the neighborhood of  $3 \times 10^{-6}$  g/g at least to 25 km. Mastenbrook (1968), employing balloon-borne frost-point hygrometers, measured the water vapor in the stratosphere over Trinidad, Washington, D.C., and Thule, Greenland during 1964 and 1965. He found that the median vertical distribution of water vapor in the stratosphere, to

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(1)

(IV.7)

1

1

a height of 28 km, ~~(100 mb - 15 km)~~ approximates a constant mixing ratio within the range  ~~$2 \times 10^{-6}$  to  $3 \times 10^{-6}$  g/g~~  $2 \times 10^{-6}$  to  $3 \times 10^{-6}$  g/g.

McKinnon and Morewood (1970) also measured the water vapor distribution in the upper troposphere and lower stratosphere from 70°N to 40°S over North and South America by an automatic tracking solar spectrometer mounted in a jet aircraft. They found that the seasonal variations were within a few per cent in the lower stratosphere and that median values <sup>above</sup> about 17.7 km, during Northern Hemisphere winter, gradually decrease from  $1.75 \times 10^{-6}$  at 65°N to  $1.25 \times 10^{-6}$  at 30°S. These values are roughly half those obtained by Mastenbrook and Williamson and Houghton. ~~Therefore,~~ In the current UCLA 3-layer GCM, a constant mixing ratio of  $2.5 \times 10^{-6}$  g/g is assumed for the stratosphere above 100 mb.

*(New paragraph)*

Although relation (IV.1) is adopted as the vertical distribution of the mixing ratio in the troposphere, there are many possible methods by which  $q$  can be estimated from (IV.1). In the following, two methods suitable for the 3-layer GCM <sup>are</sup> will be presented.

(i) Method A

Throughout the troposphere, the vertical distribution of  $q$  is assumed to be expressed by (IV.1) with a single value of  $k$ . Thus

$$q = \begin{cases} q_{st} (p/p_{st})^k & \text{for } p > p_{st} = 100 \text{ mb} \\ q_{st} = 2.5 \times 10^{-6} & \text{for } p \leq p_{st} \end{cases} \quad \text{for} \quad \text{(IV.2)}$$

By (II.3), the effective water vapor amount of  $p_2 \geq p_{st}$  is

$$u^* = \frac{\beta}{\gamma} \left[ \left( \frac{p_g}{p_{st}} \right)^\gamma - \left( \frac{p_2}{p_{st}} \right)^\gamma \right] \quad \text{(IV.3)}$$

where  $\beta = p_{st}^{\alpha+1} q_{st} \log p_{oo}^{\alpha} = 2.55/x (0.1)^{\alpha} \times 10^{-4} \text{ cm}$  gm/cm<sup>2</sup>

$$\gamma = k + \alpha + 1$$

and for  $p_z \leq p_{st}$ ,

$$u_z^* = u_{st}^* + \frac{\beta}{\alpha+1} \left[ 1 - \left( \frac{p_z}{p_{st}} \right)^{\alpha+1} \right] \quad (IV.4)$$

where  $u_{st}^*$  means  $u^*$  for  $p_z = p_{st}$ .

It is now necessary to estimate a value of  $k$  at each grid point.

From (IV.2),

$$\log(q/q_{st}) = k \log(p/p_{st}) \quad (IV.5)$$

The simplest way to obtain a single value of  $k$  for an  $N$ -layer tropospheric model may be

$$k = \frac{\sum_{j=1}^N \log(q_j/q_{st})}{\sum_{j=1}^N \log(p_j/p_{st})} = \log \left( \prod_{j=1}^N (q_j/q_{st}) \right) / \log \left( \prod_{j=1}^N (p_j/p_{st}) \right) \quad (IV.6)$$

where  $M = 2N - 1$ .  
For a 3-layer model, (IV.6) becomes

$$k = \log(q_1 \cdot q_3 \cdot q_5 / q_{st}^3) / \log(p_1 \cdot p_3 \cdot p_5 / p_{st}^3) \quad (IV.7)$$

However, if a least squares method is used,

$$k = \frac{\sum_{j=1}^N \log(q_j/q_{st}) \cdot \log(p_j/p_{st})}{\sum_{j=1}^N \log^2(p_j/p_{st})} \quad (IV.8)$$

This expression may be superior to (IV.6) but it also requires much more computer time.

If  $p_{st}$  is larger than 400 mb, the range of change of  $\log(p/p_{st})$  is less than that of  $\log(q/q_{st})$ . If  $\log(p/p_{st})$  is replaced in (IV.8) by some mean value, (IV.8) reduces to (IV.6) as a limiting case.

(ii) Method B

Two values of  $k$  are used to express the vertical distribution of mixing ratio in the troposphere. That is

$$q = q_c (p/p_c)^{k_1} \quad \text{for } p \geq p_c \quad (IV.9)$$

$$q = q_{st} (p/p_{st})^{k_2} \quad \text{for } p_{st} \leq p < p_c \quad (IV.10)$$

$$q = q_{st} = 2.5 \times 10^{-6} \quad \text{for } p \leq p_{st} = 100 \text{ mb}$$

where  $p_c$  is a critical level in the middle troposphere around 400 mb.

In this case, the effective water vapor amount can be expressed as follows:

$$u_z^* = \frac{\beta_1}{\gamma_1} \left[ \left( \frac{p_g}{p_c} \right)^{\gamma_1} - \left( \frac{p_z}{p_c} \right)^{\gamma_1} \right] \quad \text{for } p_z \geq p_c \quad (IV.11)$$

*delete  
Subscript z*

$$u_z^* = u_c^* + \frac{\beta_2}{\gamma_2} \left[ \left( \frac{p_c}{p_{st}} \right)^{\gamma_2} - \left( \frac{p_z}{p_{st}} \right)^{\gamma_2} \right] \quad \text{for } p_{st} < p_z \leq p_c \quad (IV.12)$$

$$u_z^* = u_{st}^* + \frac{\beta_2}{\alpha+1} \left[ 1 - \left( \frac{p_z}{p_{st}} \right)^{\alpha+1} \right] \quad \text{for } p \leq p_{st} \quad (IV.4)$$

where

$$\beta_1 = p_c^{\alpha+1} q_c / g p_{oo}^\alpha$$

$$\gamma_1 = k_1 + \alpha + 1$$

$$\beta_2 = p_{st}^{\alpha+1} q_{st} / g p_{oo}^\alpha$$

$$\gamma_2 = k_2 + \alpha + 1$$

To obtain  $k_1$  and  $q_c$ , the least squares method is applied to the relation

$$y_j = \alpha + k_1 x_j$$

where  $x_j = \log(p_j/p_c)$ ,  $y_j = \log q_j$ ,  $\alpha = \log q_c$ .

Then,

$$k_1 = \frac{N \sum_{j=1}^N x_j y_j - \sum_{j=1}^N x_j \sum_{j=1}^N y_j}{D} \quad (IV.13)$$

$$q_c = \exp\left(\frac{N \sum_{j=1}^N y_j \sum_{j=1}^N x_j^2 - \sum_{j=1}^N x_j y_j \sum_{j=1}^N x_j}{D}\right) \quad (IV.14)$$

$$D = N \sum_{j=1}^N x_j^2 - \left(\sum_{j=1}^N x_j\right)^2$$

Using the above value of  $q_c$ ,  $k_2$  is obtained from

$$k_2 = \log(q_c / q_{st}) / \log(p_c / p_{st}) \quad (IV.15)$$

The results of the three methods for estimating  $k$ , (IV.6) and (IV.8) of method A, and (IV.13) of method B, will now be compared to observations. Rodgers (1967) presented the zonal mean values of the mixing ratio in the lower troposphere for the northern hemisphere which <sup>Rasmusson</sup> ~~Rasmusson~~ calculated on the basis of the five year data sample of the MIT General Circulation Library. The values of the mixing ratio were given at 400, 500, 700, 850 and 1000 mb. The seasonal and latitudinal <sup>(mean of the year)</sup> mean values of  $k$  and the mixing ratio at 1000 mb estimated by the three methods, and the 1000 mb observations, are <sup>compared</sup> ~~contrasted~~ in Table IV-1. For method B,  $k_1$  and  $k_2$  are shown by numbers without and with parentheses, corresponding to values for the lower troposphere from 1000 mb to 400 mb and for the upper troposphere from 400 mb to 100 mb, respectively ( $p_c = 400$  mb).

With the exception of the lowest layer in the winter high latitudes where an inversion layer predominates, the form of relation (IV.1) is in fairly good agreement with the climatological data in the lower troposphere below 400 mb. It can be considered that  $k_1$  deduced by method B faithfully yields the actual condition for the lower troposphere. Throughout the year,  $k_1$  is larger in midlatitudes than in the tropics and high latitudes and, with the exception of high latitudes, is larger in winter than in summer.

The vertical profiles of <sup>the</sup> mixing ratio obtained by methods A and B are compared to the observations in Figure IV-1

see p. vii

IV-6

Variation of the zonal

TABLE IV.1 <sup>The</sup> Seasonal and latitudinal mean values of  $k$  in the troposphere and the mixing ratio at <sup>at</sup> the 1000 mb <sup>at</sup> estimated by the three methods. ~~The~~ Method B yields two values of  $k$  ( $k_1$  and  $k_2$ ). The values for the upper troposphere between 100 and 400 mb,  $k_2$ , are shown in parentheses.

LAT. (°N)	k				① 1000 mb (g/Kg)			
	A	A'	k <sub>1</sub> (B)	k <sub>2</sub>	A	A'	B	OBS.
January								
0	3.90	3.93	2.86	(4.31)	19.9	21.3	13.6	13.8
20	3.65	3.65	3.59	(3.67)	11.1	11.1	10.9	10.4
40	3.26	3.26	3.36	(3.22)	4.6	4.6	4.7	4.4
60	2.85	2.84	2.90	(2.82)	1.8	1.7	1.8	1.5
70	2.65	2.65	2.39	(2.75)	1.1	1.1	1.0	0.8
April								
0	3.92	3.95	3.06	(4.26)	20.8	22.3	15.2	14.9
20	3.72	3.72	3.70	(3.73)	13.1	13.1	13.0	12.1
40	3.42	3.43	3.23	(3.50)	6.6	6.7	6.2	5.9
60	3.04	3.05	2.89	(3.10)	2.8	2.8	2.6	2.5
70	2.77	2.77	2.89	(2.72)	1.5	1.5	1.5	1.2
July								
0	4.04	4.08	2.71	(4.57)	27.3	30.1	16.9	16.8
20	3.98	4.02	2.92	(4.40)	24.0	26.2	16.3	16.0
40	3.81	3.83	3.09	(4.09)	16.0	16.7	12.3	12.0
60	3.59	3.60	3.19	(3.74)	9.7	10.0	8.4	7.5
70	3.46	3.47	3.23	(3.55)	7.2	7.4	6.6	6.0
October								
0	4.02	4.06	2.66	(4.55)	26.0	28.7	15.9	16.2
20	3.90	3.92	3.15	(4.20)	19.8	20.8	15.1	14.6
40	3.54	3.55	3.25	(3.66)	8.7	8.9	7.8	7.5
60	3.17	3.17	3.12	(3.19)	3.7	3.7	3.6	3.2
70	3.01	3.02	2.70	(3.13)	2.5	2.6	2.2	2.0



The value of  $k$  obtained by method A shows a simpler seasonal and latitudinal variation. Since  $k$  is directly proportional to the ~~mixing ratio~~ <sup>log of  $H$</sup>  ~~in~~ <sup>- at fixed pressure</sup> the lower troposphere ~~as shown by the solid lines of Fig. IV-1~~,  $k$  decreases with increasing latitude and from summer to winter. This behavior does not agree with the observations.

In low latitudes throughout the year, and in midlatitudes in summer, method A overestimates  $k$  by about 1.0. As a result, the mixing ratio at 1000 mb is more than 30 per cent larger than the observations. As an extreme case, the solid and dashed curves on the right side of Fig. IV-1 demonstrate the difference between the vertical profiles of the mixing ratio obtained by method A and B, respectively, for the normal condition at the equator in July. The mixing ratio in the lowest 350 mb layer is overestimated, and that in the upper troposphere is underestimated, by method A. This may lead to an overestimation of the cooling rate rate of the tropospheric column by long wave radiation by yielding a too small net upward flux at the earth's surface and a too large net outgoing flux at the top of the troposphere.

Table IV-1 also shows the difference between  $k$  deduced by (IV.6) and (IV.8) of method A. The numbers labeled by A' and A refer to (IV.6) and (IV.8), respectively. ~~In this case~~ <sup>Since</sup> 400 mb is the highest level for which there are observed values of  $q$ . ~~Therefore~~, as previously discussed, the difference between the A' and A values is negligibly small and (IV.6) can be employed in lieu of (IV.8).

*delete* <sup>up to now</sup> Prior to September, 1971, (IV.6) of method A <sup>has been</sup> employed in the UCL A 3-layer GCM. Method B is currently undergoing testing prior to its utilization.

*Method B is employed in the current UCL A 3-layer GCM.*

*more figures to follow Table IV-1*

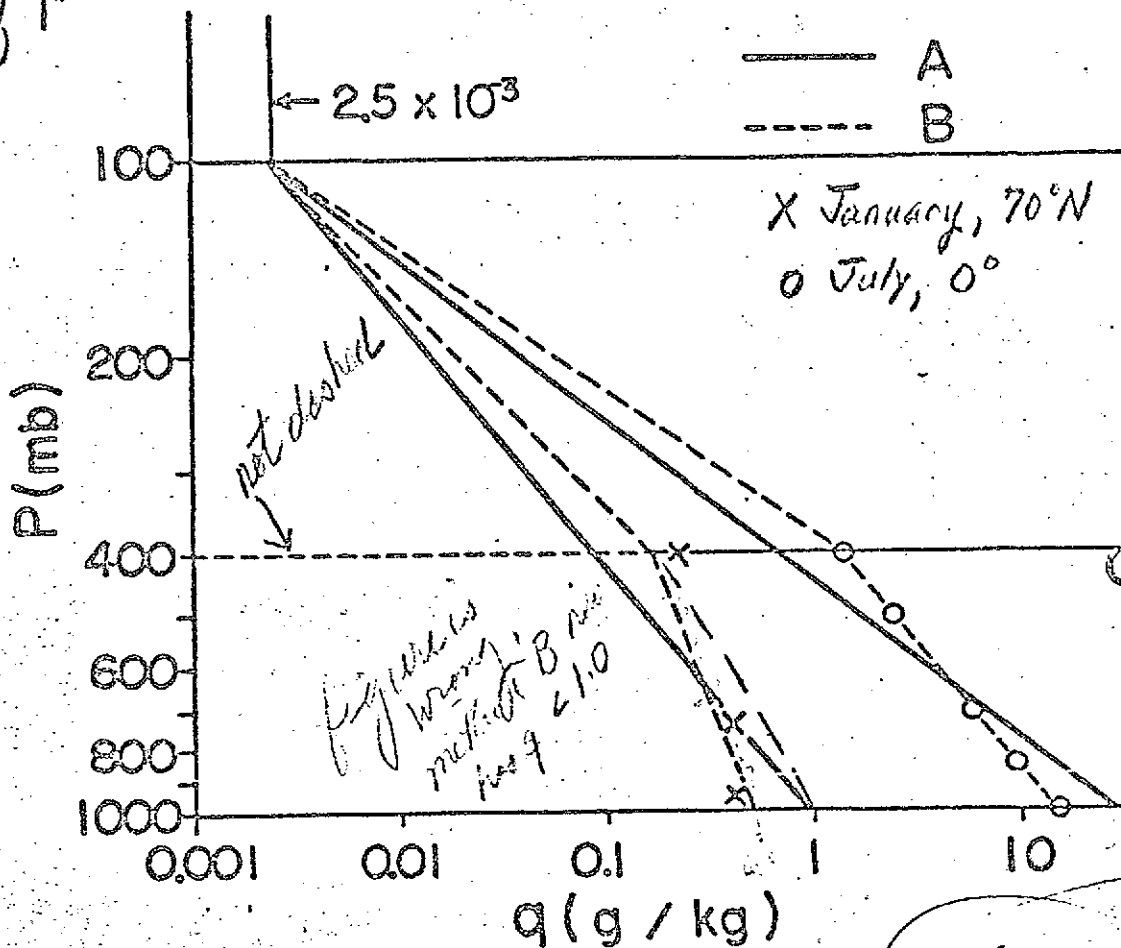


Fig IV-1. ~~Diff Figure 2~~

*(see p. v) for title*

(iii) Hemispheric mean normal values of mixing ratio

In order to save computer time, it may be desirable to approximate the interpolation factor  $m$ , discussed in section II.2, by constant values over the global domain or throughout the year. Appropriate constant values of  $m$  can be obtained by substituting mean values of the mixing ratio into (II.24) - (II.27). For this purpose, Table IV-2 presents the mean normal mixing ratio averaged over the northern hemisphere ( $0^\circ$ - $70^\circ$ N) based on Rasmuson's data. Since the semi-empirical expressions for  $m$  are functions of  $\log q$ , the hemispheric mean values shown as  $\bar{q}$  represent the mean taken with respect to  $\log q$ . The constants that are deduced via methods A and B are also shown.

TABLE IV.2. The northern hemisphere <sup>e</sup> mean normal values of mixing ratio  $\bar{q}$  and <sup>the</sup> various constants obtained by <sup>the</sup> methods A and B.   
 The mixing ratio is shown in units of ~~g/kg~~.  $g/kg$ .

		JAN	APR	JUL	OCT	ANN
$\bar{q}$	1000 mb	6.0	7.6	13.0	9.3	8.6
	850	3.9	4.9	8.4	6.1	5.6
	700	2.0	2.6	4.8	3.3	3.0
	500	0.69	0.86	1.8	1.2	1.1
	400	0.31	0.38	0.85	0.58	0.49
B	$k_1$	3.24	3.28	2.98	3.06	3.14
	$k_2$	3.51	3.66	4.23	3.94	3.83
	$q_{400}$	0.32	0.40	0.88	0.59	0.51
	$q_{1000}$	6.32	8.05	13.49	9.70	9.03
A	$k$	3.44	3.56	3.90	3.71	3.65

The annual mean vertical profile of the mixing ratio of water vapor in the troposphere as obtained from method B is

$$q = \begin{cases} 0.51 \times 10^{-3} \left(\frac{p}{400}\right)^{3.14} = 9.03 \times 10^{-3} \left(\frac{p}{1000}\right)^{3.14} & \text{for } p \geq 400 \text{ mb} \\ 2.5 \times 10^{-6} \left(\frac{p}{100}\right)^{3.83} & \text{for } 400 \text{ mb} \geq p \geq 100 \text{ mb} \end{cases} \quad (\text{IV.16})$$

(iv) Pressure scaling factor

To evaluate the effective water vapor amount, the pressure scaling factor  $\alpha$  must be specified in (II.3).

El<sup>S</sup>asser (1942), on the basis of theoretical and laboratory experimental<sup>S</sup> studies, adopted a value of  $\alpha = 0.5$  for his radiation chart. Yamamoto (1952) recommended  $\alpha = 1.0$  on the basis of studies presented after El<sup>S</sup>asser's paper. Based on the laboratory experiment by Howard, et al (1955), Manabe and Möller (1961) adopted  $\alpha = 0.6$ .

Water vapor has many absorption bands in the infra-red spectral region. Each band consists of many lines whose individual shapes can be important to the radiation transfer computation.

Consider a single absorption line. For a weak line whose center is not blacked out, the total absorption does not depend on pressure broadening, hence  $\alpha = 0$ . For a strong line whose center region is completely blacked out, theory shows that the total absorption is proportional to the pressure, that is,  $\alpha = 1$ .

In the actual atmosphere, absorption by water vapor occurs via an ensemble of weak, intermediate and strong lines, and the fractional percentage of each line changes with the amount of the absorber. Therefore, the value of  $\alpha$  may vary from 0 to 1, depending upon atmospheric conditions.

In the lower troposphere, since  $p/p_{\infty} \sim 1$ , the particular choice of  $\alpha$  does not <sup>greatly</sup> strongly affect the radiation budget. However, the radiation budget in the stratosphere may be very sensitive to the choice of  $\alpha$ , since  $p/p_{\infty} \leq 0.1$ . Consequently, if the pressure scaling approximation embodied in (11.3) is to be utilized for a GCM which includes the stratosphere, a careful selection of  $\alpha$  is required.

A value of  $\alpha = 1$  was adopted for the 2-layer moist ~~model~~ <sup>UCLA</sup> <sup>GMC.</sup> A value of  $\alpha = 0.6$  is currently <sup>being used in</sup> <sup>UCLA</sup> ~~in use with~~ the 3-layer GCM. Preliminary results indicate that the difference <sup>in</sup> <sup>between</sup> the cooling rate of a tropospheric layer, for these two values of  $\alpha$ , is about  $0.1^{\circ}\text{C}/\text{day}$ .

## 2. Effective amount of carbon dioxide

In the earth's atmosphere, the spatial and seasonal variation of carbon dioxide content is less than <sup>the</sup> 2% <sup>0 PER cent. Except for</sup> although ~~CO<sub>2</sub> content is increasing rapidly via~~ <sup>we can</sup> ~~human activity. Excluding studies of the influence of CO<sub>2</sub> on climate,~~ ~~it can~~ be assumed that the CO<sub>2</sub> ~~content~~ in the atmosphere is constant in space and time. It is assumed, in the following, that the amount of CO<sub>2</sub> is 0.032 per cent by volume (320 ppm), that is 0.0489 per cent by weight.

Following Yamamoto (1952), the pressure scaling factor  $\alpha$  for CO<sub>2</sub> is assumed to be 1.0. The effective amount of CO<sub>2</sub> from the earth's surface to a level  $p_s$ , which is usually expressed as a thickness at normal temperature (0°C) and pressure (1 atmosphere or 1000 mb), is given by dividing (II.3) by  $\rho_{\text{NTP}}$  as follows:

$$u_{\text{CO}_2}^*(p_s, p_s) = \frac{1}{g \rho_{\text{NTP}}} \int_{p_s}^{p_g} q_{\text{CO}_2} \frac{p}{p_{\infty}} dp$$

$$= h(p_s^2 - p_g^2) / p_{\infty}^2 \quad (IV.17)$$

where

$$h = \frac{q_{\text{CO}_2} p_{\infty}}{2g \rho_{\text{NTP}}} = 126 \text{ cm-NTP} \quad (IV.18)$$

and  $q_{\text{CO}_2}$  is the percentage content of CO<sub>2</sub> by weight (0.0489%),  $\rho_{\text{NTP}}$  is the density of CO<sub>2</sub> at NTP ( $1.977 \times 10^{-3} \text{ g cm}^{-3}$ ),  $p_s$  is the surface pressure and  $h$  is the total effective amount of CO<sub>2</sub> in an air column. The effective amount of CO<sub>2</sub> in the layer between  $p_i$  and  $p_j$  is then

$$u_{\text{CO}_2}^*(p_i, p_j) = 126 \frac{|p_i^2 - p_j^2|}{p_{\infty}^2} \text{ cm-NTP} \quad (IV.19)$$

If the pressure scaling factor for carbon dioxide is  $\alpha$ , the following general form is obtained instead of (IV.19):

$$u_{\text{CO}_2}^*(p_i, p_i) = \frac{2h^0}{\alpha+1} \left[ \left( \frac{p_i}{p_{\infty}} \right)^{\alpha+1} - \left( \frac{p_i}{p_{\infty}} \right)^{\alpha+1} \right] \quad (\text{IV.20})$$

where

$$h^0 = \alpha_{\text{CO}_2} p_{\infty} / 2g \rho_{\text{NTP}} = 0.126 \times (1000) \text{ cm}^2 \text{ NTP}$$

## V. SUMMARY

have been presented for calculating the short wave and long wave radiative heating of clear and cloudy air, which can be used in numerical

Simplified schemes ~~for the computation of the radiation budget in the troposphere and at the earth's surface, have been presented which are useful for~~ <sup>calculations</sup> general circulation models with a coarse vertical resolution.

## (i) Long wave radiation

(a) A bulk transmission function was introduced and its value was determined by interpolation between 1.0 and the transmission function of the layer. The interpolation factors were parameterized as a function of the thickness of the layer, the pressure, the water vapor mixing ratio at the <sup>a</sup>reference level, and the lapse rates of the temperature and mixing ratio of the layer. The interpolation factor is linearly proportional to the thickness of the layer and varies slightly with the pressure and mixing ratio at the reference level. The effect of the lapse rates of temperature and mixing ratio is relatively small and is negligible in many cases.

(b) It was assumed that any cloud occurring in a layer with a temperature less than  $-40^{\circ}\text{C}$  allows transmission of a fraction of the incident long wave radiation. This effect was roughly approximated by reducing the cloud amount by the transmissivity of the cloud.

(c) Empirical expressions for the transmission functions of water vapor and carbon dioxide were presented and the errors of the expressions were tabulated.

(d) For simplicity, a pressure scaling approximation was adopted for the radiation computation. Scaling factors of either 0.6 or 1.0 for water vapor, and 1.0 for carbon dioxide, were employed. <sup>Test</sup> Preliminary calculations showed that the radiation budget in the troposphere was not sensitive to the choice of the value <sup>of</sup> for the scaling factor.

## (ii) Solar radiation

(a) The solar spectrum was divided into ~~the parts~~, the scattered part and the absorbed part. The scattered part, with wavelength less than  $0.9 \mu$ , undergoes conservative Rayleigh scattering ~~with its~~ absorption by water vapor is negligible. The absorbed part, with wavelengths larger than  $0.9 \mu$ , undergoes absorption with negligible Rayleigh scattering. This ~~concept~~ <sup>approximation</sup> significantly simplifies the solar radiation calculation for a clear sky.

(b) A cloud layer was regarded as a fictitious layer of water vapor that absorbs the same amount of solar insolation as the cloud layer. This concept considerably simplifies the solar radiation calculation for the atmosphere with clouds. The <sup>average</sup> values of the equivalent cloud water vapor amount were estimated for four typical cloud types ~~under normal conditions~~.

(c) General schemes for the estimation of the absorption of solar radiation in an atmospheric layer, and at the earth's surface, were presented for ~~the cases of:~~ (i) a clear sky, (ii) a quasi-vertical single cloud layer, and (iii) quasi-horizontal multiple cloud layers.

## (iii) Vertical distribution of water vapor and carbon dioxide

(i) To obtain the effective water vapor and carbon dioxide amounts required by the radiation calculation, it is necessary to know the vertical distributions of the mixing ratios of water vapor and carbon dioxide throughout the entire atmospheric column. It was assumed that the mixing ratio of water vapor in the stratosphere above 100 mb has the constant value of  $2.5 \times 10^{-6}$  g/g irrespective of season or latitude. Furthermore, it was assumed that the ~~per~~ <sup>percentage</sup> amount of carbon dioxide by volume is 320 ppm throughout the ~~global~~ atmosphere. It was shown that the normal vertical profile of the water vapor mixing ratio in the troposphere is <sup>well</sup> fairly expressed by  $p^k$  and <sup>the suggestion was made</sup> it was recommended that the <sup>predicted</sup> vertical profile of the water vapor mixing ratio ~~predicted by~~ <sup>in</sup> the GCM be adjusted to  $p^k$  for the radiation calculation.



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## References

- Arakawa, A., A. Katayama and Y. Mintz, 1969: Numerical simulation of the general circulation of the atmosphere. Proc. WMO/IUGG Symposium on Numerical Prediction, Tokyo 1968, pp. IV-7 to IV-8-12.
- Callendar, G. S., 1941: Infrared absorption by CO<sub>2</sub> with special reference to atmospheric radiation. Q. J. Roy. Met. Soc., 67, 263.
- Clodman, J., 1957: Some statistical aspects of cirrus cloud. Mon. Wea. Rev., 85, 37-41.
- Coulson, K. L., 1959: Radiative flux from the top of a Rayleigh atmosphere. Ph.D. Thesis, Dept. Met., UCLA, 176 pp.
- Elsasser, W. M., 1942: Heat transfer by infrared radiation in the atmosphere. Harvard Meteorological Studies, No. 6, 107 pp. *initial caps*
- Feigel'son, E. M., 1964: Light and Heat Radiation in Stratus Clouds. Academy of Science of USSR, Inst. Phys. of Atmosphere, Moskva, 245 pp. (Translated from Russian, ~~1955~~ Jerusalem, 1966).
- Fritz, *F.F.* *S.* and J. S. Winston, 1962: Synoptic use of radiation measurement from satellite TIROS II. Mon. Wea. Rev., 90, 1-9.
- Gates, W. L., E. S. Batten, A. B. Kahle, *and* A. B. Nelson, 1971: A documentation of the Mintz-Arakawa 2-level atmospheric general circulation model. The Rand Corporation, Santa Monica, Calif., ~~(in press)~~ 408 pp. R-877-ARPA
- Howard, J. N., D. L. Burch, and D. Williams, 1955: Near-infrared transmission through synthetic atmosphere. Geophysical Research Papers. No. 40, Air Force Cambridge Research Center, 244 pp.
- Joseph, J. H., 1966: Calculation of radiative heating in numerical general circulation models. Tech. Rep. No. 1, Numerical Simulation of Weather and Climate, Dept. Met., UCLA, 60 pp.
- Katayama, A., 1966: On the radiation budget of the troposphere over the northern hemisphere (I). J. Met. Soc. Japan, SII, 44, 381-401.
- McDonald, J. E., 1960: Direct absorption of solar radiation by atmospheric water vapor. J. Meteor., 17, 317-328.

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Joseph, J. H., 1970: On the calculation of solar radiation fluxes in the troposphere. Solar Energy, 13, 251-261.

Manabe, S. and F. Möller, 1961: On the radiative equilibrium and heat balance of the atmosphere. Mon. Wea. Rev., 89, 503-532.

Mastenbrook, H. J., 1968: Water vapor distribution in the stratosphere and high troposphere. J. Atm. Sci., 25, 299-311.

Mügge, R. and F. Möller, 1932: Zur Berechnung von Strahlungsströmen and Temperaturänderungen in Atmosphären von beliebigem Aufbau. Z. für Geophys., 8, 53-64.

McKinnon, D. and H. W. Morewood, 1970: Water vapor distribution in the lower stratosphere over North and South America. J. Atm. Sci., 27, 483-493.

Murgatroyd, R. J., P. Goldsmith and W. E. H. Hollings, 1955: Some recent measurements of humidity from aircraft up to height of about 50,000 ft. over southern England. Q. J. Roy. Met. Soc., 81, 533-537.

Orvig, S. (ed.), 1970: Climates of the polar region. World Survey of Climatology, V. 14., 370 pp.

*Possely* Posey J. W. and P. F. Clapp, 1964: Global distributions of normal surface albedo. Geophysica International, 4, 33-48.

Rodgers, C. D., 1967: The radiative heat budget of the troposphere and lower stratosphere. Planetary Circulation Project, Dept. Met., MIT, Rep. No. 2, 99 pp.

Smith, W. L., 1966: Note on the relationship between total precipitable water and surface dew point. J. App. Met., 5, 726-727.

Williamson, E. J., and J. T. Houghton, 1965: Radiometric measurements of emission from stratospheric water vapor. Q. J. Roy. Met. Soc., 91, 330-338.

Yamamoto, G., 1952: On a radiation chart. Sci. Rept. Tohoku Univ., S.5, Geophysics, 4, 9-23.

Yamamoto, G. and G. Onishi, 1948: Absorption coefficient of water vapor in the far infrared region. Sci. Rept. Tohoku Univ., S.5, Geophysics, 1, 71-75.

Appendix A: The finite difference scheme for long wave radiation for the 3-layer model

The general form of the finite difference analog for the flux of long wave radiation is given by (II.45). In the current <sup>UCLA</sup> 3-layer GCM, the cloud thickness is defined in a somewhat different manner from the general case. As shown in Fig. A-1, the lowest cloud is situated between levels 4 and 5. Assuming that  $T_{\infty} = T_0$ , we obtain

$$R_0 = \sigma T_c^4 \tilde{\tau}_{\infty 0} + \tau_{\infty 0} \Delta B_{CO} + \frac{1+m_0^+ \tau_{20}}{1+m_0^+} \Delta B_{02} (1-CL_1^+)$$

$$+ \frac{\tau_{20} + \tau_{40}}{2} \Delta B_{24} (1-CL_1^+)(1-CL_3^+)$$

$$+ \left( \frac{\tau_{40} + \tau_{60}}{2} \Delta B_{46} + \tau_{60} \Delta B_{6g} \right) (1-CL_1^+)(1-CL_3^+)(1-CL_5^+)$$

$$R_2 = \left( \sigma T_c^4 \tilde{\tau}_{\infty 2} + \tau_{\infty 2} \Delta B_{CO} + \frac{1+m_2^- \tau_{02}}{1+m_2^-} \Delta B_{02} \right) (1-CL_1^+)$$

$$+ \frac{1+m_2^+ \tau_{42}}{1+m_2^+} \Delta B_{24} (1-CL_3^+)$$

$$+ \left( \frac{\tau_{42} + \tau_{62}}{2} \Delta B_{46} + \tau_{62} \Delta B_{6g} \right) (1-CL_3^+)(1-CL_5^+),$$

$$R_4 = \left( \sigma T_c^4 \tilde{\tau}_{\infty 4} + \tau_{\infty 4} \Delta B_{CO} + \frac{\tau_{04} + \tau_{24}}{2} \right) (1-CL_1^+)(1-CL_3^+)$$

$$+ \frac{1+m_4^- \tau_{24}}{1+m_4^-} \Delta B_{24} (1-CL_3^+) \quad \Delta B_{0,2}$$

$$+ \left( \frac{1+m_4^+ \tau_{64}}{1+m_4^+} \Delta B_{46} + \tau_{64} \Delta B_{6g} \right) (1-CL_5^+),$$

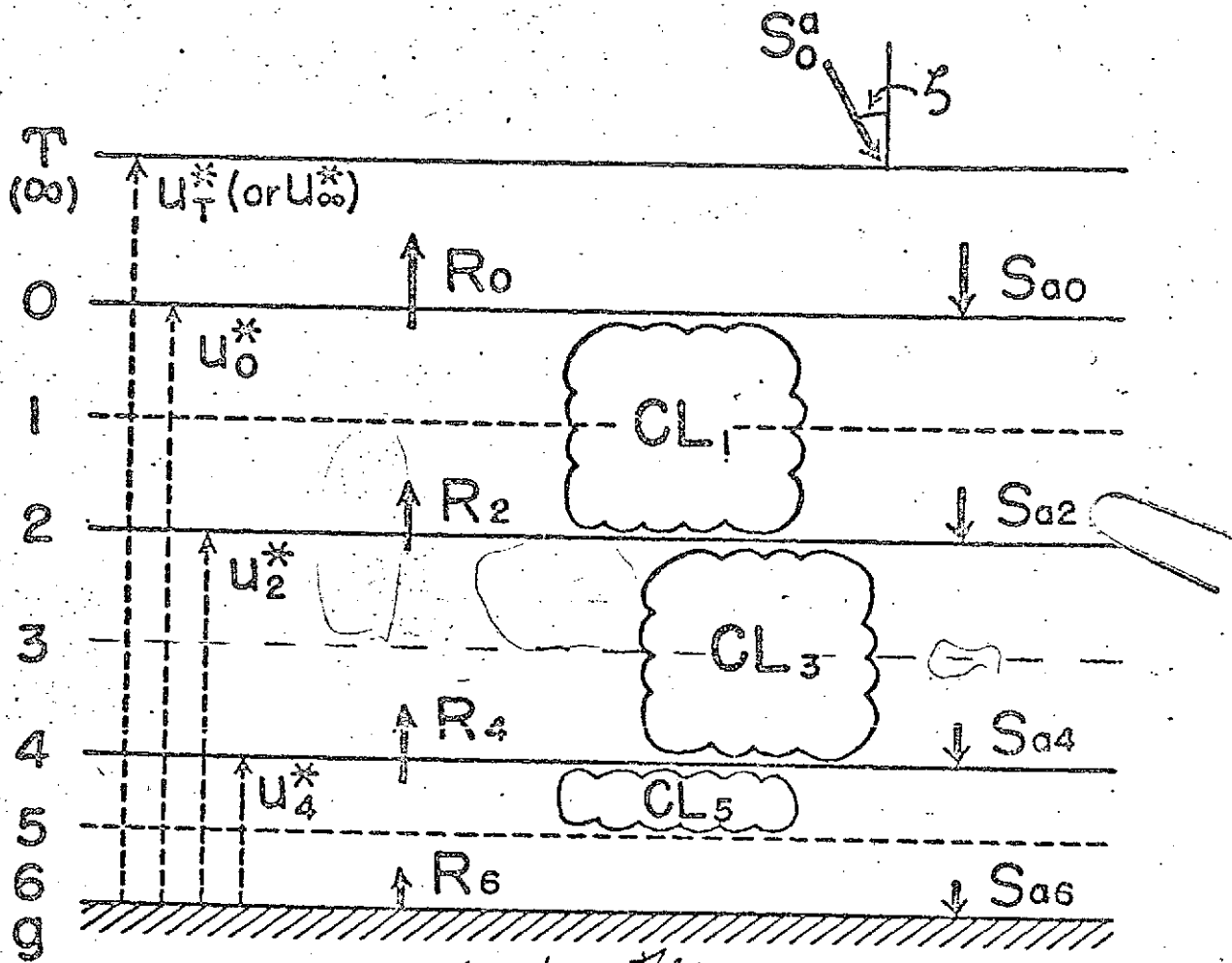


Fig. A-1. Schematic representation of <sup>the</sup> clouds and radiation fluxes in the ~~Figure 10~~ 3-level general circulation model.

$$\begin{aligned}
 R_6 = & \left( \sigma T_c^4 \bar{\tau}_{\infty 6} + \tau_{\infty 6} \Delta B_{c0} + \frac{\tau_{06} + \tau_{26}}{2} \Delta B_{02} \right) (1-CL_1^i)(1-CL_3^i)(1-CL_5^i) \\
 & + \frac{\tau_{26} + \tau_{46}}{2} \Delta B_{24} (1-CL_3^i)(1-CL_5^i) + \frac{1+m_5^- \tau_{46}}{1+m_6^-} \Delta B_{46} (1-CL_5^i) \\
 & + \frac{1+m_{55} \tau_{56}}{1+m_{56}} \Delta B_{55} CL_5^i + \Delta B_{6g}
 \end{aligned}$$

where

$$\Delta B_{ij} = \sigma T_i^4 - \sigma T_j^4$$

$$CL_i^* = \alpha_i^* CL_i$$

and  $CL_i$  is the cloudiness and  $\alpha_i^*$  is the greyness of a cloud. If the temperature in a non-convective cloud layer is less than  $-40^\circ\text{C}$ ,  $\alpha_i^* = 0.5$  is assumed. The second term from the last, in the expression for  $R_{s_j}$ , is an additional term necessitated by the assumption that the bottom of the lowest cloud is level 5;  $m_{56}$  denotes the interpolation factor  $m$  for the layer between levels 5 and 6.

Appendix B: The finite difference scheme for solar radiation for the 3-layer model

In the following, the general expressions developed in section III.4 (ii) for the solar radiation calculation will be specialized to the present UCLA 3-layer GCM.

The cloudiness  $CL_i$  is restricted to either 0 or 1. Thus, if  $l_i = 1$ ,  $CL_i = 1$  and, if  $l_i = 0$ ,  $CL_i = 0$ ; therefore,

$$l_i = CL_i \quad (B.1)$$

In the following,  $CL_i$  will be substituted for  $l_i$  everywhere.

Employing the relations

$$\begin{aligned} (1-CL_i)^{\uparrow} &= (1-CL_i) \quad \wedge \\ CL_i^{\uparrow} &= CL_i \quad \wedge \\ CL_i(1-CL_i) &= 0 \quad \wedge \end{aligned}$$

and the definition  $CR_i = CL_i R_i$ , we obtain from (III.45), (III.43), (III.37), (III.44) and (III.42):

$$w(l_i) = w'(CL_i) = 1 - CR_i \quad (B.2)$$

$$M_i = \prod_{j=1}^{i-3} (1-CL_j) \quad (B.3)$$

$$W'(l_1, \dots, l_n) = \prod_{i=1}^n (1 - CR_i) \quad (B.4)$$

$$W(l_1, \dots, l_n) = 1 \quad (B.5)$$

*for i < 4*

$$M_i = \prod_{j=1}^{i-3} (1-CL_j)$$

and

line up the = signs, as elsewhere.

## (i) Absorption in the atmosphere

Combining (III.38), (III.39) and (B.3), the optical path length of the layer between the top of the atmosphere and level  $i$ ,  $D_{Ti}$ , is:

$$D_{T0} = (u_T^* - u_0^*) \sec \zeta, \quad (B.6)$$

$$D_{T2} = D_{T0} + (1-CL_1)(u_0^* - u_2^*) \sec \zeta + 1.66 CL_1 u_{c1}^*, \quad (B.7)$$

$$D_{T4} = D_{T2} + (1-CL_3)(u_2^* - u_4^*) [(1-CL_1) \sec \zeta + 1.66 CL_1] + 1.66 CL_3 u_{c3}^*, \quad (B.8)$$

$$D_{T6} = D_{T4} + (1-CL_5) u_4^* \{ (1-CL_1)(1-CL_3) \sec \zeta + 1.66 [1 - (1-CL_1)(1-CL_3)] \} + 1.66 CL_5 (u_{c5}^* + u_5^*), \quad (B.9)$$

where the last term on the right hand side of (B.9) is an additional term due to the assumption that the base of cloud 5 is level 5 as shown in Fig. A-1.

From (III.47) and (B.4), the downward flux at level  $i$  of the absorbed part of the solar radiation is

$$S_{ai} = S_0^a \left[ \prod_{j=1}^{i-1} (1-CR_j) \right] [1-A(D_{Ti})] \quad (B.10)$$

Thus, for  $i = 0, 2, 4$  and  $6$ ,

$$S_{a0} = S_0^a [1-A(D_{T0})], \quad (B.11)$$

$$S_{a2} = S_0^a (1-CR_1) [1-A(D_{T2})], \quad (B.12)$$

$$S_{a4} = S_0^a (1-CR_1)(1-CR_3) [1-A(D_{T4})], \quad (B.13)$$

$$S_{a6} = S_0^a (1-CR_1)(1-CR_3)(1-CR_5) [1-A(D_{T6})], \quad (B.14)$$

From (III.49), the absorption in the atmospheric layers  $j = 1, 3, 5$  is

$$AS_1 = (1-CR_1) S_{a0} - S_{a2} \quad (B.15)$$

$$AS_3 = (1-CR_3) S_{a2} - S_{a4} \quad (B.16)$$

$$AS_5 = (1-CR_5) S_{a4} - S_{a6} \quad (B.17)$$



(ii) Absorption by the earth's surface

Let  $CR_i = R(CL_i) = CL_i R_i$  , (B.18) ^

$$CR_{ii} = R(CL_i, CL_i) ,$$
 (B.19) ^

$$CR_{ijk} = R(CL_i, CL_j, CL_k) ,$$
 (B.20) ^

Applying (III.46) to (III.13) <sup>and (III.14)</sup> (III.15) yields

$$CR_{ii} = 1 - \frac{(1-CR_i)(1-CR_i)}{1-CR_i CR_i} ,$$
 (B.21) ^

and

$$CR_{135} = 1 - \frac{(1-CR_1)(1-CR_3)(1-CR_5)}{1-(CR_1 CR_3 + CR_3 CR_5 + CR_5 CR_1) + 2CR_1 CR_3 CR_5} ,$$
 (B.22) ^

$$= 1 - \frac{M}{2-M-(CR_1+CR_3+CR_5) + CR_1 CR_3 CR_5} ,$$
 ^

where  $M = (1-CR_1)(1-CR_3)(1-CR_5)$ .

(a) The absorbed part

The absorbed part of the solar radiation that reaches the earth's surface can be obtained from (III.50) - (III.53). From (III.50) and (B.20),

$$S_{\alpha 6}^i = \frac{S_0 \alpha_6}{(1-\alpha_s CR_{135})} \quad \text{delete}$$
 (B.23) ^

From (III.51), (III.52) and (B.18) - (B.20), the contribution to the downward flux of solar radiation at the earth's surface from multiple reflections due to the top of cloud  $i+1$ ,  $S_{\alpha 6}^i$ , is

$$S_{\alpha 6}^3 = S_{\alpha 3} CR_3 \frac{CR_1 (1-CR_{35})}{(1-CR_1 CR_{35})(1-\alpha_s CR_{135})} ,$$
 (B.24) ^

$$S_{\alpha 6}^5 = S_{\alpha 5} CR_5 \frac{CR_{13} (1-CR_5)}{(1-CR_{13} CR_5)(1-\alpha_s CR_{135})} ,$$
 (B.25) ^

The total absorbed part of the solar radiation reaching the earth's surface is then by (III.53),

$$S'_{a6} + S_{a6}^2 + S_{a6}^4 \quad (B.26)$$

(b) The scattered part

The scattered part of the solar radiation that reaches the earth's surface can be obtained from (III.54) and (III.55),

$$S_{s6} = S_o \frac{1 - \alpha_c}{1 - \alpha_c \alpha_s} \quad (B.27)$$

$$\alpha_c = 1 - (1 - \alpha_o)(1 - CR'_{135}) \quad (B.28)$$

where  $CR'_{135}$  is the total albedo of the clouds in the entire atmospheric column;  $CR'_{135}$  is found by replacing  $CR_i$  by  $CR'_i$  in (B.22) and using  $CR'_i = CL_i R'_i$  where  $R'_i$  is the albedo for the scattered part of cloud  $i$ .

Finally, the total downward flux at the earth's surface is  $S'_{a6} + S_{a6}^2 + S_{a6}^4 + S_{s6}$ . Therefore, by (III.56), the absorption at the earth's surface is

$$\begin{aligned} S_s &= (1 - \alpha_s) (S'_{a6} + S_{a6}^2 + S_{a6}^4 + S_{s6}) \\ &= (1 - \alpha_s) \left\{ \frac{1}{1 - \alpha_s CR_{135}} \left[ S_{a6} + S_{a6} \frac{CR_3 CR_1 (1 - CR_{35})}{1 - CR_1 CR_{35}} \right. \right. \\ &\quad \left. \left. + S_{a6} \frac{CR_5 CR_{13} (1 - CR_5)}{1 - CR_{13} CR_5} \right] + S_o \frac{1 - \alpha_c}{1 - \alpha_c \alpha_s} \right\} \quad (B.29) \end{aligned}$$

In the actual programming, clouds are indexed somewhat differently from the present description. Cloud types 1, 2 and 3 refer to cloud layers 1, 3 and 5 of the present indexing scheme, respectively. The values of the equivalent cloud water vapor amount,  $u_{ci}^*$ , for cloud types 1, 2 and 3 are adopted for high, middle and low clouds, respectively, as shown in Table III-2. The surface albedos currently in use are presented in section III.5.

Appendix C: Required accuracy of  $m$ 

For simplicity, consider the layer between levels  $i-2$  and  $i$  shown in Fig. II-3. From (II.16) and (II.17), the contributions of this layer to the net flux of long wave radiation at levels  $i$  and  $i-2$  are respectively

$$C_{i-1,i} = (\pi B_i - \pi B_{i-2}) \bar{\tau}_{i-2,i}$$

and

$$C_{i-1,i-2} = (\pi B_i - \pi B_{i-2}) \bar{\tau}_{i,i-2}$$

From (II.11), the fractional contribution of the above fluxes to the heating rate of the layer between  $i-2$  and  $i$ ,  $h$ , is

$$h = \frac{g(C_{i-1,i} - C_{i-1,i-2})}{c_p \Delta p}$$

Assuming a constant temperature lapse rate,  $\Gamma = \partial T / \partial p$ ,  $h$  becomes

$$h = 4g c_p^{-1} \sigma T_i^3 \Gamma (\bar{\tau}_{i-2,i} - \bar{\tau}_{i,i-2})$$

where the following approximation has been used

$$T_i^4 - T_{i-2}^4 = 4T_i^3 (T_i - T_{i-2}) = 4\Delta p \Gamma T_i^3$$

If  $\bar{\tau}$  has a maximum error of  $(\Delta \bar{\tau})_{MAX}$ , the error of  $h$ ,  $\Delta h$ , is

$$\Delta h \leq 8g c_p^{-1} \sigma T_i^3 \Gamma (\Delta \bar{\tau})_{MAX}$$

The maximum possible value of  $\Delta h$  is then

$$(\Delta h)_{MAX} = 8g c_p^{-1} \sigma T_i^3 \Gamma (\Delta \bar{\tau})_{MAX}$$

Considering somewhat large values of  $T_i$  and  $\Gamma$ , such as  $T_i = 300^\circ K$  and  $\Gamma = 8^\circ K/100 \text{ mb}$ , we find

$$(\Delta \bar{\tau})_{MAX} = (\Delta h)_{MAX} / 7.2 \quad (C.1)$$

where the unit of  $\Delta h$  is  $^\circ K \text{ day}^{-1}$ .

Now, we consider that a 10 per cent error is permissible in the <sup>calculation</sup> ~~estimation~~ of the heating rate. The maximum cooling rate of an atmospheric layer due to long wave radiation is about  $2^{\circ}\text{K day}^{-1}$  in a clear sky. Then the maximum permissible error in the heating rate <sup>will</sup> ~~may~~ be about  $0.2^{\circ}\text{K day}^{-1}$ . From (C.1),  $\bar{\tau}$  is thus permitted to have a maximum error of about 0.03.

From (II.18) or (II.19), the error of the interpolation factor  $m$  is related to  $\Delta\bar{\tau}$  as follows:

$$\Delta m = \frac{-\Delta\bar{\tau}(1+m)^2}{\Delta\bar{\tau}(1+m)+(1-\tau)} \quad (\text{C.2})$$

The maximum permissible error in  $m$ , for  $\Delta\bar{\tau} = 0.03$ , is shown in Table C-1 for several combinations of  $m$  and  $\tau$ . The combinations shown in the Table above the heavy solid line occur in the real atmosphere only in extreme cases when  $\Delta p$  is larger than 100 mb. From this viewpoint, it is evident that the maximum permissible error in  $m$  is at least 20 per cent and increases with distance below the heavy solid line.

TABLE C.1. Required accuracy in  $m$  for the accuracy of 0.03 in  $\bar{\tau}$ .

m	$\tau$								
	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
1	0.1	0.1	0.2	0.2	0.2	0.3	0.3	0.5	0.8
2	0.3	0.3	0.3	0.4	0.5	0.6	0.7	0.9	1.4
3	0.5	0.5	0.6	0.7	0.8	0.9	1.1	1.5	2.2
4	0.7	0.8	0.9	1.0	1.2	1.4	1.7	2.1	3.0
6	1.3	1.5	1.6	1.8	2.1	2.4	2.9	3.6	4.7
8	2.1	2.3	2.5	2.8	3.2	3.6	4.3	5.2	6.6
10	3.0	3.2	3.5	3.9	4.4	5.0	5.8	6.9	8.4
15	5.6	6.0	6.5	7.1	7.8	8.7	9.9	11.3	13.4

Appendix D: The seasonal variation of the sun's declination and distance from the earth.

In a numerical simulation of the seasonal variation of the general circulation of the atmosphere, the seasonal variation of the solar declination,  $\delta$ , and the earth-sun distance,  $r_E$ , must be known.

The apparent orbital motion of the sun about the earth is shown in Fig. D-1 by the ellipse  $ABA'B'A$  with the earth located at focus  $F$ . The circle  $ACA'C'A$  represents the orbit of a fictitious sun which moves with constant speed and a one year period. Let the sun and fictitious sun be at perigee,  $A$ , at time  $t_0$  and at positions  $S$  and  $S'$  at time  $t$ , respectively.

The mean and true anomalies of the sun are shown by the angles  $M$  and  $\omega$ . The mean anomaly is given by

$$M(t) = \frac{2\pi}{T} (t - t_0)$$

where  $T$  is the one year period. The date of perigee varies annually from Jan. 2 to Jan. 5. The mean date of perigee for the years 1950-1972 was Jan. 3.36 (UT). Thus, letting  $t$  represent the time in days from the beginning of a year,  $t_0$  is

2.36 days. Assuming  $T$  equal to 365 days,  $M$  is given by

$$M(t) = 0.0172142 (t - 2.36), \text{ radians}$$

An asymptotic solution of Kepler's second law in terms of the eccentricity  $e$  of the elliptic orbit yields

$$r_E(t)/\bar{r}_E = A_0 - A_1 \cos M - A_2 \cos 2M - A_3 \cos 3M - \dots$$

and

$$\omega(t) = M + B_1 \sin M + B_2 \sin 2M + B_3 \sin 3M + \dots$$

where

$$A_0 = 1 + e^2 = 1.00027956$$

$$A_1 = e - \frac{3}{8}e^3 - \frac{5}{32}e^5 - \dots \approx 0.01671825$$

$$A_2 = \frac{1}{2}e^2 - \frac{1}{3}e^4 - \dots \approx 0.00013975$$

$$A_3 = \frac{3}{8}e^3 - \frac{135}{64}e^5 - \dots \approx 0.00000175$$

$$B_1 = 2e - \frac{1}{4}e^3 + \frac{5}{96}e^5 - \dots \approx 0.0334388$$

$$B_2 = \frac{5}{4}e^2 - \frac{11}{24}e^4 + \dots \approx 0.0003494$$

$$B_3 = \frac{13}{12}e^3 - \frac{645}{940}e^5 + \dots \approx 0.00000506$$

and  $\bar{r}_E$  is one astronomical unit. The numerical values given above are for the earth's orbit ( $e = 0.01672$ ).

Letting  $\ell$  and  $\epsilon$  denote the ecliptic longitude of the sun and the inclination of the earth's orbit ( $23^\circ 27'$ ), respectively, the solar declination  $\delta$  is given by

$$\delta(t) = \sin^{-1}(\sin \epsilon \sin \ell)$$

where

$$\ell(t) = \omega(t) + \ell_0$$

and  $\ell_0$ , the ecliptic longitude at perigee, is  $-1.3550737$  radians ( $-77.64^\circ$ ).  $\delta$  is positive for  $0 \leq \ell \leq \pi$  and negative for  $\pi \leq \ell \leq 2\pi$ .