NASA TECHNICAL NOTE

NASA TN D-7502

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ELASTOHYDRODYNAMIC FILM THICKNESS FORMULA BASED ON X-RAY MEASUREMENTS WITH A SYNTHETIC PARAFFINIC OIL

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NATIONAL AERONAUTICS AND SPACE ADMINISTRATION • WASHINGTON, D. C. • DECEMBER 1973

1. Report No. NASA TN D-7502	2. Government Accession No.	3. Recipient's Catalog No.
4. Title and Subtitle ELASTOHYDRODYNAMIC FILM THICKNESS FORMULA BASED ON X-RAY MEASUREMENTS WITH A SYNTHETIC PARAFFINIC OIL		5. Report Date December 1973.
		6. Performing Organization Code
7. Author(s) Stuart H. Loewenthal, Richard J. Parker, and Erwin V. Zaretsky		8. Performing Organization Report No. E-7041
9. Performing Organization Name and Address NASA Lewis Research Center and U.S. Army Air Mobility R&D Laboratory Cleveland, Ohio 44135		10. Work Unit No. 501-24
		11. Contract or Grant No.
12. Sponsoring Agency Name and Address National Aeronautics and Space Administration Washington, D.C. 20546		13. Type of Report and Period Covered Technical Note
		14. Sponsoring Agency Code

15. Supplementary Notes

16. Abstract

An empirical elastohydrodynamic (EHD) film thickness formula for heavily loaded contacts based upon X-ray film thickness measurements made with a synthetic paraffinic oil was developed. The deduced relation was found to adequately reflect the high load dependence exhibited by the measured minimum film thickness data at high Hertzian contact stresses, that is, above 1.04×10^{9} N/m^{2} (150 000 psi). Comparisons were made with the numerical results from a theoretical isothermal film thickness formula. The effects of changes in contact geometry, material, and lubricant properties on the form of the empirical model are also discussed.

17. Key Words (Suggested by Author(s)) Bearings; Lubrication; Elastohydrodynamic lubrication; Rolling-element bearings; Ball bearings; High temperature lubrication; Film thickness; Synthetic lubricants	18. Distribution Statement Unclassified - unlimited

19. Security Classif. (of this report)	20. Security Classif. (of this page)	21. No. of Pages	22. Price*
Unclassified	Unclassified	21	Domestic, \$2.75 Foreign, \$5.25

* For sale by the National Technical Information Service, Springfield, Virginia 22151

ELASTOHYDRODYNAMIC FILM THICKNESS FORMULA BASED ON X-RAY MEASUREMENTS WITH A SYNTHETIC PARAFFINIC OIL

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SUMMARY

An empirical elastohydrodynamic (EHD) film thickness formula for heavily loaded contacts based upon X-ray film thickness measurements made with a synthetic paraffinic oil was developed. The empirical model was formulated from test data which covered a wide and practical range of operating conditions. Maximum Hertz stresses ranged from 1.04×10^{9} to 2.42×10^{9} N/m^2 (150 000 to 350 000 psi), disk temperatures from 339 to 505 K (150° to 450° F), and mean surface speeds from 9.4 to 37.6 m/sec (370 to 1480 in./sec).

Predicted values of minimum film thickness from the deduced film thickness formula were compared to X-ray test data and to the numerical results from a theoretical isothermal EHD film thickness equation. In contrast to conventional theory, the present film thickness model did reflect the high sensitivity of film thickness to applied load exhibited by the test data. The empirical model showed improved agreement with the X-ray test data throughout the range of test conditions.

It was judged that the empirical film thickness formula can be used with reasonable certainty for rolling-element bearing and gear systems composed of steel, employing the synthetic paraffinic oil studied herein and whose contact geometry approximates that upon which the model is based.

INTRODUCTION

The importance of maintaining a sufficient elastohydrodynamic (EHD) film thickness between dynamically contacting machine elements has in recent years been more fully appreciated. The prediction of elastohydrodynamic film thickness has been the focal point of many theoretical and experimental investigations (refs. 1 and 2). The ratio of the elastohydrodynamic minimum film thickness to the composite surface roughness has become an acceptable indicator of the effectiveness of the lubricant film within the rolling-element contact zone. It has been experimentally shown to influence the fatigue life of rolling-element bearings (refs. 3 and 4). Predetermination of this lubricant parameter involving an accurate prediction of minimum film thickness will be of great value to the designer in obtaining realistic estimates of component fatigue life (ref. 5).

The bulk of the experimental work conducted in elastohydrodynamic lubrication has been confined to conditions of moderate speeds; that is, up to 25.4 m/sec (1000 in./sec), and moderate loads; that is, maximum Hertz stresses to 1.24×10^9 N/m^2 (180 000 psi) (refs. 6 to 9). The research of reference 10 has extended the elastohydrodynamic film thickness measurements to maximum Hertz stresses of 2.42×10^9 N/m^2 (350 000 psi) which includes the design operating range of most machine components such as bearings and gears. In contrast to the results obtained by previous investigators which showed reasonably good correlation at moderate speeds and loads between elastohydrodynamic theory and film thickness measurement, the data of reference 10 showed a marked deviation between prediction and experiment. In particular, at high contact stresses; that is, maximum Hertz stresses greater than 1.38×10^9 N/m^2 (200 000 psi), the sensitivity of the film thickness to load as determined experimentally is far greater than that predicted by the classical elastohydrodynamic theory of references 11 and 12.

Several attempts have been made to resolve the apparent discrepancy between theory and experiment. A critical examination of the X-ray technique itself was made (ref. 13) for possible load dependent experimental errors. However, no experimental factors were uncovered which could seriously alter the accuracy of the X-ray measurements. On the theoretical side, the influence of several possible rheological factors have been investigated, such as the effects of a non-Newtonian lubricant of the Ree-Eyring form (ref. 14), the effects of heating at the inlet of the contact region (ref. 15) and the effects of a reduced lubricant viscosity-pressure dependence using a composite exponential model (ref. 16) and using a power-law model (ref. 17).

While each of the aforementioned modifications to elastohydrodynamic theory has succeeded somewhat in improving the agreement between theory and experimental data within the heavy load regime, the resulting predicted values of film thickness differed little in magnitude from those computed using classical EHD theory. Furthermore, the modified theories do not sufficiently account for the high film thickness-load dependence to allow accurate predictions of film thickness under realistic operating conditions.

It becomes the objective of the work reported herein to (1) develop an empirical elastohydrodynamic film thickness model based on an analysis of the experimental data of reference 10 and (2) compare the empirical relation derived to that of conventional elastohydrodynamic theory.

SYMBOLS

a	minor semiaxis of Hertzian contact, m (in.)
b	major semiaxis of Hertzian contact, m (in.)
C_i	coefficient defined indirectly by eq. (1)
E_1, E_2	modulus of elasticity of elements 1 and 2, N/m^2 (psi)
E'	$\left(\frac{1 - \nu_1^2}{\pi R_1} + \frac{1 - \nu_2^2}{\pi E_2}\right)^{-1}$, N/m^2 (psi)
$\overline{H}_{min}$	dimensionless minimum film thickness parameter, $\frac{h_{min}}{R'}$
h_c	film thickness in Cheng's theory defined by eq. (8), m (in.)
h_{min}	minimum film thickness, m (in.)
K_j	coefficient defined indirectly by eq. (6)
n_i	exponent defined indirectly by eq. (1)
n_m	exponent defined indirectly by eq. (3)
$\overline{P}_{Hz}$	dimensionless stress parameter, $\frac{p_{Hz}}{E'}$
p_{Hz}	maximum Hertz stress, N/m^2 (psi)
R_1, R_2	radius of elements 1 and 2 in rolling direction, m (in.)
R'	$\left(\frac{1}{R_1} + \frac{1}{R_2}\right)^{-1}$, m (in.)
S_j	exponent defined indirectly by eq. (6)
T	local temperature, K (^oF)
T_0	disk temperature, K (^oF)
$\overline{U}$	dimensionless speed-viscosity parameter, $\frac{\mu_o u}{E'R'}$
u	$\frac{1}{2}(u_1 + u_2)$, m/sec (in./sec)
u_1, u_2	surface velocities of elements 1 and 2, m/sec (in./sec)
α	pressure-viscosity coefficient, m^2/N (in.^{2}/lbf)

μ_o	contact inlet absolute viscosity, N-sec/m^2, (lb-sec/in.^{2})
ν_1, ν_2	Poisson's ratio of elements 1 and 2

EHD FILM THICKNESS EXPERIMENTAL DATA

The film thickness model which is presented herein was developed from film thickness data (ref. 10) acquired in an X-ray rolling disk machine (fig. 1) with a synthetic paraffinic oil. This particular oil is of interest since it is one of a class of several ad-

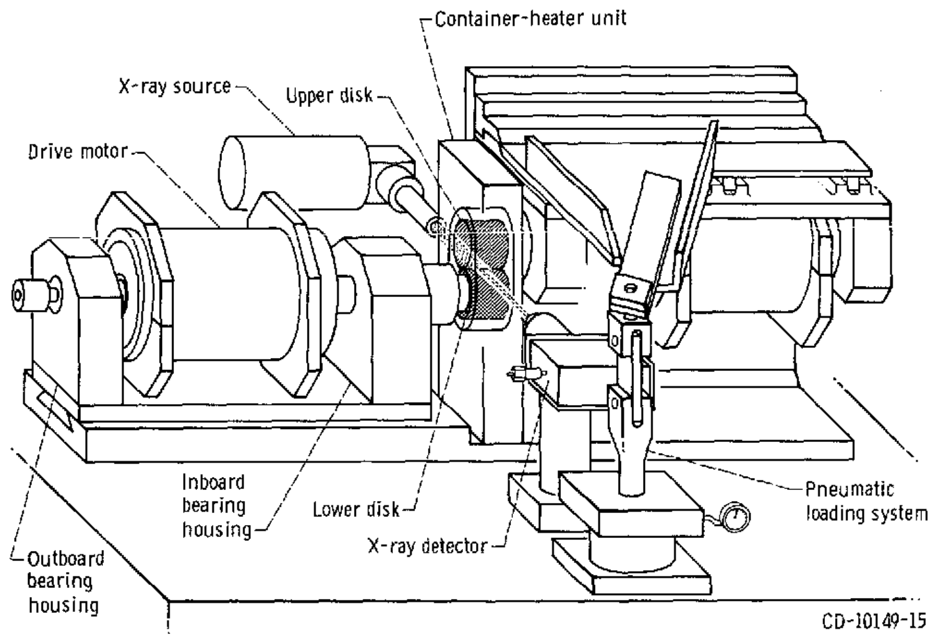


Figure 1. - X-ray rolling-contact disk machine (ref. 10).

vanced lubricants singled out for high temperature jet engine application. Its performance as a bearing lubricant has been extensively evaluated (e.g., see refs. 18 and 19).

The method of measuring film thickness with the X-ray technique comprises projecting X-rays between the surfaces of the two contacting disks and detecting the rate of X-ray transmission through the contact. Since the greatest constriction occurs at the trailing edge of the contact, the X-ray count thus becomes a measure of the lubricant's minimum film thickness.

The range of test conditions include disk temperatures from 339 to 505 K (150° to 450° F), surface speeds from 9.4 to 37.6 m/sec (370 to 1480 in./sec) corresponding to disk rolling speeds from 5000 to 20 000 rpm, and maximum Hertz stresses from 1.04×10^{9} to 2.42×10^{9} N/m^2 (150 000 to 350 000 psi). Two crowned-cone AISI M-50 steel disks each with a radius of 1.83 centimeters (0.72 in.) and a surface finish of 2.5×10^{-6} to 5.0×10^{-6} centimeter (1 to 2 μ in.) rms were used as the test specimens (fig. 2). (Both crowned disks and crowned-cone disks (with a cone angle of 10°) were tested and no significant differences were found between the two sets of data.) All the data reported herein were generated with the crowned-cone test disks. The properties of the synthetic paraffinic oil which was used in the course of this study are listed in table I.

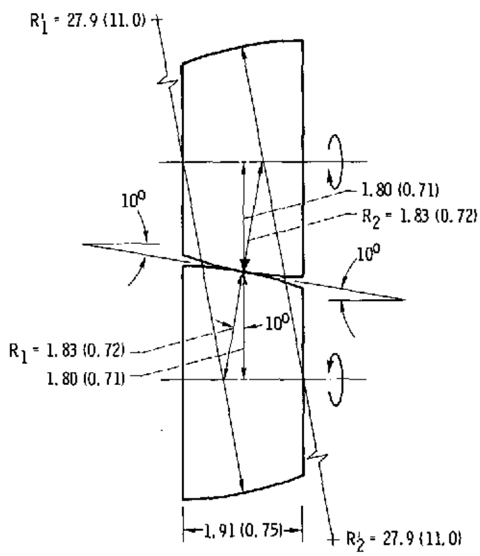


Figure 2. - Contacting disk geometry (ref. 10). (All linear dimensions in cm (in.).)

TABLE I. - PROPERTIES OF THE SYNTHETIC PARAFFINIC OIL

Kinematic viscosity, cS (or 10^{-6} m^2/sec), at -	
233 K (-40° F)	>100 000
311 K (100° F)	443
372 K (210° F)	40
478 K (400° F)	5.8
589 K (600° F)	[a]2.3
Flash point, K ($^{\circ}$F)	541 (515)
Fire point, K ($^{\circ}$F)	589 (600)
Autoignition temperature, K ($^{\circ}$F)	703 (805)
Volatility (6.5 hr at 533 K (500° F)), wt. %	14.2
Specific heat at 533 K (500° F), J/(kg)(K) (Btu/(lb)($^{\circ}$F))	2910 (0.695)
Thermal conductivity at 533 K (500° F), J/(m)(sec)(K) (Btu/(hr)(ft)($^{\circ}$F))	0.12 (0.070)
Specific gravity at 533 K (500° F)	0.71
Pressure-viscosity coefficient at 298 K (77° F), m^2/N (psi^{-1})	[b]1.3×10^{-8} (0.92×10^{-4})

[a]Extrapolated.
[b]From ref. 25.

A summary of the X-ray test results showing measured minimum film thickness h_{min} plotted against maximum Hertz stress p_{Hz} for various mean surface speeds u and disk temperatures T_0 is presented in figure 3. Within a heavily loaded contact, the film thickness dependence upon load as evidenced by the X-ray data is far greater than any current EHD theory predicts. This is illustrated by the dashed line appearing in figure 3(a) whose slope represents the typical stress exponent from a commonly used

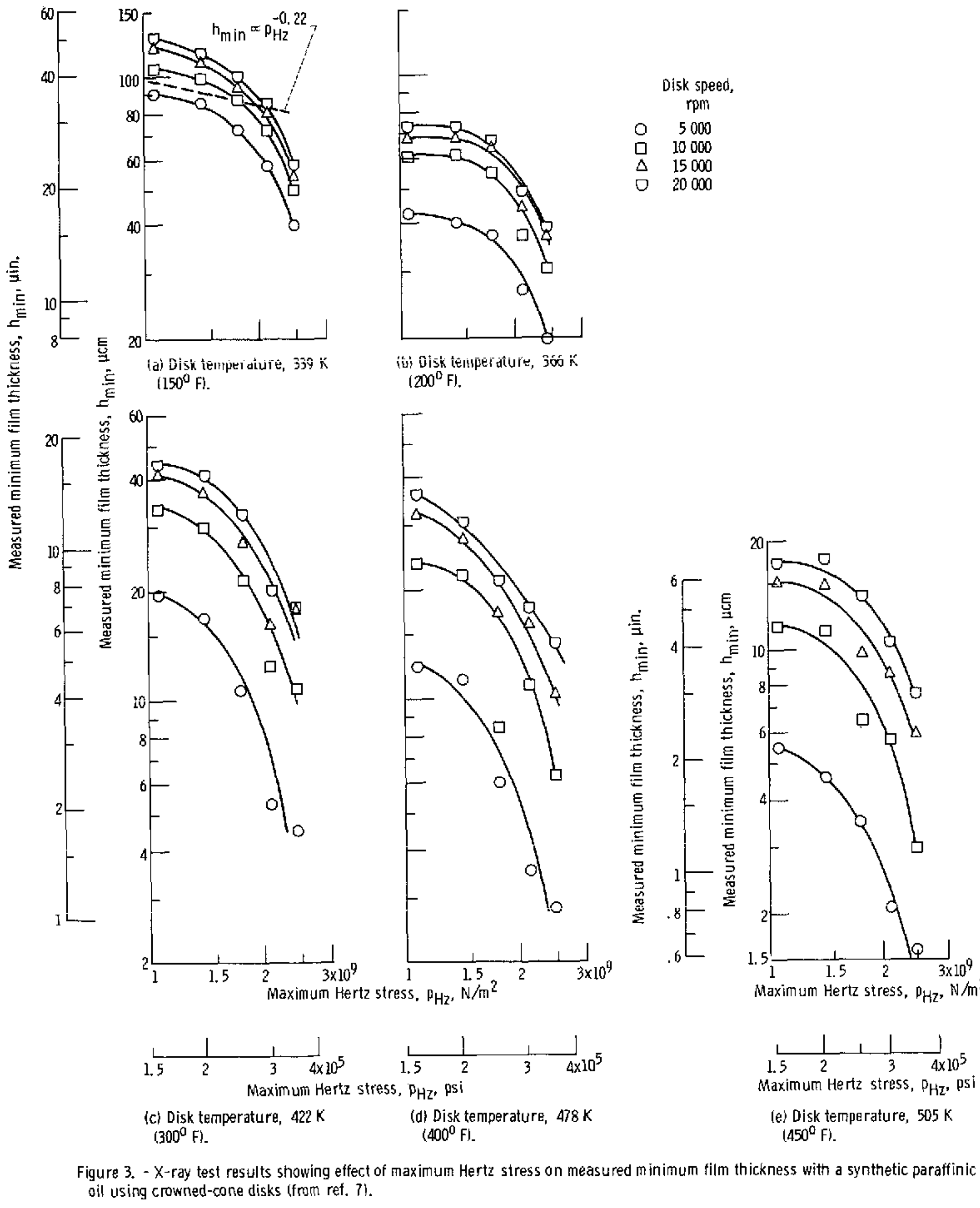


Figure 3. - X-ray test results showing effect of maximum Hertz stress on measured minimum film thickness with a synthetic paraffinic oil using crowned-cone disks (from ref. 7).

film thickness formula (ref. 16). The pronounced effect of increasing load on h_{min} is clearly apparent from these graphs.

By introducing the customary nondimensional elastohydrodynamic groupings, the measured X-ray data can be represented over the entire range of test conditions in a single plot. This is accomplished in figure 4, which is a log-log plot of the dimensionless film thickness parameter $\overline{H}_{min} = h_{min}/R'$ plotted against the dimensionless speed-viscosity parameter $\overline{U} = \mu_0 u/E'R'$ for various maximum Hertz stresses. The straight

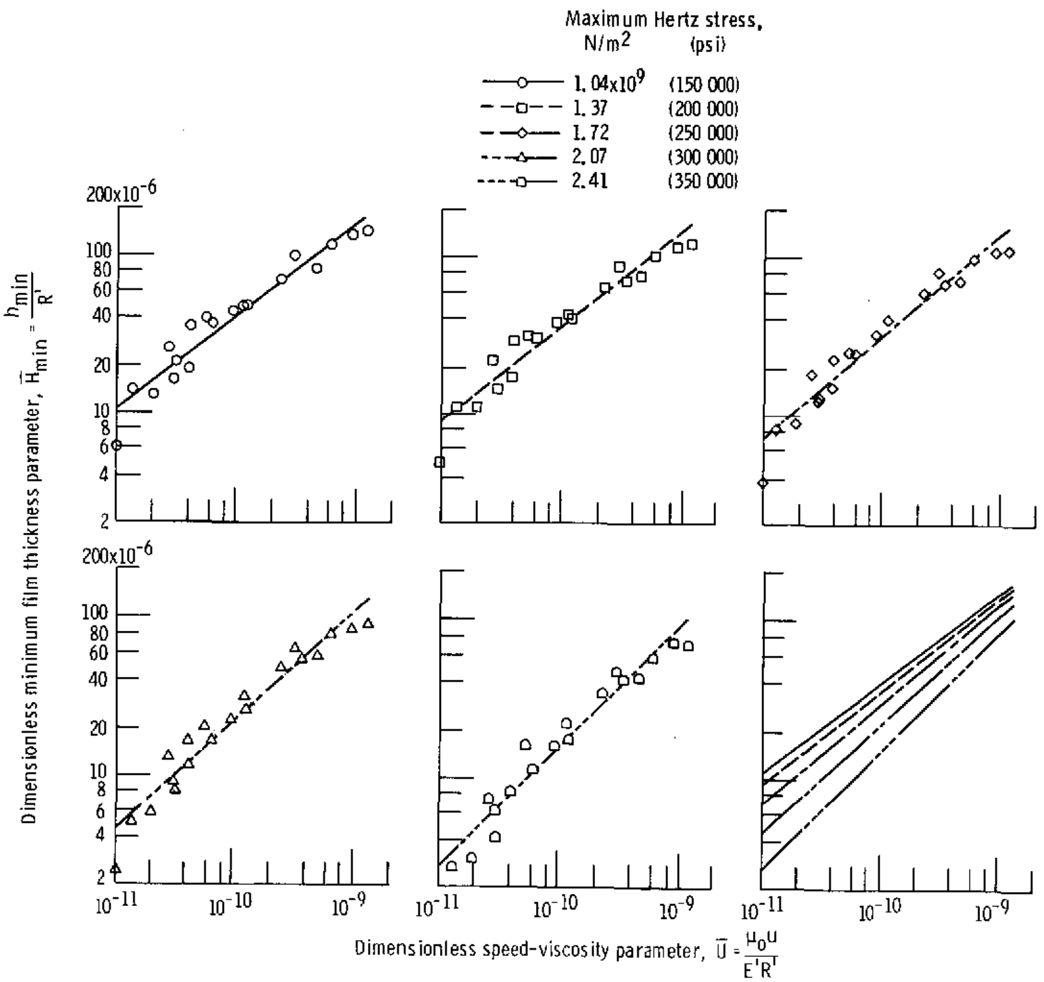


Figure 4. - Effect of maximum Hertz stress on sensitivity of minimum film thickness to changes in speed and viscosity for synthetic paraffinic oil.

lines appearing on this figure have been least-square fitted through the measured data for each test stress level and can be represented by the following simple power relation:

$$\frac{h_{min}}{R'} = C_i \left(\frac{\mu_o u}{E'R'}\right)^{n_i} \tag{1}$$

or in dimensionless groupings

$$\overline{H}_{min} = C_i \overline{U}^{n_i} \tag{2}$$

where parameters C_i and n_i, listed in table II, are some function of maximum Hertzian contact pressure, say $f(p_{Hz})$.

It is apparent from figure 5 that the measured minimum film thickness shows an enhanced sensitivity to surface speed and lubricant viscosity with increasing stress level. The variation of exponent n_i with operating conditions is not accounted for by conventional elastohydrodynamic theory. However, Westlake and Cameron (ref. 20), in studying some 35 different fluids, report a similar variation of the exponent for the nondimensional speed-viscosity parameter with contact load for their empirical midfilm thickness equation. These data were generated using an optical interferometric method for determining the amount of film generated between a rolling steel ball and a flat glass plate. It should be mentioned that no such variation of exponent with stress level was observed for their minimum film thickness data. Furthermore, the data of reference 20 was obtained from elastohydrodynamic contacts in which the lubricant was subject to shear rate levels and contact pressure levels which are considerably lower than those generated by the rolling disk contact in the X-ray experiments.

For purposes of developing an approximate film thickness expression for synthetic paraffinic oils, the complication of a pressure dependent speed-viscosity parameter exponent can be avoided without introducing serious inaccuracies by selecting some mean value of n_i, that is, n_m, so that equation (2) can be written as

$$\overline{H}_{min} = \overline{U}^{n_m} f(p_{Hz}) \tag{3}$$

where the function $f(p_{Hz})$ has been introduced to mathematically model the effect of contact stress on minimum film thickness and where it has been determined that $n_m = 0.66$. By comparing equations (2) and (3), it is clear that

$$f(p_{Hz}) = C_i \overline{U}^{(n_i - 0.66)} \tag{4}$$

TABLE II. - EMPIRICAL MINIMUM FILM THICKNESS COEFFICIENTS FOR EQUATION (1)

Dimensionless stress parameter, $\overline{P}_{Hz} = \frac{P_{Hz}}{E'}$	Maximum Hertz stress		Coefficient C_i	Exponent n_i
	N/m^2	psi		
3.09×10^{-3}	1.04×10^{9}	150 000	28.1	0.584
4.12	1.38	200 000	44.6	.609
5.15	1.72	250 000	93.5	.649
6.17	2.07	300 000	231.	.702
7.20	2.42	350 000	654.	.764

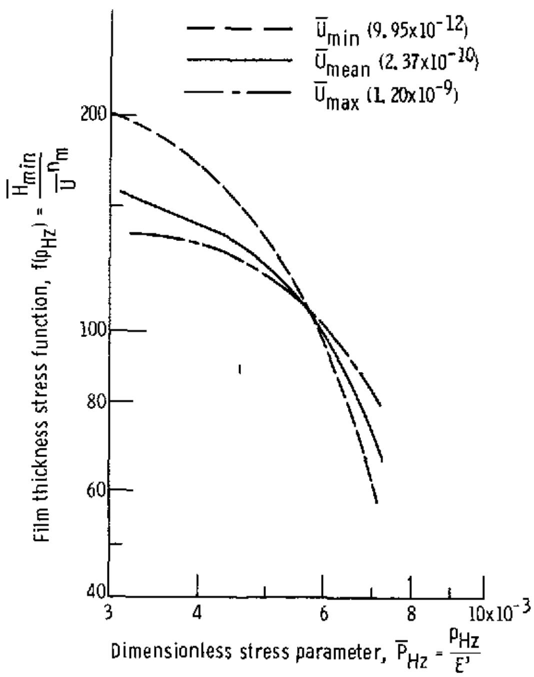


Figure 5. - Sensitivity of minimum film thickness to maximum Hertz stress as a function of surface speed and lubricant viscosity for synthetic paraffinic oil.

or more concisely

$$f(p_{Hz}) = \frac{H_{min}}{\bar{U}^{0.66}} \tag{5}$$

It is evident from equation (4) that $f(p_{Hz})$ is unavoidably an explicit function of $\bar{U}$. The extent of the dependence of $f(p_{Hz})$ with $\bar{U}$ over its experimental range is illustrated in figure 5. In this plot the broken lines represent the variation of the log of $f(p_{Hz})$ with the log of the dimensionless stress parameter $\bar{P}_{Hz}$ at the minimum and maximum experimental values of $\bar{U}$. It is apparent from this figure that the sensitivity of minimum film thickness to maximum Hertz stress will vary slightly in accordance with the operating speed and temperature condition. At the risk of introducing a small degree of error, the extent of which will become evident later, and in order to conform more closely to conventional elastohydrodynamic theory, the variation of film thickness to contact stress will be modeled for a constant mean value of the experimental dimensionless speed-viscosity parameter. This variation is represented by the solid curve in figure 5. There are obviously several ways of mathematically representing this variation of $f(p_{Hz})$ with $\bar{P}_{Hz}$. The one which is presented here is partially in keeping with conventional elastohydrodynamic theory, that is, let

$$f(p_{Hz}) = K_j P_{Hz}^{S_j} \tag{6}$$

where coefficients K_j and S_j would normally be constants independent of contact load under conventional theory but here by necessity K_j and S_j will be deemed as quasi-constant, that is, constant over specific intervals of operating contact stress as shown in table III. Thus when equations (3) and (6) are combined, they form a complete corre-

TABLE III. - MINIMUM FILM THICKNESS CORRELATION COEFFICIENTS FOR EQUATION 7

Range of operating contact stress			Coefficient K_j	Exponent S_j
Dimensionless stress parameter, $P_{Hz} = \frac{p_{Hz}}{E'}$	Maximum Hertz stress			
	N/m^2	psi		
3.09×10^{-3} to 4.12×10^{-3}	1.04×10^{9} to 1.38×10^{9}	150 000 to 200 000	31.1	-.38
4.13×10^{-3} to 5.15×10^{-3}	1.39×10^{9} to 1.72×10^{9}	201 000 to 250 000	2.81	-.72
5.16×10^{-3} to 6.17×10^{-3}	1.73×10^{9} to 2.07×10^{9}	251 000 to 300 000	.0864	-1.38
6.18×10^{-3} to 7.20×10^{-3}	2.08×10^{9} to 2.42×10^{9}	301 000 to 350 000	.00127	-2.21

lation for determining minimum film thickness over a wide range of temperature and speed conditions for heavy contact loads, that is, above a p_{Hz} of 1.04×10^9 N/m^2 (150 000 psi). This correlation can be written:

$$\overline{H}_{min} = K_j \overline{U}^{0.66} \overline{P}_{Hz}^{S_j} \tag{7}$$

where the four discrete values of coefficients K_j and S_j over the maximum Hertz stress range from 1.04×10^9 to 2.42×10^9 N/m^2 (150 000 to 350 000 psi) appear in table III.

DISCUSSION OF RESULTS

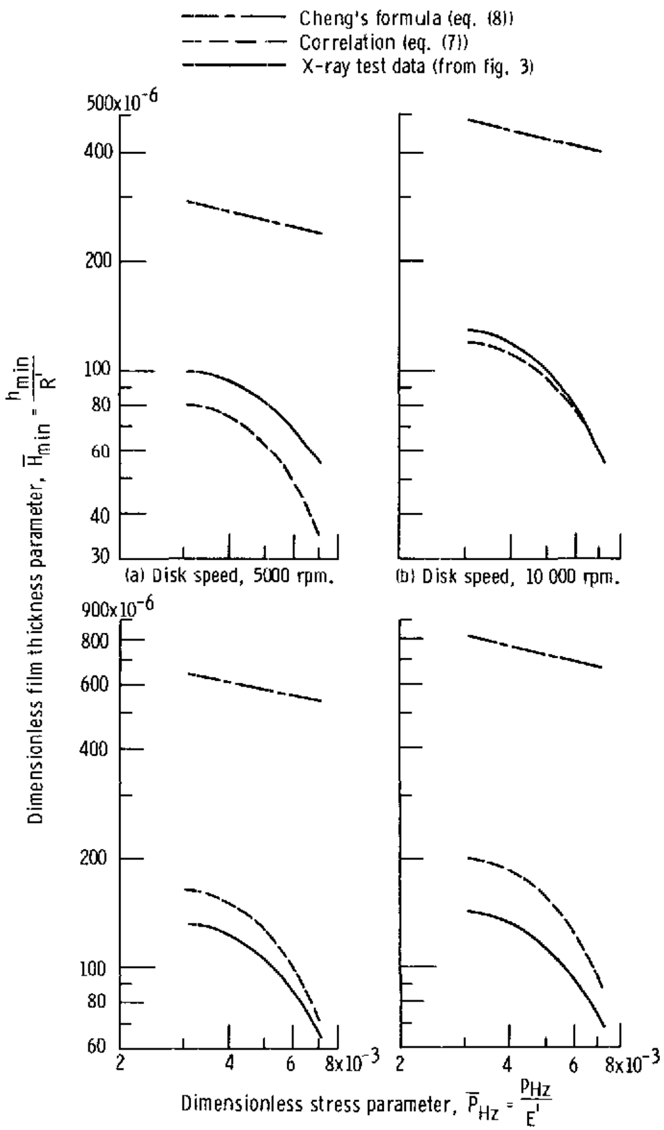
Comparison with Conventional EHD Theory

A comparison of the film thickness correlation from equation (7) with the experimental X-ray data (ref. 10) is shown in tabular form in table IV and in graphical form with the isothermal film thickness equation of Cheng (ref. 21) in figures 6, 7, and 8. Cheng's film thickness formula is considered typical of those from conventional elastohydrodynamic theory for line contact. It can be written

$$\frac{h_c}{R'} = 1.47 \left(\frac{\alpha \mu_o u}{R'}\right)^{0.74} \left(\frac{p_{Hz}}{E'}\right)^{-0.22} \tag{8}$$

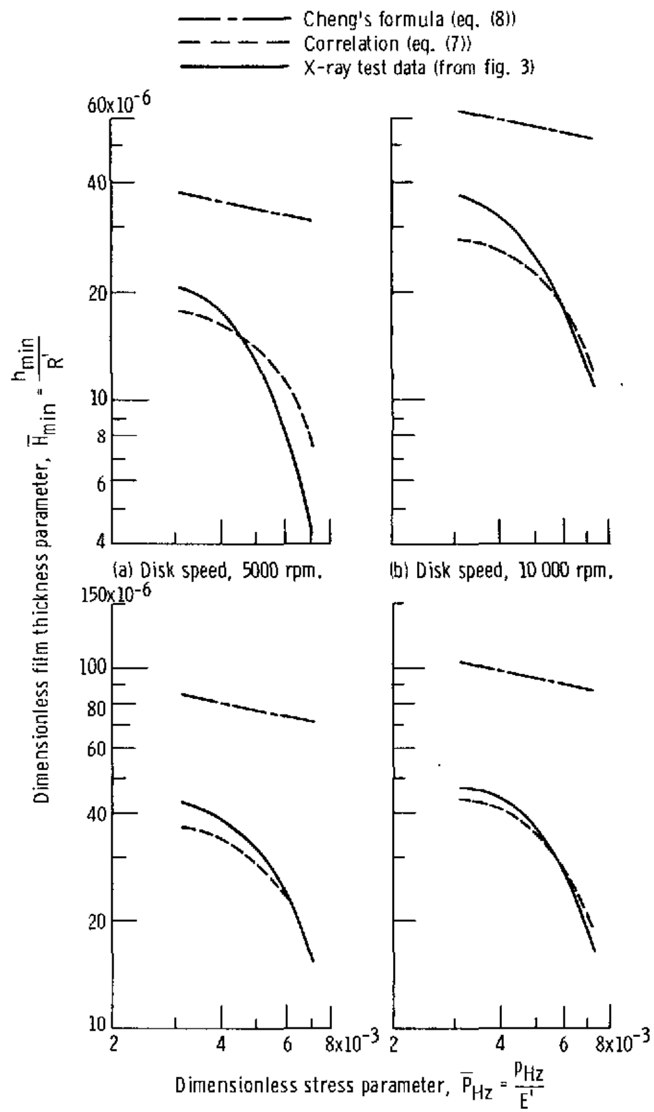
In computing Cheng's film thickness, a correction factor of 0.8 has been applied to adjust Cheng's center film thickness to minimum film thickness. Values of the pressure-viscosity coefficient α, for the synthetic paraffinic oil utilized in this computation were extracted from reference 22 and appear in table V as a function of temperature. In this report (ref. 22) several parameters representing the pressure-viscosity characteristics for this oil were presented. The α parameter deemed most appropriate for film thickness calculations to be made here was reportedly obtained by applying a numerical integration technique to the experimental data. The necessity of extrapolating the published data for α over a relatively large temperature interval (83 K (150° F)), places the value estimated in table V for α at 505 K (450° F) in some question.

As anticipated, it is apparent from the plots presented in figures 6, 7, and 8 that there is improvement in predicting actual minimum film thicknesses with the present empirical model (eq. (7)) for the system under study. This improvement is largely attributed to the ability of the deduced film thickness relation to reflect the high sensitivity of minimum film thickness to contact stress under heavy applied loads. It is evident that deviations of up to several hundred percent exist between measured and predicted values of minimum film thickness using commonly accepted EHD formulas.



(c) Disk speed, 15 000 rpm. (d) Disk speed, 20 000 rpm.

Figure 6. - Comparison of predicted minimum film thickness with X-ray test data for synthetic paraffinic oil at disk temperature T_0 = 339 K (150° F).



(c) Disk speed, 15 000 rpm. (d) Disk speed, 20 000 rpm.

Figure 7. - Comparison of predicted minimum film thickness with X-ray test data for synthetic paraffinic oil at disk temperature T_0 = 422 K (300° F).

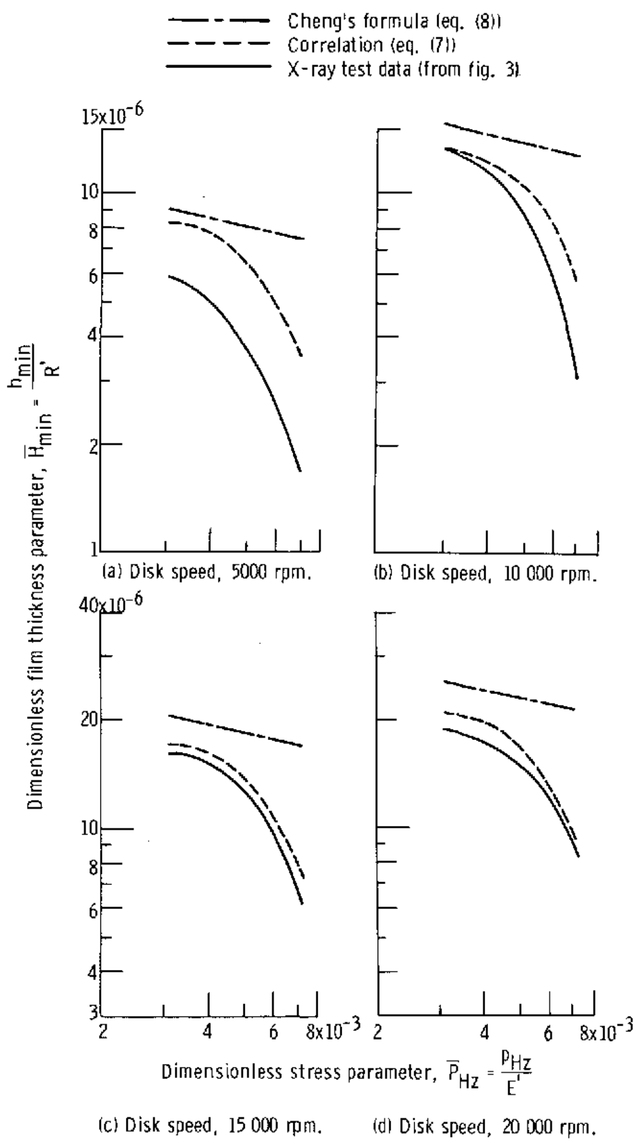


Figure 8. - Comparison of predicted minimum film thickness with X-ray test data for synthetic paraffinic oil at disk temperature $T_0 = 505$ K (450^0 F).

TABLE IV. - COMPARISON OF EMPIRICAL FILM THICKNESS MODEL WITH X-RAY DATA

FOR A SYNTHETIC PARAFFINIC OIL (REF. 7)

(a) SI units

Disk temperature, K	Disk speed, rpm	Maximum Hertz stress, N/m^2	Minimum film thickness, h_{min}, μ cm		Disk temperature, K	Disk speed, rpm	Maximum Hertz stress, N/m^2	Minimum film thickness, h_{min}, μ cm		Disk temperature, K	Disk speed, rpm	Maximum Hertz stress, N/m^2	Minimum film thickness, h_{min}, μ cm	
			X-ray	Correlation				X-Ray	Correlation				X-Ray	Correlation
339	5 000	1.04×10^9	89	74	422	5 000	1.04×10^9	20	15	505	5 000	1.04×10^9	5	8
		1.38	86	69			1.38	18	15			1.38	5	8
		1.72	71	58			1.72	10	13			1.72	5	5
		2.07	58	46			2.07	5	10			2.07	2	5
		2.42	41	33			2.42	5	8			2.42	2	2
	10 000	1.04×10^9	107	114		10 000	1.04×10^9	33	25		10 000	1.04×10^9	12	13
		1.38	99	107			1.38	30	23			1.38	10	10
		1.72	86	91			1.72	20	20			1.72	8	10
		2.07	71	71			2.07	12	15			2.07	5	8
		2.42	51	51			2.42	10	10			2.42	2	5
	15 000	1.04×10^9	119	150		15 000	1.04×10^9	41	33		15 000	1.04×10^9	15	15
		1.38	109	140			1.38	35	30			1.38	15	15
		1.72	94	119			1.72	25	25			1.72	10	13
		2.07	81	91			2.07	15	20			2.07	8	10
		2.42	53	66			2.42	15	15			2.42	5	8
	20 000	1.04×10^9	127	180		20 000	1.04×10^9	43	41		20 000	1.04×10^9	18	18
		1.38	117	168			1.38	41	38			1.38	18	18
		1.72	102	142			1.72	30	30			1.72	15	15
		2.07	84	112			2.07	20	25			2.07	10	10
		2.42	58	79			2.42	20	18			2.42	8	8
366	5 000	1.04×10^9	43	38	478	5 000	1.04×10^9	12	10					
		1.38	41	36			1.38	10	8					
		1.72	38	30			1.72	5	8					
		2.07	25	23			2.07	2	5					
		2.42	20	18			2.42	2	5					
	10 000	1.04×10^9	61	61		10 000	1.04×10^9	23	15					
		1.38	61	56			1.38	23	13					
		1.72	58	48			1.72	8	13					
		2.07	38	38			2.07	10	10					
		2.42	30	25			2.42	8	8					
	15 000	1.04×10^9	66	79		15 000	1.04×10^9	30	20					
		1.38	66	74			1.38	28	18					
		1.72	64	63			1.72	18	15					
		2.07	43	48			2.07	15	13					
		2.42	38	36			2.42	10	8					
	20 000	1.04×10^9	71	96		20 000	1.04×10^9	36	23					
		1.38	71	89			1.38	30	20					
		1.72	66	76			1.72	20	18					
		2.07	48	58			2.07	18	15					
		2.42	41	43			2.42	15	10					

TABLE IV. - Concluded. COMPARISON OF EMPIRICAL FILM THICKNESS MODEL WITH X-RAY DATA

FOR A SYNTHETIC PARAFFINIC OIL (REF. 7)

(b) U.S. customary units

Disk temperature, °F	Disk speed, rpm	Maximum Hertz stress, psi	Minimum film thickness, h_{min}, μ in.	
			X-ray	Correlation
150	5 000	1.5×10^5	35	29
		2.0	34	27
		2.5	28	23
		3.0	23	18
		3.5	16	13
	10 000	1.5×10^5	42	45
		2.0	39	42
		2.5	34	36
		3.0	28	28
		3.5	20	20
	15 000	1.5×10^5	47	59
		2.0	43	55
		2.5	37	47
		3.0	32	36
		3.5	21	26
	20 000	1.5×10^5	50	71
		2.0	46	66
		2.5	40	56
		3.0	33	44
		3.5	23	31
200	5 000	1.5×10^5	17	15
		2.0	16	14
		2.5	15	12
		3.0	10	9
		3.5	8	7
	10 000	1.5×10^5	24	24
		2.0	24	22
		2.5	22	19
		3.0	15	15
		3.5	12	10
	15 000	1.5×10^5	26	31
		2.0	26	29
		2.5	25	25
		3.0	17	19
		3.5	15	14
	20 000	1.5×10^5	28	38
		2.0	28	35
		2.5	26	30
		3.0	19	23
		3.5	16	17

Disk temperature, °F	Disk speed, rpm	Maximum Hertz stress, psi	Minimum film thickness h_{min}, μ cm	
			X-Ray	Correlation
300	5 000	1.5×10^5	8	6
		2.0	7	6
		2.5	4	5
		3.0	2	4
		3.5	2	3
	10 000	1.5×10^5	13	10
		2.0	12	9
		2.5	8	8
		3.0	5	6
		3.5	4	4
	15 000	1.5×10^5	16	13
		2.0	14	12
		2.5	10	10
		3.0	6	8
		3.5	6	6
	20 000	1.5×10^5	17	16
		2.0	16	15
		2.5	12	12
		3.0	8	10
		3.5	8	7
400	5 000	1.5×10^5	5	4
		2.0	4	3
		2.5	2	3
		3.0	1	2
		3.5	1	2
	10 000	1.5×10^5	9	6
		2.0	9	5
		2.5	3	5
		3.0	4	4
		3.5	3	3
	15 000	1.5×10^5	12	8
		2.0	11	7
		2.5	7	6
		3.0	6	5
		3.5	4	3
	20 000	1.5×10^5	14	9
		2.0	12	8
		2.5	8	7
		3.0	7	6
		3.5	6	4

Disk temperature, °F	Disk speed rpm	Maximum Hertz stress, psi	Minimum film thickness, h_{min}, μ cm	
			X-Ray	Correlation
450	5 000	1.5×10^5	2	3
		2.0	2	3
		2.5	1	2
		3.0	1	2
		3.5	1	1
	10 000	1.5×10^5	5	5
		2.0	4	4
		2.5	3	4
		3.0	2	3
		3.5	1	2
	15 000	1.5×10^5	6	6
		2.0	6	6
		2.5	4	5
		3.0	3	4
		3.5	2	3
	20 000	1.5×10^5	7	7
		2.0	7	7
		2.5	6	6
		3.0	4	4
		3.5	3	3

TABLE V. - ESTIMATED VALUES OF PRESSURE-VISCOSITY COEFFICIENT α

FOR A SYNTHETIC PARAFFINIC OIL (FROM REF. 22)

Disk temperature		Pressure-viscosity coefficient, α	
K	^{o}F	m^2/N	psi^{-1}
339	150	17.1×10^{-9}	11.8×10^{-5}
422	300	10.8	7.5
505	450	[a]4.9	[a]3.4

[a]Extrapolated.

Application of the empirical film thickness expression to systems where different lubrication conditions prevail will be discussed next.

Effects of Contact Geometry, Material, and Lubricant

The present correlation is based exclusively upon measurements made with a single test lubricant and disk geometry and material combination. The disk geometry was chosen to simulate the ball-inner race contact of a 120-millimeter bore angular contact ball bearing (ref. 23). The contact between the test disks approaches the condition of line contact with an ellipticity ratio b/a of 5.9 where b and a are, respectively, the major and minor semi-axes of the contact ellipse. The equivalent radius of curvature in the direction of rolling R' for the test disks was 0.915 centimeter (0.36 in).

It is difficult to extend the results presented herein to different contact geometries, materials, and lubricants without further experimental verification. Indeed, each of these factors are known to influence film thickness in varying degrees. For example, Cheng (ref. 21) has theoretically shown that the proportions of the contact ellipse with b/a varying from 1 to 5 have a relatively mild effect on film thickness. The dependence of film thickness upon mean surface speed u, absolute viscosity μ_0, and contact stress p_{Hz} changes little as the shape of the contact ellipse varies from point to line contact. Similarly, Archard and Cowking in reference 24 have shown that there is great similarity between the EHD lubrication of point and line contacts. In addition, the typical contacts between the races and the balls or rollers in rolling-element bearings and the contacts between gear teeth approximate the line contact case. In view of the aforementioned, the minimum film thickness correlation (eq. (7)) is suitable for most practical applications without further modifications for small differences in contact geometry.

With regard to the size of the contacting elements, elastohydrodynamic theory indicates that film thickness is moderately dependent upon the contacting element's equivalent radius of curvature in the rolling direction R'. In utilizing nondimensional groupings, the film thickness correlation implies that film thickness is a function of R' to the 0.34 power at a given contact stress level. This is reasonably in accord with what conventional EHD theory forecasts; for example, see Cheng's formula (eq. (8)) where $h_o \alpha R^{0.26}$ for p_{Hz} = constant. The sensitivity of minimum film thickness to the contracting element's R' in the heavy load regime remains to be established experimentally. Until such time, minimum film thicknesses predicted by the deduced relation in equation (7) will be most meaningful for those systems in which the R' of the contact approximates the R' of the test disks.

The effect of the material parameter E', which is termed as the equivalent modulus of elasticity, is reflected in the film thickness expression through the dimensionless pressure parameter $\overline{P}_{Hz}$ and the dimensionless speed-viscosity parameter $\overline{U}$. But due to the varying nature of the exponent of $\overline{P}_{Hz}$, the effect on calculated film thickness of materials having elastic properties different from steel would be unjustifiably great at high contact loads. Hence, the film thickness correlation presented here should only be used for those rolling-element bearing and gear systems composed of steel ($E' = 33.6\text{x}10^{10}$ N/m^2 ($48.6\text{x}10^6$ psi)).

It is expected that the most significant effect on equation (7) will be due to lubricant properties. The empirical coefficients utilized in the correlation, namely, n_m, K_j, and S_j, have been derived from the experience gained with a single test fluid. Consequently, no meaningful generalizations can be made with regard to the effects of different lubricant types or formulations on the values of these coefficients without benefit of additional experimental data. Until such time, the application of the film thickness correlation must be restricted to those systems employing the synthetic paraffinic oil upon which the deduced relationship is based.

It may be apparent that the pressure-viscosity coefficient α which is customarily used to characterize the film forming capabilities of a lubricant apart from the effects of the lubricant's absolute viscosity, is conspicuously absent from the present correlation. In the present model, the role formerly played by the pressure-viscosity coefficient has been fulfilled in part by the lubrication coefficient K_j. That is, the relative value of a lubricant's K_j and the value of its absolute viscosity μ_o at given operating condition is indicative of the amount of film thickness this particular lubricant can be expected to generate. An important distinction between α and K_j, is that coefficient α is solely temperature dependent while coefficient K_j is solely contact stress dependent. Careful examination of the X-ray data revealed that the effect of temperature on film thickness could be adequately reflected by the μ_o variable in the dimensionless speed-viscosity parameter $\overline{U}$, and thus the added complication of an additional temperature dependent variable could be avoided. Secondly, there has been some difficulty in properly characterizing the pressure-viscosity characteristics of fluids for purposes of calculating film thickness. This is due mainly to difficulties one encounters when attempting to develop a test apparatus which can accurately reproduce the very severe operating pressure and shear stress levels which a lubricant experiences within an elastohydrodynamic contact (ref. 20). Consequently, relevant test data of this nature has been generally limited. In view of the aforesaid, it was judged advantageous to dispense with α for the present film thickness model.

In conclusion, the film thickness relation developed herein represents an initial attempt at empirically modeling the effects of high contact stress (above 1.04×10^9 N/m^2 (150 000 psi) on minimum film thickness in an elastohydrodynamic contact. It is hoped that this formula, which does reflect the high load dependence exhibited by experimental data, will aid the designer in obtaining a better appraisal of the extent of lubricant film separating his contacting machine elements. As more experimental data becomes available, the film thickness model will be extended to other lubricants and different operating parameters.

SUMMARY OF RESULTS

An empirical elastohydrodynamic (EHD) film thickness formula for heavily loaded contacts based upon X-ray film thickness measurements made with a synthetic paraffinic oil was developed. The minimum film thickness formula was developed from test data which covered a wide and practical range of operating conditions. Maximum Hertz stresses ranged from 1.04×10^9 to 2.42×10^9 N/m^2 (150 000 to 350 000 psi), disk temperatures from 339 to 505 K (150° to 450° F), and mean surface speeds from 9.4 to 37.6 m/sec (370 to 1480 in./sec). Predicted values of minimum film thickness were compared to the X-ray test data for the same conditions and to the results from a theoretical isothermal EHD analysis. The effects of contact geometry, material, and lubricant factors upon predicted film thickness were considered. The following results were obtained:

1. In contrast to present elastohydrodynamic theory, the empirical film thickness model does reflect the high sensitivity of minimum film thickness to contact stress exhibited by the test data under heavy applied loads. Deviations of up to several hundred percent were observed between measured and predicted values of minimum film thickness and those forcasted by a typical film thickness formula at high contact stresses.

2. The measured minimum film thickness data were observed to display an enhanced sensitivity to mean surface speed and lubricant absolute viscosity with increasing contact stress.

3. Finally, it was judged that the empirical film thickness formula can be used with reasonable certainty for rolling-element bearing and gear systems composed of steel, employing the synthetic paraffinic oil specified herein, and whose contact geometry approximates that upon which the model is based.

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