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SPACE SHUTTLE GN&C EQUATION DOCUMENT

No. 7

(Rev. 3)

RENDEZVOUS TARGETING

By

Wayne H. Tempelman

July 1973

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for

National Aeronautics and Space Administration
Guidance and Control Systems Branch
Avionics Systems Engineering Division
Lyndon B. Johnson Space Center, Houston, Texas

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REVISION 3 CHANGES

The following is a summary of the major technical changes included in this revision:

1. A plane change maneuver has been incorporated in the program's logic.
2. In the search for an apsidal crossing, program checks have been included to update through intervals of π in the case of near circular orbits or close proximity to an apsidal point.
3. Following an astronaut overwrite of a Lambert maneuver, the state vector is updated to establish a new target vector for use in the powered flight routine.
4. Following the computation of each maneuver, the position vector of the primary vehicle is offset — to help compensate for the effects of the finite maneuver - prior to the update of the state vector to the next maneuver point.

FOREWORD

This document is one of a series of candidates for inclusion in a future revision of JSC-04217, "Space Shuttle Guidance, Navigation and Control Design Equations." The enclosed has been prepared under NAS9-10268, Task No. 15-A, "GN & C Flight Equation Specification Support", and applies to functions 1, 2, 3, and 6 of the Rendezvous Targeting Module (OG3) as defined in JSC-03690, Rev. D, "Space Shuttle Orbiter Guidance, Navigation and Control Software Functional Requirements", dated January 1973.

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NOMENCLATURE

a	Semi-major axis of a conic
a_i	Alarm code i
a_T	Estimated magnitude of thrust acceleration
a_1	Failure in fuel optimization loop
a_2	Failure in height loop
a_3	Failure in phasing loop
a_4	Failure in obtaining Lambert solution in General Maneuver Routine
a_5	Failure to find perigee/apogee in Search Routine
a_6	Incompatible altitudes and elevation angle
a_7	Failure to find time corresponding to elevation angle in Search Routine
a_8	Failure to find desired position vector in Desired Position Routine
a_9	Failure to update through θ in Update Routine
c	Iteration counter
c_h	Height iteration counter
c_p	Phase iteration counter
c_1, c_2, c_3	Intermediate variables
Δh	Delta altitude
Δr_{proj}	Delta projected position

Δt	Delta time
Δt_{max}	Maximum time step allowed in Search Routine iteration
$\Delta \underline{v}$	Maneuver velocity
$\Delta \underline{v}_{LOS}$	Maneuver in line-of-sight coordinates
$\Delta \underline{v}_{LV}$	Maneuver in local vertical coordinates
Δv_h	Δv used during height maneuver
Δv_p	Δv used during phasing maneuver
Δv_T	Delta velocity used in fuel minimization loop
Δx	Delta independent variable
$\Delta \theta$	Delta central angle
e	Error
e_c	Eccentricity
e_h	Height error
e_p	Phasing error
e_L	Elevation angle (defined in Figure 2)
g_0	Reference value for the acceleration of gravity
$\underline{i}$	Unit vector
$\underline{i}_N$	Unit normal to the plane used in powered flight guidance
i	Number of the maneuver
i_{max}	Number of the last maneuver in the maneuver sequence
m	Estimated vehicle mass
M	Rotational matrix
$\underline{n}$	Vector normal to the orbital plane
n_r	Number of revolutions

p	Partial used in Newton Raphson iteration
r	Distance ratio
$\underline{r}$	Position vector
$\underline{r}_D$	Desired position vector
$\underline{r}_{1c}$	Target vector used in powered flight guidance
s	Switch used in Desired Position Routine
s_{astro}	Astronaut overwrite switch
s_{coplan}	Coplanar switch
s_{direct}	Maneuver direction switch
s_{eng}	Engine select switch
s_{exit}	Program exit switch
s_{fail}	Failure switch
s_{man}	Maneuver switch
s_{opt}	Maneuver optimizing switch
s_{outp}	Out-of-plane switch
s_{pert}	Perturbation switch
s_{phase}	Phase match switch
s_{proj}	Projection switch
s_{rdes}	Desired position switch
s_{soln}	Solution switch
s_{search}	Search switch
s_{tar}	Target offset switch
s_{term}	Terminal constraint switch
s_{update}	Update switch

t	Time
t_F	Final time
$\underline{v}$	Velocity vector
v_v	Vertical component of velocity
v_c	Circular velocity
x	Independent variable in Iteration Routine
y_P, $\dot{y}_P$, $\dot{y}_T$	Out-of-plane parameters (see Figure 6a)
α	Radial component of velocity divided by v_C
β	Horizontal component of velocity divided by v_C
ϵ_1	Tolerance on time in fuel optimizing loop
ϵ_2	Tolerance on height in height loop
ϵ_3	Tolerance on central angle in phasing loop
ϵ_4	Tolerance on central angle in Search Routine
ϵ_5	Tolerance on elevation angle in Search Routine
ϵ_6	Altitude increment in Search Routine
ϵ_7	Tolerance on central angle in Desired Position Routine
ϵ_8	Tolerance on central angle in Update Routine
γ	Flight path angle
μ	Gravitational constant
θ	Central angle
θ_p	Perigee angle

Subscripts

F	Final
i	Number of the maneuver
I	Initial
LOS	Line-of-sight
LV	Local vertical
N	New
0	Old
P	Primary
S	Stored
T	Target
TA	Target for primary vehicle

1. INTRODUCTION

The rendezvous of the Shuttle (primary vehicle) with a target vehicle (e.g. the Space Station) is accomplished by maneuvering the Shuttle into a trajectory that intercepts the target vehicle orbit at a time that results in the rendezvous of the two vehicles. The function of rendezvous targeting is to determine the targeting parameters for the powered flight guidance for each of the maneuvers made by the Shuttle during the rendezvous sequence.

In order to construct the multimaneuver rendezvous trajectory, sufficient constaints must be imposed to determine the desired trajectory. Constraints associated with the Shuttle mission will involve such considerations as fuel, lighting, navigation, communication, time, and altitude. The function of premission analysis is to convert these—which are generally qualitative constraints–into a set of secondary quantitative constraints that can be used by the onboard targeting program. By judicious selection of the secondary constraints, it should be possible to determine off-nominal trajectories that come close to satisfying the primary constraints.

The proposed onboard rendezvous targeting program consists primarily of a main program and a generalized multiple-option maneuver subroutine. The driving program automatically and sequentially calls the maneuver subroutine to construct the rendezvous configuration from a series of maneuver segments. The main program is capable of handling rendezvous sequences involving any given number of maneuvers. Enough different types of maneuver constraints are incorporated into the subroutine to provide the flexibility required to select the best set of secondary constraints during premission planning. In addition, the astronaut has a large, well defined list of maneuver options if he chooses to modify the selected nominal rendezvous scheme.

As the new approach represents, in essence, just one targeting program, there is considerable savings in computer-storage requirements compared to former approaches in which each maneuver used in the rendezvous scheme had a separate targeting program. The programming and verification processes of this unified approach will also result in implementation efficiencies.

1.1 Number of Independent Constraints Involved in a Rendezvous Sequence

During the Gemini and Apollo flights and in the design of the Skylab rendezvous scheme various numbers of maneuvers were utilized in the rendezvous sequence. The range went from two (Apollo 14 and 15) to six (Skylab).

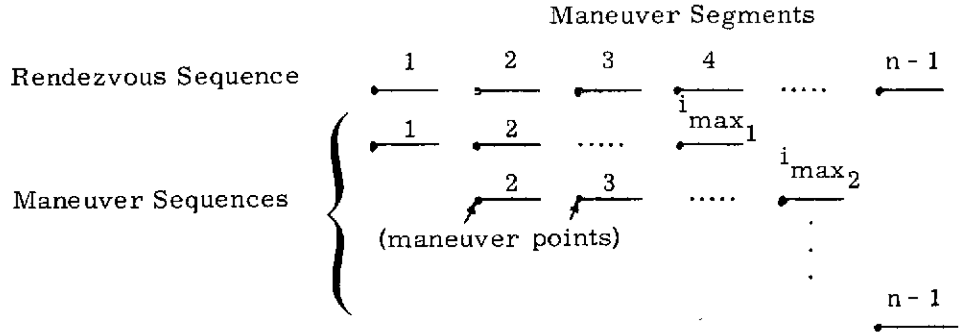
The number of independent constraints (i. e., the number of explicitly satisfied constraints) in each rendezvous sequence must equal the number of degrees of freedom implicitly contained in the sequence. To establish this number, a rendezvous configuration can be constructed by imposing arbitrary constraints until the configuration is uniquely defined. For example, a four maneuver coplanar sequence is shown in Figure 1, followed by a coast to a terminal point. Using the constraints v_i (velocity magnitude), r_i and θ_i, it is easy to establish that the total number involved is 12, assuming the time of the first maneuver has been established. Removing one maneuver will reduce the number of degrees of freedom by three. Hence, the number of independent constraints necessary to uniquely determine the maneuver sequences are

Number of maneuvers in sequence	Number of independent constraints required
1	3
2	6
3	9
4	12
etc	

If the above rendezvous are not coplanar, one additional constraint has to be added to each sequence to allow for the out-of-plane component.

In some cases the number of primary constraints may be insufficient to uniquely determine a rendezvous trajectory for the desired number of maneuvers. One way of overcoming this deficiency in constraints is by introducing sufficient variables to complete the determination of the rendezvous trajectory and then determining values for these variables by minimizing the fuel used.

In order to take advantage of updated state vectors due to navigation or ground updates, the rendezvous targeting program is called prior to each maneuver to compute the upcoming maneuver. In general, each maneuver computation will involve a multimaneuver sequence as the nature of the targeting constraints do not allow the maneuvers to be independently computed. The relationship between the rendezvous sequence involving n maneuvers and the maneuver sequences is shown below.



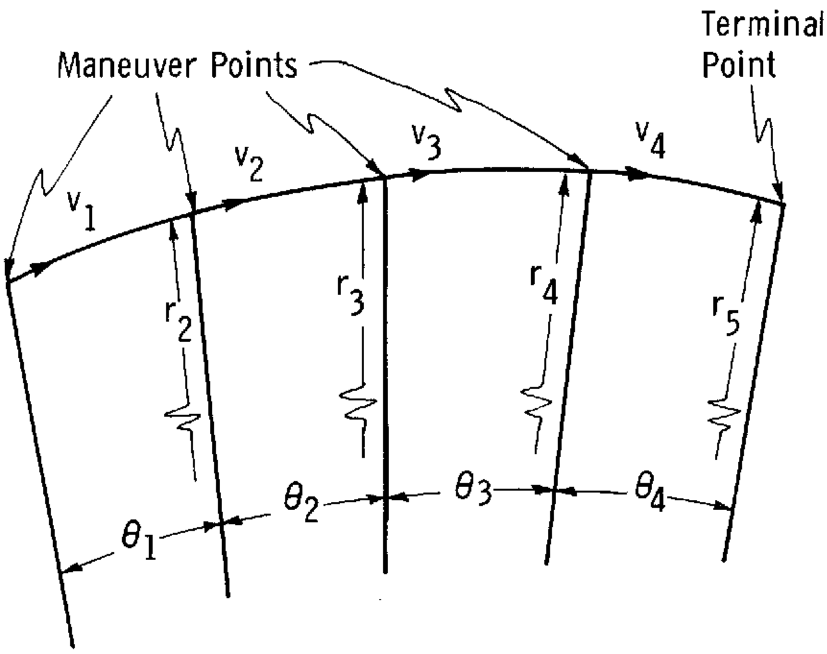


Figure 1. A Possible Set of Constraints Involved in a Four Maneuver Rendezvous Sequence

Each maneuver sequence is composed of a number of maneuver segments and is basically independent from the other maneuver sequences. These sequences must have the same number of independent constraints as tabulated above.

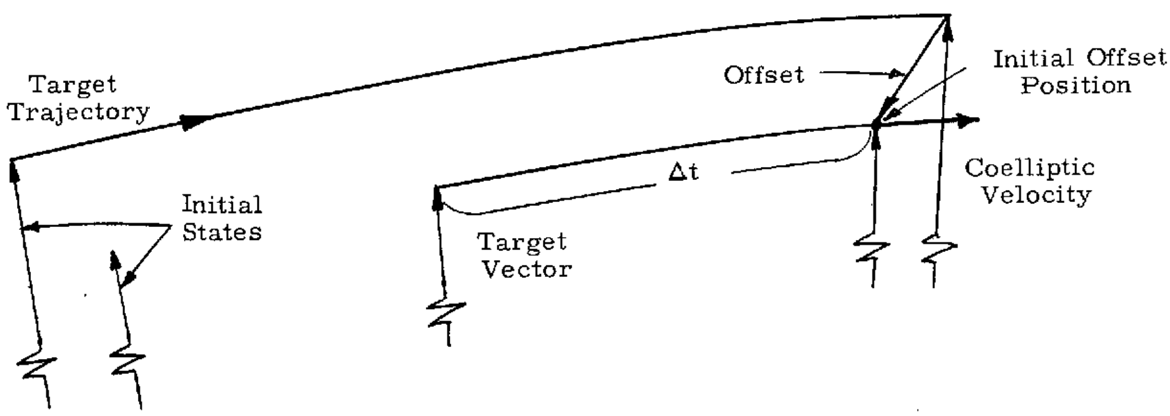
1.2 The Construction of a Maneuver Segment

Each n-maneuver rendezvous sequence can be divided into n-1 maneuver segments. Each segment involves, basically, the addition of a maneuver to the primary vehicle's velocity vector and an update of both vehicle's state vectors to the next maneuver point.

A maneuver segment is herein generated in one of three ways:

Forward generation — A maneuver $\Delta \underline{v}$ is computed and added to the velocity vector in a specified direction. The state vector of the primary vehicle is then updated through a specified amount to arrive at the next maneuver position.

Target generation — The target vehicle is updated through a specified amount and then offset to establish a target vector. An option is available at this point to compute a coelliptic velocity vector and update through Δt to establish a new target vector as shown below.



The maneuver is then computed by uniquely specifying the nature of the traverse between the primary vehicle's position and the target vector.

Integrated generation — In this case, the maneuver segment is computed as an integral part of a maneuver sequence involving more than one maneuver segment. The nature of the constraints are such that the maneuver sequence cannot be subdivided into uniquely defined maneuver segments. The maneuver segment will usually have one degree of freedom, which will generally be assumed to be the magnitude of the maneuver.

Each of the above methods is defined by specifying trajectory constraints by setting certain switches and parameter values. Specifying a trajectory constraint is equivalent to specifying one or more independent constraints. On the other hand, specifying an independent constraint can also be equivalent to specifying one or more trajectory variables. (See Ref. 9) A trajectory constraint common to all three of the above methods is the state vector update switch s_{update}. The options associated with this switch are:

$$s_{update} = \begin{cases} 1 & \text{Update from time } t \text{ to time } t_F \\ 2 & \text{Update through time interval } \Delta t \\ 3 & \text{Update through } n \text{ revolutions} \\ 4 & \text{Update through } \theta \text{ radians} \\ 5 & \text{Update to be colinear with a specified position vector} \end{cases}$$

In the next three sections, the trajectory constraints associated with each of the above methods will be listed.

1.2.1 Maneuver Options in Forward Generation of Maneuver Segment

The forward generation of a maneuver segment is accomplished in one of two ways. Either the maneuver magnitude is uniquely determined in terms of the state vector at the maneuver time or the maneuver is determined by an iterative search to satisfy a terminal constraint.

The maneuver magnitude Δv is either calculated or assumed depending on the maneuver switch s_{man}, and it is applied in a direction controlled by the direction switch s_{direct}. The options associated with the maneuver switch are:

$s_{man} =$

- 1 — Δv is assumed specified
- 2 — Δv is computed based on a post maneuver velocity vector being "coelliptic" with the state vector of the target vehicle
- 3 — Δv is computed from the conic circular velocity constraint
- 4 — Δv is computed based on a Hohmann type transfer resulting in a Δh change in altitude

The options associated with the maneuver direction switch are:

$s_{direct} =$

- -1 — Apply Δv is horizontal direction in plane of primary vehicle
- 1 — Apply Δv in horizontal direction parallel to orbital plane of the target vehicle
- -2 — Apply Δv along velocity vector in plane of primary vehicle
- 2 — Apply Δv along velocity vector parallel to orbital plane of the target vehicle

The selection of the update switch s_{update_P} determines the update of the primary vehicle's trajectory following the maneuver to the position of the next maneuver. A terminal constraint can be imposed at this point by setting the terminal switch s_{term}:

$$s_{term} = \begin{cases} 1 & \text{Terminal constraint is a height constraint} \\ -1 & \text{Terminal constraint is a phasing constraint} \end{cases}$$

Following the computation of the height/phasing error, the maneuver magnitude is varied in an iterative search to satisfy the height/phasing constraint.

1.2.2 Maneuver Options in Target Generation of Maneuver Segment

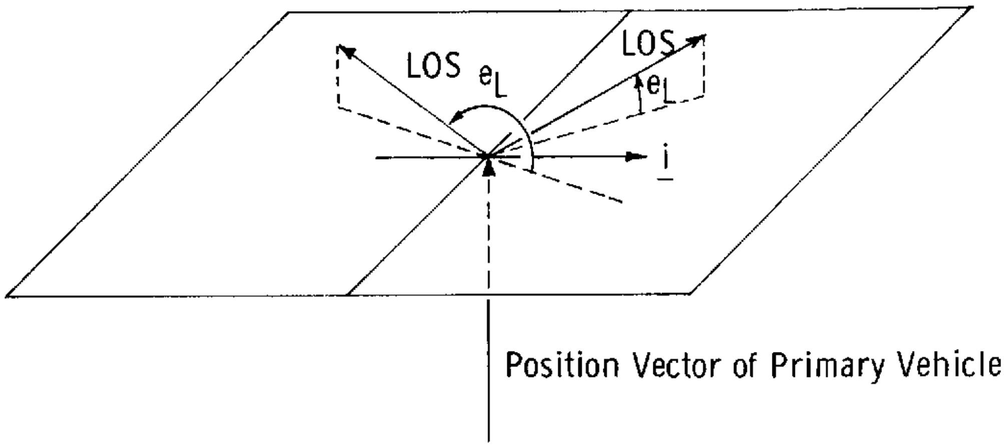
The target generation of a maneuver segment starts with the selection of the update switch for the target vehicle. If this switch equals four, θ will be augmented by the central angle between the primary and target vehicles before being used. The position of the target vehicle is then offset through either (e_L, Δh) or ($\Delta\theta$, Δh), depending on whether s_{tar} is negative or positive, to obtain a target vector. The "TPI offsets" (e_L, Δh) are discussed in Section 5 (see Figure 2 for definition of e_L). If $|s_{tar}|$ equals two, a coelliptic velocity vector is computed based on the target vector, and a new target vector is defined by updating the coelliptic state vector through Δt. The options associated with s_{tar} are:

$$s_{tar} = \begin{cases} -2 & \text{Offset target } (e_L, \Delta h)\text{. Compute coelliptic velocity and update through (negative) } \Delta t. \\ -1 & \text{Offset target } (e_L, \Delta h) \\ 0 & \text{No target offset} \\ 1 & \text{Offset target } (\Delta\theta, \Delta h) \\ 2 & \text{Offset target } (\Delta\theta, \Delta h)\text{. Compute coelliptic velocity and update through (negative) } \Delta t \end{cases}$$

The nature of the traverse between the primary vehicle's initial state vector and the target vector is controlled by the maneuver switch s_{man}:

$$s_{man} = \begin{cases} 5 & \text{Lambert - the trajectory is time constrained} \\ 6 & \text{Horizontal - the maneuver is constrained to be in the horizontal direction} \\ 7 & \text{Tangential - the maneuver is constrained to be in the direction of the velocity vector} \\ 8 & \text{Apogee/Perigee - the trajectory has an apogee/perigee occurring at the target point.} \end{cases}$$

There is a minimum Δv option associated with the above maneuvers which is controlled with the optimization switch s_{opt}:



$\underline{i}$ = Unit horizontal in forward direction for primary vehicle

LOS = Line of Sight

1. If the LOS projection on $\underline{i}$ is positive:
   a. When the LOS is above the horizontal plane, $0 < e_L < \pi/2$
   b. When the LOS is below the horizontal plane, $3\pi/2 < e_L < 2\pi$
2. If the LOS projection on $\underline{i}$ is negative:
   a. When the LOS is above the horizontal plane, $\pi/2 < e_L < \pi$
   b. When the LOS is below the horizontal plane, $\pi < e_L < 3\pi/2$

Figure 2. Definition of the Elevation Angle e_L

s_{opt} =

- -2 Minimize the sum of the magnitude of the first and the next maneuver (based on a coelliptic parting velocity) by varying Δt, the time of update of the target vehicle.
- -1 Minimize the magnitude of the first maneuver by varying Δt, the time of update of the target vehicle.
- 1 Minimize the magnitude of the first maneuver by varying Δt, the time between the next maneuver and the initial offset position. (See sketch on page 1-4)
- 2 Minimize the sum of the magnitudes of the first and the next maneuver (based on a coelliptic parting velocity) by varying Δt, the time between the next maneuver and the initial offset position (see sketch on page 1-4).

This minimization is accomplished by driving the slope (Δv / independent variable) to zero using a Newton Raphson iteration scheme.

1.2.3 Maneuver Options in Integrated Generation of Maneuver Segment

The integrated generation of a maneuver segment involves an iterative solution to determine a maneuver sequence which cannot be sequentially solved for its maneuver segment components. The maneuver is computed by guessing its magnitude, assigning a direction and plane through selection of the direction switch s_{direct}, updating the primary vehicle's state vector after selecting switch s_{update_p} and then calling additional maneuver segments until reaching the point at which the terminal constraint is to be attained. The maneuver is then iteratively determined by satisfying the terminal constraint. The number of additional maneuver segments and the nature of the terminal constraint are controlled by the terminal constraint switch s_{term}

$s_{term} =$

-2, -3, ...	The terminal constraint is a phasing constraint and it occurs at the $\lvert s_{term} \rvert$ maneuver point from the start of the maneuver segment.
2, 3, ... < 10	The terminal constraint is a height constraint and it occurs at the s_{term} maneuver point from the start of the maneuver segment.
$(10 < s_{term} < 100)$	Both a height and phasing constraint occur at the same maneuver point. The first digit n_1 of s_{term} represents a phasing constraint that occurs at the n_1 maneuver point from the start of the phasing maneuver segment. The last digit n_2 of s_{term} represents a height constraint that occurs at the n_2 maneuver point from the start of the height maneuver segment.

1.2.4 Summary of the Maneuver Segment Constraints

The maneuver and trajectory constraints that can be imposed on a maneuver segment can be divided into the following catagories (see Figure 3).

Primary vehicle update constraints
Target vehicle update constraints
Initial velocity constraints
Offset constraints
Terminal constraints
Traverse constraints

Table 1 contains a detailed listing of the constraints. The three independent constraints (four in the case of noncoplanar traverses) which govern a maneuver segment cannot be chosen arbitrarily from this list for two reasons:

(1) There is not a one-to-one correspondence between the trajectory constraints and the independent constraints.

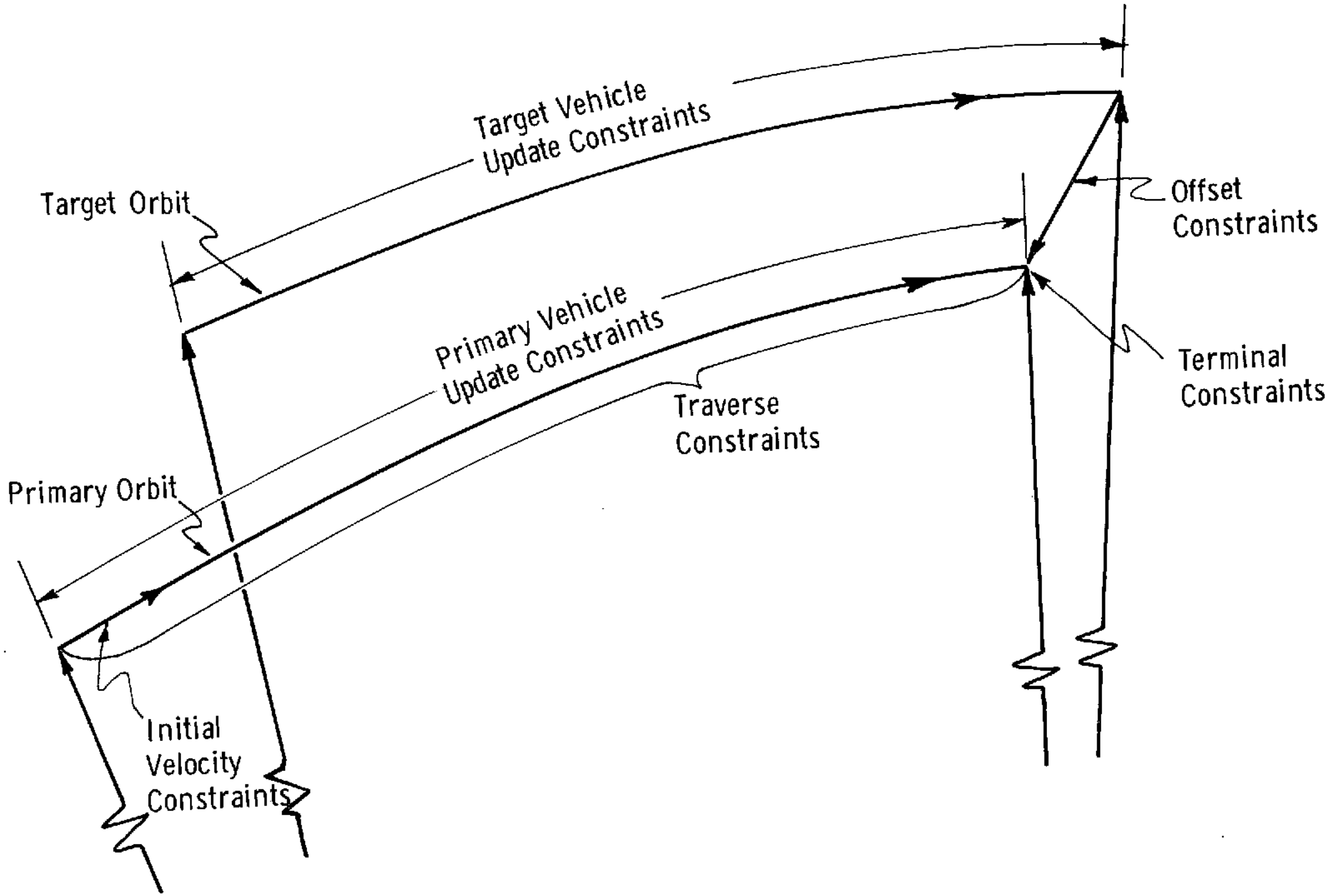


Figure 3. Constraint Catagories on a Maneuver Segment

(2) Selecting some constraints negates the need for some others (e.g. selecting a Lambert constraint negates the need for a maneuver direction constraint).

In the case of a straight forward rendezvous profile, a basic understanding of the nature of the constraints should allow the constructor of the rendezvous sequence to choose a set of trajectory constraints which determine the required number of independent constraints. For a complex rendezvous profile, such as Skylab a more formal approach such as presented in Reference 11 should be used. One of the justifications for presenting the three methods of generating a maneuver segment was to aid the constructor of the rendezvous sequence in choosing compatible sets of constraints.

TABLE 1

DETAILED LISTING OF CONSTRAINTS

(Sheet 1 of 2)

Primary and Target Vehicle Update Constraints

- Delta time
- Initial and final time
- Central angle
- Number of revolutions
- Terminal position vector

Initial Velocity Constraints

- Plane
  - Parallel to target orbit
  - Parallel to primary orbit
- Direction
  - Horizontal
  - Along velocity vector
- Magnitude
  - Circular
  - Coelliptic
  - Altitude change
  - Specified

Offset Constraints

- Angle
- Altitude
- Elevation angle

Terminal Constraints

- Height
- Phase

TABLE 1

DETAILED LISTING OF CONSTRAINTS

(Sheet 2 of 2)

Traverse Constraints

- Minimum Fuel
  - One maneuver optimization
  - Two maneuver optimization
- Apogee/Perigee designation
- Horizontal maneuver
- Tangential maneuver
- Lambert (time)

2. FUNCTIONAL FLOW DIAGRAMS

The rendezvous targeting program consists of two major parts—a generalized maneuver subroutine which basically computes a maneuver and updates the state vectors of both vehicles to the time of the next maneuver and a main program which sequentially calls the subroutine to assemble a rendezvous sequence. These programs call a number of subroutines which are briefly described below and in detail in Section 5.

Search	-	To update the state vectors to either a specified apsidal crossing, a time, or an elevation angle.
Phase Match	-	To phase match the target vehicle's state vector to the primary vehicle's position vector.
Desired Position	-	To compute an offset target vector or a desired position to be used in a phasing constraint.
Update	-	To update a state vector through a specified interval.
Coelliptic Maneuver	-	To compute a coelliptic velocity vector.
Iteration	-	To determine a new estimate of the independent variable in a Newton Raphson iteration scheme.

The functional flow diagram for the main program is shown in Figure 4. The main function of this program is to sequentially call the General Maneuver Routine to compute each maneuver segment for maneuvers numbered from i to i_{max}. There are three major options that can be exercised prior to the calculation of the first maneuver segment:

(1) A search for the time of the first maneuver. This time can be specified by:

(a) An elevation angle, which is to be attained at the maneuver time.

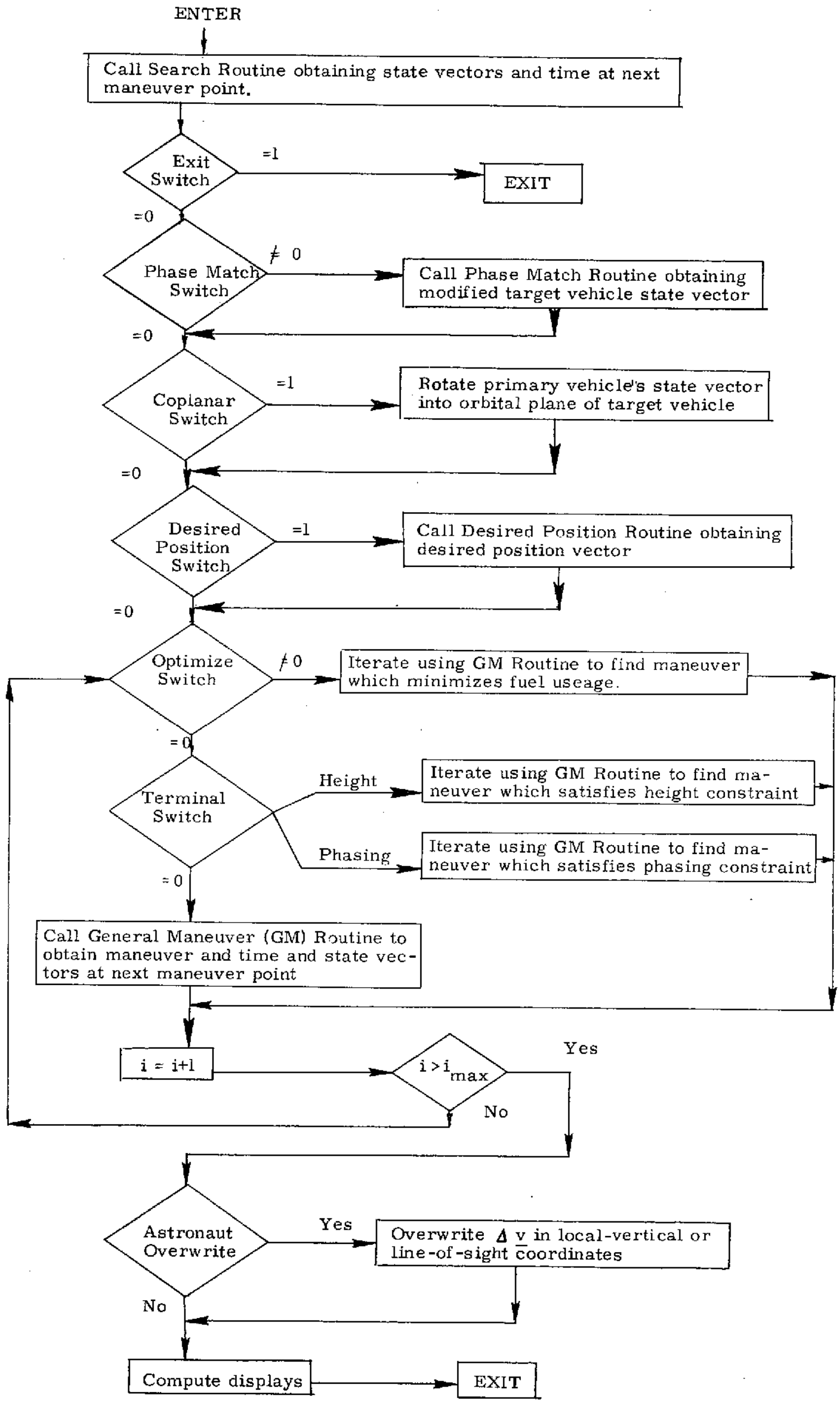


Figure 4. Main Program - Function Flow Diagram

(b) Whether the next maneuver should occur at the next apsidal crossing, the next perigee crossing or the nth apsidal crossing.

(2) A phase matching of the state vector.

(3) A rotation of the primary vehicle's state vector into the plane of the target vehicle.

There are three separate iterative loops built around the call to the general maneuver routine. One loop serves to minimize the fuel used during a maneuver segment with the options determined by the optimizing switch.

The other two iterative loops involve maneuver segments which contain constraints that do not allow the explicit calculation of the maneuver. These constraints are height and phasing constraints imposed at the end of a maneuver segment and controlled with the terminal switch. The iterative loop will involve several maneuver segments if sufficient constraints are not imposed to solve each segment uniquely.

The functional flow diagram for the general maneuver routine is shown in Figure 5. This routine generates the departure velocity at the initial point in one of two ways:

(1) As an explicit function of the initial state vectors.

(2) By defining a target vector and then computing an intercept trajectory based on a specified constraint (as indicated by the setting of s_{man}). The target vector is determined by offsetting the updated position vector of the target vehicle. Depending on the setting of the switch s_{tar}, a coelliptic velocity vector is computed at the offset point and the coelliptic state vector is updated through Δt to obtain a target vector.

Following an update of both vehicle's state vectors to the time of the next maneuver, the Δv used or the terminal height/phase errors are calculated as required.

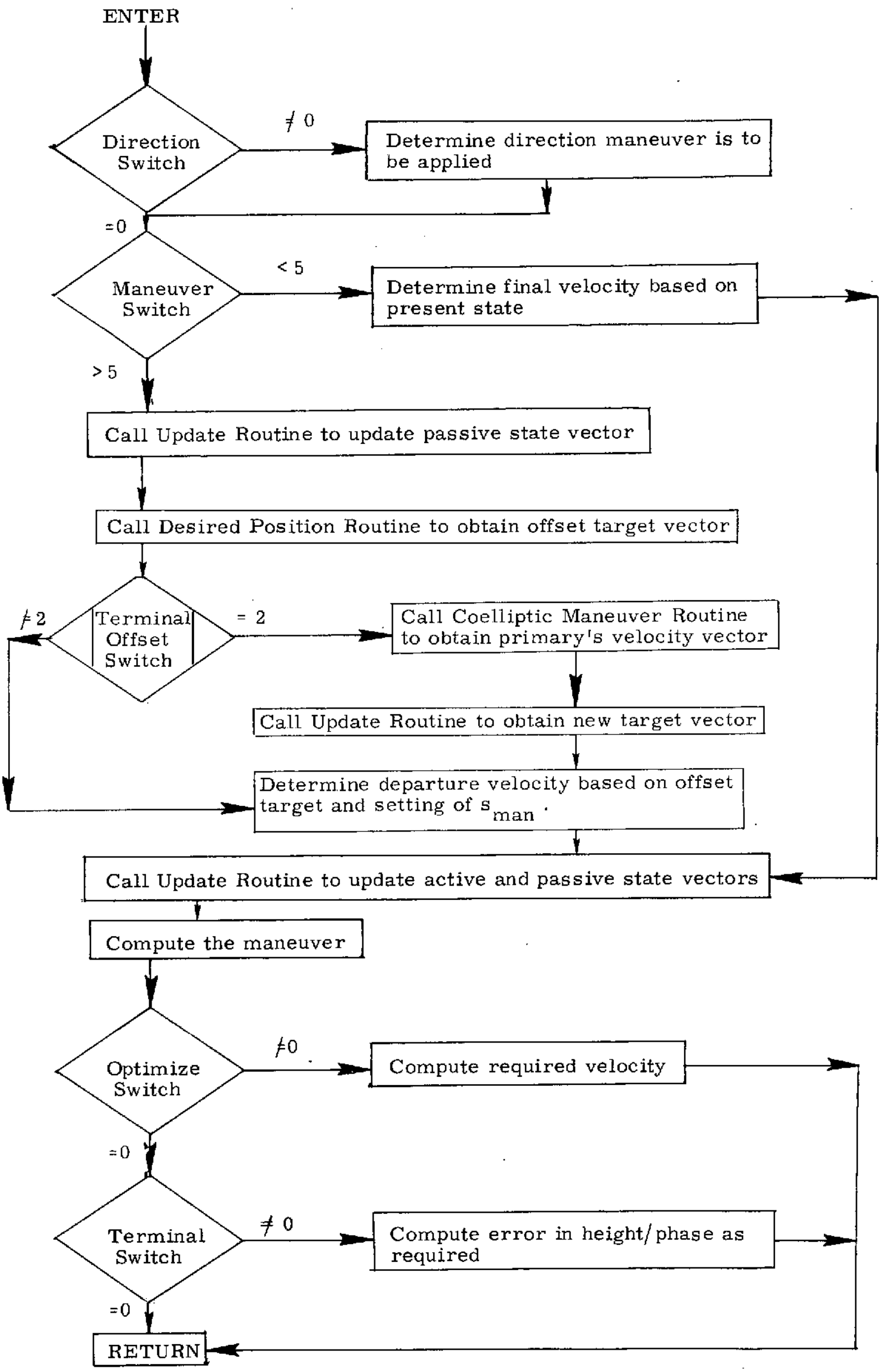


Figure 5. General Maneuver Routine - Functional Flow Diagram

3. INPUT AND OUTPUT VARIABLES

The inputs to the orbiter rendezvous targeting program can be divided into five catagories.

Pre-Maneuver Switches

Upon selecting a maneuver from the rendezvous sequence, these switches (specified for each maneuver) serve in determining the state vectors at the maneuver point, the out-of-plane parameters and the calculation of a desired position vector. These inputs can also be used in determining the time of a specified apsidal crossing or the time at which a specified elevation angle is to be attained.

Coplanar switch

$s_{coplan} =$
- 0 Bypass
- 1 Rotate primary state vector into plane of target vehicle's orbit

Exit switch

$s_{exit} =$
- 0 Bypass
- 1 Exit from routine

Out-of-plane switch

$s_{outp} =$
- 0 Bypass
- 1 Compute out-of-plane parameters
- 2 Compute out-of-plane parameters and modify maneuver by $-\dot{y}_P$

Perturbation switch

$s_{pert} =$
- 0 Do conic state vector updates
- 1 Include oblateness based on J_2
- ... Other perturbations as required

Phase match switch

$s_{phase} =$
- 0 Bypass
- 1 Phase match state vectors (target leading primary)
- 2 Phase match state vectors based on target leading primary by more than 360°
- -1 Phase match state vectors (primary leading target)
- -2 Phase match state vectors based on primary leading target by more than 360°

Desired position switch

$s_{rdes} = \begin{cases} 0 & \text{Bypass} \\ -1 & \text{Compute desired position vector based on } (e_L, \Delta h) \\ 1 & \text{Compute desired position vector based on } (\Delta\theta, \Delta h) \end{cases}$

Search switch

$s_{search} = \begin{cases} -4 & \text{Compute elevation angle} \\ -3 & \text{Search for elevation angle} \\ -2 & \text{Update to time } t_i \\ -1 & \text{Search for next perigee crossing} \\ 0 & \text{Bypass} \\ n & \text{Search for the nth apsidal crossing } (n>0) \end{cases}$

Maneuver Switches

These switches (specified for each maneuver) set the constraints employed in determining the maneuver segments.

Direction switch

$s_{direct} = \begin{cases} -2 & \Delta v \text{ in direction of primary's velocity vector, parallel to primary's orbital plane} \\ -1 & \Delta v \text{ in horizontal direction, parallel to primary's orbital plane} \\ 0 & \text{Bypass} \\ 1 & \Delta v \text{ in horizontal direction, parallel to target's orbital plane} \\ 2 & \Delta v \text{ in direction of primary's velocity vector, parallel to target's orbital plane} \end{cases}$

Maneuver switch

$s_{man} = \begin{cases} 1 & \Delta v \text{ is specified} \\ 2 & \Delta v \text{ is based on coelliptic velocity} \\ 3 & \Delta v \text{ is based on circular velocity} \\ 4 & \Delta v \text{ is based on altitude change} \\ 5 & \text{Lambert maneuver to offset target vector} \\ 6 & \text{Horizontal maneuver to offset target vector} \\ 7 & \text{Tangential maneuver to offset target vector} \\ 8 & \text{Perigee/apogee insertion at offset target vector} \\ 9 & \text{Plane change} \end{cases}$

Maneuver optimizing switch

$$s_{opt} = \begin{cases} 0 & \text{Bypass} \\ 1 & \text{Minimize } \Delta v_i \\ 2 & \text{Minimize } \Delta v_i + \Delta v_{i+1} \end{cases}$$

Multi-revolution solution switch

$$s_{soln} = \begin{cases} -1 & \text{Solution with smallest initial flight path angle (measured from local vertical)} \\ 1 & \text{Solution with largest initial flight path angle} \end{cases}$$

$$s_{tar} = \begin{cases} -2 & \text{Offset target } (e_L, \Delta h)\text{. Compute coelliptic velocity and update through (negative) } \Delta t \\ -1 & \text{Offset target } (e_L, \Delta h) \\ 0 & \text{No target offset} \\ 1 & \text{Offset target } (\Delta\theta, \Delta h) \\ 2 & \text{Offset target } (\Delta\theta, \Delta h)\text{. Compute coelliptic velocity and update through (negative) } \Delta t \end{cases}$$

Terminal constraint switch

$$s_{term} = \begin{cases} n & (<0) \text{ Compute phasing error and back up } -(n+1) \text{ maneuvers for start of phase loop} \\ 0 & \text{Bypass} \\ n & (0 < n < 10) \text{ Compute height error and back up } n - 1 \text{ maneuvers for start of height loop} \\ n & (10 < n < 100) \text{ Phase and height loop terminate on same maneuver. For phase loop back up } x - 1 \text{ (where x is first digit of n) maneuvers for start of phase loop. For height loop back up } y - 1 \text{ (where y is last digit of n) maneuvers for start of height loop} \end{cases}$$

Update switch

$$s_{update} = \begin{cases} 1 & \text{Update through } t_F - t \\ 2 & \text{Update through } \Delta t \\ 3 & \text{Update through } n_r \\ 4 & \text{Update through } \theta \\ 5 & \text{Update to be colinear with } \underline{r}_D \end{cases}$$

Parameter Values

The parameter values (specified for each maneuver) are values for the constrained parameters.

Δh	Delta altitude
Δh_F	Delta altitude, final
$\Delta \theta$	Delta central angle
Δt	Delta time
Δv	Maneuver magnitude
n_r	Number of revolutions
t_F	Final time
e_L	Elevation angle

Post-Maneuver Switch

This switch determines the options available following the calculation of the maneuver.

Astronaut overwrite switch

$$s_{astro} = \begin{cases} 0 & \text{Bypass} \\ 1 & \text{Overwrite maneuver in local vertical coordinates} \\ 2 & \text{Overwrite maneuver in line-of-sight coordinates} \end{cases}$$

Maneuver Call Variables

The maneuver call variables have to be specified for each call to the maneuver sequence.

$\underline{r}_P, \underline{v}_P$	State vector of the primary vehicle
$\underline{r}_T, \underline{v}_T$	State vector of the target vehicle
i	Maneuver number
t	Current time
t_i	Time of the i^{th} maneuver (if $s_{search} \neq -2$, then t_i is not specified)
s_{eng}	Engine select switch
m	Estimated vehicle mass

Depending on the rendezvous sequence, there may also be some switches that have to be modified as a function of the maneuver number.

Excluding the maneuver call variables, all the input variables can be set prior to the flight.

The output parameters for the initial maneuver in the sequence are more complete than for the succeeding maneuvers.

Output Parameters for the Initial Maneuver

Δv_i	Maneuver magnitude
$\Delta \underline{v}_{LOS\,i}$	Maneuver in line of sight coordinates
$\Delta \underline{v}_{LV\,i}$	Maneuver in local vertical coordinates
$\underline{r}_{1c}$	Target vector used in Powered Flight Guidance Routine (See Ref. 5)
$\underline{i}_N$	Unit normal to plane used in same routine
t_T	Time associated with $\underline{r}_{1c}$

Other parameters such as delta altitude, phasing angle, elevation angle and perigee altitude can be computed as required.

Output Parameters for the Other Maneuvers in the Sequence

t	Time of the maneuver
Δv	Maneuver magnitude

Illustration of Inputs

Table 2 contains a set of inputs for the Shuttle targeting program based on the five maneuver Skylab rendezvous configuration. The following switches and parameters are not used as inputs to the Orbiter program:

$$s_{astro},\ s_{exit},\ s_{opt},\ s_{outp},\ s_{soln},\ \Delta\theta,$$

The inputs in Table 2 are set prior to the mission so they will not have to be inserted by the astronaut. The astronaut will have to modify the following quantities upon resetting the maneuver number as well as inserting the time of the next maneuver.

i = 2: $s_{term_2} = 0,\ s_{term_4} = 32$

i = 3: $s_{man_3} = 5,\ \Delta t_3 = -\Delta t_{NSR-TPI}$

i = 4: $s_{term_4} = 0$

TABLE 2

INPUT VARIABLES FOR SKYLAB RENDEZVOUS CONFIGURATION

Input Variable	Maneuver 1 (NC1)	2 (NC2)	3 (NCC)	4 (NSR)	5 (TPI)
s_{coplan}	1	1	0	1	0
s_{direct}	1	1	1	0	0
s_{man}	1	1	1	2	5
s_{pert}	1	1	1	0	1
s_{phase}	1	1	0	0	0
s_{rdes}	-1	-1	0	0	0
s_{search}	-2	-2	-2	-2	-3
s_{tar}			-2		0
s_{term}	0	1	0	42	0
s_{update_P}	3	3	2	1	2
s_{update_T}			1		4
e_L	e_L	e_L	e_L		e_L
i_{max}	4	4	3	4	5
n_r	$n_{rNC1-NC2}$	$n_{rNC2-NCC}$			
t_F	t_{TPI}	t_{TPI}	t_{TPI}	t_{TPI}	
θ					$\theta_{TPF-TPI}$
Δt			$\Delta t_{NSR-NCC}$		
Δh		Δh_{NCC}	Δh_{TPI}	Δh_{TPI}	
Δh_F	Δh_{TPI}	Δh_{TPI}			
Δv	Δv_{NC1}	Δv_{NC2}	Δv_{NCC}		

4. DESCRIPTION OF EQUATIONS

The only equations contained in this document which are not trivial are those involved in computing the traverse between two specified position vectors. The required equations can be derived from the equation of the conic expressed in the form

$$r = r_F / r_I = \beta_I^2 / \left[1 + e_c \cos(\theta + \theta_P) \right]$$

where

$$e_c = \left[\alpha_I^2 \beta_I^2 + (\beta_I^2 - 1)^2 \right]^{1/2}$$

$$\theta_p = \cos^{-1} \left[(\beta_I^2 - 1)/e_c \right] \quad \text{(perigee angle)}$$

$$v_c = (\mu / r_I)^{1/2}$$

α_I and β_I are the normalized (with respect to v_c) radial and horizontal components of velocity.

The above equation can be expressed

$$p_s/r_I = \beta_I^2 = (\alpha_I \beta_I \sin\theta - c_2) / c_1 \qquad (1)$$

where

$$c_1 = \cos\theta - 1/r$$

$$c_2 = 1 - \cos\theta$$

$$p_s = \text{semilatus rectum}$$

For a maneuver that is constrained to be in a horizontal direction, Eq. (1) can be solved for β_I as a function of the specified α_I.

$$\beta_I = \left[\alpha_I \sin\theta \pm (\alpha_I^2 \sin^2\theta - 4 c_1 c_2)^{1/2} \right] / 2 c_1$$

As there has to be both a positive and negative β_I solution to this equation (one trajectory in each rotational direction), the sign choice is resolved in favor of plus β_I.

For a maneuver that is applied along the velocity vector, the flight path angle γ_1 is to be held fixed. Using Eq. (1)

$$\tan \gamma_I = \alpha_I / \beta_I = (c_1 \beta_I^2 + c_2)/\beta_I^2 \sin \theta$$

Therefore,

$$\beta_I = [\, c_2/(\sin \theta \tan \gamma_I - c_1)]^{1/2}$$

By interchanging the I and F subscripts, Eq. (1) can be expressed

$$p_s = r_F (\alpha_F \beta_F \sin \theta - c_2)/(\cos \theta - r)$$

Combining with Eq. (1) using the apogee/perigee constraint $\alpha_F = 0$ results in the required horizontal and radial components of velocity for apogee/perigee designation maneuvers.

$$\beta_I = [r\, c_2/(r - \cos \theta)\,]^{1/2}$$

$$\alpha_I = \beta_I (1 - 1/r)/\tan(\theta/2)$$

The derivation of the equation

$$\theta = \cos^{-1} [\, r_P \cos (e_L')/r_T] - e_L'$$

where

$$e_L' = \begin{cases} e_L \text{ if } e_L \le \pi \\ e_L - \pi \text{ if } e_L > \pi \end{cases}$$

for computing the desired central angle θ between two positions ($\underline{r}_P$, $\underline{r}_T$) which satisfies the TPI constraints (e_L, Δh) is discussed in Ref. 12. This equation is used in the Desired Position Routine.

5. DETAILED FLOW DIAGRAMS

Figures 6 and 7 contain the detailed flow diagrams of the main Shuttle rendezvous targeting program and the general maneuver routine, respectively. The following six routines are called by these two programs.

Iteration Routine

This routine contains a Newton Raphson iterative driver based on numerically computed partials. The routine computes a new estimate of the dependent variable x and returns the old values of the error e and x. If the iteration counter c exceeds 15, a convergence switch s_{fail} is set equal to one.

Coelliptic Maneuver Routine

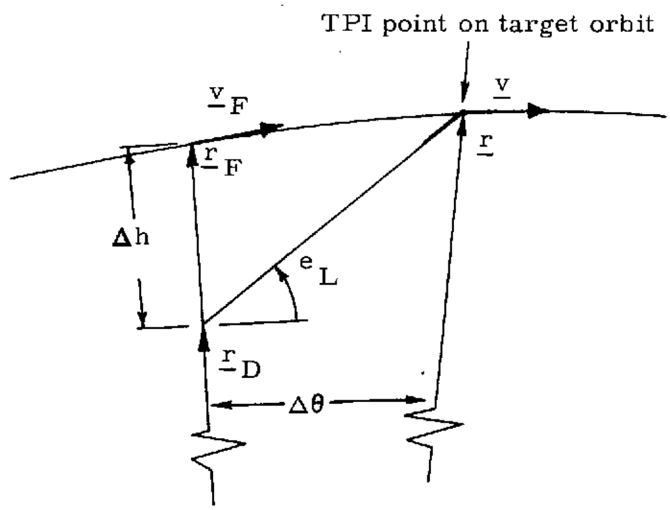
This routine computes a coelliptic velocity vector $\underline{v}_N$ based on a target vehicle's state vector and a delta altitude.

Phase Match

This routine phase matches the target state vector to the primary state vector. The controlling switch (s_{phase}) equals two if the leading vehicle leads the other vehicle by more than one revolution: otherwise the switch equals one. If the primary vehicle leads to target vehicle, the switch is negative.

Desired Position Routine

This routine updates a specified state vector to the time t_F and then offsets the updated state vector through either ($\Delta\theta$, Δh) or (e_L, Δh), depending on the setting of the switch s, to obtain $\underline{r}_D$. The routine contains an iterative search to solve the (e_L, Δh) offset problem, where e_L is defined in Figure 2 and Δh (positive when the target orbit is above the primary) is defined as shown below. (This represents the TPI geometry used in Apollo and Skylab.)



Update Routine

This routine updates a state vector based on the update switch s_{update}

s_{update}
- $= 1$ Updates through the time $t_F - t$
- $= 2$ Updates through the time Δt
- $= 3$ Updates through n_r revolutions
- $= 4$ Updates through the angle θ
- $= 5$ Updates to where the orbit intersects the line defined by $\underline{r}_D$

Search Routine

This routine makes the following computations depending on the setting of the search switch s_{search}

s_{search}
- $= n$ (>0) Finds the time of the n^{th} apsidal crossing and updates the state vector to that time
- $= -1$ Finds the time of the next perigee crossing and updates the state vector to that time
- $= -2$ Updates the state vector through the time $t_F - t$ and computes the elevation angle
- $= -3$ Finds the time associated with a specified elevation angle and updates the state vector to that time

The detailed flow charts for these routines are shown in Figures 8 to 13. The iterative algorithm used to determine the time associated with the elevation angle is described in Ref. 8.

Each input and output variable in the routine and subroutine call statements can be followed by a symbol in brackets. This symbol identifies the notation for the corresponding variable in the desired description and flow diagrams of the called routine. When identical notation is used, the bracketed symbol is omitted.

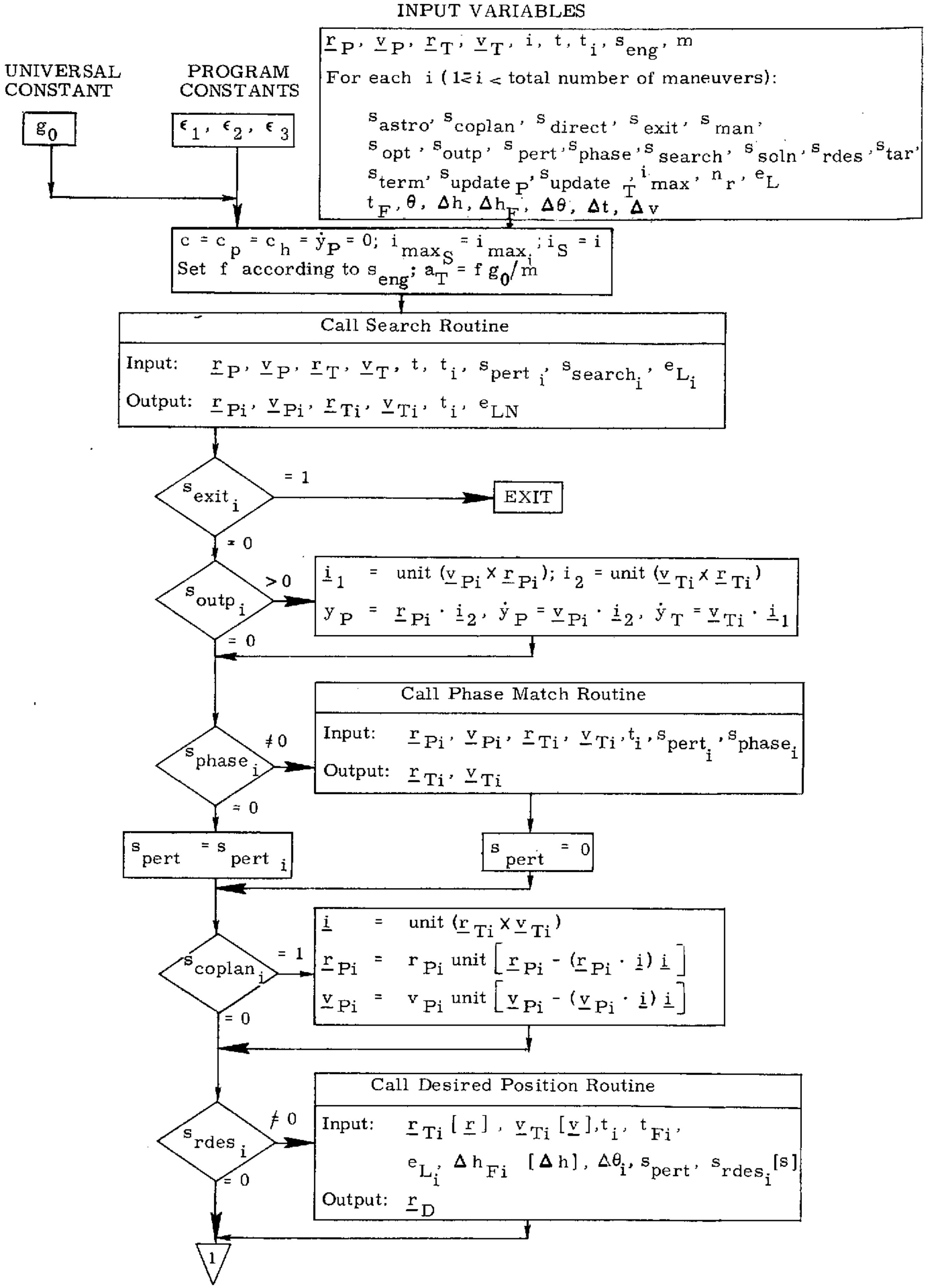


Figure 6a. Main Program - Detailed Flow Diagram

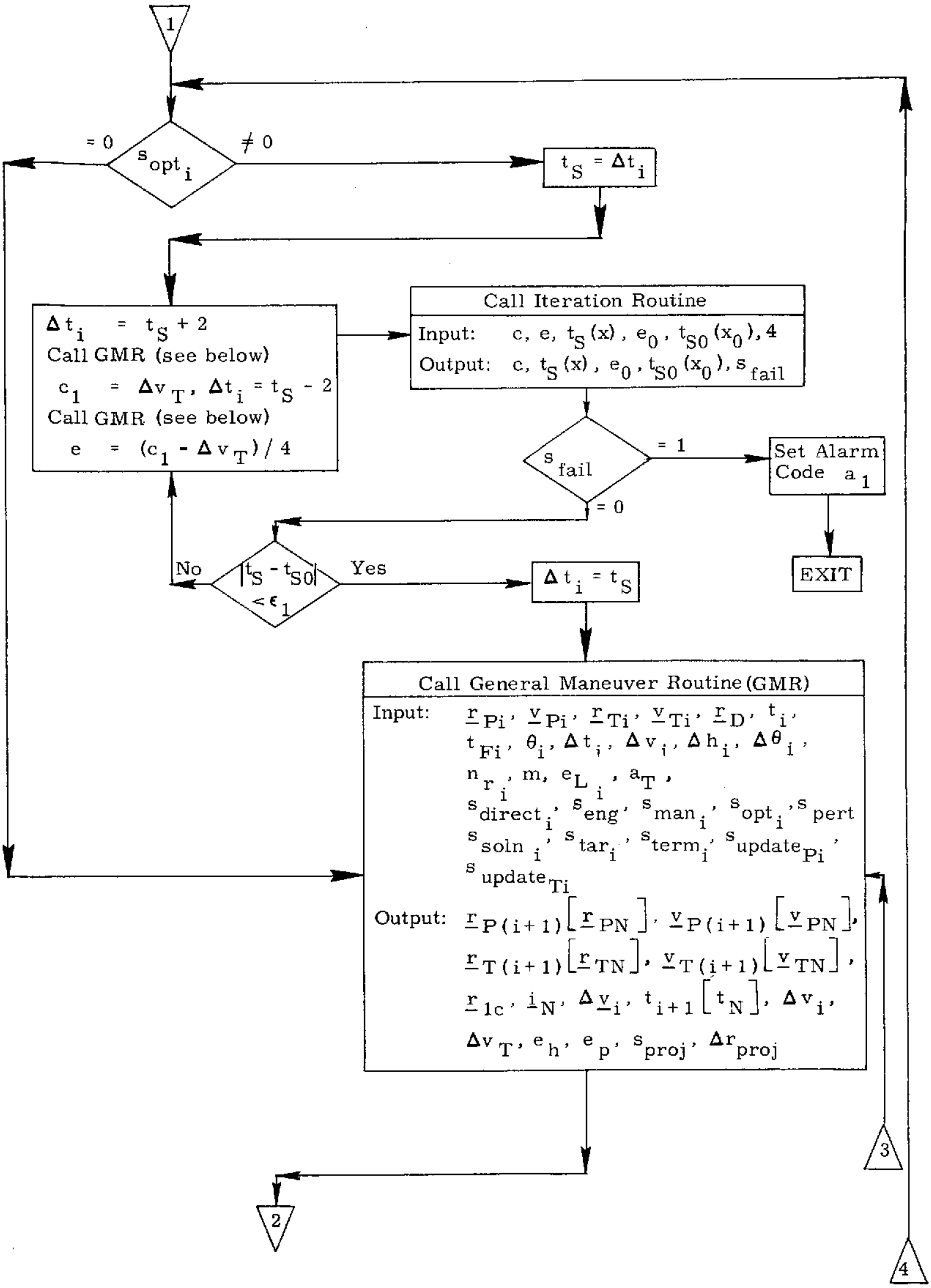


Figure 6b. Main Program - Detailed Flow Diagram

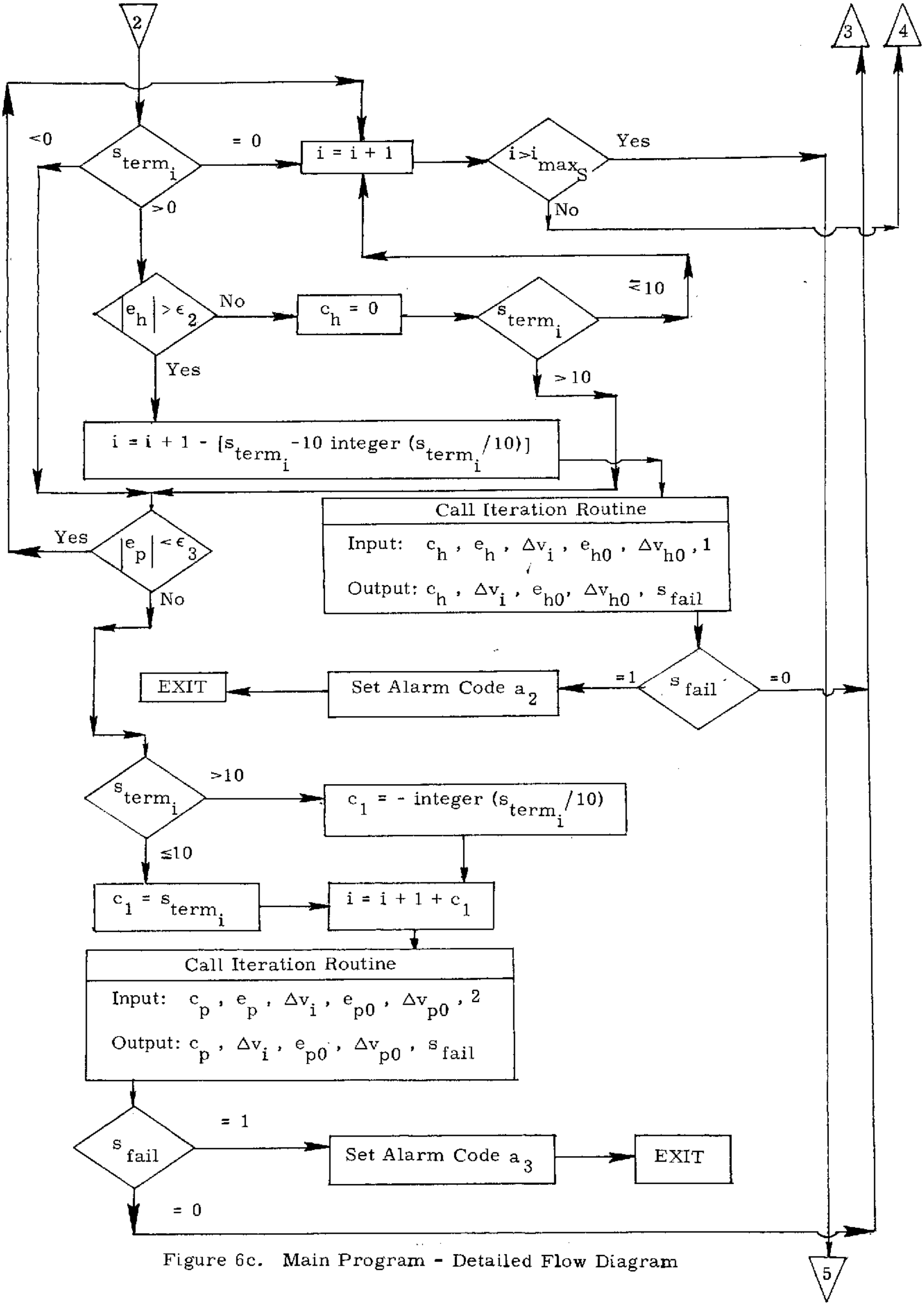


Figure 6c. Main Program - Detailed Flow Diagram

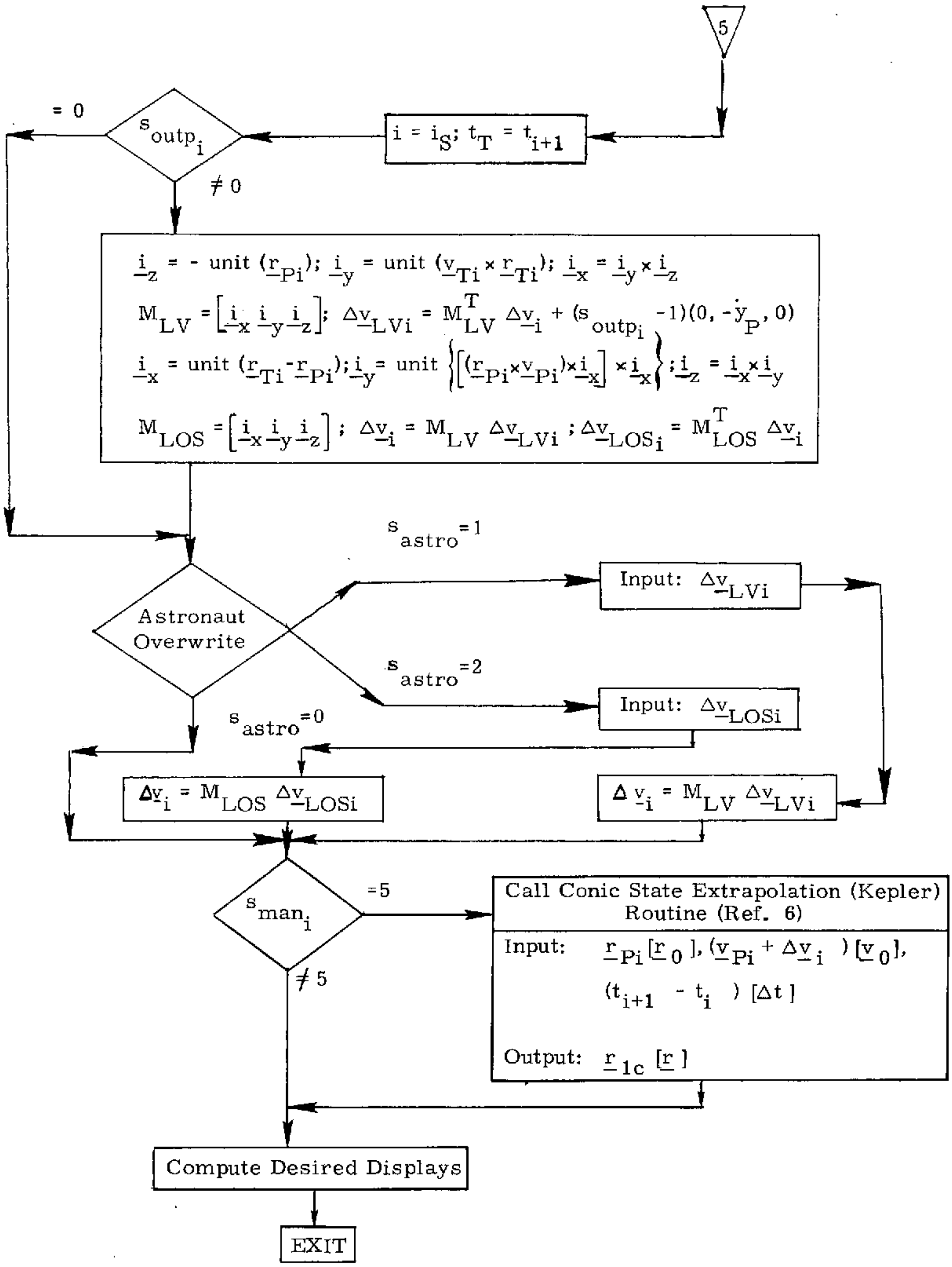


Figure 6d. Main Program - Detailed Flow Diagram

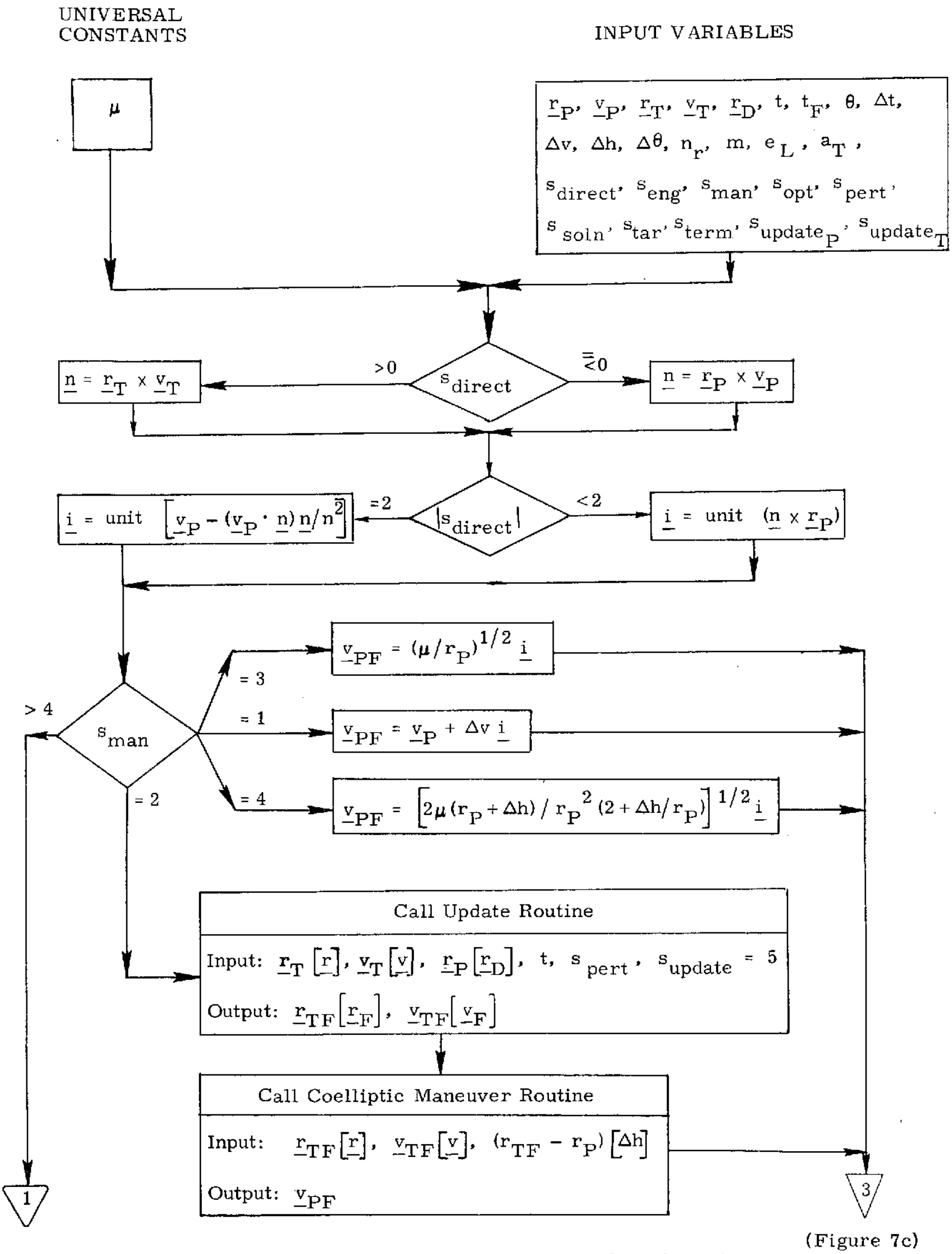


Figure 7a. General Maneuver Routine - Detailed Flow Diagram

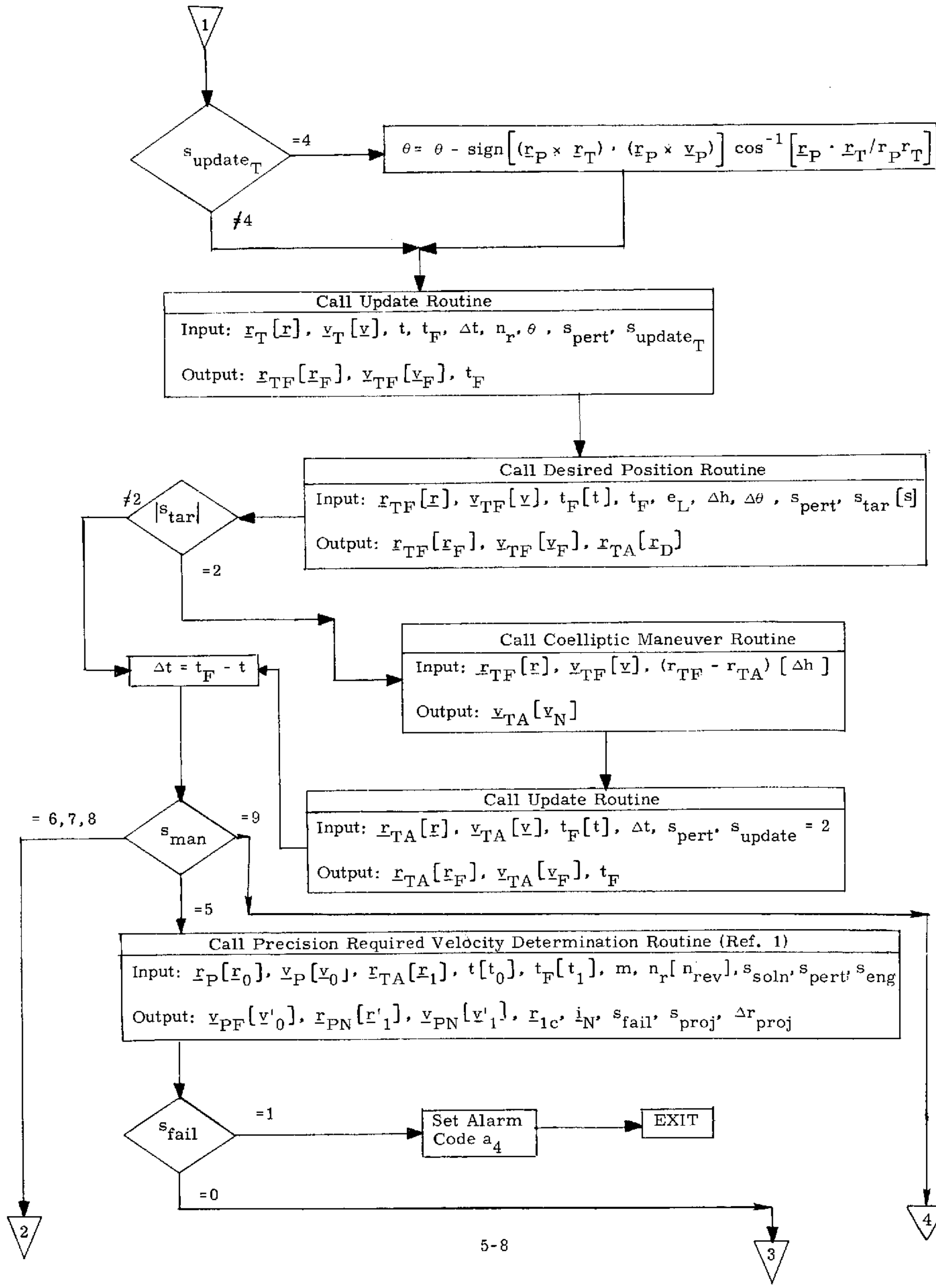


Figure 7b. General Maneuver Routine - Detailed Flow Diagram

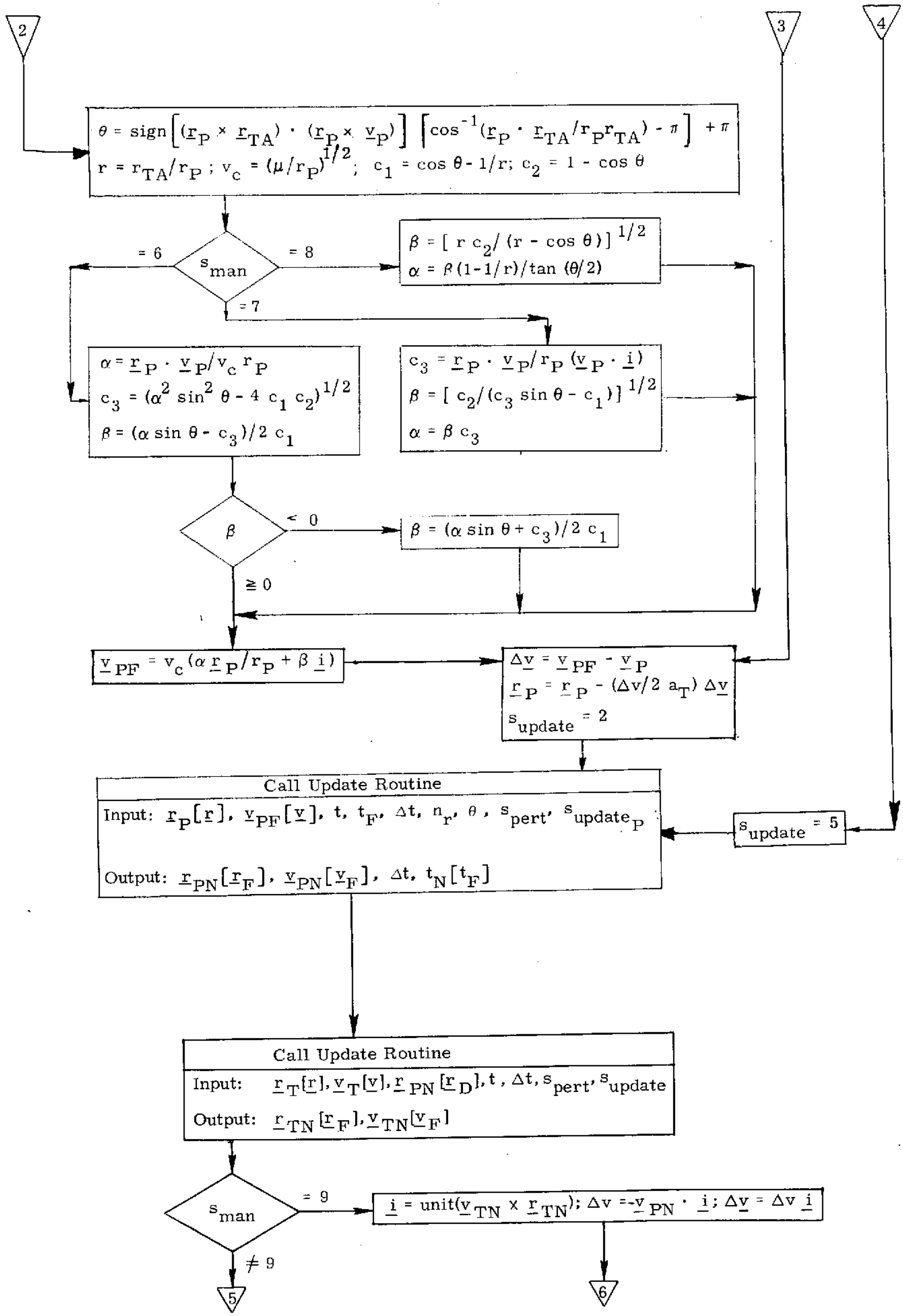


Figure 7c. General Maneuver Routine - Detailed Flow Diagram

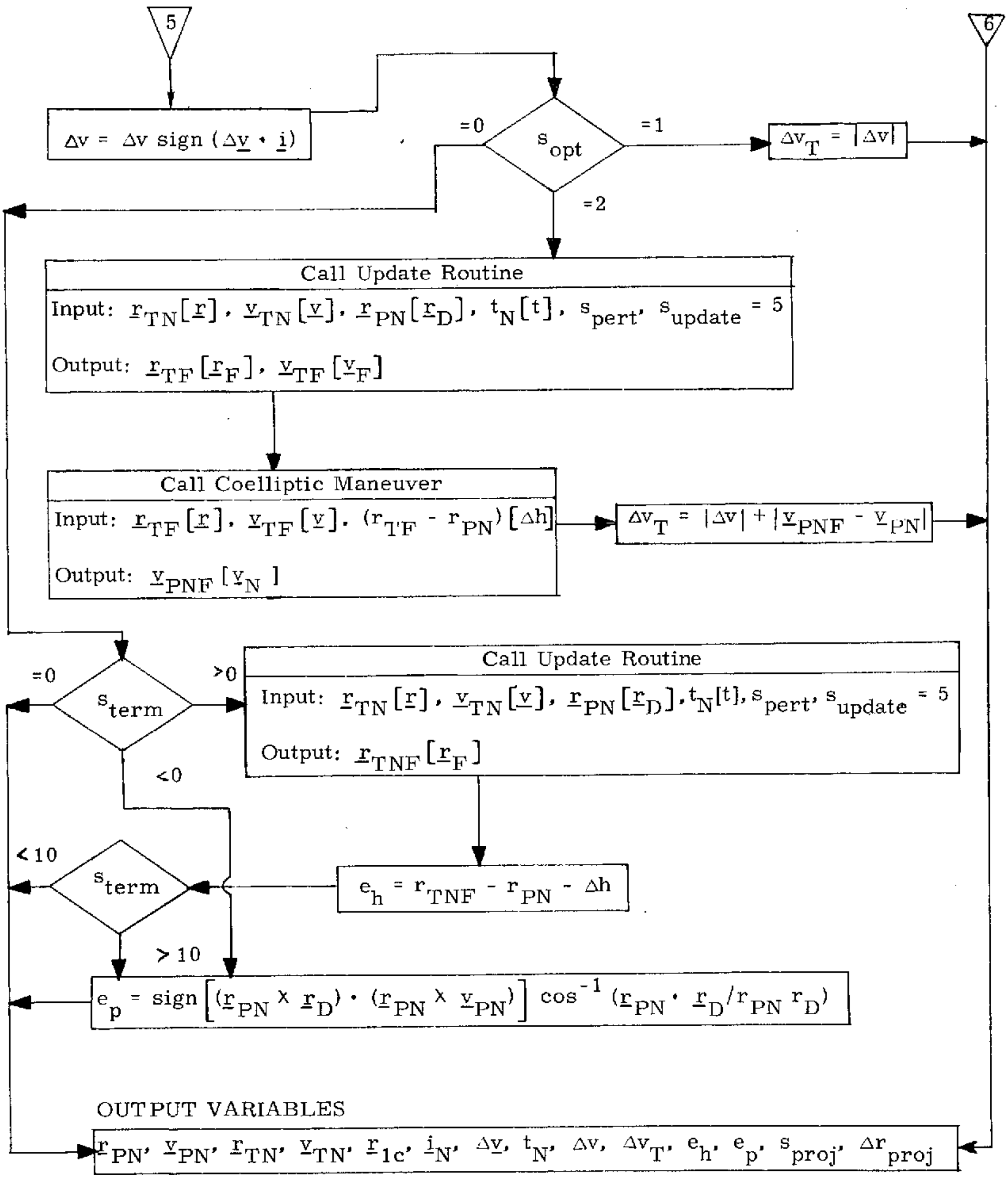


Figure 7d. General Maneuver Routine - Detailed Flow Diagram

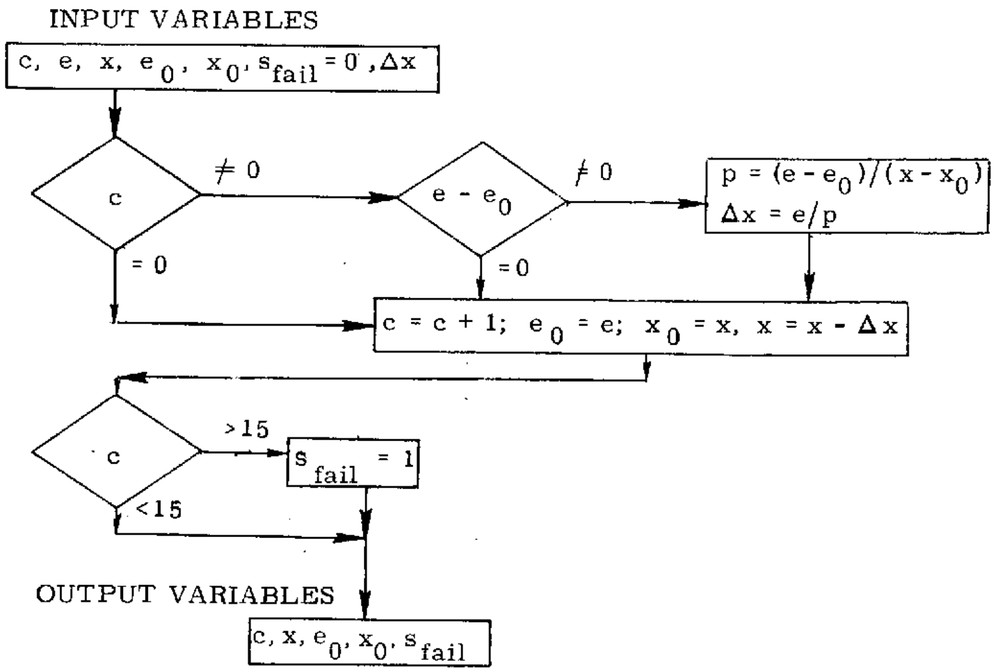


Figure 8. Iteration Routine - Detailed Flow Diagram

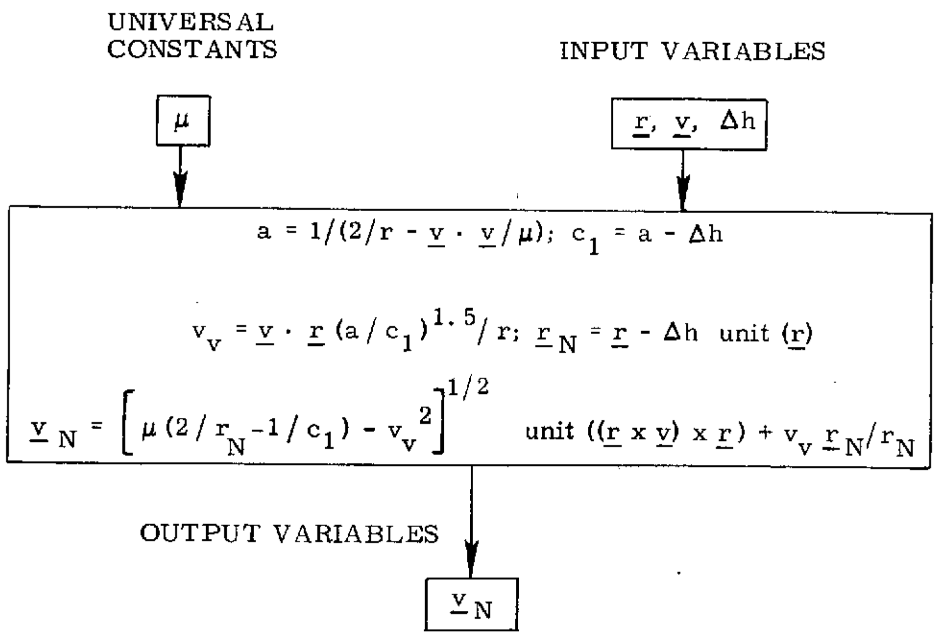


Figure 9. Coelliptic Maneuver Routine - Detailed Flow Diagram

INPUT VARIABLES

$\underline{r}_P, \underline{v}_P, \underline{r}_T, \underline{v}_T, t, s_{pert}, s_{phase}$

$$\underline{i} = \text{unit} \; (\underline{r}_T \times \underline{v}_T)$$

$$\underline{r}_D = \text{unit} \left\{ \left[(\underline{r}_P \times \underline{v}_P) \times \underline{r}_P \right] \times \underline{i} \right\}$$

$$c_1 = \text{sign} \left[(\underline{r}_D \times \underline{r}_T) \cdot \underline{i} \right]$$

$$\theta = \text{sign}(s_{phase}) \left\{ c_1 \left[\pi - \cos^{-1} (\underline{r}_D \cdot \underline{r}_T / r_T) \right] - (\left| s_{phase} \right| - 1)(1 + c_1) \pi - \pi \right\}$$

Call Update Routine

Input: $\underline{r}_T [\underline{r}], \underline{v}_T [\underline{v}], t, \theta, s_{pert}, s_{update} = 4$

Output: $\underline{r}_T [\underline{r}_F], \underline{v}_T [\underline{v}_F], \Delta t$

Call Update Routine

Input: $\underline{r}_T [\underline{r}], \underline{v}_T [\underline{v}], -\Delta t, s_{pert} = 0, s_{update} = 2$

Output: $\underline{r}_T [\underline{r}_F], \underline{v}_T [\underline{v}_F]$

OUTPUT VARIABLES

$\underline{r}_T, \underline{v}_T$

Figure 10. Phase Match Routine - Detailed Flow Diagram

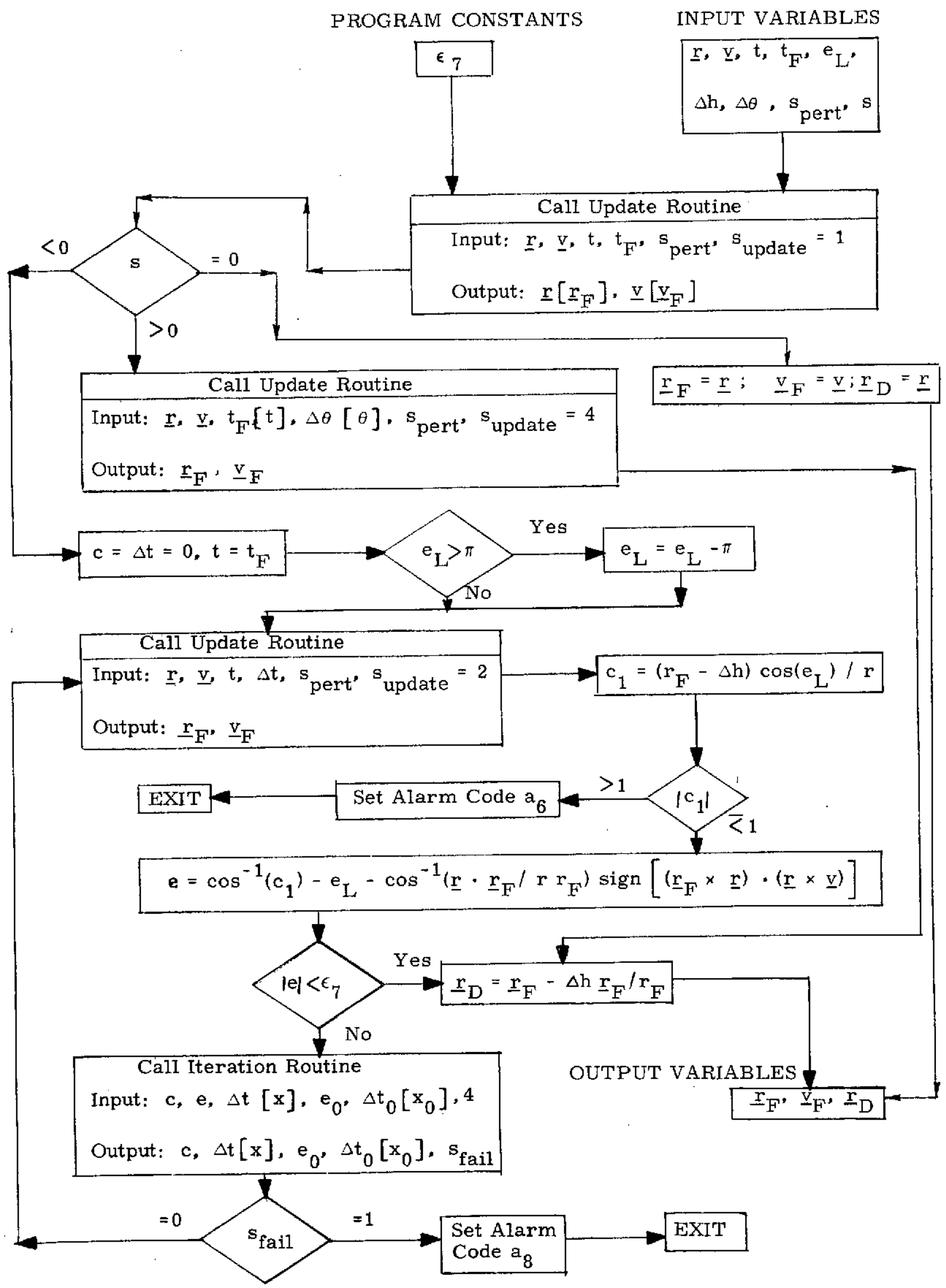


Figure 11. Desired Position Routine - Detailed Flow Diagram

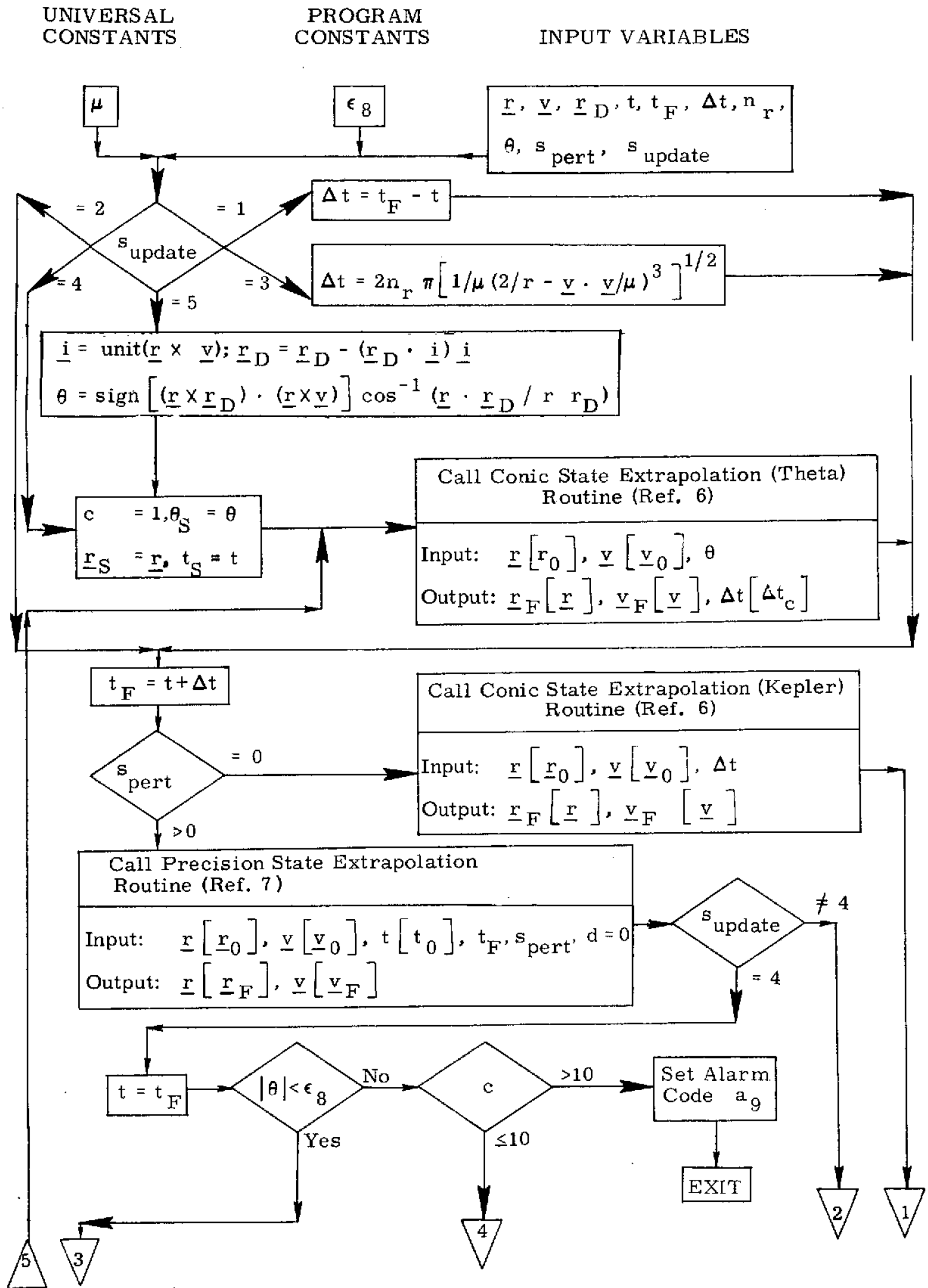


Figure 12a. Update Routine - Detailed Flow Diagram

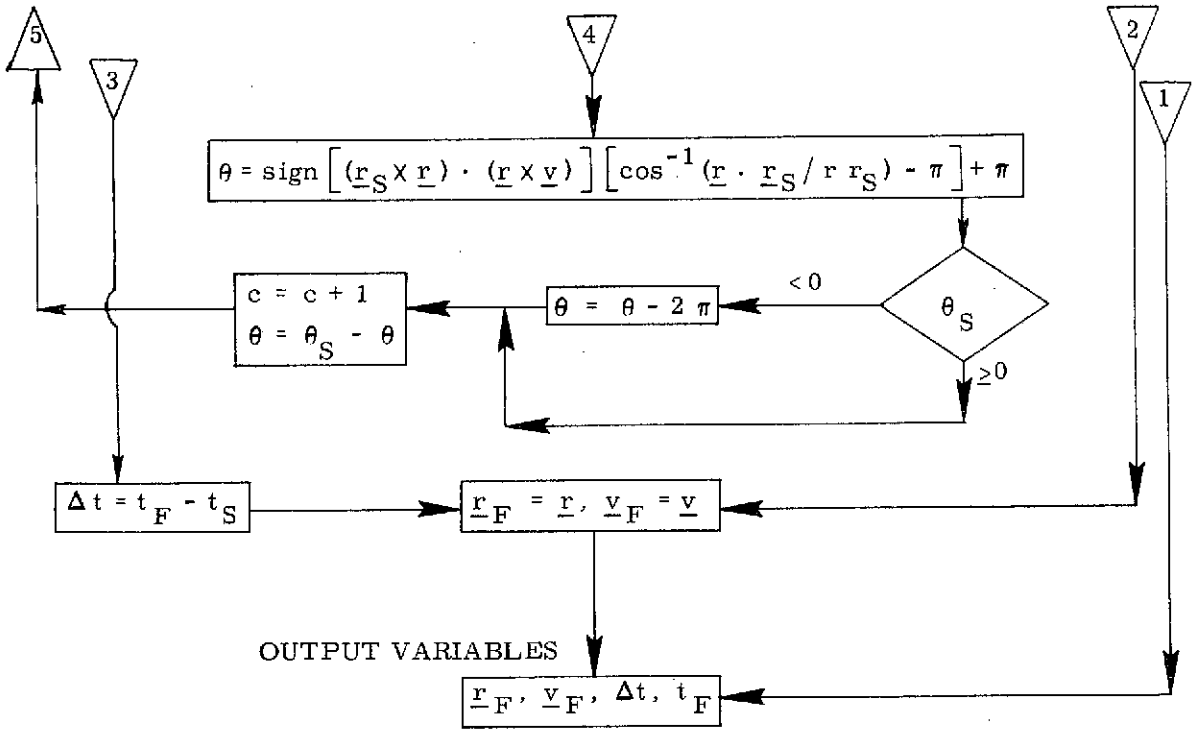


Figure 12b. Update Routine - Detailed Flow Diagram

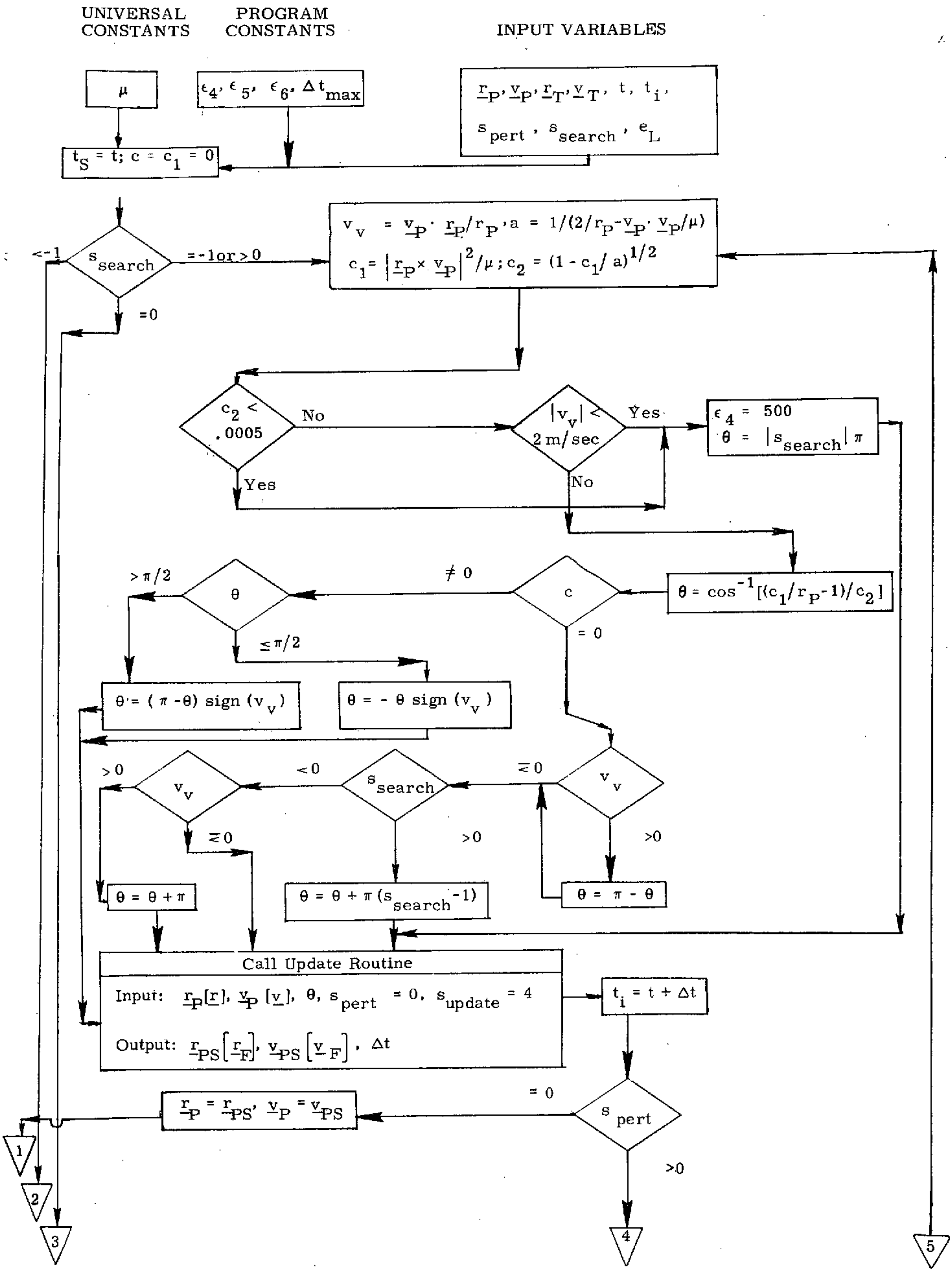


Figure 13a. Search Routine - Detailed Flow Diagram

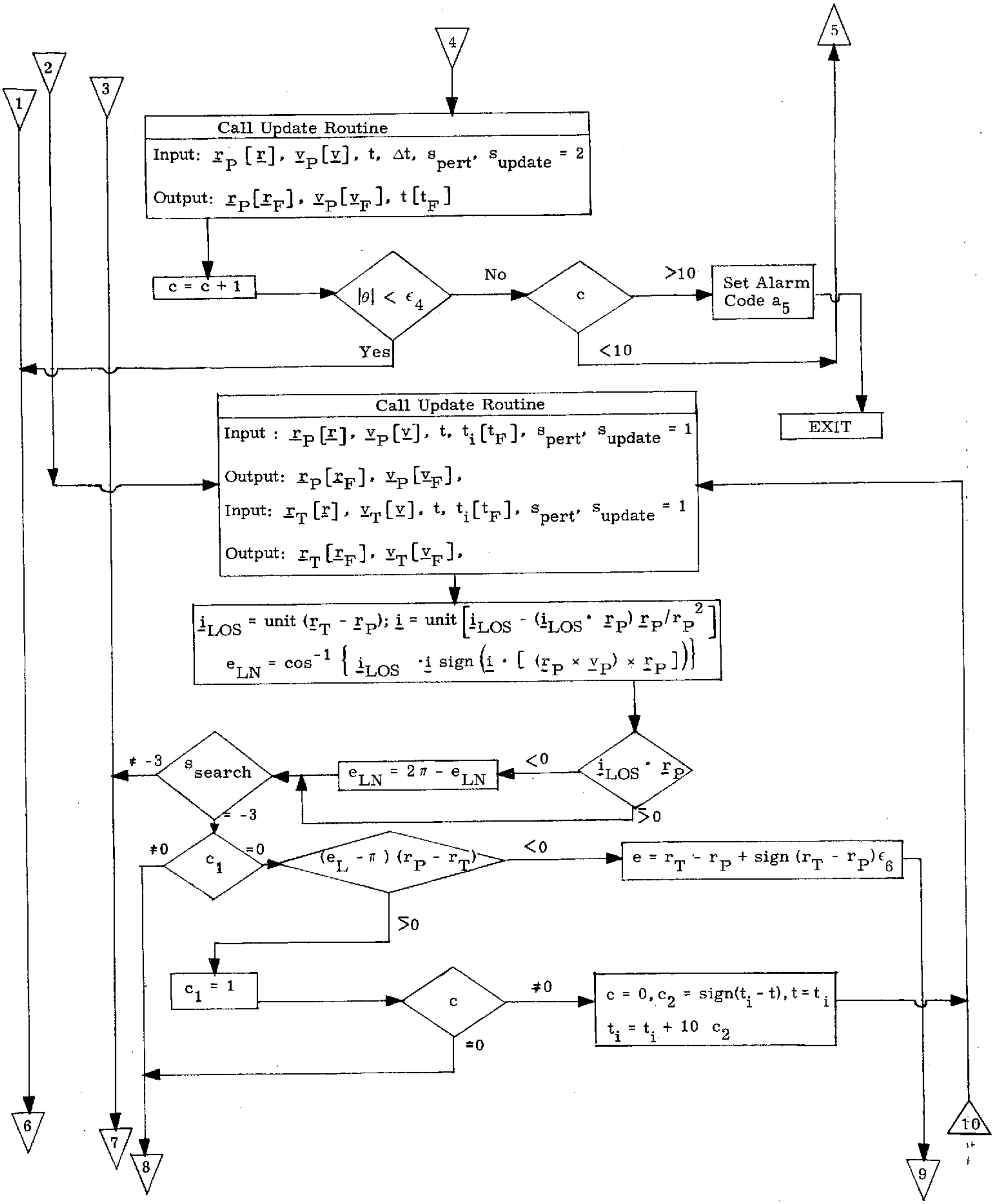


Figure 13b. Search Routine - Detailed Flow Diagram

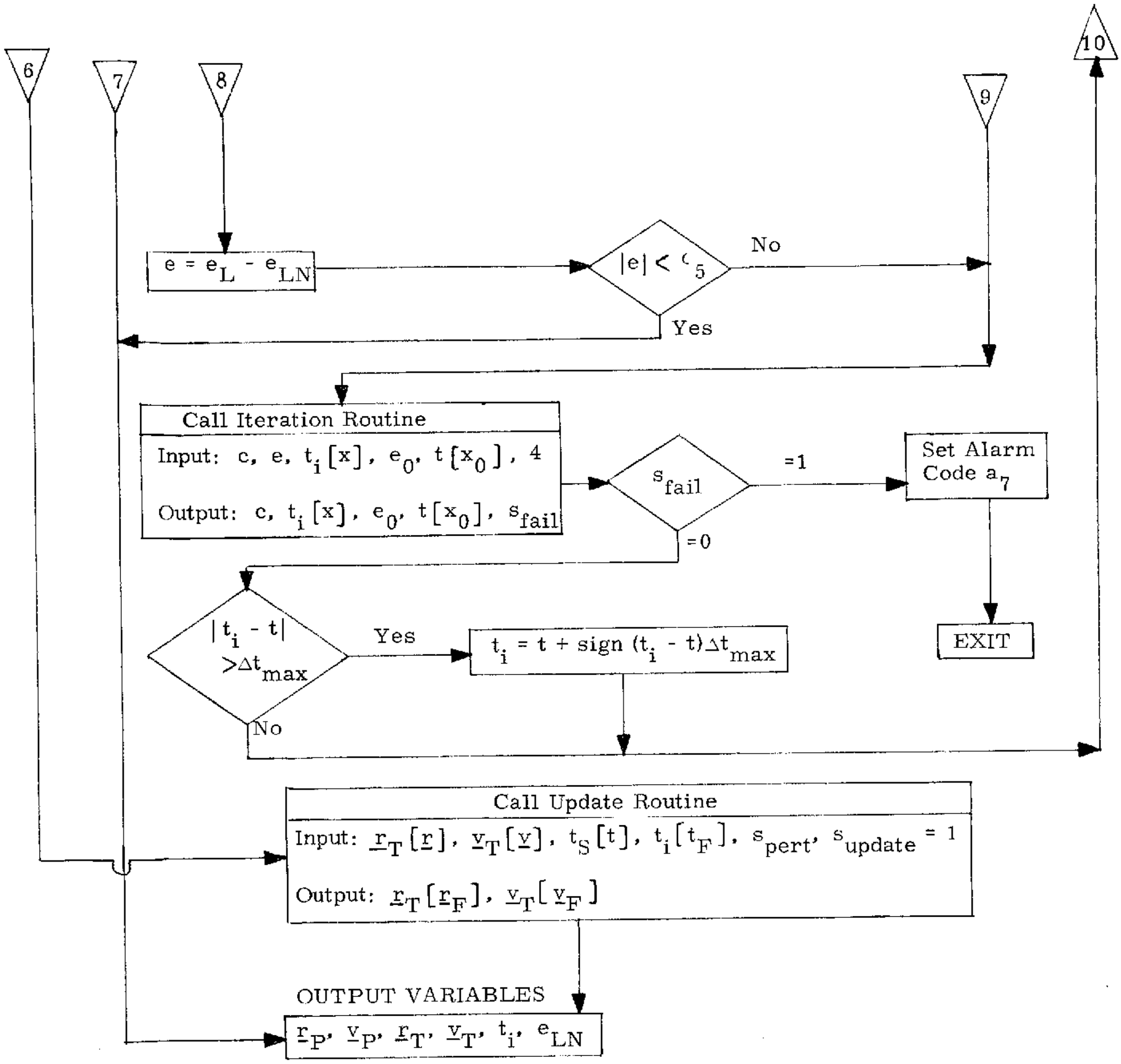


Figure 13c. Search Routine - Detailed Flow Diagram

6. SUPPLEMENTARY INFORMATION

The Shuttle rendezvous targeting program proposed herein uses the basic targeting philosophy employed in Apollo and Skylab. (See Ref. 3 and 4.) The Shuttle program represents, to some degree, a general solution to the rendezvous targeting problem. The program has the capability of solving the Apollo and Skylab rendezvous configurations as well as many other configurations that can be determined by specifying sets of secondary constraints. The constraints contained in this document may require modification as experience is gained in generating sets of secondary constraints based on a variety of Shuttle missions. Reference 9 contains a more detailed discussion of the nature and number of the secondary constraints employed in the maneuver sequences. Reference 11 is a users' guide to the Shuttle targeting program.

The targeting program logic has been verified by a computer simulation that disclosed no iterative convergence problems for the range of trajectories considered. References 10 and 13 contain simulations of the Skylab five-maneuver rendezvous configuration and the six maneuver (through TPI)"Mission 2" shuttle rendezvous, respectively. This program is currently being integrated with the Orbiter navigation programs to verify their combined effectiveness on a number of Orbiter missions.

Some of the maneuvers described in this document were based on conic orbit assumptions although the conic/precision switch was set to indicate that oblateness should be considered. One example is the computation of the "circular orbit" maneuver based on the conic circular velocity vector. This calculation can be modified to provide a maneuver that would result in a minimum altitude change orbit in an oblate gravity field. (See Ref. 2.)

As TPI is generally desired at the midpoint of darkness, a stored value for the sun's position vector could be used to generate a display giving the angular distance between TPI and the midpoint of darkness. This vector could also be used to directly define the location of the TPI point (using the Update Routine option of updating to be colinear with a specified vector).

A decision has not been reached on a final listing of the desired astronaut displays. As no program alarms are set for low perigees (following Skylab practice), the perigee altitudes for each maneuver segment contained in the maneuver sequence can be displayed. (If the orbits and maneuver magnitudes are approximately nominal, the astronaut can probably assume that the perigee altitudes will be acceptable.)

Some standard orbit shaping maneuvers (e.g. circularization, altitude change) are included as rendezvous targeting maneuvers in this document. Hence, the rendezvous targeting program as herein contained has a limited capability to perform orbit shaping maneuvers assuming a dummy target vehicle state vector is used. The program can be expanded to allow the computation of the orbit shaping maneuvers for the shuttle mission by eliminating the need for the dummy state vector and adding additional options for orbit shaping maneuvers. However, the additional program complexity and the large number of inputs that have to be set nominally equal to zero do not seem to justify a unified approach to the calculation of the rendezvous targeting and orbit shaping maneuvers. Once all the orbit shaping maneuvers are defined, a separate Space Shuttle GN&C Equation Document will be issued for orbit shaping targeting. This document might utilize some of the routines herein contained.

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13. Tempelman, W., "The Computation of the Mission 2 Rendezvous Profile Using the Shuttle Rendezvous Targeting Program", STS Memo No. 35-73, Draper Laboratory, May 2, 1973.