

CHARLES STARK DRAPER LABORATORY


# SPACE SHUTTLE GN\&C EQUATION DOCCMENT 

# No. 24 <br> Unified Powered Flight Guidance <br> By 

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The publication of this report does not constitute approval by the National Aeronautics and Space Administration of the findings or the conclusions contained therein. It is published only for the exchange and stimulation of ideas.

## PREFACE

This document reflects a complete revision of the ortiter powered flight guidance acheme. A unifled approech to powered light guidance has been taken to accommodate all phases of exo-atmospheric orbiter powtered flight, from ascent through deorbit. The guidance scheme has been changed from the previous modified version of the Lambert Aim Point Maneurer Mode used in Apollo to one that employs Linear Tangent Guidary e concepts. As such it supersedes the previous document. "Powered Flight Guidance". GN\&C Equation Document No. 11, Rev. 2, April 1972. In addition, this documint replaces the previons ascent phase equation document titled "Powered Ascent Guidance", GNeC Equation Document No. 1-71, January 1971.

## FOREWORD

This document is one of a series of candidates for inclusion in a suture revision of JSC-04217 "Space Shuttle Guidance. Navigation and Control Design Equations". The enclosed has been prepared under NAS9-10268. Task No. 15-A. "GN\&C Flight Equations Specification Support". and applies to functions 1 through 4 of the Orbit Insertion Guidance Module (OG1) and to functions 2 through 6 of the Power Flight Guidance Module (OG2) as defined in JSC-03690, Rev. D. "Space Shuttle Orbiter Guidance, Navigation and Control Software Functional Requirements". dated 'anuary 1973.


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## NOMENCLATLRE

| ${ }^{2}$ | Specific force lamit during SSME mancusers |
| :---: | :---: |
| ${ }^{\text {a }}$. 1 | Assutited thrus: acceleration for ${ }^{\text {t }}$ ( ${ }^{\text {phase }}$ |
| $C 1$ $\therefore$ | Constants used to detine iarget (entry interface) velocity consiraint. $y_{i}=C_{1} \cdot C_{2} i_{h}$ |
| $C_{10}$ | Intermediate variable used to determine sensiticith of $\mathrm{E}_{\mathrm{a}}{ }^{\text {to }} \mathrm{I}_{1}$ |
| 'sime <br> 'OMS <br> ${ }^{1} \mathrm{HC}$ |  |
| ${ }^{1} \mathrm{~T} .1$ | Totai assumed thrust for $\mathrm{i}^{\text {th }}$ phase |
| $\pm$ | Gravit bector |
| H | Thrus: acceleration inteural. $\int^{t}\left(f T^{i(a)}\right) i^{2}(a)$ |
| $\mathrm{H}_{\mathrm{i}}$ |  |
| $\underline{1}$ | Unit thrust vector |
| $\left.\begin{array}{c}i \\ \vdots \\ \vdots \\ \vdots \\ -2\end{array}\right\}$ | Lnit vectors relative to desired trajector, $\beta$ binne : $\underline{i}_{\text {, }}$ is radial along $\underline{r}_{d}, \underline{y}$, is normal to de:sueciajectory planc, and $\underline{i}_{2}$ is in the downrange direction. $\underline{i}_{z}=\underline{i}_{x} \times \underline{x}_{1}$ |
| $\underline{1}$. | I nit ventor in the direction of the orotal anculat momentum: vector normal to the transfer plane (set Ref. 5) |
| $\stackrel{1}{-r t}$ | U'nit rector in direction of $\underline{r}_{1}$ |

i. j Element subscript variables

J
$\mathbf{J}_{i}$
$K_{i}$
k
$L$
$L_{i}$
m
${ }^{m} 0, i$
$\left.\begin{array}{l}\dot{m}_{\text {SSME }} \\ \dot{\mathrm{m}}_{\mathrm{OMS}} \\ \dot{\mathrm{m}}_{\mathrm{RCS}}\end{array}\right\}$
$\dot{m}_{i}$
n *ismber of thrust phases
$\mathbf{n}_{\text {rev }}$

P
$\mathbf{P}_{i}$

Q
$Q_{i}$
$i^{\text {th }}$ element of $J . \int_{t_{g o, i-1}}^{t g o, i}{ }_{\left(f_{T} / m\right) t d t}^{0}$

Maximum throttle setting of SSME
$i^{\text {th }}$ element of $\left.L, \int_{t_{\text {go, i-1 }}}^{t} \mathrm{go,i}_{\mathrm{T}} / \mathrm{m}\right) \mathrm{dt}$
Current e. imated venicle mass

Mass at beginning of $i^{\text {th }}$ phase
i Total mass flow rate, $\mathrm{i}^{\text {th }}$ phase

Number of revolutions of coast (see Ref. 5)
. $i^{\text {th }}$ eiement of P. $\int_{t g o, i-1}^{i g o, i} \int_{g o, i-1}^{t}\left(f_{T} / m\right) s^{2} d s d t$
Thrust acceleration integral,
$i^{\text {th }}$ element of $Q . \int_{t}^{t}$ go,i-1

Thrust acceleration integral, $\int_{0}^{t_{\text {go }}}\left(f_{T} / m\right) t d t$

SSME throttle setting. $i^{\text {th }}$ phase ( $1.0=100 \%$ )

Subscript variable referring to current phase number
Thrust acceleration integral, $\int_{0}^{t}{ }_{0}\left(f_{T} / m\right) d t$

Mass flow rate of single SSME, OMS, or RCS engine, respectively

Thrust acceleration ircegral, $\int_{0}^{t} \int_{0}^{t}\left(f_{T} / m\right) s^{2} d s d t$
$\int_{\mathrm{gO}_{\mathrm{i}} \mathrm{i}-1}^{\mathrm{t}}\left(\mathrm{f}_{\mathrm{T}} / \mathrm{m}\right) \mathrm{s} d s \mathrm{dt}$

| $\underline{r}$ | Vehicie position vector |
| :---: | :---: |
| $\underline{\underline{r a s}}$ tias | A position bias to account for effects of a rotating ihrusi .. . :or $\underline{(r}_{\text {bias }}=\underline{r}_{\text {go }}-\underline{r}_{\text {thrust }}$ ) |
| $\underline{r}_{\text {c }}$ | Vehicle position vector at beginning of gravity computation coast segment |
| $\underline{r}_{\text {c }}$ | Vehicle position vector at end of granty computation coast segment |
| $\underline{r}_{\text {d }}$ | Desired terminal (cutoff) position |
| $r_{d}$ | Desired radius magnitude at terminal (cutoff) position |
| rgo | Position-to-be-gained including bias ( $\underline{r}_{\text {thrust }}$ reflects irue position-to-be-gained) |
| $\underline{r}$ ¢0xy | Projection of $\underline{r}_{\text {go }}$ on plane defined by $\underline{-}_{x}$ and $i_{y}$ |
| $\mathrm{r}_{\text {goz }}$ | Component of $\underline{r}_{\text {go }}$ along $\underline{i}_{z}$ (downrange) |
| $\underline{E r g v}^{\text {gra }}$ | Second integral of central force field gravitational acceleration over thrusting maneuver |
| $\underline{E}_{p}$ | Predicted terminal (cutof) position |
| $\underline{r}_{\text {ref }}$ | Position on reference trajectory at tirre ${ }^{\text {tref }}$ |
| $\mathrm{r}_{\mathrm{t}}$ | Target position in inertial coordinates |
| $\underline{1}$ tei | Entry interface target in Earth fixed coordinates |
| $\underline{\mathbf{r}}$ thrust | Second antegral of thrust acceleration vector over thrusting maneliver |


| S | Thrust acceleration integral, $\int_{0}^{t_{g o}} \int_{0}^{t}\left(f_{T} / m\right) d s d t$ |
| :---: | :---: |
| engoff | Switch indicating whether engine-off command has been issued, $\left\{\begin{array}{l}=0, \text { command not issued } \\ =1, \text { command issued }\end{array}\right.$ |
| sguess | Swritch set by Lambert routine indicating type of solution (see Ref. 5) |
| $\mathbf{S}_{\mathbf{i}}$ | ith element of $S, \int_{t_{g o, i-1}}^{t_{g o, i}} \int_{\mathrm{t}_{\mathrm{go,i}, \mathrm{i}}}^{t}\left(\mathrm{f}_{\mathrm{T}} / \mathrm{m}\right) \mathrm{ds} \mathrm{dt}$ |
| $S_{\text {mode }}$ | Maneuver mode $\begin{cases}1 & \text { Ascent, standard } \\ 2 & \text { Ascent, reference trajectory } \\ 3 & \text { Ascent, Lambert } \\ 4 & \text { Ascent, once-arourd abort } \\ 5 & \text { Ascent, return-to-launch-site abcrt } \\ 6 & \text { On-orbit, external delta-v } \\ 7 & \text { On-orbit, Lambert } \\ 8 & \text { On-orbit, deorbit }\end{cases}$ |
| $s_{\text {pass } 1}$ | $\begin{cases}=1 & \text { first active guidance call } \\ =0 & \text { not first \&ctive guidance call }\end{cases}$ |
| $s_{\text {pert }}$ | Gravity perturbation switch ( $>0$ for non-Keplerian model) |
| $s_{\text {phase, }}$ i | $\left\{\begin{array}{l} =0, \text { constant tinrust during phase } i \\ =1, \text { constant acceleration during phase } i \end{array}\right.$ |
| $s_{\text {pre }}$ | $\begin{array}{ll} \text { Prethrust switch } & \mid=1 \text {, prethrust } \\ & =0 \text {, active guidance } \end{array}$ |
| ${ }^{\text {proj }}$ | Switch indicating whether the initial and target position vectors are to be projected into the plane defined by $-N$ (see Ref. ) |
| Ssoln | Lambert solution type switch (see Ref. 5) |
| $\begin{aligned} & { }^{s_{S S M E, i}} \\ & { }^{s^{\prime}} \mathrm{OMS,i} \\ & \mathrm{~s}_{\text {RCS, }} \end{aligned}$ | Number of SSME., OMS, RCS engines,respectively, assumed thrusting during $i^{\text {th }}$ phase |


| $t$ | Time associated with $\underline{\mathbf{r}}, \underline{\mathrm{V}}$ |
| :---: | :---: |
| ${ }^{\text {t }}$, i | Estimated burn time remainirg in phase i |
| $t_{c}$ | Coast time between last and next-to-last phasc for tank separation |
| ${ }^{\text {tref }}$ | Nominal cut-off time for reference trajectors mowle |
| $\mathrm{t}_{\mathrm{go}}$ | Time-to-go until end of maneuver |
| ${ }^{\prime}$ go | $\mathrm{t}_{\text {go }}$ of previous call |
| $\mathrm{t}_{\mathrm{go,i}}$ | Time-to-go until end of ith phase |
| ${ }^{\text {tig }}$ | Ignition time of first phase |
| ${ }^{\text {prev }}$ | t of previous guidance cycle |
| $t_{t}$ | Time at target (point where terminal constraint defined) |
| u | Iteration variable determined in Lambert Routine (see Rei. 5) |
| $\underline{v}$ | Vehicle velocity vector |
| $\underline{V}_{\text {bias }}$ | A velocity bias te account for effects of a rotati:le thisest vector ( $\underline{-b}_{\text {hias }}=\underline{V}_{\text {go }}-\underline{V}_{\text {thrust }}$ ) |
| $\mathrm{v}_{\mathrm{c}} 1$ | Vehicle velocity vector at beginning of gravity computation coast segment |
| $\mathrm{v}_{\mathrm{c}} \mathbf{2}$ | Vehicle velocity vector at end of gravity computation roasi segment |
| $\stackrel{\square}{-}$ | Desirea velority vector at terminal (cutoff) masition |
| $\mathrm{v}_{\mathrm{d}}$ | Desire.l velocity magnitude at terminal (eutaril maition |
| $\underline{\because r} \mathrm{~d}$ | Value of $\underline{v}_{a}$ resulting from perturbed time of flight |


| $v_{\text {ex, }}$ | Effective exhaust gas velocity for phase 1 |
| :---: | :---: |
| $\underline{V g o}^{\text {O }}$ | Velocity-to-be-gained including bias. (V) thrust refleci: true velocity-to-be-gained) |
| ${ }^{\mathbf{V}} \mathrm{goz}$ | Component of $\underline{v}_{\text {go }}$ along $\underline{i}_{z}$ (downrange) |
| $\underline{v}_{\mathbf{g o}}^{\prime}$ | Recomputed $\underline{V}_{\text {go }}$ to satisfy terminal constraints |
| $\underline{v}_{\text {grav }}$ | First integral of central force field gravity acceleration over thrusting maneuver |
| $\underline{-r e f}^{\text {r }}$ | Velocity on reference trajectory at time $t_{\text {ref }}$ |
| $\underline{\mathbf{v}} \mathbf{t}$ | Velocity vector at target |
| $\underline{V}{ }^{\prime}$ | Value of $\underline{\underline{U}}_{t}$ resulting from perturbed time of flight |
| $\mathbf{v}_{\text {th }}$ | Projection of $\underline{v}_{t}$ on horizontal plane |
| $\underline{-1}$ |  |
| $\underline{\text { V }}$ thrust | First integral wi thrust acceleration vector over thrusting maneuver |
| $\underline{v} \mathbf{t v}$ | Projection of $\underline{v}_{t}$ on vertical |
| $\underline{V}^{\text {'tv }}$ | Projection of $\underline{v}^{\prime} \mathbf{t}$ on vertical |
| $\gamma_{d}$ | Desired inertial flight path angle at terminal (cutoff) position |
| $\Delta \underline{E}$ | Position offset used in gravity computation |
| $\Delta r_{z}$ | Dowrrange component of terminal position error tusec during reference trajectory mode) |
| $\Delta t$ | Guidance cycle time step |
| $\therefore$ catoff | Value of $t_{g o}$ used to define time to issue engine cutoff command and terminate active steering computations |


| $\Delta t_{\text {go }}$ | Change in $\mathrm{t}_{\text {go }}$ used in throttling computations |
| :---: | :---: |
| $\Delta t_{10}$ | Time interval before $\mathrm{t}_{\mathbf{i g}}$ to start active guidance calls |
| $\Delta \mathbf{V}_{C}$ | Impulse velocity increment used in gravity omputations |
| $\underline{U V}_{\mathbf{g o}}$ | Change in $\mathrm{V}_{\text {go }}$ |
| $\Delta \underline{v}_{\text {sensed }}$ | Total velocity change accumulated on accelerometers since last reading |
| $\triangle \nabla^{\text {OMS }}$ | Total characteristic velocity to be imparted during the last ascent phase by the OMS engines |
| $8 t$ | Perturbation in coast time used in deorbit rem :ired $v$.locity computations to determine sensitivity to coast time |
| ¢ cone | Sine of half cone angle of ex.'usion (see Ref. 5) |
| ${ }^{*}$ vgo | Value of $\\|_{\underline{g} \mathbf{g}} \mid$ defining prethrust convergence limit |
| $\underline{\lambda}$ | Unit vector in direction of $\underline{\mathrm{g}}_{\mathrm{go}}$ |
| i | Time derivative of unit vector coincident with $\underline{\lambda}$ but rotating wir. lesired thrust vector turning rate, $\boldsymbol{w}_{\mathbf{f}}$ |
| $\rho$ | namping factor used in determining the change in $\mathrm{JV}_{\mathrm{g}} \mathrm{g}$ |
| $\sigma$ | Scaling factor in required velocity computations |
| ${ }^{\boldsymbol{T}} \mathbf{i}$ | Hatio of mass to mass flow rate frr , .h phase |
| - | - |

- Time rate of change of
$u_{i} \quad$ Desired thrust eector turning rate


## 1. INTRODUCTION

### 1.1 Objectives and Requirements

The primary objective of powered flight guidance is to issue proper steering and if necessary throttle commands during the thrusting portions of a mission such that the desired objectives of the maneuver are satisfied in a reasonably efficient manner. In addition, the navigated vehicle state vector must be maintained through the maneuver. These are obvious requirements of the powered flight guidance program. Additional objectives or requirements, which influence the overall design. are listed below:
(1) The guidanc. program should be applicable to all powered maneuvers. This objective assumes that a single guidance routine is less expensive to design. code, and maintair. in a flight computer than several smaller routines. This guidance scheme, referred to as Unified Powered Flight Guidance (UPFG), can be used for all exo-atmospheric orbiter: thrusting maneuvers, from ascent through decrbit. Future versions of this scheme should probably include the atmospheric solid rocket toost phase for completness and to minimize any difficulties in transition from one guidance phase to the next. A complete description of the many types of maneuvers possible with this UPFG routime is includedi in the following section.
(2) The guidance program should be simple and fiexible. Iny guidance program which must handle the manv types oi m.aneuvers required oi the space shuttle orbiter, irom the relatively high accelerations during ascent to the vis: ion accelerations experienced with a single Orbital Maneurnran System (OMS) engine, can probably not be classified as truly simple. However, by properly structuring the routin:the various requirements can te handled efficiently and the impact of new requirements can be minimized. This LP $\quad$ C, routine is structured such that communality of basic computations is maintained with specialized operations performed ancording to the mission phase and desired maneuver objectives. In addition, to minimize program size. maximum use is made of subroutines required for other GAN functions.
(3) The premaneu i 1 prediction of the maneuver $\Delta v$ require ment should be reasonahly accurate. In order to assist the premaneuver targeting process and mahe mission critical decisions, the powered iligh: gcicance scheme must accurately predict the maneu:er $\& \mathfrak{V}$. especially for long, iow acceleration orbital manewers. This requirerrent can have a very significant effect upon the geidance algorithm design. Many guidance scheines are based upon approximations which become more accu!: $:$ as the man :ir ar progresses. Although these schem = may sausfac'. ily valculate steering commanes duriny the course of the maneuver, they may not adequately predict the maneuver $\Delta x$. This unifleq guicance scheme is based upon modifications to the Linear Tangent Guidance (LTG) concept which improve overall accuracy for both premaneuver targeting and guidance. This impro:ement in the LTG concept uas aiso required for one of the asrent guidance modes so that engine throttle changes, which are based upon an accurate prediction of the terminal (cutofi) state of the rehicle. could be properly calculated.
(4) The gcidance algorithm should satisfy primary maneuver objectives for nominal and perturbed conditions. When practical, the guidance scheme should close the guidance loop around the true naneuver constraints, rather tian a set of artificial constraints based upon nominai conditions. For example, during many on-orbit maneuvers the true constraints lie on the coasting trajectnry subsequent to the maneuver. During the terminal phase rendezvous maneuver the objective is to intercept the target vehicle. During the deorbit maneuve: the true mane uver constraints are ai entry interface, where a pre-dctermined relationshif betueen entry range, velocity, and fight path angle mus: be satisfied. It is possible to determine a set of artificial constraints. : tefined at thrust cutoff. which wili satisfy the maneuver objectives. However, the process of accurately valculatir: these artificial constraints can significan:ly complicate the targetins process. In addition, the artificial constraints unuld necessarily have to je based upon a nominal coast trajectors aiter thrust eutoff. Iny perturbations during the maneuver
> could result in a loss of performance (i.e. the true constraints are only partially satisfied) or an increase in total maneuver $\Delta v$. caused by trying to force the solution onto the nominal coast trajectory at thrust cutoff. Furthermore. if the guidance system satisfies true maneuver constraints, then much of the analysis necessary to determine guidance software performance under perturbed conditions can be eliminated.
> (5) The guidance algorithm should produce near fuel optimal maneuvers for nominal and per sbed conditions. The Linear Tangent Guidance (LTG) Equations which form the basis for the UPFG scheme are based upon classical optimization theory and appear to give excellent performance. However. it should be noted that even a truly optimal scheme will use excessive fuel if the ignition time is poorly chosen or the maneuver constraints result in an unnecessarily over-constrained maneuver which could be more efficiently performed by using more than one maneuver.

### 1.2 Eypes of Maneuvers

From a guidance viewpoint, the orbiter maneuvers can be conveniently separated into two classes. The first class, ascent, is characterized by a constraint on the vehicle altitude at thrust termination. The second class of maneuvers, onorbit, does not require any constraint on the position at thrust termination. On-orbit maneuvers are typically intended to place the vehicle onto any coasting trajectory which satisfies a rendezvous intercept constraint or deorbi entry-interface condition. In these cases any type of thrust cutoff position constraint would probably increase the maneuver $\Delta v$ unnecessarily.

Each of these classes of maneuvers can be further subdivided into individual maneuver modes with a particular set of objectives and a specified set of constraints which meet these objectives. These individual modes are listed below:
(I) Standard Ascent Maneuver

This mode is intended for use during the ascent phase of most miss ins. The insertion conditions are specified preflight, and defined by a desired terminal (cutoff) altitude, velocity. flight path angle, and orbital plane. Thus all components of tue terminal state are specified except the downrange component of position.

## (f) Ascent to Coast Reference Trajectory

This mode is intended for use on Mission 3B, a time critical mission involving ascent, rendezvous, satellite retrieval, and deorbit with return to the launch site one revolution after liffoff. To satisfy the objectives of this mission in the presence of perturbations and small launch delays. the main engine throttles are used to improve control over the ascent and subsequent coasting trajectory. Introducing the throttie command as one additional degree of freedom in the guidance algorithni makes it possible to insert the orbiter onto a coast reference trajectory at cutoff. This coast reference trajectory can be determined preflight to provide proper closing relocity with the satellite for the rendezvous braking maneuvers.
(3) Lambert Ascent Maneuver

This mode can be used to insert the orbiter at a specified altitude onto a coasting trajectory which intercepts a specified position (target) at a specified time. It is similar to the coast reference trajectory mode, except that throttle commands for trajectory control are not used. Therefcre, although the resulting coasting irajectory doez intercept the target, it does not guarantee the prr er closing velocity for sucessful braking. This mode can ve used during the latter phases of ascent to a coast reference trajectory, when g-limiting considerations may override throttling. Since most perturbe"'ions occur well prior to $g$-limiting, the coast reference trajectory mode is designed to complete compensatic 7 i.r perturbations prior to $g$-limiting. During $s$-iimiting, the Lambert ascent mode will mainto : 4 intercept trajectory with only small deviations in terminal rendezvous closing velocity. It shouid be noted that ne additional code is required to support this guidance mode, since it evolves as a natural result of two other modes, the on-orbit Lambert maneuver and the standard ascent.

## (4) Once-Around Abort

This mode is intended for use during the latter portion of ascent in the event of an engine failure. The resulting coasting trajectory insures that proper entry interface condition (range, velocity, and flight path angle) are achieved for sucessful reentry. As with other guidance modes. the insertion conditions necessary to satisfy true maneuver constraints, such as at entry interiace. are recomputed every guidance cycle. Very little additional code is required to support this mode. since it evolves from a combination of the standard ascent and deorbit guidance modes.
(5) Return-to-Launsh-Site Avort
(Studies are currently in process to determine desired thrust termination conditions).
(b) Externa: Delta-V Maneuver

This maneuver is designed to guide the vehicle through a constant attitude maneuver which achieves a specified velocity change. This mode is used for small on-orbit maneuvers, such as rendezvous phasing maneuvers. It is similar to the Apollo External Delta-V maneuver mode.
(7) Lambert On-orbit Maneuver

This mode is designed to insert the vehicle onto a coasting trajectory which intercepts a specified position (target) at a specifi $\sim$ d time. This is typically a constant attitude maneuver and is intended primarily for rendezvous terminal phase and automatic braking. It is similar to the Lambert Ascent maneuver, except that no constraint is placed upon the vehicle altitude at thrust termination.
(8) Deorbit Maneuver

This mode is designed to place the vehicle onto a coasting trajectory which satisfies entry interface conditions. These entry interface conditions are assumed to be defined by a prescribed (possibly functional) relationship between entry range, velocity, and night path angle. This mode is similar to the once-around abort maneuver. except that noc constraint is placed upon the vehicle altitude at thrust termination.

The functional flow of the program will be described in the next section to aid in understanding the general method employed in this quidance scheme. Following this, the rarticular equations and flow charts relating to the various modes are presented in greater detail.

## 2. FUNCTIONAL FLOW DIAGRAM

UPFG is designed to be called by a Servicer Routine at appropriate times during a powered maneuver. The first call can be made at any time prior to active guidance calls. This first, or prethrust, call is different from the active guidance calls in that certain required parameters are initialized and the compatibility of targeting information is checked and revised if necessary. The first call for active guidance is made at a specified time interval before ignition with following calls made at periodic intervals through-out the maneuver.

During the prethrust call, an iteration takes place which recycles the routine, without advancing the state vector, until convergence of the required velocity-to-be-gained takes place. During astive guidance, however, a single pass through the routine is made each guidance cycle call, incorporating sersed velocity changes and updating accordingly.

This section will present the general functional flow of computations performed 1 : UPFG. No attempt will be made here to define the actal equations :ised or to differentiate between the various modes of operation (other than to point out that several functional areas are not required by the External Delta-V mode). Detailed equations and flow charts will be covered in later sections.

The computational flow normally proceeds through nine distinct functional groupings or blocks of computations after each entry into UPFG. (see Figure 2-1) External Delta-V maneuvers, being less . 'mplicated, by-pass two of these functional blocks. Each functional block of comp utations results in the determination of the values of specific variables or commands used by the following functional blocks. The equations employed within the blocks may vary depending upon the various maneuver modes but the resulting output list from each block is the same.

The first block of computations encountered is either initialization (Block 1) or update (Block 2) depending upon whether a prethrust or active guidance cycle call is being made. In Block 1 the initial values of required variables are set and the state is advanced on a coasting trajectory to the time when the first active guidance call will be made. In Block 2, after the accelerometers are read, the state is advanced based upon the sensed velocity change over the time elapsed since the last accelenometer reading. The sensed velocity change is also used to update certain other variables. Logic is included to exit the routine from this point if either (1) an active guidance call is being made prior to ignition or (2) an active guidance call is 'eing made after an engine-off command has been issued. Prethrust flow does not use this exit path.

In the next block of computations (Block 3) the time-to-go remaining for the maneuver is computed. This computation involves solving for the phase elements of one of the thrust integrals. To avoid duplication these elements are saved to be sumined .n the next block.

Several scalar integrals relating to the thrust acceleration are computed in Block 4. These integrals are needed in following computations to predict the cutoff state and thrust direction.

Block 5 incorporates an input thrust turning rate or determines one for ascent and abort modes. The turning rate is utilized to determine a unit thrust direction according to I.TG concepts. In addition, actual contributions to velocity and position due to incorporating the turning rate are determined and biases are computed which reflect the differences between actual contributions and the necessary velocity-to-be-gained and pos:tion-to-be-gained vectors. The biases are used in the routine to improve prediction.

The steering biock (Block 6, remains to be determined. It must, however, set up steering commands based upon the unit thrust directivas: amputed previously, taking into ace unt autopilot requirements. This block of equations must differ ntiate between prethrust and active guidance calls when making these computations.

Block 7 and 8, which are skipped in the External Delta- $\Gamma^{「}$ Mode, estimate the effects of gravity over the thrusting trajectory and incorporate these efferts in the prediction of terminal conditions. Using these terminal conditions the velocity-tc-be-gained is revised to force the predicted target conditions to satisfy the sperified target conditions. This revision is necessary to prepare for the next pass through the routine.

Block 9, which determines and issues engine commands, is also to be determined at a later date when engine characteristics are better defined. It also must differentiate between prethrust and active guidance calls when formulating these commands.

This completes one pass, prethrust or active guidance, through the $r$ utine. From this point, if a prethrust call is being made and a change fo velncity-to-legained greater than some limiting value has been computed, the flow returns to the beginning of llock 3. If the velocity-to-be-gained change has converged to licas than the limiting value, or if an active guidance call is being made, the flow exits to the Servicer Routine to await the next call.


Figure 2-1. Functional Flow Liagram

## 3. INPUT AND OUTPUT VARIABLES

The Unified Powered Flight Guidance program is not designed to run independently through-out the maneuver. It must be called periodically by the Servicer Routine, finst in prethrust and subsequently at the beginning of each active guidance cycle. Sonse variables require input values at each call and some variables are modified internally requiring only initial input values. Thus, the input list for UPFG can become complicated. Furthermore, certain variables optionally require input values (initially and/or subsequently) depending upon the maneuver mode desired. In an effort to alleviate some confusion the input will be listed below in two catagories: input reçuired on the prethrust call and input required each active guidance call. The first catagory will be further broken down into two groupings: those inputs required by all maneuver modes and those inputs optionally required by some modes and not by others. Variables that are modified internally only, and u hose values affect future UPFG computations, will not be listed is input.

In addition to input which may be changed each call, a certain number of constants are required to be preset. These constants are not modified during program execution. They may be broken down into two catagories: universal constants and program constants. Universal constants are those constants not related to UPFG such as gravitational constants, earth radius, expected thrust levels,etc. Program constants are physical parameters which are urique to UPFG such as convergence criterion, iteration limits, etc.

Sensors provide another source of input. The values of certain variables are updated by sensors on a continuous or semi-continuous basis. When a program uses the value of this variable it obtains the current value existing at the time the storage location for that variable is read. Sensors read and the variables they control are listed below.

Many variables are modified and assigned new values during the prethrust and active guidance calls. With common storage locations they could all be termed "output". The list below, however, contains only those variables, known to affect other routines, that have been assigned new values by UPFG. Those that only effert subsequent calls to UPFG have been omitted.

For on-orbit maneuvers (modes 6,7 , and 8) the desired thrust vector turning rate $\omega_{f}$ is required as input to UPFG. It is anticipated that this quantity will be set to zero for External Delta-V and Lambert maneuvers, and computed in the deorbit targeting program (Ref. 3) to minimize $\Delta v$ for the deorbit maneuver. If necessary, at a later date, an optimal iurning rate for the Lambert maneuver could be determined.

Input - Prethrust Call Only (All Modes)

| Symbol | Definition |  |
| :---: | :---: | :---: |
| $S_{\text {pre }}$ | Prethrust switch | 11, prethrust \| 0 , active guidance |
| $S_{\text {mode }}$ | Maneuver mode | $\begin{cases}1 & \text { Ascent, standard } \\ 2 & \text { Ascent, reference trajectory } \\ 3 & \text { iscent, Lambert } \\ \ddagger & \text { Ascent, once-around abort } \\ 5 & \text { Ascent, return-to-launch-site abort } \\ 6 & \text { On-orbit, external } \Delta v \\ 7 & \text { On-orbit, Lambert } \\ 8 & \text { On-orbit, deorbit }\end{cases}$ |

$n \quad$ Number of thrust phases
${ }^{\mathrm{t}} \mathrm{ig}$ Ignition time, first phase
$t$
r $\quad$ State vector
$\underline{v}$

| $s_{S S M E, i}$ |  |
| :--- | :--- |
| $s_{\text {OMS. } i}$ | 1 Number of engines (SSME, OMS, RCS) for $i^{\text {th }}$ phase |

${ }^{5}$ RCS. i
${ }^{3}$ phase, i Phase switch, ith phase $\quad \left\lvert\, \begin{aligned} & 0, \text { constant thrust } \\ & 11, \text { constant acceleration }\end{aligned}\right.$
$\underline{v}_{\text {go }} \quad$ Estimated velocity-to-be-gained vector
$m_{0, i} \quad$ Mass at beginning of phase i

Mode Required

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $x$ | $x$ | $x$ | $x$ |  |  |  |  |
| $x$ | $x$ | $x$ | $x$ |  |  |  |  |
| $x$ | $x$ | $x$ | $x$ |  |  |  |  |
| $x$ | $x$ | $x$ | $x$ |  |  |  |  |
| $x$ | $x$ | $x$ | $x$ |  |  |  |  |
| $x$ | $x$ | $x$ | $x$ |  |  |  |  |
| $x$ |  | $x$ | $x$ |  |  |  |  |
| $x$ |  |  |  |  |  |  |  |
| $x$ |  |  |  |  |  |  |  |
|  | $x$ |  |  |  |  |  |  |
|  | $x$ |  |  |  |  |  |  |
| $x$ | $x$ | $x$ | $x$ |  |  |  |  |
|  |  | $x$ | $x$ |  |  | $x$ | $x$ |
| $x$ |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |
|  |  | $x$ | $x$ |  |  | $x$ | $x$ |
|  |  |  |  |  |  |  |  |
|  |  | $x$ | $x$ |  |  | $x$ | $x$ |
|  |  |  |  |  |  |  |  |
|  |  | $x$ |  |  |  | $x$ |  |
|  |  |  | $x$ |  |  |  | $x$ |
|  | $x$ | $x$ | $x$ |  |  | $x$ | $x$ |
|  |  |  | $x$ |  |  |  | $x$ |
|  |  |  | $x$ |  |  |  | $x$ |
|  |  |  |  |  | $x$ | $x$ | $x$ |

Symbol
${ }^{t} c$
$t_{b, i}$
$\Delta v_{\text {OMS }}$
$a_{L}$
$\stackrel{i}{y}_{\mathbf{y}}$
$\underline{r}_{d}$
$r_{d}$
$\mathbf{v}_{\mathrm{d}}$
$\gamma_{d}$
$\underline{v}_{d}$
$t_{\text {ref }}$
$K_{1}$
${ }_{-}^{i} N$
${ }^{n}$ rev
soln
$\underline{r}_{t}$
$\underline{r}_{\text {tef }}$
$\left.\begin{array}{l}t_{t} \\ c_{1} \\ c_{2} \\ \omega_{f}\end{array}\right\}$

## Definition

Coast time between last and next to last phase
Time remaining in $i^{\text {th }}$ phase
Velocity change to be imparted by OMS phase
Specific force limit during SSME maneuvers
Unit vector normal to desired tra ectory
Desired cutoff position
Desired radius magnitude of $\underline{r}_{\mathbf{u}}$
Desired velocity magnitude at cutoff
Desired fligit path angle at cutoff
Desired velocity vector at cutoff
Desired cutoff time
Desired throttle setting for SSME for $\mathrm{i}^{\text {th }}$ phase
Lambert unit normal to projection plane
(see Ref. 5)
Lambert number of revolutions (see
Ref. 5)
Lambert solution type switch (see
Ref. 5!
Target position - inertial coordinates
Target (entry interface) - Earth fixed coordinates
Time at target
Constants in target (entry interface) velocity
constraint, $v_{v}=C_{1}+C_{2} v_{h}$
Desired thrust vector turning rate

Inputs for Mode 5 are to be determined.

## Input Required - Ictive Guir ice Call (1ll Modes) <br> Symbol <br> Definition <br> sSSME \& / No. of engines (SSME, OMIS, RCS) to be considered on ${ }^{\text {somb.k }}$ f for current phase (k th phase) <br> ${ }^{s}$ RCS.:

Program Consiants
Symbol Definition

| $\epsilon$ cone | Lambert required sine of half cone angle of exclusion |
| :--- | :--- |
| $\epsilon_{\text {vgo }}$ | Value of $\Delta \underline{v}_{\text {go }} \mid$ नieĩining prethrust convergence limit |

$\Delta t_{t 0} \quad$ Time interval before $t_{i g}$ io start active guidance calls
$\delta \mathrm{t}$ Offset in coast time used in deorbit required velocity com- putations to determine sensitivity to conast time
$\Delta t_{\text {cutoff }}$ Value of $t_{g o}$ used to define time to issue engine cutoff command and terminate active steering computations
$S_{\text {pert }} \quad$ Gravity perturbation switch

## Univer: al Constants

| $\mu$ | Gravitational constant |
| :---: | :---: |
| $\begin{aligned} & \mathrm{f}_{\mathrm{SS} M \mathrm{E}} \\ & \mathrm{f}_{\mathrm{OSIS}} \\ & \mathrm{f}_{\mathrm{RCS}} \end{aligned}$ | Ful! thrust of single SSML, On? , lis engine |
| $K_{\text {max }}$ | . Haximum throttle setting of SSME |
| $\begin{aligned} & \dot{\mathrm{m}}_{\mathrm{SSME}} \\ & \dot{\mathrm{~m}}_{\mathrm{OMSS}} \\ & \mathrm{~m}_{\mathrm{RCS}} \end{aligned}$ | Mass flow rate of single SSME, OMS, RCS engine at full thrust |

## Sensors Required

## Sensed Variahle

## Definition

| $\Delta \underline{y}$ sensed | Total velocity change accumulated on accelerometers |
| :--- | :--- |
| since last reading |  |
| Actual time when accelerometers are read iwili be |  |
|  | associated with state vector |

## Output

Output frc.a this prograni will be in the form of attitude and engine cormands and navicai:on state. These include:

Symbol
Definition

| - | Steering commanc (TBD) |
| :--- | :--- |
| - | Ignition (engine-on) command (TBD) |
| $\kappa_{k}$ | Throttie command |
| - | Cutoff (engine-off) command (TBD) |
| $t$ |  |
| $\underline{v}$ |  |

## 4. DESCRIPTION OF DQUATIONS

The guidance scheme presented in this document evolved from a rather extensive modification of the Linear Tangent Guidance (LTG) concept described in Ref. 1. The original LTG concept was designed for an orbiter ascent maneuver in which tie thrust cutoff altitude, orbital plane, velocity, and fight path angle were constrained to specified values determined prior to the flight. The current unified scheme retains that basic guidance mode, the standard ascent mode as described in the introduction. However, to adapt the LTG concept to the various shuttle maneuvers, from ascent through deorbit. several important changes were made.

First. the progran 2 was restructured to efficiently accomodate the various maneuver modes and to permit the calculation oi steering commands early in the guidance cycle, thus minimizing the computational lag. Second, the atdition of a required velocity calculation toward the end of each guidance cycle was made. The basic standard ascent mode does not constrain either the downrange component of position or the time of thrust cutoff, and therefore the subsequent coasting trajectory can vary considerably depending upon ignition delays, atmospheric effects during boost, engine perturbations or failures, and any other factors which might effect the trajectory. By including a required velocity calculation every guidance cycle, based upon the precicted cutoff position, the true maneuver objectives can be satisfied and the effects of perturbations minimized. The third important change to the LTG concept involved the elimination of any cutoff position constraint for on-orbit maneuvers. An aliernate equation, which uses the input vehicle turning rate, simplifies premaneuver targeting and minimizes maneuver $\Delta V$. The fourth change involved the design of a new scheme for the prediction of the effects of gravity on the powered trajectory. The original LTG scheme was not suitable for long, low thrust orbital maneurers such as deoroit. A new scheme, based upon a conic ccasting trajectory, appears to give jood performance for all manelive-s. Finally, to support the ascent to reference trajectory guidance mode, equations were added to account for a rotating thrust rector. These additional equations improve the prediction of both the maneuver time and cutoff position. They are required for proper calculation of throttling commands during ascert, and they provide increased premaneuve. prediction accuracy in all modes.

As described in Section 2, the Unified Powered Flight Guidance (UPFG) Iioutine consists of wine tlocks of in-line computations connected by the logic necessary to perform either the prethrust function or the active guidance function for the eight possible maneuver modes. A single entry and exit point are maintained for these computations. This section will describe the computations required for one pass, entry to exit, through UPFG.

Figure 5-1 shows the seque cing of the major computational block. Inımediately after entry into UPFG a test is made on the value of the switch. spre. If $S_{\text {pre }}=1$, a prethrust call is being made: $\mathbf{s p r e}=0$ implies an active guidance sall. For neretirust calls, initialization in flock 1 is performed. For acije guidance calls, Block 1 is bypassed arid an update in Block 2 is performed.

## 4. 1 Initialization (Block 1 )

This blcek of computations provide; the necessary one-time initialization of certain variabies required so set-up the prethrust calcuistions. These variables include $k$, the maneuver phase rounter. It should be noted that the first phase of each maneurer is denoted " 1 ". Sereral switches are a: so set and the values of variables which may be defined by input, depending upon maneuver mode, are initialized. An initial value of $\underline{r}_{\text {grav }}$ is required, since the calculation of $r$ go ir. Block 5 occurs prior to the calculation of gravity effects in Rlork i. To estimate $\underline{r g r a x}^{\text {g }}$ it is assumed tiat the acceleration of gravity will he equal to its present value over the entire meneuver. Then, if a one second maneurer is ass.mad for simpiicity.

$$
\underline{E}_{\mathrm{gra}}=-\frac{1}{2} \frac{\mu r}{|\underline{r}|^{3}}
$$

 for the estimated maneuver time as follows.

$$
\underline{r}_{\text {grav }}(\text { nex })=\left[\frac{t_{\text {go }}(\text { new }}{t_{g o}(\text { previoust }}\right]^{2} \text { Eqrav }^{\text {(previous) }}
$$

where $t_{g o}$ (previous) has beer set to 1 . On subsequent passes through Block 5 a previous $t g$, will have been set in the previous pass and the revision to rav will proceed normaliv.

Finally, the effective engine paramete $s$ for the array of engiaes to be used during each phase, $i$ : 1 to $n$, are determined. These engine parimeters include the total thrust $\mathcal{T}_{\mathrm{T}, \mathrm{i}}$, mass-flow-rate $\dot{m}_{i}$. the exhaust velocity "ex,i , the initial acceleration ${ }_{T_{,}}$, and the ratio of mass to mass-flow-rate $T_{i}$ inr each phase of the manewrer.

 This is accon, plished by calling the Precisinn State Fxtrapolati - Rc tine (Ref. i). It should be noted that this sets un the prethrust opridions as if thrusting is ex-
 expected to significantly affect the results and it simplifies the program, in a!-
ternative would be to extrapolate the state to $t_{\text {ig }}$ in the prethrust call and each active guidance call prior to $t_{i g}$. It should be coted that this method of initialization, although satisfactory for on-orbit maneuvers, is not applicable to ascent. In future revisions of this document a scheme which is satisfactory for both ascent and on-orbit maneuvers will be included.

The computations now proceed to Block 3 since the update (Block 2) is skipped during the prethrust call.

### 4.2 Uipdate (Block 2)

Plock 2, which is skipped in the prethrust call. is the first block encountered during each active guidance call. This block acquires the velocity change sensed by the accelerometers and accoumts for that velocity change by updating affected pariables to the time when the accelerometers were read. The routine which reads the accelerometers and clock has not been defined brt it is assumed that it will require some sort of intialization on the first call such that velocity changes on succeeding calls can be obtained by differencing from a previous reading. Thus, on the first call the velocity change is assumed to be zero.

To allow for a variable guidance cycle time step, the time step is computed each cycle as follows,

$$
\Delta t=t-t \text { prev }
$$

where $t$ is the carrent clocis reading at the time the accelerometers are read and $t_{\text {prev }}$ is the previous time the acceierometers were read. On the first active guidance call $\Delta t$ will be $-=-\mathrm{ly}$ zero since $t_{\text {prev }}$ was set in prethrust to $t_{\text {ig }}-\Delta t_{t 0^{\circ}}$ the expected time of the first active guidance call. The variable $t_{\text {prev }}$ is set to $t$ for use later.

The velocity-to-be-gained is updated using the velocity change sensed by the accelerometers since the last guidance call as follows.

$$
\underline{v}_{g o(\text { new })}=\underline{v}_{\sim}(\text { (previous) })-\Delta \underline{v}_{\text {grised }}
$$

This gives exrellent estimate of the velocity-to-be-gained for subsequent guidance calculations. Near the end of the guidance routine this value of $\underline{q}$ go is adjusted s!ightiv to account tor any changes in desired cutoff velocity and to prepare for the next guidance cycle.

It should be noted that on the first active guidance call (denoted by $\mathbf{s}_{\text {pass }}$ - ${ }^{\text {1) }}$ $\Delta \underline{v}_{\text {sensed }}$ is assumed to be zero. Therefore, updating $\underline{v}_{g o}$ is bypassed under these circumstances and

$$
\begin{aligned}
& \hat{v}_{\text {sensed }}=0 \\
& \mathbf{s}_{\text {passi }}=0
\end{aligned}
$$

Next. the state, $\underline{r}, \underline{y}$, is advanced to the time of the latest accelerometer reading by calling the Powered Flight Navigation Routine (Ref. 6).

The remainder of the update block, which is bypassed when $t \leq t_{i g}$, results in decrementing the ${ }^{t}$ go values for each phase, $i$, by $\Delta t$

$$
t_{\text {go, i (new) }}-t_{\text {go, i (previous) }}-\Delta t
$$

during thrusting. Also decremented are the current mass, m, and current phase burn time $t_{b}$

$$
\begin{aligned}
& m(\text { new })=m(\text { previ us })-\dot{m}_{k} \Delta t \\
& t_{b, k}(\text { new })
\end{aligned}
$$

If $t_{b, k}$ le comes less than or equal to zero the end of that phase has been reached. Under thore circumstances a change of phase is performed by setting the $t$ go for the current phase to zero and then incrementing the phase number by one.

$$
\begin{aligned}
& t_{\text {go, } k}=0 \\
& k(\text { new })=k \text { 'previous }+1
\end{aligned}
$$

It shculd be noted at this point that mass, $m$, has not been adjusied to reflect step changes in this variable that may occur during a phase change due to such things as tank jettison or staging. It is expected thit some routine external to UPFG will maintain current mass following phase changes for use by both Guidance and Control.

During the $r$ sast period between the last and next-to-last phases of an ascent maneurer the coast time, $t_{c}$, is decremented $b: \Delta t$.

$$
t_{c(n e w)}=t_{c(\text { previous })}-\Delta t
$$

No changes to mass which would account for externai tank jettison have been made here as this function is assumed to be performed by a routine external to CPFG. Burn times remaining are also assumed to be constant during the roast time. Block 3 normally follows Block 2 during active guidance $c \cdot l l$ unless $t<t_{i g}$ or unless the th ist-off command has been issued as indicated by sengoff $=1$.

If either of these two conditions exists, all further computations are byrassed and LFFG is exited.

### 4.3 Time-To-Go

The UPFG program is designed to provide guioance for all exo-atmospheric maneuvers, including ascent-io-orbit. The complexity of the time-to-go computations is dependeat upon the number of distinct thrust phases in the maneuver. All orbital maneuvers nominally have only one distinct thrust phase (OMS constant thrust), while an ascent maneuver will have at least $t$ hree distirct thrust phases (i.e. (1) SSME constant thrust prior to g-limiting . (2) SSME constant acceieration during g-limiting and (3) ONS constaat thrust after tank seperation). The equations that are described in this section are the equations required to compute the time-to-go until maneuver completion for a multi-phase ascent maneuver. The equations required for any single-phase maneuver are merely a subset of these equations: in particular, they are the same equations that will be solved during an ascent maneuver when actually in the final thrust phase of the maneuver (OMS constant thrust phase).

To compute the time-to-go it is first necessary to compute estimates of the current values of thrust magnitude $f_{T, k}$, mass flow rate $\boldsymbol{m}_{k}$, thrust acceleration $a T_{, k}$. effective exhaust velocity $v_{e x, k}$; and mass to mass now rate ratio, $T_{k}$. where $k$ is an index refering to the current thrust phase. Estimates of $f_{T, k}$ and $\dot{m}_{k}$ are given by

$$
\begin{aligned}
& \dot{m}_{k}=K_{k} s_{\text {SSME, } k} \dot{m}_{\text {SSME }}{ }^{+s_{\text {OMS, }}}{ }^{\dot{m}_{O M S}}{ }^{+s_{\text {RCS, }}}{ }^{\dot{m}_{R C S}}
\end{aligned}
$$

If $\dot{m}_{k} \neq 0, a_{T, k} \cdot v_{e x, k}$ and $T_{k}$ are computed as follows

$$
\begin{aligned}
\mathbf{a}_{\mathrm{T}, \mathrm{k}} & =\mathrm{f}_{\mathrm{T}, \mathrm{k}} / \mathrm{m} \\
\mathbf{v}_{\mathrm{ex}, \mathrm{k}} & =\mathrm{f}_{\mathrm{T}, \mathrm{k}^{\prime} \dot{\mathrm{m}}_{\mathrm{k}}} \\
\mathrm{~T}_{\mathrm{k}} & =\mathrm{v}_{\text {ex,k}} / \mathrm{a}_{\mathrm{T}, \mathrm{k}}
\end{aligned}
$$

where $m$ is the current estimated mass of thi vehicle. If $\dot{m}_{k}=0$, the last three equations are bypassed and the previous estimates of $a_{T, k^{*}} v_{e x, k}$ and $T_{k}$ are used.

If there are two or more thrust phases remaining (including the current phase), the assumed burn times, $t_{b, i}$ of each phase are used to compute the velocity change due to thrust, $L_{i}$, that will be $\varepsilon$ pplied during each phase $i$ from the current phase to the $n-2$ phase ( $n$ denotes the number of phases in the maneuver). If phase i has constant thrust,

$$
L_{i}=-\nabla_{e x, i} \ln \left(\frac{T_{i}-t_{b_{i}} i}{T_{i}}\right),
$$

and if phase i has constant acceleration

$$
t_{i}={ }^{2}{ }_{1} t_{b, i}
$$

where $a_{L}$ is the SSME acceleration limit.
Since the velocity change to be applied in the $n^{\text {th }}$ phase, $L_{n}$. (after tank stagingl is predefined, the velocity change to be applied in the $n-1$ phase can be determined by

$$
L_{j}=\left|\underline{v}_{g o}\right|-\sum_{i=k}^{n-2} L_{i}-I .
$$

where $\mathbf{j}=\mathbf{n - 1}$ and $\underline{Y}_{g o}$ is the relocity-tc be-gained vector computed in Block 2. If there are onlv two phases remaining $\mathrm{f}=\mathrm{n}-\mathrm{n}$ ), thr n

$$
I_{j}=I_{\underline{v}_{0}} \mid-L_{n}
$$

where again $j=n-1$. During the final phase of the maneuver $L_{j}$ is given by

$$
L_{j}=\left|\underline{v}_{g o}\right|
$$

where, in this case, $j=n$.
Having determined $L_{j}$, the burn time remaining in the $j^{\text {th }}$ phase can be computed. If phase $j$ has constant thrust

$$
t_{b, j}=T_{j}\left(1-e^{-i I_{j} j v} e v_{, j}\right)
$$

or if phase $\mathbf{i}$ has constant acceleration

$$
i_{0, j}=L_{j} / a_{1} .
$$

The time-to-go until the end of phase $\vdots, t_{g o, i}$, for $:=k, \ldots, n$ is required in Block 4 in order th evaluate the tirusi integrals I., I, S. Q. II, and P. These times are given by
and wher:

$$
t_{g o, i}=t_{g o, i-1}+t_{b, i} \text { ior } i<n
$$

and weri

$$
t_{g o, i-1}=0 \quad \text { for } i, k
$$

The timn-lo-gn until the end of the $n^{\text {th }}$ phase, which is the total time-to-gn until the end of the maneuver, is given by

$$
t_{g o} \quad t_{g o, n}=t_{g o, n-1} b, n+t_{c}
$$

where $t_{c}$ is the coast time during external task separation between phase $n-1$ and phase $n$. Although $t_{c}$ is not actually part of the maneuver time, it is added to the time-to-go until the end of phase $n$ in order to maintain continuity of the steering commands.

### 4.4 Integrals of Thrust

The LTG guidance concept requires the evaluation of several thrust integrals. These integrals are defined as follows

$$
\begin{aligned}
& L=\int_{0}^{t g o}(f / m) d t \\
& S=\int_{0}^{t} g o\left[\int_{0}^{t}(f / m) d s\right] d t \\
& J=\int_{0}^{t g o}(f / m) t d t \\
& Q=\int_{0}^{t}\left[\int_{0}^{t}(f / m) s d s\right] d t \\
& H=\int_{0}^{t g o}(f / m) t^{2} d t \\
& P=\int_{0}^{t g o}\left[\int_{0}^{t}(f / m) s^{2} d s\right] d t
\end{aligned}
$$

where ( $\mathrm{f} / \mathrm{m}$ ) is the thrust acceleration, $t_{g o}$ is the time-to-go until the end of the maneuver, and $t$ and $s$ are variabes of integration. For the space shuttle vehicle, it is assumed that either ( $f / \mathrm{m}$ ) is corstant or $f(t h r u s t)$ and $\dot{m}$ (mass flow rate) are constant, therefore, these integrals can be integrated in closed form.

The UPFG program is designed to accomodate multi-thrust-phase maneuvers as well as single-thrust-phase maneuvers. Therefore, the above integrals must be evaluated piece by piece since each thrust phase has a distinct thrust profile. This is accomplished by evaluating the thrust integrals separately ior each phase and then summing them up. The thrust integrals for each phase i are defined as follows

$$
\begin{aligned}
& L_{i}=\int_{t_{g o, i-1}}^{t_{T o, i}}\left(f_{T, i} / m_{i}\right) d t \\
& S_{i}=\int_{t_{g o, i-1}}^{t_{g o, i}}\left[\int_{t_{g o, i-1}}^{t}\left(f_{T_{, i}} / m_{i}\right) d s\right] d t \\
& J_{i}=\int_{t_{g o, i-1}}^{t_{g o, i}}\left(f_{I, i} / m_{i}\right) t d t \\
& H_{i}=\int_{t_{g o, i-1}}^{t_{g o, i}}\left(f_{T, i} / m_{i}\right) t^{2} d t
\end{aligned}
$$

where $\left(f_{T, i} / n_{i}\right)$ defines the thrust profile for phase $i$, and $t_{g o, i}$ is the time-to-go until the end of phase $i$ (note: $t_{\text {go, } i-1}=0$ for $i \leq k$ ).

The computation of the thrust integrals for each phase is performed in two steps. This is done in order to minimize computer memory requirements and computation time. In the first step, the equations vary depending upon the type of thrust phase, while for the second step, the equations are identical for both types of phases. Aiso, it should be noted that $H_{i}$ is not explicitly computed. It can be shown using integration by parts tinat $H$, which is the sum of the $H_{i}$ 's. can te computed directly as a function of the time-to-go and the integrals $J$ and $Q$. The evaluation of $L_{i}$ has already been described in Sectior 4. in because it is required in the computati, in of time-to-go. However, for the sake of rompleteness, it will be described again in this section.

Step 1 in evaluating the thrust integrals is to compute $L_{i}$ and then $t_{1}$..ormpute part of $I_{i}, S_{i}, Q_{i}$, and $P_{i}$. If phase $i$ has constant thrust then

$$
\begin{aligned}
& L_{i}=-v_{e x, i} \ln \left(\frac{T_{i}-t_{b, i}}{T_{i}}\right) \\
& J_{i}=L_{i} T_{i}-v_{e x, i} t_{b, i} \\
& S_{i}=-J_{i}+t_{b, i} L_{i} \\
& Q_{i}=S_{i}\left(T_{i}+t_{g o, i-1}\right): \frac{1}{2} v_{e x, i} t_{b, i}^{2} \\
& P_{i}=Q_{i}\left(r_{i}+t_{g o, i-1}\right)-\frac{1}{2} v_{e x, i} t_{b, i}^{2}\left(\frac{1}{3} t_{b, i}+t_{g o, i-1}\right) .
\end{aligned}
$$

and if phase $i$ has constant acceleration then

$$
\begin{aligned}
& L_{i}=a_{L} t_{b, i} \\
& J_{i}=\frac{1}{2} L_{i} t_{b, i} \\
& S_{i}=J_{i} \\
& Q_{i}=S_{i}\left(\frac{1}{3} t_{t, i}+t_{g o, i-1}\right) \\
& P_{i}=\frac{1}{6} S_{i}\left(t_{g o, i}^{2}+2 t_{g n, i} t_{g n, i-1}+3 t_{g o, i-1}^{2}\right)
\end{aligned}
$$

In step 2, the remainder of $J_{i}, S_{i}, Q_{i}$ and $P_{i}$ are computed as follows

$$
\begin{aligned}
& J_{i}=J_{i}+L_{i} t_{g o, i-1} \\
& S_{i}=S_{i}+L t_{b, i} \\
& Q_{i}=Q_{i}+J t_{b, i} \\
& P_{i}=P_{i}+H t_{b, i}
\end{aligned}
$$

where $L$, $J$, and $H$ are the total thrust integrals from the current phase to the $i-1$ th phase. If $i=n$, the effects of the coast time, $t_{c}$, between phases $n-1$ and $n$ are then added in as follows

$$
\begin{aligned}
& S_{i}=S_{i}+L t_{c} \\
& Q_{i}=Q_{i}+J t_{c} \\
& P_{i}=P_{i}+H t_{c}
\end{aligned}
$$

Having evaluated the thrust integrals for each phase, the total thrust integrals are given by

$$
\begin{array}{ll}
L=\sum_{i=k}^{n} L_{i}, & J=\sum_{i=k}^{n} J_{i} \\
S=\sum_{i=k}^{n} S_{i}, & Q=\sum_{i=k}^{n} Q_{i} \\
P=\sum_{i=k}^{n} P_{i}, & H=t_{g o} J-Q
\end{array}
$$

It should be pointed out that $H$ must be evaluated as the total thrust integral from the current phase to the $i^{\text {th }}$ phase for each r'iase because it is required in the computation of each $P_{i}$.

### 4.5 Turning Rate

The main results of this block of computations are the desired unit thrust direction, $\frac{1}{f}$, and a vector, $\dot{\boldsymbol{\lambda}}$, which is associated with the thrust turning rate.

The first operation in this block is to define $\underline{\lambda}$. a vector in the direction of $\underline{\forall} \mathrm{g}^{\prime}$, the velocity-to-be-gained. The unit vector, $\boldsymbol{\lambda}$, is the vector about which an expansion is later ruade to deter mine $i_{i}$. From this point one of four different methods will be employed depending upon $s_{\text {mode }}$.

The first operation in this block is to define $\boldsymbol{\lambda}$. a vector in the direction of $\underline{\mathbf{v}}_{\mathrm{g} \boldsymbol{g}}$, the velocity-to-be-gained. The unit vector, $\underline{\lambda}$. is the vector about which an expansion is later made to determine $i_{i}$. From this point one of four different methods will be employed depending upon $s_{\text {mode }}$ -

Modes 1 through 5, ascent and abort, follow closely the LTG method of determining the rate $\dot{\underline{i}}$, by first using a tesired burn-out position, $\underline{r}_{d}$, to estimate $\underline{r}_{\boldsymbol{g}}$ o

$$
\underline{r}_{g o}=\underline{r}_{\mathrm{d}}-\left(\underline{r}+\underline{v}_{\mathrm{to}}+\underline{\mathrm{r}}_{\mathrm{grav}}\right)
$$

The effect of gravity on this $\underline{r}_{\text {go }}$ is given by $\underline{\mathbf{r}}_{\text {grav }}$ which is estimated using $\underline{r}_{\text {grav }}$ from the previous call as follows,

$$
\underline{r}_{\text {grav(new) }}=\left(\frac{t_{\text {go(new) }}}{t_{\text {go(previous) }}}\right)^{2} \underline{r}_{\text {grav(prevans) }}
$$

The projection of $\underline{r}_{g o}$ on the plane normal to the downrange direction, $\underline{r}_{\text {goxy }}$, is given by

$$
\begin{aligned}
\underline{i}_{z} & \left.=\operatorname{unit}^{( } \underline{r}_{d} \times \underline{i}_{\underline{y}}\right) \\
\underline{\underline{r}}_{\text {goxy }} & =\underline{\underline{r}}_{\text {go }}-\left(\underline{i}_{z} \cdot \underline{r}_{g o}\right) \underline{i}_{z}
\end{aligned}
$$

Using the integral, $S$, computed previously, the downrange component of $E$ go can be modified by the LTG reiationship

$$
r_{g o z}=\frac{\left(S-\underline{\lambda} \cdot \underline{r}_{g o x y}\right)}{\underline{\lambda} \cdot \underline{i}_{z}}
$$

and a new $\underline{r}_{\text {go }}{ }^{\text {is thas found to be }}$

$$
\underline{\mathbf{r}}_{\text {go }}=\underline{r}_{\text {goxy }}+\mathbf{r}_{\text {goz }} \underline{\mathbf{i}}_{z}+\underline{\mathbf{r}}_{\text {hias }}
$$

In this equation the effects of a rotating thrust vector are included by the term, $\underline{r}_{\text {bias, }}$ whinh was computed on the previous guidance cycle in Block 5. The rate, $\underline{\dot{\lambda}}$, which crirresponds to the velucity of the tip of a unst vecter coincident with $\underline{\lambda}$ but rotating with the desired unit thrust vector rotation rate is now obtained using the integrals, $L, J, S$, and $Q$ :

$$
\underline{\dot{\lambda}}=\frac{\left(\underline{r}_{g_{0}}-S \lambda\right)}{(Q-S J / \bar{L})}
$$

note ua:

$$
\dot{\lambda} \neq \frac{d}{d t}(\underline{\lambda})
$$

For modes $6-8$, a rotation rate, $\omega_{f}$, is input and $\boldsymbol{\lambda}$ is determined by

$$
\dot{\lambda}=\omega_{f} \text { unit }\{(\underline{\lambda} \times \underline{r}) \times \underline{\lambda}]
$$

The predicted unit thrust direction at time, $t$, is given ty

$$
\underline{i}_{f}=\operatorname{unit}[\underline{\lambda}-(J / L) \underline{\underline{\lambda}}]
$$

It is recognized that the results of integrating a rotating thrust vector can be significantly different from $\underline{r}_{g o}$ and $\underline{\mathbf{g}}_{\mathrm{go}}$ if the rotation angle is large.

The angle between $\underline{\lambda}$, a unit vector in the direction of $\underline{v}_{\mathrm{go}}$, and $\underline{i}_{f}$, the unit thrust direction, is given by

$$
\phi=\cos ^{-1}\left(\underline{i}_{\mathrm{f}} \cdot \underline{\lambda}\right)
$$

Since the linear tangent guidance equations are designed to align $\boldsymbol{\lambda}$ and the thrust direction at the time $J / L(J / L$ is approximately the midpoint of the maneuver), then

$$
\dot{\varphi}=-\quad \mathrm{L} / \mathrm{J}
$$

Based upon $\Phi$ and $\dot{\phi}$ the first and second integrals of the thrust acceleration are given by

$$
\begin{aligned}
\underline{v}_{\text {thrust }} & =\int_{0}^{t_{g o}^{g o}} \frac{f}{m}\left[\underline{\lambda} \cos (\phi+\dot{\phi} t)+\frac{\dot{\lambda}}{|\underline{\lambda}|} \sin (\phi+\dot{\phi} t)\right] d t \\
\underline{r}_{\text {thrust }} & \left.=\int_{0}^{t} g o \int_{0}^{t} \frac{f}{m}\left[\underline{\lambda} \cos (\phi+\dot{\phi} t)+\frac{\underline{\dot{\lambda}}}{|\underline{\lambda}|} \sin o+\dot{o} t\right)\right] d \_d t
\end{aligned}
$$

This can be simplified by assuming that

$$
\sin (\phi+\dot{\phi} t) \approx \phi+\dot{\phi} t
$$

and

$$
\cos (\varphi+\dot{\phi} t) \approx 1-(\partial+\dot{\phi} t)^{2} / 2
$$

Using the thrust integrals computed in Block 4, the actual (considering rotation) change in position and velocity due to thrust is computed as follows,

$$
\begin{aligned}
v_{\text {thrust }}= & \left(L-\frac{1}{2} L \phi^{2}-J \phi \dot{\phi}-\frac{1}{2} H \dot{\phi}^{2}\right) \underline{\lambda} \\
& -(L \phi+J \dot{\phi}) \text { unit }(\underline{\dot{\lambda}}) \\
r_{\text {thrust }}= & \left(S-\frac{1}{2} S \phi^{2}-Q \phi \dot{\phi}-\frac{1}{2} P \dot{\phi}^{2}\right) \underline{\lambda} \\
& -(S \phi+Q \dot{\phi}) \text { unit }(\underline{\lambda})
\end{aligned}
$$

It may be noted that $\underline{v}_{\text {thrust }}$ and $\underline{r}_{\text {thrust }}$ are resolved into components parallel and normal to $\underline{\lambda}$ (parallel to $\underline{\lambda}$ ).

Biases to thrust cutnff velocity and position are computed by

$$
\begin{aligned}
& \underline{v}_{\text {bias }}=\underline{v}_{\text {go }}-\underline{v}_{\text {thrust }} \\
& \underline{\underline{r}}_{\text {bias }}=\underline{\underline{r}}_{\text {go }}-\underline{\underline{r}}_{\text {thrust }}
\end{aligned}
$$

## 4. 6 Steering Commands

This block of computations is to be determined at iater date. It will receive the vector, $\underline{\lambda}$ and $\underline{\boldsymbol{\lambda}}$ and determine steering commands based upon the unit thrust direction as determined by an equation of the furm

$$
\underline{i}_{f}=\text { unit }[\underline{\lambda}-(J / L) \underline{\dot{\lambda}}]
$$

In order to issue steering commands to the autopilot, lead ternis may be added as required.

At the end of the prethrust call steering commands will take the form $\sim f$ a command to maneuver to the ignition attitude but during active guidance calis a turning rate may be implied. Logic will be included to incorporate this and to activate and deactivate steering at the proper times.

## 4. 7 Prediction of Gravity Effects (Block i,

The solution of the LTG equations requires a prediction of both the first and second integrals of gravity over the thusting maneuver. The technique originally devised for ascent, and described in Ref. 1, was not appropriate for the long, lowthrust on-orbit maneuvers such as deorbit. Therefure a new technique has been devised which is applicavle to all maneuvers. This technique is based upon a coasting trajectory which is constructed such that it remains 'close' to the powered trajectory thre"shout the maneuver. The effects of gravity on the powered trajectory are then assumed to approximate the effects of gravity on the coasting trajectory. Thus the Kepler (Conic State Extrapolation) Rcutine can be used to determine the required integrals of gravity. Figure 4-1 illustrates this concept.

## Initial Coasting Trajectory



Figure 4-1. Prediction of Gravity Effects

To construct this special coasting trajectury, assume for the moment that the maneuver takes place in field free space. The initial conditions ( $t=0$ ) for the powered trajectory are defined $b_{j} \underline{I}$ and $\underline{V}$. The . .ist and second integrals of the thrust acceleration over the powered maneuver. Ethrust and $I_{\text {thrust }}$. are described in Section 4.5. Therefore, at thrust cutoft he position and velocity on the powered trajectory are given by

$$
\begin{aligned}
& \underline{r}_{\text {cutoff }}=\underline{r}^{+} \underline{t_{g o}}{ }^{+} \underline{\underline{t}}_{\text {thrust }} \\
& \underline{E}_{\text {cutoff }}=\mathbf{E}^{+} \mathbf{E}_{\text {thrust }} \\
& \text { I }
\end{aligned}
$$

Where $t_{0}$ is the maneuver time. The resulting trajectories in field free space are illustrated in Figure $\mathbf{4} \mathbf{- 2}$, where coasting trajecories simply appear as straight limes.


Figure 4-2. Prediction of Gravity Effects - Field Free Space
 in field free space are completely defined. A cubic equation can be used to model the state vector on the powered trajectory as a function of time.

$$
\begin{aligned}
& \underline{r}_{P}(t)=\underline{A}+\underline{B} t+\underline{C} t^{2}+\underline{D} t^{3} \\
& \underline{\underline{P}}_{P}(t)=\underline{B}+2 \underline{C} t+3 \underline{D} t^{2}
\end{aligned}
$$

where $\underline{r}_{P}(t)$ is the position. The velocity $\Psi P^{(t)}$ is equal to $\mathbf{d} \underline{r}_{P}(t) / d t$. The four vector coefificients. $\mathbf{A}, \underline{B}, \underline{C}$, and $\underline{\mathbf{E}}$ can be determined such that the boundary conditions on position and velocity at the initial and final times are satisifed. It shouid be noted that in actual practice it is not necessary to actually solve for these coefficients. They are merely used to aid in describing the concept.

A coasting trajectory can now be constructed which remains 'close' to the powered trajectory. The position on this coasting trajectory $\underset{f}{ }(t)$ is defined by the linear equation

$$
\underline{r}_{c}(\mathbf{t})=\underline{A}^{\prime}+\underline{B}^{\prime} t
$$

The velocity $\underline{U}_{c}(t)$ is constant and equal to $\underline{B}^{\prime}$.
To determine $A^{\prime}$ and $\underline{B}^{\prime}$. the following integrals must be satisfied.

$$
\begin{aligned}
& \left.\int_{0}^{t} \underline{r}_{c}(t)-\underline{r}_{p}(t)\right] d t=0 \\
& \int_{i}^{t} g 0 \quad\left[r_{c}(t)-\underline{r}_{p}(t)\right]\left[t_{g o}-t\right] d t=0
\end{aligned}
$$

Since gravity is strictly a function of position, the first integral insures that the average position difference (or error) is zero. In addition, since errors in the initial position (and gravity) have more time to propagate and thus have more influence on the total position er.or, the second integral weights the error as a function of time. Using these integrals, the initial position $A^{\prime}$ and the initial velocity $\underline{B}^{\prime}$ on the coasting trajectory can be easily computed. Using $\underline{A}^{\prime}$ and $\underline{B}^{\prime}$, the initial conditions for the $c<$ r-ting trajectory. $\underline{r}_{c 1}$ and $\underline{v}_{c 1}$, reduce to the following simple form:

$$
\begin{aligned}
& \underline{r}_{c 1}=\underline{r}-\frac{1}{10} \underline{r}_{\text {thrust }}-\frac{1}{30} \underline{v}_{\text {thrust }} t_{\text {go }} \\
& \underline{v}_{c 1}=\underline{v}+\frac{6}{5} \frac{\underline{r}_{\text {thrust }}}{t_{g o}}-\frac{1}{19} \underline{v}_{\text {thrust }}
\end{aligned}
$$

The Kepler Koutine is used to extrapolate these in.tial conditions through the time $t_{g o}$. thus obtaining $\underline{r}_{c 2}$ and $\underline{r c z}_{c}$. Then the effects of gravitr on the coasting trajectory, which approximate the effects on the powered trajectory, are given by

$$
\begin{aligned}
& \underline{v}_{g r a v}=\underline{v}_{c 2}-\underline{v}_{c 1} \\
& \underline{r}_{g r a v}=\underline{E}_{c 2}-\underline{r}_{c 1}-\underline{v}_{c 1} t_{g o}
\end{aligned}
$$

### 4.8 Velocity-to-be-Gained

A more accurate prediction of the cut-cff position is now obtained as follows.

$$
\underline{r}_{p}=\underline{r}+\underline{v} t_{g o}+\underline{r}_{g r a v}+\underline{r}_{\text {thrust }}
$$

For on-orbit Lambert and deorbit maneuvers ( $s_{\text {mode }}=7.81$ a desired cutoff pcsition is given by

$$
\underline{r}_{d}=\underline{r}_{p}
$$

For aborts and ascents (other than reference trajectory) the thrust . .utnff altitude is constrained, therefore

$$
\underline{r}_{d}=r_{d} \text { unit }\left(\underline{r}_{p}\right)
$$

After determination of $\underline{r}_{d}$ this block splits into three branches according to mode: standard ascent, ascent to reference trajectory ard the remaining modes.

### 4.8.1 Standard Ascent, $S_{\text {mode }}=1$

The required velocity at burnout. $\underline{V}_{d}$, to satisfy the terminal constraints for this mode is a function of the inputs: $\underline{i}_{y}, \mathbf{r}_{\mathbf{d}}, \mathbf{v}_{\mathbf{d}}, \boldsymbol{\gamma}_{\mathbf{d}}$. Since $r_{d}$ was a!ready used to determine $\underline{r}_{d}$. the remaining three variables (1 vector and 2 scalars) are utilized in the following manner

$$
\begin{aligned}
& \underline{i}_{x}=\text { unit }\left(\underline{r}_{d}\right) \\
& \underline{i}_{z}=\underline{i}_{x} \times \underline{\underline{i}}_{y} \\
& \underline{v}_{d}=v_{d}\left[\begin{array}{c}
\underline{i}_{x} \\
\underline{i}_{y} \\
\underline{i}_{z}
\end{array}\right]^{T}\left[\begin{array}{c}
\sin \gamma_{d} \\
0 \\
\cos :_{d}
\end{array}\right]
\end{aligned}
$$

Vote that $1_{1}$ and $i_{2}$ define the radial and downrange directions with respect in the cutoff state. The original LTG equations, described in Ref. 1, used the current veh:cle position to define these directions. This resulted in continuous rotation of the desired terminal velocity. The input norrial to the transfer plane, 1 . , is directed in the opposite sense to the current orbital angular velocity vector.

### 4.8.2 Ascent to Reference Trajectory ( s mode $=2$ )

The objective of this mode is to intercept a coast reference trajectory for the single orbit rendezvous on Mission 3B. Throttling of the Space Shattie Main Engines is used to increase the number of guidance conirol variables (i. e., degrees of freedom) by one, and thereby enable the guidance algorithm to control the downrange component of position at insertion.

The following process is used to accomplish this. First, the Conic State Extrapolation Routine (Ref. 4) is used to extrapolate the reference trajectory state
 should be noted that for proper operation of the guidarice equations during the initial pass through the program, it is assumed that the input reference state (ref Yref) corresponds to the nominal time of thrust cutoff determined from prenight simulation). The extrapolated state ( $\mathbf{r} \mathbf{d}^{\mathbf{N}} \underline{\mathrm{V}}_{\mathrm{d}}$ ) can be used to determine the error in the downrange component of position at cutoff $\left[\underline{i}_{2} \cdot\left(\underline{r}_{d}-\underline{r}_{p}\right)\right]$. The other components of the extrapolated state ( $\underline{r}_{d} \underline{E}_{d}$ ) are used to update $\underline{E}_{g 0}$ and $\underline{r}_{g 0}$ on the subsequent guidance cycle.

Using this error in downrange position, it is necessary to determine a change in the time-to-go which drives this error to zero. Based upon the simplifying assumption of a flat earth and constant acceleration, it can be shown that the change in relative position $\Delta r_{z}$ resulting from changes in $t_{g o}$ is given by

$$
\frac{d \Delta r_{z}}{d t_{g o}}=\frac{-\underline{i}_{z} \cdot \underline{\underline{g}}_{g o}}{2}
$$

Then the change in $t_{g o}$ necessary to drive the position error to zero can be determined from

$$
\Delta t_{g o}=-2 \frac{\underline{i}_{z} \cdot\left(\underline{r}_{d}-\underline{r}_{p}\right)}{\underline{i}_{z} \cdot \underline{g}_{g o}}
$$

The new SSME throttle setting necessary to achieve this change in time-togn is based upon the assumption that the maneaver velocity-to-be-gained is fairly insensitive to changes in throttle setting. Thus

$$
K_{k}=\frac{K_{k} t_{b, k}}{t_{h, k}+\Delta t_{g o}}
$$

where $t, k$ is the burn time remaining in the current maneuver phase. The throttle setting $K_{k}$ is limited to a maximum value $K_{\max }$.

On the last guidance cjcle prior to the coastant acceleration (g-limited) phase, the maneuver is converted to the Lambert ascent mode ( $s_{\text {mode }}=3$ ) and ascent is completed in that mode. This is necessary since throttling for trajectory control and g-limiting may be incompatible. However, use of an artifical g -limit sl'ghtly less than the true limit, or a slight relaxation of the g-limit constraint could make it possible (with minor changes) to maintain the reference trajectory throttling mode throughout the second maneuver phase prior to tank separation. The Lambert ascent mode, however, will insure intercept with the target satellite with only minor variation in closing veiocity.

### 4.8.3 Modes Requiring Lambert Solutions, $s_{\text {mode }}=\mathbf{3 , 4 . 7 . 8}$

The remaining modes (with the exception of $s_{\text {mode }}=6$. External Delta-V. which does not enter this block) require calls to the Conic Required Velocity Determination Routine (Ref. 5). This routine solves a Lambert problem to determine the initial velocity required to satisfy certain terminal constraints.

The straight Lambert modes, ascent and on-orbit (s mode $=3,7$ ) require one call to the Conic Required Velocity Routine to determine. $\underline{v}_{d}$. the desired velocity to coast from the end of the maneuver to the target, $\underline{r}_{t}$. in the given time interval, $t_{t}-\left(t+t_{g o}\right)$.

Mrdes 4, 5, 8, abort and deorbit require atmospheric reentry with the entry point at a constant location relative to the rotating earth. It is assumed that a constraint of the form mentioned earlier is required of the vertical, viv and horizontal, $\mathbf{v}_{\text {th }}$, components of entry interface velocity $\mathrm{F}_{\mathbf{t}}$.

$$
\nabla_{t v}=C_{1}+C_{2} \nabla_{t h}
$$

For small entry angles ( -1.0 to $\mathbf{- 1 . 7}$ degrees) and entry velocities around 26.000 ft/sec this constraint is nearly equivalent to expressing the entry angle as a linear function of the entry velocity.

The only remaining variable that can be adjusted to cause the entry point to satisfy this terminal constraint is the coast time between cutoff and entry interface, $t_{t}-\left(t+t_{g o}\right)$. In other words, since $t$ and $t_{g o}$ are fixed by previnus computations, $t_{t}$, the time at entry interface, must be adjusted. Using the currently - ssumed $t_{t}$ and entry interface point relative to earth. $\underline{r}_{\text {tef }}$ an inertial entry interface position, $\underline{r}_{t}$, is determined by a call to the Eiarth Fixed to Inertial Routine. Then $\underline{r}_{d}, \underline{r}_{t}$, and the coast time ( $t_{t}-t-t_{g o}$ ) are input to the Conic Required Velocity Routine to wrain the desired velocity at cutoff. $\underline{v}_{d}$, and velocity at entry interface, $\underline{\mathbf{v}}_{\boldsymbol{t}}$. The entry interface velocity, $\underline{\underline{v}}_{t}$, will not in general lie on the constraint defined above but at some other point as shown in Figure 4-3.


Figure 4-3. Entry Interface Constraint

The components of $\mathrm{I}_{\mathbf{t}}$ are determined as follows:

$$
\begin{aligned}
& \underline{\mathbf{i}}_{\mathbf{r t}}=\operatorname{unit}^{\left(\underline{r}_{t}\right)} \\
& \boldsymbol{v}_{\mathbf{t v}}=\underline{\underline{i}}_{\mathbf{r t}} \cdot \underline{v}_{t} \\
& \boldsymbol{v}_{\mathbf{t h}}=\left|\underline{\underline{t}}_{\mathbf{t}}-\boldsymbol{v}_{\mathbf{t v}} \underline{i}_{\mathbf{r t}}\right|
\end{aligned}
$$

By evaluating $C_{1 c}$

$$
c_{1 c}=\nabla_{t v}-c_{2} \nabla_{t h}
$$

we find that, in general $C_{1 c} \neq C_{1}$ and therefore, $v_{t}$ does not lie on the constraint. Thus we must vary $t_{t}$ by some value such that $C_{1 c}$ goes to $C_{1}$. In order to de?ermine the sensitivity of $C_{1 c}$ to changes in $t_{t}$ we repeat the call to the Conic Required Velocity Routine with $t$ perturbed ho amall $6 t$. In a manner similar to before a new point ( $\boldsymbol{v}^{\prime} \mathrm{th}^{\text {, }} \mathrm{V}^{\prime}$ tv) is found on Figare 4-3. The factor

$$
\sigma=\frac{C_{1}-C_{1 c}}{\left(v_{t v}-v_{t v}\right)-C_{2}\left(v_{t h}-v_{t h}\right)}
$$

represents the required change in $C_{1 c}$ (necessary to extrapolate $v_{t v}$ and $\nabla_{\text {th }}$ to the constraint line) divided by the change in $C_{1 c}$ experienced by perturbing the entry interface time by ot. It is used to extrapmlate values for the entry interface time, $t_{t}$ and desired cutoff velocity, $\underline{I}_{d}$, which should result in near satisfaction of the constraint.

$$
\begin{aligned}
& t_{t(\text { new }}=t_{t \text { (previous })}+\sigma \delta t \\
& \Psi_{d(\text { new }}=\Psi_{d(\text { previous })}+\sigma\left[\sigma_{d}^{\prime}-\Psi_{d}(\text { previous })\right]
\end{aligned}
$$

The accuracy of this technique depends upon how much the region over which the extrapolation takes place varies from the linearized assumptions. Each guidance cycle, however, should bring the extrapolated point closer to the constraint.

### 4.8.4 Revising -go

Changes in desired terminal velocity. \#d, affect the predicted terminal position (and desired terminal velocity) on the subsequent gu dance cycie. If this effect is ignored, the desired terminal velocity is over corrected and a sman! oscillation in desired terminal velocity is induced. To elinuinate this over correction, a damping factor, $p$, is introduced for modes 4, 5, 8 as follows:

$$
\Delta \underline{g}_{g o}=\rho\left(\mathbf{v}_{g o}^{\prime}-\underline{V}_{g o}\right)
$$

where $\boldsymbol{V}^{\prime}$ go is the new velocity-to-be-gained assuming no damping and is given by

The damping factor is computed by making use of the appioximate partial derivative of desired velocity with respect to terminal position

$$
\frac{\partial \underline{v}_{d}}{\partial \underline{r}_{d}} \approx\left[\begin{array}{ccc}
-\frac{1}{t_{t}-t-t_{g o}} & 0 & 0 \\
0 & -\frac{1}{t_{t}-t-t_{g o}} & 0 \\
0 & 0 & -\frac{1}{t_{t}-t-t_{g o}}
\end{array}\right]
$$

where $t_{t}-t-t_{g o}$ is the coast time from cutoff to entry interface. It can be shown that

$$
\rho=\frac{1}{1+\frac{\mathrm{g}_{\mathrm{go}}}{2\left(t_{t}-t-t_{g o}\right)}}
$$

Note that initially $0<\rho<1$ and $\rho$ goes to one at the end of the burn.
Velocity-to-be-gained for the next pass through the routine is now revised by

$$
\underline{\Psi}_{g 0}=\Psi_{g o}^{\prime}+\Delta \nabla_{g o}
$$

## 4. 9 Throtile Commands

The throttle setting. $K_{k}$. for the Space Shuttie Main Bagine and the engineon/off commands are issued by the last and as yet to be determined block of equations. The throttle setting will be assumed to be input each cycle for the acceleration limited phases or will be computed in the required velocity block of the previous cycle for Mode 2 (Reference Trajectory). The array of switches, sssmL, i
 in each phase.

During the prethrust call, on the last iteration to convergence engine ignition commands for ig will be issued. The appropriate time to issue these commands will be determined by the conditions: $s_{\text {pre }}=0, \Delta t=0$.

## 5. DETAILED FLOW DIAGRAMS

This section contains detailed flow diagrams of the comprataions and logic used for Uniffed Powered Flight Guidance. The overall flow is illustrated on the first diagram (Figure5-1) which shows the mine blocks of computations connected by the necessary logic to form a single rontine, UPFG. One entry and ane exit point exist for UPFG with internal branching, as indicated, to perform prethrust or active guidance for the various maneuver modes.

The following diagrams (Figares 5-2 through 5-10) further detail each of the nine blocks of in-line computations of UPFG.


Figure 5-1a. Main Routine


Figure 5-1b. Main Routine (Cont.)


Figure 5-2. block 1 - Initialization


Figure 5-3a. Block 2 - Update


Figure 5-3b, Block 2 - Update (Coni, )


Figure 5-4a. Block 3-Time-To-Go


Figure 5-4b. Block 3-Time-To-Go (Cont.)


Figure 5-5. Block 4 - Integrals of Thnist


EXIT Block 5

Figure 5-6. Block 5-Turning Rate


Figure 5-7. Block 6-Steering Command


Figure 5-8. Block 7 - Gravity Effects

## ENTER Block 8



Figure 5-9a. Block 8 - Velocity-To-Be-Gained


Figure 5-9b. Block 8 - Velocity-To-Ec Gained (Cont.)


Figure 5-9c. Block 8 - Velocity-To-Be-Gained (Cont.)


Figure 5-10. Block 9 - Throttle Commands

## 6. SUPPLEMENTARY INFORMATION

Several details of the UPFG program require further study or definition. They are listed below, not necessarily in order of importance.

## 6. 1 Targeting Assistance

Since the deorbit maneuver may be a long, low acceleration burn with a single OMS engine, traditional targeting techniques based upon an impulsive maneuver are not adequate. The powered flight guidance routine must assist the targeting to determine the $\Delta V$ required for the finite thrust maneuver and compute the optimal ignition time. Thus an additional logical path through UPFG program, very similar to the prethrust path, should be included in future revisions. This path could be used iteratively by the deorbit targeting program to search for the optimal ignition time resulting in minimum $\Delta v$.

### 6.2 Return-to-Launch-Site Abort

Considerable effort wiil be required to define the thrust cutoff condition for successful RTLS abort maneuvers. Requirements on dynamic pressure at external tank separation, orbiter glide back capability, maximium loads, maximum vehicle turning rates, fuel depletion requirements, and other factors enter into the development of RTLS powered maneuver constraints.

## 6. 3 Ascent Initialization

The problems of efficiently initializing UPFG during an ascent maneuver have not been addressed in sufficient detail. It is probably desirable to include the boost phase guidance in the overall UPFG scheme. This should simplify somewhat the initialization and transition from the relatively simple atmospheric phase guidance to the explicit LTG concept used in UPFG. The problems associated with changing maneuver objectives during ascent due to engine failures have not been explored. The best method of switching from a standard ascent maneuver to some abort mode must be determined. The reader should note that the prethrust process described in this report is primarily tailored for the on-orbit maneuvers.

## 6. 4 External Tank Disposal

The implications and requirements of the external tank disposal require further study. In this document, it is assumed that tank separation will take place during a short coast phase after the ascent velocity-to-be-gained has been driven to a prespecified value ( $<150 \mathrm{fps}$ ). Then the OMS engines complete the ascent maneuver. If this technique does not insure a suf?iciently accurate external tank impact, then further refinement will have to be considered.

If the OMS maneuver following tank separation is sufficiently large ( $\approx 300 \mathrm{fps}$ ) such that the length of the OMS maneuver is comparable in length to the phase from SRM separation to tank separation, then the guidance equations must be modified. The multi-phase LTG equations, which result in a linear vehicle pitch rate, are not optimal for an acceleration profile with a high initial acceleration and a very low final acceleration. The final OMS maneuver could probably be treated as a constant attitude phase without unduly complicating the equations.

### 6.5 Compensation for Non-Keplerian Gravity Effects

No attempt has been made to include a technique for compensation of nonKeplerian gravity effects in this initial version of UPFG. Further work is required to determine whether this compensation should be accomplished during premaneuver targeting or during the maneuver. A combination of premaneuver compensation with small adjustments during the maneuver will probably produce good accuracy with I .imal code.

## 6. 6 Ascent Mareuver Phase Changes

The ascent maneuver is divided into several maneuver phases, which may include a constant thrust phase, a constant acceleration phase, a final OMS phase, and possibly a phase with an assumed SSME failure. Transition from one phase to the next may be a function of acceleration, time, or velocity-to-he-gained. To solve for several integrals of the thrust acceleration over the total maneuver time-to-go, the times at which phase changes occur must be prespecified or calculated. Since mission perturbations may alter these times, it may he desirable to modify them during the maneuver based upon sensed acceleration.

### 6.7 Steering Commands

The development of steering emations to generate comimands for the control system is incomplete. The steering equations will combine desired rehicle attitude, desired vehicle rate, and sensed acceleration to produce a steering command.

### 6.8 Throttle Commands

An algorithm to estimate current acceleration and calculate an engine throttle setting for the constant acceleration ( $g$-limited) phast of $a$ iceat has not been develoned. The frequency with which these commands must be issued, to prevent a sawtonth profile, and the engine response to commands should be cvaluated to develop this algorithm.

### 6.9 Vehicle Mass Estimate

The $n$ aintenance of the estimated vehicle mass should be accomplished external to the UPFG program since this information is also required by the contro! system software. In addition, step changes in the vehicle mass due to events such as tank disposal or satellite deployment will obviously have to he handled elsewhere. However, for completeness, an equation has ieen included in this document (Block 2) to decrement vehicle mass for simulation purposes.

## 6. 10 Engine Failure

The failure of an engine during any mission phase could possibly be detected through changes in sensed acceleration. however it is assumed that more reliable information will be available from the performance monitoring system. Therefore, the UPFG equations have been mritten with the assumption that an external routine will notify the UPFG routine of changes in enginc statue through the input switches.

### 6.11 Throttle Lag

The ascent to reference trajectory guidance mode requires use of the SSME throttles. To make the proper adjustments in engine throttle settings, the currently cominanded throttle settings are used in comtination with nominal engine performance data. To compensate for engine performance perturbation and throttle non \& zarity, it ma; be desirable to use sensed acceleration information. However, this will introduce problems due to both guidance computational delays and eng'ne throttle response. Thus this problem requires further study.

## 52 Aternate Input Schemes

The input list for each maneuver mode, described in Section 3, has been designed with the intent to minimize the size of the flight program. A certain amount of ground calculation is required for the ascent guidance modes, however on-orbit input is consistant with current CSDL targeting concepts. Future revisions coulci include alternate input schemes for ascent maneuvers which may be easier to use but place additional calcula. ion burdens upon the guidance computer.

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