



# Study of the Technique of Stellar Occultation

by

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prepared for

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

Contract No. NAS 1 9958

November 30, 1973

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Ann Arbor, Michigan

N74-13568

Unclas  
G3/30 24552

OF STELLAR OCCULTATION (Michigan Univ.)  
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152 P HC \$9.75  
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NASA CR-112197

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## 1. Introduction

This report describes the results of a study of the stellar occultation technique for measuring the composition of the atmosphere. The work reported here was supported by NASA Contract NAS 1 9958.

The stellar occultation technique is based upon classical absorption spectroscopy, utilizing spacecraft-borne photometers as the detector and a star as the light source. During occultation the ultra-violet stellar spectrum of the star shows absorption features which are uniquely related to various atmospheric constituents. In particular, the spectral region between  $1200\text{\AA}$  and  $2000\text{\AA}$  is related to absorption by molecular oxygen and the region between  $2000\text{\AA}$  and  $3500\text{\AA}$  is related primarily to absorption by ozone. This is illustrated in Figure 1 where the photo absorption cross sections of both molecular oxygen and ozone are shown.

The intensity of starlight was monitored during the occultation using the Wisconsin stellar ultraviolet photometers aboard the Orbiting Astronomical Observatory (OAO-A2). A schematic diagram of an occultation is shown in Figure 2 where the change in intensity at a given wavelength is illustrated. The vertical projection of the attenuation region is typically 60 km. deep for molecular oxygen and 30 km. deep for ozone.

Intensity profiles obtained during various occultations were analyzed by first determining the tangential column density of the absorbing gases, and then Abel inverting the column densities to obtain the number density profile. Errors are associated with each step in the inversion scheme and have been considered as an integral part of this study.

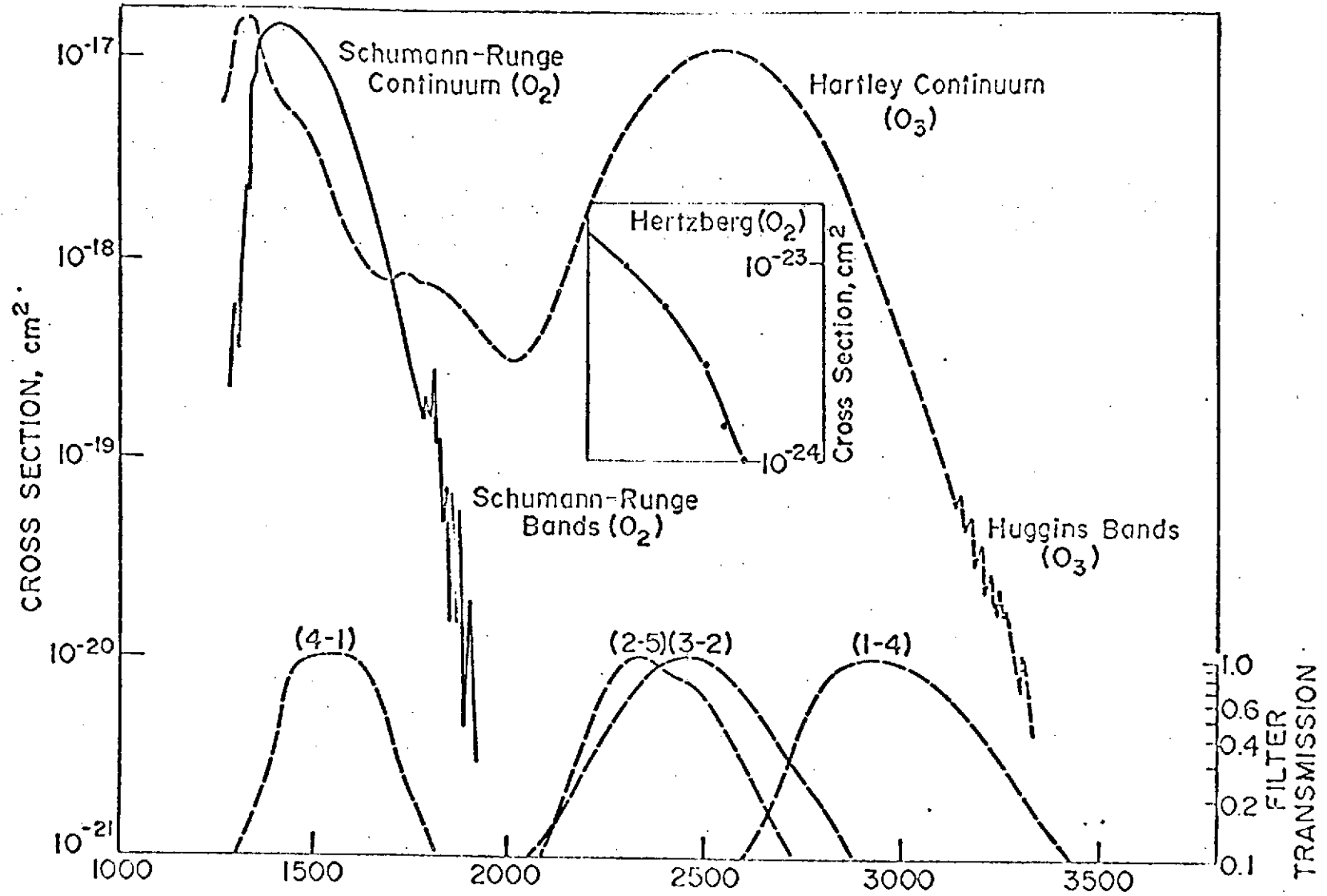
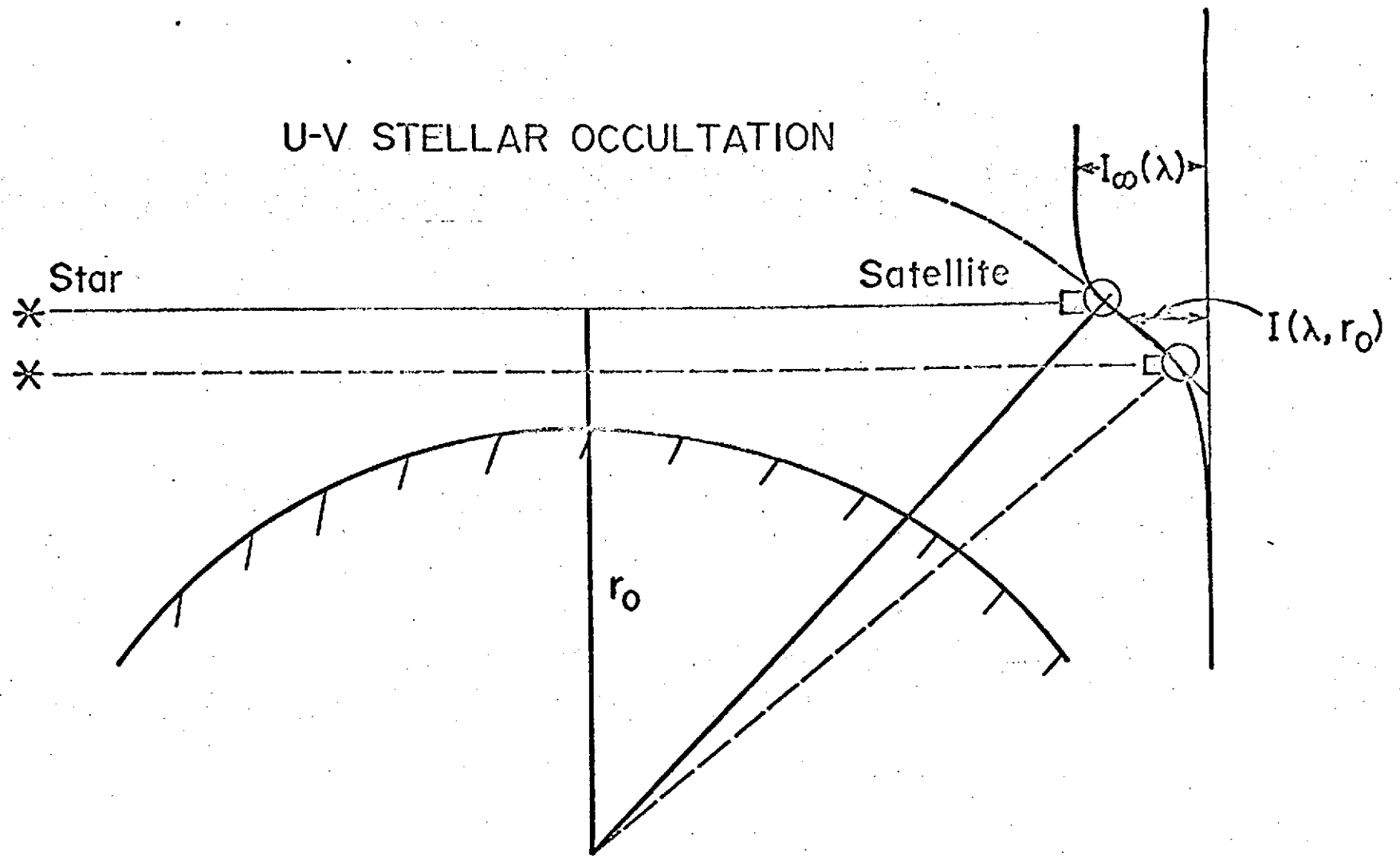


Figure 1. Oxygen and ozone cross sections with OAO filters transmission functions shown.



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Figure 2. Schematic view of occultation.

Most of the work carried out under this contract has been published in the open literature, thus, we include these papers in this report where appropriate. A general description of this study and the results were published by Hays et al. (1972) and is included as Appendix I of this report.

## 2. Data Procurement

The data used in this study were obtained through the courtesy of the O. A. O. Wisconsin Experimenters and N. A. S. A. Goddard Space Flight Center. The principle difficulty associated with the data procurement centered on obtaining intensity information at high rates. It was possible to sample the stellar photometer accumulator at high frequency, but only bits 7 through 14 of this register were available in this mode of operation. During this procedure the photometers continued to accumulate for 128 periods of 0.54080 seconds. Upon completion of this process the register was locked until a new command was received, at which time the cycle was repeated. Unfortunately, a finite period of time was required for the new command to be sent and a gap appears in the data every 128 samples. This caused many occultations to be rejected due to re-set blanks appearing in the middle of the attenuation period.

A typical data record is shown in Figure 3, we note that the bottom of each record corresponds to 64 accumulated counts and the top to 16,384 accumulated counts. The overflow within this sub-set is obvious. The data analyses utilized differences between individual samples and a smooth interpolation to determine the overflow and count accumulated during a sample period. The record labeled Channel 44 in Figure 3 is from the  $1500\overset{\circ}{\text{A}}$  filter used to measure molecular oxygen.



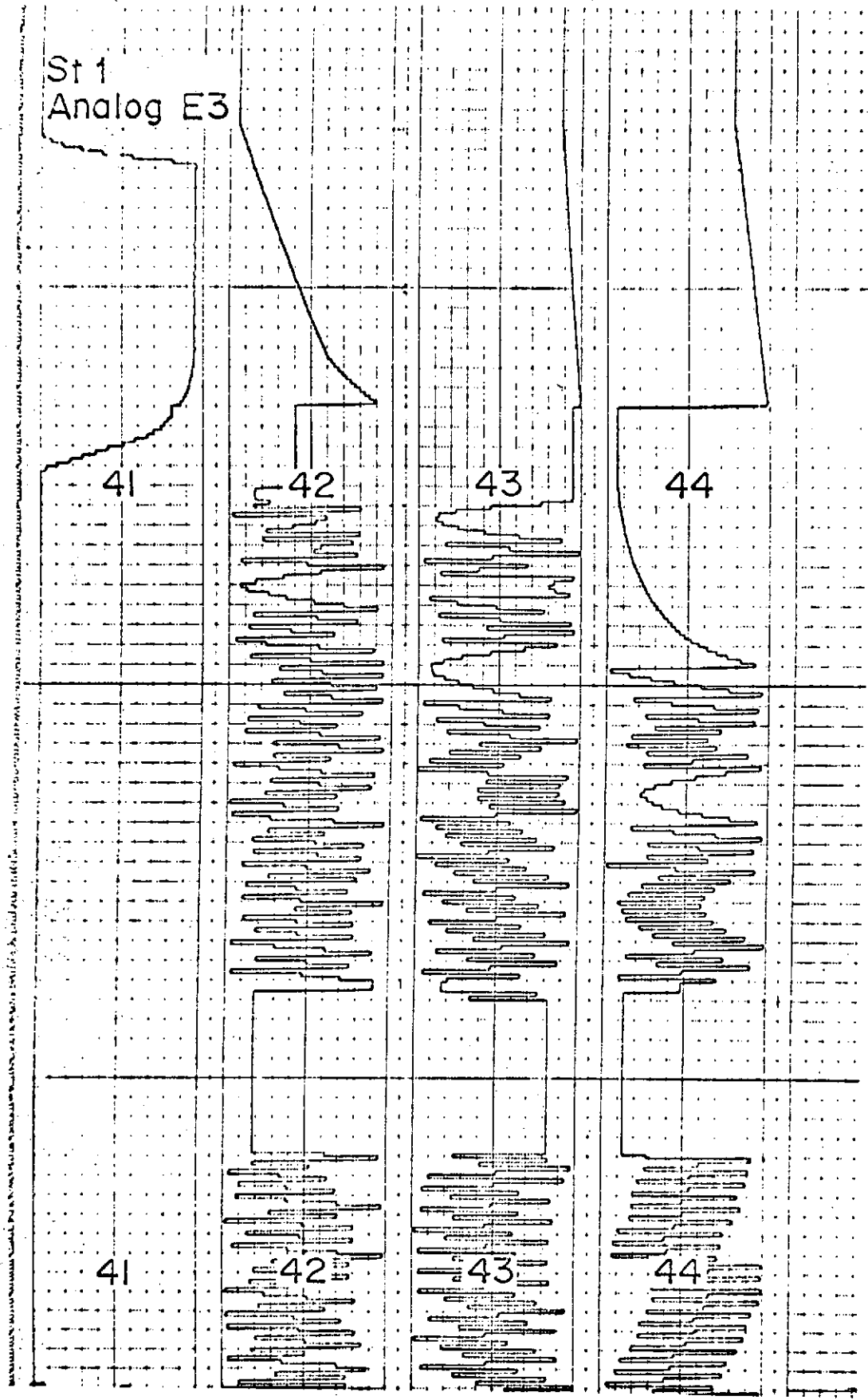


Figure 3. Typical occultation row data.

The stellar photometers have broad band filters which typically cover a  $200\text{\AA}$  wide region in the spectrum. The relative transmission of these filters are shown at the bottom of Figure 1. Due to the broad spectral band of these filters it was necessary to measure the spectrum of all stars used in the study. This was done prior to the occultation using the O. A. O. spectrometer.

In addition to intensity and spectral information it was necessary to determine the altitude of the tangent ray. This requires an accurate knowledge of the spacecraft positions as well as the coordinates of the star. We acknowledge the considerable effort of the NASA Goddard Space Flight Center who carefully re-checked the satellite ephemeris in order to achieve the accuracy required for our study.

### 3. Tangent Ray Geometry

Throughout an occultation period, the tangent ray point follows a curved path above the surface of the earth. The coordinates used to describe this path are shown in Figure 4. In this system, the x-axis points toward the First Point of Aries, the y-axis is also in the equatorial plane pointing  $90^\circ$  east of Aries, and the z-axis points toward the North Pole. The time of scan enters into the coordinate system through the Greenwich hour angle of the x-axis,  $\lambda_g$ .

To obtain the height of the tangent point, the angle  $\beta$  at the spacecraft between rays to the star and to the center of the earth must be evaluated in terms of the direction numbers of the star,

$$l = \cos \alpha \cos \zeta$$

$$m = \sin \alpha \cos \zeta$$

$$n = \sin \zeta$$

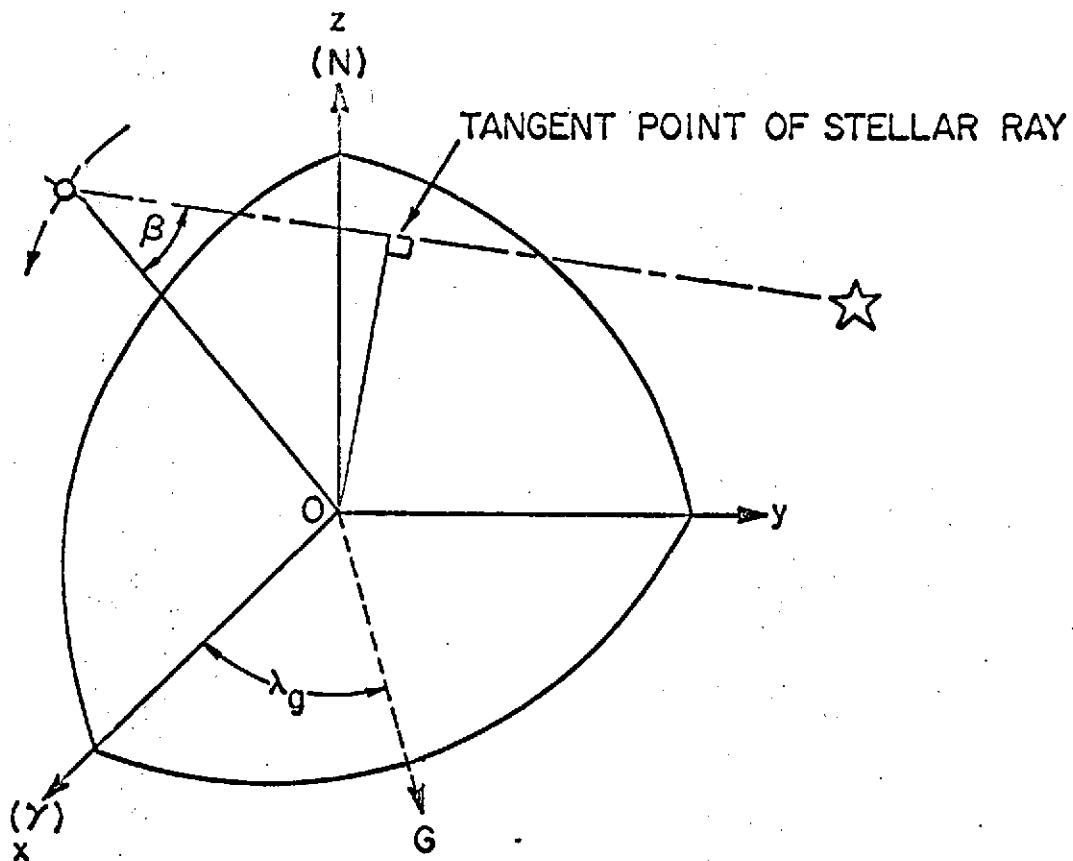


Figure 4. Geometry of occultation for any stellar object.

and the direction cosines of the satellite position,

$$a = \cos \lambda_s \cos \phi_s \cos \lambda_g - \sin \lambda_s \cos \phi_s \sin \lambda_g$$

$$b = \sin \lambda_s \cos \phi_s \cos \lambda_g + \cos \lambda_s \cos \phi_s \sin \lambda_g$$

$$c = \sin \phi_s$$

where  $\alpha$  = right ascension

$\delta$  = declination

$\lambda$  = longitude

$\phi$  = latitude

Subscripts s refers to the spacecraft and subscript o will refer to the tangent ray point. Then utilizing the above quantities,

$$r_o = r_s \sin \beta \quad (1)$$

where  $\beta = \pi - \cos^{-1} [a + bm + cn]$  ,

$$\phi_o = \tan^{-1} [z_o (x_o^2 + y_o^2)^{-\frac{1}{2}}] \quad (2)$$

$$\lambda_o = \cos^{-1} [x_o (x_o^2 + y_o^2)^{-\frac{1}{2}}] - \lambda_g \quad (3)$$

If  $y_o > 0$ ,  $0 \leq \lambda_o \leq 180$ ; if  $y_o < 0$ ,  $180 \leq \lambda_o \leq 360$ .

The additional formulae needed to evaluate Equations 1-3 are

$$x_o = r_s (a + l \cos \beta)$$

$$y_o = r_s (b + m \cos \beta)$$

$$z_o = r_s (c + n \cos \beta)$$

Stellar refraction is neglected in our analysis due to the low index of refraction at the altitudes considered in this study.

#### 4. Data Processing and Error Analysis

The technique of data processing and the analysis of resulting errors has been reported during this study by Roble and Hays (1972). This paper is included as Appendix II and is a complete description of the

results of this study. A complete description of the computer routine used to evaluate the occultation results obtained in this study is included as Appendix III of this report. The reader is referred to these detailed appendices for further information on the results of our study. A separate discussion of influence of orbital errors is included in Appendix IV.

## 5. Data Analysis

A very large number of scans have been reduced in the course of this contract period. Table I summarizes these results and includes all orbits which have yielded at least a partial result e. g., a molecular oxygen profile or an ozone profile. To provide information on quality in the individual channels, the following code is used

c = processing completed

+ = resulting number density profile,  $n(r_0)$ , acceptable

- =  $n(r_0)$  not acceptable

? =  $n(r_0)$  questionable

Other information in the table refers to the starting time (GMT and local) of the scan, the location of the ray tangent point, the target star, and the nature of some of the difficulties. Starting times are only approximate, as they vary up to 20 seconds from channel to channel and they must be entered to the nearest half-second for acceptable accuracy in height of ray tangent point.

These results have been analyzed from the point of view of their geophysical information and are to be published in journals during the next few months. All papers are in press at this time. Detailed computer output containing tabular input data and results are available to the contracting agency on request, but are too lengthy to include in this report.

Table I

orbit	date	GMT		local		Long. (deg)	Lat. (deg)	star	ozone		O <sub>2</sub>	remarks on data loss
		hr	min	hr	min				2390Å	2460Å		
5778	1/13/70	5	28	23	52	-84	48	γ Peg	-		+	O <sub>3</sub> filter
5779	1/13/70	7	8	23	51	-109	48	γ Peg	-		+	O <sub>3</sub> filter
8884	8/17/70	4	55	5	55	15	4	α Lup			+	O <sub>3</sub> reset
8898	8/18/70	4	19	5	51	23	9	β Lup	?	?	?	Height
8899	8/18/70	5	59	5	51	-2	9	β Lup	?	?		Height, O <sub>2</sub> unevenness
8913	8/19/70	5	22	5	54	8	11	β Lup			+	O <sub>3</sub> reset
8942	8/21/70	5	55	4	27	-22	24	τ Sco	+	-	?	O <sub>3</sub> data cutoff, O <sub>2</sub> reset
8943	8/21/70	7	35	4	23	-48	24	τ Sco			+	
8956	8/22/70	5	18	4	26	-14	24	τ Sco	+			O <sub>2</sub> reset
8957	8/22/70	6	58	4	22	-39	25	τ Sco	+	-	+	Height
8971	8/23/70	6	20	4	44	-24	27	δ Sco	+	+	-	O <sub>2</sub> reset
8985	8/24/70	5	45	4	13	-23	24	τ Sco		+	+	
8986	8/24/70	7	25	4	13	-48	24	τ Sco		+	+	
9000	8/25/70	6	47	5	17	-32	27	δ Sco	+	+	+	
9014	8/26/70	6	10	4	30	-25	26	δ Sco	+	+		
10795	12/28/70	2	31	21	55	-69	42	i Her			+	€ Aur spectrum used
10796	12/28/70	4	12	22	12	-91	43	i Her	-	-	-	£ Aur spectrum used Height
10902	1/4/71	13	29	22	38	-223	-16	α And	+		+	
11795	3/7/71	15	59	0	33	128	-34	δ Ori	+		+	
12178	4/3/71	6	8	22	33	-114	36	i Ori		+	+	
14579	9/17/71	5	42	2	51	-43	26	⊙ Oph	+		-	O <sub>2</sub> reset
14580	9/17/71	6	22	1	50	-68	26	⊙ Oph	?		+	O <sub>3</sub> values high in bulge

In addition, orbits 13161, 13187, 13189, and 13195 of June 10 and June 12, 1971, provided high quality data which are awaiting accurate satellite position reports.

## 6. Geophysical Results

Interpretations of the results obtained under this contract have been presented in papers published in the open literature. Molecular oxygen results were compared with theory and previous measurements by Hays and Roble (1972b). Ozone results were discussed by Hays and Roble (1972c), Roble and Hays (1972b) and Roble and Hays (1972c). These papers are included as Appendices V through VIII.

## 7. Summary of Study

The present study of the stellar occultation technique of recovering atmospheric composition has led to the following conclusions:

- a) Any constituent which causes significant absorption in a stellar spectrum can be monitored by using the ultraviolet occultation technique.
- b) The error associated with the inversion of occultation data limits the altitude region for which results of high quality can be obtained. In most cases the high quality data results from regions where the intensity lies between 10% to 90% of the unattenuated flux. Finite difference errors are responsible for limiting the structural detail which can be recovered. Orbital errors are serious and are reflected in the altitude scale of the recovered density profiles.
- c) Serious geophysical studies can be carried out using as principal data the density profiles of molecular oxygen and ozone obtained from stellar occultation. The following major geophysical features have been identified during this study:
  - (1) Molecular oxygen shows a strong solar cycle variation in the lower thermosphere. This probably results from changes in

the constant pressure level surfaces at the base of the thermosphere.

- (2) Molecular oxygen in equatorial regions shows strong magnetic storm variations due to large scale dynamic motions resulting from polar region heating.
- (3) Ozone in the mesopause region is controlled by wet chemistry and supports modern photochemical theories.



APPENDIX I.

Terrestrial Atmospheric Composition from Stellar Occultations

## Terrestrial Atmospheric Composition from Stellar Occultations

**Abstract.** *Stellar ultraviolet light transmitted through the earth's upper atmosphere is strongly absorbed by ozone and molecular oxygen. The stellar ultraviolet photometers aboard the Orbiting Astronomical Observatory (OAO-2) satellite have measured the intensity changes of several stars during occultation of the star by the earth's atmosphere. From the occultation data the nighttime vertical number density profiles of molecular oxygen at altitudes from 120 to 200 kilometers and of ozone at altitudes from 60 to 100 kilometers have been obtained.*

In the earth's upper atmosphere, stellar ultraviolet light is strongly absorbed in the Schumann-Runge continuum of  $O_2$  and the Hartley continuum of  $O_3$ . By monitoring the intensity of ultraviolet starlight in these continuum regions from a satellite as the star is occulted by the earth's atmosphere, we are able to obtain information on the number density profile of  $O_2$  in the lower thermosphere and  $O_3$  in the upper mesosphere (1, 2). In this report we describe the technique used to obtain the number density profiles from stellar occultation measurements. The data were obtained by the University of Wisconsin stellar photometers aboard the Orbiting Astronomical Observatory (OAO-2) satellite. We discuss here the inversion process and also show the  $O_2$  and  $O_3$  distributions determined from a typical occultation scan.

In the occultation technique classical

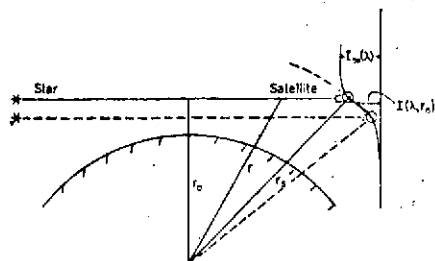


Fig. 1. Geometry of ultraviolet stellar occultation.

absorption spectroscopy is used to determine the number density profile of the absorbing species in the upper atmosphere. The star is the source of ultraviolet light, the OAO stellar photometers are the detectors, and the atmosphere between them is the absorption cell. During the occultation process, the ultraviolet light is selectively absorbed in spectral regions for which  $O_2$  and  $O_3$  have large absorption cross sections. The intensity of the transmitted ultraviolet light is related to the number density along a tangential column of the absorbing species ( $N_i$ ) by Beer's law

$$I(\lambda, r_0) = I_\infty(\lambda) \exp \left[ - \sum_i \sigma_i(\lambda) \cdot N_i(r_0) \right] \quad (1)$$

where  $I_\infty(\lambda)$  is the unattenuated intensity above the atmosphere of the star at wavelength  $\lambda$ ,  $\sigma_i(\lambda)$  is the absorption cross section of the  $i$ th absorbing species, and  $N_i(r_0)$  is the tangential column number density of the  $i$ th absorbing species at a tangential ray height  $r_0$ . The star's spectrum is measured above the atmosphere by the OAO ultraviolet spectrometer. By also knowing the absorption cross section, one can relate  $N_i(r_0)$  to the intensity of the transmitted ultraviolet light. In practice, Eq. 1 must be integrated with respect to wavelength because of the finite passband of the ultraviolet filter. The best

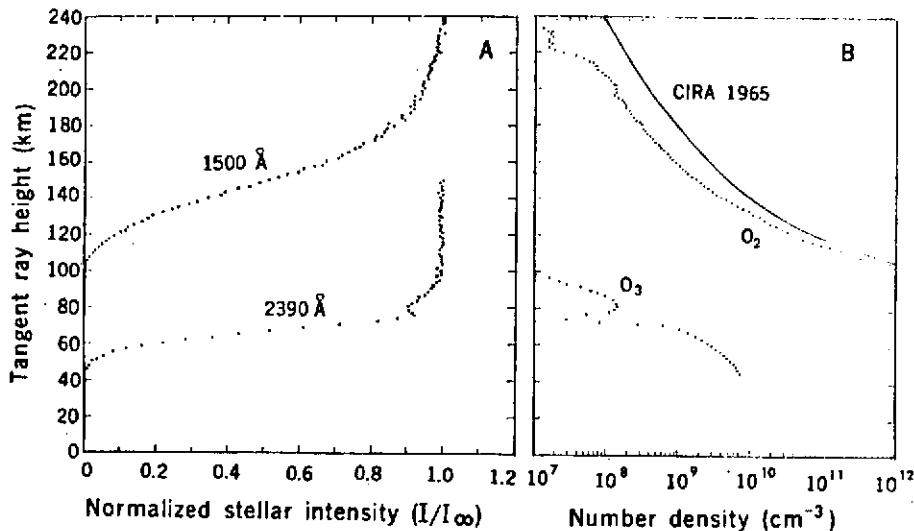


Fig. 2. (A) Normalized intensity of a star as a function of the tangent ray height during occultation of the star by the earth. The curves correspond to the intensity measured in the 1500-Å and 2390-Å channels of the University of Wisconsin stellar photometers aboard the OAO-2 satellite [22 August 1970, 04 hours 22 minutes (local time), 25°N, 39°E]. (B) Number densities of O<sub>2</sub> and O<sub>3</sub> as a function of tangent ray height determined from the occultation measurements. The solid line is the O<sub>2</sub> number density from the CIRA 1965 model atmosphere.

results are thus obtained when a single species dominates the absorption process, and the OAO filters were, therefore, selected accordingly.

Once  $N_i(r_0)$  is known, it is a simple matter to invert the data and obtain the vertical number density profile of the absorbing species at the occultation tangent point. A simple geometrical argument (Fig. 1) shows that  $N_i(r_0)$  along the ray path for a spherically stratified atmosphere can be written as

$$N_i(r_0) = 2 \int_{r_0}^{\infty} \frac{n_i(r) r dr}{(r^2 - r_0^2)^{1/2}} \quad (2)$$

where  $n_i(r)$  is the number density of the  $i$ th absorbing species at radius  $r$ . Equation 2, the Abel integral equation (1, 2), is easily inverted to give the number density of the absorbing species at a tangent ray height  $r$

$$n_i(r) = \frac{d}{dr} \left[ -\frac{1}{\pi} \int_{r_0}^{\infty} \frac{r N_i(r_0) dr_0}{(r_0^2 - r^2)^{1/2}} \right] \quad (3)$$

Thus, the stellar occultation technique can be used to obtain the vertical density profile of any absorbing atmospheric species which can be spectrally isolated.

Hays and Roble (2) have calculated the tangential ultraviolet transmission of the earth's upper atmosphere. Their results show that both the strong atmospheric absorption of O<sub>2</sub> in the Schumann-Runge continuum near 1500 Å and the strong atmospheric absorption

of O<sub>3</sub> in the Hartley continuum near 2500 Å occur in spectrally isolated regions in which the stellar ultraviolet absorption is primarily due to a single species. The absorption cross sections of O<sub>2</sub> and O<sub>3</sub> at these wavelengths have a peak around 10<sup>-17</sup> cm<sup>2</sup>. Therefore, we are able to determine the distribution of these species near altitudes at which  $N_i$  is approximately 10<sup>17</sup> cm<sup>-2</sup>. If a spectral region away from the peak cross section is utilized, one is able to observe higher tangential column number densities or, equivalently, to measure the number density at lower altitudes within the atmosphere. The ultraviolet light in the wavelength interval from 1400 to 1600 Å is absorbed primarily by O<sub>2</sub> at altitudes from 130 to 230 km. In the wavelength interval from 2400 to 2600 Å, the ultraviolet light is absorbed primarily by O<sub>3</sub> at altitudes from 60 to 100 km (2).

The normalized intensity data obtained during one of the many occultations are shown in Fig. 2A for two stellar photometers having filters centered at 1500 and 2390 Å. The normalized intensity is obtained as a function of time; however, by knowing the star's position and the orbital elements of the satellite, we are able to relate time to the tangent ray height of the occulting star. Because of high-altitude absorption, the normalized intensity in the O<sub>2</sub> channel at 1500 Å decays first. Then the intensity in the

2390-Å channel decays rapidly at altitudes where O<sub>3</sub> absorption becomes important. The excellent quality of the data obtained by the OAO-2 stellar photometers allows determination of detailed structure. The data in the 1500-Å channel were inverted, and the results are shown in Fig. 2B where the O<sub>2</sub> number density profile is shown as a function of height. For comparison, the O<sub>2</sub> profile of the Committee on Space Research (COSPAR) CIRA 1965 (3) model atmosphere is shown also. These data are only a sample of the approximately 20 stellar occultation scans that have been reduced thus far, but they illustrate the quality of the data obtained from the occultation measurements.

Most of the O<sub>3</sub> scans obtained with the 2390-Å and 2460-Å filters aboard the OAO-2 satellite have a normalized intensity scan similar to the one shown in Fig. 2A. The intensity of the occulting star decreases as the starlight penetrates into the atmosphere until a slight increase in the intensity curve occurs at a tangent ray height near 70 km. These data, when inverted, give the number density profile of O<sub>3</sub> shown in Fig. 2B. The structure in the measured intensity curve is caused by a bulge in the nighttime O<sub>3</sub> number density profile, with the peak occurring near 82 km and a minimum near 75 km for this particular scan. The stellar occultation measurements clearly define the structure of the nighttime O<sub>3</sub> profile at high altitudes where no previous measurements had been made.

The occultation technique has also been used with the sun as the light source. The measurements, however, are restricted to sunrise and sunset, and a general review of the subject has been given by Link (4).

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#### References and Notes

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2. ———, *J. Atmos. Sci.* **25**, 1141 (1968).
3. *COSPAR International Reference Atmosphere* (North-Holland, Amsterdam, 1965).
4. F. Link, *Eclipse Phenomena in Astronomy* (Springer-Verlag, New York, 1969).
5. The National Center for Atmospheric Research is sponsored by the National Science Foundation.

10 January 1972

APPENDIX II.

A Technique for Recovering the Vertical  
Number Density Profile of Atmospheric  
Gases from Planetary Occultation Data

# A TECHNIQUE FOR RECOVERING THE VERTICAL NUMBER DENSITY PROFILE OF ATMOSPHERIC GASES FROM PLANETARY OCCULTATION DATA

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(Received 29 March 1972)

**Abstract**—The occultation technique of determining the properties of the atmosphere using absorption spectroscopy is examined. The intensity of a star, in certain atmospheric absorption bands, is monitored by a satellite tracking the star during occultation by the Earth's atmosphere. The intensity data in certain wavelength intervals, where absorption is attributed to a single species, are related to the tangential column number density of the absorbing species through Beer's law. The equation for the tangential column number density is the Abel integral equation which is inverted to obtain the number density profile of the absorbing species at the occultation tangent ray point. Two numerical schemes for inverting the Abel integral equation for signals of low intensity with statistical noise superimposed are presented; one for determining the number density profile of atmospheric species that decrease exponentially with height, and the second for determining the profile of constituents having a more complex vertical structure, such as ozone. The accuracy of retrieving the number density distribution from planetary occultation data is examined. A theoretical analysis of the errors in determining the number density from occultation data of very low signal intensity is also presented. The errors in retrieving the number density profile are related to the intensity of the source, the number of data points per scan, and the degree of data smoothing required before inversion. As a specific example, calculations are made of the errors in retrieving the molecular oxygen and ozone number density profiles from occultation intensity data in the Schumann-Runge continuum of molecular oxygen at 1450 Å and the Hartley continuum of ozone at 2450 Å.

## 1. INTRODUCTION

Absorption spectroscopy is an important technique for determining the composition and number density profiles of constituents in the upper atmosphere (Watanabe, 1958; Hinteregger, 1962). Most of the measurements have been made from rockets monitoring the absorption of the solar u.v. flux in certain atmospheric bands (Johnson *et al.*, 1951; Bryam *et al.*, 1957; Kupperian *et al.*, 1959; Jursa *et al.*, 1963, 1965; Carver *et al.*, 1964, 1966; Weeks and Smith, 1968; Opal and Moos, 1969; Wildman *et al.*, 1969; Quessette, 1970; Brannon and Hoffman, 1971). Satellites have also been used to measure the intensity of certain u.v. sources (i.e., Sun, Moon, stars) during occultation of the source by the Earth's atmosphere (Fig. 1). The intensity data measured during occultation are then used to obtain information about the absorbing species in the upper atmosphere (Venkateswaran *et al.*, 1961; Rawcliffe *et al.*, 1963; Miller and Stewart, 1965; Thomas *et al.*, 1965; Fesenkov, 1967; Thomas and Norton, 1967; Norton and Warnock, 1968; Stewart and Wildman, 1969; Hinteregger and Hall, 1969; Lockey *et al.*, 1969; Link, 1969; Reid and Withbroe, 1970; May, 1971; Reid, 1971; Roble and Norton, 1972; Hays *et al.*, 1972).

Basically, the occultation technique is similar to the classical technique of absorption spectroscopy. The Sun, Moon, or a star are used as a source of light and the satellite

\* The National Center for Atmospheric Research is sponsored by the National Science Foundation.

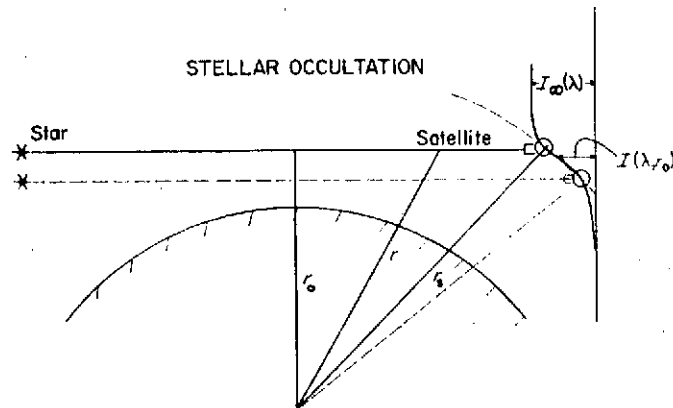


FIG. 1. GEOMETRY OF STELLAR OCCULTATION.  $I_{\infty}(\lambda)$  IS THE INTENSITY OF THE UNATTENUATED STAR ABOVE THE ATMOSPHERE AND  $I(\lambda, r_0)$  IS THE MEASURED INTENSITY OF THE STAR AT TANGENT RAY HEIGHT  $r_0$ .

photometers as a detector, with the atmosphere between acting as the absorption cell. As the satellite moves in its orbit, the source ultimately is occulted by the Earth. During the occultation process, the intervening atmosphere absorbs progressively more of the light due to the strong photo-absorption features of the atmospheric gases. The occultation intensity data can be related to the tangential column number density of the absorbing species if the absorption cross-sections are known. The equation for the tangential column number density of the absorbing species is the Abel integral equation which is easily inverted to give the number density profile at the occultation tangent point. The occultation technique works best for a point source, such as a star, where the vertical resolution at the tangent point is very sharp (Hays and Roble, 1968b). For a finite-size source, such as the Sun, the light rays emitted from the upper and lower limb can have tangent ray heights separated by about 26 km when observed from normal satellite altitudes. This complicates the problem of relating the transmitted solar intensity to the tangential column number density (May, 1971; Roble and Norton, 1972). However, once the tangential column number density is obtained, it again becomes easy to invert the Abel integral equation and obtain the local number density at the tangent ray point. For simplicity, we therefore assume that the light source is small, such as a star (Hays *et al.*, 1972) or a small portion of the solar disk (Hinteregger and Hall, 1969; Reid and Withbroe, 1970; Reid, 1971).

In the present paper, we describe a technique for retrieving the number density profile of the absorbing species from occultation intensity data. The data-reduction procedure is similar to the method of computing the radial emission distribution of a cylindrical plasma source, as treated by the plasma physicists. The similarity exists because both techniques reduce to the problem of solving the Abel integral equation (Hays and Roble, 1968a, b; Barr, 1962).

In practice, the numerical solution of the Abel integral equation requires special handling because of its sensitivity to small random errors in the data. Therefore, we develop two separate techniques of numerically inverting the Abel integral equation to obtain the number density distribution of the absorbing species. The first utilizes an exponential approximation for atmospheric species which decrease nearly exponentially with altitude; the second, a quadratic approximation which works better for atmospheric species having a more complex vertical distribution such as atmospheric ozone (Hays *et al.*,

1972). The retrieval accuracy of both schemes is examined, and a theoretical analysis of errors is presented which enables us to determine the accuracy of the measurement in the presence of random statistical errors inherent in light sources of very low intensity. The inversion technique and analysis of errors, although examined with respect to molecular oxygen and ozone, are general and can be used to determine the accuracy of any occultation measurement if the signal level and data-acquisition rate are known.

## 2. OCCULTATION TECHNIQUE

The intensity of a star, as measured by a telescope aboard a satellite, during occultation of the star by the Earth's atmosphere (Fig. 1) is expressed as

$$F^*(r_0) = \varphi \int_0^\infty T(\lambda) I(\lambda, r_0) d\lambda \quad (1)$$

where  $r_0$  is the tangent ray height of the light ray passing tangentially to the Earth's surface,  $\varphi = A Q_e T_0$ ,  $A$  is the aperture area of the telescope,  $Q_e$  is the quantum efficiency of the photo-multiplier,  $T_0$  is the optical transmission of the system,  $T(\lambda)$  is the broadening function or transmission of the dispersive element at wavelength  $\lambda$  and  $I(\lambda, r_0)$  is the intensity of the star at wavelength  $\lambda$  and tangent ray height  $r_0$ .

The intensity of starlight passing through the Earth's upper atmosphere is determined by considering the combined attenuation of all of the absorbing species along the ray path (Beer's law).

$$I(\lambda, r_0) = I_\infty(\lambda) \exp \left( -\sum_i \sigma_i(\lambda) N_i(r_0) \right) \quad (2)$$

where  $I_\infty(\lambda)$  is the stellar intensity above the atmosphere and  $\sigma_i(\lambda)$  is the absorption cross-section of the  $i$ th atmospheric species. In the case of a spherically stratified atmosphere, the tangential column number density  $N_i(r_0)$  is

$$N_i(r_0) = 2 \int_{r_0}^\infty \frac{n_i(r) r dr}{\sqrt{r^2 - r_0^2}} \quad (3)$$

where  $n_i(r)$  is the number density of the  $i$ th absorbing species at a distance  $r$  from the center of the Earth.

Hays and Roble (1968b) calculated the tangential u.v. transmission for the mesosphere and the lower thermosphere. They show that in certain specific wavelength intervals the u.v. starlight is absorbed by a single species. These include a broadband region near 1500 Å where molecular oxygen acts as a single absorber in the lower thermosphere and also a region near 2500 Å where ozone is a single absorber in the mesosphere. In both of these regions, Hays and Roble (1968b) show that refractive attenuation, Rayleigh scattering, and absorption by other minor atmospheric constituents are negligible over most of the altitude range where absorption occurs. In addition, they point out that only the relative intensity is important for the occultation measurement. The stellar occultation data are therefore normalized as

$$F(r_0) = \frac{\int_0^\infty T(\lambda) I_\infty(\lambda) \exp(-\sigma(\lambda) N(r_0)) d\lambda}{\int_0^\infty T(\lambda) I_\infty(\lambda) d\lambda} \quad (4)$$

The interference filter used in the occultation measurements is selected so that  $T(\lambda)$  includes only a wavelength region where absorption is caused by the single species of

interest. By knowing the filter-broadening function, the stellar spectra above the atmosphere, and the absorption cross-section, the normalized signal is then only a function of the tangential column number density  $F(r_0) = F(N(r_0))$ . The intensity data can be inverted to give the tangential column number density  $N(r_0) = N(F(r_0))$ . Equation (3) for the tangential column number density is the Abel integral equation (c.f. Hays and Roble, 1968a, b) which is directly inverted to give the number density of the absorbing species at radius  $r$

$$n_s(r) = -\frac{1}{\pi} \int_r^\infty \frac{[dN(r_0)/dr_0] dr_0}{\sqrt{r_0^2 - r^2}} \quad (5)$$

In the next sections, we develop two separate numerical inversion schemes; one applicable for determining the number density profile of atmospheric species decreasing nearly exponentially with altitude, such as molecular oxygen; and the second for atmospheric species having a more complex number density profile, such as ozone.

### 3. DATA REDUCTION

(a) *Exponential form.* The intensity of a star measured by photometers aboard a satellite during occultation is generally weak and usually obtained in digital form (Hays *et al.*, 1972). At low intensities, pulse-counting systems are used with a finite integration time between data points. Therefore, the intensity data are obtained at a series of discrete tangent ray heights  $F(r_0)$ . The normalized intensities are converted to the tangential column number density of the absorbing species along the ray path  $N(r_0)$  as described in the previous section. Thus, a set of discrete values of the tangential column number density is obtained as a function of tangent ray height during occultation. We approximate the tangential column number density between data points in the interval  $r_i < r < r_{i+1}$  by

$$N(r) = \alpha_i \exp(-\beta_i(r - r_i)) \quad (6)$$

where  $r_i$  and  $r_{i+1}$  are the  $i$  and  $i + 1$  tangent ray heights of the data points, respectively. The coefficients  $\alpha_i$  and  $\beta_i$  are determined from a least squares fit to the data in the vicinity of the  $i$ th data point by minimizing the variance of Equation (6) with the data and smoothing  $M$  adjacent points (c.f. Roble and Norton, 1972).

$$\delta_i = \sum_{j=K_1}^{K_2} (N_j - \alpha_i \exp(-\beta_i(r_j - r_i)))^2 \quad (7)$$

where  $K_1 = i - M/2$ ,  $K_2 = i + M/2$ ,  $N_j$  is the tangential column number density at the  $j$ th data point. Then by differentiating Equation (6) and replacing the integral in Equation (5) by a finite sum, we obtain

$$n(r_i) = \frac{1}{\pi} \sum_{i=i}^{\infty} \alpha_i \beta_i \int_{r_i}^{r_{i+1}} \frac{\exp(-\beta_i(r - r_i))}{\sqrt{r^2 - r_i^2}} dr \quad (8)$$

Expanding the denominator and integrating, we obtain

$$\begin{aligned} n(r_i) = \frac{1}{\pi} \sum_{i=i}^{\infty} \frac{\alpha_i \beta_i}{(r_i + r_i)^{1/2}} & \left\{ \left( 1 + \frac{1}{2} \frac{(r_i - r_i)}{(r_i + r_i)} - \frac{1}{4\beta_i} \frac{1}{(r_i + r_i)} \right) \left( \frac{\pi}{\beta_i} \right)^{1/2} \right. \\ & \times [\operatorname{erf}(\beta_i(r_{i+1} - r_i))^{1/2} - \operatorname{erf}(\beta_i(r_i - r_i))^{1/2}] \exp(\beta_i(r_i - r_i)) \\ & \left. + \frac{1}{2\beta_i(r_i + r_i)} [ (r_{i+1} - r_i)^{1/2} \exp(-\beta_i(r_{i+1} - r_i)) - (r_i - r_i)^{1/2} ] \right\} \quad (9) \end{aligned}$$

where  $\operatorname{erf}(x)$  is the error function of  $x$ .



Thus, the number density of the absorbing species at radius  $r_i$  is determined directly from Equation (9) using the calculated values of  $\alpha_i$  and  $\beta_i$ .

(b) *Quadratic form.* When the absorbing constituent has a number density height profile which is not exponentially decreasing with altitude, but varies over the scan altitude, such as for ozone (Fig. 2), then the exponential scheme developed in the previous section may not be

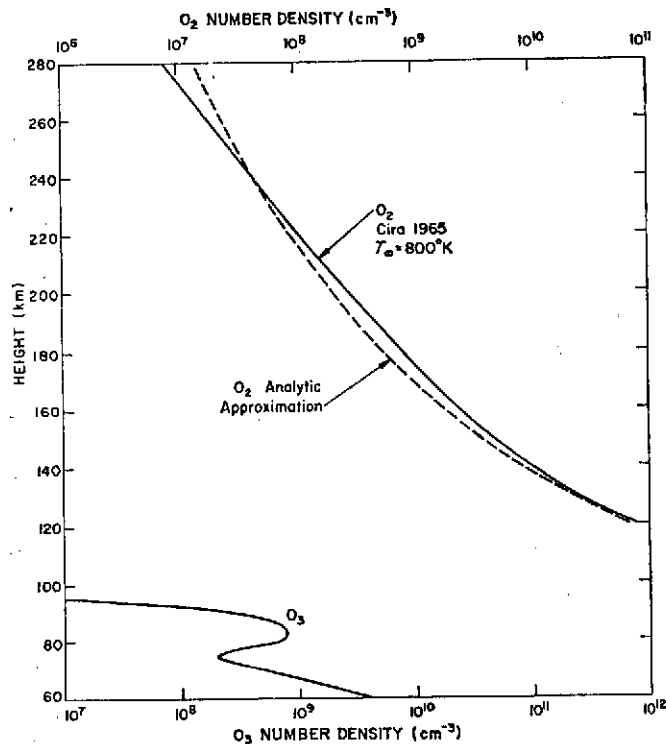


FIG. 2. THE MOLECULAR OXYGEN AND OZONE NUMBER DENSITY AS A FUNCTION OF HEIGHT. THE SOLID CURVE FOR  $O_2$  IS THE NUMBER DENSITY DISTRIBUTION FROM THE MEAN CIRA 1965 MODEL ATMOSPHERE AND THE DASHED CURVE IS AN ANALYTIC APPROXIMATION TO THE DISTRIBUTION. THE  $O_3$  NUMBER DENSITY DISTRIBUTION IS AN ANALYTIC APPROXIMATION OF A NIGHT-TIME PROFILE AND IT IS GIVEN BY EQUATION (17).

best suited for the data-reduction process. In this case, we approximate the tangential column number density in the altitude region  $z_i < z < z_{i+1}$  by the quadratic relationship

$$N(z) = Az^2 + Bz + C \quad (10)$$

where  $z = r - r_0$  and  $r_0$  is the radius of the Earth, 6371 km.

The coefficients  $A$ ,  $B$  and  $C$  are again determined by minimizing the variance of equation (10) with the data and smoothing  $M$  adjacent points

$$\delta_i = \sum_{j=K_1}^{K_2} (N_j - (A_i z_j^2 + B_i z_j + C_i))^2 \quad (11)$$

Minimizing Equation (11) and solving directly for  $A_i$ ,  $B_i$  and  $C_i$  we obtain

$$\begin{aligned} A_i &= \frac{(M(\overline{Nz}) - (\overline{N})(\overline{z}))(M(\overline{z^3}) - (\overline{z})(\overline{z^2})) - (M(\overline{Nz^2}) - (\overline{N})(\overline{z^2}))(M(\overline{z^3}) - (\overline{z})(\overline{z}))}{(M(\overline{z^3}) - (\overline{z^2})(\overline{z}))(M(\overline{z^3}) - (\overline{z})(\overline{z^2})) - (M(\overline{z^4}) - (\overline{z^2})(\overline{z^2}))(M(\overline{z^2}) - (\overline{z})(\overline{z}))} \\ B_i &= \frac{(M(\overline{Nz}) - (\overline{z})(\overline{N})) - (M(\overline{z^3}) - (\overline{z^2})(\overline{z}))A_i}{(M(\overline{z^2}) - (\overline{z})(\overline{z}))} \\ C_i &= \frac{(\overline{N}) - (\overline{z^2})A_i - (\overline{z})B_i}{M} \end{aligned} \quad (12)$$

where

$$(\overline{\quad}) = \sum_{j=K_1}^{K_2} (\quad)_j.$$

As in the previous section, by differentiating Equation (10) and replacing the main integral of equation (5) with a finite sum and then integrating between data points, we obtain

$$\begin{aligned} n(r_i) = -\frac{1}{\pi} \sum_{i=1}^{\infty} \left[ 2A_i((r_{i+1}^2 - r_i^2)^{1/2} - (r_i^2 - r_{i-1}^2)^{1/2}) \right. \\ \left. + (B_i - 2A_i r_i) \log_e \frac{(r_{i+1} + (r_{i+1}^2 - r_i^2)^{1/2})}{(r_i + (r_i^2 - r_{i-1}^2)^{1/2})} \right]. \end{aligned} \quad (13)$$

Thus, the number density of the absorbing species as radius  $r_i$  is determined from equation (13) using the coefficients  $A_i$  and  $B_i$  calculated for each data point.

#### 4. ATMOSPHERIC MODEL

The molecular oxygen number density profile in the lower thermosphere, obtained from the mean CIRA 1965 model atmosphere, is shown in Fig. 2. For mathematical simplicity, we approximate this profile by an atmosphere with a constant scale height gradient (Nicolet, 1960),

$$H = H_s + \beta(r - r_s) \quad (14)$$

where  $H_s$  is the scale height of the atmosphere at radius  $r_s$ , and  $\beta$  is the vertical gradient of the scale height. The molecular oxygen number density is then represented as

$$n(r) = n_s \left[ \frac{H_s}{H_s + \beta(r - r_s)} \right]^{(1+\beta)/\beta} \quad (15)$$

where  $n_s = 2 \times 10^{12} \text{ cm}^{-3}$ ,  $r_s = 6471 \text{ km}$ ,  $H_s = 5 \text{ km}$ , and  $\beta = 0.23$  to give the approximation to the CIRA 1965 molecular oxygen distribution shown in Fig. 2. The tangential column number density of molecular oxygen along the ray path is determined by inserting Equation (15) into Equation (3), expanding the denominator, and integrating to obtain

$$\begin{aligned} N(r_0) = \left(\frac{2}{r_0}\right)^{1/2} n_s \left[ \frac{H_s}{H_s + \beta(r_0 - r_s)} \right]^{1/\delta} \left\{ \left[ \frac{\beta}{H_s + \beta(r_0 - r_s)} \right]^{-3/2} B\left(\frac{3}{2}, \left(\frac{1}{\delta} - \frac{3}{2}\right)\right) \right. \\ \left. + r_0 \left[ \frac{\beta}{H_s + \beta(r_0 - r_s)} \right]^{-1/2} B\left(\frac{1}{2}, \left(\frac{1}{\delta} - \frac{1}{2}\right)\right) \right\} \end{aligned} \quad (16)$$

where  $\delta = \beta/(1 + \beta)$  and  $B(x, y)$  is the Beta function,  $B(x, y) = \gamma(x)\gamma(y)/\gamma(x+y)$  and  $\gamma(x)$  is the Gamma function. The normalized intensity as a function of tangent ray height is calculated from Equation (2) and is shown in Fig. 3 for absorption at a single wavelength

1450 Å in the Schumann-Runge continuum where the molecular oxygen absorption coefficient is approximately  $2 \times 10^{-17} \text{ cm}^2$ .

Hays *et al.* (1972) presented a typical night-time ozone distribution determined from occultation measurements made by the Orbiting Astronomical Observatory (OAO-2). The

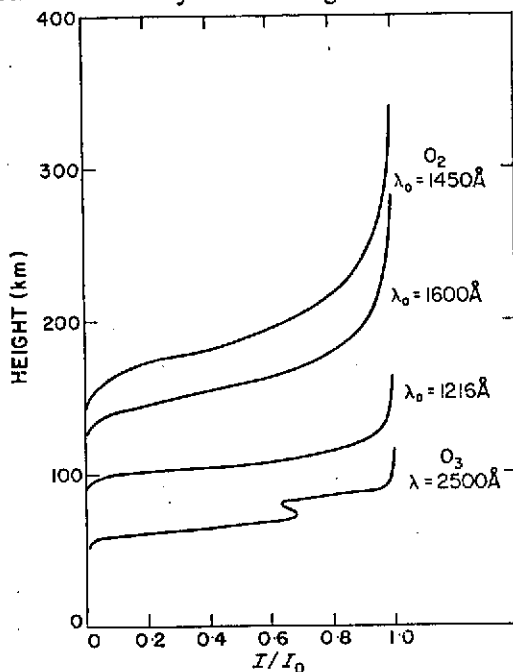


FIG. 3. THE NORMALIZED TANGENTIAL TRANSMISSION OF THE EARTH'S ATMOSPHERE AT VARIOUS WAVELENGTHS AS A FUNCTION OF HEIGHT.

ozone number density profile given in their paper can be reasonably represented as a sum of exponential and Gaussian functions

$$n(r) = n_b \exp(-(r - r_b)/H_b) + n_c \exp(-((r - r_c)/H_c)^2) \quad (17)$$

where  $n_b = 5 \times 10^{10} \text{ cm}^{-3}$ ,  $r_b = 50 \text{ km}$ ,  $H_b = 4.34 \text{ km}$ ,  $n_c = 8 \times 10^8 \text{ cm}^{-3}$ ,  $r_c = 83 \text{ km}$ , and  $H_c = 5 \text{ km}$ . The ozone number density distribution obtained from equation (17) is shown in Fig. 2.

The coefficients used in this study give an ozone number density slightly higher than those for a typical night-time ozone distribution determined by Hays *et al.* (1972). This difference, however, is not important because the profiles are similar and the main features of the ozone analysis are preserved in either case.

The total columnar number density along a ray path is obtained by inserting equation (17) into equation (3), expanding the denominator, and integrating to give

$$\begin{aligned} N(r_0) = & 2n_b \exp(r_b/H_b) r_0 K_1(r_0/H_b) + 2n_c \exp(-((r_0 - r_c)/H_c)^2) \\ & \times (2r_0)^{-1/2} \left( \frac{2}{H_c^2} \right)^{-3/4} \gamma\left(\frac{3}{2}\right) \exp((r_0 - r_c)^2/(2H_c^2)) \\ & \times D_{-3/2}((2)^{1/2} H_c^{-1}(r_0 - r_c)) + r_0 \left( \frac{2}{H_c^2} \right)^{-1/4} \gamma\left(\frac{1}{2}\right) \\ & \times \exp((r_0 - r_c)^2/(2H_c^2)) D_{-1/2}((2)^{1/2} H_c^{-1}(r_0 - r_c)) \end{aligned} \quad (18)$$

where  $K_1(x)$  is the modified Bessel function of the first kind,  $\gamma(x)$  is the Gamma function, and  $D_{-1}(x)$  is the parabolic cylinder function. The normalized intensity as a function of tangent ray height is obtained from Equation (18) and Equation (2), and is shown in Fig. 3 for absorption at a single wavelength in the Hartley continuum of ozone at 2500 Å. At this wavelength, ozone has an absorption cross-section of about  $10^{-17}$  cm<sup>-2</sup>.

### 5. SAMPLING ERROR

The accuracy in retrieving the number density profile with the data-reduction scheme depends upon (a) the data-acquisition rate or number of data points per scan, (b) the statistical noise inherent in a signal of low intensity, (c) the smoothing parameter  $M$  used in the inversion process, (d) knowledge of the satellite position during occultation, and (e) departures from spherical symmetry. The first three items are examined in this paper. Item (d) depends on the satellite tracking network and the degree of accuracy of the orbital

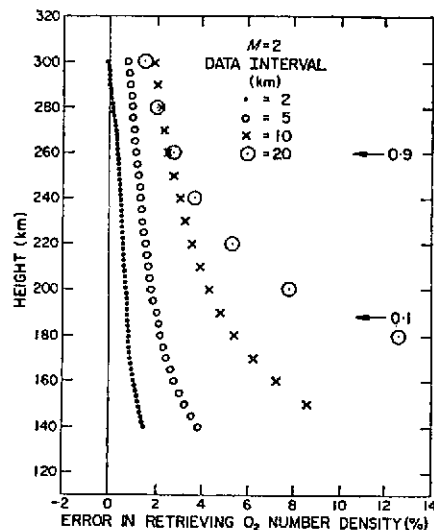


FIG. 4. THE ERROR IN RETRIEVING THE ANALYTIC MOLECULAR OXYGEN NUMBER DENSITY AS A FUNCTION OF HEIGHT FOR VARIOUS DATA-ACQUISITION INTERVALS.

elements. The intensity of the star is measured as a function of time, but by knowing the star's position and the satellite orbital elements, we can relate the intensity to the tangent ray height of the occulting star. Thus, errors in satellite time and position appear as errors in the tangent ray height and will not be discussed in this paper. The last item may be important for certain atmospheric species which change rapidly with time, as perhaps during sunrise or sunset; however, in this analysis, spherical symmetry is assumed.

The error in retrieving the molecular oxygen number density profile using the exponential data-reduction scheme is plotted as a function of height in Fig. 4 for various data-acquisition rates. The tangential column number density for the analytic molecular oxygen distribution is calculated at the various altitudes using Equation (16). These data are inverted using the exponential data-inversion scheme, and the retrieved molecular oxygen number density is compared to the number density given by the analytic model. The errors in retrieving the number density are then plotted in Fig. 4. The results show that the errors are smallest when a large number of data points per occultation scan are used in the

inversion process. The smoothing parameter  $M$  is also a factor in the retrieval accuracy. The smoothing parameter is primarily used to smooth the analytic representation of the tangential column number density through noisy data points and its importance will be evident in the next section. But for noise-free data, an increase in the smoothing parameter decreases the retrieval accuracy for a number density profile having a slight curvature.

At high altitudes, the retrieval accuracy would decrease due to the practical requirement of truncating the inversion integral at an upper boundary. It is difficult to convert the normalized intensity to tangential column number density for the upper part of the scan where the normalized intensity approaches 1.0, especially in the presence of statistical noise.

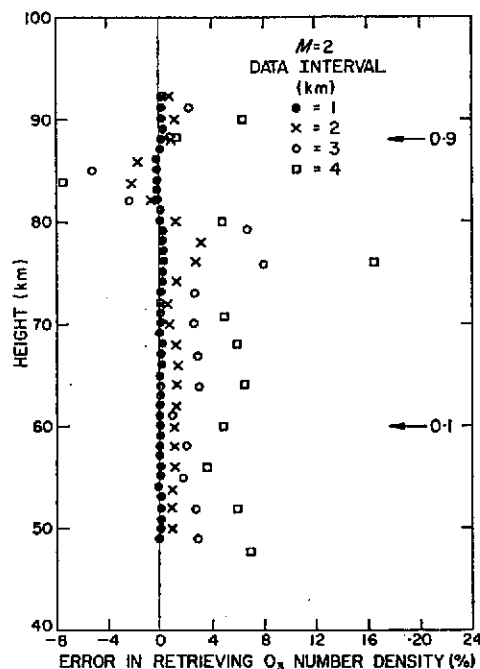


FIG. 5. THE ERROR IN RETRIEVING THE ANALYTIC OZONE NUMBER DENSITY AS A FUNCTION OF HEIGHT FOR VARIOUS DATA-ACQUISITION INTERVALS.

To reduce this error and extend the upper limit of the occultation scan, it is convenient to utilize a model atmosphere and calculate the tangential column number beyond the uppermost data point for use in the inversion integral.

The error in retrieving the model ozone number density profile using the analytic expression for the tangential column number density given by Equation (18) and also using the quadratic data-reduction scheme is shown in Fig. 5. The retrieval is good for a high data-acquisition rate, but as the data-acquisition rate decreases, the accuracy in defining the vertical structure and in particular the ozone bulge near 83 km also decreases. The lowest data-acquisition rate considered in Fig. 5 shows considerable error in retrieving the ozone bulge. These results were obtained using a smoothing parameter of  $M = 2$ . As the smoothing parameter is increased, the accuracy of retrieving the vertical structure depends critically on the data-acquisition rate. For a particular application, there is a tradeoff between the data-acquisition rate and the smoothing parameter.

For simplicity, the results presented here assume absorption at a single wavelength.

However, the technique which has been developed is general and can be applied to any broadband filter by including a wavelength integration.

### 6. THEORETICAL ANALYSIS OF STATISTICAL ERRORS

For occultation scans of very low intensity, the statistical noise which is superimposed upon the basic signal intensity becomes an appreciable part of the signal. The errors in retrieving the number density profile of the absorbing species from occultation data having random statistical noise are considered in this section.

(a) *Molecular oxygen*. The solid curve in Fig. 6 shows the calculated normalized intensity variation of a star during occultation using Equations (16) and (2). The peak signal

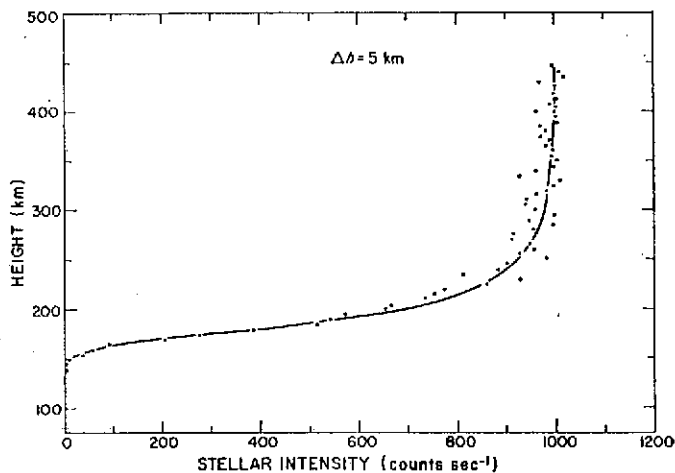


FIG. 6. AN OCCULTATION SCAN IN THE SCHUMANN-RUNGE CONTINUUM OF MOLECULAR OXYGEN AT 1450 Å WITH RANDOMLY SCATTERED STATISTICAL NOISE SUPERIMPOSED. THE STANDARD DEVIATION OF THE NOISE IS PROPORTIONAL TO THE SQUARE ROOT OF THE BASIC SIGNAL LEVEL.

level in this example is approximately 1000 counts sec<sup>-1</sup> and the calculations are made at a single wavelength of 1450 Å. Also shown in Fig. 6 is the basic scan with a superimposed Poisson noise distribution having a standard deviation proportional to the square root of the signal level. The molecular oxygen number density profiles shown in Fig. 7 for the two scans given in Fig. 6 have been retrieved using the data-reduction scheme described in the previous sections. Other statistical noise distributions yield an ensemble of retrieved molecular oxygen number density profiles which can be statistically analyzed.

If we assume that the tangential column number density is  $N = N_0 + \Delta N$  where  $N_0$  is the basic tangential column number density without statistical noise and  $\Delta N$  is the tangential column number density error due to the statistical noise, then in the approximation given by Equation (6) we expand  $\alpha$  and  $\beta$  as

$$\begin{aligned}\alpha &= \alpha_0 + \Delta\alpha \\ \beta &= \beta_0 + \Delta\beta.\end{aligned}\tag{19}$$

Inserting these relationships into Equation (7), expanding the exponential, and subtracting the basic unscattered state, we obtain

$$\delta_i^* = \sum_{j=K_1}^{K_2} [\Delta N_j - \Delta\alpha_j \exp(-\beta_{0j}(z_j - z_i)) + \alpha_{0j} \Delta\beta_j(z_j - z_i) \exp(-\beta_{0j}(z_j - z_i))]^2.\tag{20}$$

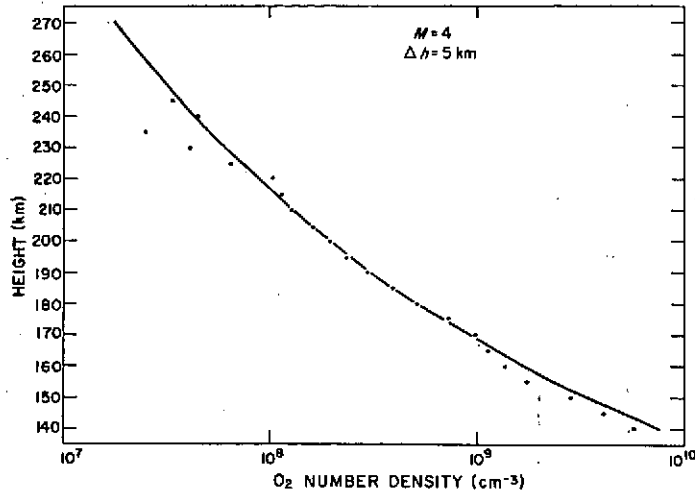


FIG. 7. THE RETRIEVED MOLECULAR OXYGEN NUMBER DENSITY PROFILE FOR THE OCCULTATION SCAN WITH RANDOMLY SCATTERED NOISE SHOWN IN FIG. 6. THE SOLID CURVE GIVES THE ANALYTIC MOLECULAR OXYGEN NUMBER DENSITY DISTRIBUTION.

The coefficient errors  $\Delta\alpha_i$  and  $\Delta\beta_i$  are calculated in the least squares sense from the relationships

$$\frac{\partial \delta_i^*}{\partial \Delta\alpha_i} = \frac{\partial \delta_i^*}{\partial \Delta\beta_i} = 0 \quad (21)$$

and are given in the Appendix.

The intensity of the star, considering single wavelength absorption, is calculated from Beer's law

$$F_0 = F(\lambda, r_0) = F_\infty(\lambda) \exp(-\sigma(\lambda)N(r_0)) \quad (22)$$

where  $F_\infty(\lambda)$  is the signal measured above the atmosphere in counts  $\text{sec}^{-1}$ . The linear variation in tangential column number density due to a signal with statistical noise  $F = F_0 + \Delta F$  is obtained by introducing the value of  $N$  into Equation (22) and expanding the exponential to obtain

$$\Delta N = -\frac{1}{\sigma(\lambda)} \frac{\Delta F}{F(\lambda, r_0)}. \quad (23)$$

If a Poisson random noise with a standard deviation for  $\Delta F$  proportional to the square root of the basic signal ( $F_0$ )<sup>1/2</sup> is introduced into the expressions for the coefficients  $\Delta\alpha_i$  and  $\Delta\beta_i$ , the mean square errors for the coefficients  $\langle \Delta\alpha_i \rangle$  and  $\langle \Delta\beta_i \rangle$  are determined using the propagation of statistical errors theorem and they are presented in the Appendix.

The errors in the number density of the absorbing species at the tangent ray point are determined from the integral

$$\Delta n(r_t) = -\frac{1}{\pi} \int_{r_t}^{\infty} \frac{[d \Delta N/dr] dr}{\sqrt{r^2 - r_t^2}}. \quad (24)$$

The tangential column number density of the absorbing species is determined by inserting Equation (19) into Equation (6). The noise in the number density of the absorbing

species at the tangent ray point is obtained by expanding the exponential, subtracting the basic state, and differentiating the tangential column number density error  $\Delta N$  to give

$$\Delta n(r_i) = \frac{1}{\pi} \sum_{i=1}^{\infty} (\beta_{0i} \Delta \alpha_i + \alpha_{0i} \Delta \beta_i) \int_{r_i}^{r_{i+1}} \frac{\exp(-\beta_{0i}(r-r_i))}{\sqrt{r^2-r_i^2}} dr - \frac{1}{\pi} \sum_{i=1}^{\infty} \alpha_{0i} \beta_{0i} \Delta \beta_i \int_{r_i}^{r_{i+1}} \frac{(r-r_i) \exp(-\beta_{0i}(r-r_i))}{\sqrt{r^2-r_i^2}} dr. \quad (25)$$

Integrating, we obtain

$$\Delta n(r_i) = \frac{1}{\pi} \sum_{i=1}^{\infty} (R_i \Delta \alpha_i + S_i \Delta \beta_i) \quad (26)$$

where  $R_i$  and  $S_i$  are coefficients given in the Appendix. The standard deviation of the number density error at the tangent ray point due to statistical noise is calculated using the propagation of statistical errors theorem

$$\langle \Delta n(r_i) \rangle = T_i F_{\infty}^{-1/2} \quad (27)$$

where

$$T_i = \frac{1}{\pi} \left[ \sum_{i=1}^{\infty} (R_i X_i + S_i Y_i)^2 \right]^{1/2}$$

and  $X_i$  and  $Y_i$  are given in the Appendix.

(b) *Ozone*. The errors in the ozone number density caused by statistical noise superimposed upon the basic signal are determined from the quadratic data-reduction scheme. The signal intensity variations are related to the tangential column number density variations through Equation (23) and the number density error of ozone at the tangent ray point is obtained from Equation (24). The tangential column number density with superimposed variations caused by statistical noise is  $N = N_0 + \Delta N$ . The coefficients in Equation (10) are assumed to have a variation  $A_i = A_{0i} + \Delta A_i$ ,  $B_i = B_{0i} + \Delta B_i$ , and  $C_i = C_{0i} + \Delta C_i$ . Introducing these variations into Equation (11) and subtracting the basic state, we have

$$\delta_i^* = \sum_{j=K_1}^{K_2} (\Delta N_j - \Delta A_i z_j^2 - \Delta B_i z_j - \Delta C_i)^2. \quad (28)$$

Solving for the coefficients  $\Delta A_i$ ,  $\Delta B_i$  and  $\Delta C_i$ , we obtain relationships similar to Equation (12) with the exception that  $N_j$  is replaced by  $\Delta N_j$ ,

$$\Delta A_i = \sum_{j=K_1}^{K_2} Q_j \Delta N_j, \quad (29)$$

$$\Delta B_i = \sum_{j=K_1}^{K_2} Q_j^* \Delta N_j,$$

where  $Q_j$  and  $Q_j^*$  are given in the Appendix. The number density error at the tangent ray point is obtained by inserting Equation (29) into Equation (24) and integrating to give

$$\Delta n(r_i) = -\frac{1}{\pi} \sum_{i=1}^{\infty} \left[ \sum_{j=K_1}^{K_2} ((2W_{1i} - 2r_i W_{2i}) Q_j + W_{2i} Q_j^*) \cdot \frac{\Delta F_j}{\sigma F_{0j}} \right] \quad (30)$$

where

$$W_{1i} = (r_{i+1}^2 - r_i^2)^{1/2} - (r_i^2 - r_i^2)^{1/2}$$

$$W_{2i} = \log_e \left( \frac{r_{i+1} + (r_{i+1}^2 - r_i^2)^{1/2}}{r_i + (r_i^2 - r_i^2)^{1/2}} \right).$$



The standard deviation of the number density error at the tangent ray point is calculated using the propagation of statistical errors theorem and assuming that the standard deviation of the noise is proportional to the square root of the basic signal

$$\langle \Delta n(r_i) \rangle = E_i \cdot F_\infty^{-1/2} \tag{31}$$

where

$$E_i = \frac{1}{\pi} \left\{ \sum_{j=1}^{\infty} \sum_{k=1}^{K_1} \left[ [(2W_{1i} - 2r_s W_{2i}) Q_j + W_{2i} Q_j^*] \frac{1}{\sigma} \left( \frac{F_\infty}{F_{0j}} \right)^{1/2} \right]^2 \right\}^{1/2}$$

7. ANALYSIS OF STATISTICAL ERRORS

The error in retrieving the molecular oxygen number density from occultation scans having a random statistical noise superimposed upon the basic signal is defined as

$$e(r_i) = \frac{\langle \Delta n(r_i) \rangle}{n(r_i)} \tag{32}$$

The numerator is the standard deviation of the retrieved number density, given by equation (27) for molecular oxygen and equation (31) for ozone, and  $n(r_i)$  is the number density at radius  $r_i$ . The standard deviation of the retrieved number density is inversely proportional to the square root of the unattenuated signal level for both molecular oxygen and ozone. Therefore, in Fig. 8  $T_i(n(r_i))^{-1}$  is plotted as a function of altitude for various smoothing parameters  $M$ . The error in retrieving the molecular oxygen number density for

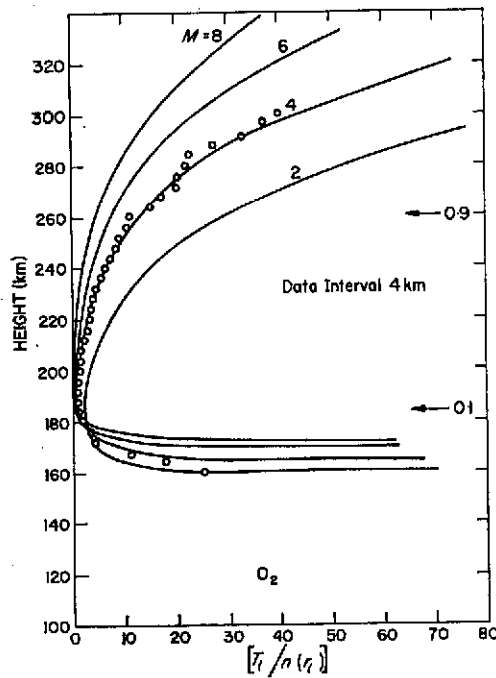


FIG. 8. THEORETICAL ANALYSIS OF ERRORS IN RETRIEVING THE MOLECULAR OXYGEN NUMBER DENSITY FROM OCCULTATION SCANS WITH RANDOM STATISTICAL NOISE SUPERIMPOSED. THE RETRIEVAL ERROR IS OBTAINED BY DIVIDING THE PARAMETER  $T_i(n(r_i))^{-1}$  BY  $F_\infty^{1/2}$ , THE SQUARE ROOT OF THE UNATTENUATED BASIC SIGNAL LEVEL. THE CIRCLES REPRESENT THE RESULTS OF A NUMERICAL ANALYSIS OF ERRORS FOR  $M = 4$ .

any signal level is obtained by dividing the parameter  $T_i(n(r_i))^{-1}$  by  $F_{\infty}^{1/2}$ . An increase in the smoothing parameter  $M$  decreases the error in retrieving the molecular oxygen number density. However, as discussed earlier, the accuracy of retrieving the basic molecular oxygen number density profile from unscattered occultation scans decreases as the smoothing parameter  $M$  increases. Therefore, there exists a value of  $M$  which gives a minimum in the overall error in retrieving the molecular oxygen number density profile at a given data-acquisition interval. The results of a numerical analysis of errors are shown in Fig. 8 for 100 occultation scans with different randomly scattered noise distributions. There is general

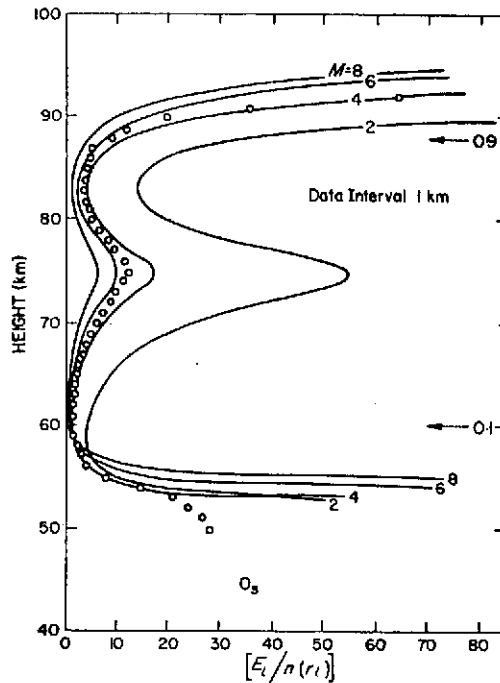


FIG. 9. THEORETICAL ANALYSIS OF ERRORS IN RETRIEVING THE OZONE NUMBER DENSITY FROM OCCULTATION SCANS WITH RANDOM STATISTICAL NOISE SUPERIMPOSED. THE RETRIEVAL ERROR IS OBTAINED BY DIVIDING THE PARAMETER  $E_i(n(r_i))^{-1}$  BY  $F_{\infty}^{1/2}$ , THE SQUARE ROOT OF THE UNATTENUATED BASIC SIGNAL LEVEL. THE CIRCLES REPRESENT THE RESULTS OF A NUMERICAL ANALYSIS OF ERRORS FOR  $M = 4$ .

agreement between the results of the theoretical and numerical analysis of errors. Also shown in the figure are arrows indicating the altitudes of the 0.1 and 0.9 normalized intensity points. The retrieval of the molecular oxygen number density is best for data lying between these normalized intensity ratios.

The standard deviation of the retrieved ozone number density is also inversely proportional to the square root of the unattenuated signal level. Therefore, in Fig. 9 the parameter  $E_i(n(r_i))^{-1}$  is plotted as a function of height for various smoothing parameters. In general, the error in retrieving the ozone bulge and the number density below about 70 km is low. However, in the region of the ozone minimum near 75 km the retrieval error increases. The data-acquisition interval in this case is 1 km and an increase in the smoothing parameter  $M$  decreases the error in retrieving the ozone number density. As the data-acquisition interval increases, the error in retrieving the ozone number density increases.

For a given data-acquisition interval, there also is a value of the smoothing parameter  $M$  which gives a minimum in the overall error in retrieving the ozone number density. The circles in Fig. 9 again are the results of a numerical analysis of the errors for 100 ozone occultation scans with different randomly scattered noise distributions. The general agreement between the theoretical and numerical analysis of errors enables us to theoretically calculate the error coefficients and thus determine accuracy of an occultation scan for any signal intensity.

#### 8. DISCUSSION

We have presented a numerical technique for retrieving the number density profile of absorbing atmospheric gases from planetary occultation data having random statistical noise superimposed upon the basic signal. Because of the sensitivity of the Abel integral equation to random noise, some smoothing is required before evaluating the inversion integral. An exponential technique was developed for atmospheric species whose number density profile decreases exponentially with height, whereas the quadratic scheme works better for atmospheric species having a more complex vertical profile. These techniques have been used to reduce the OAO-2 stellar occultation data and obtain the night-time molecular oxygen and ozone distribution in the upper atmosphere (Hays *et al.*, 1972). With a modification to account for the finite size of the solar disk, these techniques have also been used to reduce the SOLRAD-8 solar occultation data (Roble and Norton, 1972). The retrieval errors are determined by knowing the approximate signal level and acquisition rate of the occultation data. For each specific case, there is a minimum in the retrieval error determined by the degree of smoothing for the appropriate data-acquisition rate and basic signal level. Generally, the best accuracy is achieved between the normalized intensity limits of 0.1–0.9 where the derivative in the numerator of the Abel integral equation changes rapidly. Figure 2 shows that by selecting the appropriate spectrally isolated wavelength interval, the number density distribution at practically any altitude may be obtained in the region where the tangential column number density is approximately equal to the inverse of the effective cross-section.

The analysis was performed for single wavelength absorption, whereas measurements are made for some finite passband determined by the characteristics of the monitoring spectrometer or photometer. In this case, an additional integration over wavelength is required in the analysis, but it does not fundamentally alter the results as long as the absorption process is spectrally isolated. Actually, a wavelength interval selected to cover a continuum region where the absorption cross-section is smoothly increasing or decreasing allows deepest penetration into the atmosphere. The altitude range of the occultation data is greater in this case than for the case of a constant cross-section over the selected wavelength interval.

*Acknowledgments*—We are grateful for the assistance of Messrs. K. Hansen, J. Hastings and M. Graves who provided the numerical calculations and plotting. A. Lundberg typed the manuscript.

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APPENDIX

Several of the coefficients discussed in the text are presented in this Appendix.

$$\Delta\alpha_i = \frac{\sum_{j=K_1}^{K_2} \left[ \frac{\exp(-\beta_{0i}(z_j - z_i))}{\{\bar{z}_i^1\}} - \frac{(z_j - z_i) \exp(-\beta_{0i}(z_j - z_i))}{\{\bar{z}_i^2\}} \right] \Delta N_j}{\frac{\{\bar{z}_i^0\}}{\{\bar{z}_i^1\}} - \frac{\{\bar{z}_i^1\}}{\{\bar{z}_i^2\}}}$$

$$\Delta\beta_i = \frac{1}{\alpha_{0i}} \left\{ \frac{\sum_{j=K_1}^{K_2} \left[ \frac{\exp(-\beta_{0i}(z_j - z_i))}{\{\bar{z}_i^0\}} - \frac{(z_j - z_i) \exp(-\beta_{0i}(z_j - z_i))}{\{\bar{z}_i^1\}} \right] \Delta N_j}{\frac{\{\bar{z}_i^2\}}{\{\bar{z}_i^1\}} - \frac{\{\bar{z}_i^1\}}{\{\bar{z}_i^0\}}}$$

where

$$\{\bar{z}_i^0\} = \sum_{j=K_1}^{K_2} \exp(-2\beta_{0i}(z_j - z_i))$$

$$\{\bar{z}_i^1\} = \sum_{j=K_1}^{K_2} (z_j - z_i) \exp(-2\beta_{0i}(z_j - z_i))$$

$$\{\bar{z}_i^2\} = \sum_{j=K_1}^{K_2} (z_j - z_i)^2 \exp(-2\beta_{0i}(z_j - z_i))$$

$$\overline{\langle \Delta\alpha_i \rangle} = \frac{X_i}{F_\infty^{1/2}}$$

where

$$X_i = \left[ \sum_{j=K_1}^{K_2} \left\{ \frac{\left[ \frac{\exp(-\beta_{0i}(z_j - z_i))}{\{\bar{z}_i^1\}} - \frac{(z_j - z_i) \exp(-\beta_{0i}(z_j - z_i))}{\{\bar{z}_i^2\}} \right] \frac{1}{\sigma} \left( \frac{F_\infty}{F_{0j}} \right)^{1/2}}{\frac{\{\bar{z}_i^0\}}{\{\bar{z}_i^1\}} - \frac{\{\bar{z}_i^1\}}{\{\bar{z}_i^2\}}} \right\}^2 \right]^{1/2}$$

and

$$\overline{\langle \Delta\beta_i \rangle} = \frac{Y_i}{F_\infty^{1/2}}$$

where

$$Y_i = \left[ \frac{1}{\alpha_{0i}} \sum_{j=K_1}^{K_2} \left\{ \frac{\left[ \frac{\exp(-\beta_{0i}(z_j - z_i))}{\{\bar{z}_i^0\}} - \frac{(z_j - z_i) \exp(-\beta_{0i}(z_j - z_i))}{\{\bar{z}_i^1\}} \right] \frac{1}{\sigma} \left( \frac{F_\infty}{F_{0j}} \right)^{1/2}}{\frac{\{\bar{z}_i^2\}}{\{\bar{z}_i^1\}} - \frac{\{\bar{z}_i^1\}}{\{\bar{z}_i^0\}}} \right\}^2 \right]^{1/2}$$

$$R_i = \beta_{0i} \epsilon^{-1/2} \left\{ \left( 1 + \frac{1}{2} \frac{\Delta}{\epsilon} - \frac{1}{4\beta_{0i}\epsilon} \right) \left( \frac{\pi}{\beta_{0i}} \right)^{1/2} [\operatorname{erf}((\beta_{0i}\gamma)^{1/2}) - \operatorname{erf}((\beta_{0i}\Delta)^{1/2})] \exp(\beta_{0i}\Delta) \right. \\ \left. + \frac{1}{2\beta_{0i}\epsilon} [\gamma^{1/2} \exp(-\beta_{0i}\eta) - \Delta^{1/2}] \right\}$$

where  $\Delta = r_i - r_i$

$$\gamma = r_{i+1} - r_i$$

$$\epsilon = r_i + r_i$$

$$\eta = r_{i+1} - r_i$$

$$\begin{aligned}
S_i = & \left( \frac{\pi}{\beta_{0i}} \right)^{1/2} [\operatorname{erf}((\beta_{0i}\gamma)^{1/2}) - \operatorname{erf}((\beta_{0i}\Delta)^{1/2})][\alpha_{0i}\epsilon^{-1/2}(1 + \frac{1}{2}\Delta\epsilon^{-1} - (4\beta_{0i}\epsilon)^{-1}) \\
& - \alpha_{0i}\beta_{0i}\epsilon^{-1/2}((4\beta_{0i})^{-1}(2 + \Delta\epsilon^{-1}) - \frac{1}{2}(2\Delta + \Delta^2\epsilon^{-1}) - \frac{3}{8}\beta_{0i}^{-2}\epsilon^{-1})] \\
& \cdot \exp(\beta_{0i}\Delta) + (2\beta_{0i})^{-1}[y^{1/2}\exp(-\beta_{0i}\eta) - \Delta^{1/2}] \\
& \cdot [\alpha_{0i}\epsilon^{-3/2} + \alpha_{0i}\beta_{0i}\epsilon^{-1/2}(2 + 2\Delta\epsilon^{-1})] \\
& - \alpha_{0i}\beta_{0i}(4\beta_{0i}^2\epsilon^{3/2})^{-1}[y^{1/2}(2\beta_{0i}\gamma + 3)\exp(-\beta_{0i}\eta) - \Delta^{1/2}(2\beta_{0i}\Delta + 3)]
\end{aligned}$$

$$Q_i = \left[ \left( \frac{P}{K} \bar{z}^2 - \frac{L(\bar{z})}{K} \right) + \frac{LM}{K} z_i - \frac{PM}{K} z_i^2 \right]$$

$$Q_i^* = \left[ \left( \frac{L^2}{PK} (\bar{z}) - \frac{(\bar{z})}{P} - \frac{L}{K} \bar{z}^2 \right) + \left( \frac{M}{P} - \frac{L^2M}{PK} \right) z_i + \frac{LM}{K} z_i^2 \right]$$

$$L = (M(\bar{z}^3) - (\bar{z})(\bar{z}^2))$$

$$P = (M(\bar{z}^2) - (\bar{z})(\bar{z}))$$

$$K = [(M(\bar{z}^3) - (\bar{z})(\bar{z}^2))(M(\bar{z}^3) - (\bar{z})(\bar{z}^2)) - (M(\bar{z}^2) - (\bar{z})(\bar{z}))(\bar{z})(\bar{z})].$$

## Appendix III a.

This Appendix gives some details of data processing which is employed to obtain molecular oxygen profiles and ozone profiles from occultation data. The details are divided into three parts, or processes, dealing with (1) the computation of tangential column number density for molecular oxygen,  $N(O_2, r_o)$ ; (2) the computation of tangential column number density for ozone,  $N(O_3, r_o)$ ; and (3) the retrieval of number density of either gas. Computer programs with accompanying notes are included.

### Process 1

To obtain  $N(O_2, r_o)$ , the raw modulus 256 occultation data on the strip chart or digital printout are converted to normalized signal intensity by a program filed in "PLOT". The input to this program is the sequence of counts taken at an interval of  $\frac{1}{2}$  sec spacecraft time. The measure of intensity which is used is the difference between successive counts. The output consists of a graph of normalized signal intensity v. time and the corresponding punched cards.

Following the conversion of the data to normalized intensity, a program filed in "OAOPR" is activated to complete the process. First, the ray tangent point location is computed. Then an interpolation table is formed consisting of normalized signal intensity v. tangential column number density. The table is then used to find  $N(O_2, r_o)$  corresponding to the normalized, measured signal intensity.

### Process 2

To obtain  $N(O_3, r_o)$ , the raw occultation data on the strip chart or digital printout are converted to normalized signal intensity by the "PLOT" program, and the "OAOPR" program is activated as for  $O_2$ . However, in this case corrections to  $N(O_3, r_o)$  are made for the effects of the  $O_2$  Herzberg bands and Rayleigh scattering upon ozone.

### Process 3

Once the tangential column number density,  $N$ , is computed for all  $m$  data levels, a parabolic fit is forced upon  $N(h)$  and the smoothed values are then inverted by the finite differencing scheme,

$$\begin{aligned}n(r_j) &= -\frac{2}{\pi} \frac{d}{dr} \int_r^\infty \frac{r}{r_0} (r_0^2 - r^2)^{-\frac{1}{2}} N(r_0) dr_0 \\ &= -\frac{2}{\pi} \int_r^\infty (r_0^2 - r^2)^{-\frac{1}{2}} \frac{dN(r_0)}{dr_0} dr_0\end{aligned}$$

Details of the smoothing technique and inversion for various species is given in the attached paper on inversion techniques.



\$SIG SAFF C=510 'MAURICE GRAVES'  
 \*\*LAST SIGNON WAS: 09:36.59 12-29-71  
 USER "SAFF" SIGNED ON AT 21:18.13 ON 12-29-71  
 \$LIST PLOT

```

1      DIMENSION AY(150),Y(150)
2      DIMENSION BY(150)
2.05   DIMENSION YINT(150)
2.1    READ(5,861) (BY(K),K=1,80)
2.15   PRINT 861, (BY(K),K=1,80)
3      READ(5,894) N,MAX
3.5    PRINT 894, N,MAX
4      READ 893, (AY(I),I=1,N)
4.1    LMT=5
4.2    PRINT 893, (AY(I),I=1,N)
5      DO 882 K=1,LMT
6      882 Y(K)=AY(K)
6.1    JA=LMT-1
6.2    DO 1 K=1,JA
6.3    1 YINT(K)=Y(K)-Y(K+1)
7      J=N-LMT
7.1    WRITE(6,861) (BY(K),K=1,80)
8      WRITE(6,899)
9      899 FORMAT(20X,8HEXPECTED,9X,6HACTUAL,14X,10HDIFFERENCE,10X,
10     18HRECORDED,12X,9HINTENSITY,/,22X,5HVALUE,9X,5HVALUE,35X,
11     25HVALUE//)
11.1   PZ=0.15*MAX
12     DO 883 K=1,J
12.1   Y(K)=AY(K)
12.2   DO 2 MP=1,JA
12.3   2 Y(K+MP)=Y(K+MP-1)-YINT(K+MP-1)
12.41  YB=(Y(K)+Y(K+1)+Y(K+2)+Y(K+3)+Y(K+4))*0.2
12.42  AI=(Y(K)-2.*Y(K+1)+2.*Y(K+3)-Y(K+4))/(-12.)
12.43  BI=(4.*Y(K)+Y(K+1)+Y(K+3)+4.*Y(K+4)-10.*YB)/14.
12.44  CI=(Y(K)-8.*Y(K+1)+8.*Y(K+3)-Y(K+4))/12.
12.45  B=(-9.*AI)+BI
12.46  C=(27.*AI)-(6.*BI)+CI
12.47  D=- (27.*AI)+9.*BI-3.*CI-2.*BI+YB
12.5   P1=216*AI+36*B+6*C+D
12.52  L=K+2
13     A=(Y(L)-2.*Y(L+1)+Y(L+2))/2.
14     B=(-5.*Y(L)+8.*Y(L+1)-3.*Y(L+2))/2.
15     C=3.*Y(L)-3.*Y(L+1)+Y(L+2)
20     P=16.*A+4.*B+C
20.2   P=(P+P1)/2.
21     X=AY(K+LMT)
22     L=0
22.1   PRINT 3,K,J,X,P,P1
22.2   3 FORMAT(T2,2I5,3F20.6)
23     886 IF (ABS(X-P)-160.) 884,885,885
24     884 IF ((P-X).GT.100.) X=X+MAX
24.2   PW=0.08*YINT(K+LMT-2)
24.4   IF (PW.LT.PZ) PW=PZ
24.42  IF (PW.GT.160.) PW=160.
24.5   IF ((Y(K+LMT-2)-Y(K+LMT-1)).GT.(Y(K+LMT-1)-X+PW)) X=X-MAX
24.8   Y(K+LMT)=X
25     B=X-P
26     C=Y(K+LMT-1)-Y(K+LMT)
26.1   YINT(K+LMT-1)=C
27     WRITE(6,895) K,P,X,B,AY(K+LMT),C
28     895 FORMAT(I5,5F20.0)
  
```

```

29      883  CONTINUE
30      J=N-1
32.05   WRITE (7,861) (BY(K),K=1,80)
32.1    WRITE (7,865) (YINT(K),K=1,J)
33      WRITE (6,898)
34      898  FORMAT ('1          GRAPH OF INTENSITY VS. TIME'//)
35      READ (5,860) DY,CY,EY
36      860  FORMAT (3A1)
36.1    PRINT 861, (BY(K),K=1,80)
37      DO 692 K=1,12
38      692  BY(K)=10*K
39      WRITE (6,681) (BY(K),K=1,12)
40      681  FORMAT (/3X,12F10.0)
41      DO 690 K=1,150
42      690  BY(K)=CY
43      DO 691 K=1,25
44      691  BY(5*(K-1)+1)=EY
45      WRITE (6,861) (BY(K),K=1,120)
46      BY(1)=DY
47      DO 870 K=2,150
48      870  BY(K)=BY(K-1)
49      N=N-1
49.1    JB=1
49.2    DO 741 J=1,N
49.3    IF (YINT(J) .GT. YINT(JB)) JB=J
49.4    741  CONTINUE
50      DO 888 K=1,N
51      BY(1)=CY
52      JA=K/5
53      JA=JA*5
54      IF (JA .EQ. K) BY(1)=EY
54.1    L=YINT(K)*100./YINT(JB)
55      BY(L)=EY
57      PRINT 861, (BY(J),J=1,120),L
58      861  FORMAT (120A1,I5)
59      BY(L)=BY(L+1)
60      888  CONTINUE
63      865  FORMAT (12F6.0)
64      893  FORMAT (14F5.0)
65      894  FORMAT (2I5)
66      887  CALL EXIT
67      RETURN
68      885  L=L+1
69      X=AY(K+LMT)-MAX*L
69.1    PRINT 4,K,L,MAX,X
69.2    4    FORMAT (T2,3I5,F20.6)
70      IF (L-400) 886,887,887
71      END

```

END OF FILE

Part 1: Line numbers 1-6

In this part, an informative comment card is read (line 2. 1), the number of data values (N) and the maximum counts on calibration (MAX) are read (line 3), the raw data (AY) are read (line 4), and the first 5 values are set in Y (line 6). These five values must be monotonic estimates of values on the curve of actual (not normalized) intensity.

Part 2: Line numbers 6. 1-12. 5)

In this part, an allowable departure of a predicted value of intensity from the actual value is established. This departure is  $\pm 0.15$  (line 11. 1). Then the coefficients (A1, B, C, D) for cubic curve-fitting with five points are calculated (lines 12-12. 47), and a sixth point is predicted (line 12. 5).

Part 3: Line numbers 12. 52-20. 2

In this part, a parabolic curve-fitting is carried out for the third, fourth, and fifth points, yielding a new prediction of the sixth point (line 20). An average of the two predictions becomes the final estimate (line 20. 2) of the sixth point. This procedure continues within loop 883 until the entire set of data points is exhausted.

Part 4: Line numbers 21-29

In this part, the modulo conversion is accomplished, based upon some intuitive conditions regarding signal intensity. The resulting signal intensities (C) are written out (line 27).

Part 5: Line numbers 30-71

In this part, the signal intensities are punched (line 32. 1) for the data input to the final processing program. The intensity values are also plotted (line 57) after normalization (line 54. 1).

1ST OADR

```

1 C THIS IS THE PARABOLIC LEAST SQUARE INVERSION FOR OCCULTATION DATA
2 C THIS PROGRAM FOR SETTING STAR ONLY.****
3 IMPLICIT REAL*8(A-H,O-F,N-Z)
4 DIMENSION PHIC(150),ALAMC(150)
5 COMMON Z(200),ZN(200),A(200),B(200),C(200),ALT,WLL(100),FILT(100)
6 1FIC(100),SG(100),SGO2(100),SGM(100),CCM(200),NP,MPX
7 DIMENSION DA(200),FOF(200),TNO(200)
8 DIMENSION CF(200),CO2(200),RRF(200)
9 DIMENSION R(200),FN(200),IN(200),FNO(200),ZP(200)
10 DIMENSION DEN(200),ZN(200)
11 DIMENSION Q(20),T(150)
12 DIMENSION RO(200)
13 DIMENSION UD(200),UZ(200)
14 DIMENSION UT(200)

```

C\*\*\*\*\*

C\* PART-1

```

17 C*
18 C* THIS PART CALCULATES TANGENT RAY HEIGHT POINT. INPUT IS 5 VALUES
19 C* OF SPACECRAFT LONG. LAT. AND ALT. IN 10 SEC. GMT INTERVAL, 5 VALUES
20 C* OF GHA OF ARIES IN 10MIN. GMT INTERVAL, DEC. AND R.A. OF OCCULT- *
21 C* ING STAR, THE STARTING TIME IN HR., MIN., AND SEC., AND NUMBER OF
22 C* DATA POINTS NEEDED

```

C\*\*\*\*\*

C1=1./57.2957795131

WRITE(6,109)

```

26
27
28 109 FORMAT(//10X,' S/C POSITICN, LONG. LAT., AND ALT. IN 10 SEC.'
29 1 ' STEPS AND',//,10X,' GHA. OF ARIES IN TEN MINUTE STEPS')

```

READ 11,Y1,Y2,Y3,Y4,Y5

PRINT 11, Y1,Y2,Y3,Y4,Y5

CALL SET(Y1,Y2,Y3,Y4,Y5,A4,B4,C4,D4)

READ 11,Y1,Y2,Y3,Y4,Y5

PRINT 11, Y1,Y2,Y3,Y4,Y5

CALL SET(Y1,Y2,Y3,Y4,Y5,A2,B2,C2,D2)

READ 11,Y1,Y2,Y3,Y4,Y5

PRINT 11, Y1,Y2,Y3,Y4,Y5

CALL SET(Y1,Y2,Y3,Y4,Y5,A3,B3,C3,D3)

READ 11,Y1,Y2,Y3,Y4,Y5

PRINT 11, Y1,Y2,Y3,Y4,Y5

CALL SET(Y1,Y2,Y3,Y4,Y5,A5,B5,C5,D5)

READ 11,Y1,Y2,Y3,Y4,Y5

PRINT 71, Y1,Y2,Y3

```

44 71. FORMAT(///5X,' STAR DECLINATION',F5.1,' DEGREES',F5.1,' MINUTES',
45 1F7.2,' SECONDS')

```

DEC=Y1+(Y2/60.)+(Y3/3600.)

READ 11,Y1,Y2,Y3,Y4,Y5

PRINT 72, Y1,Y2,Y3

```

49 72. FORMAT(5X,' STAR RIGHT ASCENSION',F5.1,' HOURS',F5.1,' MINUTES',
50 1F7.2,' SECONDS')

```

RA=(Y1+(Y2/60.)+(Y3/3600.))\*15.00

DEC=DEC/57.29578

RA=RA/57.29578

C TRIGONOMETRIC COMPUTATIONS START HERE.

DCL=DCOS(DEC)\*DCOS(RA)

DCM=DCOS(DEC)\*DSIN(RA)

DCN=DSIN(DEC)

READ 612,T1,T2,T3,NP

```

59 612 FORMAT(3F10.5,I5)
60 PRINT 641, T1,T2,T3
61 641 FORMAT(5X,'STARTING TIME ',F5.1,'HR',F5.1,'MIN.',F7.4,'SEC.'
62 1 ' AND TIME IS INCREASING')
63 T1=T1+(T2+(T3/60.))/60.
64 NPF=NP
65 I=0
66 73 IF (T2 .LT. 10.) GO TO 74
67 T2=T2-10.
68 GO TO 73
69 74 L=T3/10.
70 TX=T2+(L*10./60.)
71 75 IF (T3 .LT. 10.) GO TO 7
72 T3=T3-10.
73 GO TO 75
74 7 S=1.+(T3/10.)
75 C* SPACECRAFT POSITIONS AND GHA OF ARIES ARE CALCULATED AT DIFFERENT
76 C* GMT TIME BY USING THE FUNCTION CAL WHICH USES THE CURVE
77 C* Y=A*T*T + B*T + C
78 PHIS=CAL(S,A2,E2,C2,D2)*C1
79 ALT=CAL(S,A3,B3,C3,D3)
80 ALAMS=CAL(S,A4,B4,C4,D4)*C1
81 IF (ALAMS .LT. 0.) ALAMS=6.28318+ALAMS
82 X=3.+(TX+(T3/60.))/10.
83 ALAMG=CAL(X,A5,B5,C5,D5)*C1
84 I=I+1
85 M=NP-I+1
86 PA=DCOS(ALAMS)
87 EB=DCOS(PHIS)
88 PC=DCOS(ALAMG)
89 D=DSIN(ALAMS)
90 E=DSIN(ALAMG)
91 DCA=PA*EB*PC-D*PB*E
92 DCB=D*PB*PC+PA*PB*E
93 DCC=DSIN(PHIS)
94 CALL HEIGHT(PHIS,RSAT,RR)
95 13 XS=RSAT*DCA
96 YS=RSAT*DCB
97 ZS=RSAT*DCC
98 RS=DSQRT(XS*XS+YS*YS+ZS*ZS)
99 ARG=DCA*DCL+DCB*DCM+DCC*DCN
100 BETA=3.1415926536-DARCOS(ARG)
101 W=DCOS(BETA)
102 XO=XS+RSAT*W*DCL
103 YO=YS+RSAT*W*DCM
104 IF(YO.LT.0..AND.ARG.GE.0.) BETA=6.293186-BETA
105 IF(YO.LT.0..AND.ARG.LT.0.) BETA=3.141593+BETA
106 ZO=ZS+RSAT*W*DCN
107 ROO=DSQRT(XO*XO+YO*YO+ZO*ZO)
108 PHIO(M)=DATAN(ZO/DSQRT(XO*XO+YO*YO))
109 ALAMO(M)=DARCCOS(XO/DSQRT(XO*XO+YO*YO))-ALAMG
110 QF=DSIN(BETA)
111 DCLS=XO/(RSAT*QF)
112 DCMS=YO/(RSAT*QF)
113 DCNS=ZO/(RSAT*QF)
114 CPER=DCL*DCLS+DCM*DCMS+DCN*DCNS
115 CALL HEIGHT(PHIO(M),RP,RR)
116 HO=RCC-RR
117 ZP(M)=HO*100000.
118 R(M)=RCC*100000.

```

```

119      RO(N)=RR*100000.
120      PHIO(N)=PHIO(N)*57.295
121      ALAMO(N)=ALAMO(N)*57.295
122      PSI=(1.5707963268-BE1A)/C1
123      T3=T3+(62.975/116.34)
124      IF(I.LT. NP) GO TO 7
125      11  FORMAT(5F10.5)
126      C*****
127      C*          PART-2
128      C*
129      C* THIS PART TAKES AS INPUT THE FILTER TRANSMISSION, CROSS SECTION
130      C* AND STAR SPECTRUM. THEN A TABLE OF TOTAL INTEGRATED NUMBER DENSITY
131      C* TN VS. NORMALISED SIGNAL. FNC, IS COMPUTED
132      C*****
133      TX=ALAMO(NP)/15.0
134      T1=T1+TX
135      IF (T1 .LE. 0.) T1=T1+24.0
136      IF (T1 .GT. 24.0) T1=T1-24.0
137      L1=T1
138      T1=(T1-L1)*60.
139      L2=T1
140      T1=(T1-L2)*60.
141      NTAB=106
142      C          INPUT DATA
143      C  FILTER DATA IN STEPS OF 20A.
144      READ 351,FX,FY,IO2
145      351  FORMAT(2F5.0,I5)
146      MPX=1.001+(FY-IX)/20.
147      DO 302 I=1,MPX
148      302  WLL(I)=FX+(I-1)*20.
149      IF (IO2 .NE. 2) GO TO 1260
150      MPX=MPX+13
151      READ 1201,(WLL(I),I=25,MPX)
152      1201  FORMAT(8F10.3)
153      1260  READ 300,(FILT(I),I=1,MPX)
154      C  CROSS-SECTION DATA
155      READ 600,(SG(I),I=1,MPX)
156      C  STAR SPECTRUM IN STEPS OF 20A.
157      READ 303,(FIO(I),I=1,MPX)
158      300  FORMAT(11F5.3)
159      303  FORMAT(11F5.0)
160      600  FORMAT(6E10.2)
161      SUN=0.
162      DO 101 J=2,MPX
163      101  SUN=SUN+((FILT(J)*FIO(J)+FILT(J-1)*FIO(J-1))/2.)*(WLL(J)-WLL(J-1))
164      C THIS SECTION MAKES A TABLE OF TOTAL INTEGRATED NUMBER
165      C DENSITY VS. NORMALISED SIGNAL.
166      DO 800 IHC=1,NTAB
167      READ(3,1216) TT
168      1216  FORMAT(5E16.6)
169      IF (IO2 .NE. 2) GO TO 1261
170      READ(2,1216) (SG(I),I=25,MPX)
171      1261  CONTINUE
172      TN(IHC)=TT
173      SUN=0.
174      DO 1000 J=2,MPX
175      1000  SUM=SUN+((FILT(J)*FIO(J)*DEXP(-SG(J)*TT)+FILT(J-1)*FIO(J-1)*
176      1DEXP(-SG(J-1)*TT))/2.)*(WLL(J)-WLL(J-1))
177      FNR=SUN/SUN
178      IF (FNR .LT. 1.D-20) FNR=1.D-20

```

```

179      FNO(IHO)=FNP
180      CONTINUE
181      C*****
182      C*          PART-3
183      C*
184      C* INPUT TO THIS PART IS THE MEASURED SIGNAL. THEN NORMALISED SIGNAL
185      C* FOF IS USED TO DETERMINE TOTAL INTEGRATED NUMBER DENSITY TNO
186      C* BY INTERPOLATION IN THE TABLE COMPUTED IN PART-2
187      C*****
188      READ 1001, (Q(I),I=1,10)
189      PRINT 1001, (Q(I),I=1,10)
190      1001  FORMAT(10A8)
191      1002  FORMAT(I3)
192      C OCCULTATION DATA
193      READ 750, (DA(I),I=1,NPP)
194      750   FORMAT(12F6.0)
195      DMIA=DA(1)
196      DO 857 I=1,NPP
197      IF (DA(I).LT.DMIA) DMIA=DA(I)
198      857  CONTINUE
199      DO 861 I=1,NPP
200      861  DA(I)=DA(I)-DMIA
201      NMXY=NPP-4
202      SUC=0.
203      DO 761 I=NMXY,NPP
204      761  SUC=SUC+DA(I)
205      DAO=SUC/5.
206      DO 762 I=1,NPP
207      762  FOF(I)=DA(I)/DAO
208      WRITE(6,1004)
209      1004  FORMAT(//'MODIFIED INPUT DATA'//)
210      PRINT 1005, (DA(I),I=1,NPP)
211      1005  FORMAT(10F8.0)
212      C THIS SECTION CALCULATES TOTAL INTEGRATED NUMBER DENSITY;
213      C USING MEASURED SIGNAL AND PREVIOUSLY COMPUTED TABLE.
214      DO 611 J=1,NTAB
215      Z(J)=DLOG(FNO(J))
216      XN(J)=DLOG(TN(J))
217      611  CONTINUE
218      609  FORMAT(T5,2D15.6)
219      M=4
220      NP=NTAB
221      CALL PARAB(M)
222      DO 630 L=1,NPP
223      IF (FOF(L).LE.0.) FOF(L)=1.D-5
224      DO 792 I=1,NTAB
225      IF (FOF(L).GE.FNO(I).AND.FOF(L).LT.FNO(I+1)) GO TO 613
226      792  CONTINUE
227      613  J=I
228      X=DLOG(FOF(L))
229      TNO(L)=A(J)*X*X+B(J)*X+C(J)
230      TNO(L)=DEXP(TNO(L))
231      614  FORMAT(T5,2D15.6)
232      630  CONTINUE
233      IF (IO2.NE.0) GO TO 2501
234      C*****
235      C*          PART-3A
236      C* IN THIS PART, THE OZONE COLUMN DENSITY IS CORRECTED FOR
237      C* HERZBERG AND RAYLEIGH SCATTERING
238      C*****

```

```

238 READ 2006, (SGC2(I), I=1, NPX)
239 2006 FORMAT (7E10.2)
240 DO 2005 I=1, NPX
241 SGM(I)=3.981D-28*((1890.+(I-1)*20.)*1.D-4) ** (-4.05)
242 2005 CONTINUE
243 814 FORMAT (1X, 6E15.4)
244 WRITE (6, 814)
245 815 FORMAT (//3X, 'WAVE L', 7X, 'FILTER', 10X, 'STAR INT.', 5X,
246 '03 X-SECTION', 3X, '02 X-SECTION', 3X, 'RAYLEIGH X-SECTION'//)
247 DO 877 I=1, NPX
248 WRITE (6, 814) WLL(I), FILT(I), FIO(I), SG(I), SGO2(I), SGM(I)
249 877 CONTINUE
250 READ 2000, ITEST, EPSC
251 READ 2000, MZZ, ZZO, DZZ
252 2000 FORMAT (15, 2E12.4)
253 WRITE (6, 3001) ITEST, EPSC, MZZ, ZZO, DZZ
254 3001 FORMAT (/T4, 'ITEST = ', I4, ' EPSC = ', F7.4, ' MZZ = ', I5,
255 ' ZZO = ', D10.3, ' DZZ = ', D10.3/)
256 READ 2009, (CCM(I), I=1, NZZ)
257 2009 FORMAT (T2, 7E9.2)
258 DO 2500 II=1, NPP
259 I=NPP+1-II
260 ITT=0
261 KK=(ZP(I)-ZZO)/DZZ+1
262 IF (KK.LE.1) KK=1
263 CO2(I)=CCM(KK)*DEXP((ZP(I)-(ZZO+DZZ*(KK-1)))*DLOG(CCM(KK+1)
264 1/CCM(KK))/DZZ)
265 C3(I)=CC2(I)/0.2095
266 SUN=FAT(0.0, CC2(NPP), CM(NPP))
267 FATO=DLOG(FOP(I))+SUN
268 IF (FATO.LE.1.0E-5) FATO=1.0E-5
269 C1=FNO(I)
270 F1=FAT(C1, CO2(I), CM(I))
271 C2=C1*0.75
272 IF (C2.LE.1.0D10) C2=1.0D10
273 2002 F2=FAT(C2, CO2(I), CM(I))
274 IF (DABS(F2-F1).LE.1.0D-10) GO TO 2004
275 C3=C1+(FATO-F1)*(C2-C1)/(F2-F1)
276 IF (DABS(C3).GE.1.0E35) GO TO 2004
277 ITT=ITT+1
278 IF (DABS((C3-C2)/C2).LE.EPSC) GO TO 2004
279 IF ((ITT-ITEST).GT.1) GO TO 2004
280 IF (DABS((F1-FATO)/(F2-FATO)).LE.1.0) GO TO 2003
281 F1=F2
282 C1=C2
283 2003 C2=C3
284 GO TO 2002
285 2004 CONTINUE
286 TNO(I)=C3
287 IF (C3.LT.0.0) TNO(I)=0.0
288 2500 CONTINUE
289 C*****
290 C*
291 C* PART-4
292 C*
293 C* THIS PART FITS A PARABOLA THROUGH TANGENT RAY HEIGHT ZP COMPUTED
294 C* IN PART-1 VS. TOTAL INTEGRATED NUMBER DENSITY TNO COMPUTED IN PART
295 C* -3. THEN USING LEAST SQUARE COEFFICIENTS, INVERSION IS CARRIED ON
296 C* TO COMPUTE THE NUMBER DENSITY. THIS NUMBER DENSITY IS, THEN, USED
297 C* IN THE CALCULATION OF MOLECULAR OXYGEN TEMPERATURE.
C*****

```



```

298 2501 NP=NEF
299 DO 1211 I=1,NPF
300 Z(I)=ZP(I)
301 1211 XN(I)=TNC(I)
302 PI=3.1415926536
303 C THIS SECTION OBTAINS THE COEFFICIENTS FOR LEAST SQUARE PARABOLA
304 M=8
305 WRITE(6,21) M
306 21 FORMAT('1',' SMOOTHING WITH M=',I3//)
307 WRITE(6,123)
308 CALL PARAB(M)
309 123 FORMAT(' IGNORE TEMP. IN OZONE RESULTS'//)
310 12 FORMAT(5X,6E12.4)
311 N1=NP-1
312 DO 4 J=1,N1
313 DEN(J)=0.0
314 DO 5 I=J,N1
315 D=2.*A(I)
316 5 DEN(J)=DEN(J)+D*(DSQRT(R(I+1)**2-R(J)**2)-DSQRT(R(I)**2-R(J)**2)
317 1+(B(I)-2.*A(I)*RO(I))*DLCG((R(I+1)+DSQRT(R(I+1)**2-R(J)**2))/
318 2(R(I)+DSQRT(R(I)**2-R(J)**2)))
319 DEN(J)=-DEN(J)/PI
320 4 CONTINUE
321 WRITE(6,601) L1,L2,T1
322 601 FORMAT(5X,'LOCAL STARTING TIME',I6,' HOUR',I6,' MINUTE',
323 1F7.3,'SRCCNDS.')
324 WRITE(6,23)
325 23 FORMAT(/10X,'HEIGHT',9X,'NUMBER',9X,'TEMP.',10X,'LAT.',11X,
326 1'LONG.',10X,'COLUMN',10X,'NORMALIZED',10X,'O2 COLUMN DENSITY',
327 2/11X,'IN CM.',8X,'DENSITY',8X,'IN DEG. K.',5X,'IN DEG.',8X,
328 3'IN DEG.',8X,'DENSITY',9X,'INTENSITY',11X,'DENSITY',/)
329 N1=N1-1
330 DO 977 J=1,N1
331 T(J)=0.0
332 DO 150 I=J,N1
333 A2=DEN(I+1)/(R(I+1)**2)
334 A1=DEN(I)/(R(I)**2)
335 A1=DABS(A1)
336 A2=DABS(A2)
337 150 T(J)=T(J)+(A2-A1)/DLOG(A2/A1)*(Z(I+1)-Z(I))
338 T(J)=(3.8038D-4)*T(J)/DEN(J)
339 T(J)=T(J)*RO(I)*RO(I)
340 977 WRITE(6,22) Z(J),DEN(J),T(J),PHIO(J),ALAMO(J),XN(J),FOF(J),CO2
341 22 FORMAT(4X,6E15.6,E18.6,E19.6)
342 STOP
343 978 DO 911 J=1,N1
344 UD(J)=DEN(J)
345 UZ(J)=Z(J)
346 UT(J)=T(J)
347 911 IF (UD(J) .LE. 0.0) UD(J)=1.0E 01
348 UYMIN=7.0
349 UYEX=0.5
350 UXMN=40.0E 05
351 UDX=20.0E 05
352 IF (102 .EQ. 0) GO TO 1531
353 CALL PLTOFS(100.0,100.0,UXMN,UDX,1.5,1.0)
354 UPX=100.
355 UPDX=100.
356 CALL PAXIS(1.5,1.,16HTEMP. IN DEG. K.,-16,10.,0.,UPX,UPDX,-1.)
357 CALL PAXIS(1.5,1.,15HALTITUDE IN CM.,15,8.,90.,UXMN,UDX,-0.5)

```

```

353 CALL PGRID(1.5,1.0,0.5,0.5,20,10)
354 CALL PLINE(UT(1),UZ(1),N1,1,1,1,2,1)
355 CALL PSYMB(1.5,9.5,-0.1,Q(1),0.0,80)
356 IF (M.EQ.4) GO TO 1530
357 CALL PSYMB(1.5,9.25,-0.1,3HM=8,0.0,3)
358 GO TO 1532
359 1530 CALL PSYMB(1.5,9.25,-0.1,3HM=4,0.0,3)
360 1532 CALL PLEND
361 1531 CONTINUE
362 CALL PLTOPS(UYMIN,UYEX,UXMIN,UDX,1.5,1.0)
363 CALL PLGAAS(1.5,1.0,14HNUMBER DENSITY,-14,10.0,0.0,UYMIN,-UYEX)
364 CALL PAXIS(1.5,1.0,15HALTITUDE IN CM.,15,8.0,90.0,UXMIN,UDX,-0.5)
365 CALL PLGGRD(1.5,1.0,1.0/UYEX,10.0,8.0,0.0)
366 CALL PGRID(1.5,1.0,10.0,0.5,1,15)
367 CALL PLYCS(2)
368 CALL PLINE(UD(1),UZ(1),N1,1,1,1,2,1)
369 CALL PLEND
370 CALL PSYMB(1.5,9.5,-0.1,Q(1),0.0,80)
371 IF (M.EQ.4) GO TO 912
372 CALL PSYMB(1.5,9.25,-0.1,3HM=8,0.0,3)
373 GO TO 913
374 912 CALL PSYMB(1.5,9.25,-0.1,3HM=4,0.0,3)
375 913 CALL PLEND
376 6 CONTINUE
377 CALL EXIT
378 END
379 C*****
380 C*
381 C* PART-5
382 C*
383 C* SUBROUTINE SET TAKES 5 EQUIDISTANT VALUES AND FITS A LEAST SQUARE
384 C* PARABOLA Y=P*X*X + Q*X + R. COEFFICIENTS P,Q AND R ARE RETURNED
385 C*****
386 SUBROUTINE SET(Y1,Y2,Y3,Y4,Y5,P,Q,R,S)
387 IMPLICIT REAL*8(A-H,O-Z)
388 COMMON Z(200),KN(200),A(200),B(200),C(200),ALT,WLL(100),FILT(100)
389 1FIO(100),SG(100),SGO2(100),SGM(100),CCM(200),NP,MPX
390
391 P=((2.*Y1)-Y2-(2.*Y3)-Y4+(2.*Y5))/14.
392 Q={(-140.*Y1)+(40.*Y2)+(120.*Y3)+(74.*Y4)-(92.*Y5)}/140.
393 R={ (126.*Y1) - (56.*Y3) - (42.*Y4) + (42.*Y5) }/70.
394 RETURN
395 END
396 C*****
397 C*
398 C* PART-6
399 C*
400 C* SUBROUTINE HEIGHT CALCULATES THE RADIUS OF EARTH RR, AND DISTANT
401 C* RP OF A POINT FROM CENTER OF EARTH; WHEN LATITUDE PHI, AND ALTITUDE
402 C* ALT, OF THE POINT ARE KNOWN.
403 C*****
404 SUBROUTINE HEIGHT(PHI,RP,RR)
405 IMPLICIT REAL*8(A-H,O-Z)
406 COMMON Z(200),KN(200),A(200),B(200),C(200),ALT,WLL(100),FILT(100)
407 1FIO(100),SG(100),SGO2(100),SGM(100),CCM(200),NP,MPX
408 REAL*8 KAPPA
409
410 EC=.0818202
411 R=6378.387
412 DENOM=DSQRT(1.-(EC*EC*DSIN(PHI)*DSIN(PHI)))

```

```

418   RCOS=DCOS (PHI) *R/DENOM
419   RSIN=S*(1.-EC*EC)*DSIN (PHI)/DENOM
420   RR=DSQRT (RCOS*RCOS+RSIN*RSIN)
421   PHIP=DARSIN (RSIN/RR)
422   KAPPA=3.1415926536
423   RP=DSQRT (RR*RR+ALT*ALT-2.*RR*ALT*DCOS (KAPPA))
424   RETURN
425   END

```

```

426 C*****
427 C*
428 C*

```

PART-7

```

429 C* SUBROUTINE PARAB FITS THE LEAST SQUARE PARABOLA THROUGH M POINTS
430 C* AT A TIME IN THE CURVE OF Z VS. XN. THE PROCESS IS CONTINUED FOR
431 C* NP POINTS AND COEFFICIENTS A,B AND C ARE RETURNED TO MAIN PROGRAM
432 C* NOTE THAT XN=A*Z*Z + B*Z + C
433 C*****

```

```

434   SUBROUTINE PARAB (M)
435   IMPLICIT REAL*8 (A-H, C-Z)
436   COMMON Z (200), XN (200), A (200), B (200), C (200), ALT, WLL (100), FILT (100)
437   1PIO (100), SG (100), SGO2 (100), SGM (100), CCM (200), NP, MPX
438   L=M/2
439   DO 2 I=1, NP
440   IZ=NP+1-I
441   K=1
442   IF (I.GT.L.AND.I.LT.IZ) K=I-L
443   IF (I.GE.IZ) K=NP-M
444   ZB=0.
445   Z2B=0.
446   Z3B=0.
447   Z4B=0.
448   XNB=0.
449   ZNB=0.
450   Z2NB=0.
451   K2=K+M
452   DO 3 J=K, K2
453   ZB=ZB+Z (J)
454   Z2B=Z2B+Z (J) **2
455   Z3B=Z3B+Z (J) **3
456   Z4B=Z4B+Z (J) **4
457   ZNB=ZNB+Z (J) *XN (J)
458   Z2NB=Z2NB+XN (J) *(Z (J) **2)
459   3 XNB=XNB+XN (J)
460   J=I
461   M1=M
462   M=M+1
463   A (J) = ((M*ZNB-XNB*ZB) * (M*Z3B-ZB*Z2B) - (M*Z2NB-XNB*Z2B) * (M*Z2B-ZB**2)
464   1) / ((M*Z3B-Z2B*ZB) * (M*Z3B-ZB*Z2B) - (M*Z4B-Z2B**2) * (M*Z2B-ZB**2))
465   D=A (J)
466   B (J) = ((M*ZNB-ZB*XNB) - (M*Z3B-ZB*Z2B) *D) / (M*Z2B-ZB*ZB)
467   E=B (J)
468   C (J) = (XNE-D*Z2B-E*ZB) / M
469   M=M1
470   2 CONTINUE
471   RETURN
472   END
473   FUNCTION CAL (X, A, B, C, D)
474   IMPLICIT REAL*8 (A-H, O-Z)
475   CAL = (A*X**2) + (B*X) + C
476   RETURN
477   END

```

FUNCTION FAT(CO3,CO2,CM)  
 IMPLICIT REAL\*8 (A-H,O-T,W-Z)

C  
C  
C  
C  
C

THIS FUNCTION GENERATES THE SIGNAL OF A STAR AS SEEN THROUGH AN  
 ATMOSPHERE WITH COLUMN DENSITIES CO3,CO2,CM AND VIEWED THROUGH  
 FILTER WITH TRANSMISSION FILT(I) AT WAVELENGTH WLL(I)

```

COMMON Z(200),XN(200),A(200),B(200),C(200),ALT,WLL(100),FILT(10
1FIO(100),SG(100),SGO2(100),SGM(100),CCM(200),NP,MPX
X=SG(1)*CC3+SGC2(1)*CO2+SGM(1)*CM
IF(DABS(X).GT.50.) X=50.
X1=FILT(1)*FIO(1)*DEXP(-X)
FAT=0.0
DO 10 I=2,MPX
X=SG(1)*CO3+SGO2(I)*CO2+SGM(I)*CM
IF(DABS(X).GT.50.) X=50.
X2=FILT(1)*FIO(I)*DEXP(-X)
FAT=FAT+0.5*(X1+X2)*(WLL(I)-WLL(I-1))
10 X1=X2
FAT=DLOG(FAT)
RETURN
END
  
```

10

500	160.12	160.59	161.26	161.84	162.42						
501	-26.32	-27.06	-27.30	-27.53	-27.76						
502	778.6	778.7	778.9	779.0	779.1						
503	298.4833	300.9916	303.4983	306.005	308.5116						
504	28.0	55.0	26.01582								
505	0.0	6.0	53.1338								
506	13.0	29.0	32.0	065							
507	1800.3000.	00									
508	.020	.021	.022	.024	.026	.030	.035	.040	.048	.055	.060
509	.070	.080	.090	.100	.130	.160	.200	.260	.360	.420	.520
510	.620	.740	.840	.940	.990	1.000	.980	.940	.880	.840	.800
511	.730	.740	.680	.640	.600	.500	.400	.350	.300	.240	.200
512	.170	.140	.120	.100	.080	.060	.050	.042	.040	.037	.035
513	.013	.030	.028	.025	.023	.020					
514	7.80E-19	7.50E-19	7.00E-19	6.50E-19	5.80E-19	5.20E-19	5.80E-19	5.80E-19	5.20E-19	5.20E-19	5.20E-19
515	4.70E-19	4.30E-19	3.80E-19	3.40E-19	3.40E-19	3.40E-19	3.40E-19	3.20E-19	3.20E-19	3.20E-19	3.20E-19
516	3.30E-19	3.60E-19	4.30E-19	5.20E-19	5.20E-19	5.20E-19	5.20E-19	6.60E-19	6.60E-19	8.80E-19	8.80E-19
517	1.10E-18	1.40E-18	1.80E-18	2.20E-18	2.20E-18	2.20E-18	2.20E-18	2.70E-18	2.70E-18	3.20E-18	3.20E-18
518	3.90E-18	4.50E-18	5.20E-18	5.20E-18	5.80E-18	5.80E-18	5.80E-18	6.60E-18	6.60E-18	7.40E-18	7.40E-18
519	8.00E-18	9.00E-18	1.00E-17	1.01E-17	1.01E-17	1.01E-17	1.01E-17	1.10E-17	1.10E-17	1.20E-17	1.20E-17
520	1.20E-17	1.15E-17	1.10E-17	1.10E-17	1.08E-17	1.08E-17	1.08E-17	1.05E-17	1.05E-17	1.00E-17	1.00E-17
521	9.80E-18	9.00E-18	8.20E-18	7.50E-18	7.50E-18	7.50E-18	7.50E-18	6.80E-18	6.80E-18	6.00E-18	6.00E-18
522	5.00E-18	4.50E-18	4.00E-18	4.00E-18	3.40E-18	3.40E-18	3.40E-18	2.70E-18	2.70E-18	2.30E-18	2.30E-18
523	1.90E-18	1.50E-18	1.20E-18	1.20E-18	9.00E-19	9.00E-19	9.00E-19	7.00E-19	7.00E-19	5.40E-19	5.40E-19
524	3.95E-19										
525	18	20	20	18	20	20	22	22	22	22	24
526	24	26	26	26	28	30	32	32	34	36	36
527	40	38	38	38	40	36	34	34	32	30	32
528	30	32	30	34	34	30	32	32	32	32	34
529	38	34	34	34	36	34	36	36	36	36	36
530	36	36	38	38	38	40					
531	1.00E-19	8.00E-20	5.00E-20	3.00E-20	3.00E-20	1.30E-20	1.30E-20	3.00E-22	3.00E-22	1.35E-22	1.35E-22
532	7.40E-23	4.80E-23	3.30E-23	2.55E-23	2.55E-23	2.10E-23	2.10E-23	1.75E-23	1.75E-23	1.50E-23	1.50E-23
533	1.30E-23	1.17E-23	1.05E-23	9.55E-24	8.00E-24	8.00E-24	8.00E-24	7.40E-24	7.40E-24	6.65E-24	6.65E-24
534	6.20E-24	5.65E-24	5.15E-24	4.60E-24	4.15E-24	4.15E-24	4.15E-24	3.55E-24	3.55E-24	2.20E-24	2.20E-24
535	2.70E-24	2.30E-24	1.95E-24	1.70E-24	1.70E-24	1.45E-24	1.45E-24	1.28E-24	1.28E-24	1.12E-24	1.12E-24
536	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
537	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.

538	0.	0.	0.	0.	0.	0.	0.
539	0.	0.	0.	0.	0.		
540	100	.0100					
541	11b	23.D5	1.D5				

OF FILE

Part 1: Line numbers 1-125

In this part, the spacecraft position values are entered for use in interpolation. There are five such values for longitude, latitude, and altitude furnished by GSFC several weeks after the occultation. The first of these five values corresponds to the time (given at a 10-second interval) which is less than but nearest to the starting time. The Greenwich hour angle of Aries is then read in for five times spaced ten minutes apart and centered upon the time less than but nearest to the starting time. Star position information is also entered, followed by the starting time and the number of data points. The position of the data point is computed from the above inputs, using subroutines HEIGHT and SET and function CAL. Program comments for Part I give additional details.

Notation

DEC = declination of star, in radians  
RA = right ascension of star, in radians  
NP, NPP = number of data points in scan  
PHIS = latitude of subsatellite point in radians  
ALT = altitude of spacecraft, in cm  
ALAMS = longitude of subsatellite point, in radians  
ALAMG = Greenwich hour angle of Aries, in radians  
RSAT, RP = distance of subsatellite point from Earth's center, in cm  
ROO, R = distance of data point from Earth's center, in cm  
HO, ZP = height of data point, in cm  
RO, RR = radius of Earth along a line to the data point, in cm  
PHIO = latitude of data points, in degrees  
ALAMO = longitude of data point, in degrees  
T1, T2, T3, TX, X, S = temporal variables

Part 2: Line numbers 126-180

In this part, the filter transmission values, the appropriate absorption cross section, and the star spectrum are entered for the pertinent wave band at a 20 Å interval. A normalization factor for the signal intensity is computed and a table of normalized signal intensity v. tangential column density (integrated number density) is formed. The latter values are already stored in file CROSSDAT.

Notation

NTAB	=	number of tabular points
FX	=	beginning wavelength of band, in Å
FY	=	final wavelength of band, in Å
IO2	=	switch for gaseous species and absorption cross section (1: ozone, 2: O <sub>2</sub> , 3: O <sub>2</sub> with Hudson's cross section)
MPX	=	number of wavelength intervals in band
WLL	=	wavelength
FILT	=	normalized value of filter transmission
SG	=	absorption cross section, in cm <sup>2</sup>
FIO	=	spectral star intensity, in digital voltmeter counts/time interval
SUN	=	normalizing factor, in counts/time interval
TT, TN	=	tabular tangential column density, in cm <sup>-2</sup>
SUM	=	filtered signal intensity integrated over wavelength, in counts/time interval
FNR, FNO	=	tabular, normalized signal intensity corresponding to TT

Part 3: Line numbers 181-233

In this part, a comment card is read which identifies the occultation data. The data are then entered, the background noise (minimum signal) is removed, and the data are normalized. Using (m+1) smoothing points, a logarithmic interpolation is performed in subroutine PARAB to get the least squares coefficients for the column densities corresponding to the "modified input data."

Notation

DA	=	star intensity observed during occultation, in counts/time interval
DMIA	=	minimum value of DA
FOF	=	normalized, measured signal intensity
TNO	=	column density along stellar ray path in $\text{cm}^{-2}$
A, B, C	=	least squares coefficients for a parabolic fit
M	=	one less than the number of smoothing points taken for a least squares fit

Part 3A: Line numbers 234-288 (for ozone only)

In this part, a correction to the ozone column density is made for the removal of Herzberg and Rayleigh scattering. To accomplish this, a functional inversion of  $F(r)$  is necessary, where

$$F(r) = \frac{\int_0^{\infty} T(\lambda) I_o(\lambda) \exp[-\sqrt{o_2}(\lambda) N_{o_2}(r) - \sqrt{M}(\lambda) N_M(r) - \sqrt{o_3}(\lambda) N_{o_3}(r)] d\lambda}{\int_0^{\infty} T(\lambda) I_o(\lambda) d\lambda}$$

F	=	normalized signal intensity
T	=	filter transmission
$I_o$	=	spectral stellar intensity
$\sqrt{i}$	=	cross-sectional value of $i^{\text{th}}$ species or of molecular scattering (M), in $\text{cm}^2$
$N_i$	=	tangential column density of $i^{\text{th}}$ species ( $N_M$ = total for all species), in $\text{cm}^{-2}$

The desired quantity is  $N_{o_3}(r)$ . To carry out this operation,  $\sqrt{o_2}(\lambda)$  is read in for the ozone band being considered e. g., 1800-3000 Å, with the Herzberg continuum being included.  $\sqrt{n}(\lambda)$  is then computed for the same wavelength region from Rayleigh's formula,

$$\sqrt{R}(\lambda) = 3.981 \times 10^{-28} (10^{-4} \lambda)^{-4.05} \text{cm}^2 \quad (\lambda \text{ in } \text{Å})$$



Some representative values of O<sub>2</sub> tangential column density are read in at a 5-km interval from 10 km to 310 km, and exponential interpolation is employed to find this quantity at each data point height. It is then converted to total tangential column density for all species and the final value of N<sub>O<sub>3</sub></sub> is reached through Newton's iterative method. In this method, an initial guess of N<sub>M</sub> is made for the value of N<sub>O<sub>3</sub></sub><sup>(0)</sup> and function "FAT" produces a signal F<sub>1</sub> which is depleted by passage through an atmosphere with tangential column densities N<sub>O<sub>2</sub></sub>, N<sub>M</sub> and through the filter. The second guess N<sub>O<sub>3</sub></sub><sup>(1)</sup> reduces N<sub>O<sub>3</sub></sub><sup>(0)</sup> by one quarter, and if the resulting signal F<sub>2</sub> is close enough to F<sub>1</sub> we have the N<sub>O<sub>3</sub></sub><sup>(j)</sup> which is sought. Otherwise, a new N<sub>O<sub>3</sub></sub><sup>(j)</sup> is computed by the formula

$$N_{O_3}^j = N_{O_3}^{j-2} + (F_0 - F_{j-1})(N_{O_3}^{j-1} - N_{O_3}^{j-2}) / (F_j - F_{j-1})$$

where F<sub>0</sub> is the sum of the "FAT" functional evaluation for the case where N<sub>O<sub>3</sub></sub> = 0 (actually a new normalizing factor) and the log of the signal corresponding to N<sub>M</sub>. A second tolerance test is provided by  $|(N_{O_3}^j - N_{O_3}^{j-1}) / N_{O_3}^{j-1}| \leq \epsilon$  and a limit is placed on the number of iterations. The data points are taken from top to bottom and certain safeguards are included for computational instability encountered at the lowest levels.

<u>Notation</u>	
SGO2	= O <sub>2</sub> cross section ( $\bar{\sigma}_{O_2}$ ), in cm <sup>2</sup>
SGM	= Rayleigh scattering cross section ( $\bar{\sigma}_M$ ), in cm <sup>2</sup>
ITEST	= maximum number of iterations
ITT	= iteration counter
EPSC	= tolerance
MZZ	= number of values in interpolation table for O <sub>2</sub> column density
ZZO	= base height of interpolation table, in cm
DZZ	= vertical interval in table, in cm

Notation (cont'd)

CCM = tabular O<sub>2</sub> column density, in cm<sup>-2</sup>  
CO<sub>2</sub> = interpolated O<sub>2</sub> column density (NO<sub>2</sub>), in cm<sup>-2</sup>  
CM = total column density (N<sub>M</sub>), in cm<sup>-2</sup>  
SUN = new normalizing factor  
FATO(F<sub>O</sub>), C1(N<sub>O<sub>3</sub></sub><sup>j-2</sup>), C2(N<sub>O<sub>3</sub></sub><sup>j-1</sup>), C3(N<sub>O<sub>3</sub></sub><sup>j</sup>), F1(F<sub>j-1</sub>), F2(F<sub>j</sub>)  
= parameters in Newton's method

Part 4: Line numbers 289-383

Using subroutine PARAB, the coefficients are obtained for fitting a curve of total tangential column density against tangent ray height, and the mathematical inversion is performed to get number density. Please refer to the notes on the O<sub>2</sub> error analysis for details on this procedure as well as the computation of temperature.

A STOP statement (line 342) negates CALCOMP plotting. When STOP is removed, the plotter produces number density graphs for two smoothing values. When molecular oxygen is being investigated, it draws two additional graphs for temperature.

Notation

Z = height, in cm  
XN = tangential column density, in cm<sup>-2</sup>  
DEN = number density, in cm<sup>-3</sup>  
T = molecular temperature, in °K

Parts 5-7: Line numbers 384-499

Subroutine SET returns the coefficients after fitting a least squares parabola through five equidistant points, whereas subroutine PARAB fits any number of points which need not be equidistant. Additional information is available in the program comments and in notes by Mr. A. Shah.

Part 8: Line numbers 500-541

Sample data are listed for an ozone run. They are:

<u>Line</u>	
500	Longitude of spacecraft at a 10-sec interval, beginning with the value nearest to, but less than, the starting time of the scan, in deg
501	Latitude of spacecraft, ditto, in deg
502	Altitude of spacecraft, ditto, in km
503	Greenwich hour angle of Aries at a 10-min interval, centered around the value which is nearest to, but less than, the starting time of the scan, in deg
504	Declination of star in deg, min, sec
505	Right ascension of star in hr, min, sec
506	Starting time in hr, min, sec and number of data points
507	Beginning and ending wavelengths, in Å, and species identifier
508-513	S2F5 filter
514-524	Ozone cross section, in cm <sup>2</sup>
525-530	Stellar spectral values of intensity over the same wavelength intervals
531-539	Molecular oxygen cross section over the same wavelength intervals
540-541	Control values for corrections in Part 3A

### Appendix III b.

This Appendix explains the procedure of finding a random number from a Poisson distribution as an additive scattering term. The procedure is made necessary by the fact that a Gaussian error distribution with a variance equal to the signal intensity causes negative values to occur at times when the scattered signal is superposed upon a basic signal as low as 5 counts/ $\Delta t$ .

The Poisson distribution gives the probability of the number of occurrences of an event within a given time interval, e. g., 1 sec, and it is quite pertinent to the present case. A Poisson distribution, as described by the expression

$$p(n) = \exp [-t] \frac{t^n}{n!}$$

has the values shown in figure when  $t = 10, 1, \text{ and } 0.1$ .  $n$  must be an integer, but  $t$ , the mean signal intensity, is a real number. Since we seek a random number from a Poisson distribution to be added to the signal as random noise, an inversion of the formula is desired. In lieu of this difficult inversion, we can search for a random number,  $y, 0 < y < 1$ , which will permit an  $n$  to be generated such that  $p(n) < y < p(n+1)$ . When the inequality succeeds,  $n$  is accepted as the simulated signal intensity.

The procedure is employed successfully when the signal is between  $10^{-1}$  and 10 counts/ $\Delta t$ . At lower values of the basic signal, the computer search for Poisson random variables becomes too time-consuming so the signal is then left intact.

### Appendix IIIc.

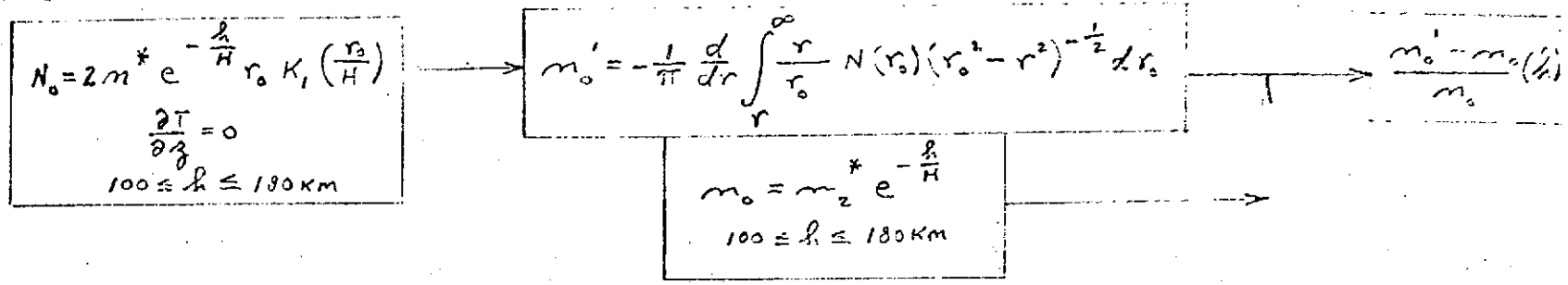
This Appendix uses a simple flow chart and copies of error analysis programs with notes to give the details of the numerical error analysis for each of the species studied. Both single wavelength and broad waveband cases are considered.

The first flow chart applies to molecular oxygen, which is assumed to have a monotonically decreasing number density with increasing height. Following this figure is a program, "THREE", which handles the broadband filter case. Explanatory notes may be found at the end of the program.

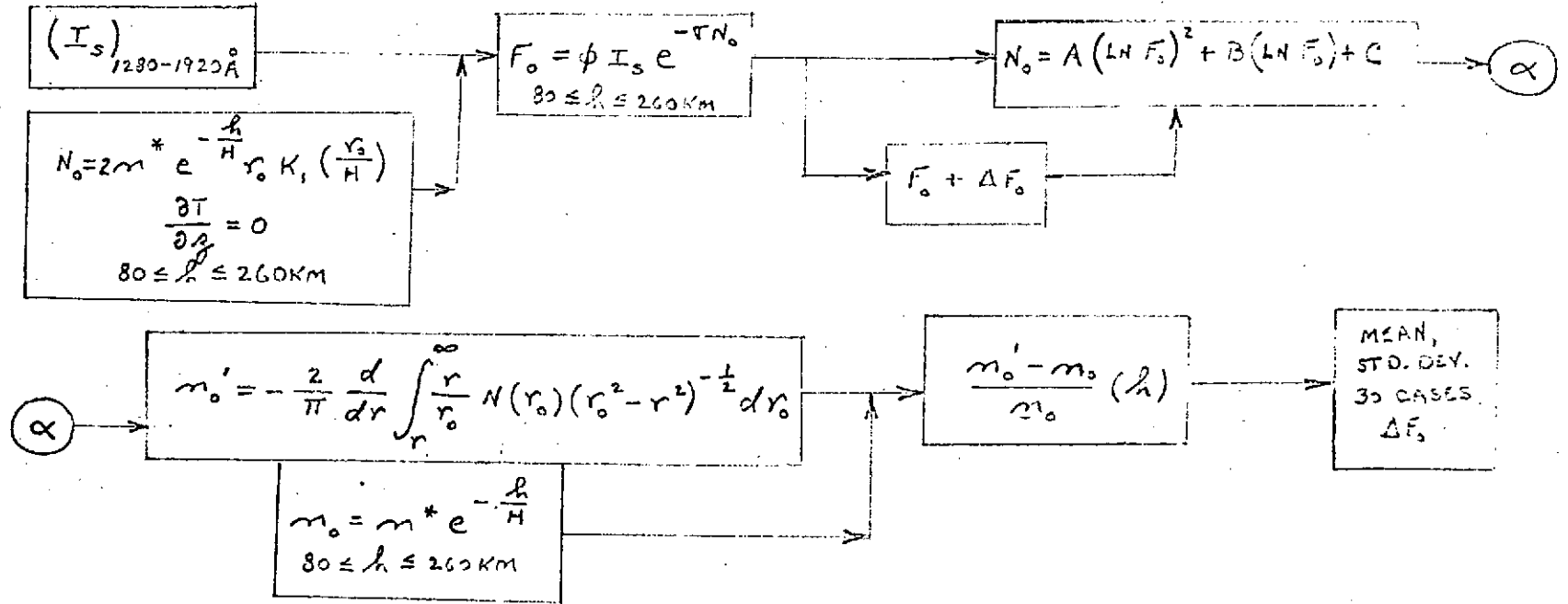
The second flow chart applies to ozone, which is assumed to have a Gaussian-peaked feature superimposed upon a monotonically decreasing number density with increasing height. Following this figure is a program "OZNUM", which applies to the broadband filter. Explanatory notes for this program terminate the Appendix.

1. NUMERICAL SIMULATION

a. SINGLE WAVELENGTH



b. BROADBAND



61

T = TEMPERATURE  
 SUBSCRIPT 0 → DATA POINT HEIGHT  
 N = COLUMN DENSITY  
 m\* = NUMBER DENSITY AT A REFERENCE LEVEL  
 h = HEIGHT ABOVE A REFERENCE LEVEL  
 H = SCALE HEIGHT OF AN ISOTHERMAL ATMOSPHERE

r = DISTANCE FROM EARTH'S CENTER  
 I\_s = STELLAR RADIATION INTENSITY  
 F = SIGNAL INTENSITY  
 phi = FILTER FACTOR  
 tau = ABSORPTION CROSS-SECTION  
 A B C = COEFFICIENTS FOR LEAST SQUARES FITTING

IC SAFE 'MAURICE GRAVES'  
 LAST SIGNON WAS: 08:59.31 01-22-71  
 SER "SAFE" SIGNED ON AT 18:32.05 ON 01-23-71  
 IST THREE(1,50)

```

1 C MOLECULAR OXYGEN NUMERICAL ERROR ANALYSIS, BROADBAND FILTER
2 C
3 IMPLICIT REAL*8(A-F,D-Z)
4 DIMENSION SG(100),WL(100),FIL(100),FILT(100),GRAND(400),KHL(400)
5 IDEN(400),DERR(400),ZKM(400),DEBRA(400),DERRSQ(400),LMEAN(400),
6 ZSIGMA(400),P(16)
7 COMLEN FSGE(400)
8 COMMON Z(400),XN(400),A(400),B(400),C(400),XP(4),FNC(400),
9 IFCF(400),TKK(400),TAC(400),FC,XNC,RE,NP
10 DATA DEBRA,DERRSQ/800*0./
11 SF=10.05
12 VA=1.0
13 WLO=5000.0-E
14 TF=21000.
15 FCO=1.785E15
16 P1=3.1415927
17 RE=6371.05
18 FC=80.05
19 FP=80.05
20 XNC=8.72E13
21 NF=211
22 NPP=181
23 NFH=NFH
24 FLP=.5D5
25 NN=2
26 NH=1
27 J1YMA=6
28 J1ZMAX=6
29 J1X=0
30 J1Y=0
31 J1Z=0
32 C
33 C WHEN MM=MMM, THE PROGRAM COMPUTES MEAN AND STANDARD DEVIATION
34 C OF PERCENTAGE ERROR AT 2KM SPACING FOR MM CASES. WHEN
35 C MM .NE. MMM, THE PROGRAM COMPUTES PERCENTAGE ERROR AND TEMP.
36 C AT VARIOUS SPACINGS WITH RANDOM ERROR(NN=2) OR WITHOUT
37 C RANDOM ERROR(NN=1).
38 C
39 MM=9
40 MMM=30
41 IF(MM.EQ.MMM)WRITE(6,20)MM
42 20 FORMAT(//21F STATISTICS FOR MM=,I3/)
43 IF(MM.EQ.MMM)NH=2
44 C
45 C INPUT C2 CROSS SECTION(SG),WAVELENGTH INTERVAL(WL(J)),
46 C S4F1 FILTER TRANSMISSION(FILT)
47 C
48 READ(60,(SG(I),I=1,32)
49 600 FORMAT(6D10.2)
50 DO 300 I=1,32
51 300 WL(I)=1280.+(I-1)*20.
52 READ(61,(FILT(I),I=1,32)
53 601 FORMAT(11D5.3)
54 C
55 C COMPUTE BLACKBODY STAR SPECTRUM AND ADJUST TO GET DESIRED
56 C MAXIMUM COUNT RATE

```

```

57 C
58 DO 301 I=1,52
59 FIO(I)=DEXP(-(.921*V1+19.3875))*(WL(I)**5)/((WL(I)*1.D-8)**5)*(DEX
60 11.4387/(WL(I)*F)-1.)/(DEXP(1.4387/(WL(I)*1.D-8*F))-1.)*(1.D-7)
61 FIO(I)=FIO(I)*FIO
62 301 CONTINUE
63 C
64 C COMPUTE NORMALIZATION FACTOR
65 C
66 SUM=0.
67 DO 302 J=2,52
68 302 SUM=SUM+((FILT(J)*FIO(J)+FILT(J-1)*FIO(J-1))/2.)*(WL(J)-WL(J-1))
69 WRITE(6,303)SUM
70 303 FORMAT(F10.4)
71 I0=210.D5/(N0-1)
72 C
73 C FORM A TABLE OF COLUMN DENSITY(TT) VERSUS SIGNAL INTENSITY
74 C (SUM) BASED UPON AN INTEGRATION OF THE FORMULA FOR TT WHICH
75 C ASSUMES SPHERICAL STRATIFICATION AND AN ISOTHERMAL ATMOSPHERE
76 C
77 DO 800 IHO=1,NP
78 SUM=0.
79 RO=RE+HO+(IHO-1)*HD
80 ZO=RO-RF
81 IF(IHO.EQ.182) XNO=5.D13
82 IF(IHO.EQ.183) XNO=3.D13
83 IF(IHO.EQ.184) XNO=1.D13
84 DO 701 I=185,210
85 XK=(I-184)/2.
86 701 IF(IHO.EQ.1) XNO=1.D13/1.D1**XK
87 TT=XNO*DEXP(-(ZO-80.D5)/SH)*DSQRT(1.57079*SH*RO)
88 I*(1.+3.*SH/(8.*RO)-15./32.*(SH/RO)**2+315./216.*(SH/RO)**3)
89 IF(IHO.EQ.211) TT=1.D-10
90 TWA(IHO)=TT
91 DO 1000 J=2,32
92 1000 SUM=SUM+((FILT(J)*FIO(J)*DEXP(-SG(J)*TT)+FILT(J-1)*FIO(J-1)*
93 1DEXP(-SG(J-1)*TT))/2.)*(WL(J)-WL(J-1))
94 FNO(IHO)=SUM/SUN
95 PSUB(IHO)=SUM
96 800 CONTINUE
97 IF(NN.EQ.2)GO TO 302
98 C
99 C LOOP FOR NP RANDOM ERRORS IN SIGNAL INTENSITY AND VARIOUS
100 C DATA POINT SPACINGS
101 C
102 DO 2 JIZ=1,JIZMAX
103 DO 602 IH=1,NPP,NH
104 FOF(IH)=FNO(IH)
105 WRITE(6,501)IH,IHW(IH),FNO(IH),FOF(IH)
106 501 FORMAT(I2,I5,3D15.6)
107 602 CONTINUE
108 GO TO 806
109 802 CONTINUE
110 IF(MM.NE.MMM)GO TO 803
111 C
112 C LOOP FOR ACCRUING ERRORS IN D2 DENSITY RESULTING FROM
113 C RANDOMLY SCATTERED SIGNAL INTENSITY, FOR MANY CASES,
114 C WITH A SINGLE DATA POINT SPACING
115 C
116 DO 18 JIX=1,MM

```



```

117 803  CONTINUE
118 C
119 C  SET UP FOR GAUSS SSP AND RANDU SSP
120 C
121 IX=203+100*(JIX-1)
122 IF(MM.NE.MMM)IX=201
123 JX=IX
124 AM=0.
125 P(1)=0.
126 IF(MM.EQ.MMM)GO TO 804
127 C
128 C  LOOP FOR RANDOM ERRORS IN SIGNAL INTENSITY AND VARIOUS
129 C  DATA POINT SPACINGS
130 C
131 DO 28 JIY=1,JIYMAX
132 804  DO 810 IH=1,NPP,NI
133  SUM=FSUB(IH)
134  IF(SUM.LT.10.)GO TO 9
135  SD=DSORT(SUM)
136  CALL GAUSS(IX,SD,AM,V)
137  FOF(IH)=(SUM+V)/SUM
138 503  IF(FOF(IH).GE.1.)FOF(IH)=1.-1.D-5
139  IF(FOF(IH).LT.FNO(1))FOF(IH)=FNO(1)+1.D-12
140  N=100
141  GO TO 809
142 9  IF(SUM.LT..1)FOF(IH)=FNO(IH)
143  IF(SUM.LT..1)N=C
144  IF(SUM.LT..1)GO TO 809
145 C
146 C  DETERMINE A RANDOM NUMBER FROM A POISSON DISTRIBUTION
147 C  WHEN .1<SIGNAL INTENSITY<10.
148 C
149 NMAX=SUN+5.
150 F=DEXP(-SUM)
151 P(2)=SUM*F
152 2  GSUM=P(2)
153  CALL RANDU(JX,JY,YFL)
154  DO 1 N=2,NMAX
155  IF(N.EQ.2)GO TO 10
156  U=SUM** (N-1)*F
157  D=1.
158  II=N-2
159  DO 3 J=1,II
160 3  D=(N-J)*D
161  GSUM=GSUM+U/D
162  P(N)=GSUM
163 10  IF(P(N).GE.YFL.AND.P(N-1).LE.YFL)GO TO 12
164  IF(MM.NE.MMM)WRITE(6,91)N,P(N-1),P(N),YFL
165 91  FORMAT(15,3F12.5)
166 1  CONTINUE
167 13  JX=JY
168  GO TO 2
169 12  JX=JY
170  FOF(IH)=(N-2)/SUM
171  IF(FOF(IH).LT.1.D-10)FOF(IH)=1.D-10
172 809  IF(MM.NE.MMM)WRITE(6,805)IH,FOF(IH),FNO(IH),N,YFL
173 805  FORMAT(15,2D15.6,15,F10.5)
174 810  CONTINUE
175 806  HDP=HDP*2.
176  IF(MM.EQ.MMM)HDP=2.D5

```

```

177 NPH=(NPH-1)/2+1
178 IF(J1Y.EQ.1)NPH=181
179 IF(J1Z.EQ.1)NPH=181
180 IF(MM.EQ.MMM)NPH=91
181 C
182 C COMPUTE O2 NUMBER DENSITY FROM AN EXPONENTIAL MODEL
183 C ATMOSPHERE FOR LATER COMPARISON WITH RETRIEVED VALUES
184 C
185 DO 801 IHO=1,NPH
186 RO=RE+HO+(IHO-1)*HDP
187 CALL ATMOD(RO)
188 ONUMD(IHO)=XP(2)
189 RHTO(IHO)=RO
190 ZKM(IHO)=RHTO(IHO)-RE
191 801 CONTINUE
192 C
193 C INTERPOLATE IN THE TABLE TO FIND COLUMN DENSITY(TNO) FROM
194 C A SIGNAL INTENSITY(FOF) WHICH IS A SIMULATION OF THE
195 C MEASURED SIGNAL
196 C
197 DO 502 J=1,NP
198 Z(J)=DLOG(FNO(J))
199 XN(J)=DLOG(TWV(J))
200 502 CONTINUE
201 M=8
202 IF(NN.EQ.1)M=2
203 CALL PARAB(M)
204 DO 630 L=1,NPP,NH
205 DO 612 I=1,NP
206 IF(FOF(L).GE.FNO(I).AND.FOF(L).LT.FNO(I+1))GO TO 613
207 612 CONTINUE
208 613 J=I
209 X=DLOG(FOF(L))
210 TNO(L)=A(J)*X*X+B(J)*X+C(J)
211 TNO(L)=DEXP(TNO(L))
212 630 CONTINUE
213 C
214 C INVERT COLUMN DENSITY(TNO) TO GET O2 NUMBER DENSITY(DEN)
215 C
216 DO 700 J=1,NPH
217 Z(J)=ZKM(J)
218 K=J*NH-(NH-1)
219 700 XN(J)=TNO(K)
220 M=8
221 IF(NN.EQ.1)M=2
222 CALL PARAB(M)
223 N1=NPH-1
224 DO 4 J=1,N1
225 DEN(J)=0.
226 DO 5 I=J,N1
227 D=2.*A(I)
228 5. DEN(J)=DEN(J)+D*(DSQRT(RHTO(I+1)**2-RHTO(J)**2)-DSQRT(RHTO(I)**
229 1RHTO(J)**2))+(B(I)-2.*A(I)*RE)*DLOG((RHTO(I+1)+DSQRT(RHTO(I+1)
230 2-RHTO(J)**2))/(RHTO(I)+DSQRT(RHTO(I)**2-RHTO(J)**2)))
231 DEN(J)=-DEN(J)/PI*2.
232 C
233 C COMPUTE ERROR AND ACCRUE ERROR SUMS
234 C
235 DERR(J)=(DEN(J)-ONUMD(J))/ONUMD(J)
236 DERRSQ(J)=DERRSQ(J)+DERR(J)*DERR(J)

```

```

237 DERPA(J)=DERRA(J)+DERR(J)
238 4 CONTINUE
239 WRITE(6,85) JIZ,JIY,JIX
240 85 FORMAT(/3I5)
241 IF(MM.EQ.MMM)GO TO 18
242 WRITE(6,24)NPH
243 24 FORMAT(/26H NUMBER OF DATA POINTS =,I3/)
244 WRITE(6,25)NP
245 25 FORMAT(/29H NUMBER OF TABULAR POINTS =,I3//)
246 26 WRITE(6,23)
247 23 FORMAT(/15X,1HZ,14X,1HT,14X,3HNO2,12X,4HCNO2,14X,4HDERK//)
248 C
249 C COMPUTE TEMPERATURE FROM A FORM OF THE HYDROSTATIC LAW
250 C
251 N1=N1-1
252 DO 6 J=1,N1
253 I=0.
254 DO 7 I=J,N1
255 A2=DEN(I+1)/(RHTO(I+1)**2)
256 A1=DEN(I)/(RHTO(I)**2)
257 A1=DABS(A1)
258 A2=DABS(A2)
259 7 T=T+(A2-A1)/DLOG(A2/A1)*(ZKM(I+1)-ZKM(I))
260 T=(3.8038E-4)*T/DEN(J)
261 T=T*RHTO(I)*RHTO(I)
262 IF(MM.NE.MMM)WRITE(6,22)ZKM(J),T,DEN(J),ONUMD(J),DERR(J)
263 22 FORMAT(10X,5D15.6)
264 6 CONTINUE
265 NH=NH*2
266 IF(MM.EQ.MMM)NH=2
267 IF(NN.EQ.1)GO TO 8
268 IF(MM.EQ.MMM)GO TO 18
269 28 CONTINUE
270 IF(NN.FC.2.AND.MM.NE.MMM)GO TO 83
271 18 CONTINUE
272 IF(MM.EQ.MMM)GO TO 79
273 8 CONTINUE
274 IF(MM.NE.MMM)GO TO 80
275 79 WRITE(6,82)
276 82 FORMAT(/78X,1HZ,10X,10HMEAN ERROR,7X,5HSIGMA,10X,5HSUMSQ,
277 111X,3HSUM/)
278 C
279 C COMPUTE STATISTICS FROM ACCRUED SUMS
280 C
281 DO 80 J=1,N1
282 DMEAN(J)=DERRA(J)/(MM*1.)
283 SIGMA(J)=DSQRT(DERRSQ(J)/(MM*1.)-DMEAN(J)*DMEAN(J))
284 WRITE(6,81)ZKM(J),DMEAN(J),SIGMA(J),DERRSQ(J),DERRA(J)
285 81 FORMAT(5D15.6)
286 80 CONTINUE
287 83 CALL EXIT
288 END
289 SUBROUTINE PARAB(M)
290 C
291 C PARAB FINDS THE SECOND ORDER LEAST SQUARES COEFFICIENTS
292 C A,B, AND C FOR INPUT VECTORS Z AND XN, USING M+1 SMOOTHING
293 C POINTS
294 C
295 C IMPLICIT REAL*8(A-H,O-Z)
296 COMMON FSUB(400)

```

```
COMMON Z(400),XN(400),A(400),B(400),C(400),XP(4),FND(400),  
IFUF(400),TWW(400),TNO(400),HO,XNO,RE,NP
```

```
L=M/2
```

```
DO 2 I=1,NP
```

```
IZ=NP+1-L
```

```
K=1
```

```
IF(I.GT.L.AND.I.LT.IZ)K=I-L
```

```
IF(I.GE.IZ)K=NP-M
```

```
ZB=0.
```

```
Z2B=0.
```

```
Z3B=0.
```

```
Z4B=0.
```

```
XNB=0.
```

```
ZNB=0.
```

```
Z2NB=0.
```

```
K2=K+M
```

```
DO 3 J=K,K2
```

```
ZB=ZB+Z(J)
```

```
Z2B=Z2B+Z(J)**2
```

```
Z3B=Z3B+Z(J)**3
```

```
Z4B=Z4B+Z(J)**4
```

```
ZNB=ZNB+Z(J)*XN(J)
```

```
Z2NB=Z2NB+XN(J)*(Z(J)**2)
```

```
3 XNB=XNB+XN(J)
```

```
J=I
```

```
M1=M
```

```
M=M+1
```

```
Y=(M*Z3B-Z2B*ZB)*(M*Z3B-ZB*Z2B)-(M*Z4B-Z2B**2)*(M*Z2B-ZB**2)
```

```
IF(DABS(Y).LT.1.D-40) A(J)=0.
```

```
IF(DABS(Y).LT.1.D-40) GO TO 1
```

```
A(J)=((M*ZNB-XNB*ZB)*(M*Z3B-ZB*Z2B)-(M*Z2NB-XNB*Z2B)*(M*Z2B-ZB*  
1) / ((M*Z3B-Z2B*ZB)*(M*Z3B-ZB*Z2B)-(M*Z4B-Z2B**2)*(M*Z2B-ZB**2))
```

```
1 D=A(J)
```

```
YY=M*Z2B-ZB**2
```

```
IF(DABS(YY).LT.1.D-40) B(J)=0.
```

```
IF(DABS(YY).LT.1.D-40) GO TO 11
```

```
B(J)=((M*ZNB-ZB*XNB)-(M*Z3B-ZB*Z2B)*D)/(M*Z2B-ZB*ZB)
```

```
11 E=B(J)
```

```
C(J)=(XNB-D*Z2B-E*ZB)/M
```

```
M=M1
```

```
2 CONTINUE
```

```
RETURN
```

```
END
```

```
SUBROUTINE ATMOD(RW)
```

```
C
```

```
C
```

```
C
```

```
ATMOD FINDS THE O2 NUMBER DENSITY AT ALTITUDE ZW
```

```
IMPLICIT REAL*8(A-H,O-Z)
```

```
COMMON FSUB(400)
```

```
COMMON Z(400),XN(400),A(400),B(400),C(400),XP(4),FND(400),  
1FUF(400),TWW(400),TNO(400),HO,XNO,RE,NP
```

```
PE=6371.D5
```

```
ZH=PW-RE
```

```
XP(2)=8.72013*DEXP(-(ZW-80.D5)/10.D5)
```

```
RETURN
```

```
END
```

2.50E-19	5.00E-19	1.50E-18	2.30E-18	8.00E-18	1.30E-17
1.40E-17	1.41E-17	1.42E-17	1.30E-17	1.26E-17	1.10E-17
1.00E-17	8.20E-18	7.00E-18	6.00E-18	4.50E-18	3.50E-18
2.50E-18	1.80E-18	1.20E-18	8.00E-19	5.50E-19	3.50E-19

357	2.00E-19	1.60E-19	1.00E-19	8.00E-20	5.00E-20	3.00E-20				
358	1.30E-20	7.00E-21								
359	.040	.080	.120	.160	.230	.320	.440	.600	.700	.920
360	.990	1.000	.990	.980	.970	.960	.880	.780	.640	.500
361	.320	.260	.200	.170	.110	.080	.060	.040	.030	.020

END OF FILE

Part I: Line numbers 1-62

In this part, the initial parameter values are set and the input data are read.

Notation

- SH =  $O_2$  scale height for an isothermal atmosphere, in cm
- VM = visual magnitude of star
- WLO = reference wavelength for stellar spectrum, in cm
- TF = Effective blackbody stellar temperature, in  $^{\circ}K$
- FOO = adjusting factor for obtaining a desired fictitious count rate at the top of the scan
- RE = average radius of the earth, in cm
- HO, HP = base height of the  $O_2$  profile, in cm
- XNO =  $O_2$  number density at the base height, in molecules/cm<sup>3</sup>
- NP = number of points in a formulated table of normalized signal v. column density. Note: column density = mass of species in a 1 cm<sup>2</sup> channel centered on the stellar ray, also called the total integrated number density.
- NPP, NPH = number of data points in the profile
- HDP = halfstepsize for computing  $O_2$  densities in an exponential atmosphere, in cm
- NN = switch for introducing random error into the signal intensity
- NH = stepsize for computing  $O_2$  number densities, in number of tabular data points
- JIYMAX = limiting value of the number of data point spacings used, with random error in signal
- JIZMAX = limiting value of the number of data point spacings used, without random error in signal

JIX, JIY, JIZ =counters

MM, MMM = switch for selecting one of three paths (see program comments)

SG = O<sub>2</sub> absorption cross sectional values in cm<sup>2</sup>;  
the values are listed at the end of the program

WL = wavelength in cm

FILT = normalized values of S4F1 filter transmission; the values are listed after the SG values at the end of the program

FIO = stellar spectral intensity, in digital voltmeter counts/ sec

Part II: Line numbers 63-96

In this part, a normalization factor for the signal intensity is computed and a table of normalized signal intensity v. column density (see NP under Notation, Part I) is formed. In making the table, arbitrary values of  $O_2$  number density are selected which cover the expected range of values of the species profile. Then the expression for the column density,

$N = 2 \int_{r_0}^{\infty} n(r) r (r^2 - r_0^2)^{-1/2} dr$  is integrated numerically by an expansion of the modified Bessel function of the first kind, assuming an isothermal atmosphere:

$$N_0 = 2 n_0^* e^{-\frac{r_0 - r_0^*}{H}} r_0 K_1\left(\frac{r}{H}\right) = 2 n_0^* e^{-\frac{r_0 - r_0^*}{H}} \left(\frac{\pi H r_0}{2}\right)^{1/2} \left(1 + \frac{3}{8} \frac{H}{r_0} + \dots\right)$$

$n_0$  = number density

$r_0$  = distance to the earth's center

H = scale height

$K_1$  = modified Bessel function of the first kind

Superscript \* refers to the reference height, which is 80 Km in this case.

Then the signal intensity corresponding to N is computed from the expression,

$$I = \int_0^{\infty} T(\lambda) I_{\infty}(\lambda) e^{-\sigma(\lambda) N} d\lambda \left[ \int_0^{\infty} T(\lambda) I_{\infty}(\lambda) d\lambda \right]^{-1}$$

T = filter transmission

$\lambda$  = wavelength

$\sigma$  = species cross section

$I_{\infty}$  = integrated stellar spectral intensity without atmospheric attenuation

Complete notes on the derivation of the formula for  $N_0$  are included on the pages preceeding Part III. There are some minor differences in notation.



## Notation

- SUN = integration factor, or the integrated stellar spectral intensity,  
in counts/sec
- HD = stepsize for altitude of the ray's tangent point used in the  
calculation of the column densities, in cm
- IHO = counter and altitude setter; in lines 81-89 it is used to arbi-  
trarily reduce the column density to near zero at the upper  
end of the scan
- SUM = filtered signal intensity integrated over wavelength, in  
counts/sec
- RO = distance from the center of the earth to the ray's tangent  
point, in cm
- ZQ = altitude of the ray's tangent point, in cm
- TT, TWW = column density for the table, computed at altitude ZQ, in  $\text{cm}^{-2}$
- FNO = integrated, normalized signal intensity corresponding to TT
- FSUB = storage vector for the integrated, unnormalized signal  
intensity corresponding to TT, in counts/sec

Part III: Line numbers 97-174

In this part, the simulated signal intensity is obtained. If no random error is to be admitted, lines 98-108 select certain tabular values of the integrated, normalized signal intensity as the simulated signal intensity. If random error is to be introduced, there are two possibilities: (1) scatter the signal values with a set of random numbers from a Gaussian distribution with its mean = 0 and its variance = the signal intensity; (2) find positive integral values of the signal from a Poisson distribution having its parameter = signal intensity.

A Poisson distribution is characterized by the probability function,

$$P(n) = 0, \text{ n not an integer}$$

$$P(n) = \frac{I^n e^{-I}}{n!}, \text{ n} = 0, 1, 2, \dots$$

I = mean signal intensity, the Poisson parameter in this case

To find an n which belongs to a Poisson distribution, the procedure employed (lines 123, 125, and 145-171) is this:

1. Seek a random number (YFL) between 0 and 1 which permits a random variable, n, to be found such that  $P(n) < YFL < P(n+1)$ .

a. The search begins with n = 0 and proceeds with increasing integral values. This permits the probabilities to be cumulated, i. e.,

$$P(3) = \frac{I^3 e^{-I}}{3!} + P(2) + P(1).$$

b. An upper limit on n of I + 5 is imposed for computing economy.

c. If n = 0 is the outcome, substitute a small positive value for 0 ( $n=10^{-10}$ ) to allow subsequent computation to continue.

2\* Assign n as the value of the signal

In practice, the Poisson random variables were sought when  $0.1 < I < 10$ . At the lower limit,  $P(n \geq 1) = 0.12$ , and the search procedure is quite time-consuming at lesser values of I. Below 0.1, the tabular number is accepted as the simulated signal, but a different smoothing procedure in the interpolation subroutine produces a number density unlike that for the no-scatter case. At the upper limit and beyond, the Gaussian distribution yields a real number which is added to the signal in a rapid computer operation.

#### Notation

- JIZ = loop counter for no-scattering case
- FOF = simulated (measured) signal intensity, in counts/sec
- JIX = loop counter for any quantity of scattering cases with a single data-point spacing
- IX = GAUSS subroutine argument; it must be changed manually at the start of a run if new random numbers are desired; it is an odd, positive integer
- JX = RANDU subroutine argument; it must be changed manually before each entry to avoid repetition
- AM = GAUSS subroutine argument: the mean of the distribution
- P = probability of a Poisson random variable equalling the integral value of the subscript minus 1
- JIIY = loop counter for one scattering case with variable data-point spacing
- SD = GAUSS subroutine argument: the standard deviation of the distribution

---

\* n-2 in the program corresponds to the n discussed here.

V = GAUSS subroutine argument: the value returned  
N = indicator of the program path taken and the Poisson integer  
NMAX = upper limit on the Poisson integer  
U = numerator in the Poisson formula  
D = denominator in the Poisson formula

Part IV: Line numbers 175-238

In this part, O<sub>2</sub> number densities from an exponential atmosphere pinned at 80 Km are computed (lines 175-191) for later comparison with retrieved number densities. The former values are furnished by subroutine ATM0D.

In order to carry out a mathematical inversion leading to retrieved number density, the column density must first be obtained. This is achieved through a logarithmic interpolation scheme as follows:

1. The table (Part III) is converted to the natural logarithms of signal intensity (FNO) and column density (TNO).
2. Using subroutine PARAB, a least squares parabola is fitted to M + 1 values of ln(FNO). This is done at each tabular value of ln(FNO) to obtain the coefficients (A, B, C).
3. The simulated signal (FOF) is located between tabular values of FNO, its logarithm is found, and the column density (N) is computed by 
$$N_i = \exp \left[ A_i \ln I)^2 + B_i \ln I = C_i \right]$$
.

Tests with various numbers of smoothing points indicate that 3(M = 2) is the optimum number for unscattered signal intensities and 9 (M = 8) is optimum for randomly scattered data.

The mathematical inversion is represented by

$$m'_c = -\frac{2}{\pi} \frac{d}{dr} \int_{r_0}^{\infty} \frac{r}{r_0} N(r_0) (r_0^2 - r^2)^{-\frac{1}{2}} dr,$$

and the error analysis may be expected to invert  $N(r_0)$  according to an expression such as

$$m(r_0) = -\frac{2}{\pi} \sum_{i,j}^{m-1} \left\{ \frac{N_{i+1} - N_i}{r_{i+1} - r_i} \ln \left[ \frac{r_{i+1} + (r_{i+1}^2 - r_j^2)^{1/2}}{r_i + (r_i^2 - r_j^2)^{1/2}} \right] \right\}$$

However, as may be seen in the ensuing notes, the random noise being superposed is

$$\Delta m(r_0) = -\frac{2}{\pi} \int_{r_0}^{\infty} \frac{d\Delta N}{dr} (r^2 - r_0^2)^{-\frac{1}{2}} dr,$$

and smoothing of  $\Delta N$  is necessary, i. e.,  $\Delta N = A(r-r_0)^2 + B(r-r_0) + C$ .

The resulting formula is

$$\Delta n(r_j) = -\frac{2}{\pi} \sum_{i=j}^{\infty} \left\{ A_i (r_{i+1}^2 - r_j^2)^{1/2} - (r_i^2 - r_j^2)^{1/2} + (B_i - 2A_i r_0) \ln \left[ \frac{r_{i+1} + (r_{i+1}^2 - r_j^2)^{1/2}}{r_i + (r_i^2 - r_j^2)^{1/2}} \right] \right\}$$

The coefficients  $A_i$  and  $B_i$  are again supplied by subroutine PARAB, the arguments of the table becoming altitude (ZKM) and column density (TNO).

The error,  $(n_0' - n_0) / n_0$  is finally computed at each altitude level and its sums and squares of sums are accrued for multiple cases (lines 232-238).

#### Notation

ONUMD, XP(2) =  $O_2$  number density from the exponential model atmosphere,  
in  $\text{cm}^{-3}$

RHTO, RO = distance of the ray tangent point from the earth's center, in cm

ZKM = altitude of the ray tangent point, in cm

Z = abscissa of the table

XN = ordinate of the table

M = one less than the number of smoothing points taken for a  
least squares fit in subroutine PARAB

X = entry value for a logarithmic interpolation

DEN = retrieved number density, in  $\text{cm}^{-3}$

DERR = error in number density

DERRSQ = sum of squares of DERRs

DERRA = sum of DERRs

A, B, C, = least squares coefficients for a parabolic fit

Part V: Line numbers 239-288

In this part, the molecular temperature (T) is computed from the expression

$$T_{O_2} = \frac{m \bar{z}}{k n_{O_2}(z)} \int_z^{\infty} n_{O_2}(p) dp$$

which is derived from the hydrostatic approximation (see the following notes).

If multiple cases are being handled, the mean and standard deviation are then computed for the errors at each altitude level.

Notation

T = molecular temperature, in °K

MM, MMM = number of cases for error statistics

DMEAN = mean error in number density

SIGMA = one standard deviation of number density error

Part VI: Line numbers 353-361

In this part, the  $O_2$  scattering cross sectional values are listed at an interval of  $20 \text{ \AA}$  (lines 353-358).

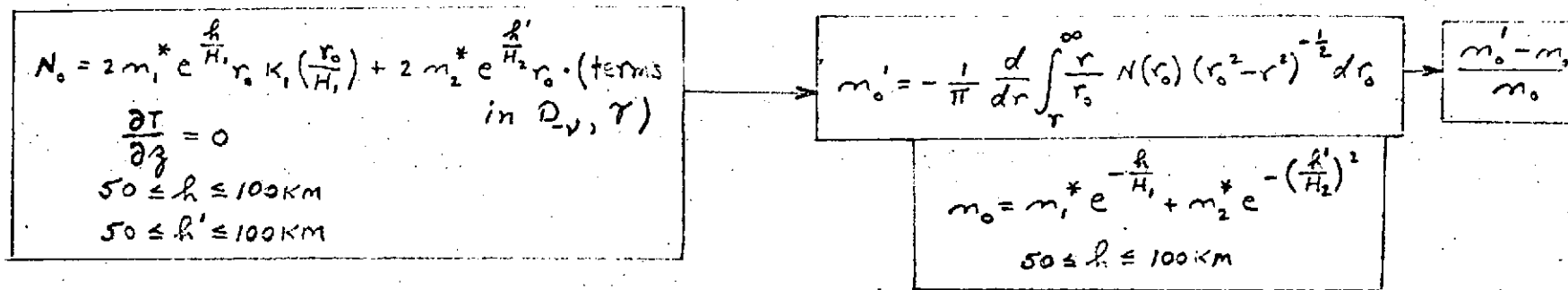
The S4F1 filter transmission values, adjusted to a maximum value of 1.0, are also listed at the same wavelength interval (lines 359-361).



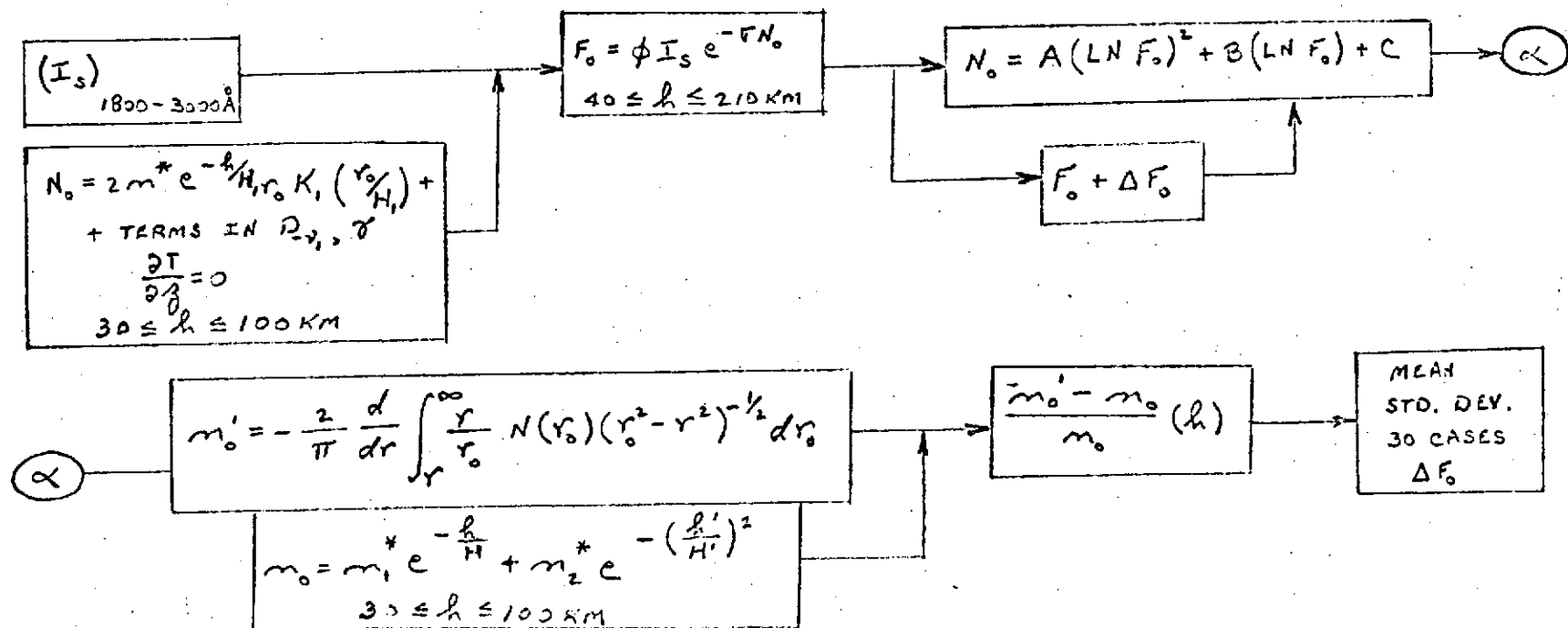
O<sub>3</sub>

1. NUMERICAL SIMULATION

a. SINGLE WAVELENGTH



b. BROADBAND



$h'$  = HEIGHT ABOVE "GAUSSIAN" REFERENCE LEVEL  
 $H_1$  = O<sub>3</sub> SCALE HEIGHT  
 $H_2$  = O<sub>3</sub> " " AROUND THE "GAUSSIAN" ALT.  
 $m_1^*$  = NUMBER DENSITY AT A REFERENCE LEVEL

$m_2^*$  = NUMBER DENSITY AT THE "GAUSSIAN" ALT  
 $D_{-1}$  = WEBER PARABOLIC CYLINDER FUNCTION  
 $\gamma$  = GAMMA FUNCTION  
 REST OF NOTATION SAME AS FOR O<sub>3</sub>

SIG SAFF T=10 F=10 MAURICE GRAVES  
 \*LAST SIGNON WAS: 16:25.53 07-06-71  
 USER "SAFF" SIGNED ON AT 15:32.47 ON 07-12-71  
 LIST CZNUM

```

1 C OZONE NUMERICAL EFFECT ANALYSIS
2 C
3 IMPLICIT REAL*8 (A-H,C-Z)
4 DIMENSION SG(100),WL(100),FIC(100),FILT(100),CNUMD(400),RHTO(400)
5 1DEN(400),DERR(400),ZKM(400),TNOX(400),P(16),LERRA(400),
6 2DERRSQ(400),LMEAN(400),SIGMA(400)
7 CCMCN Z(400),XN(400),A(400),B(400),C(400),XP(4),FNO(400),
8 1FOP(400),TWW(400),TNC(400),HC,XNO,RE,NE
9 LATA LERRA,DERRSQ/800*0./
10 SH=4.3415
11 XNOC3=8.18
12 ZWOO=83.D5
13 SEOO=5.15
14 VM=1.
15 WLO=5000.E-8
16 TF=19000.
17 FCO=.981E16
18 EI=3.141592653589793
19 FE=6.71.E5
20 HC=30.D5
21 BE=HC
22 XNO=5.D10
23 NE=171
24 NEP=141
25 MEH=NEP
26 HLP=5.D4
27 MN=2
28 MH=1
29 JIY=0
30 JIZ=0
31
32 C
33 C WHEN MM=MMM, THE PROGRAM COMPUTES MEAN AND STANDARD DEVIATION
34 C OF PERCENTAGE ERROR AT HCP SPACING FOR MM CASES. WHEN
35 C MM .NE. MMM, THE PROGRAM COMPUTES PERCENTAGE ERROR AND TEMP.
36 C AT VARIOUS SPACINGS WITH RANDOM ERROR (NN=2) OR WITHOUT
37 C RANDOM ERROR (MM=1).
38 C
39 MM=10
40 MMM=10
41 IF (MM.EQ.MMM) WRITE (6,20) MM
42 20 FORMAT (//21H STATISTICS FOR MM=,I3/)
43 C
44 C INPUT OF CROSS SECTION (SG), WAVELENGTH INTERVAL (WL), FILTER
45 C TRANSMISSION (FILT)
46 C
47 READ (5,600) (SG(I),I=1,61)
48 600 FORMAT (6D10.2)
49 DC 300 I=1,61
50 WL(I)=1800.+(I-1)*20.
51 C
52 FILTER S2F5 TRANSMISSION
53 C
54 READ (5,601) (FILT(I),I=1,61)
55 601 FORMAT (11F5.3)
56 C
57 COMPUTE EIACKEDY STAR SPECTRUM AND ADJUST TO GET DESIRED

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55 C MAXIMUM COUNT RATE
56 C
57 DC 301 I=1,61
58 FIO(I)=DEXP(-(.921*VM+19.3875))*(WLO**5)/((WL(I)*1.D-8)**5)*(DEX
59 11.438/(WLC*TF))-1.)/(DEXP(1.438/(WL(I)*1.D-8*TF))-1.)*(1.D-7)
60 FIO(I)=FIC(I)*FOO
61 301 CCNTINUE
61.1 C
61.2 C COMPUTE NORMALIZATION FACTOR
61.3 C
62 SUN=C.
63 IC 302 J=2,61
64 302 SUN=SUN+1*(FILT(J)*FIC(J)+FILT(J-1)*FIO(J-1))/2.*(WL(J)-WL(J-1))
65 WRITE(6,303) SUN
66 303 FCRMA1(/L15.6/)
67 HL=170.D5/(NP-1)
68 C
69 C FROM A TABLE OF COLUMN DENSITY(TT) VERSUS SIGNAL INTENSITY
70 C (SUM) BASED UPON AN INTEGRATION OF THE FORMULA FOR TT WHICH
71 C ASSUMES SPHERICAL STRATIFICATION AND AN ISOTHERMAL ATMOSPHERE
72 C
73 DC 800 IHC=1,NE
74 SUM=0.
75 RC=RE+HC+(IHC-1)*HE
76 ZC=RC-RE
77 TT=XNC*DEXP(-(ZC-50.D5)/SH)*DSQRT(1.57079*SH*RO)
78 1*(1.+3.*SH/(E.*RO)-15./32.*(SH/RO)**2+315./216.*(SH/RC)**3)
79 C1=2./(SHCO*SHCC)
80 VA=ZC-ZWCC
81 VE=VA*1.4142/SHCO
82 IF(DAES(VE).GT.12.6) TTG=C.
83 IF(DAES(VE).GT.12.8) GC TC 14
84 CYL1=PCFN(-.5,VB)
85 CYL2=ECFN(-1.5,VE)
86 TTG=(XNCC3*DEXP(-(VA/SHOO)**2)/DSQRT(2.*RC))*(.8862269/(C1
87 1**75)*DEXP(VA*VA/(2.*SHCC*SHCO))*CYL2+RO/(C1**25)*1.7724538*
88 2DEXP(VA*VA/(2.*SHOO*SHOO))*CYL1)
89 14 TNOX(IHC)=TT+TTG
90 IF(IHC.GT.162) IT=10.**((NE-IHC)/5.)
91 IF(IHC.EQ.171) IT=1.D-10
92 TWW(IHC)=IT
93 IC 1000 J=2,61
94 1000 SUM=SUM+((FILT(J)*FIC(J)*DEXP(-SG(J)*TWW(IHO))+FILT(J-1)*
95 1FIO(J-1)*DEXP(-SG(J-1)*TWW(IHC)))/2.)*(WL(J)-WL(J-1))
96 FNC(IHC)=SUM/SUN
97 WRITE(6,504) IHC,TWW(IHO),FNC(IHO),TTG
98 504 FORMAT(T2,I5,4D15.6)
99 800 CCNTINUE
100 IF(NN.EQ.2) GO TO 900
101 C
102 C LOOP FOR NO RANDOM ERRORS IN SIGNAL INTENSITY AND VARIOUS
103 C DATA POINT SPACINGS
104 C
105 JIZMAX=5
106 DC 8 JI2=1,JIZMAX
107 900 DO 801 IHO=1,NPP,NH
108 SUM=0.
109 IC 1001 J=2,61
110 1001 SUM=SUM+((FILT(J)*FIC(J)*DEXP(-SG(J)*TNOX(IHC))+FILT(J-1)*
111 1FIO(J-1)*DEXP(-SG(J-1)*TNCX(IHO)))/2.)*(WL(J)-WL(J-1))

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```

112 FCF(IHO)=SUM/SUN
113 WRITE(6,501)IHC,TNOX(IHC),FCF(IHO)
114 501 FORMAT(T2,I5,2D15.6)
115 801 CONTINUE
116 IF(NN.EQ.1)GO TO 806
117 IF(MM.NE.MMM)GC TC 803
118 C
119 C LOOP FOR ACCRUING ERRORS IN C3 DENSITY RESULTING FROM
120 C RANDOMLY SCATTERED SIGNAL INTENSITY, FOR MANY CASES,
121 C WITH A SINGLE DATA FCINT SPACING
122 C
123 DC 18 JIY=1,MM
124 803 CONTINUE
125 C
126 C SET UP FOR GAUSS SSP AND RANDU SSP
127 C
128 IX=203+100*(JIX-1)
129 IF(MM.NE.MMM)IX=201
130 JX=IX
131 AM=0.
132 F(1)=0.
133 IF(MM.EQ.MMM)GC TC 804
134
135.1 C
136 C LOOP FOR RANDOM ERRORS IN SIGNAL INTENSITY AND VARIOUS
137 C DATA FCINT SPACINGS
138 C
139 JIYMAX=1
140 DC 28 JIY=1,JIYMAX
141 804 DO 810 IH=1,NPP,NH
142 SUM=FCF(IH)*SUM
143 IF(SUM.LT.10.)GO TO 9
144 SD=DSQRT(SUM)
145 CALL GAUSS(IX,SD,AM,V)
146 FCF(IH)=(SUM+V)/SUM
147 503 IF(FCF(IH).GE.1.)FOF(IH)=1.-1.D-5
148 IF(FCF(IH).LT.FNO(1))FOF(IH)=FNO(1)+1.D-12
149 N=100
150 GC TC 809
151 9 IF(SUM.LT..1)FCF(IH)=FNO(IH)
152 IF(SUM.LT..1)N=0
153 IF(SUM.LT..1)GO TO 809
154 C
155 C DETERMINE A RANDOM NUMBER FROM A POISSON DISTRIBUTION
156 C WHEN .1<SIGNAL INTENSITY<10.
157 C
158 NMAX=SUM+5.
159 F=DEXP(-SUM)
160 F(2)=SUM*F
161 2 GSUM=F(2)
162 CALL RANDU(JX,JY,YFL)
163 DC 1 N=2,NMAX
164 IF(N.EQ.2)GC TC 10
165 U=SUM**(N-1)*F
166 D=1.
167 II=N-2
168 DC 3 J=1,II
169 3 D=(N-J)*I
170 GSUM=GSUM+U/D
171 F(N)=GSUM
172 10 IF(P(N).GE.YFL.AND.P(N-1).LE.YFL)GO TO 12

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```

173 1 CCNTINUE
174 13 JX=JY
175 CC TO 2
176 12 JX=JY
77 FCF(IH) = (N-2) / SUN
178 IF (FCF(IH) .LT. 1.D-10) FOF(IH) = 1.D-10
179 809 IF (MM.EQ.MMM) WRITE(6,805) IH, FOF(IH), FNO(IH), N, YFL
180 805 FORMAT(15,2D15.6,15,F10.5)
181 810 CCNTINUE
182 806 HDE=HLE*2.
183 IF (MM.EQ.MMM) HLE=.5DE
184 NPH=(NEH-1)/2+1
185 IF (J1Y.EQ.1) NPH=141
186 IF (J1Z.EQ.1) NPH=141
186.1 IF (MM.EQ.MMM) NPH=261
187 C
188 C COMPUTE C2 NUMBER DENSITY FROM AN EXPONENTIAL MODEL
188.1 C ATMOSPHERE WITH A SUPERPOSED GAUSSIAN CURVE FOR LATER
189 C COMPARISON WITH RETRIEVED VALUES
190 C
191 DC 807 IHC=1, NPH
192 RC=RE+HC + (IHC-1) *BLE
193 CALL ATMCE(RO)
194 CNUMD(IHC)=XP(2)
195 RHIC(IHC)=RO
196 ZKM(IHC)=RHIC(IHC)-RE
197 807 CCNTINUE
198 C
199 C INTERPOLATE IN THE TABLE TO FIND COLUMN DENSITY (TNO) FROM
200 C A SIGNAL INTENSITY (FOF) WHICH IS A SIMULATION OF THE
201 C MEASURED SIGNAL
202 C
203 DC 502 J=1, NP
204 Z(J)=DLCC(FNO(J))
205 XN(J)=DICG(TWK(J))
206 502 CCNTINUE
207 M=8
208 IF (NN.EQ.1) M=2
209 CALL FARAP(M)
210 DC 630 I=1, NPH, NH
211 IF (FCF(I) .LT. FNO(1)) FOF(I) = FNC(1) + 1.D-12
212 DC 612 I=1, NP
213 IF (FOF(I) .GE. FNO(I) .AND. FCF(I) .LT. FNO(I+1)) GC TO 613
214 612 CCNTINUE
215 613 J=I
216 X=DLCC(FCF(I))
217 TNO(L) = A(J) * X * X + B(J) * X + C(J)
218 TNO(I) = DEXP(TNC(L))
219 630 CCNTINUE
220 C
221 C INVERT COLUMN DENSITY (TNC) TO GET O2 NUMBER DENSITY (DEN)
222 C
223 DC 700 J=1, NPH
224 Z(J)=ZKM(J)
225 K=J*NH - (NH-1)
226 700 XN(J)=TNC(K)
227 M=8
228 IF (NN.EQ.1) M=2
229 CALL FARAP(M)
230 N1=NPH-1

```

```

231      EC 4 J=1,N1
232      DEN(J)=C.
233      IC 5 I=J,N1
234      E=2.*A(I)
235      IT=DAES(RHTO(I)**2-RHTO(J)**2)
236      5      DEN(J)=DEN(J)+D*(DSQRT(RHTC(I+1)**2-RHTO(J)**2)-DSQRT(DT))
237      1+(B(I)-2.*A(I)*RE)*DLOG((RHTC(I+1)+DSQRT(RHTO(I+1)**2
238      2-RHTO(J)**2))/(RHTC(I)+DSQRT(DT)))
239      DEN(J)=-DEN(J)/PI*2.
240      C
241      C      COMPUTE ERROR AND ACCRUE EFFECT SUMS
242      C
243      DERR(J)=(DEN(J)-ONUM(J))/CNUMD(J)
244      LEERRSQ(J)=DERRSQ(J)+LEERR(J)*DERR(J)
245      DERRA(J)=DERRA(J)+LEERR(J)
246      4      CCNTINUE
247      85      WRITE(6,85)JIZ,JIY,JIX
248      FORMAT(/3I5)
249      IF(MM.EQ.MMM)GC TO 18
250      WRITE(6,24)NPE
251      24      FORMAT(/26H      NUMBER OF DATA POINTS =,I3/)
252      WRITE(6,25)NP
253      25      FORMAT(/29H      NUMBER OF TABULAR POINTS =,I3//)
254      WRITE(6,23)
255      23      FORMAT(/18X,1HZ,13X,3HNC3,12X,4HCNO3,12X,4HDERR//)
256      N1=N1-1
257      IC 6 J=1,N1
258      IF(MM.NE.MMM)WRITE(6,22)ZKM(J),DEN(J),CNUMD(J),DERR(J)
259      22      FORMAT(10X,4D15.6)
260      6      CCNTINUE
261      29      NH=NH*2
262      IF(MM.EQ.MMM)NH=1
263      IF(MM.EQ.1)GC TO 8
264      IF(MM.EQ.MMM)GC TO 18
265      28      CCNTINUE
266      IF(MM.EQ.2.AND.MM.NE.MMM)GC TO 83
267      18      CCNTINUE
268      IF(MM.EQ.MMM)GC TO 79
269      8      CCNTINUE
270      IF(MM.NE.MMM)GC TO 80
271      79      WRITE(6,82)
272      82      FORMAT(/8X,1HZ,10X,10HMEAN ERROR,7X,5HSIGMA,10X,5HSUMSQ,
273      111X,3ESUM/)
274      C
275      C      COMPUTE STATISTICS FROM ACCRUED SUMS
276      C
277      LC 80 J=1,N1
278      DMEAN(J)=DERRA(J)/(MM*1.)
279      SIGMA(J)=DSQRT(DERRSQ(J)/(MM*1.)-DMEAN(J)*DMEAN(J))
280      81      WRITE(6,81)ZKM(J),DMEAN(J),SIGMA(J),DERRSQ(J),DERRA(J)
281      80      FORMAT(5I15.6)
282      83      CCNTINUE
283      CALL EXIT
284      END
285      SUBROUTINE PARAB(M)
286      C
287      C      PARAB FINDS THE SECOND ORDER LEAST SQUARES COEFFICIENTS
288      C      A,B, AND C FOR INPUT VECTORS Z AND XN, USING M+1 SMOOTHING
289      C      FCINTS

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```

290      IMPLICIT REAL*8 (A-H,O-Z)
291      COMMON Z(400),XN(400),A(400),E(400),C(400),XP(4),FNO(400),
292      1FCF(400),TWW(400),TNC(400),HC,XNO,RE,NP
293      I=M/2
294      DO 2 I=1,NP
295      IZ=NP+1-I
296      K=1
297      IF (I.GT.L.AND.I.LT.IZ) K=I-1
298      IF (I.GE.IZ) K=NP-M
299      ZE=0.
300      Z2B=0.
301      Z3B=0.
302      Z4B=0.
303      XNB=0.
304      ZNB=0.
305      Z2NB=C.
306      K2=K+M
307      DO 3 J=K,K2
308      ZB=ZB+Z(J)
309      Z2B=Z2B+Z(J)**2
310      Z3B=Z3B+Z(J)**3
311      Z4B=Z4B+Z(J)**4
312      ZNB=ZNB+Z(J)*XN(J)
313      Z2NB=Z2NB+XN(J)*(Z(J)**2)
314      3 XNB=XNB+XN(J)
315      J=I
316      M1=M
317      M=M+1
318      Y=(M*Z3B-Z2B*ZE)*(M*Z3B-ZE*Z2B)-(M*Z4B-Z2B**2)*(M*Z2B-ZB**2)
319      IF (DAES(Y).LT.1.D-40) A(J)=0.
320      IF (LAES(Y).LT.1.D-40) GO TC 1
321      A(J)=((M*ZNB-XNB*ZB)*(M*Z3B-ZB*Z2B)-(M*Z2NB-XNB*Z2B)*(M*Z2B-ZB*
322      1)/((M*Z3B-Z2B*ZB)*(M*Z3B-ZE*Z2B)-(M*Z4B-Z2B**2)*(M*Z2B-ZB**2))
323      1 I=A(J)
324      YY=M*Z2B-ZB**2
325      IF (DAES(YY).LT.1.D-40) B(J)=0.
326      IF (LAES(YY).LT.1.D-40) GO TC 11
327      E(J)=((M*ZNB-ZB*XNB)-(M*Z3B-ZE*Z2B)*D)/(M*Z2B-ZB*ZB)
328      11 E=B(J)
329      C(J)=(XNB-D*Z2B-E*ZB)/M
330      M=M1
331      2 CCNTINUE
332      RETURN
333      END
334      SUBROUTINE ATMOD(RW)
335      C
336      C      ATMOD FINDS THE O3 NUMBER DENSITY AT ALTITUDE ZW FOR AN
337      C      ISOTHERMAL ATMOSPHERE WITH A SUPERPOSED GAUSSIAN CURVE
338      C
339      IMPLICIT REAL*8 (A-H,C-Z)
340      COMMON Z(400),XN(400),A(400),B(400),C(400),XP(4),FNO(400),
341      1FCF(400),TWW(400),TNC(400),HC,XNO,RE,NP
342      RE=6371.D5
343      ZW=RW-RE
344      XP(2)=5.D10*DEXP(-(ZW-50.D5)/4.34D5)+8.D8*DEXP(-(ZW-83.D5)
345      1/5.D5)**2)
346      RETURN
347      END
348      SUBROUTINE GAMMA(XY,GX,IEE)
349      IMPLICIT REAL*8 (A-H,O-Z)

```

```

350      IF (XX-57.) 6,6,4
351      IER=2
352      GX=1.D7E
353      RETURN
354      6      X=XX
355      ERR=1.0E-6
356      IER=0
357      GX=1.0
358      IF (X-2.C) 50,50,15
359      10      IF (X-2.C) 110,110,15
360      15      X=X-1.0
361      GX=GX*X
362      GO TO 1C
363      50      IF (X-1.) 60,120,110
364      60      IF (X-ERR) 62,62,80
365      62      Y=FICAT (IDINT (X)) -X
366      IF (LAES (Y) -ERR) 130,130,64
367      64      IF (1.-Y-ERR) 130,130,70
368      70      IF (X-1.) 80,80,110
369      80      GX=GX/X
370      X=X+1.
371      GO TO 7C
372      110     Y=X-1.
373      GY=1.0+Y* (-0.5771017+Y* (0.9858540+Y* (-0.8764218+Y* (0.8328212+
374      1Y* (-0.5664729+Y* (0.2548205+Y* (-0.05149930))))))
375      GX=GX*GY
376      120     RETURN
377      130     IER=1
378      RETURN
379      END
380      FUNCTION FCFN (V, X)
381      IMPLICIT REAL*8 (A-H, C-Z)
382      SQRTPI=1.77245385090551
383      SQRT2=1.41421356237309
384      FAC1=2.** (-0.5*V-0.5) *SQRTPI*DEXP (-0.25*X*X)
385      CALL GAMMA (.5*V+1., GAM1, IEF)
386      CALL GAMMA (.5*V+0.5, GAM2, IER)
387      FAC2=CFHG (.5*V+0.5, 0.5, 0.5*X*X) /GAM1-SQRT2*X*CFHG (.5*V+
388      11., 1.5, 0.5*X*X) /GAM2
389      FCFN=FAC1*FAC2
390      RETURN
391      END
392      FUNCTION CFHG (A, C, X)
393      IMPLICIT REAL*8 (A-H, O-Z)
394      SUM=1.
395      TERM=1.
396      S=1.
397      1      TERM=TERM*X*(A+S-1.) / (S*(C+S-1.))
398      SUM=SUM+TERM
399      S=S+1.
400      IF (SUM.EQ.0.) GO TO 1
401      IF (LAES (TERM/SUM) .GE..00001) GO TO 1
402      CFHG=SUM
403      RETURN
404      END

```

FILE



SLIST TEMP

1	7.80D-19	7.50D-19	7.00E-19	6.50D-19	5.80E-19	5.20D-19					
2	4.70E-19	4.30D-19	3.80D-19	3.40D-19	3.20D-19	3.20D-19					
3	3.30D-19	3.60D-19	4.30E-19	5.20D-19	6.60D-19	8.80D-19					
4	1.10E-18	1.40D-18	1.80E-18	2.20D-18	2.70D-18	3.20D-18					
5	3.90D-18	4.50D-18	5.20E-18	5.80D-18	6.60E-18	7.40D-18					
6	8.00E-18	9.00D-18	1.00D-17	1.01D-17	1.10D-17	1.20D-17					
7	1.20D-17	1.15D-17	1.10E-17	1.08D-17	1.05D-17	1.00D-17					
8	9.80E-18	9.00D-18	8.20E-18	7.60D-18	6.80D-18	6.00D-18					
9	5.00D-18	4.50D-18	4.00E-18	3.40D-18	2.70D-18	2.30D-18					
10	1.90E-18	1.50D-18	1.20D-18	9.00D-19	7.00D-19	5.40D-19					
11	3.20E-18										
12	.020	.021	.022	.024	.026	.030	.035	.040	.048	.055	.060
13	.070	.080	.090	.100	.130	.160	.200	.260	.360	.420	.520
14	.620	.740	.840	.940	.990	1.000	.980	.940	.880	.840	.800
15	.780	.740	.680	.640	.600	.500	.400	.360	.300	.240	.200
16	.170	.140	.120	.100	.080	.060	.050	.042	.040	.037	.035
17	.033	.030	.028	.025	.023	.020					

END OF FILE

Part I: Line numbers 1-61

In this part, the initial parameter values are set and the input data are read.

Notation

SH	=	O <sub>3</sub> scale height for an isothermal atmosphere, in cm
XNOO3	=	O <sub>3</sub> number density at Gaussian peak, in molecules/cm <sup>3</sup>
ZWOO	=	altitude of Gaussian peak, in cm
SHOO	=	half-depth of Gaussian layer, in cm
VM	=	visual magnitude of star
WLO	=	reference wavelength for stellar spectrum, in cm
TF	=	effective blackbody stellar temperature, in °K
FOO	=	adjusting factor for obtaining a desired fictitious count rate at the top of the scan
RE	=	mean radius of the earth, in cm
HO, HP	=	base height of the O <sub>3</sub> profile, in cm
XNO	=	O <sub>3</sub> number density at the base height in molecules/cm <sup>3</sup>
NP	=	number of points in a formulated table of normalized signal intensity v. column density. Note: column density = mass of species in a 1 cm <sup>2</sup> channel centered on the stellar ray, also called the total, or integrated, number density
NPP, NPH	=	number of data points in the profile
HDP	=	halfstepsize for computing O <sub>3</sub> densities in an exponential atmosphere, in cm
NN	=	switch for introducing random error into the signal intensity
NH	=	stepsize for computing O <sub>3</sub> number density, in number of data point intervals skipped

MM, MMM = test for selecting one of three paths (see program comments)

SG =  $O_3$  absorption cross sectional values, in  $cm^2$ ; the values are listed at the end of the program

WL = wavelength, in cm

FILT = normalized values of S2F5 filter transmission; the values are listed at the end of the program

FIO = stellar spectral intensity, in digital voltmeter counts/time interval

Part II: Line numbers 61.1 - 99

A table of normalized signal intensity v. column density is formed in the same way as for O<sub>2</sub> (p. 10 of O<sub>2</sub> notes). The reference height is now 50 Km.

In addition, a representation of the mesospheric O<sub>3</sub> peak is obtained by use of the Weber cylinder function. To avoid data roughness at the altitudinal limits of the superposed Gaussian curve, the cylinder function is forced between 40 Km and 130 Km.

Notation

- SUN = integration factor, or the integrated stellar spectral intensity; in counts/time interval
- HD = stepsize for altitude of the ray's tangent point used in the calculation of column density, in cm
- IHO = counter and altitude setter; in lines 90-91 it is used to arbitrarily reduce the column density to near zero at the upper end of the scan
- RO = distance from the center of the earth to the ray's tangent point, in cm
- ZQ = altitude of the ray's tangent point, in cm
- TT, TWW = column density for the table, computed at altitude ZQ, in cm<sup>-2</sup>
- VB = parameter for Weber cylinder function
- C1, CYL1, CYL2 = parameters for column density in Gaussian superimposed curve
- TTG = column density in Gaussian contribution, in molecules/cm<sup>3</sup>
- TNOX = total column density, in molecules/cm<sup>3</sup>
- SUM = filtered signal intensity integrated over wavelength, in counts/time interval
- FNO = integrated, normalized signal intensity corresponding to TT in the table

Part III: Line numbers 100 - 181

This part is essentially unchanged from the O<sub>2</sub> program.

A signal is computed from

$$I = \int_0^{\infty} \tau(\lambda) I_{\infty}(\lambda) e^{-\tau(\lambda)N} d\lambda \left[ \int_0^{\infty} \tau(\lambda) I_{\infty}(\lambda) d\lambda \right]^{-1},$$

which has the Gaussian superimposed column densities incorporated in "N".  
If the signal is scattered, a Poisson distribution is available at the lower end of the scan (0.1 < I < 10.) If the signal intensity is less than 0.1, the scattering is bypassed.

Notation

- JIZ, JIZMAX = loop counter and its limiting value for no scattering case  
FOF = simulated (measured) signal intensity, in counts/time interval  
JIX = loop counter for MM scattering cases with a specific data point spacing  
IX, JX, AM, P, SD, V, N, NMAX, U, D = GAUSS and RANDU subroutine arguments; Poisson parameters as in O<sub>2</sub> program  
JIY, JIYMAX = loop counter and its limiting value for a scattering case with variable data-point spacing

Part IV: Line numbers 182 - 245

The procedure and notation are identical to those used in the O<sub>2</sub> error analysis. However, subroutine ATMOD yields the model O<sub>3</sub> through solution of the expression

$$n(O_3) = n^* \exp[-(h-h^*)/H] + n' \exp[-(h-h')/H']^2$$

where

$n$  = number density, in molecules/cm<sup>3</sup>

$n^*$  = number density at a reference altitude, in molecules/cm<sup>3</sup>

$h$  = altitude, in cm

$h^*$  = reference altitude, in cm

$H$  = scale height of O<sub>3</sub>, in cm

$n'$  = O<sub>3</sub> number density at Gaussian peak, in molecules/cm<sup>3</sup>

$h'$  = altitude of Gaussian peak, in cm

$H'$  = half-depth of Gaussian layer, in cm

Numerical values are inserted into ATMOD for the above parameters.

Part V: Line numbers 246 - 283

Temperature computations are omitted.

If multiple cases are being handled, the mean and standard deviation are computed for the error at each altitude level.

Notation

MM, MMM = number of cases for error statistics

DMEAN = mean error in number density, in percent

SIGMA = one standard deviation of number density error, in percent

Part VI: Line numbers 1 - 17

$O_3$  cross sectional values are listed row by row at an interval of  $20 \text{ \AA}$  (lines 1 - 11).

S2F5 filter transmission values, adjusted to a maximum value of 1.0, are also listed at the same wavelength interval (lines 12 - 17).



#### Appendix IV. Orbital Errors

The effects of the uncertainty in satellite position have been traced by imposing small input errors upon the satellite coordinates and mapping the error departures of the ray tangent point. The symbols used are as in Section 3, Section 3, with the addition of star-superscripts to denote reference, or zero-error quantities. A special case is assumed, for convenience, which places the spacecraft and the stellar ray in the equatorial plane, and the source star at the First Point of Aries in the same plane.

Tangent ray errors are shown in Figures 1-4 which have distance scales on the abscissae to supplement degree indicators of longitude and latitude. They disclose that altitude errors in tangent ray point nearly match vertical satellite displacements; they amount to a few meters when a satellite is displaced latitudinally 10 Km; and they reach magnitudes of a few Km when a satellite is displaced longitudinally 10 Km. Thus, as expected the tangent ray heights are quite sensitive to the longitudinal accuracy of spacecraft positioning.

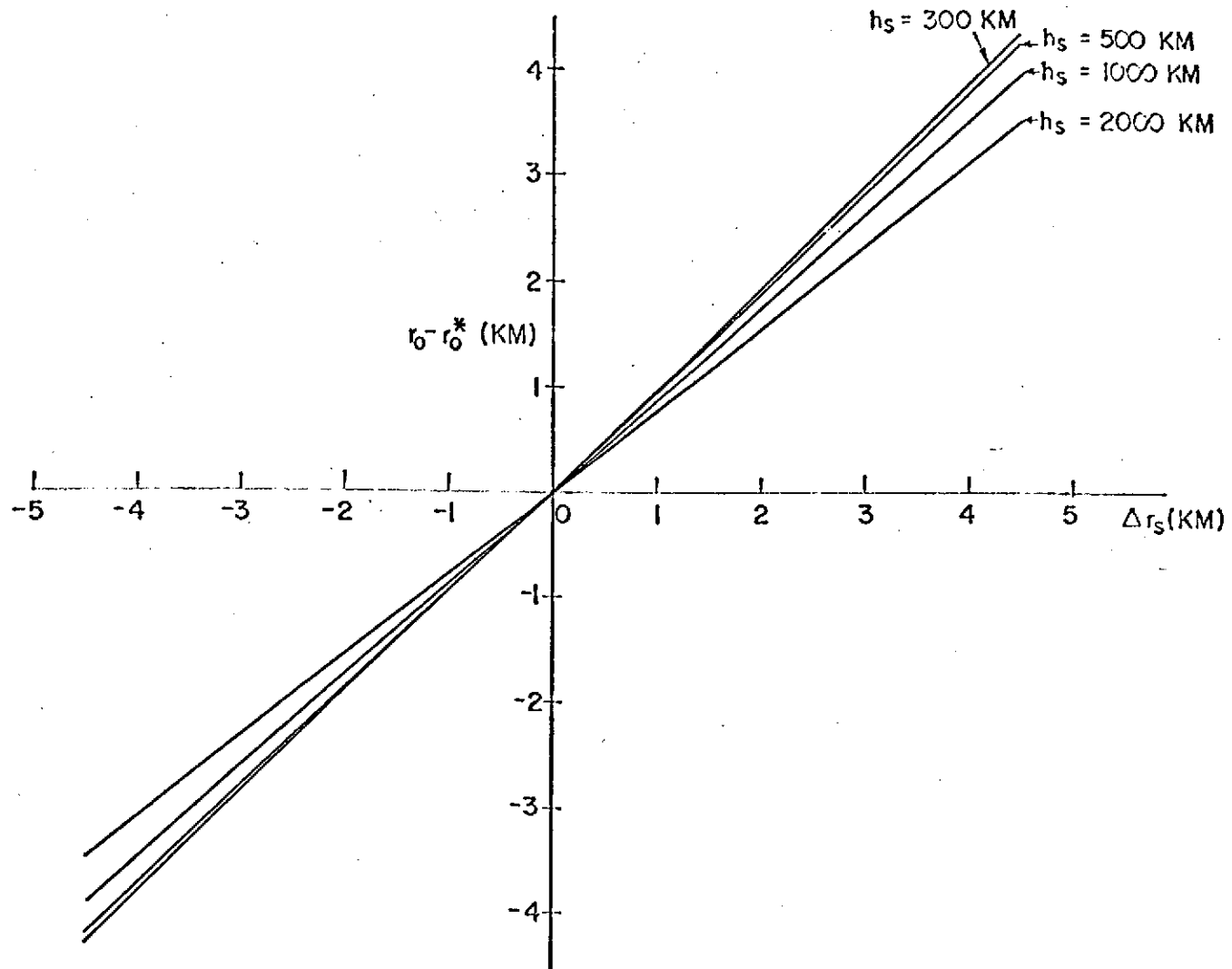


Figure 1. Tangent ray height error as a function of satellite height error and altitude.

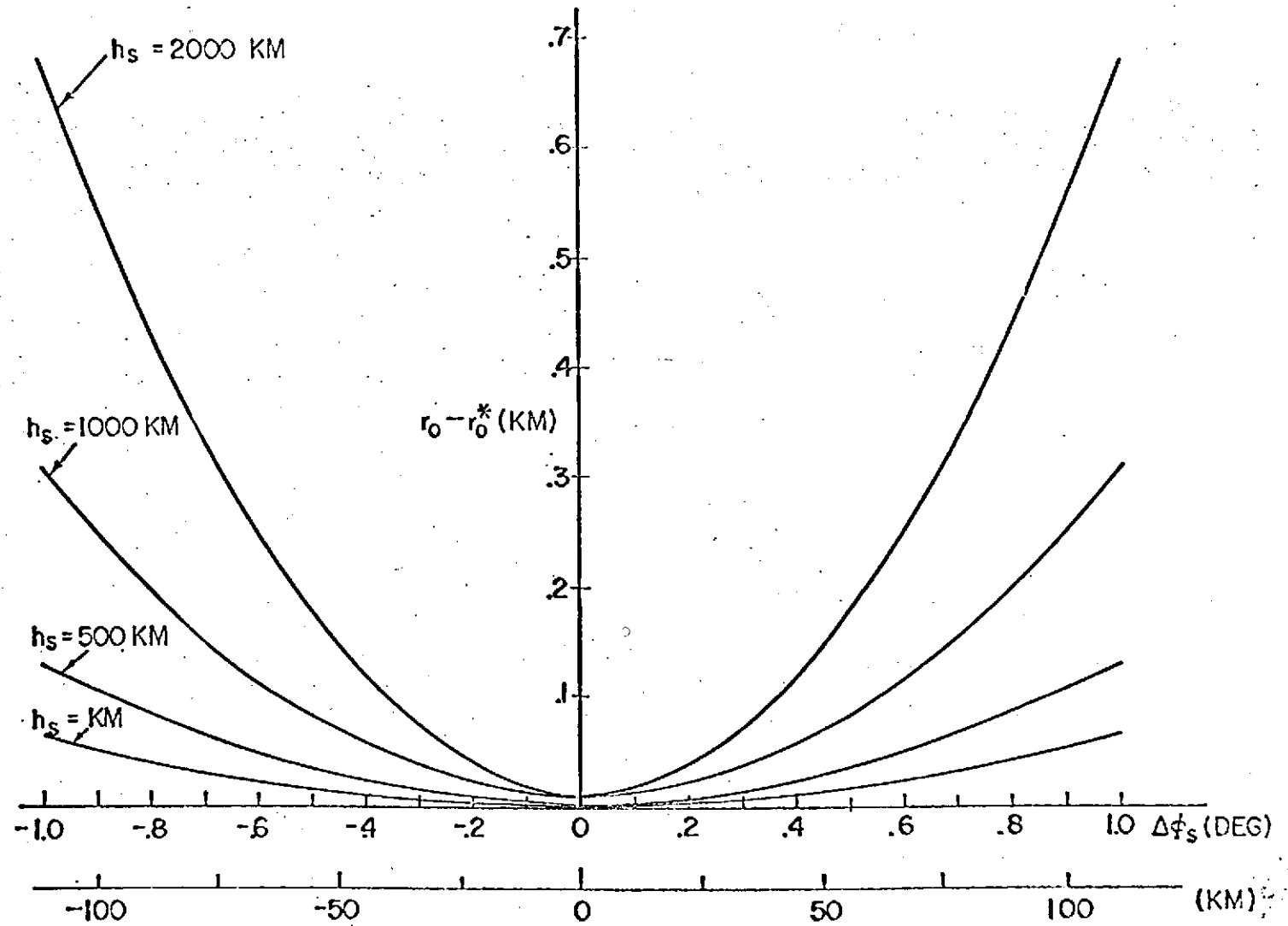


Figure 2. Tangent ray height error as a function of satellite latitude error.

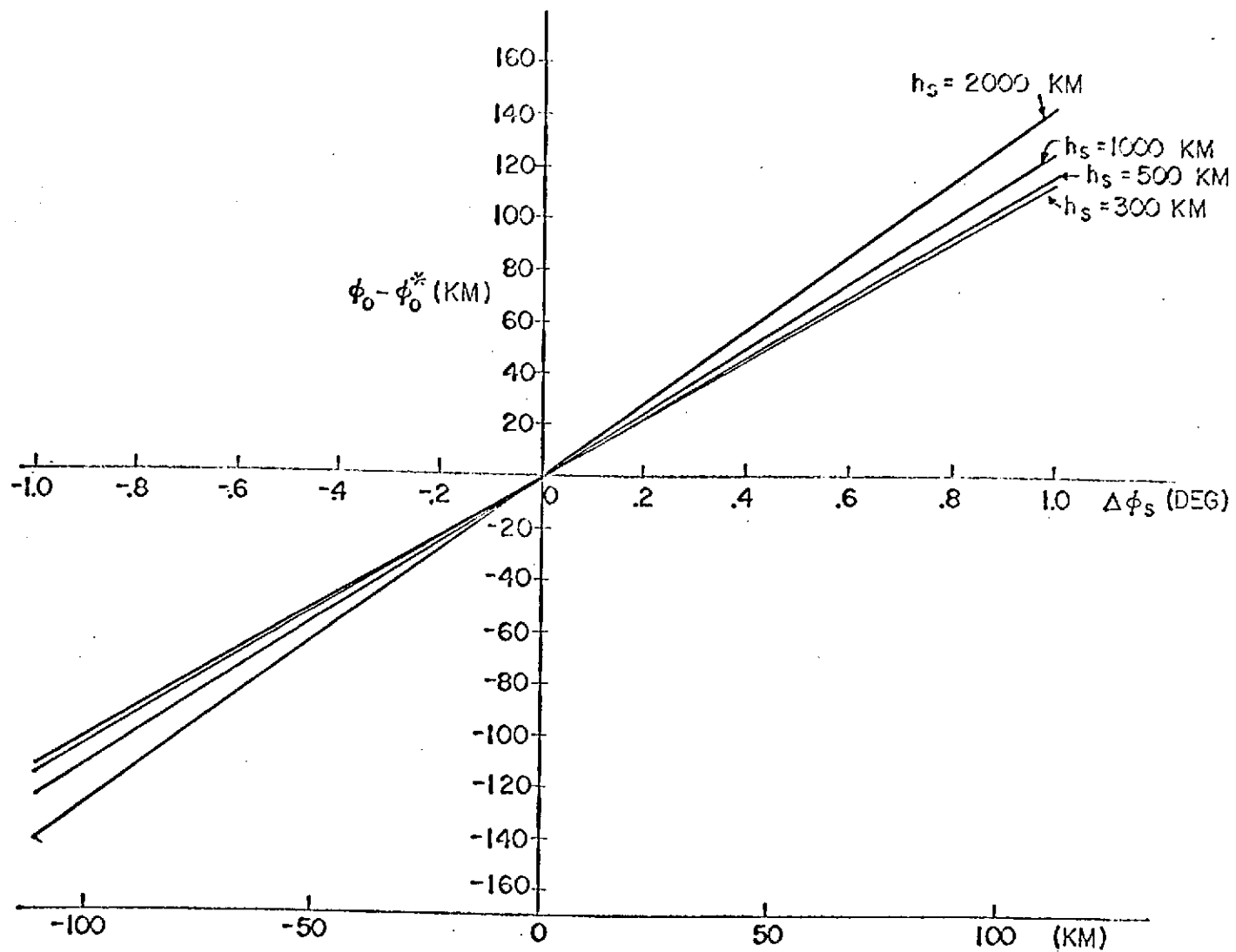


Figure 3. Tangent point longitude error as a function of satellite longitude error.

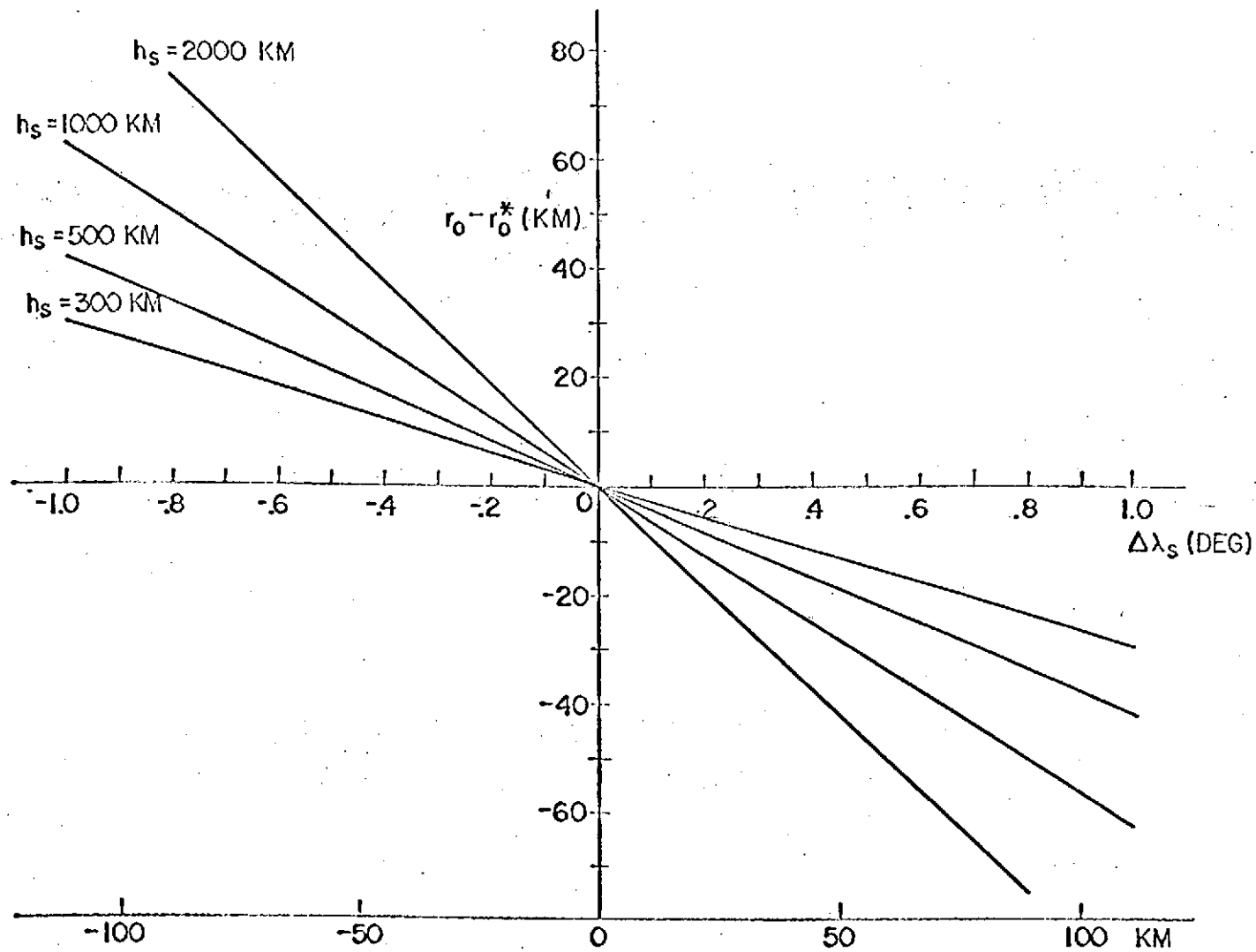


Figure 4. Satellite tangent ray height error as a function of satellite longitude error.

APPENDIX V.

Stellar Occultation Measurements of Molecular  
Oxygen in the Lower Thermosphere

# STELLAR OCCULTATION MEASUREMENTS OF MOLECULAR OXYGEN IN THE LOWER THERMOSPHERE

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(Received in final form 8 August 1972)

**Abstract**—Stellar ultraviolet light near 1500 Å is attenuated in the Earth's upper atmosphere due to strong absorption in the Schumann–Runge continuum of molecular oxygen. The intensity of stars in the Schumann–Runge continuum region has been monitored by the University of Wisconsin stellar photometers aboard the OAO-2 satellite during occultation of the star by the Earth's atmosphere. These data have been used to determine the molecular oxygen number density profile at the occultation tangent point. The results of 14 stellar occultations obtained in low and middle latitudes are presented giving the night-time vertical number density profile of molecular oxygen in the 140–200 km region. In general, the measured molecular oxygen number density is about a factor of 2 lower than the number densities predicted by the CIRA 1965 model. Also, the number density at a given height appears to decrease with decreasing solar activity. Measurements taken at low latitudes during the August 1970 geomagnetic storm showed a decrease in the molecular oxygen number density at a given height several days after the peak of the storm followed by a slow recovery to pre-storm densities.

## 1. INTRODUCTION

The first attempt at determining the molecular oxygen concentration in the upper atmosphere by u.v. absorption spectroscopy was made on a V-2 rocket experiment in 1949 (Friedman *et al.*, 1951). The molecular oxygen concentration was obtained from the solar u.v. absorption measurements made at various altitudes by a spectrometer aboard the rocket. Since then, numerous rocket flights have been made to examine the molecular oxygen distribution and its variations, in addition to determining the u.v. spectra of the Sun (Byram *et al.*, 1955; Kupperian *et al.*, 1959; Jursa *et al.*, 1963; Hall *et al.*, 1963; Hinteregger *et al.*, 1965; Week and Smith, 1968; Opal and Moos, 1969; Quessette, 1970; Brannon and Hoffman, 1971).

The molecular oxygen distribution in the altitude range 100–200 km has also been determined from mass spectroscopic measurements (Schaefer and Nichols, 1964; Nier *et al.*, 1964; Hedin and Nier, 1966; Schaefer, 1968; Krankowsky *et al.*, 1968; von Zahn and Gross, 1969). In addition to molecular oxygen, the other major and minor constituents of the upper atmosphere are likewise determined as a function of altitude during the rocket flight.

More recently, satellites have been used to determine the properties of the upper atmosphere from u.v. absorption measurements. The satellites have monitored the attenuation of solar u.v. radiation in various isolated wavelength intervals during occultation at orbital sunrise and sunset. These data have been used to retrieve the neutral air density (Thomas *et al.*, 1965; Kreplin, 1965; Venables, 1967; Landin *et al.*, 1965; Landin *et al.*, 1967) and the molecular oxygen distribution in the lower thermosphere (Thomas and Norton, 1967; Norton and Warnock, 1968; Link, 1969; Stewart and Wildman, 1969; Lockey

*et al.*, 1969; Reid and Withbroe, 1970; May, 1971; Roble and Norton, 1972). These measurements, however, are limited only to sunrise and sunset.

Hays and Roble (1968a) suggested that u.v. stars may be used as source for occultation measurements to determine the night-time distribution of molecular oxygen and ozone in the lower thermosphere and upper mesosphere. Their calculations showed that in the spectral region near  $1500 \text{ \AA}$  attenuation is primarily due to absorption by molecular oxygen, and that Rayleigh scattering and absorption by other minor constituents can be neglected.

During the past few years, we have used the Orbiting Astronomical Observatory (OAO-2) to obtain u.v. stellar occultation data in various spectral intervals. These data have been used to obtain the night-time molecular oxygen number density and neutral gas temperature in the lower thermosphere from about 140–200 km. The purpose of this paper is to describe the experimental technique and discuss the results which were obtained during quiet and disturbed geomagnetic conditions.

## 2. EXPERIMENTAL TECHNIQUE

The general details of the stellar occultation technique (Fig. 1) have been described by Hays and Roble (1968a, b), Hays *et al.* (1972) and Roble and Hays (1972). Here we describe the specifics of the occultation measurements made by the OAO-2 satellite.

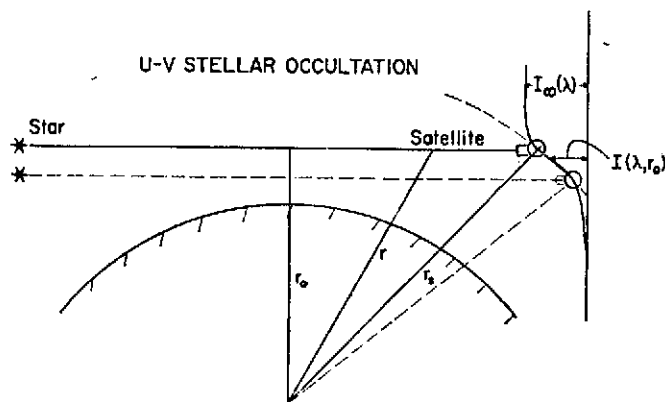


FIG. 1. GEOMETRY OF STELLAR OCCULTATION,  $r_0 = r_t$ .

The OAO-2 satellite has one 16 in. dia u.v. telescope, four 8 in. dia u.v. telescopes, and an u.v. spectrometer having a resolution of approximately  $5 \text{ \AA}$ . The University of Wisconsin optical package, used in conjunction with the telescopes, consists of a series of u.v. filters which are used for stellar photometry. Filter (4-1) shown in Fig. 2 has a broadband transmission function centered near  $1450 \text{ \AA}$  in the Schumann continuum region of molecular oxygen. This filter was used to obtain the molecular oxygen distributions from the stellar occultation measurements. The detection systems of the u.v. telescopes have a variable time integration range. The data shown in Fig. 4, which is typical of the high data rate occultation scans, has approximately a 1.5 km altitude resolution between data points.

Prior to occultation of the star by the Earth, an unattenuated u.v. spectrum of the star is obtained from the spectrometer. As an example, the spectrum for a typical bright u.v. star is shown in Fig. 3 for the wavelength region of the molecular oxygen u.v. filter. Although only a measurement of the relative change of the u.v. stellar intensity is required during occultation (Hays and Roble, 1968a), the spectral distribution of the star's energy



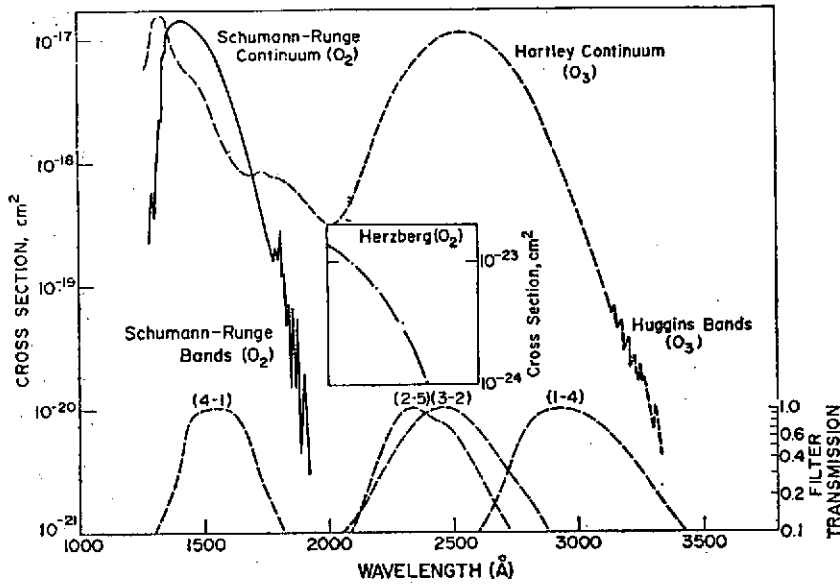


FIG. 2. ABSORPTION CROSS SECTIONS FOR MOLECULAR OXYGEN AND OZONE IN THE SPECTRAL REGION EXTENDING FROM 1000 TO 3500 Å. DASHED CURVES ARE THE O.A.O. STELLAR PHOTOMETER FILTER TRANSMISSION CURVES USED IN THIS STUDY.

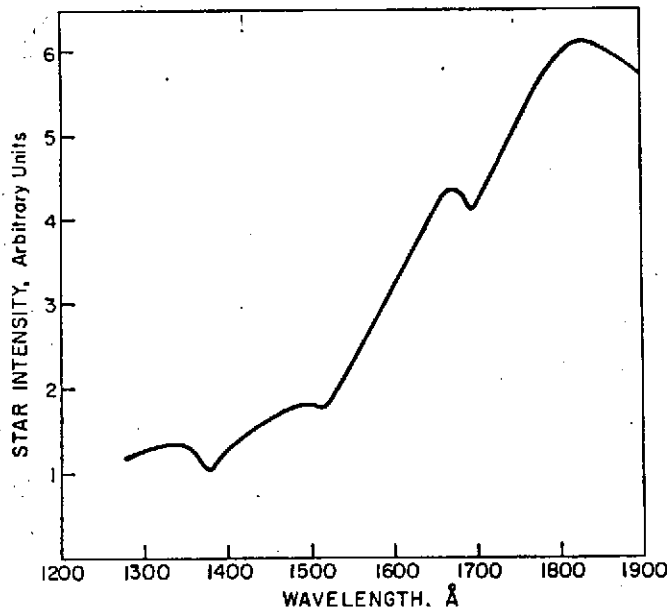


FIG. 3. TYPICAL SPECTRUM OF A STAR USED AS AN OCCULTATION LIGHT SOURCE.

flux is required to determine the transmission through the broadband filter. Thus, a detailed stellar spectrum covering the spectral passband of the filter is necessary for data reduction.

A schematic diagram of an occultation of a star by the Earth is shown in Fig. 1. The satellite acquires the star in its telescopes prior to occultation above the absorbing atmosphere. As the satellite moves in its orbit, the source is ultimately occulted by the Earth's atmosphere and the geometry at two satellite positions is illustrated schematically in Fig. 1.

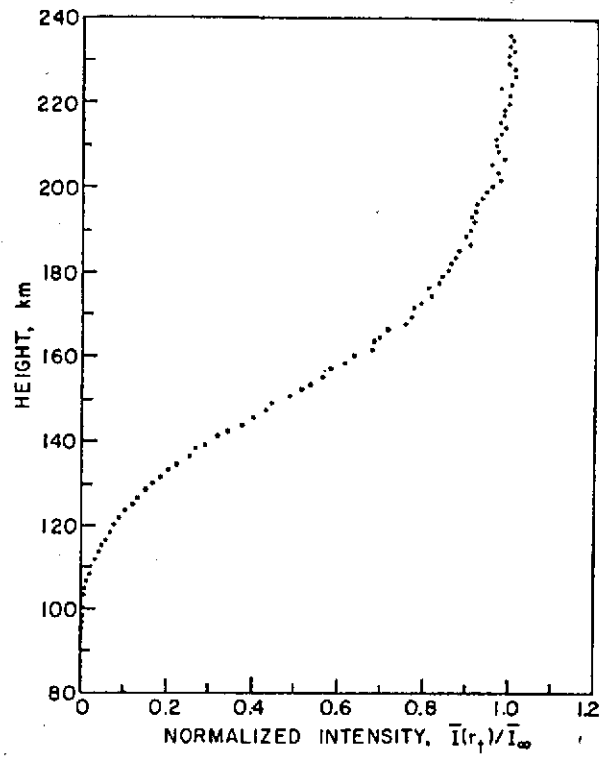


FIG. 4a. NORMALIZED STELLAR INTENSITY AS A FUNCTION OF TANGENT RAY HEIGHT DURING OCCULTATION.

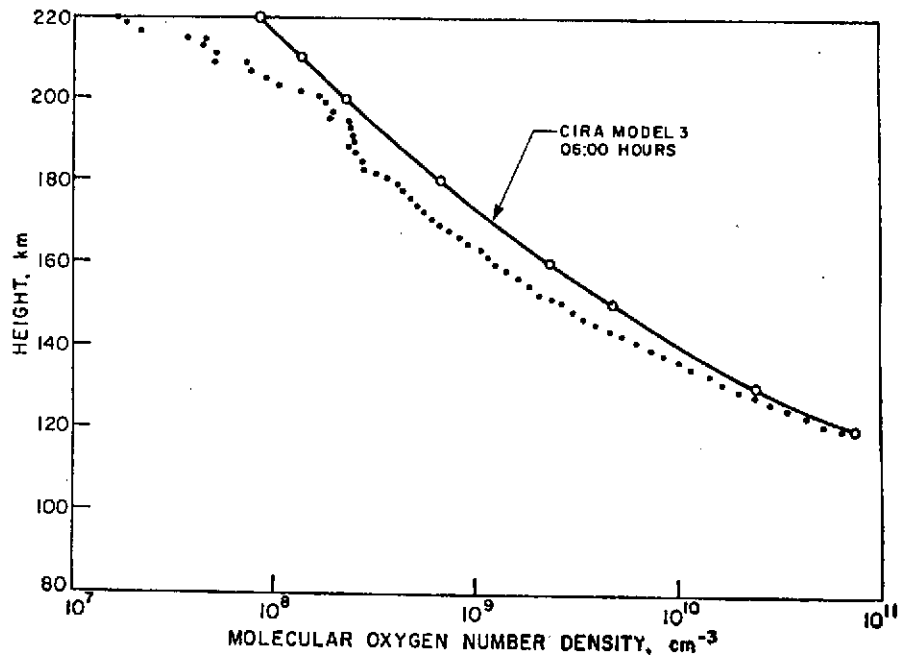


FIG. 4b. MOLECULAR OXYGEN DENSITY VS. HEIGHT FROM DATA SHOWN IN FIG. 4a.

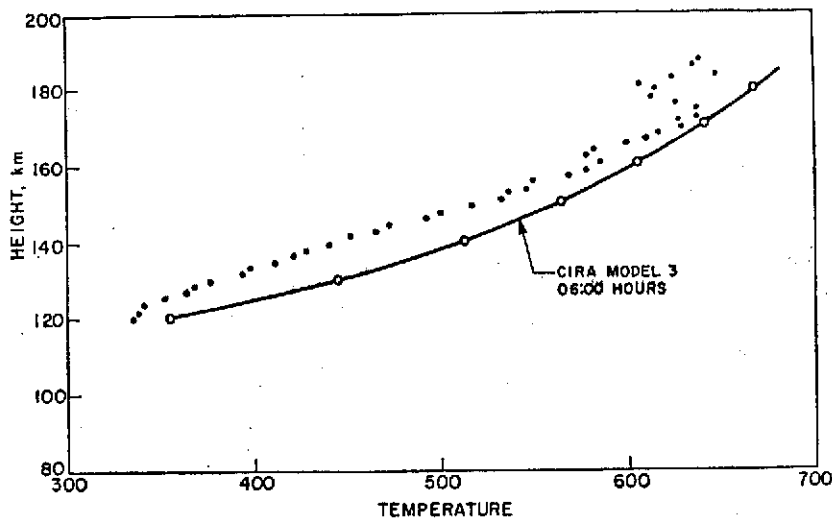


FIG. 4c. TEMPERATURE VS. HEIGHT DEDUCED FROM THE MOLECULAR OXYGEN NUMBER DENSITY DATA SHOWN IN FIG. 4b ASSUMING DIFFUSIVE EQUILIBRIUM.

The intensity of the star is measured as a function of time during occultation by the Earth. By knowing the satellite position as a function of time and the star's position, we can relate the star's intensity to the tangent ray height of the star during occultation. By also knowing the position of the star, satellite, and the time, we obtain from geometry the geographic position of the tangent ray point.

The intensity data are normalized to the intensity of the star above the atmosphere and the normalized data are related to the tangential column number density of the absorbing species, here  $O_2$ , by the integral relationships expressing Beer's law

$$I(r_t)/I_\infty = \left\{ \int_0^\infty T(\lambda) I_\infty(\lambda) \exp(-\sigma_{O_2}(\lambda) N_{O_2}(r_t) d\lambda) \right\} \left\{ \int_0^\infty T(\lambda) I_\infty(\lambda) d\lambda \right\}^{-1} \quad (1)$$

where from the geometry in Fig. 1, assuming a spherically stratified atmosphere

$$N_{O_2}(r_t) = 2 \int_{r_t}^\infty \frac{n_{O_2}(r) r dr}{\sqrt{r^2 - r_t^2}}. \quad (2)$$

Here  $I(r_t)$  is the photometer count rate at tangent ray height  $r_t$  and  $I_\infty$  is the photometer count rate of the stellar signal above the atmosphere.  $T(\lambda)$  is the filter transmission and  $I_\infty(\lambda)$  is the intensity of the star at wavelength  $\lambda$  above the atmosphere determined by the spectrometer;  $\sigma_{O_2}(\lambda)$  is the absorption cross-section of molecular oxygen shown in Fig. 2 (Ditchburn and Young, 1962; Hudson *et al.*, 1969; Ackerman, 1970) and  $N_{O_2}(r_t)$  is the tangential column number density of molecular oxygen at tangent ray radius  $r_t$ ; and  $n_{O_2}(r)$  is the local number density at the radius  $r$ .

Molecular oxygen is the sole absorbing species in the wavelength interval determined by filter 4-1, therefore the normalized intensity data are converted to tangential column number density data using Equation (1). The basic measurements are thus converted to data giving the tangential column number density of molecular oxygen as a function of tangent ray height. Equation (2) for the tangential column number density is the Abel integral

equation (cf. Hays and Roble, 1968a, b) which is inverted to obtain the local number density at the tangent ray point assuming spherical symmetry,

$$n_{O_2}(r) = \frac{1}{2\pi} \frac{d}{dr} \left( \int_r^\infty \frac{r' N_{O_2}(r') dr'}{r' \sqrt{r'^2 - r^2}} \right). \quad (3)$$

The actual numerical techniques for accomplishing this inversion are described in detail in a paper by Roble and Hays (1972).

The neutral gas temperature is derived by assuming diffusive equilibrium above 130 km for the  $O_2$  number density. Thus,

$$T_{O_2}(r) = \frac{m_{O_2}}{k n_{O_2}(r)} \int_r^\infty g(r') n_{O_2}(r') dr' \quad (4)$$

where  $m_{O_2}$  is the mass of molecular oxygen,  $k$  is Boltzmann's constant, and  $g$  is the acceleration of gravity.

### 3. RESULTS

A relatively large number of absorption profiles were obtained during the time period extending from January 1970 to August 1971. Of these, command errors and tracking errors have caused serious question about many profiles. The final result of carefully selecting only those profiles where altitude is accurately known and where there are no command interrupts during the occultation yields 14 useful occultations. Of these, approximately 6 are in geomagnetically quiet periods and one series of 8 profiles was obtained during the magnetic storm of August 1970. Almost all of these profiles were obtained in low latitudes due to the low inclination of the orbital plane of the OAO-2 satellite. In addition to the molecular oxygen occultation scans, measurements were also made with a filter located in the Hartley continuum of ozone. (Filters 2-5 and 3-2 shown in Fig. 2). A single night-time ozone number density profile was presented by Hays *et al.* (1973) and the results of the other scans are presented by Hays and Roble (1973).

#### (a) Molecular oxygen profiles during geomagnetically quiet periods

The geographic position, date, and local time of all of the scans analyzed are given in Table 1. Six of these were taken during geomagnetically quiet conditions. The normalized intensity measurements obtained on orbit 8884 on 17 August 1970 are shown in Fig. 4a. In Figs. 4b and 4c the retrieved molecular oxygen number density and temperature are shown and they are compared with *CIRA* 1965 (model 3, 06:00 hr.). In general, the retrieved molecular oxygen number density distribution in the 140–180 km region is about a factor of two lower than the number densities predicted by the *CIRA* 1965 model atmosphere at the same local time as the measurements. The composite of all of our quiet time data is shown in Fig. 5 along with the results of Krankowsky *et al.* (1968), Weeks and Smith (1968), and Roble and Norton (1972). The envelope encompassing the ozone number density profiles obtained from Hays and Roble (1972) and the molecular oxygen number density given by the mean *CIRA* 1965 atmosphere are also shown. The small number of stellar occultation profiles makes detailed seasonal variations difficult to discern. However, there does appear to be a marked decrease in the molecular oxygen number density corresponding to the decrease in solar activity during the period from the beginning of 1970 to late 1971. The monthly mean of the solar F10.7 flux during this period decreased from 153 to 120 ( $\times 10^{-22}$ ) W cm<sup>-2</sup>. The daily solar F10.7 emission for each of the individual

TABLE 1

Orbit	Date	Universal time		Local time		Long.*	Lat.	Daily solar F10.7 emission
		(hr)	(min)	(hr)	(min)	(deg)	(deg)	( $\times 10^{-22}$ W cm $^{-2}$ )
5778	1/13/70	5	28	23	52	-84	48	172.9
8884	8/17/70	4	55	5	55	15	4	151.6
8898	8/18/70	4	19	5	51	23	9	149.0
8913	8/19/70	5	22	5	54	8	11	145.4
8942	8/21/70	5	55	4	27	-22	24	142.8
8943	8/21/70	7	35	4	23	-48	24	142.8
8957	8/22/70	6	58	4	22	-39	25	141.7
8971	8/23/70	6	20	4	44	-24	27	146.2
8985	8/24/70	5	45	4	13	-23	24	140.2
8986	8/24/70	7	25	4	13	-48	24	140.2
9000	8/25/70	6	47	5	17	-32	27	136.5
10902	1/ 4/71	13	29	22	38	-223	-16	140.2
11795	3/ 7/71	15	59	0	33	128	-34	105.2
14580	9/17/71	6	22	1	50	-68	26	116.5

\* Measured positive eastward from Greenwich.

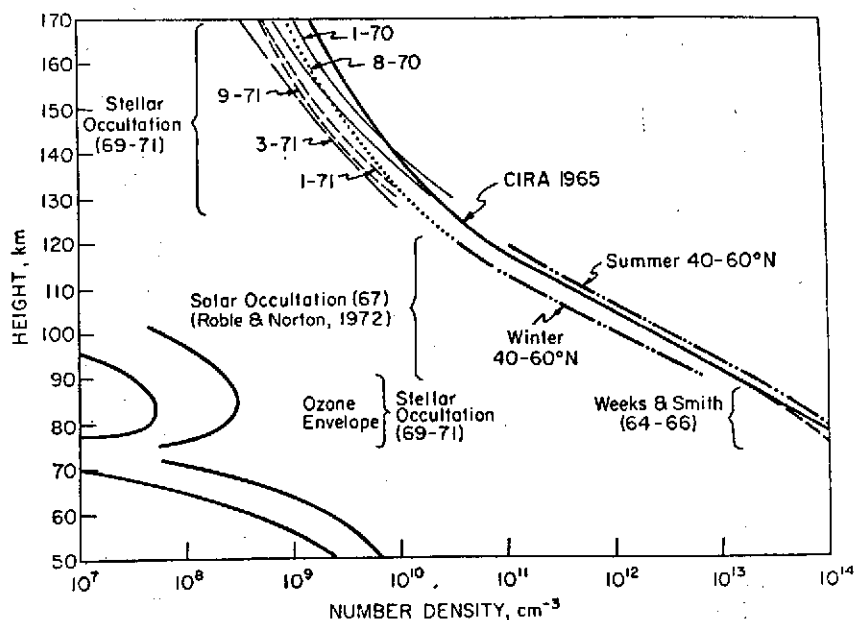


FIG. 5. COMPOSITE MOLECULAR OXYGEN DENSITY RESULTS FOR QUIET TIMES.  
 .... Krankowsky *et al.*, 1968.

scans are given in Table 1. The apparent decrease in the thermospheric molecular oxygen at a fixed height probably implies that there is a general decrease in the altitude of the constant pressure surface at the base of the thermosphere as well as a decrease in the thermospheric temperature. This suggests a solar cycle influence in the lower thermosphere.

#### (b) Molecular oxygen profiles during geomagnetically disturbed periods

A series of observations was obtained during the magnetically disturbed period at the end of August 1970. These data are presented in Fig. 6 where density and temperature at

fixed heights are shown as a function of time. For comparison, we show the daily sum of magnetic  $K_p$  index on this same figure and give the latitude and longitude of the tangent ray point and local time of each occultation scan in Table 1. We note the marked decrease in the molecular oxygen number density following the storm at a time when the temperature was significantly enhanced. This is obviously in disagreement with the concept of a diffusively stable atmosphere. The decrease in the molecular oxygen density is apparently due to

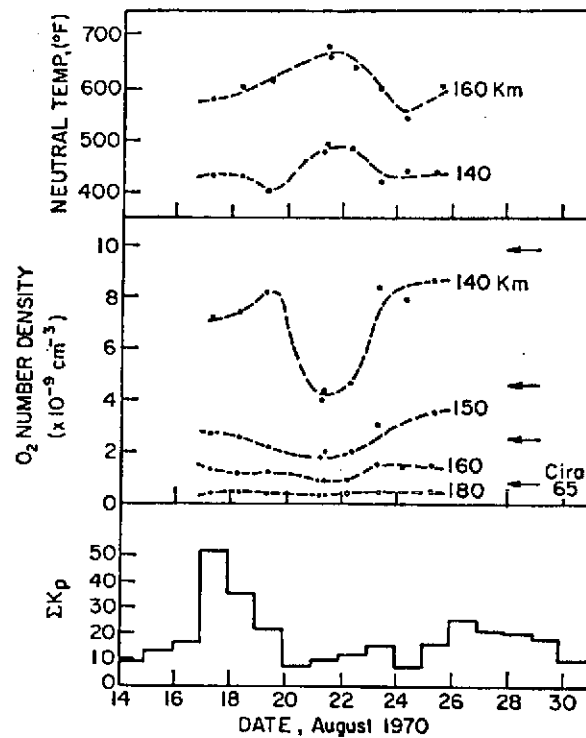


FIG. 6. MOLECULAR OXYGEN DENSITY AND TEMPERATURE VARIATIONS DURING THE MAGNETIC STORM OF AUGUST 1970.

the existence of a global-scale circulation system which upwells over the polar regions and subsides in the equatorial zone. Such a large horizontal scale overturning causes air with more atomic than molecular oxygen to move downward over the equator resulting in a change in the composition of the lower thermosphere. This result is consistent with the observation that energy deposited in the polar thermosphere is redistributed globally resulting in a global thermospheric temperature enhancement at low latitudes following a large magnetic storm (Hays *et al.*, 1972b). The daily solar F10.7 emission for the days on which occultation scans were made is given in Table 1. During the period the daily solar F10.7 emission decreased from 151.6 to 136.5 ( $\times 10^{-22}$  W cm $^{-2}$ ). However, the molecular oxygen number density during this period first decreased, but then returned to pre-storm levels several days after the storm. This behavior also suggests an atmospheric response different from that expected from short term solar e.u.v. changes.

## 4. SUMMARY

The stellar occultation technique for measuring molecular oxygen in the Earth's thermosphere has demonstrated the following general behavior: (a) Thermospheric molecular oxygen decreases with decreasing solar activity. This may result from a general cooling of the lower thermosphere in addition to the cooling observed in the upper thermosphere.

(b) Magnetic storms result in strong global circulation systems which upwell in the polar regions and sink in equatorial regions. This results in a temporary decrease in the molecular constituents in the equatorial thermosphere.

*Acknowledgement*—We wish to acknowledge the support of Grant NGR 23-005-360, and NASA Contract Nas 1 9958. The National Center for Atmospheric Research is sponsored by the National Science Foundation. We wish to acknowledge the helpful support of Dr. Houk, and the O.A.O. Wisconsin and NASA experimental team.

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APPENDIX VI.

Observation of Mesospheric Ozone  
at Low Latitudes

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## OBSERVATION OF MESOSPHERIC OZONE AT LOW LATITUDES

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(Received in final form 21 August 1972)

**Abstract**—Stellar ultraviolet light near 2500 Å is attenuated in the Earth's upper atmosphere due to strong absorption in the Hartley continuum of ozone. The intensity of stars in the Hartley continuum region has been monitored by the University of Wisconsin stellar photometers aboard the OAO-2 satellite during occultation of the star by the Earth's atmosphere. These data have been used to determine the ozone number density profile at the occultation tangent point. The results of approximately 12 stellar occultations, obtained in low latitudes, are presented, giving the nighttime vertical number density profile of ozone in the 60- to 100-km region. The nighttime ozone number density has a bulge in its vertical profile with a peak of 1 to  $2 \times 10^8 \text{ cm}^{-3}$  at approximately 83 km and a minimum near 75 km. The shape of the bulge in the ozone number density profile shows considerable variability with no apparent seasonal or solar cycle change. The ozone profiles obtained during a geomagnetic storm showed little variation at low latitudes.

### 1. INTRODUCTION

The first measurement of the ozone concentration in the upper atmosphere was made by Johnson *et al.* (1951) using u.v. absorption spectroscopy. The ozone number density distribution was determined from the u.v. absorption measurements made at various altitudes by a spectrometer aboard a rocket. Since then daytime measurements of the ozone number density distribution have been made from rockets using the Sun as the u.v. source (Van Allen and Hopfield, 1952; Johnson *et al.*, 1952; L'vova *et al.*, 1964; Poloskov *et al.*, 1966; Nagata *et al.*, 1967; Weeks and Smith, 1968; Krueger, 1969) and nighttime measurements have been made using the Moon as a source of u.v. light (Carver *et al.*, 1967; Carver *et al.*, 1972). The high altitude ozone number density distribution has also been determined from solar occultation measurements made from a satellite (Rawcliffe *et al.*, 1963; Miller and Stewart, 1965). These measurements have determined the ozone number density up to an altitude of about 70 km.

Several other techniques have been used to obtain the ozone number density distribution in the upper atmosphere which include (a) satellite eclipse photometry (Venkateswaren *et al.*, 1961; Fesenkov, 1967), (b) nighttime airglow spectral photometry (Reed, 1968), (c) spectral analysis of backscattered solar radiation as observed from a satellite (Rawcliffe and Elliot, 1966; Anderson *et al.*, 1969), (d) chemi-luminescent ozone sondes (Hilsenrath, 1971) and (e) rocket and ground based observations of the molecular oxygen emission  $\text{O}_2(^1\Delta_g)$  at  $1.27 \mu$  (Evans and Llewellyn, 1970; 1972; Evans *et al.*, 1970). In the latter technique, the ozone number density profile is calculated from the measured altitude profiles of the  $\text{O}_2(^1\Delta_g)$  emission at  $1.27 \mu$  using photochemical theory. Their results give the ozone number density profile at twilight up to 100 km and preliminary observations indicate a strong seasonal variation of the upper ozone layer at high latitudes. The peak ozone number density at 85 km varied between  $1.3 \times 10^8 \text{ cm}^{-3}$  in midwinter to less than  $0.3 \times 10^8 \text{ cm}^{-3}$  in midsummer.

Hays and Roble (1968a) suggested that the nighttime distribution of ozone in the upper mesosphere may be obtained from satellite measurements of the intensity of u.v. stars during occultation by the Earth's atmosphere. During the past few years, we have used the Orbiting Astronomical Observatory (OAO-2) to obtain u.v. stellar occultation data in various spectral regions. The data from the u.v. filters centered at 2390 and 2460 Å have been used to obtain the nighttime ozone number density distribution from 60 to 100 km at low latitudes. In this paper we describe the experimental technique and present the results which were obtained during quiet and disturbed geomagnetic conditions.

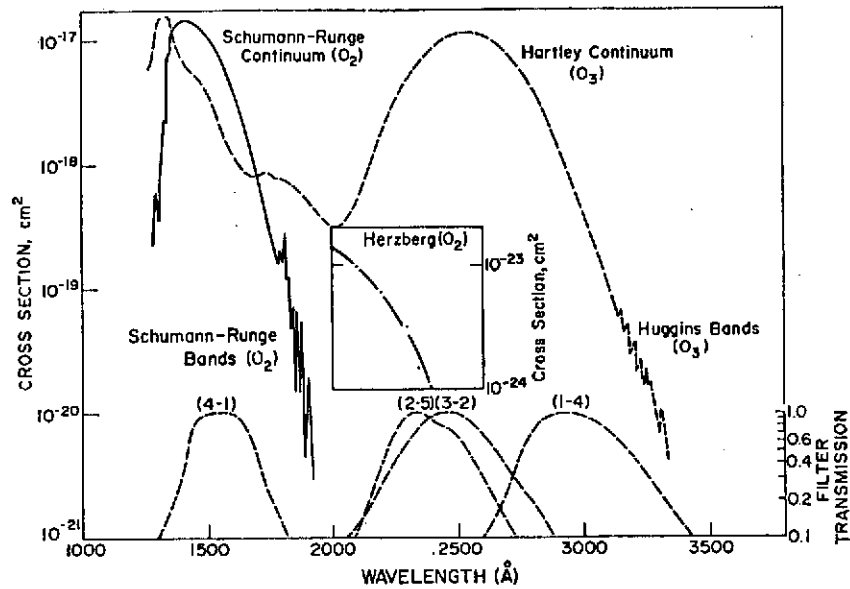


FIG. 1. ABSORPTION CROSS SECTIONS FOR MOLECULAR OXYGEN AND OZONE IN THE SPECTRAL REGION EXTENDING FROM 1000 TO 3500 Å.

Dashed curves are the OAO-2 stellar photometer filter transmission curves used in this study.

## 2. EXPERIMENTAL TECHNIQUE

The general details of the stellar occultation technique have been described by Hays and Roble (1968a, b; 1972), Hays *et al.* (1972) and Roble and Hays (1972). Here we describe the specifics of the occultation measurements made by the OAO-2 satellite as they apply to the determination of the nighttime ozone number density.

The OAO-2 satellite has one 16 in.-dia u.v. telescope and four 8 in.-dia u.v. telescopes and an u.v. spectrometer having a resolution of approximately 5 Å. Filter 2-5 and 3-2 shown in Fig. 1, are the two filters in the University of Wisconsin optical package that are used for the ozone stellar occultation measurements. The transmission function of these filters is located in the Hartley continuum of ozone (Fig. 1) with the peak transmission at 2380 and 2460 Å. The detection systems of the u.v. telescope have a maximum data acquisition rate of  $\frac{1}{8}$  sec time integration resulting in a high altitude resolution at the tangent point. The intensity data are obtained as a function of time, and by knowing the star's position is orbital elements of the satellite, we can relate the intensity to the tangent ray heights during occultation by the Earth's atmosphere. Figure 2 shows the normalized intensity data as a function of tangent ray height for a typical occultation scan. Hays and Roble (1972) describe the technique used to relate the normalized intensity

data to the tangential column number density of the absorbing species along the ray path. This technique allows for the broadband characteristics of the u.v. transmission function and can be used as long as absorption is due to a single species. Hays and Roble (1968b) have calculated the u.v. transmission of the Earth's upper atmosphere and have shown that the stellar light in the wavelength regions covered by the filter transmission functions, shown in Fig. 1, is primarily absorbed by ozone. However, at

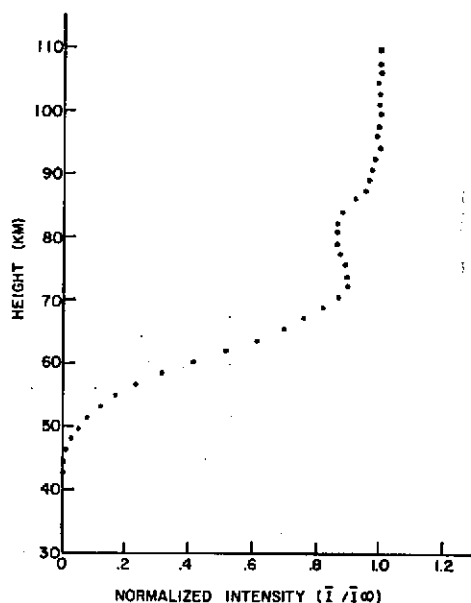


FIG. 2. NORMALIZED INTENSITY PROFILE FOR A TYPICAL OZONE OCCULTATION.

altitudes below about 70 km the stellar ultraviolet light is also absorbed by molecular oxygen in the Herzberg continuum. The absorption contribution in the Herzberg continuum is calculated using the molecular oxygen number density distribution obtained from the *CIRA* 1965 model atmosphere. This absorption contribution is removed from the stellar intensity data and the corrected normalized intensity is related to the ozone tangential column number density (Hays and Roble, 1972). Once the tangential column number density is known as a function of the tangent ray height, the data are inverted, using the technique described by Roble and Hays (1972), to obtain the local number density at the tangent ray point.

### 3. RESULTS

Stellar occultation measurements using the two filters centered in the Hartley continuum of ozone were made during the period extending from January 1970 through August 1971. From these data we have obtained 12 orbits on which one or both of the filters could be used to determine the ozone density in the mesosphere. These results, corrected for molecular oxygen absorption in the Herzberg continuum, and rayleigh scattering are presented in Figs. 3-5 where four separate profiles are illustrated in each figure. According to the analysis of Roble and Hays (1972), the ozone number density is best retrieved from the occultation data between tangent ray heights of 55 to 95 km. The retrieved ozone number density data above and below this altitude interval are less reliable.

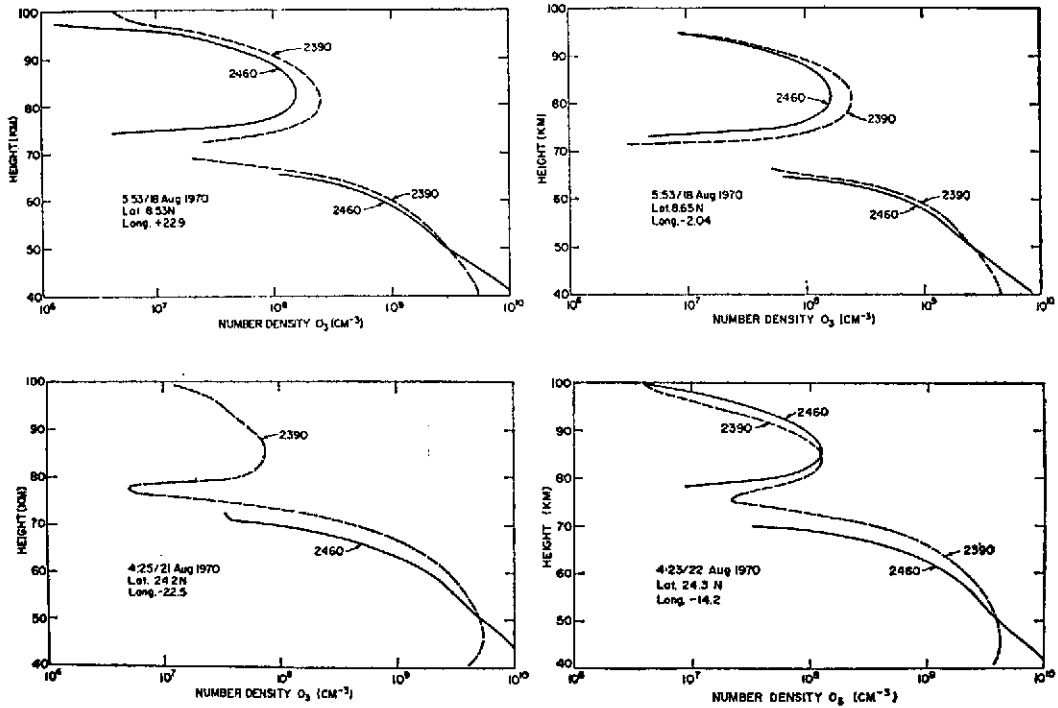


FIG. 3. ALTITUDE PROFILES FOR ATMOSPHERIC OZONE OBTAINED USING THE STELLAR OCCULTATION TECHNIQUE.

Geographic position refers to the tangent ray point of the occultation measurements (time as LMT; longitude measured positive eastward from Greenwich).

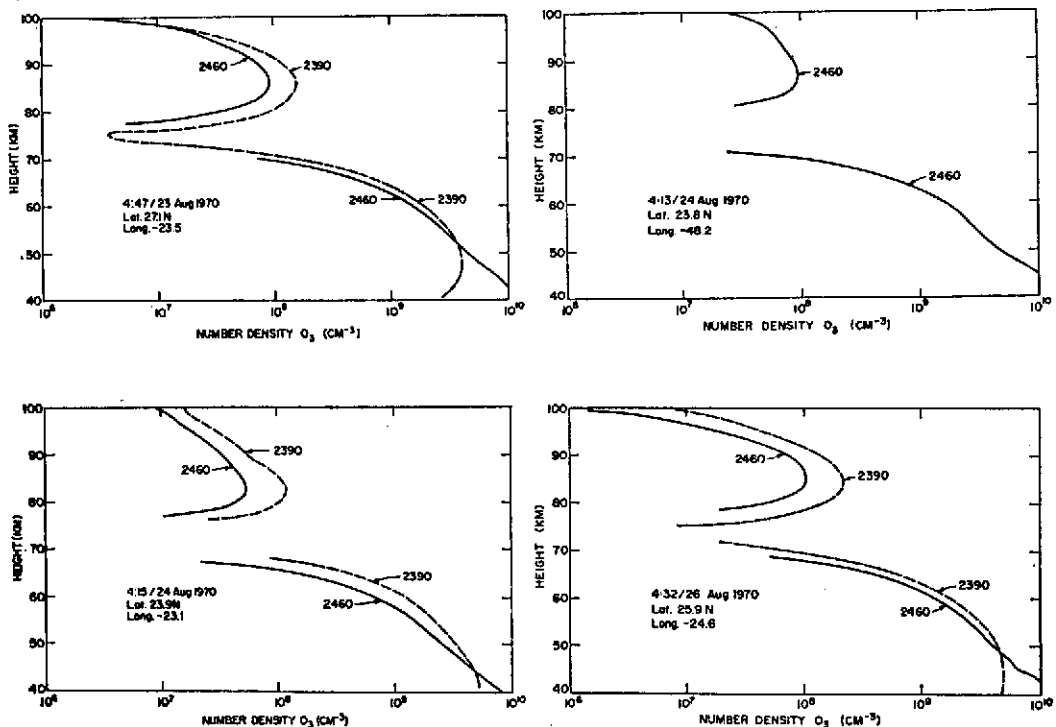


FIG. 4. SAME CAPTION AS IN FIG. 3.

A careful examination of these ozone profiles in the mesosphere does not show any striking systematic seasonal or diurnal pattern in the equatorial regions. There does appear to be a systematic increase in the altitude of the high altitude ozone bulge with increasing latitude. However, due to the small number of scans, which were made at nearly random local time, season, and latitude, it is difficult to place great weight on the

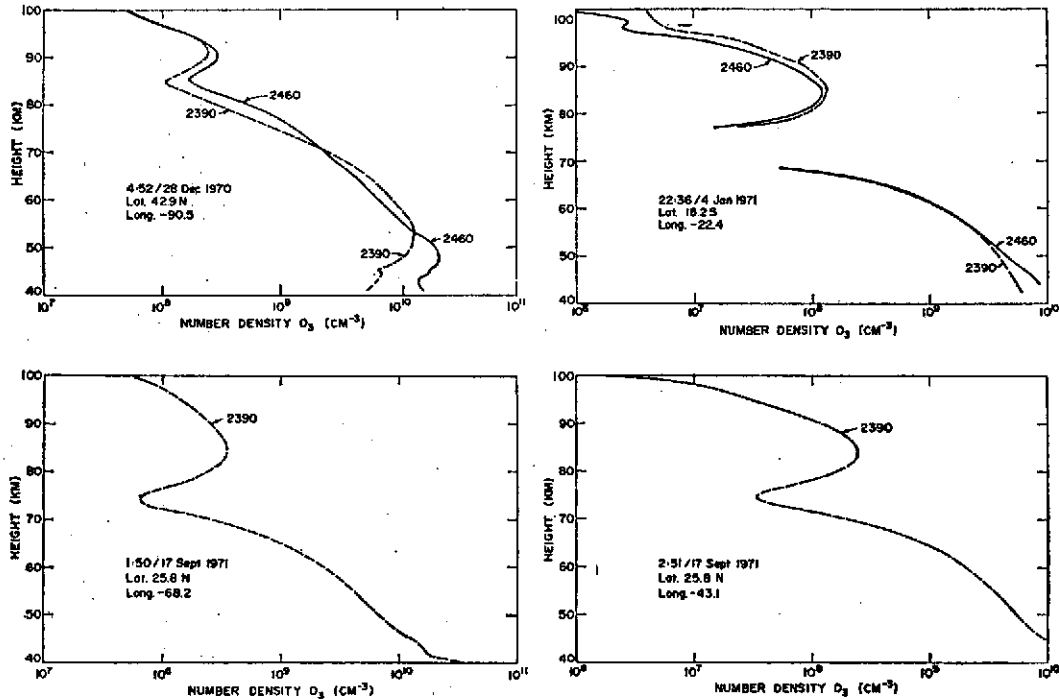


FIG. 5. SAME CAPTION AS IN FIG. 3.

slight variations observed. The major conclusion is that ozone varies very little between 55 and 100 km during the course of the night at low latitudes. The main feature is the expected bulge in density which occurs at approximately 85 km and the depletion of ozone just below that altitude. This behavior is predicted by most recent theoretical studies which incorporate the hydrogen chemistry in their model.

Numerous theoretical studies of the chemistry of ozone in a moist atmosphere have followed the early discussion of Bates and Nicolet (1950) of the influence of hydrogen compounds (Hampson, 1964; Hunt, 1966a; Hunt, 1966b; Hesstvedt, 1968; Leovy, 1969; Bowman *et al.*, 1970; Hunt, 1971; Shimazaki and Laird, 1972; Strobel, 1972). The result of these studies is somewhat confusing due to the large number of choices of possible rate coefficients, photodissociation rates, boundary conditions, and eddy mixing rates used by these authors. The theoretical results do indicate the general features observed in the OAO-2-A2 stellar occultation measurement. This is illustrated in Fig. 6 where three representative theoretical profiles (Hesstvedt, 1968; Shimazaki and Laird, 1970; Hunt, 1971) are compared with our envelope curve. Hunt (1971) appears to agree best with the observations, but it is difficult to assess whether this is fortuitous or the result of a correct choice of the multitude of possible variable factors.

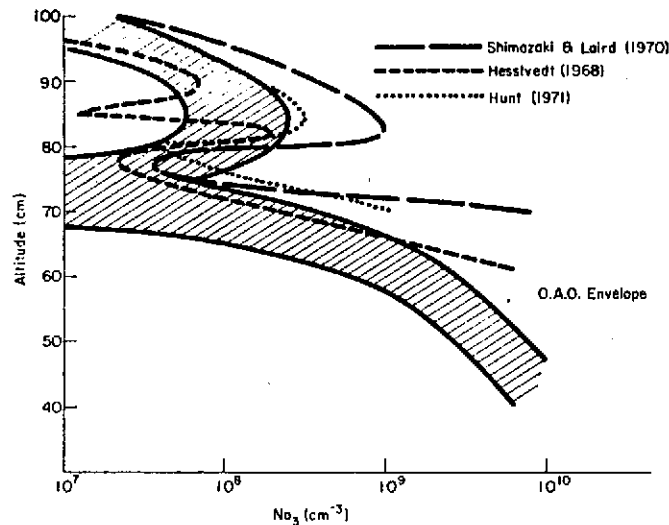


FIG 6. COMPARISON OF MEASURED OZONE ENVELOPE WITH THEORETICAL CALCULATIONS.

It should be pointed out that during the period of time in which these observations were taken one series of measurements was made while the large magnetic storm of August 1970 was in progress. There does not appear to be any significant correlation between the ozone density in the mesosphere and storm in agreement with the prediction of Maeda and Aiken (1968). This is not surprising, but relatively large variation in  $O_2$  at higher altitudes was observed (Hays and Roble, 1972) at the same time.

#### SUMMARY

The stellar occultation measurements of ozone in the mesosphere indicate the following conclusions:

1. Mesospheric ozone varies by as much as a factor of 4 at high altitudes, but does not show any clear seasonal or diurnal nighttime pattern. A slight increase in the altitude of the 85 km bulge appears to be associated with increasing latitude.
2. The observations are generally in agreement with results of theoretical predictions which utilize a moist atmosphere in which hydrogen compounds are considered in the chemistry.
3. There is no apparent relationship between mesospheric ozone and geomagnetic activity at low latitudes.

*Acknowledgement*—This work was supported by NASA Grant NGR 23 005-360 and NASA Contract NAS 1 9958. The National Center for Atmospheric Research is sponsored by the National Science Foundation. We wish to acknowledge the helpful support of Dr. Houk, and the O.A.O. Wisconsin and NASA experimental team.

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APPENDIX VII.

The Nighttime Distribution of Ozone  
in the Low-Latitude Mesosphere

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THE NIGHTTIME DISTRIBUTION OF OZONE  
IN THE LOW-LATITUDE MESOSPHERE

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<sup>1</sup>The National Center for Atmospheric Research is sponsored by the National Science Foundation.

In press, Pure and Applied Geophysics, 1973.

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## ABSTRACT

The intensity of stars at wavelengths in the Hartley continuum region of ozone has been monitored by the University of Wisconsin stellar photometers aboard the OAO-2 satellite during occultation of the star by the earth's atmosphere. These occultation data have been used to determine the ozone number density profile at the occultation tangent point. The nighttime ozone number density profile has a bulge in its vertical profile with a peak of 1 to  $3 \times 10^8 \text{ cm}^{-3}$  at approximately 83 km and a minimum near 75 km. The ozone number density at high altitudes varies by as much as a factor of 4, but does not show any clear seasonal variation or nighttime variation. The retrieved ozone number density profiles define a data envelope that is compared with other nighttime observations of the ozone number density profile and also the results of theoretical models.

Calculations are also presented that illustrate the difference in retrieving the bulge in the ozone number density profile from stellar and solar occultation data.

## 1. Introduction

Stellar intensity measurements, in certain ultraviolet atmospheric absorption bands, made by a satellite during occultation of the star by the earth's atmosphere, can be used to obtain the local number density of the absorbing species at the occultation tangent point (Hays and Roble, 1968a;b). During the past few years the Orbiting Astronomical Observatory (OAO-2) has been used to make these occultation intensity measurements using bright ultraviolet stars (Hays, et al., 1972). The occultation technique is based on classical ultraviolet absorption spectroscopy. The star is the source of ultraviolet light and the stellar photometers aboard the OAO-2 satellite are the detectors. The atmosphere between the star and the detectors acts as the absorption cell. During the occultation process, the stellar light passes through progressively denser regions of the atmosphere and the ultraviolet light is absorbed due to the strong absorption features of certain atmospheric gases. The basic measurement is thus the stellar intensity as a function of time, but by knowing the star's position and the satellite orbital elements, the intensity is related to the tangent ray height of the occulting star. If absorption is due to a single species, the normalized stellar intensity is directly related to the tangential column number density of the absorbing species through Beer's Law (Hays and Roble, 1972a). The data giving the tangential column

number density as a function of tangent ray height are directly inverted, using the technique described by Roble and Hays (1972) to obtain the local number density profile of the absorbing species at the occultation tangent point.

Occultation measurements made in the Schumann - Runge continuum of molecular oxygen near 1500 Å have been used to obtain the molecular oxygen number density profile in the lower thermosphere. These measurements have been discussed by Hays and Roble (1972a). The nighttime number density profile of ozone in the mesosphere has also been determined from stellar intensity measurements made in the Hartley continuum region of ozone near 2500 Å. The results of these measurements are discussed by Hays and Roble (1972b).

In this paper, we compare the results of the OAO-2 ozone measurements with theory and other nighttime ozone measurements. We also present the results of a single occultation scan and discuss the difference in retrieving the ozone number density bulge near 80 km from solar and stellar occultation data.

## 2. Stellar occultation scan

The ozone and molecular oxygen absorption cross sections and the transmission function of the various ultraviolet filters used with the OAO-2 stellar photometers for the occultation measurements are shown in Fig. 1. Filter (4-1) is located in the Schumann - Runge continuum region of molecular oxygen and is used to obtain the molecular oxygen number density in the lower thermosphere. Filters (2-5) and (3-2) are in the Hartley continuum region of ozone and are used to obtain the ozone number density profile in the mesosphere. Filter (1-4) is used to measure the ozone number density in the stratosphere.

The results presented in this paper were determined from occultation data obtained by the OAO-2 stellar photometers using filters (2-5) and (3-2). Hays and Roble (1968b) have shown that stellar ultraviolet light near  $2500 \text{ \AA}$  is primarily absorbed by ozone in the earth's atmosphere and stellar occultation measurements within this wavelength region can be used to obtain the ozone number density profile in the 60 to 100 km altitude interval. In the lower portion of the altitude interval rayleigh scattering and molecular oxygen absorption in the Hertzberg continuum have small contributions to the total attenuation. These contributions are eliminated from the data as discussed by Hays and Roble (1972a).

The intensity spectrum of the star above the earth's atmosphere

is measured by an ultraviolet spectrometer aboard the OAO-2 satellite. A typical star spectrum, obtained on orbit 12178 on April 3, 1971, is shown in Fig. 2. The solar spectrum is also shown in Fig. 2 for comparison. The normalized signal of this star, measured during occultation using the OAO-2 stellar photometer with filter (3-2), is shown in Fig. 3a. Below 100 km the signal decreases due to absorption by ozone. The decrease in the signal continues until an altitude of about 83 km, then a slight increase in the signal occurs reaching a maximum around 75 km. Below 75 km the signal decreases again until occultation near 45 km. The ozone number density profile at the occultation tangent point is obtained from the intensity data using the inversion scheme described by Roble and Hays (1972) and Hays and Roble (1972a) and is shown in Fig. 3b. The dip in the normalized intensity curve near 80 km, shown in Fig. 3a, is due to a bulge in the ozone number density distribution at that altitude. The peak ozone number density at the bulge is  $3 \times 10^8 \text{ cm}^{-3}$  at 83 km. The minimum occurs near 73 km, however the magnitude of the number density at that altitude is difficult to determine. A complete discussion of accuracy of the inversion scheme and an analysis of errors in the occultation measurements have been given by Roble and Hays (1972).

### 3. Stellar vs. solar occultation

The ozone number density profile has also been determined from solar occultation measurements made from a satellite (Rawcliffe et al., 1963; Miller and Stewart, 1965). If the intensity from the entire solar disk is used during occultation, the effect of the finite size of the sun must be considered in analyzing the data. In this section we examine whether the bulge in the ozone number density near 80 km can be determined from solar occultation measurements when the entire solar disk is used as the source. For simplicity a model of the ozone number density profile is used in these calculations. The ozone profile is approximated by a sum of gaussian and exponential functions (Roble and Hays, 1972) and the profile is shown in Fig. 3b. The normalized intensity calculations for a stellar occultation, using OAO-2 filter (3-2), are shown in Fig. 3a. The intensity calculations for the broadband filter require a wavelength integration over the effective passband of the filter (Hays and Roble, 1972a). Therefore, we use the stellar spectrum, shown in Fig. 2, the ozone absorption cross-section and the filter transmission functions, shown in Fig. 1, to calculate the normalized occultation signal for the model ozone number density distribution. The calculated normalized intensity profile is shown in Fig. 3a and it has approximately the same shape as the OAO-2 occultation data. The dip in the intensity curve defines the bulge in the ozone number density profile.



For a solar occultation, the finite size of the sun must be considered because the solar energy emitted from different portions of the solar disk is transmitted through the earth's atmosphere at different tangent ray heights. The normalized intensity for a solar occultation is calculated using the technique of Roble and Norton (1972) and is shown in Fig. 3a. For these calculations, we used the solar energy spectrum, shown in Fig. 2, the ozone cross-section and the filter (3-2) transmission function shown in Fig. 1. The dip in the intensity curve that occurs in the stellar occultation case does not appear in the solar occultation case. From normal satellite altitudes the tangent ray heights of the upper and lower limb of the solar disk are separated by approximately 25 km. Therefore, a smearing of the intensity variation caused by the bulge in the ozone number density profile occurs when the entire solar disk is used during occultation and a dip in the intensity profile is not evident. The calculated normalized intensity variation across the solar disk at a tangent ray height of 83 km for the center of the solar disk is shown in Fig. 4. The tangent ray height of the upper limb is 95 km and the lower limb is 70 km. The intensity variation caused by the bulge in the ozone number density profile at 83 km occurs near the center of the solar disk. The intensity structure across the solar disk

is not evident when the overall intensity from the entire solar disk is measured during occultation. In theory, the structure in the ozone number density profile can be retrieved from solar occultation data using the entire solar disk, with an iterative technique. However, in practice, the iterative scheme is difficult to use especially in the presence of statistical noise. Better results would be obtained if a small portion of the solar disk is used for the solar occultation measurements.

#### 4. Comparison of results

The ozone number density profiles from 12 stellar occultations have been presented by Hays and Roble (1972b). The measurements indicate that mesospheric ozone has a variation of as much as a factor of 4 at high altitudes, but does not show any clear seasonal or nighttime variation. A slight increase in the altitude of the bulge in the ozone number density profile appears to be associated with increasing latitude. These measurements plus a few additional ones are used to define an envelope representing the ozone number density profiles determined from the OAO-2 stellar occultation measurements. The envelope is shown in Fig. 5 along with nighttime and twilight ozone profiles determined by other experimenters. At high altitudes there is good agreement with the ozone number density profiles determined by the OAO-2 satellite and the profiles deduced by Evans and Llwelllyn (1970; 1972). Between 60 and 75 km, the data of Evans and Llwelllyn (1970; 1972), Evans et al. (1970), Weeks and Smith (1968), Miller and Stewart (1965), Tisone (1972) and Rawcliffe et al. (1963) are clustered along the upper limb of the OAO-2 ozone data envelope. Below 60 km, the OAO-2 ozone data envelope does not agree with the results of Carver, et al. (1967; 1972), Reed (1968) and Hilsenrath (1971). This departure from the other observations is probably due to the difficulty of retrieving the ozone number density at low signal levels in the presence of statistical noise.

Roble and Hays (1972) show that the ozone number density is best retrieved between normalized occultation data values of 0.1 - 0.9. Therefore, the ozone number density profile determined from the OAO-2 occultation measurements are limited to the 60 - 95 km altitude interval. In Fig. 6 the OAO-2 ozone data envelope is also compared with the ozone number density profile determined from various theoretical models. In general, there is a large variation between the various theoretical models. However, the general features of the OAO-2 ozone measurements appear to be present in the results of theoretical predictions which utilize a moist atmosphere in which hydrogen compounds are considered in the chemistry.

#### ACKNOWLEDGMENT

We thank K. S. Hansen for programming assistance and A. N. Shah for obtaining the data. This work was supported in part by NASA Grant NGR 23 005-360 and NASA contract NAS 19958. We wish to acknowledge the helpful support of Dr. Houk and the OAO Wisconsin and NASA experimental team.

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## FIGURE LEGENDS

- Fig. 1. Absorption cross-sections for molecular oxygen and ozone. The dashed curves give the transmission functions for the filters used with the OAO-2 stellar photometers. Filter (4-1) is used to obtain the molecular oxygen number density in the lower thermosphere and Filters (2-5), (3-2) and (1-4) are used to determine the ozone number density in the mesosphere.
- Fig. 2. Solid curve is the stellar intensity as a function of wavelength measured by the OAO-2 spectrometer prior to occultation on orbit 12178. This star was used to obtain the occultation data shown in Fig. 3. The dashed curve is the solar spectrum obtained from Ackerman (1970).
- Fig. 3. a) The dots give the normalized intensity as a function of tangent ray height measured by the OAO-2 stellar photometers using the 2460 Å filter. The geographic position of the occultation tangent point is 35°N, 114°W and time is 22:33 LMT on April 3, 1971. The solid and dashed curves are the calculated normalized intensities for a stellar and solar occultation respectively using the model ozone profile in Fig. 3b. b) The dots give the retrieved ozone number density profile from the OAO-2 occultation data shown in Fig. 3a. The solid curve is the model ozone number density profile used to calculate the stellar and solar intensity profiles in Fig. 3a.

Fig. 4. The calculated normalized intensity distribution across the solar disk at a tangent ray height of 83 km. The satellite altitude at occultation is 715 km. The OAO-2 filter transmission function for the 2460 Å filter is used in the calculations. The tangent ray heights for the upper and lower solar limbs are also given in the figure.

Fig. 5. Comparison of the OAO-2 ozone data envelope with the nighttime ozone number density profiles determined by various observers.

Fig. 6. Comparison of the OAO-2 ozone data envelope with the ozone number density profile determined from various theoretical models.

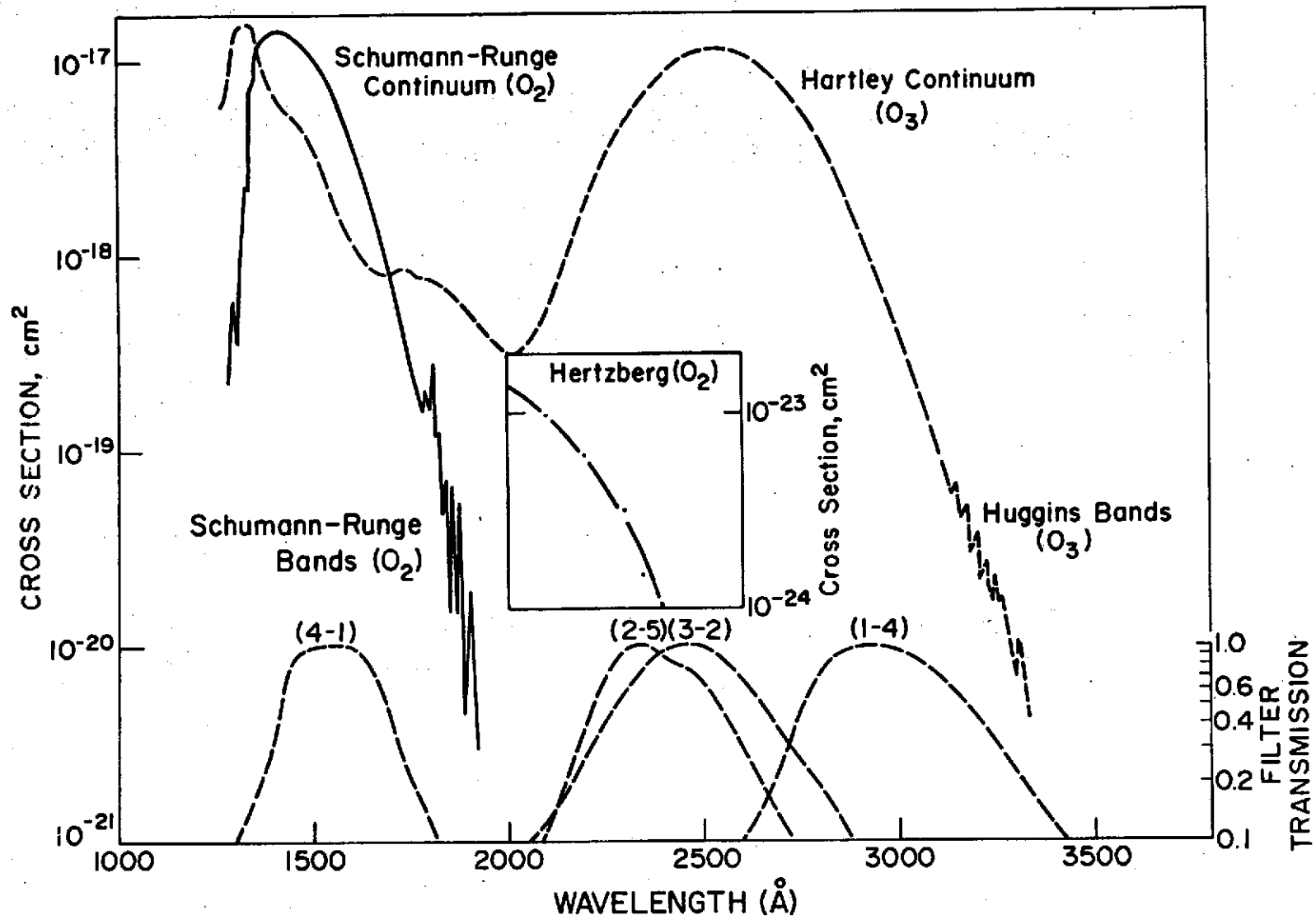


Figure 1.

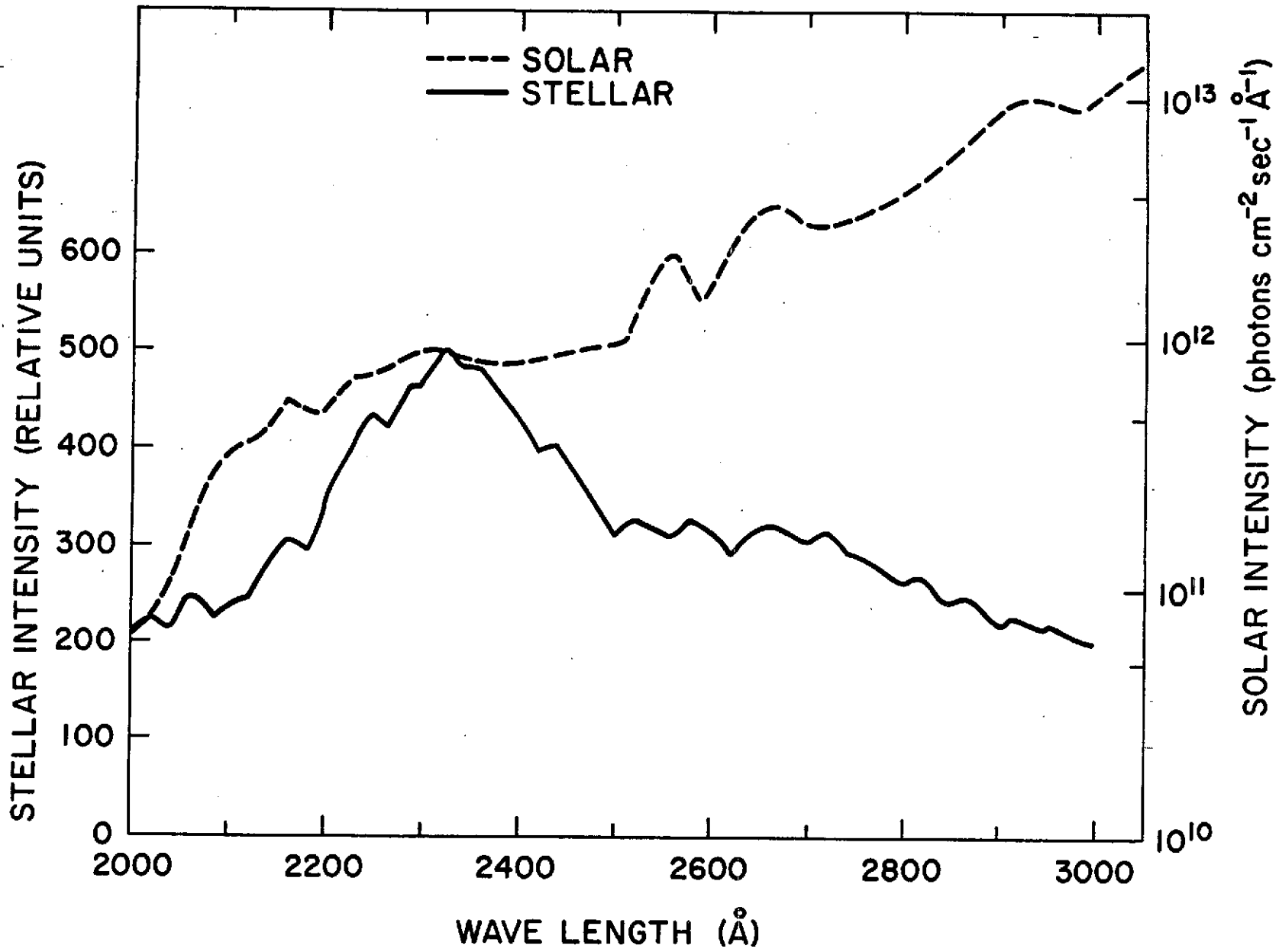


Figure 2.

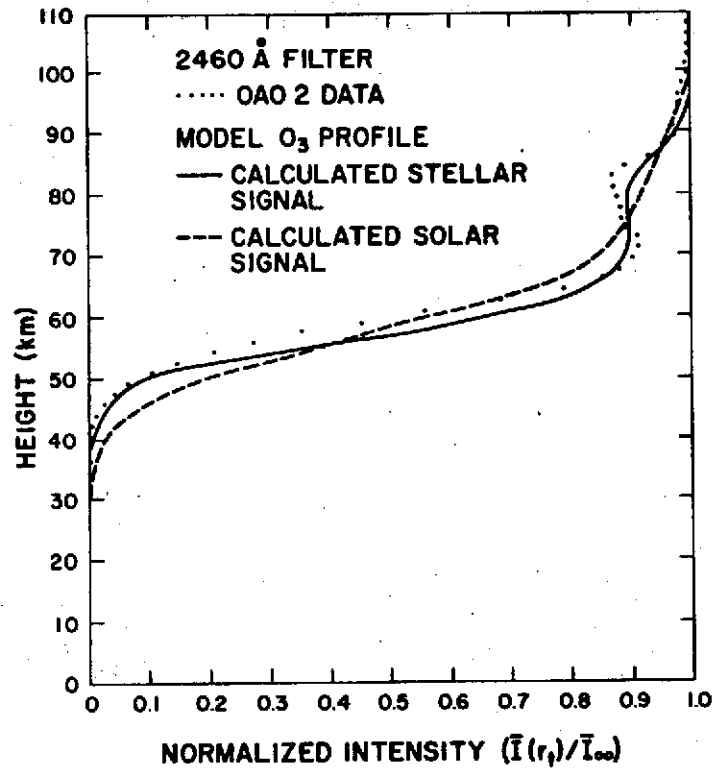


Figure 3a.

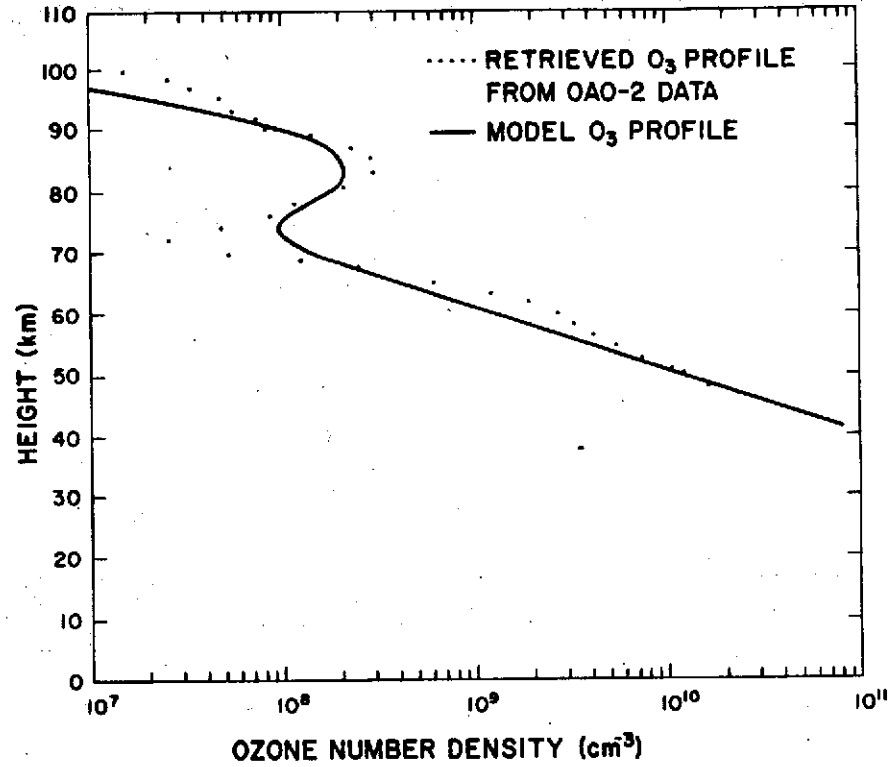


Figure 3b.

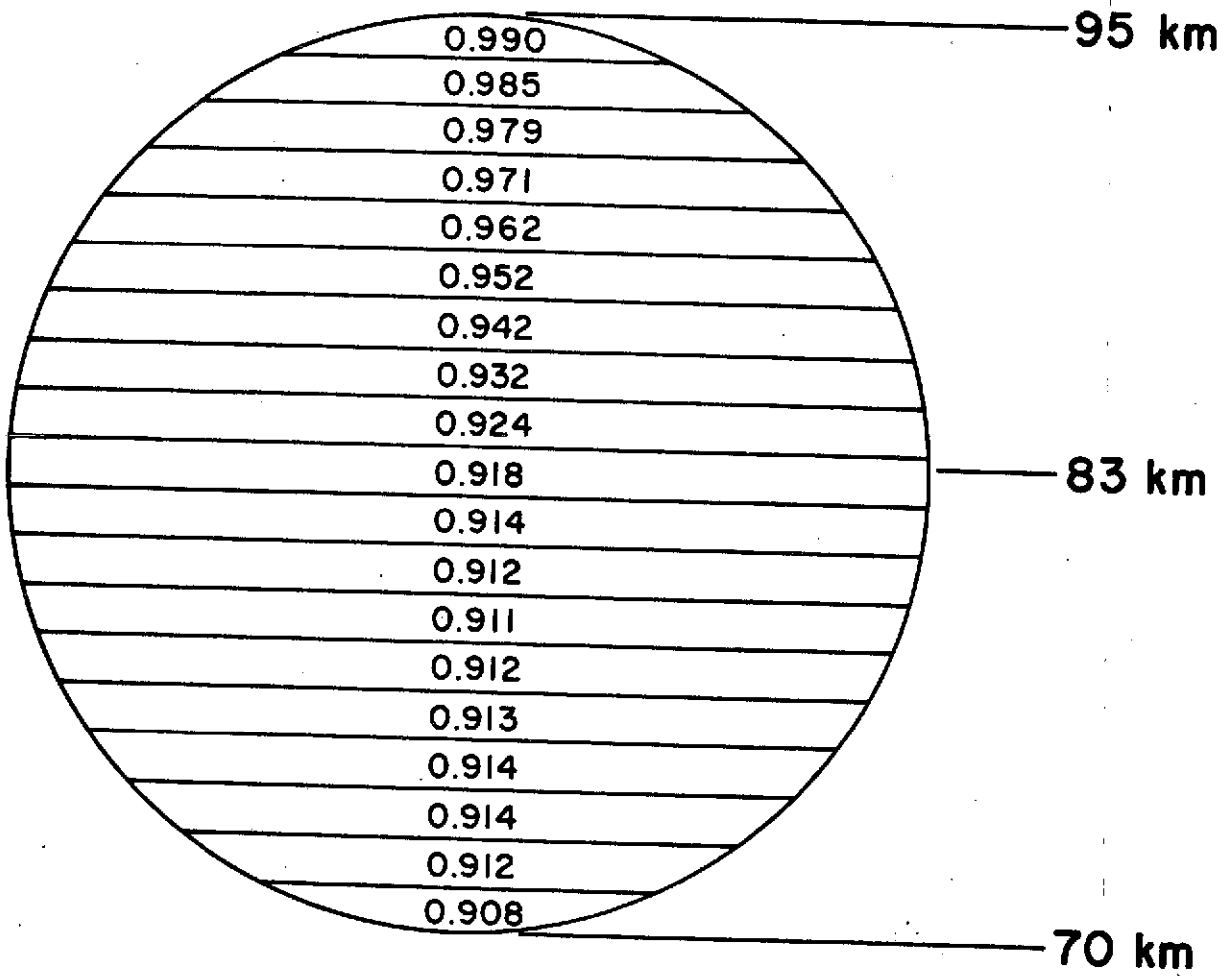


Figure 4.

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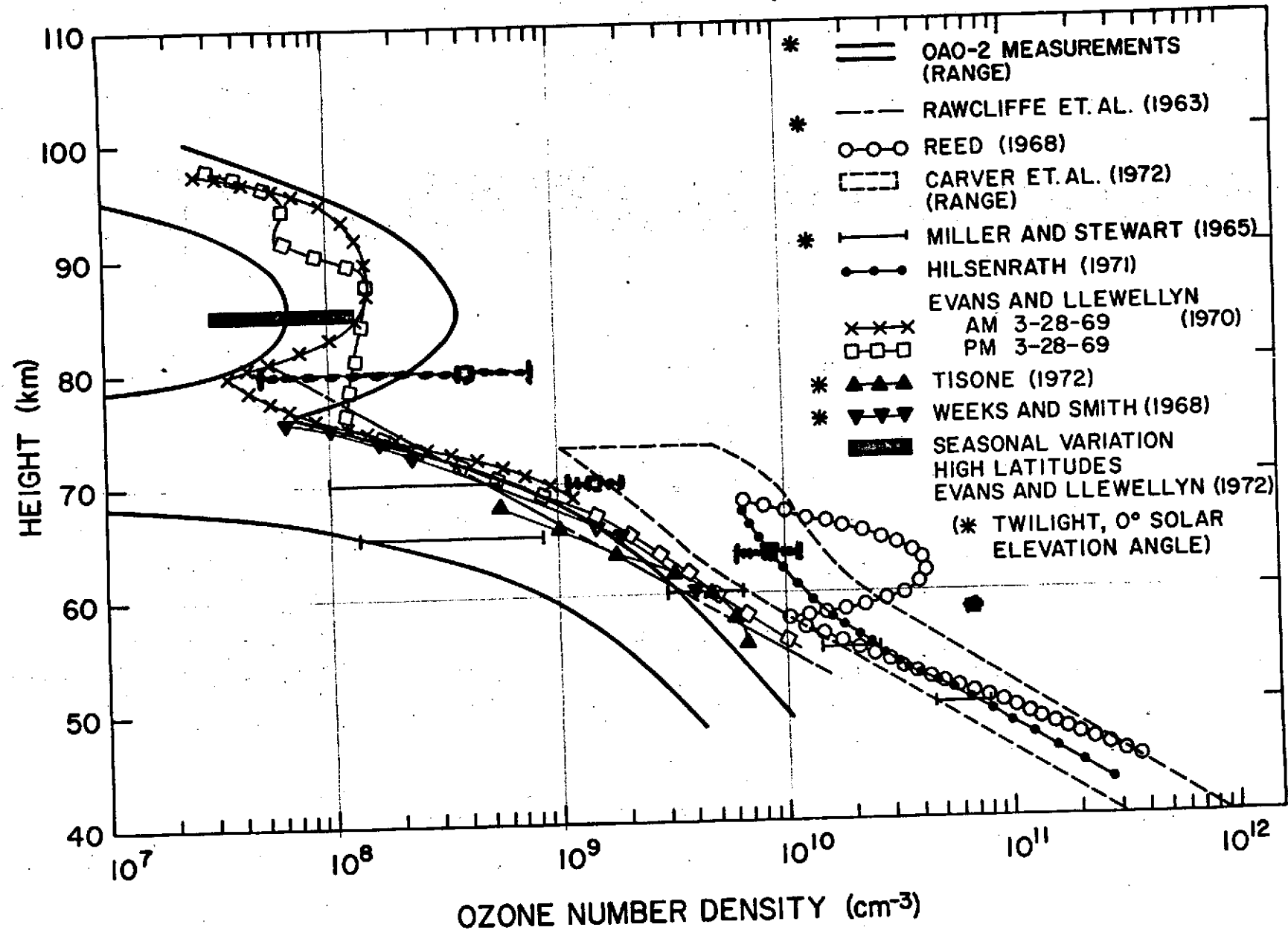


Figure 5.



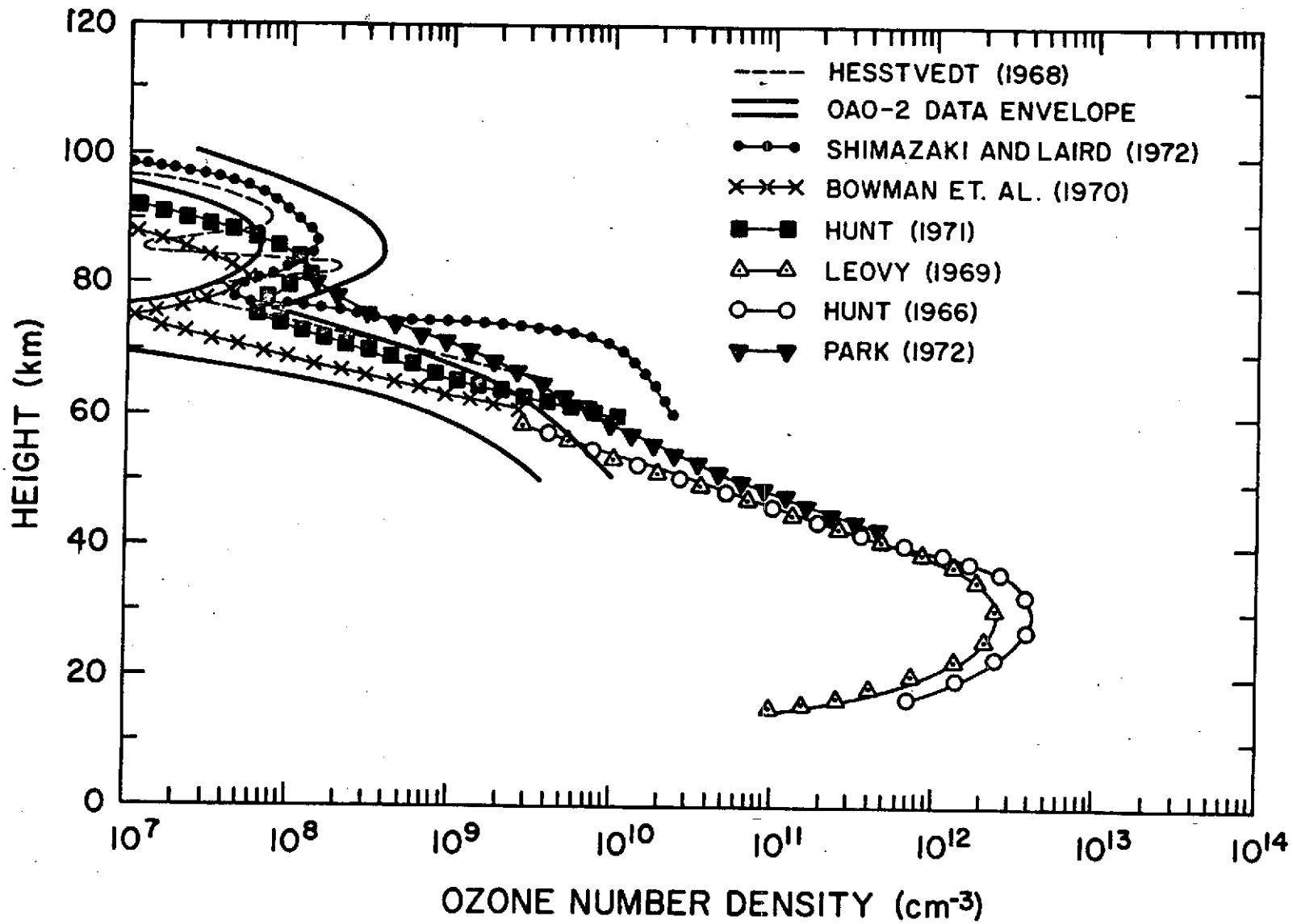


Figure 6.