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ANALYSIS OF STALL FLUTTER OF A HELICOPTER ROTOR BLADE

by Peter Crimi

Prepared by
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for Langley Research Center

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16. Abstract

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SUMMARY

A study of rotor blade aeroelastic stability was carried out, using an analytic model of a two-dimensional airfoil undergoing dynamic stall and an elastomechanical representation including flapping, flapwise bending and torsional degrees of freedom. Results for a hovering rotor demonstrated that the models used are capable of reproducing both classical and stall flutter. The minimum rotor speed for the occurrence of stall flutter in hover was found to be determined from coupling between torsion and flapping. Instabilities analogous to both classical and stall flutter were found to occur in forward flight. However, the large stall-related torsional oscillations which commonly limit aircraft forward speed appear to be the response to rapid changes in aerodynamic moment which accompany stall and unstall, rather than the result of an aeroelastic instability. The severity of stall-related instabilities and response was found to depend to some extent on linear stability. Increasing linear stability lessens the susceptibility to stall flutter and reduces the magnitude of the torsional response to stall and unstall.

INTRODUCTION

Aeroelastic stability of a helicopter rotor blade is a multifaceted problem because of the extreme variations of the aerodynamic environment within the flight envelope of the aircraft. In hovering flight, a blade can undergo classical binary flutter (Ref. 1) or stall flutter (Ref. 2). In forward flight, the linear instability experienced by systems with periodically varying parameters can occur (Ref. 3). While these types of instability are not normally encountered with blades of current design, due to the relatively low disc loading and weak coupling of translational and rotational degrees of freedom, they are certainly not precluded from new designs, particularly those intended to extend present performance capabilities. Of immediate concern, however, in both design and operation, is the occurrence of large-amplitude torsional oscillations and excessive control-linkage loads associated with blade stall on the retreating side of the rotor disc at high forward speed or gross weight, effectively limiting aircraft performance. This problem has prompted a number of recent studies of dynamic stall and the effects of stall on blade dynamics (Refs. 4-8).

While stall has been identified as a causal element of the problem, the nonlinearity of the stall process, coupled with the unsteady aerodynamic environment, has precluded an analysis to the depth required to gain a thorough understanding of the mechanisms involved. In particular, it has not been clear whether the blade undergoes a true aeroelastic instability, a simple forced response, or some hybrid phenomenon which takes on the character of one or the other extreme, depending on flight conditions and blade vibrational characteristics.

Stall flutter for axial flight is amenable to analysis by empirical methods similar to those developed for analyzing stall flutter in cascades (Ref. 9). The flutter mechanism for that case has been identified as deriving from the extraction of energy from the free stream by the periodic variation of the aerodynamic moment. Analogous methods applied to the forward-flight problem (Refs. 10 and 11) have been inconclusive, however, the primary difficulty possibly being in applying empirical methods without a clear definition of the underlying mechanism of the problem.

A method was recently developed for analyzing dynamic stall of an airfoil undergoing arbitrary pitching and plunging motions which provides an ideal tool for analyzing the stall problem in forward flight. The method, which is described in detail in Ref. 7, employs models for each of the basic flow elements contributing to the unsteady stall of a two-dimensional airfoil. Calculations of the loading during transient and sinusoidal pitching motions are in good qualitative agreement with measured loads. Dynamic overshoot, or lift in excess of the maximum static value, as well as unstable moment variation, are in clear evidence in the computed results.

This study was directed to analyzing the aeroelastic stability of a helicopter rotor, particularly as it relates to stall, using the method of Ref. 7 to compute aerodynamic loading. The representation of the elastomechanical system includes flapping and flapwise bending degrees of freedom as well as torsion. A listing of the computer program used to perform the calculations is given in Appendix A.

SYMBOLS

b	blade semichord, m
$\bar{C}_L$	mean lift coefficient, ratio of time average of l to $\rho \Omega^2 R^2 b$
C_l	lift coefficient, $C_l = c_l /(\rho U^2 b)$
$C_{m\,c/4}$	moment coefficient referred to quarterchord, $C_{m\,c/4} = m_{c/4}/(2 \rho U^2 b^2)$
c	blade chord, m
f_θ	mode shape of first uncoupled torsional mode, unit tip deflection
f_ϕ	mode shape of first uncoupled flapwise bending mode, unit tip deflection
h_β	tip deflection due to flapping, semichords
h_ϕ	tip deflection due to bending, semichords
h_i	translational coordinates of 2-D system ($i = 1, 2$), semichords
I_o	moment of inertia of 2-D system about pitch axis, kg - m
I'_θ	blade moment of inertia about elastic axis per unit span, kg - m
k_i	translational spring stiffnesses of 2-D system ($i = 1, 2$), N/m^2
k_θ	torsional spring stiffness of 2-D system, N/rad
l	lift per unit span at aerodynamic reference radius, N/m
l_{s_i}	offsets of springs from pitch axis of 2-D system ($i = 1, 2$), m
M_b	total blade mass, kg
m	blade mass per unit span, kg/m
$m_{c/4}$	aerodynamic moment per unit span at aerodynamic reference radius, N

m_1	masses of 2-D system, kg/m
R	rotor radius, m
r_o	inner radius of blade lifting surface, m
r_R	aerodynamic reference radius, m
U	instantaneous free-stream speed at aerodynamic reference section, m/sec
U_o	reference speed, $U_o = \Omega \ r_R$, m/sec
x_m	distance aft of elastic axis of blade section mass center, m
$\overline{x}$	distance aft of pitch axis of mass center of m_1, m
Z_β	generalized coordinate of 2-D system, equivalent to h_β , semichords
Z_ϕ	generalized coordinate of 2-D system, equivalent to h_ϕ, semichords
α	angle of attack, deg
δ	flapping hinge offset, m
θ_o	collective pitch angle, deg or rad
θ_1	blade tip torsional deflection, rad
$\tilde{\theta}$	angle of zero restraint of 2-D system torsion spring, rad
μ	advance ratio, ratio of forward speed to Ω R
ρ	free-stream density, kg/m^3
τ	dimensionless time, $\tau = U_o \ t/b$
ψ	blade azimuth angle measured from downwind direction, deg or rad
Ω	rotor rotational speed, rad/sec
Ω^*	dimensionless rotor speed, $\Omega^* = \Omega R/(\omega_{\theta_o} b)$
ω_f	flutter frequency, rad/sec

ω_{θ_o}	frequency of first uncoupled, nonrotating torsion mode, rad/sec
ω_{ϕ_o}	frequency of first uncoupled, nonrotating flapwise bending mode, rad/sec

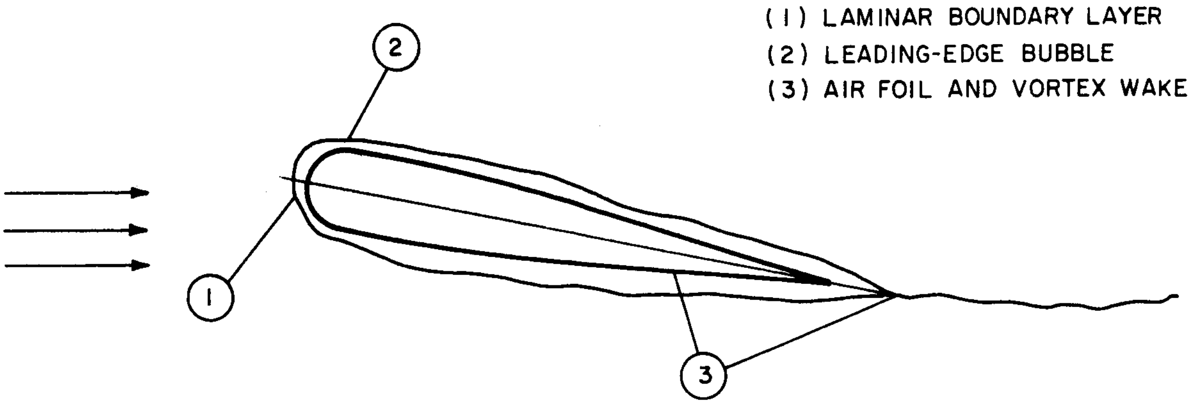
PROBLEM FORMULATION

Aerodynamic Loading

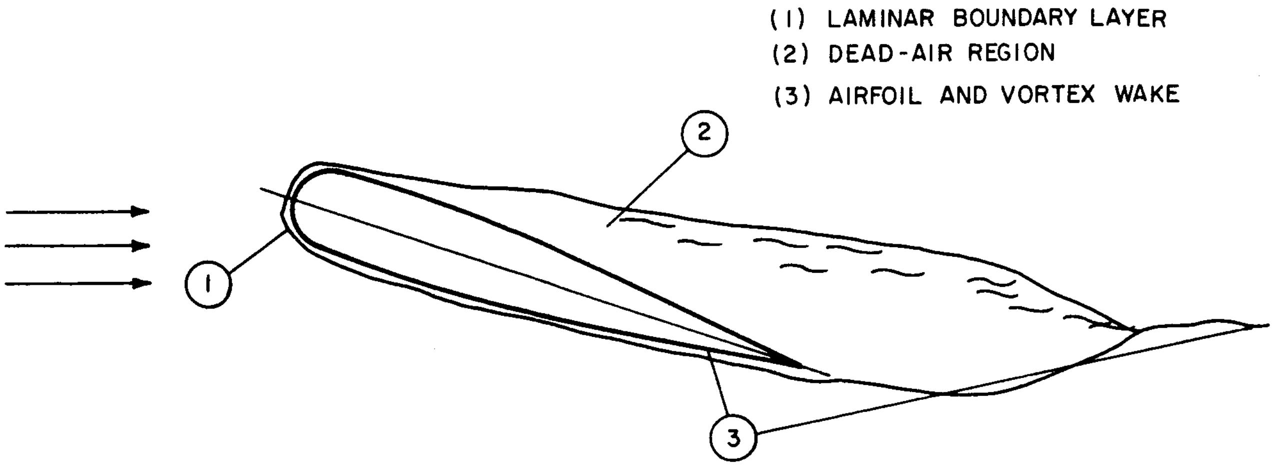
In the flutter analysis, only leading-edge stall was considered, so the following relates specifically only to that type, even though the basic method can treat trailing-edge stall as well. When the airfoil is not stalled, the flow elements represented are (see Figure 1a): (1) the laminar boundary layer from the stagnation point to separation near the leading-edge, (2) the small leading-edge separation bubble; and, (3) a potential flow, including a vortex wake generated by the variation with time of the circulation about the airfoil. When the airfoil is stalled, as indicated in Figure 1b, the flow elements are: (1) the laminar boundary layer, (2) a dead-air region extending from the separation point to the pressure recovery point; and, (3) a potential flow external to the airfoil and dead-air region, again including a vortex wake. The analytic representations of these elements are described briefly below. Details are given in Ref. 7.

Potential Flow.—Given the airfoil section characteristics and motions, together with the distribution of pressure in the dead-air region if the airfoil is stalled, the flow and pressure over the airfoil must be determined to compute the integrated load and analyze the boundary layer. The problem was formulated by imposing linearized boundary conditions of flow tangency and pressure, using a perturbation velocity potential derived from source and vortex distributions. The resulting coupled set of singular integral equations is solved by casting the singularity distributions in series form and solving for the unknown coefficients by imposing boundary conditions at prescribed points.

Boundary Layer.—Because the relative importance of the individual elements of the boundary layer flow as they affect dynamic stall could not be established in advance, the representation in Ref. 7 was made as general as possible. The method of finite differences for unsteady flows with variable step size in both streamwise and normal directions, was employed, with the error in each finite-difference approximation the order of the square of the step size. It was determined from preliminary calculations performed for this study that, at least for leading-edge stall, results are virtually unaffected by assuming quasi-steady flow in the boundary layer. That assumption was therefore employed for all flutter computations, to take advantage



(a) ATTACHED FLOW



(b) LEADING-EDGE STALL

Figure 1 FLOW ELEMENTS

of the resulting substantial savings in computer storage requirements and computing time.

Dead-Air Region.—The function of the model of the dead-air region is to define the streamwise distribution of pressure in that region, given the locations of the separation and recovery points and the pressure at the recovery point. The dead-air region is assumed to consist of a laminar constant-pressure free shear layer from separation to transition, a turbulent constant-pressure mixing region, and a turbulent pressure-recovery region. The laminar shear layer is analyzed by the method of Ref. 12, assuming quasi-steady flow. The turbulent mixing and pressure-recovery regions are analyzed using the steady-flow momentum integral and first moment equations. Profile parameters in these regions are assumed to be universal functions of a dimensionless streamwise coordinate, with those functions derived from an exact viscous-inviscid interaction calculation. Matching of approximate solutions for the mixing and pressure-recovery regions at their interface completes the analysis.

Leading-Edge Bubble.—The leading-edge bubble on an unstalled airfoil is analyzed using the same basic relations employed for the dead-air region. Given the boundary-layer parameters at separation, the length of the bubble and the amount of pressure rise possible, for that length, in the pressure recovery region, are computed. That pressure rise is compared with the rise in pressure in the potential flow over the length of the bubble. If the latter is greater than the former, the bubble is assumed to have burst, and the stall process is initiated.

Loading Calculation Procedure.—Calculations proceed by forward integration in time, using the blade motions derived by integrating the equations of motion of the elastomechanical system. If, at a given instant, the airfoil is not stalled, the potential flow is computed, and the boundary layer and leading-edge bubble are analyzed to check for bubble bursting. If the airfoil is stalled, the pressure distribution in the dead-air region is computed, the potential flow evaluated, and the boundary layer is analyzed to locate the separation point. The last two steps are repeated iteratively until assumed and computed separation points agree. Rate of growth of the dead-air region is determined from an estimate of the rate of fluid entrainment derived from the potential-flow solution. Unstall is determined by first postulating its occurrence and analyzing the leading-edge bubble which would then form to ascertain whether that event did in fact occur.

During unstall, the dead-air region is washed off the airfoil at the free-stream speed.

Elastomechanical Representation

The equations of motion for a rotor blade with flapping, flapwise bending and torsional degrees of freedom can be written in the form (Ref. 3)

$$\frac{d^2 h_\beta}{d\tau^2} + \frac{R}{b}\,\frac{M_{\beta\theta}}{M_{\beta\beta}}\,\frac{d^2\theta_1}{d\tau^2} + \bar{\omega}_\beta^{\,2}\, h_\beta - \frac{R}{b}\,\bar{\Omega}^2\,\frac{T_{\beta\theta}\,\theta_1}{M_{\beta\beta}}$$

$$= \frac{Rb}{U_o^{\,2}}\,\frac{F_\beta}{M_{\beta\beta}}$$

$$\frac{d^2 h_\phi}{d\tau^2} + \frac{M_{\phi\theta}}{b\,M_{\phi\phi}}\,\frac{d^2\theta_1}{d\tau^2} + \bar{\omega}_\phi^{\,2}\, h_\phi - \bar{\Omega}^2\,\frac{T_{\phi\theta}}{M_{\phi\phi}}\,\theta_1$$

$$= \frac{b}{U_o^{\,2}}\,\frac{F_\phi}{M_{\phi\phi}}$$

$$\frac{d^2\theta_1}{d\tau^2} + \frac{b}{R}\,\frac{M_{\beta\theta}}{M_{\theta\theta}}\,\frac{d^2 h_\beta}{d\tau^2} + \frac{b\,M_{\phi\theta}}{M_{\theta\theta}}\,\frac{d^2 h_\phi}{d\tau^2} + \bar{\omega}_\theta^{\,2}\,\theta_1$$

$$- \frac{b}{R}\,\bar{\Omega}^2\,\frac{T_{\beta\theta}}{M_{\theta\theta}}\,h_\beta - \bar{\Omega}^2\,\frac{b\,T_{\phi\theta}}{M_{\theta\theta}}\,h_\phi$$

$$= \frac{b^2\,F_\theta}{U_o^{\,2}\,M_{\theta\theta}}$$

where h_β and h_ϕ are tip displacements due to flapping and bending, respectively, in semichords, θ_1 is torsional displacement at the blade tip and the frequencies* are the following functions of rotational speed:

$$\bar{\omega}_\beta^{\,2} = -\bar{\Omega}^2 \frac{T_{\beta\beta}}{M_{\beta\beta}}, \qquad \bar{\omega}_\phi^{\,2} = \bar{\omega}_{\phi_o}^{\,2} - \bar{\Omega}^2 \frac{T_{\phi\phi}}{M_{\phi\phi}},$$

$$\bar{\omega}_\theta^{\,2} = \bar{\omega}_{\theta_o}^{\,2} - \bar{\Omega}^2 \frac{T_{\theta\theta}}{M_{\theta\theta}}$$

The inertial and centrifugal-force coefficients are given by

$$M_{\beta\beta} = \int_\delta^R (r+\delta)^2 \, m dr, \qquad M_{\phi\phi} = \int_\delta^R m \, f_\phi^{\,2} \, dr,$$

$$M_{\theta\theta} = \int_\delta^R I_\theta' \, f_\theta^{\,2} \, dr,$$

$$M_{\beta\theta} = -\int_\delta^R m \, x_m \, (r-\delta) \, f_\theta \, dr,$$

$$M_{\phi\theta} = -\int_\delta^R m \, x_m \, f_\phi \, f_\theta \, dr,$$

$$T_{\beta\beta} = -\int_\delta^R r \, (r-\delta) \, m \, dr,$$

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*Barred quantities are dimensionless frequencies, U_o/b being reference frequency; e.g., $\bar{\Omega} = \Omega \, b/U_o$.

$$T_{\phi\phi} = -\int_{\delta}^{R} f_{\phi}'^{\,2} \left\{ \int_{r}^{R} r_1\, m\,(r_1)\, d\, r_1 \right\} dr,$$

$$T_{\theta\theta} = -M_{\theta\theta}, \qquad T_{\beta\theta} = -M_{\beta\theta},$$

$$T_{\phi\theta} = \int_{\delta}^{R} (r-\delta)\, f_{\phi}'\; f_{\theta}\, m\, x_m\, dr$$

The complexity of the aerodynamic representation precludes evaluation of the generalized forces F_β , F_ϕ and F_θ by the usual strip approximation. It was felt essential, however, to retain both translational degrees of freedom in the investigation of the forward-flight problem, so a simple two-dimensional model of the dynamics could not be used. Therefore, a two-dimensional airfoil suspended in such a way as to have three degrees of freedom was analyzed. Inertial and stiffness parameters were assigned to make the coupled natural frequencies of the two-dimensional system match those of the rotor blade.

The system analyzed is shown schematically in Figure 2. The matching of the two-dimensional system with the blade dynamics proceeds as follows. Three generalized coordinates are first defined to correspond to those of the blade. Clearly, angular displacement θ_1 should correspond to blade torsional displacement at the blade tip. The counterparts of flapping and bending, Z_β and Z_ϕ, respectively, are defined by

$$Z_\beta = A_1\, h_1 + Bh_2, \quad Z_\phi = A_2\, h_1 - Bh_2$$

$$\text{where} \quad A_1 = \frac{\bar{\omega}_\beta^{\,2} - \bar{\omega}_2^{\,2}}{\bar{\omega}_\phi^{\,2} - \bar{\omega}_\beta^{\,2}}, \quad A_2 = \frac{\bar{\omega}_2^{\,2} - \bar{\omega}_\phi^{\,2}}{\bar{\omega}_\phi^{\,2} - \bar{\omega}_\beta^{\,2}},$$

$$B = \frac{(\bar{\omega}_2^{\,2} - \bar{\omega}_\phi^{\,2})(\bar{\omega}_2^{\,2} - \bar{\omega}_\beta^{\,2})}{(\bar{\omega}_\phi^{\,2} - \bar{\omega}_\beta^{\,2})\,\bar{\omega}_2^{\,2}} \qquad (1)$$

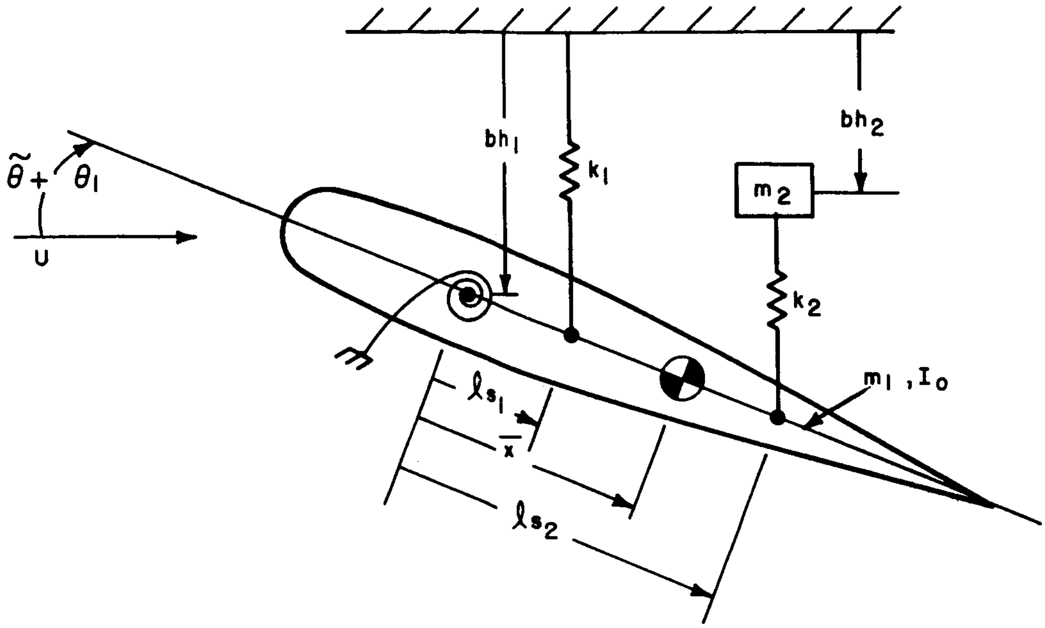

Figure 2 TWO-DIMENSIONAL ELASTOMECHANICAL SYSTEM

and $$\bar{\omega}_i^2 = (k_i/m_i)(b/U_o)^2, \quad i = 1, 2.$$

With the above definitions, $Z_\beta + Z_\phi = -h_1$, to give the correct translational correspondence. It can further be shown that the uncoupled natural frequencies of the two-dimensional system match those of the blade, provided

$$\left(\frac{k_\theta + k_1 l_{s_1}^2 + k_2 l_{s_2}^2}{I_o}\right)\left(\frac{b}{U_o}\right)^2 = \bar{\omega}_\theta^2$$

while $\bar{\omega}_1^2$ and $\bar{\omega}_2^2$ satisfy

$$\bar{\omega}_1^2 \; \bar{\omega}_2^2 = \bar{\omega}_\phi^2 \; \bar{\omega}_\beta^2 ,$$

$$\bar{\omega}_1^2 + (1 + m_2/m_1) \, \bar{\omega}_2^2 = \bar{\omega}_\phi^2 + \bar{\omega}_\beta^2 \qquad (2)$$

By comparing the generalized masses of the two systems, it follows that

$$m_1 \, b^2/I_o = - A_1 \, M_{\beta\beta} \, b^2/(M_{\theta\theta} \, R^2)$$

$$A_2/A_1 = M_{\beta\beta} \, /(M_{\phi\phi} \, R^2) \equiv \lambda_m$$

The last relation, together with Eqs. (1) and (2), fixes m_2/m_1:

$$m_2/m_1 = \frac{(1 + \lambda_m)(\bar{\omega}_\phi^4 + \lambda_m \, \bar{\omega}_\beta^4)}{(\lambda_m \, \bar{\omega}_\beta^2 + \bar{\omega}_\phi^2)^2} - 1$$

Equating the corresponding coefficients of the characteristic equations of the two systems provides three additional relations, which can be solved for the coupling parameters $\bar{x}$, l_{s_1}, l_{s_2}. That calculation is outlined in Appendix B.

To complete the matching, quasi-steady approximations to the damping terms of the flapping equations are equated with the result that

$$m_1 \; R/(-A_1) \;=\; 4 \, \frac{r_R}{R} \, \frac{M_{\beta\beta}}{R^2 \left[1 - (r_o/R)^4\right]}$$

$$U/U_o \;=\; 1 + \frac{4}{3} \left[\frac{1 - (r_o/R)^2}{1 - (r_o/R)^4} \right] \mu \sin \psi$$

where $\Omega r_R = U_o$. The aerodynamic reference radius r_R was selected to be .75R.

The angle of zero restraint in torsion was varied periodically to approximate the effects of cyclic pitch variation in forward flight, according to the formula

$$\tilde{\theta} \;=\; \theta_o \; \left[1 - 2 \, (R/r_R) \;\; \mu \sin \psi\right]$$

This variation gives nominally constant lift.

The equations of motion were solved by integrating analytically, using linear extrapolations to approximate the variation of lift and aerodynamic moment over the interval of integration. This scheme was found to give satisfactory results, provided the time interval of integration is no longer than about one fifth of the period of the coupled mode having the highest natural frequency.

RESULTS OF COMPUTATIONS

Configurations Analyzed

Vibrational and aerodynamic characteristics of the blade analyzed were selected to correspond to those of the model rotor blade described in Ref. 2. That blade is untwisted, of constant chord, with offset flapping hinge. Pertinent dimensionless parameters of the model blade are listed in Table 1.

TABLE 1

BLADE PARAMETERS FOR NOMINAL CONFIGURATION

Parameter	Value
b/R	.0435
δ/R	.0543
r_o/R	.174
$\omega_{\theta_o}/\omega_{\phi_o}$	3.69
$\rho R\, b^2/M_b$	.00431
x_m/b	.216
$m\, R/M_b$	1.055
$I'_\theta/M_b\, R$	3.51×10^{-4}

Two elastomechanical configurations in addition to the nominal one were analyzed. One of these had $\omega_{\theta_o}/\omega_{\phi_o} = 2.5$, with all other parameters as listed in Table 1. The third configuration had $x_m/b = .108$, with the remaining parameters as listed in Table 1.

The bending mode shape, which was computed by a finite-element method, was found not to vary appreciably over the range of rotational speeds of interest. The mode shape for $\omega_{\phi_o}/\Omega = 1.26$, which is plotted in Figure 3, was used for all computations. The torsional mode shape for the nonrotating blade, also shown in Figure 3, was used to evaluate torsional inertia parameters.

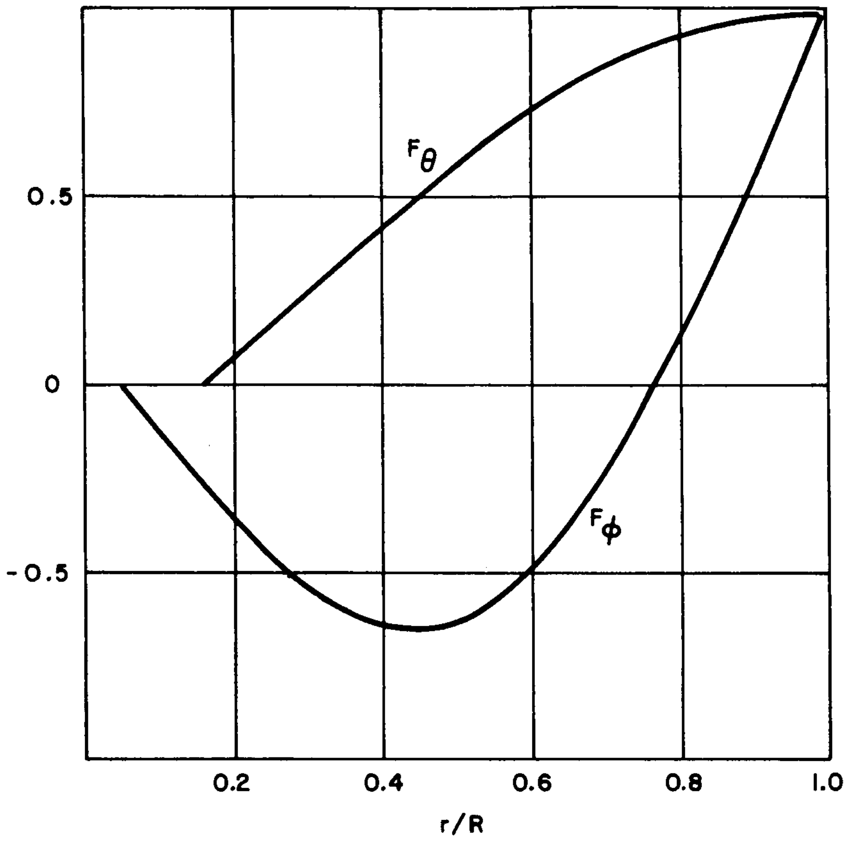

Figure 3 BENDING AND TORSION MODE SHAPES

The test blade had a NACA 23012 section. The variation of static lift and moment coefficients with angle of attack for this section were computed from a series of transient pitch calculations, and are shown in Figure 4, together with the measured section characteristics, from Ref. 13. The aerodynamic model is seen to give nearly the correct maximum lift, but at a slightly lower angle of attack, and, as indicated from the variation of $C_{m\ c/4}$, the computed center of pressure is somewhat further aft than that of the actual airfoil section below the stall angle.

Stability in Hover

Initial calculations were performed for hovering flight, with the nominal configuration, to allow a direct comparison with the test results of Ref. 2. First, rotor speed was varied parametrically, with the collective pitch at a value well below the stall incidence. A classical bending-torsion instability was encountered at $\Omega^* \equiv \Omega R/(\omega_{\theta_0} b) = 5.3$ with $\omega_f/\omega_{\theta_0} = .803$. The variation of bending, flapping, and torsional displacements with azimuth angle at flutter onset are shown in Figure 5. By way of comparison, tests (Ref. 2) yielded classical flutter at about $\Omega^* = 7.1$ with $\omega_f/\omega_{\theta_0} = .72$.

It should be noted that since the system stability was analyzed by direct simulation, a precise point of linear instability was not computed. The values of Ω^* at onset of a linear instability, both for hover and forward flight, were obtained by successively increasing or decreasing rotor speed, in small steps, until the transient response changed from convergent to divergent, or visa versa. The maximum error in the value of flutter speed, for the results presented here, is estimated to be about three percent.

Susceptibility of the system to stall flutter was investigated next. It was found that a torsional limit cycle, at approximately the highest coupled natural frequency of the system, could be triggered for Ω^* as low as 3.4. Computed blade motions for stall flutter at Ω^* of 3.5 are shown in Figure 6.

For Ω^* below 3.4, a limit cycle could not be set up, regardless of the initial conditions or the collective pitch angle. Severe oscillations involving repeated stall and unstall could be made to occur by imposing a large initial bending deflection. However, the flapping response modulated the torsional response, and caused continuous stall and/or unstall of the blade over a significant portion of

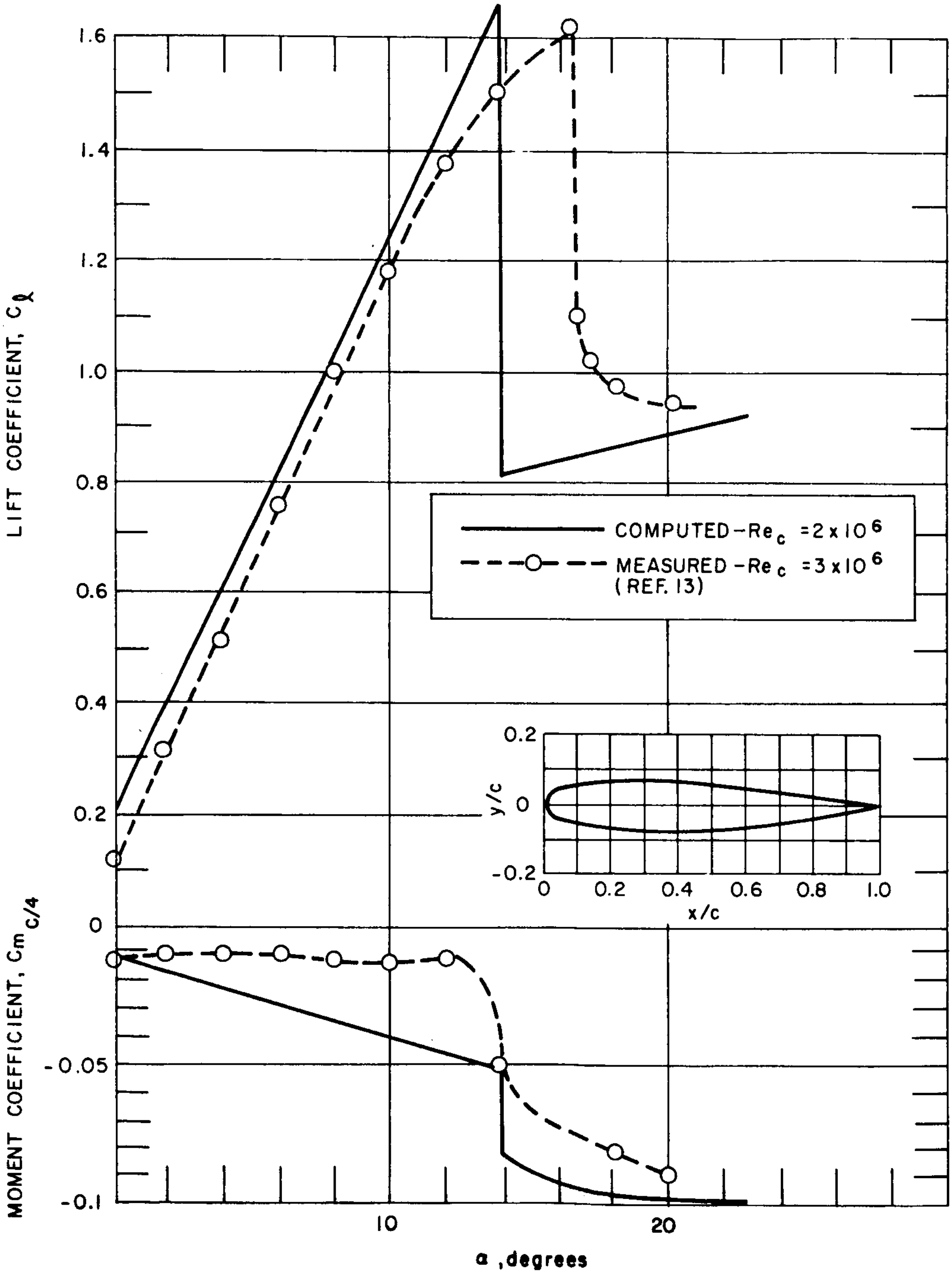

Figure 4 AIRFOIL SECTION CHARACTERISTICS FOR NACA 23012

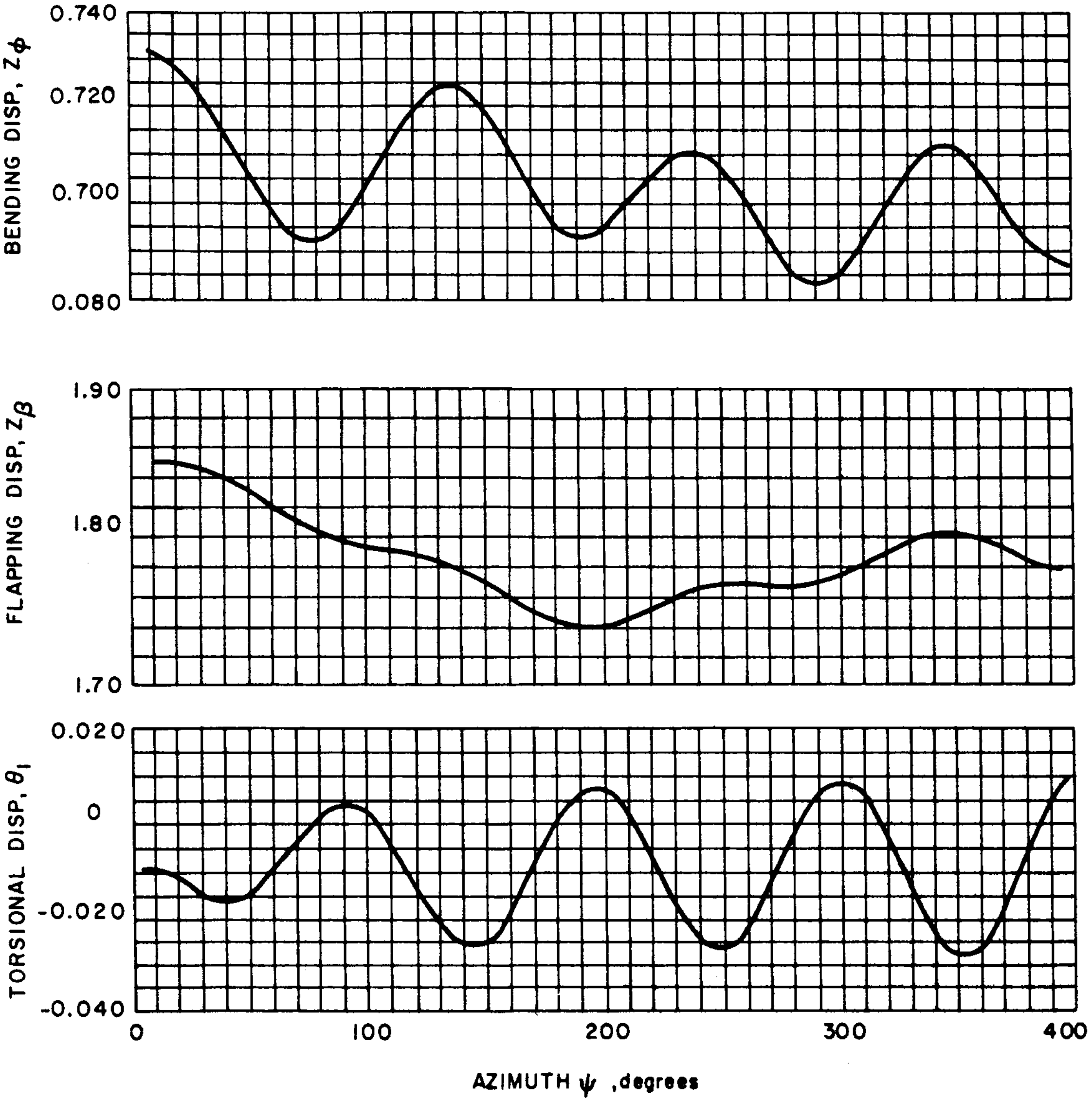

Figure 5 DISPLACEMENT TIME HISTORIES AT CLASSICAL FLUTTER ONSET
$\Omega^* = 5.3, \theta_0 = 11$ deg, $\mu = 0$

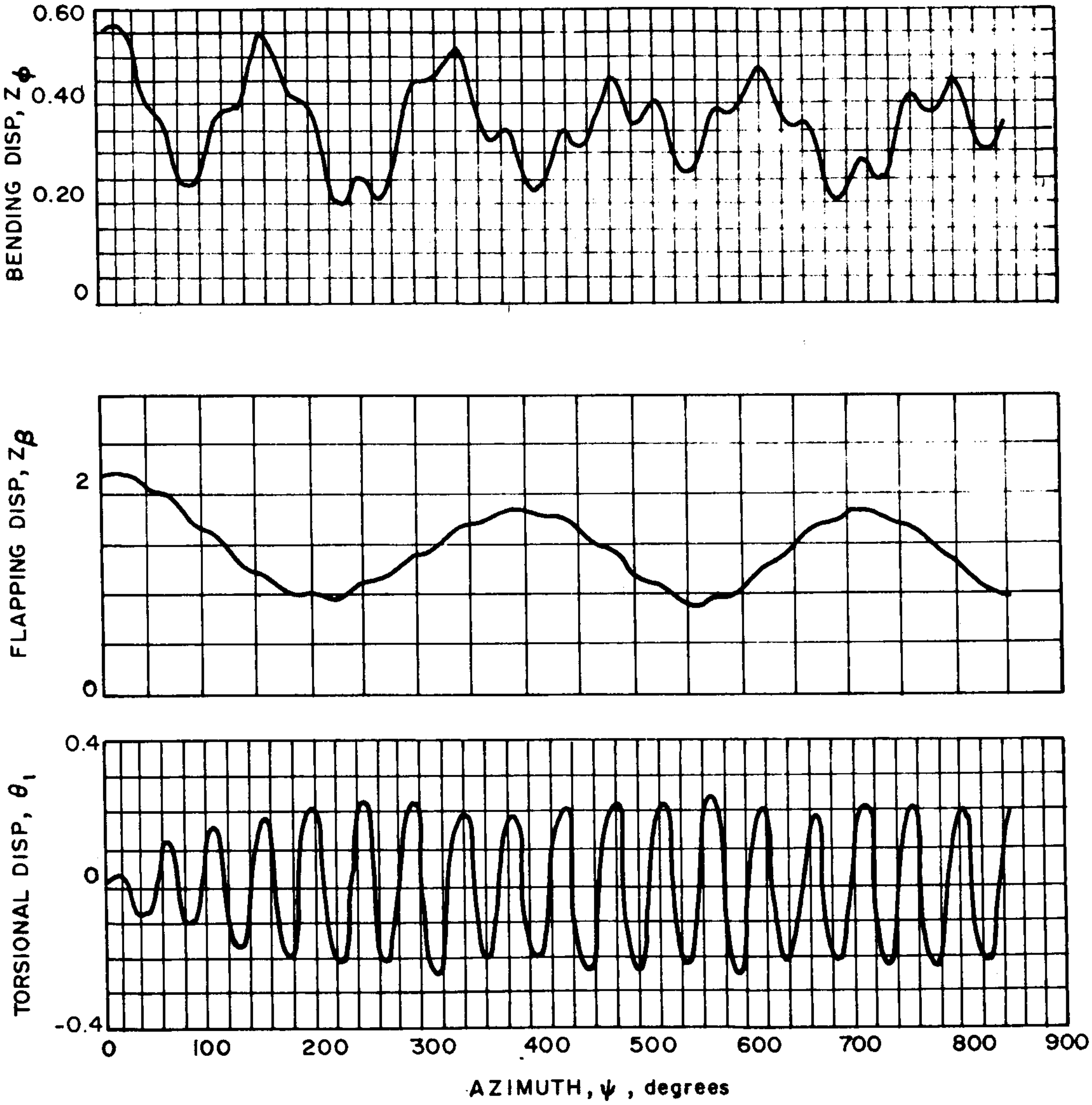

Figure 6 DISPLACEMENT TIME HISTORIES FOR STALL FLUTTER
$\Omega^* = 3.5$, $\theta_0 = 15.0$ deg, $\mu = 0$

a revolution, due to the large plunging rate generated by the flapping motion. An example of this occurrence is shown in Figure 7. Thus, while stall flutter involves only the rotational degree of freedom, the results obtained indicate that the minimum speed for its occurrence is determined by coupling with a translational degree of freedom.

Results for the hovering case are summarized in Figure 8, which compares computed and measured flutter speed and frequency, plotted against collective pitch angle. No upper limit in collective pitch angle for the occurrence of stall flutter was calculated, since that limit would depend strongly on initial conditions, and so would be arbitrary. Quantitative differences between the computed and measured stability boundaries of Figure 8 can be attributed in large part to the use of a two-dimensional aerodynamic model, which cannot precisely reproduce the aerodynamic coupling between the rotational and translational degrees of freedom.

From the basic similarity of the computed and measured stability boundaries and the character of the computed instabilities (Figures 5 and 6) it can be concluded that the aerodynamic and dynamic models formulated are capable of reproducing both classical and stall flutter as experienced by a rotor blade, and so can be employed to investigate the forward-flight problem.

Stability in Forward Flight

The nominal configuration was analyzed next for an advance ratio of .1. Computations were carried out in the same sequence as for hovering. First, the rotational speed at which classical flutter occurs was determined. Then, stall-related instabilities were investigated.

A linear bending-torsion instability of the Floquet type (Ref. 14) was encountered at $\Omega^* = 5.2$. Blade motions as a function of azimuth angle at flutter onset are shown in Figure 9. The torsional and bending displacements are seen to display the aperiodic character typical of this type of instability. The flapping motion is the steady-state response to the cyclic pitch variation.

An instability analogous to stall flutter in hover was found to occur for Ω^* as low as about 4.4, with collective pitch angle greater than 12 deg. Blade motions for $\Omega^* = 4.8$ are shown in Figure 10. The torsional displacement time history, while not strictly periodic, is nonetheless

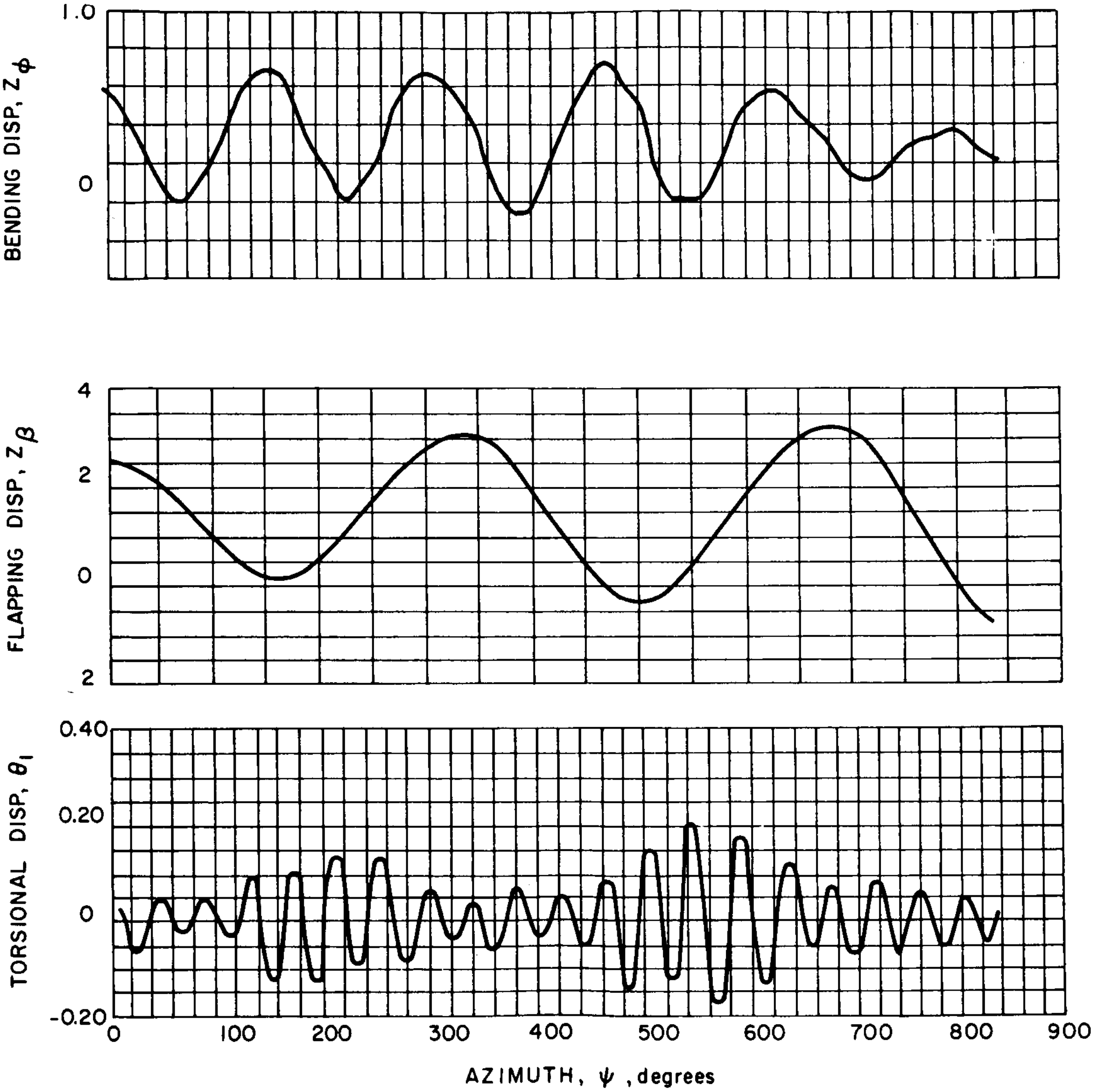

Figure 7 BLADE RESPONSE BELOW STALL FLUTTER BOUNDARY
$\Omega^* = 3.1$, $\theta_0 = 15.0$ deg, $\mu = 0$

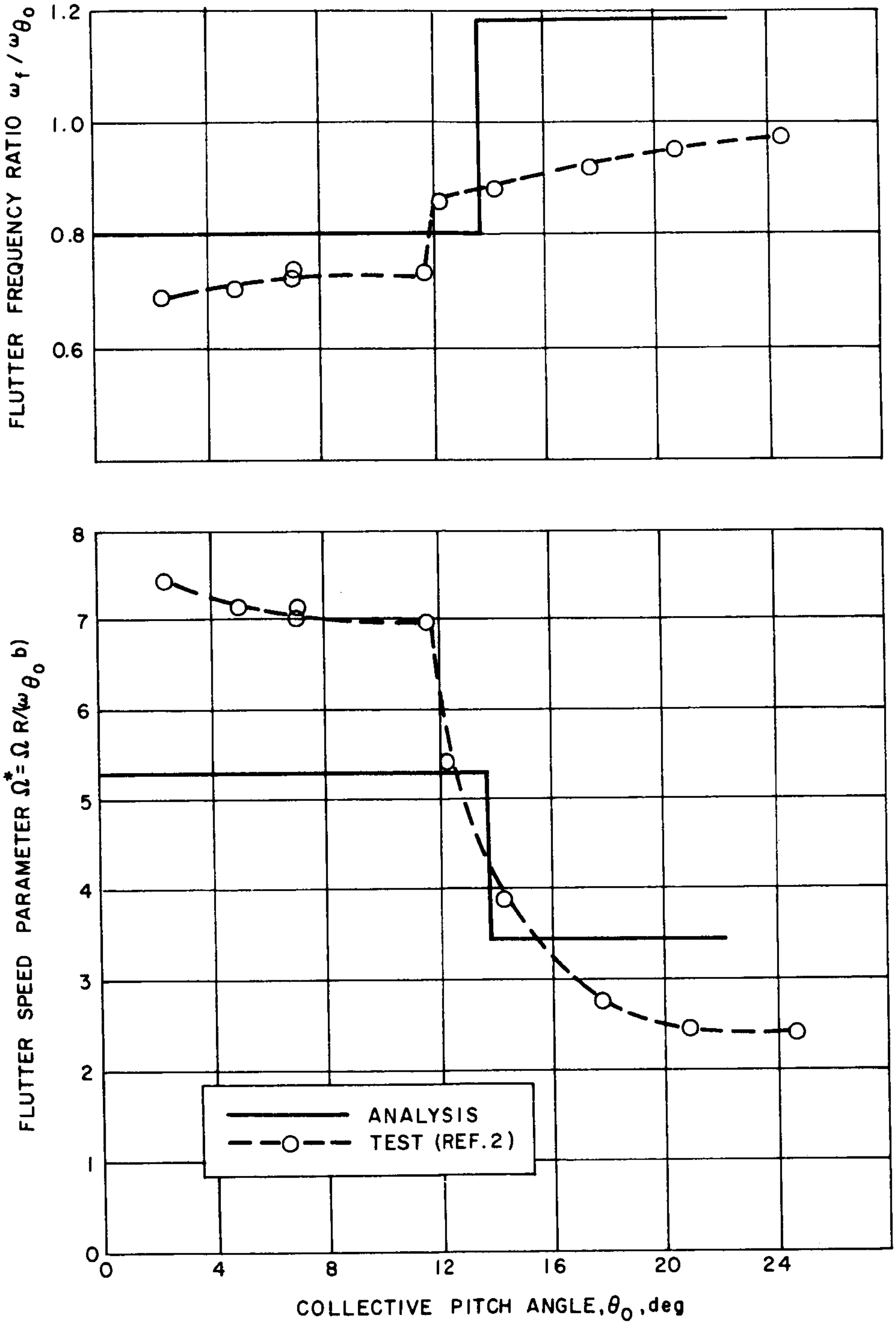

Figure 8 FLUTTER SPEED AND FREQUENCY VARIATION WITH COLLECTIVE PITCH ANGLE FOR A HOVERING ROTOR

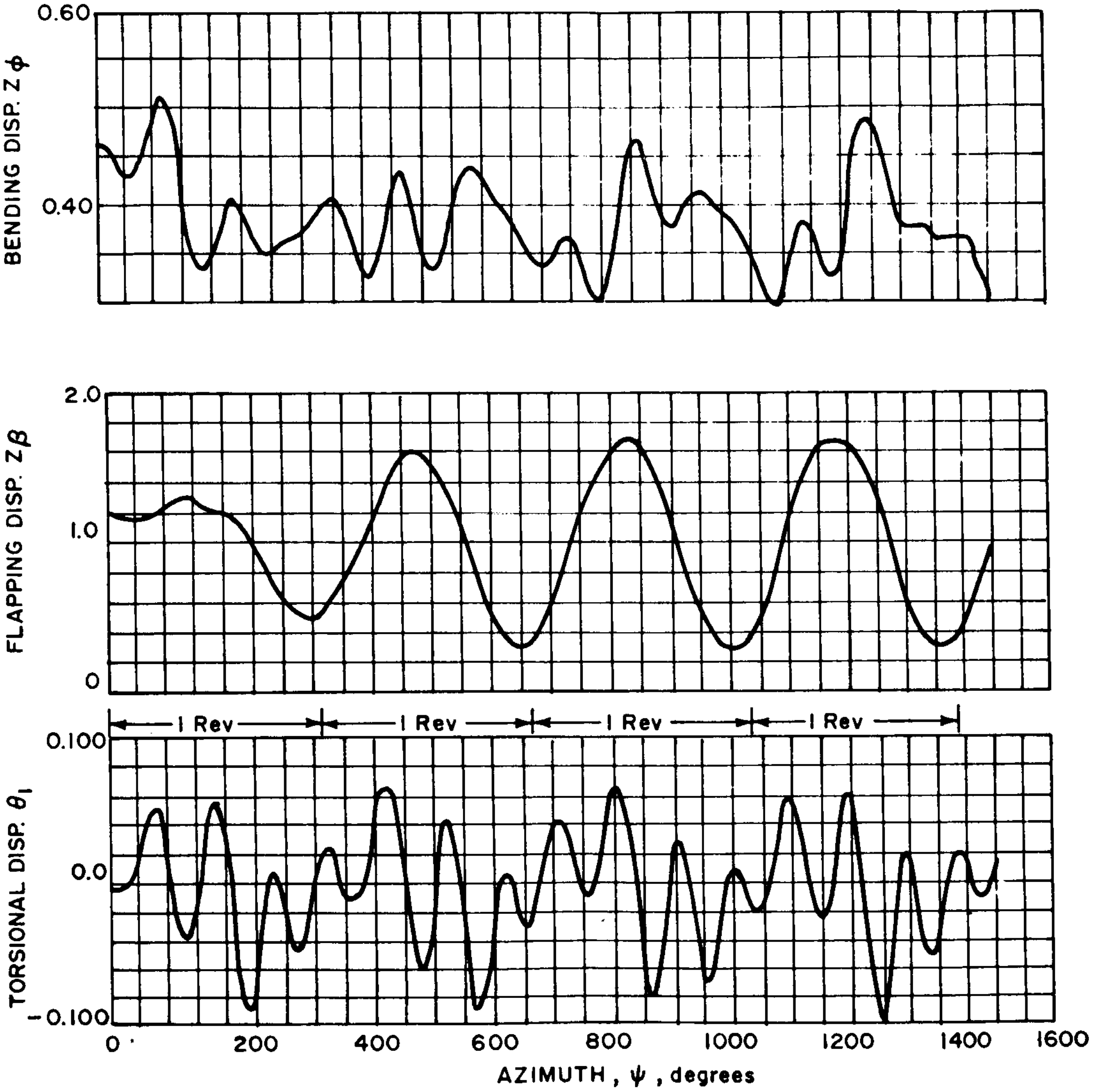

Figure 9 DISPLACEMENT TIME HISTORIES AT LINEAR INSTABILITY ONSET
$\Omega^* = 5.2$, $\theta_0 = 6$ deg, $\mu = 0.1$

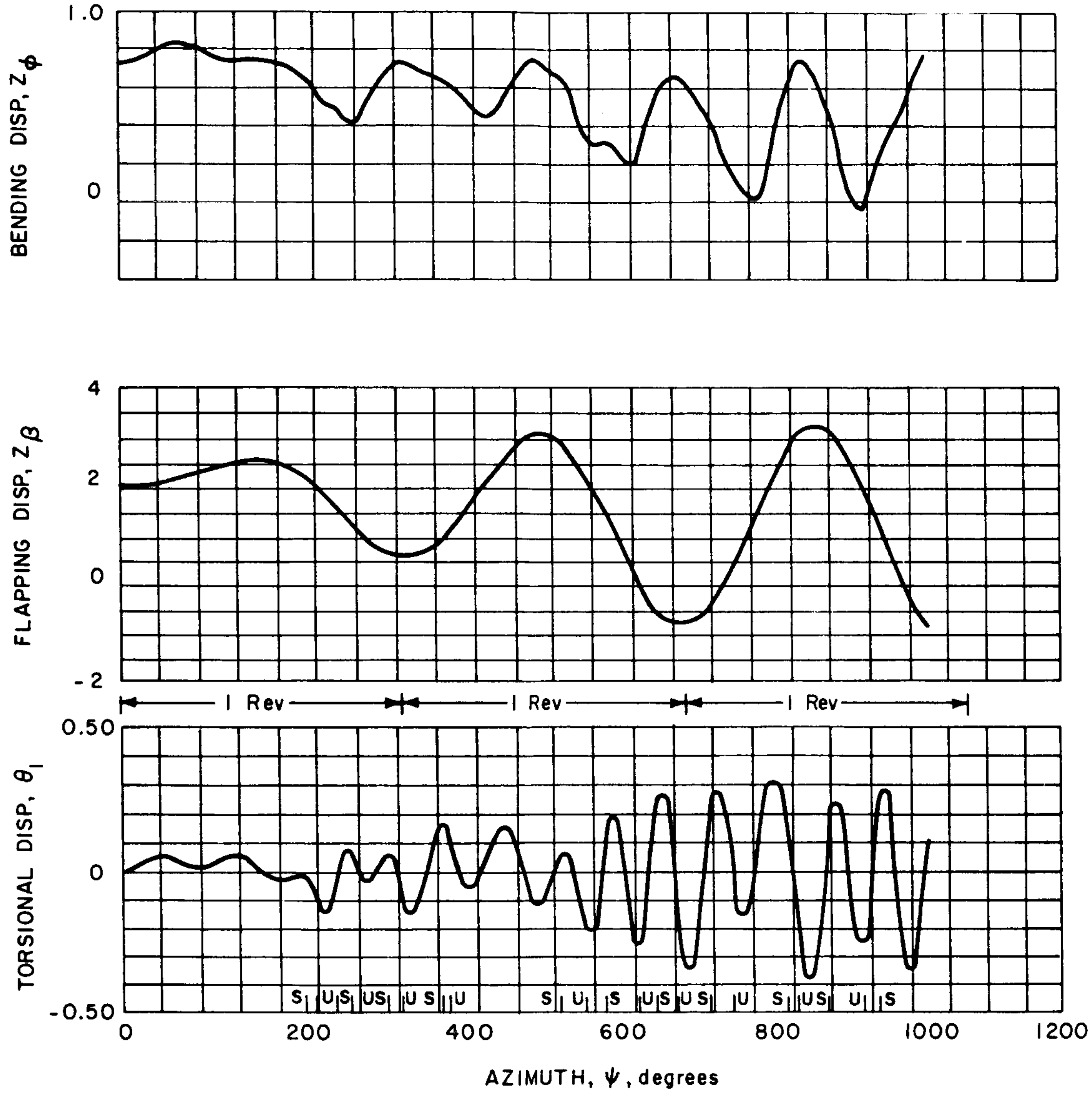

Figure 10 DISPLACEMENT TIME HISTORIES FOR STALL FLUTTER
$\Omega^* = 4.8, \theta_0 = 13$ deg, $\mu = 0.1$

brought about by successive stall and unstall. The azimuth positions at which those events occur are marked by (S) and (U), respectively, on the ψ -scale.

The blade motions for the type of instability shown in Figure 10 are not of the same character as those of particular concern in the limiting of helicopter performance, in that the excessive torsional displacements shown in Figure 10 persist over a complete revolution of the blade. The control load time history, taken from flight test (Ref. 6), shown in Figure 11 illustrates the type of stall-related blade motions usually encountered at a thrust level or forward speed near the upper limit of an aircraft. Large oscillations in the control loads, presumably deriving from blade torsional oscillations, are seen from Figure 11 to persist only between about ψ = 270 deg and ψ = 400 deg, rather than throughout a complete revolution of the blade.

A torsional displacement time history closely resembling the variation of control loads in Figure 11 was obtained for Ω^* less than 4.4, for collective pitch angles between 12 and 13 deg. Results for two typical cases are shown in Figures 12 and 13. The occurrences of stall and unstall are indicated on the abscissas. The large oscillations in torsion are clearly related to stall, but their persistence is not the result of successive stalling and unstalling, as would be the case for true stall flutter. The blade appears to be responding to the sudden changes in aerodynamic moment at stall onset and unstall, as can be seen by comparing the variation of moment coefficient shown in Figures 12 and 13 with that of torsional displacement, and noting the azimuth positions at which stall and unstall occur. There is some cyclic stall-unstall within the stall zone evident in the results, particularly at the higher rotor speed (Ω^* = 4.15, Figure 13). However, the major contributors to the oscillations appear to be the initial and final pulses associated with stall and unstall upon entering and leaving that zone. There are, in general, two cycles of torsional oscillation of excessive amplitude after the blade unstalls the last time on a given revolution. The response can be regarded as transient, on a localized time scale, or forced, when viewed on a scale of several rotor revolutions. The severity of the response is apparently due in part to the suddenness of load changes at stall and unstall, and partly to the relative lack of aerodynamic damping in pitch, particularly when the blade is not stalled.

If the collective pitch angle is increased, the blade does undergo stall flutter, as seen from the time history plotted in Figure 14. These results are for the same rotor

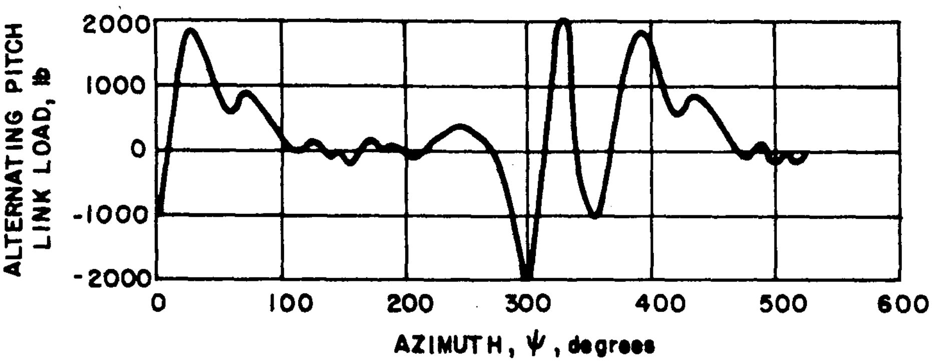

Figure 11 VARIATION OF PITCH LINK LOAD IN FLIGHT TEST OF CH-47 AT 123 KNOTS (from Ref. 6)

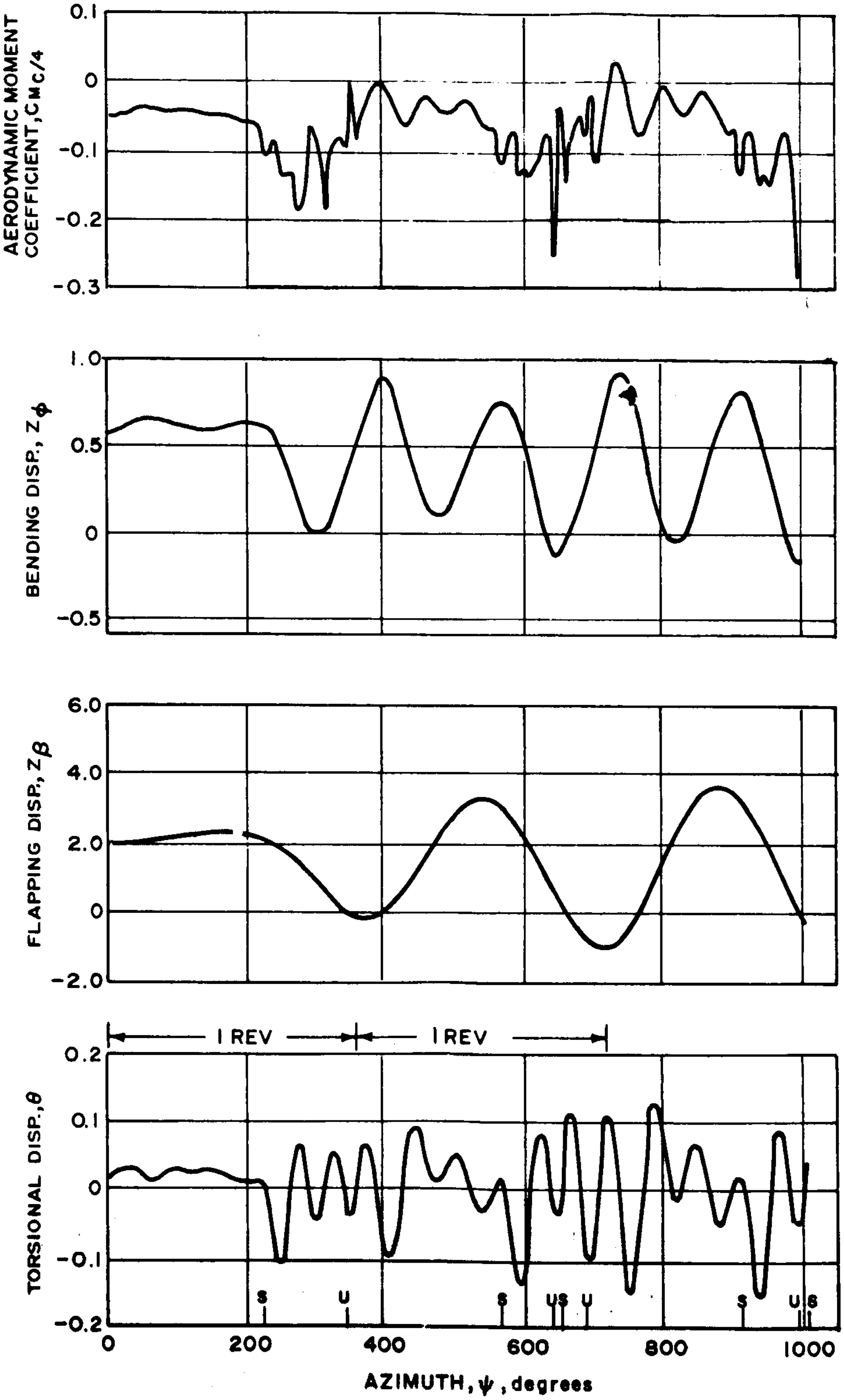

Figure 12 DISPLACEMENT AND MOMENT TIME HISTORIES FOR EXCESSIVE TORSIONAL RESPONSE

$\Omega^* = 3.89$, $\theta_0 = 12$ deg, $\mu = 0.1$

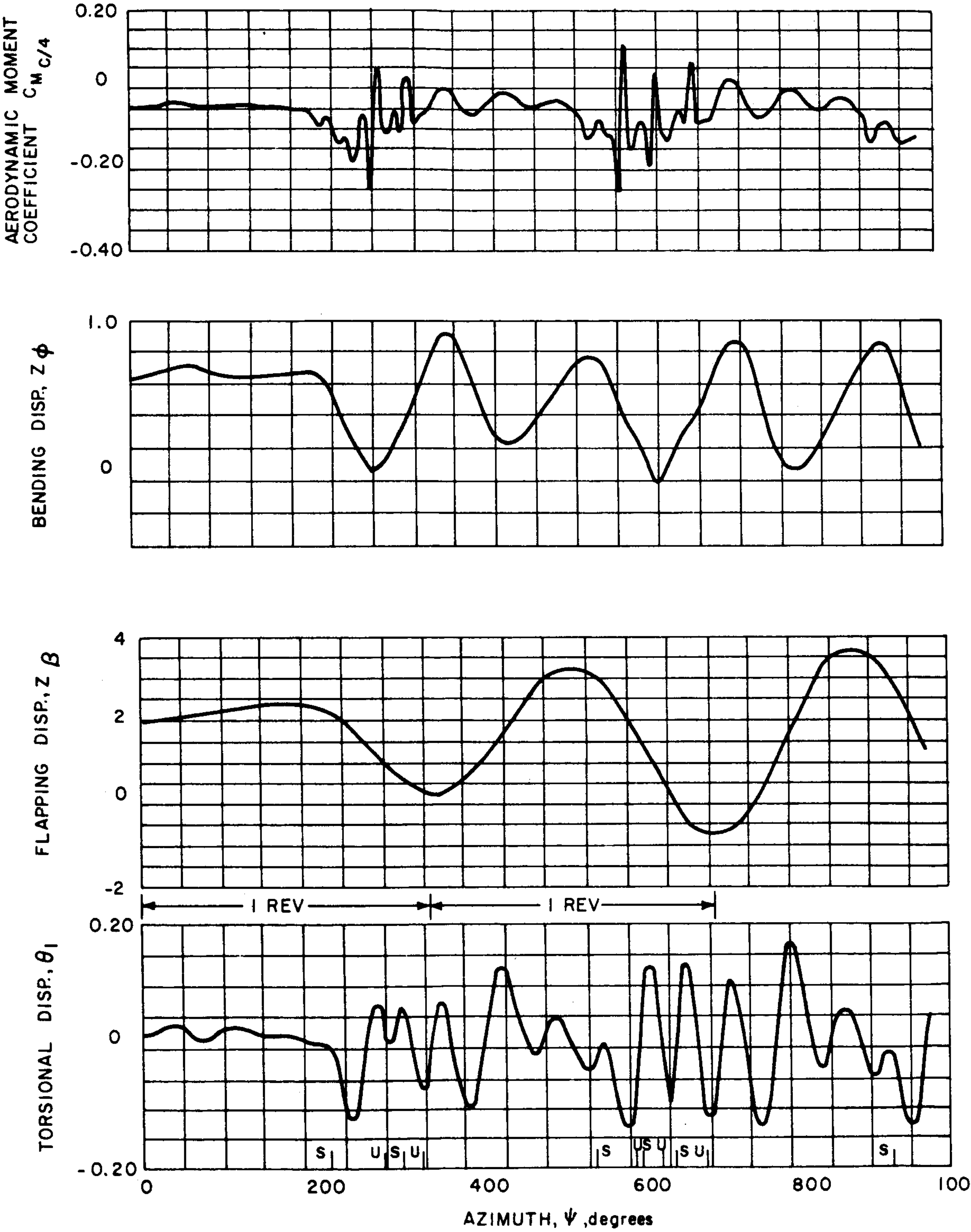

Figure 13 DISPLACEMENT AND MOMENT TIME HISTORIES FOR EXCESSIVE TORTIONAL RESPONSE
$\Omega^* = 4.15$, $\theta_0 = 12$ deg, $\mu = 0.1$

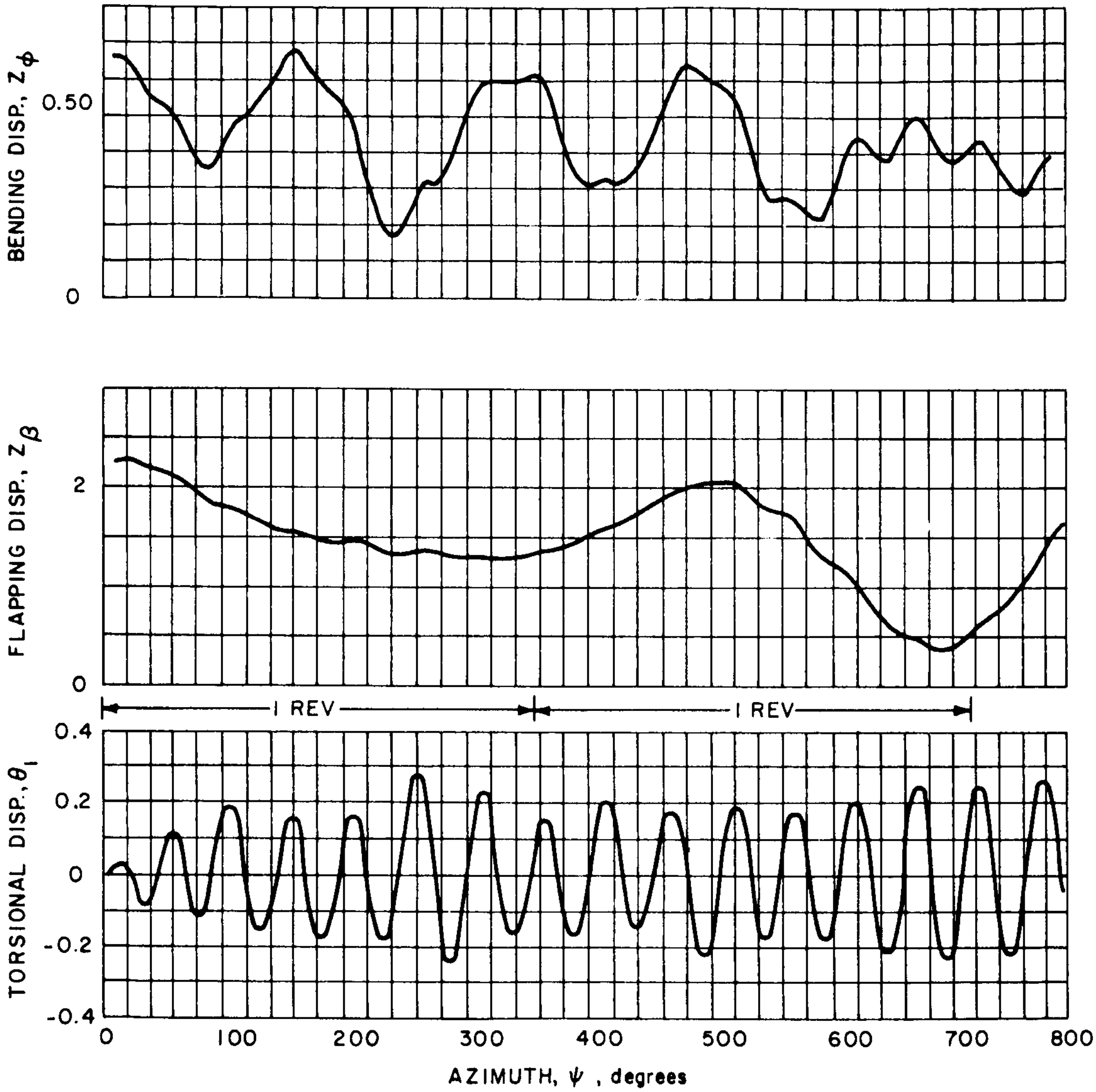

Figure 14 DISPLACEMENT TIME HISTORIES FOR STALL FLUTTER AT LOW ROTOR SPEED $\Omega^* = 3.89, \theta_0 = 14.3$ deg, $\mu = 0.1$

speed as those of Figure 12, but with θ_0 increased from 12 deg to 14.3 deg. Successive stall and unstall persists over the whole revolution of the blade for this case.

It could be argued that the blade torsional oscillations of Figures 12 and 13 are still a manifestation of stall flutter, even though successive stall and unstall is not taking place, since the aerodynamic moment can undergo unstable variations when the blade remains stalled throughout a cycle (Ref. 4). It may, in fact, be the case that the large deflections do result partly from that effect, so choosing to term them as simply a response may be somewhat misleading. On the other hand, the solutions are distinctly different from what is definitely stall flutter obtained both in hover (Figure 6) and in forward flight (Figures 10 and 14) so that label would seem to be even less appropriate. Further, the persistence of the oscillations after exit from the stall zone is clearly symptomatic of a response, so, for lack of a more precise term, solutions of the type shown in Figures 12 and 13 are identified in what follows as excessive response.

Linear Stability Boundaries

The value of Ω^* at the onset of linear instability was determined for the three configurations considered, for advance ratios of 0, .1, .2, and .3. The effects of advance ratio and torsion-bending frequency ratio on linear stability are shown in Figure 15, where Ω^* is plotted against μ for two different frequency ratios. Increasing advance ratio is seen to cause some decrease in flutter rotational speed, with most of the decrease occurring between advance ratios of .1 and .2. The substantial decrease in frequency ratio, from 3.69 to 2.5, caused only about a 4 percent reduction in flutter speed over the range of advance ratios considered. The insensitivity to frequency ratio can be attributed to the large chordwise mass imbalance, which produces the same effect in classical binary flutter of a wing (Ref. 15).

The effect of chordwise mass imbalance on linear stability is shown in Figure 16, where Ω^* at flutter onset is plotted against μ for values of x_m of .216 and .108 semichords. As one would expect, the reduction in x_m, and hence in the coupling between bending and torsion, causes a substantial increase in the flutter rotational speed.

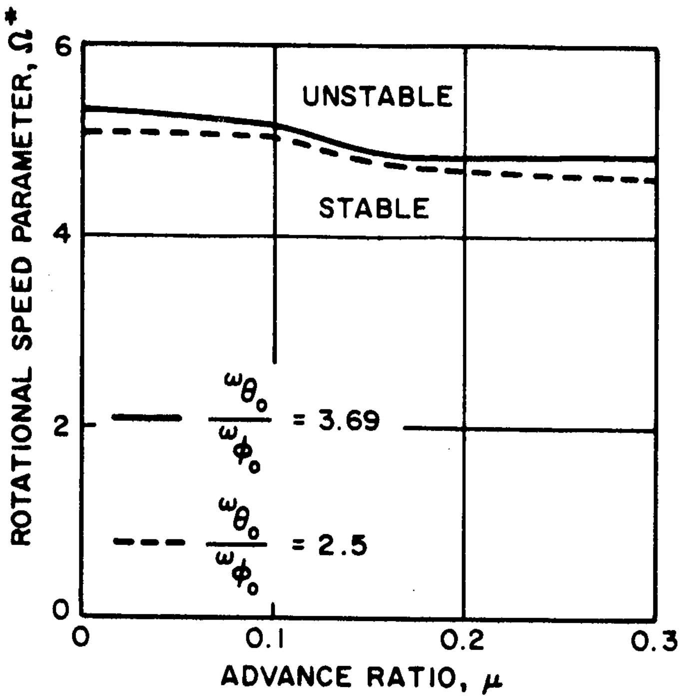

Figure 15 EFFECT OF ADVANCE RATIO AND TORSION-BONDING FREQUENCY RATIO ON LINEAR STABILITY - Xm/b = 0.216

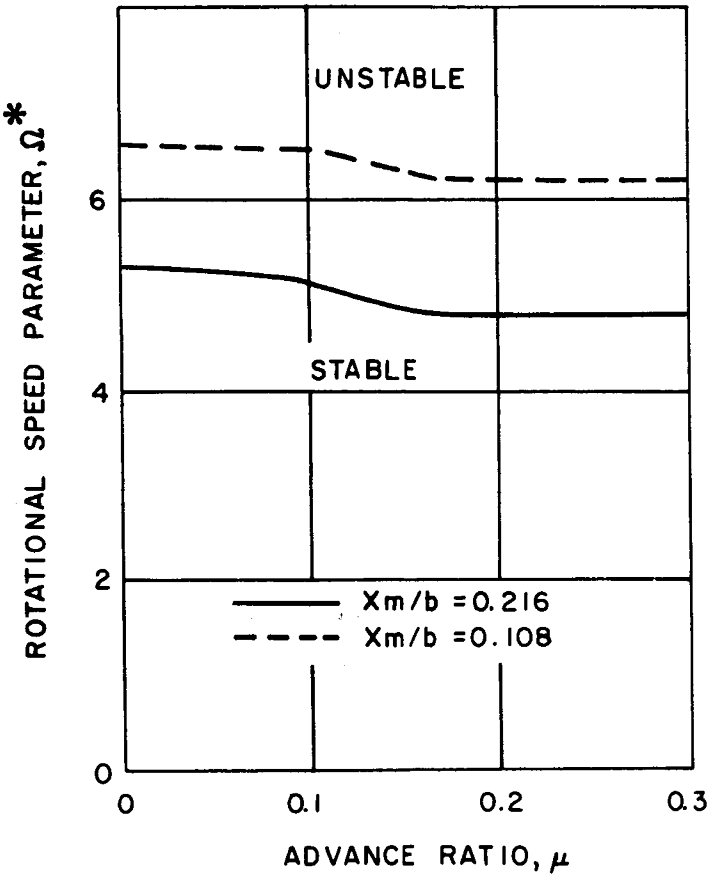

Figure 16 EFFECT OF Xm ON LINEAR STABILITY - $\omega_{\theta_0}/\omega_{\phi_0} = 3.69$

Stall Flutter and Response Boundaries

The effect of forward speed on stall-related instabilities for the three configurations was investigated by systematically varying the collective pitch angle and advance ratio, with Ω^* equal to 3.89. In order to relate the results to rotor performance, a mean lift coefficient $\bar{C}_L$ is defined, according to

$$\bar{C}_L \equiv \frac{\bar{l}}{\rho\, \Omega^2 R^2 b}$$

where $\bar{l}$ is the time-averaged lift per unit span at the aerodynamic reference radius. This coefficient is, to a good approximation, directly proportional to the thrust coefficient (see Ref. 16). The two-dimensional aerodynamic model does not provide a good measure of $\bar{C}_L$ when the rotor is partially stalled, so $\bar{C}_L$ was computed assuming it varies linearly with the collective pitch angle, using the formula

$$\bar{C}_L = a(\mu)(\theta_o + .0217)$$

The slope a and zero-lift collective pitch angle of -.0217 rad were obtained from calculations of $\bar{C}_L$ for the nominal configuration with stall precluded. The variation of a with μ is shown in Figure 17.

The results obtained for the nominal configuration are summarized in Figure 18 as a plot of $\bar{C}_L$ vs μ. As thrust is increased at a given μ, the rotor is seen to first encounter a region of excessive response, of the type discussed previously, and then, for μ of .2 or less, a region where stall flutter occurs. Increasing advance ratio has the effect of suppressing the tendency for stall flutter. At $\mu = .2$, stall flutter occurs at $\bar{C}_L = .85$, but a further increase in $\bar{C}_L$ results in excessive response again. At $\mu = .3$ a limit-cycle type of oscillation could not be triggered at all. As a result, stall flutter is confined to a region somewhat as indicated by the shaded area in Figure 18.

The suppression of stall flutter at high advance ratio is apparently caused by an effect similar to the one encountered at low rotor speed in hover, whereby the flapping motion prevented a limit cycle from occurring. This can be seen from the blade motions obtained for $\mu = .3$ and

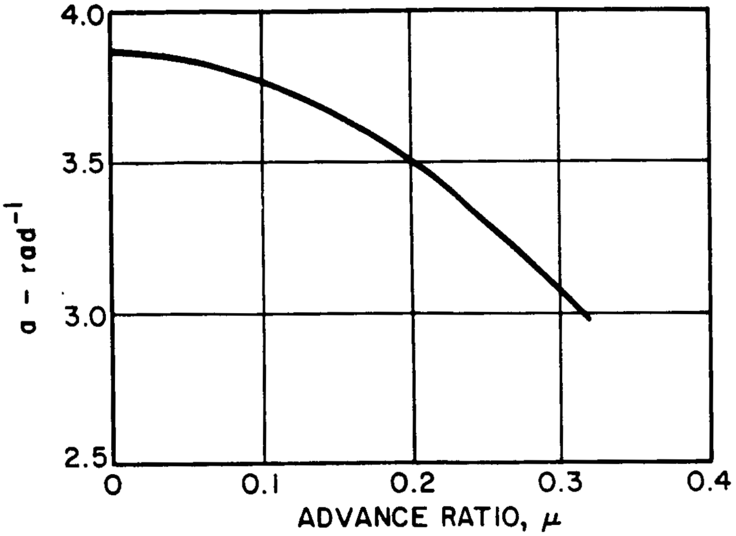

Figure 17 VARIATION OF $a = d\bar{C}_L/d\theta_0$ WITH ADVANCE RATIO

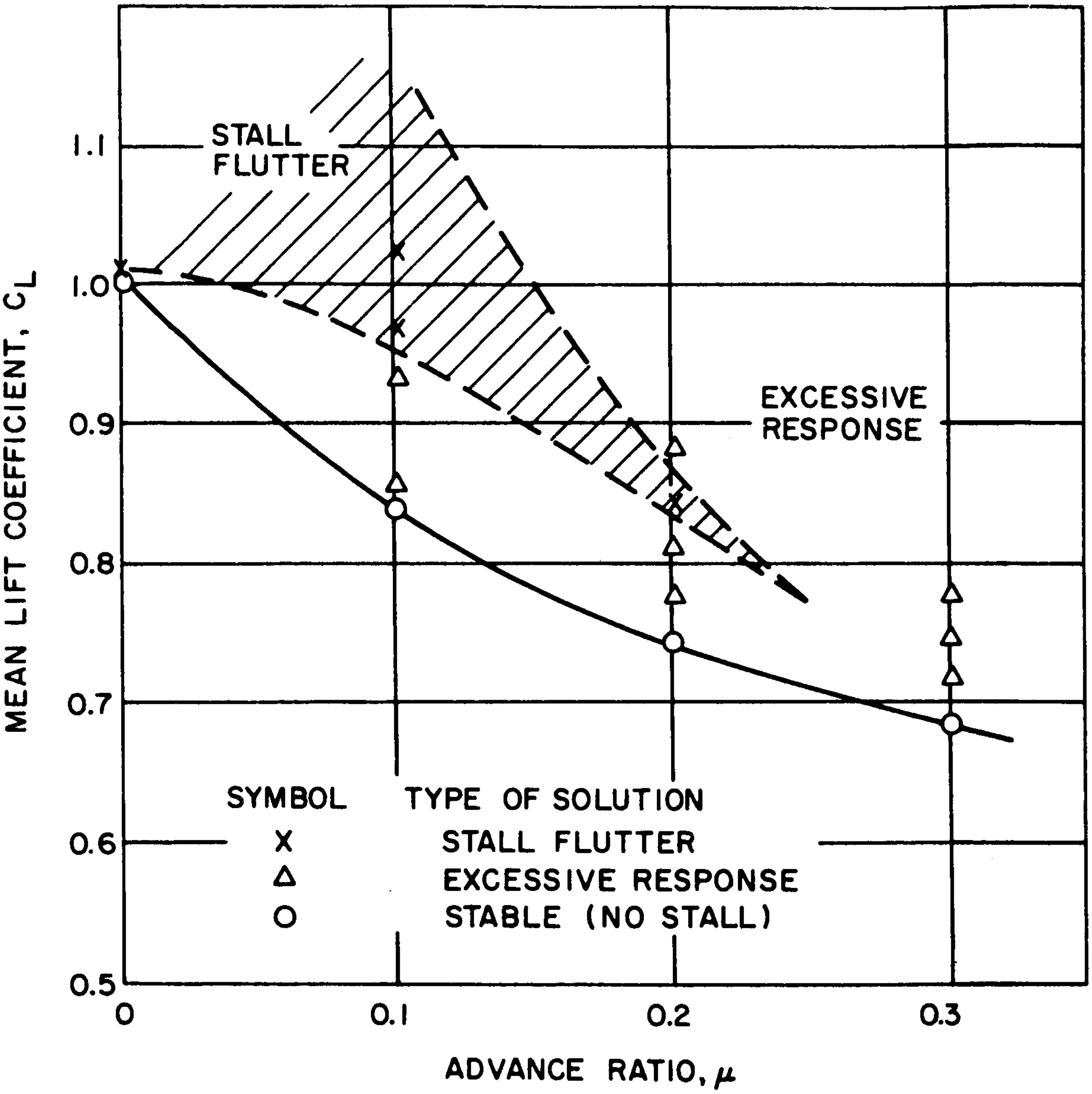

Figure 18 STALL STABILITY BOUNDARIES FOR $\Omega^* = 3.89$, $\omega_{\theta_0}/\omega_{\theta_0} = 3.69$ AND Xm/b = 0.216

$\overline{C}_L$ = .78, plotted in Figure 19. On the first revolution, as the blade enters the stall zone on the retreating side, it appears that a limit cycle is being set up, with repeated stall and unstall occurring. However, at about ψ = 420 deg, the flapping motion has built up in response to the large cyclic pitch changes, producing a negative plunging rate sufficient to keep the blade unstalled over the remainder of its passage on the advancing side. Then, when the blade again enters the stall zone, the large positive flap-induced plunging rate precludes unstall until exit from the stall zone at about ψ = 670 deg. As a result, the blade subsequently undergoes excessive torsional response, rather than stall flutter.

The effect of torsion-bending frequency ratio on stall-related instabilities can be seen from Figure 20, where $\overline{C}_L$ is plotted against μ for $\omega_{\theta_0}/\omega_{\phi_0}$ = 2.5. No instance of excessive torsional response occurred with this configuration for an advance ratio of .2 or less. Instead, limit-cycle type oscillations were set up, with almost no evidence of suppression by the flapping motion, even at relatively high values of $\overline{C}_L$ with μ = .2. At μ = .3, however, only excessive response was obtained, similar to the results for $\omega_{\theta_0}/\omega_{\phi_0}$ = 3.69.

The marked deterioration in stability at the lower frequency ratio is apparently associated with the lessened linear stability of the system. The configuration with x_m/b = .108, which is more stable, in the linear sense, than the nominal one, exhibited a trend opposite to the one resulting from a decrease in frequency ratio. The results for the smaller mass center offset, shown in Figure 21, are similar to those of the nominal configuration, Figure 18, but the region in which stall flutter occurs is somewhat reduced, there being no occurrence of stall flutter at an advance ratio of .2. Also, the amplitude of the torsional oscillations in the region of excessive response is considerably reduced, as evidenced by comparing the blade motions plotted in Figure 22, which are for μ = .1, $\overline{C}_L$ = .95 and x_m/b = .108, with those of the nominal configuration plotted in Figure 12.

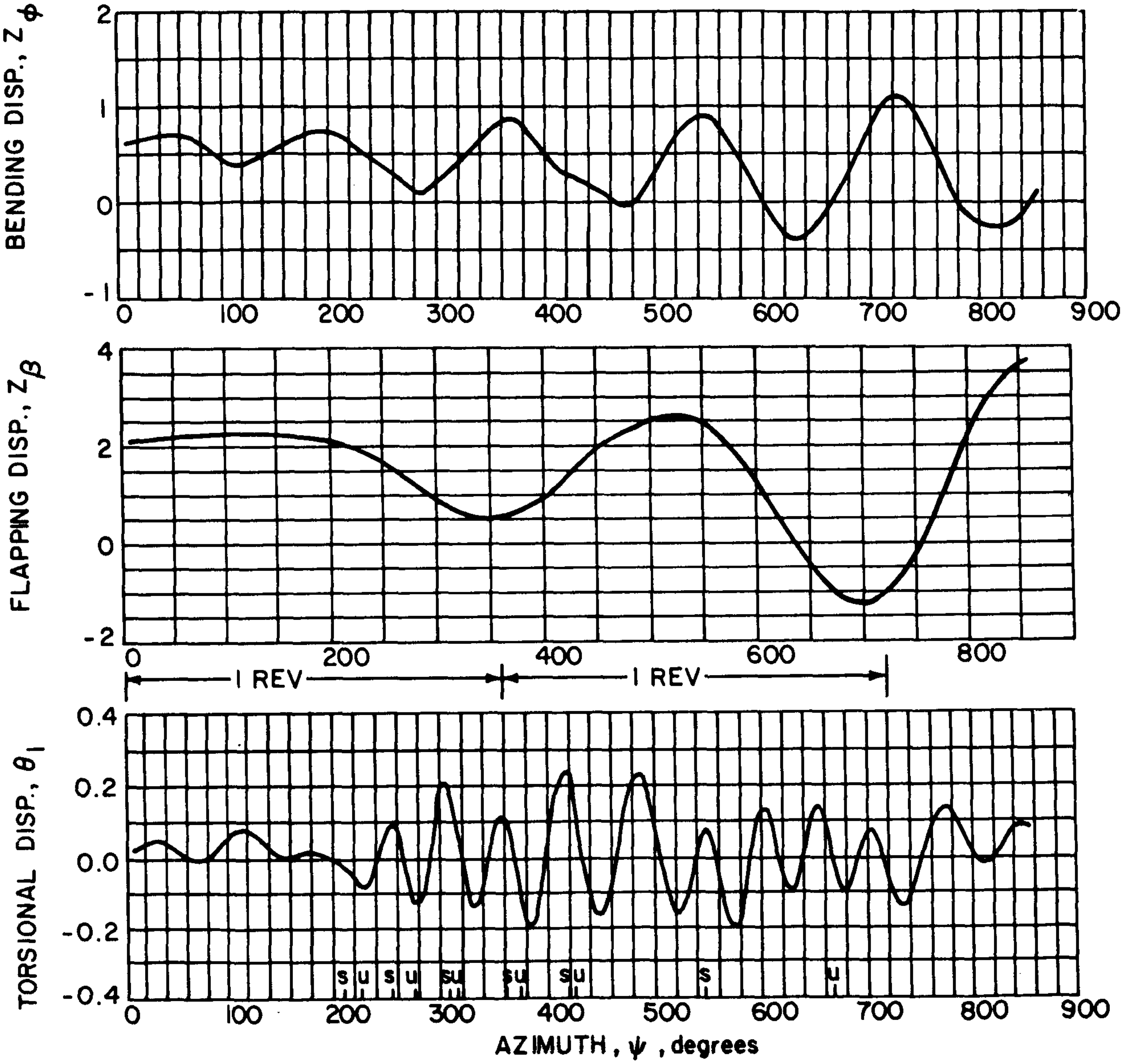

Figure 19 DISPLACEMENT TIME HISTORIES AT HIGH ADVANCE RATIO – $\Omega^* = 3.89$, $\bar{C}_L = 0.78$, $\mu = 0.3$

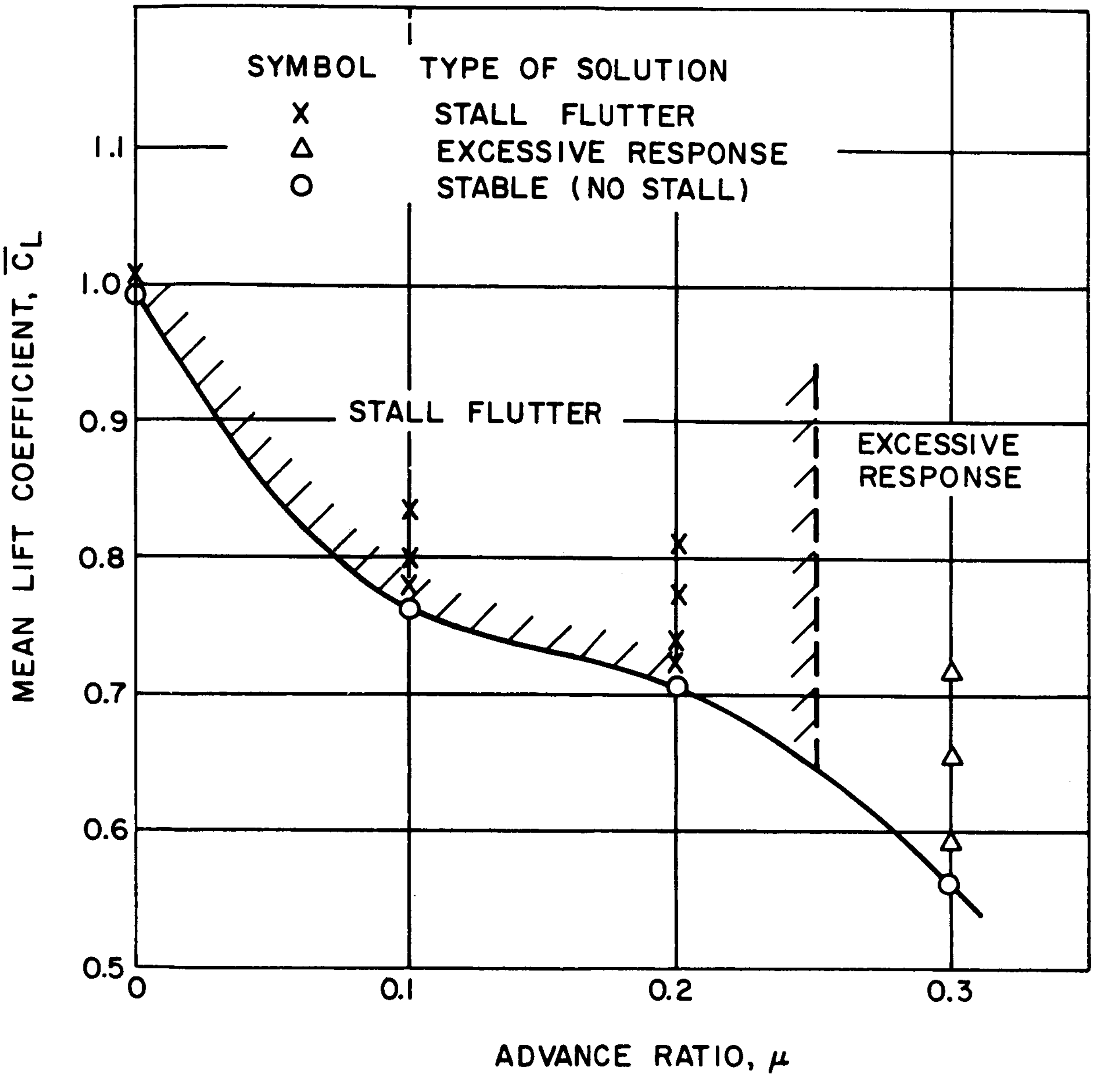

Figure 20 STALL STABILITY BOUNDARIES FOR $\Omega^* = 3.89$, $\omega_{\theta_0}/\omega_{\phi_0} = 2.5$ AND $X_m/b = 0.216$

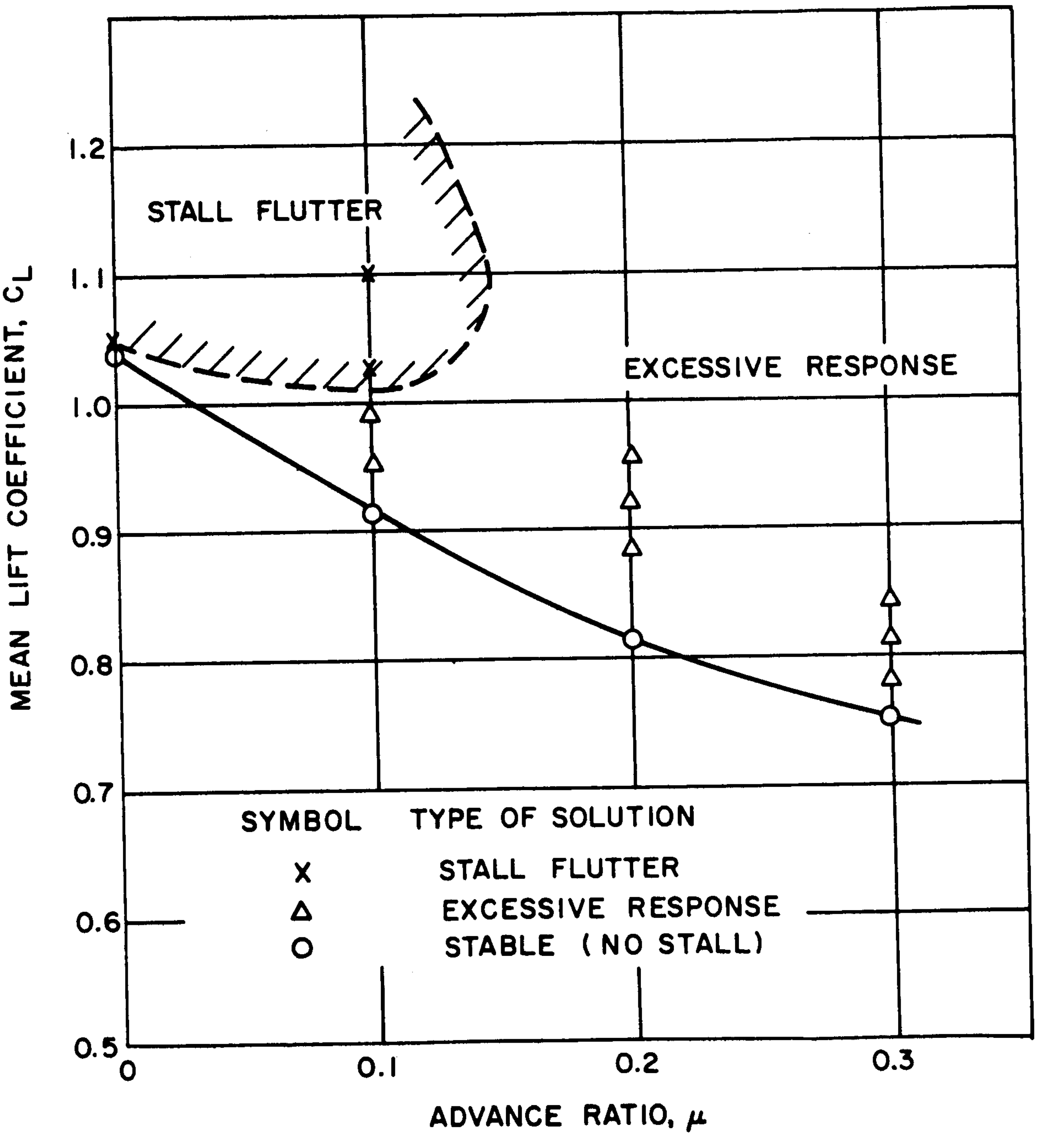

Figure 21 STALL STABILITY BOUNDARIES FOR $\Omega^* = 3.89$, $\omega_{\theta_0}/\omega_{\phi_0} = 3.69$ AND $X_m/b = 0.108$

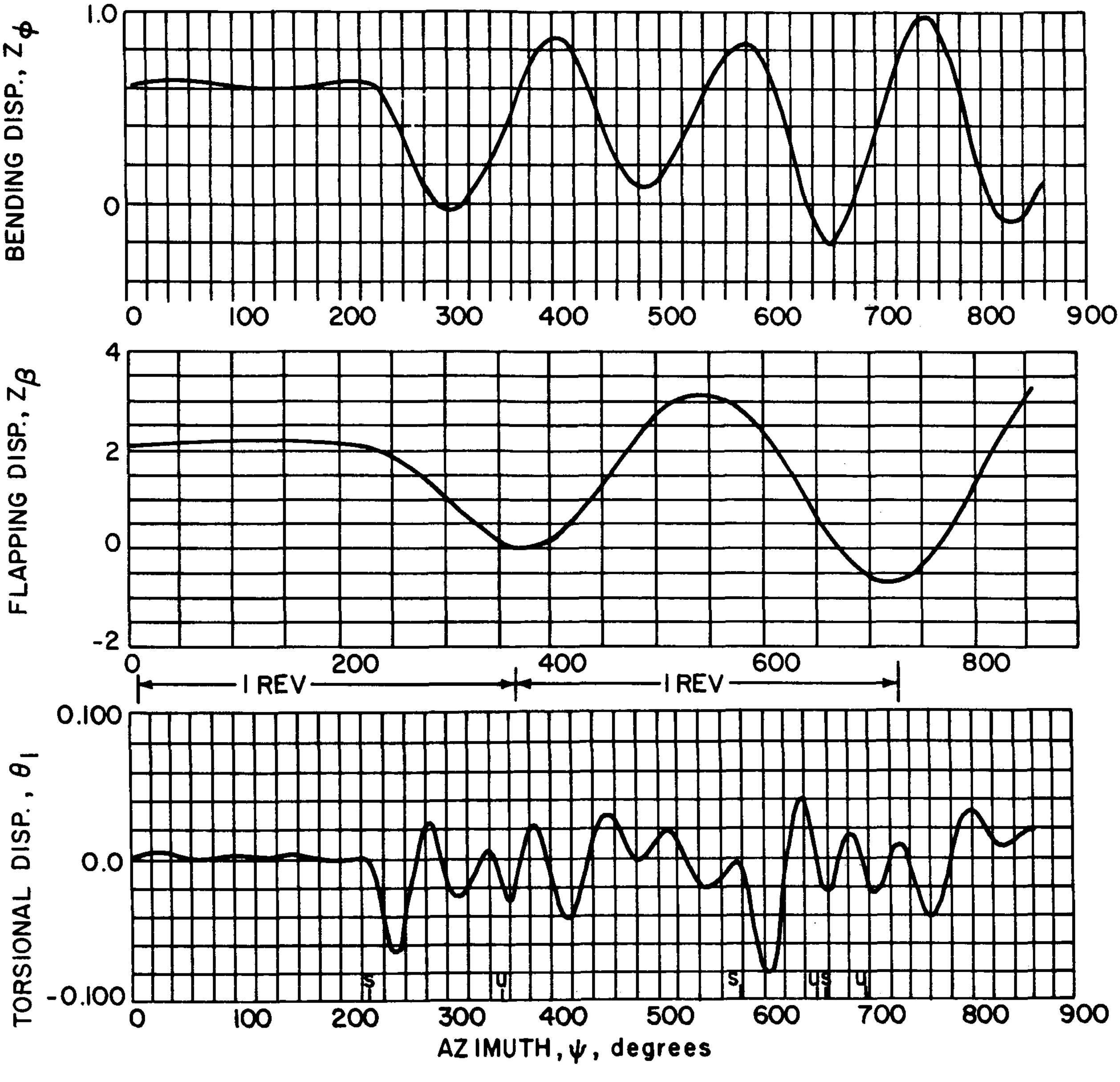

Figure 22 DISPLACEMENT TIME HISTORIES FOR EXCESSIVE TORSIONAL RESPONSE. $\Omega^* = 3.89$, $\bar{C}_L = 0.95$, $\mu = 0.1$, AND $Xm/b = 0.108$

CONCLUSIONS

An analysis has been performed of the aeroelastic stability of a helicopter rotor blade in hovering and forward flight. An analytical model of an airfoil undergoing unsteady stall and an elastomechanical representation including flapping, flapwise bending and torsional degrees of freedom were employed in the study. The following conclusions can be drawn from the results obtained.

1. Analysis of aeroelastic stability for a hovering rotor demonstrated that the aerodynamic and dynamic representations developed are capable of reproducing classical and stall flutter.

2. While stall flutter is an instability involving a single rotational degree of freedom, the minimum rotational speed for its occurrence, in hover, is determined from coupling with a translational degree of freedom.

3. In forward flight, the rotor can undergo a linear instability analogous to classical flutter and a stall-induced flutter which, while not manifested by a strictly periodic limit cycle, has the same basic mechanism for its occurrence as stall flutter of a hovering rotor.

4. The large stall-related torsional oscillations which limit forward speed and thrust are primarily the response to the rapid changes in aerodynamic moment which accompany stall and unstall, rather than the result of an aeroelastic instability.

5. Linear stability is relatively insensitive to advance ratio for advance ratios as large as .3.

6. While excessive response due to stall occurs at high advance ratio, stall flutter is precluded by the large flap-induced plunging rates.

7. The severity of stall-related instabilities and response depends to some extent on linear stability. Increasing linear stability lessens the susceptibility to stall flutter and reduces the magnitude of the torsional response to stall and unstall.

APPENDIX A

PROGRAM LISTING

APPENDIX A

PROGRAM LISTING

A listing of the FORTRAN coding of the computer program follows. The program was written in FORTRAN IV for use on an IBM 360/75 computer.

```
C                                                                       MAIN   2
C  PROGRAM TO ANALYZE UNSTEADY AIRFOIL STALL                            MAIN   3
C                                                                       MAIN   4
      CCMMON /BL1/      NTIME, NDIMC , ISTD
         CCMMON  /CLCMBL / CLVB ,  CMVB , CMPAVB
C                                                                       SETUPS17
      COMMON /INPTVB/    FTVB(64),  FPVB(64),  FPPRVB( 64),  DIDRVB(64), SETUPS18
     A      XMVB(64),    DELVB,     XMUVB,     FOVB,   XMUAVB,          SETUPS19
     B      ATOVB,       ATCVB,      ATSVB,      ROVB,        RVB(64),  SETUPS20
     C      MVB(64),     NVB                                            SETUPS21
C                                                                       SETUPS22
      COMMON /INPUTS/    NSBL,      NZ,        NOFF,      NGAM,      NSIG,  SETUPS23
     A         NCOI,     NCORD,     LOWER,     MSTOP,     MAXT,      MOTR,  SETUPS24
     B         NOTBL,    INDV,      ELSIG,     DXI,       REB,       RDBB,  SETUPS25
     C         FRZ,      ARR,       AMPLU,     FREQU,     ALPH1,     ALPH2, SETUPS26
     D         HEAVE,    AROT,      FREQF,     PHIH,      NY,        RY1,   SETUPS27
     E         DRY,      Y(100),    TEST,      UPRIM,     XU( 30),   YU( 30),SETUPS28
     F         XL(30),   YL(30),    ER1,       ER2,       ER3,       BD3R,  SETUPS29
     G         RRDBR                                                    SETUPS30
     H,   CMPA, CMPAS, BARG, EM1, HVOR, NVOR, SSPA, SVOR, TORF, X1VOR
     I, PLOTOP, PSILOW, PSIUP
     J , NOUT
         COMMON/ ZZZ/ Z(3)
C                                                                       SETUPS31
C
C     DIMENSION USAV(300,100) ,SCALS(300)                               MAIN   5
      DIMENSION USAV(1   ,1   ),SCALS(300)                              MAIN   5
      DIMENSION CAMBR(24) ,THICK(24)                                    MAIN   6
      DIMENSION XGAM(30),XSIG(100),XSIGA(100),XSIGB(100),XC(300),X(300),MAIN   7
     1SBL(300),XBSIG(100)                                               MAIN   8
      DIMENSION ACAP(30,3),BCAP(100,3),ASZ(30),AS(30,30),BS(30, 30),ASHZMAIN   9
     1(100),ASH(30,30 ),BSH(30,30   ),AR(30),ARH(100),UE(300,3)         MAIN  10
      DIMENSION ALAM(30),VZIP(30),FPRES(100),GAMAW(1000),XIW(1000)      MAIN  11
      DIMENSION BLAM(30),FLAM(10),XFLAM(10)                             MAIN  12
      DIMENSION SCALE(300,2),U(1,1,1     ),UC(100,3),V(100,2)           MAIN  13
     1 , P(200,7)
C
C
      DOUBLE PRECISION CMAT(60,60   ),RMAT(130)                         MAIN  15
C
C
         DATA IN, MOUT, NF/ 5,6, 24/
      DATA PI,TIME,UINF,RENEL,USTOP/3.14159,0.,1.,4.75E4,2.8/           MAIN  18
      DATA  FLAM           /1.75,1.75,1.724,1.527,1.354,1.,.663,.452,.25MAIN  19
     14,.21/                                                            MAIN  20
      DATA  XFLAM           /-100.,-11.26,-7.01,-3.48,-1.766,0.,1.888,4.MAIN  21
     103,6.77,7.19/                                                     MAIN  22
      DATA  DEGRES /1.74 53292 51994 330D-2/                            SUPPL 38
C
C
      EQUIVALENCE (CMAT(1),USAV(1)),(ASH(1),SCALS(1))                   MAIN  16
C
C
C
C IF ISTD =1      TIME DERIVATIVES NOT USED
```

```
          ISTD= 1
      RAD = 180. /PI
           IL= 8888
      NDIMC= 60
      CALL SETUPS                                                                  MAIN    4
       IF(ISTD .EQ. 1) GO TO  40
      DO 100 J = 1,300
      SCALS(J) =0.
      DO 100 I = 1,100
100   USAV (J,I) =0
 40     :  CONTINUE
C
        CALL READIN ( IL,& 60)
C                                                                                  MAIN   65
C  NOTE - OFFSETS ARE PUT IN AS LISTED IN  THEORY OF WING SECTIONS,  I.EMAIN   66
C         AS A FRACTION OF TOTAL CHORD, X1 BEING MEASURED FROM THE              MAIN   67
C         LEADING EDGE.  BE SURE NF IS AN EVEN NUMBER.                          MAIN   68
C                                                                                  MAIN   69
        TIME=0.
      NTIME=0
        NWAKE= 999
         ISEP=0
          ISEPT =0
         IWASH =2
           UINF  =1.
        L=0
      INDV=INDV+1                                                                  MAIN   59
      WRITE(MOUT,6)                                                                MAIN   72
      PITCH = ALPH1
      IF(INDV + MOTR .LE. 2) PITCH = PITCH - ALPH2
      IF(INDV .EQ. 2)
     X   AMPLU = 1.33333* XMUAVB * (1.-ROVB**3) / (1. - ROVB**4)
         IF( INDV.EQ. 2) FREQU= BDBR/RRDBR
      IF(INDV .GE. 2) GO TO 343
      WRITE(MOUT,25) NVOR,SVOR,HVOR,BARG,X1VOR,EMI,TORF,SSPA                   MAIN   75
      RY=RY1                                                                       MAIN   64
      HVOR=HVOR**2                                                                 MAIN   76
      BARG=BARG/6.2832                                                             MAIN   77
 343  CALL SECT(XU,YU,XL,YL,NOFF,NF,RDBB,TMDBB,CMDBB,THICK,CAMBR)               MAIN   78
      DO 7875 N=1,NF                                                               MAIN   79
      CAMBR(N)=CAMBR(N)*CMDBB                                                      MAIN   80
 7875 THICK(N)=THICK(N)*TMDBB                                                      MAIN   81
      WRITE(MOUT,4)                                                                MAIN   82
      WRITE(MOUT,7) AMPLU,FREQU,ALPH1,ALPH2,HEAVE,AROT,FREQF,RDBB,REB            MAIN   83
      WRITE(MOUT,8)                                                                MAIN   84
      WRITE(MOUT,9) (N,CAMBR(N),THICK(N),N=1,NF)                                   MAIN   85
      MX=NSBL+NZ-1                                                                 MAIN   86
      CALL SCAL(SBL,NSBL,FRZ,ARR,RDBB)                                             MAIN   87
      CALL CORDX(NSBL,NZ,RDBB,SBL,X,XC)                                            MAIN   88
      DO 2420 M=1,MX                                                               MAIN   89
      IF(XC(M)-1.) 2420,2419,2419                                                  MAIN   90
 2419 MEND=M-1                                                                     MAIN   91
      GO TO 2421                                                                   MAIN   92
 2420 CONTINUE                                                                     MAIN   93
 2421 MX=MEND                                                                      MAIN   94
```

```
      MXM1=MX-1                                                         MAIN  95
      UE(MX+1,1)=1.                                                     MAIN  96
      EPSLE=2.*(X(NZ)-X(NZ-1))                                          MAIN  97
      EPSTE=X(MX)-X(MX-1)                                               MAIN  98
      ALTC=8.36E4/SQRT(REB)                                             MAIN  99
           IF( ISTD.EQ. 1) GO TO 50
      DO 2422 M=1,MX                                                    MAIN 100
      SCALE(M,1)=0.                                                     MAIN 101
      SCALE(M,2)=0.                                                     MAIN 102
      DO 2422 N=1,NY                                                    MAIN 103
      U(M,N,1)=0.                                                       MAIN 104
 2422 U(M,N,2)=0.                                                       MAIN 105
 50          CONTINUE
      NSIGA=NSIG                                                        MAIN 106
      NSIGB=NSIG                                                        MAIN 107
      NSIG1=NSIG+1                                                      MAIN 108
      MOTR=MOTR+1                                                       MAIN 109
      NOTBL=NOTBL+1                                                     MAIN 110
      XMAX=1.-ELSIG                                                     MAIN 111
      CONA=.375*PI/DXI                                                  MAIN 112
      ANGS=PI/FLOAT(NSIG)                                               MAIN 113
      CALL SETSX(NSIG1,1.1,2.,XSIG,ANGS)                                MAIN 114
      XSEP=1.1                                                          MAIN 115
      DO 2430 N=1,NSIG1                                                 MAIN 116
      XSIGB(N)=XSIG(N)                                                  MAIN 117
 2430 XSIGA(N)=XSIG(N)                                                  MAIN 118
      DO 2431 N=1,NSIG                                                  MAIN 119
      DO 2431 NU=1,3                                                    MAIN 120
 2431 BCAP(N,NU)=0.                                                     MAIN 121
      PINT=2./FLOAT(NCORD)                                              MAIN 122
      NCP1=NCORD+1                                                      MAIN 123
      THXI=1.5/DXI                                                      MAIN 124
      NGP1=NGAM+1                                                       MAIN 125
      NWM1=NWAKE-1                                                      MAIN 126
      COUNT=0.                                                          MAIN 127
      DO 8456 N=1,NWAKE                                                 MAIN 128
      GAMAW(N)=0.                                                       MAIN 129
      XIW(N)=1.+COUNT                                                   MAIN 130
 8456 COUNT=COUNT+DXI                                                   MAIN 131
      ANGLE=PI/FLOAT(NGAM)                                              MAIN 132
      COUNT=0.                                                          MAIN 133
      DO 1002 M=1,NGP1                                                  MAIN 134
      PHIM=COUNT*ANGLE                                                  MAIN 135
      XGAM(M)=COS(PHIM)                                                 MAIN 136
      DOUNT=2.                                                          MAIN 137
      DO 1001 N=2,NGAM                                                  MAIN 138
      AS(M,N)=COS(DOUNT*PHIM)                                           MAIN 139
 1001 DOUNT=DOUNT+1.                                                    MAIN 140
 1002 COUNT=COUNT+1.                                                    MAIN 141
      CALL WASH(XGAM,NGAM,TIME,ALPH1,ALPH2,HEAVE,AROT,FREQF,PHIH,UINF,CAMAIN 142
     1MBR,NF,VZIP,1,1)                                                  MAIN 143
      DO 8458 M=1,NGP1                                                  MAIN 146
      CMAT(M,1)=1.                                                      MAIN 147
      TEMP=2.*VZIP(M)                                                   MAIN 148
      RMAT(M)=TEMP                                                      MAIN 149
```

```
      CMAT(M,2)=XGAM(M)                                                 MAIN 150
      DO 8457 N=3,NGP1                                                  MAIN 151
 8457 CMAT(M,N)=AS(M,N-1)                                               MAIN 152
 8458 CONTINUE                                                          MAIN 153
      CALL ALSOL(NGP1,CMAT,RMAT)                                        MAIN 154
      DO 8459 N=1,NGP1                                                  MAIN 155
      ACAP(N,1)=RMAT(N)                                                 MAIN 156
      ACAP(N,3)=RMAT(N)                                                 MAIN 156
 8459 ACAP(N,2)=ACAP(N,1)                                               MAIN 157
      DO 2784 M=1,MX                                                    MAIN 158
      SIGN=1.                                                           MAIN 159
      IF(M-NZ) 2774,2775,2775                                           MAIN 160
 2774 SIGN=-SIGN                                                        MAIN 161
 2775 CALL QECAL(ISEP,NGAM,NSIG,NF,XSIG,ACAP,BCAP,THICK,RCBB,GAMAW(1),UIMAIN 162
     1NF,XC(M),UF(M,1),SIGN)                                            MAIN 163
 2784 UF(M,2)=UE(M,1)                                                   MAIN 164
      DO 1004 M=2,NGAM                                                  MAIN 165
 1004 BLAM(M)=(1.125*XGAM(M)+.1875*(1.+XGAM(M))*(1.-3.*XGAM(M))*ALOG((1.MAIN 166
     1+XGAM(M))/(1.-XGAM(M))))/DXI                                      MAIN 167
      BLAM(NGP1)=-1.125/DXI                                             MAIN 168
      CALL CLCM(NCOI,ISEP,NGAM,XSIG,NSIG,XSIGA,NSIGA,XSIGB,NSIGB,ACAP,BCMAIN 504
     1AP,THICK,RDBB,GAMAW,UINF,UDOT,DXI,AROT,CMPA)                      MAIN 505
         IF ( INDV .EQ. 2)
     1CALL SUPPL                                                        MAIN
C                                                                       MAIN 169
C  INDEXING IN TIME IS CARRIED OUT AT THIS POINT.                       MAIN 170
C                                                                       MAIN 171
 9999 CCNTINUE                                                          MAIN 172
          CALL ACUCPU( IACU )
         IF( IACU .LT.  35000  ) GO TO   99
C                                                                       MAIN 175
C  NOTE - FOR READ-IN CF FCIL MOTIONS, MAKE ALPH1 = ALPHA,              MAIN 176
C         ALPH2 = ALPHA-DOT, AND HEAVE = H-DOT.                         MAIN 177
C                                                                       MAIN 178
      IF(MCTR .EQ. 2)
     XREAD(IN,2,END=8989) ALPH1,ALPH2,HEAVE                             MAIN 174
 198  NITS=1                                                            MAIN 182
      TIME=TIME+DXI                                                     MAIN 183
      NTIME=NTIME+1                                                     MAIN 184
      NWAKE=NTIME+2                                                     MAIN 185
      IF(NWAKE-998) 202,201,201                                         MAIN 186
 201  NWAKE=998                                                         MAIN 187
 202  IF(MAXT-NTIME) 8989,8800,8800                                     MAIN 188
 8800 SAVFU=UINF                                                        MAIN 189
            L= L+1
         P(L,1) =  BCBR / RRDBR * TIME   ●  RAD
        PSI360=  AMOD(  P(L,1) ,  360.)
      UINF=1.+AMPLU*SIN(FREQJ*TIME)                                     MAIN 190
      IF(INDV .EQ. 2)
     XCALL SUPP1(UINF)                                                  MAIN
      PITCH = ALPH1
      IF(INDV + MOTR .LE. 2) PITCH = PITCH - ALPH2*COS(FREQF*TIME)      MAIN 475
      UDOT=FREQU*AMPLU*COS(FREQU*TIME)                                  MAIN 191
      STEPX=.5*DXI*(UINF+SAVEU)                                         MAIN 192
      DO 1003 J=2,NWAKE                                                 MAIN 193
```

```
      JC=NWAKE-J+2
      GAMAW(JC)=GAMAW(JC-1)
 1003 XIW(JC)=XIW(JC-1)+STEPX
      IF(ISEP) 2009,2009,2007
 2007 DO 2008 N=1,NSIG
      BCAP(N,3)=BCAP(N,2)
 2008 BCAP(N,2)=BCAP(N,1)
      DO 4433 N=1,NSIG1
      XSIGB(N)=XSIGA(N)
 4433 XSIGA(N)=XSIG(N)
      GO TO 2010
 2009 DEADL=0.
      ELDOT=UINF
 2010 DO 1014 M=1,MX
      UE(M,3)=UE(M,2)
 1014 UE(M,2)=UE(M,1)
      DEAD1=DEADL
      ELD1=ELDOT
      ALAM(1)=(1.125+.75*ALOG(STEPX*.5))/DXI
      DO 1005 M=2,NGP1
 1005 ALAM(M)=BLAM(M)+.75*(1.+(1.-XGAM(M))/STEPX)*ALOG((1.+STEPX-XGAM(M)
     1)/(1.-XGAM(M)))/DXI
      DO 2006 M=1,NGP1
      ACAP(M,3)=ACAP(M,2)
 2006 ACAP(M,2)=ACAP(M,1)
      AFACT=8.*(ACAP(1,2)+.5*ACAP(2,2))-2.*(ACAP(1,3)+.5*ACAP(2,3))
      ALPHS=VZIP(1)
      CALL WASH(XGAM,NGAM,TIME,ALPH1,ALPH2,HEAVE,AROT,FREQF,PHIH,UINF,CA
     1MBR,NF,VZIP,MOTR,INDV)
      DO 1006 M=1,NGP1
      ASZ(M)=1.+2.*ALAM(M)
      AS(M,1)=XGAM(M)+ALAM(M)
      SUM=0.
      DO 4343 J=2,NWM1
 4343 SUM=SUM+(GAMAW(J)+(GAMAW(J+1)-GAMAW(J))*(XGAM(M)-XIW(J))/(XIW(J+1)
     1-XIW(J)))*ALOG((XIW(J+1)-XGAM(M))/(XIW(J)-XGAM(M)))
      ELX=1.-XGAM(M)
      IF(M-1) 1006,2130,1006
 2130 ELX=1.
 1006 AR(M)=2.*VZIP(M)+ALAM(M)*AFACT/3.+(SUM-GAMAW(2)*(1.-XGAM(M))*ALOG(
     1(1.+STEPX-XGAM(M))/ELX)/STEPX)/PI
C
C     THE FOLLOWING CALCULATIONS, THROUGH STATEMENT 4444, ARE PERFORMED
C      ONLY IF THE AIRFOIL IS STALLED.  THE AIRFOIL IS DESIGNATED TO BE
C       STALLED IF INTEGER ISEP IS NONZERO.
C
      IF(ISEP) 3247,4444,3247
 3247 GO TO (3344,3345),IWASH
 3344 XSEP=XSEP+DXI
      IF(XSEP-XMAX) 3248,3347,3347
 3347 IWASH=2
      ISEP=0
      XSEP=1.1
      DO 3015 K=1,3
      DO 3015 N=1,NSIG
```

```
3015 BCAP(N,K)=0.
     GO TO 4444
3345 IF(INDT) 3348,3348,3248
3348 IF(NITS-1) 3248,3349,3248
3349 IF(INDV.EQ.2) GO TO 6349
     IF(VZIP(1)-ALPHS) 6349,6348,6348
6348 NITS=2
     GO TO 3248
6349 CALL UNPOP(NGAM,AR,ALAM,AFACT,RMAT,CMAT,XGAM,AS,ACAP,MX,NZ,NF,XSIG
    1,BCAP,THICK,RDBB,UINF,XC,UE)
     GO TO 2785
3248 XATT=XSEP+DEAD1+.5*(ELD1+ELDOT)*DXI
     DEADL=XATT-XSEP
     DIFF=1.-XATT
         XTEST = XSEP + 3. * EPSLE
     CALL SETSX(NSIG1,XSEP,XATT,XSIG,ANGS)
     DO 4434 N=1,NSIG
4434 XBSIG(N)=.5*(XSIG(N)+XSIG(N+1))
     DO 3086 M=1,NGP1
     DO 3086 N=1,NSIG
3086 BS(M,N)=0.
     DO 3087 M=1,NGP1
     IF(XGAM(M)-XSEP) 3088,3088,3089
3089 IF(XATT-XGAM(M)) 3187,3087,3091
3091 DO 3092 I=1,NSIG1
     IF(XGAM(M)-XSIG(I)) 3093,3092,3092
3093 MARK=I
     GO TO 3094
3092 CONTINUE
3094 WIDES=XSIG(MARK)-XSIG(MARK-1)
     BS(M,MARK-1)=(XSIG(MARK)-XGAM(M))/WIDES
     BS(M,MARK)=(XGAM(M)-XSIG(MARK-1))/WIDES
     BS(M,1)=SQRT((XGAM(M)-XSEP)/(XATT-XGAM(M)))
3088 IF(DIFF-1.E-6) 3087,3098,3098
3098 BS(M,1)=BS(M,1)+DIFF**(-1.5)*SQRT(DEADL)*(2.*DIFF+(SQRT((1.-XGAM(M
    1))/(XATT-XGAM(M)))-1.)*(4.*XGAM(M)-1.-3.*XATT))
     GO TO 3087
3187 BS(M,1)=DIFF**(-1.5)*SQRT(DEADL)*(3.+    XATT-4.*XGAM(M))
3087 CONTINUE
C
C    SET-UP OF THE SECOND SET OF EQUATIONS STARTS HERE.
C
     DO 4350 K=1,NSIG
     IF(XBSIG(K)-1.) 4348,4349,4349
4348 COSK=XBSIG(K)
     SINK=SQRT(1.-COSK*COSK)
     THETK=ARCT(COSK)
     TANT=SIN(.5*THETK)/COS(.5*THETK)
     ASHZ(K)=TANT+CONA*(1.+COSK)*(1.-3.*COSK)/UINF+THXI*(PI-THETK+SINK+
    1CONA*(1.+COSK)*SINK**2)/UINF
     ASH(K,1)=.5*(ASHZ(K)-TANT)+SINK
     COUNT=1.
     DO 4355 N=2,NGAM
     COUNT=COUNT+1.
4355 ASH(K,N)=SIN(COUNT*THETK)+.75*(SIN((COUNT+1.)*THETK)/(COUNT+1.)-SI
```

```
     1N((COUNT-1.)*THETK)/(COUNT-1.))/(DXI*UINF)
      GO TO 4350
 4349 ASHZ(K)=0.
      DO 4359 N=1,NGAM
 4359 ASH(K,N)=0.
 4350 CONTINUE
      IF(DIFF-1.E-6) 5005,5006,5006
 5005 PREC=0.
      GO TO 5007
 5006 CALL ATTPR(PREC,XSIG,NSIG,ASZ,AS,AR,CMAT,RMAT,NGAM,NF,ACAP,THICK,R
     1DBB,GAMAW,UINF,UDOT,DXI,BCAP)
 5007 CALL MIXER(FPRES,PREC,UINF,UDOT,THICK,NF,XBSIG,NSIG,INDT,DEL1,THET
     11,REB,USEP,X4,CP1)
      CPCT=CP1
      DO 4800 K=1,NSIG
      CORD=XBSIG(K)
      BSH(K,1)=-1.+THXI*BINT(XSEP,XATT,CORD)/UINF
      DO 4808 N=2,NSIG
 4808 BSH(K,N)=FB(XSIG(N-1),XSIG(N),XSIG(N+1),CORD)+THXI*GB(XSIG(N-1),XS
     1IG(N),XSIG(N+1),CORD)/UINF
      CALL ESIGI(2,NSIGA,XSIGA,BCAP,CORD,VAL1)
      CALL ESIGI(3,NSIGB,XSIGB,BCAP,CORD,VAL2)
      ARH(K)=FPRES(K)+(2.*VAL1-.5*VAL2)/(DXI*UINF)
      IF(CORD-1.) 5008,4800,4800
 5008 CALL EGAMI(2,NGAM,ACAP,BCAP(1,2),XSIGA(1),XSIGA(NSIGA+1),GAMAW(2),
     1CORD,VAL1)
      CALL EGAMI(3,NGAM,ACAP,BCAP(1,3),XSIGB(1),XSIGB(NSIGB+1),GAMAW(3),
     1CORD,VAL2)
      ARH(K)=ARH(K)+(2.*VAL1-.5*VAL2)/(DXI*UINF)+.0625*AFACT*PI*(1.+CORD
     1)*(1.-3.*CORD+THXI*(1.-CORD*CORD))/(DXI*UINF)
 4800 CONTINUE
 4444 CONTINUE
C
C   CALCULATIONS FROM THIS POINT ON COMBINE THE
C   CASES OF STALLED AND UNSTALLED AIRFOILS.
C
      DO 6500 M=1,NGP1
      RMAT(M)=AR(M)
      CMAT(M,1)=ASZ(M)
      DO 6485 N=1,NGAM
 6485 CMAT(M,N+1)=AS(M,N)
      IF(ISEP) 6486,6500,6486
 6486 DO 6499 N=1,NSIG
      NGG=N+NGP1
 6499 CMAT(M,NGG)=BS(M,N)
 6500 CONTINUE
      IF(ISEP) 6502,6501,6502
 6501 NTOT=NGP1
      GO TO 6751
 6502 DO 6750 K=1,NSIG
      KK=K+NGP1
      RMAT(KK)=ARH(K)
      CMAT(KK,1)=ASHZ(K)
      DO 6748 N=1,NGAM
 6748 CMAT(KK,N+1)=ASH(K,N)
```

```
      DO 6750 N=1,NSIG                                                  MAIN 360
      NGG=N+NGP1                                                        MAIN 361
 6750 CMAT(KK,NGG)=BSH(K,N)                                             MAIN 362
      NTOT=NSIG+NGP1                                                    MAIN 363
 6751 CALL ALSOL(NTOT,CMAT,RMAT)                                        MAIN 364
      DO 6800 N=1,NGP1                                                  MAIN 365
 6800 ACAP(N,1)=RMAT(N)                                                 MAIN 366
      IF(ISEP) 6805,6820,6805                                           MAIN 367
 6805 DO 6810 N=1,NSIG                                                  MAIN 368
      NGG=N+NGP1                                                        MAIN 369
 6810 BCAP(N,1)=RMAT(NGG)                                               MAIN 370
 6820 CONTINUE                                                          MAIN 371
      GAMAW(1)=GAM1(ACAP,DXI,PI)                                        MAIN 372
        IF( PSI360  .GE. PSILOW .AND. PSI360  .LE. PSIUP) GO TO 1736
      DO 1785 M=1,MX                                                    MAIN 373
      SIGN=1.                                                           MAIN 374
      IF(M-NZ) 1780,1785,1785                                           MAIN 375
 1780 SIGN=-SIGN                                                        MAIN 376
 1785 CALL QECAL(ISEP,NGAM,NSIG,NF,XSIG,ACAP,BCAP,THICK,RDBB,GAMAW(1),UIMAIN 377
     1NF,XC(M),UE(M,1),SIGN)                                            MAIN 378
 2785 DO 8886 I=1,2                                                     MAIN 379
      US2=UE(1,1)                                                       MAIN 380
      DO 8886 M=1,MXM1                                                  MAIN 381
      US1=UE(M,1)                                                       MAIN 382
      UE(M,1)=(US1+US2+UE(M+1,1))/3.                                    MAIN 383
 8886 US2=US1                                                           MAIN 384
      GO TO (8351,8353),IWASH                                           MAIN 386
 8351 DO 8352 M=1,MX                                                    MAIN 387
 8352 SCALS(M)=0.                                                       MAIN 388
      GO TO 1786                                                        MAIN 389
 8353 CALL YSET(RY1,Y(2),NY,Y)                                          MAIN 390
      RY=RY1                                                            MAIN 391
      DO 8354 M=1,MX                                                    MAIN 392
 8354 SCALS(M)=0.                                                       MAIN 393
      IF(INDV.EQ.2) GO TO 8370                                          MAIN 395
      IF(ISEP.EQ.0.AND.VZIP(1).LT.ALPHS) GO TO 1786                     MAIN 396
 8370 CALL STAG(MX,NY,MSTOP,MST,DXI,RY,DRY,X,Y,UE,UC,V,USAV,SCALS,ISEP) MAIN 397
      LAMQ=1                                                            MAIN 398
      XSEPS=XSEP                                                        MAIN 399
      DXX=DXI                                                           MAIN 400
      IF(ISEP.EQ.1.AND.ISEPT.EQ.0.AND.NITS.EQ.1) DXX=1.E30              MAIN 401
 8367 CALL BLC(X,Y,MST,MEND,NY,RY,DRY,DXX,REB,UPRIM,FLAM,XFLAM,TEST,U,SCMAIN 402
     1ALE,UE,UC,V,XSEP,USEP,DISP,THETA,LOWER,LAMQ,MSEP,XC,USAV,SCALS,NITMAIN 403
     1S,NTIME, NOTBL , XTEST, NZ, NOUT)
      IF(XSEP-XMAX) 7736,7735,7735                                      MAIN 405
 7735 IF(ISEP) 1786,1786,7736                                           MAIN 406
 7736 DEL1=DISP                                                         MAIN 407
      THET1=THETA                                                       MAIN 408
      INDT=1-LAMQ                                                       MAIN 409
      IF(INDT.EQ.1.AND.NOTBL.EQ.2) GO TO 1786                           MAIN 410
      WRITE(MOUT,23) XSIG(1),CPCT,XSEP                                  MAIN 411
      IF(INDT) 8462,8462,8463                                           MAIN 412
 8462 IF(ISEP) 8562,8562,8563                                           MAIN 413
 8563 IF(NITS-1) 8562,8562,8562                                         MAIN 414
 8562 IF(ISEPT) 7742,7742,8562                                          MAIN 415
```

```
8562 CALL BUBB(DEL1,THET1,RFB,XSEP,USEP,XC5,DCP,DEL5,X,XC,MX,NZ,X5,U5,U
    1F,ALTC,RENEL,USTOP)
     USEP=USEP+.002046*USEP**3
     PDIFF=(USEP-U5)*(USEP+U5)
     WRITE(MOUT,22) PDIFF,DCP
     IF(DCP-PDIFF) 8263,8366,8366
8263 ISEPT=0
     GO TO 8463
8366 IF(ISEP) 8368,8368,8369
8369 IF(ISEPT) 8467,8467,8368
8467 IWASH=1
     NITS=2
     GO TO 3344
8368 GO TO (8168,1786),NOTBL
8168 CALL REATT(UC,V,X,Y,MX,NY,RY,DRY,UE,X5,DEL5,MST,REB)
     LAMQ=0
     GO TO 8367
8463 IF(ISEP) 7741,7741,7742
7741 ISEP=1
     NITS=NITS+1
     IF(INDT) 7743,7743,7643
7643 ISEPT=1
     DXSEP=1.-XSEP
     XSEP=.6*XSEP+.4
     CALL CPC(ISEP,NGAM,NF,XSIG,NSIG,XSIGA,NSIGA,XSIGB,NSIGB,ACAP,BCAP,
    1THICK,RDBB,GAMAW,UINF,UDOT,1.,XSEP,DXI,CP1)
     GO TO 3248
7742 CALL ELDER(BCAP,XSIG,NSIG,UINF,ELDOT,SIGSUM,YMX)
     IF(ISEP.EQ.1.AND.ISEPT.EQ.0.AND.NITS.EQ.1) GO TO 9210
     IF(XSEP+.5) 7841,7842,7842
7841 EPS=EPSLE
     GO TO 7843
7842 EPS=EPSTE
7843 DXSEP=ABS(XSEP-XSEPS)
     IF(DXSEP-EPS) 7834,7834,9210
7834 IF(XSEP-XMAX) 1786,1786,7835
7835 ISEP=0
     ISEPT=0
     DO 7836 K=1,3
     DO 7836 N=1,NSIG
7836 BCAP(N,K)=0.
     GO TO 1786
9210 NITS=NITS+1
     IF(NITS.EQ.2.AND.INDT.EQ.0) XSEPS=XSEP
     IF(NITS-4) 9211,9211,1786
9211 IF(XSEP-XSEPS) 9305,9305,9306
9305 XSEP=.6*XSEPS+.4*XSEP
     GO TO 9307
9306 XSEP=.6*XSEP+.4*XSEPS
9307 IF(XSEP-XMAX) 9212,9212,7835
9212 CALL CPC(ISEP,NGAM,NF,XSIG,NSIG,XSIGA,NSIGA,XSIGB,NSIGB,ACAP,BCAP,
    1THICK,RDBB,GAMAW,UINF,UDOT,1.,XSEP,DXI,CP1)
       IF( NOTBL .EQ. 2 .AND. XSEP .GT. 0.) XSEP=-.98
     GO TO 3248
7743 IF(NITS-1) 7737,7737,3248
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 7737 NITS=NITS+1                                                          MAIN 470
      ELDOT=ELD1                                                           MAIN 471
      GO TO 3248                                                           MAIN 472
 1786 WRITE(MOUT,20) NTIME                                                 MAIN 473
      WRITE(MOUT,26) XIVOR                                                 MAIN 477
      PITC  = PITCH * 180. / PI
 209  WRITE(MOUT,10) TIME,UINF,XSEP,XATT,PITC                              MAIN 478
            ALDEG= ALPH1/DEGRES                                            SUPPL349
      WRITE(6, 9001)  Z,ALDEG,ALPH1 ,  ALPH2, HEAVE                        SUPPL350
        IF( PSI360  .GE. PSILOW .AND. PSI360  .LE. PSIUP) GO TO 101
            IF( NOUT .EQ. 0)
     1WRITE(MOUT,11)                                                       MAIN 479
            IF( NOUT .EQ. 0)
     1WRITE(MOUT,12) (N,XGAM(N),VZIP(N),AR(N),ACAP(N, 1),XIW(N ),GAMAW(N),MAIN 480
     2N=1,NGP1)                                                            MAIN 481
      IF(ISEP) 7432,7433,7432                                              MAIN 482
 7432       IF( NOUT .EQ. 0)
     1WRITE(MOUT,13)                                                       MAIN 483
            IF( NOUT .EQ. 0)
     1WRITE(MOUT,17) (N,XBSIG(N),FPRES(N),ARH(N),BCAP(N, 1),N=1,NSIG)      MAIN 484
      WRITE(MOUT,14) ELDOT                                                 MAIN 485
      WRITE(MOUT,18) XSIG(1),CPOT,X4,CPOT,XATT,PREC                        MAIN 486
 7433 WRITE(MOUT,15)                                                       MAIN 487
      XPC=-1.                                                              MAIN 488
      DO 7102 N=1,NCP1                                                     MAIN 489
      CALL QECAL(ISEP,NGAM,NSIG,NF,XSIG,ACAP,BCAP,THICK,RCBB,GAMAW(1),UIMAIN 490
     1NF,XPC,QEL,-1.)                                                      MAIN 491
      CALL QECAL(ISEP,NGAM,NSIG,NF,XSIG,ACAP,BCAP,THICK,RCBB,GAMAW(1),UIMAIN 492
     1NF,XPC,QEU,1.)                                                       MAIN 493
      CALL CPC(ISEP,NGAM,NF,XSIG,NSIG,XSIGA,NSIGA,XSIGB,NSIGB,ACAP,BCAP,MAIN 494
     1THICK,RDBB,GAMAW,UINF,JDOT,1.0,XPC,DXI,CPU)                          MAIN 495
      CALL CPC(ISEP,NGAM,NF,XSIG,NSIG,XSIGA,NSIGA,XSIGB,NSIGB,ACAP,BCAP,MAIN 496
     1THICK,RDBB,GAMAW,UINF,UDOT,-1.,XPC,DXI,CPL)                          MAIN 497
      IF(N-1) 7546,7545,7546                                               MAIN 498
 7545 CPL=CPU                                                              MAIN 499
 7546 DLIFT=CPL-CPU                                                        MAIN 500
      WRITE(MOUT,16) XPC,QEL,CPL,QEU,CPU,DLIFT                             MAIN 501
 7102 XPC=XPC+PINT                                                         MAIN 502
 101        CONTINUE
      CMPAS=CMPA                                                           MAIN 503
      CALL CLCM(NCOI,ISEP,NGAM,XSIG,NSIG,XSIGA,NSIGA,XSIGB,NSIGB,ACAP,BCMAIN 504
     1AP,THICK,RDBB,GAMAW,UINF,UDOT,DXI,AROT,CMPA)                         MAIN 505
            P(L,2) =  PITC
            P(L,3) =  Z(3)
            P(L,4) =  Z(1)
            P(L,5) =  Z(2)
            P(L,6) =  CLVB
            P(L,7) =  CMPA
            IF( L .LT.  200 )  GO TO    98
            CALL    PLOTSB( PLOTOP  ,  P  ,  L )
            L= 0
  98      CONTINUE
          IF(ISTD .EQ. 1) GO TO    9999
      DO 7950 M=1,MX                                                       MAIN 506
      SCALE(M,2)=SCALE(M,1)                                                MAIN 507
```

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      SCALE(M,1)=SCALS(M)                                               MAIN 508
      DO 7950 N=1,NY                                                    MAIN 509
      U(M,N,2)=U(M,N,1)                                                 MAIN 510
 7950 U(M,N,1)=USAV(M,N)                                                MAIN 511
      GO TO 9999                                                        MAIN 512
 8989 CONTINUE                                                          MAIN 513
  99      CONTINUE
            CALL    PLOTSB( PLOTOP   ,   P   ,   L  )
          CALL ACUCPU( IACU )
         IF( IACU .LT.   35000   ) GO TO  60
          GO TO 40
 60         CONTINUE
             IF( PLOTOP.EQ. 0.)   CALL EXIT
        CALL PLTND
        CALL  EXIT
           RETURN
C
C
C
C
 1    FORMAT(13I5)                                                      MAIN  23
 2    FORMAT(3F10.4)                                                    MAIN  24
 3    FORMAT(2F10.4)                                                    MAIN  25
 4    FORMAT(1H1//)                                                     MAIN  26
 5    FORMAT(6F10.4)                                                    MAIN  27
 6    FORMAT(1H1,50X,34HANALYSIS OF UNSTEADY AIRFOIL STALL///)          MAIN  28
 7    FORMAT(8X,6HUBAR =E13.5/7X,7HUFREQ =E13.5//3X,11HALPHA ONE =E13.5/MAIN  29
     13X,11HALPHA TWO =E13.5/8X,6HHBAR =E13.5/11X,3HA =E13.5/8X,6HFREQ =MAIN  30
     1E13.5//8X,6HRO/B =E13.5//9X,5HREB =E13.5///)                      MAIN  31
 8    FORMAT(29X,1HN,25X,4HC(N),26X,4HT(N)/)                            MAIN  32
 9    FORMAT(I30,2E30.5)                                                MAIN  33
 10   FORMAT(5X,3HT =E13.5/5X,3HU =E13.5/4X,4HXS =E13.5/4X,4HXO =E13.5/4MAIN  34
     1X,4HPA =E13.5////)                                                MAIN  35
 11   FORMAT(///4X,1HN,11X,1HX,14X,5HVZ(X),12X,5HRN(X),12X,4HA(N),21X,3HMAIN  36
     1XIW,14X,5HGAMMA/)                                                 MAIN  37
 12   FORMAT(I5,4E17.5,8X,2E17.5)                                       MAIN  38
 13   FORMAT(1H1,8X,1HN,20X,1HX,21X,5HFP(X),22X,5HRH(N),21X,4HB(N)/)    MAIN  39
 14   FORMAT(//54X,9H  L-DCT =E13.5///51X,27HPRESSURES IN SEPARATED FLOWMAIN  40
     1//55X,1HX,19X,2HCP/)                                              MAIN  41
 15   FORMAT(1H1,11X,1HX,16X,3HQEL,15X,3HCPL,15X,3HQEU,15X,3HCPU,13X,9HCMAIN  42
     1PL - CPU/)                                                        MAIN  43
 16   FORMAT(6E18.5)                                                    MAIN  44
 17   FORMAT(I10,4E25.5)                                                MAIN  45
 18   FORMAT(3(40X,2E20.5/))                                            MAIN  46
 19   FORMAT(I5,5F10.4)                                                 MAIN  47
 20   FORMAT(1H1,50X,12HTIME STEP NOI3//)                               MAIN  48
 22   FORMAT(///40X,26HINCREASE IN CP REQUIRED ISE13.5//40X,26HINCREASE MAIN  49
     1IN CP POSSIBLE ISE13.5)                                           MAIN  50
 23   FORMAT(///45X,23HPOTENTIAL FLOW     XS =E12.4/60X,8HCP(XS) =E12.4/MAIN  51
     1/45X,23HBOUNDARY LAYER     XS =E12.4)                             MAIN  52
 24   FORMAT(I5,4F10.4/5F10.4)                                          MAIN  53
 25   FORMAT(12X,4HNV =I2,3X,3HS =E12.4,3X,3HH =E12.4,3X,3HG =E12.4,3X,4MAIN  54
     1HX1 =E12.4//12X,4HMI =E12.4,3X,4HWT =E12.4,3X,4HPA =E12.4///)     MAIN  55
 26   FORMAT(4X,4HX1 =E13.5)                                            MAIN  56
 9001 FORMAT('0', T50,  'EQUIVALENT ROTOR BLADE RESPONSE'               SUPPL380
```

```
9001A  //   T 5,  'FLAP DISP =',        G14.5                                SUPPL381
9001B   ,   T47,  'BENDING DISP =',  G14.5                                   SUPPL382
9001C   ,   T39,  'TORSIONAL DISP =', G14.5                                  SUPPL383
9001D   /   T38,  'SECTION PITCH ANGLE =', F9.3, ' DEGREES OR',              SUPPL384
9001E                                        F9.4, ' RADIANS '               SUPPL385
9001F   /   T21,  'SECTION PITCH RATE =',   G14.5                            SUPPL386
9001G   ,   T71,  'SECTION PLUNGING RATE =',  G14.5    //)                   SUPPL387
     END                                                                     MAIN 515
```

```
      SUBROUTINE SUPPL
      IMPLICIT REAL*8 (A-H,O-Z)
      REAL*8 FR1S, FR2S, FR3S, ANSX, OMS
C
      REAL*4             CLVB, CMVB,  CMPAVB
     1 , DUMMY, PLOTOP
      REAL   FTVB,  FPVB,  FPPRVB,  DIDRVB,  XMVB,  DELVB,  XMUVB,
     A      FOVB,  XMUAVB,  ATOVB,  ATCVB,  ATSVB,  ROVB,  RVB,  MVB,
     C      WDXI,  PSI, UINF
      REAL   ELSIG,  DXI,  REB,  RDBB,  FRZ,  ARR,  AMPLU,  FREQU,
     A   ALPH1,  ALPH2,  HEAVE,  AROT,  FREQF,  PHIH,        RY1,  DRY,
     B   X,  TEST,  UPRIM,  XU,  YU,  XL,  YL,  ER1,  ER2,  ER3,  BDBR,
     C     RRDBR
      REAL   SUM(8), YCLD(8), YNEW(8), DEL(3,3), CMPA(3), CL(3), G(3),
     A   Z   , ZPR(3), SMALLG(3), Y(3,3), YPR(3,3),GCAP(3,3)
      COMMON /BL1/       NTIME, NDIMC
      COMMON /CLCMBL/    CLVB, CMVB,  CMPAVB
         COMMON/ ZZZ/ Z(3)
      COMMON /INPTVB/    FTVB(64),  FPVB(64),  FPPRVB(64),  DIDRVB(64),
     A         XMVB(64),    DELVB,      XMUVB,      FOVB,     XMUAVB,
     B         ATOVB,       ATCVB,       ATSVB,       ROVB,        RVB(64),
     C         MVB(64),     NVB
      COMMON /INPUTS/    NSBL,       NZ,         NOFF,       NGAM,       NSIG,
     A            NCOI,       NCORD,      LOWER,      MSTOP,      MAXT,       MOTR,
     B            NOTBL,      INDV,       ELSIG,      DXI,        REB,        RDBB,
     C            FRZ,        ARR,        AMPLU,      FREQU,      ALPH1,      ALPH2,
     D            HEAVE,      AROT,       FREQF,      PHIH,       NY,         RY1,
     E            DRY,        X(100),     TEST,       UPRIM,      XU(30),     YU(30),
     F            XL(30),     YL(30),     ER1,        ER2,        ER3,        BDBR,
     G            RRDBR
     H , DUMMY(10),PLOTOP
      DIMENSION DELTA(3,3)
      DIMENSION ALPHA(3,3),BETA(3,3),GAMMA(3,3),OMS(3),OMEGA(3),CHK(3)
      DIMENSION AA(10),AB(10),ANB(20),ANT(20),AAX(10),ANSX(20),SORT(3)
     1  , TOT(2)
      CF4(X)=F4-B4+(B4*C6-C4)*X*X
      Z1(X)=HB*(CF4(X)/GB)**2+(CF4(X)*FR1S+(1.-C6*X*X)*B2-F2)*X*X
      Z2(X)=(FZ/FR1S+FR1S*CF4(X)-F2+(1.-C6*X*X)*(B2-BZ/FR1S))*X*X
      S1(X)=(2.*HB*CF4(X)/GB**2+(FR1S-FR2S)*X*X)*GA
      S2(X)=(FR1S-FR2S)*GA*X*X
      FUN(X)=(R1*Z2(X)-R2*Z1(X))**2+(R1*S2(X)-R2*S1(X))*(Z2(X)*S1(X)-Z1(
     1X)*S2(X))
      DATA BBS,REL,NPOL/1.E-7,1.E-6,3/
C
C  MASSES AND H'S ARE NCNDIMENSIONAL, WITH BLADE MASS AND RADIUS
C  AS REFERENCES.  NONROTATING NATURAL FREQUENCIES ARE
C  DIMENSICNLESS, USING ROTOR SPEED AS REFERENCE.  DISTANCES XBAB, S1LB,
C  AND S2LB ARE FRACTICNS OF SEMICHORD.  XBAR, S1L, AND S2L ARE
C  FRACTIONS OF ROTOR RADIUS.
C
      NDIMC=3
      DO  63 K = 1, 8
      SUM(K) = 0.
   63 YNEW(K) = 0.
      DO 69  I = 1, NVB
```

```
      DO 66 K = 1, 8
   66 YOLD(K) = YNEW(K)
      CALL YVB(YNEW,I)
      IF(I .LE. 1) GO TO 69
      DO 67 K = 1, 8
   67 SUM(K) = (YNEW(K) + YOLD(K)) * (RVB(I) - RVB(I-1)) / 2. + SUM(K)
   69 CONTINUE
      EM11 = SUM(1)
      EM22 = SUM(2)
      EM33 = SUM(3)
      EM13 = SUM(4)
      EM23 = SUM(5)
      H11 = SUM(6)
      H22 = SUM(7)
      H33 = - EM33
      H13 = -EM13
      H23 = SUM(8)
      BDBRR=BDBR/RRDBR
      BDS=BDBRR**2
      T11=H11*BDS
      T22=H22*BDS
      T33=H33*BDS
      T13=H13*BDS
      T23=H23*BDS
      FR1S=BDS*ER1**2-T11/EM11
      FR2S=ER2**2*BDS-T22/EM22
      FR3S=ER3**2*BDS-T33/EM33
      FR1=DSQRT(FR1S)
      FR2=DSQRT(FR2S)
      FR3=DSQRT(FR3S)
      RATM=EM11/EM22
      ZETA=(1.+RATM)*(RATM*FR1S**2+FR2S**2)/(RATM*FR1S+FR2S)**2
      RM=ZETA-1.
      SUMS=FR1S+FR2S
      HIGHS=(SUMS+DSQRT(SUMS**2-4.*ZETA*FR1S*FR2S))/(2.*ZETA)
      SMALS=FR1S*FR2S/HIGHS
      DEN=FR2S-FR1S
      A1=-(HIGHS-FR1S)/DEN
      A2=-1.-A1
      B=-A1*A2*DEN/HIGHS
      SLAM1=EM11*BDBR**2/EM33
      SLAMZ=-A1*SLAM1
      SLAM2=-SLAMZ/A2
      SUM3=SUMS+FR3S
      ADD2=FR1S*(FR2S+FR3S)+FR2S*FR3S
      ADDZ=FR1S*FR2S*FR3S
      BBAR=1.-(EM13**2/EM11+EM23**2/EM22)/EM33
      B4=SUM3+(2.*EM23*T23/EM22+2.*EM13*T13/EM11-FR1S*EM23**2/EM22-FR2S*
     1EM13**2/EM11)/EM33
      B4=B4/BBAR
      B2=ADD2+(2.*FR2S*EM13*T13/EM11+2.*FR1S*EM23*T23/EM22-T13**2/EM11-T
     123**2/EM22)/EM33
      B2=B2/BBAR
      BZ=ADDZ-(FR2S*T13**2/EM11+FR1S*T23**2/EM22)/EM33
      BZ=BZ/BBAR
```

```
      C6=(EM11*A1**2+EM22*A2**2)/EM33                                   SUPPL107
      F4=SUM3                                                           SUPPL108
      C4=(FR2S*EM11*A1**2+FR1S*FM22*A2**2)/ FM33                        SUPPL109
      GA=2.*EM11*A1/EM33                                                SUPPL110
      GB=2.*EM22*A2/EM33                                                SUPPL111
      F2=ADD2                                                           SUPPL112
      HA=EM11/FM33                                                      SUPPL113
      HB=EM22/EM33                                                      SUPPL114
      FZ=ADDZ                                                           SUPPL115
      R1=-HA-HB*(GA/GB)**2                                              SUPPL116
      R2=HA*(FR2S/FR1S-1.)                                              SUPPL117
      ZLAM=F4-B4                                                        SUPPL118
      TWLAM=B4*C6-C4                                                    SUPPL119
      FZHAT=HB*( ZLAM/GB)**2                                            SUPPL120
      F2HAT=B2-F2+FR1S*ZLAM+2.*ZLAM*TWLAM*HB/GB**2                      SUPPL121
      F4HAT=-C6*B2+FR1S*TWLAM+HB*(TWLAM/GB)**2                          SUPPL122
      G2HAT=B2-F2+(FZ-BZ)/FR1S+FR1S*ZLAM                                SUPPL123
      G4HAT=-C6*(B2-BZ/FR1S)+FR1S*TWLAM                                 SUPPL124
      SIGZ=2.*HB*ZLAM*GA/GB**2                                          SUPPL125
      SIG2=GA*(FR1S-FR2S+2.*HB*TWLAM/GB**2)                             SUPPL126
      GAM2=GA*(FR1S-FR2S)                                               SUPPL127
      UZ=-R2*FZHAT                                                      SUPPL128
      U1=R1*G2HAT-R2*F2HAT                                              SUPPL129
      U2=R1*G4HAT-R2*F4HAT                                              SUPPL130
      U3=-R2*SIGZ                                                       SUPPL131
      U4=R1*GAM2-R2*SIG2                                                SUPPL132
      U5=SIGZ*G2HAT-GAM2*FZHAT                                          SUPPL133
      U6=SIGZ*G4HAT+SIG2*G2HAT-GAM2*F2HAT                               SUPPL134
      U7=SIG2*G4HAT-GAM2*F4HAT                                          SUPPL135
      AAX(1)=UZ**2                                                      SUPPL136
      AAX(2)=2.*UZ*U1+U3*U5                                             SUPPL137
      AAX(3)=U1**2+2.*UZ*U2+U3*U6+U4*U5                                 SUPPL138
      AAX(4)=2.*U1*U2+U3*U7+J4*U6                                       SUPPL139
      AAX(5)=U2**2+U4*U7                                                SUPPL140
      CALL POLLY(4,BBS,REL,ANSX,AAX)                                    SUPPL141
      XBAR=1.E25                                                        SUPPL142
      DO 86 I=1,4                                                       SUPPL143
      IP=2*I                                                            SUPPL144
      IM=IP-1                                                           SUPPL145
      IF(DABS(ANSX(IM)).GT.1.D-10) GO TO 86                             SUPPL146
      IF(ANSX(IP).LE.0.) GO TO 86                                       SUPPL147
      XBART=DSQRT(ANSX(IP))                                             SUPPL148
      IF(XBART.LT.XBAR) XBAR=XBART                                      SUPPL149
 86   CONTINUE                                                          SUPPL150
      IF(XBAR.LT..5E25) GO TO 88                                        SUPPL151
      WRITE(6,87)                                                       SUPPL152
 87   FORMAT(1H1,10X,'NC SOLUTICN FOR XBAR')                            SUPPL153
      STOP                                                              SUPPL154
 88   CONTINUE                                                          SUPPL155
 15   ALOW=(R1*Z2(XBAR)-R2*Z1(XBAR))/(R1*S2(XBAR)-R2*S1(XBAR))          SUPPL156
      ALOW=ALOW/XBAR                                                    SUPPL157
      BLOW=(CF4(XBAR)-GA*ALOW*XBAR)/(XBAR*GB)                           SUPPL158
      XI=-ALOW-BLOW                                                     SUPPL159
      ETA=(BLOW*A1-ALOW*A2)/(A1-A2)                                     SUPPL160
      S2L=FTA/(B*HIGHS)                                                 SUPPL161
```

```
      S1L=(XI-RM*HIGHS*S2L)*HIGHS/(FR1S*FR2S)
      WRITE(6,4) ER1,ER2,ER3,RM
      WRITE(6,721) FR1,FR2,FR3,ALOW,BLOW
      WRITE(6,5) EM11,EM22,EM33,EM13,EM23
      WRITE(6,6) H11,H22,H33,H13,H23
      C13=ALOW/BDBR
      C23=BLOW/BDBR
      XBAB=XBAR/BDBR
      S1LB=S1L/BDBR
      S2LB=S2L/BDBR
      WRITE(6,41) BDBR,RRDBR
      WRITE(6,7) XBAR,XBAB,S1L,S1LB,S2L,S2LB,SMALS,HIGHS
      AA(1)=B7
      AA(2)=B2
      AA(3)=B4
      AA(4)=1.
      CALL POLLY(NPOL,BBS,REL,ANB,AA)
      SSX=SLAMZ*XBAB
      DIV=1.-SLAMZ*XBAB**2
      BETA(3,1)=(SLAM1*C13+SSX*FR1S)/DIV
      BETA(3,2)=(SLAM2*C23+SSX*FR2S)/DIV
      BETA(3,3)=(FR3S+SSX*(C13+C23))/DIV
      AXB=A1*XBAB
      BETA(1,1)=FR1S-AXB*BETA(3,1)
      BETA(1,2)=-AXB*BETA(3,2)
      BETA(1,3)=C13-AXB*BETA(3,3)
      AAXB=A2*XBAB
      BETA(2,1)=-AAXB*BETA(3,1)
      BETA(2,2)=FR2S-AAXB*BETA(3,2)
      BETA(2,3)=C23-AAXB*BETA(3,3)
      AB(4)=1.
      AB(3)=BETA(1,1)+BETA(2,2)+BETA(3,3)
      AB(2)=BETA(1,1)*(BETA(2,2)+BETA(3,3))+BETA(2,2)*BETA(3,3)-BETA(3,2
     1)*BETA(2,3)-BETA(1,2)*BETA(2,1)-BETA(1,3)*BETA(3,1)
      AB(1)=BETA(1,1)*(BETA(2,2)*BETA(3,3)-BETA(3,2)*BETA(2,3))-BETA(2,1
     1)*(BETA(1,2)*BETA(3,3)-BETA(3,2)*BETA(1,3))+BETA(3,1)*(BETA(1,2)*B
     1ETA(2,3)-BETA(1,3)*BETA(2,2))
      CALL POLLY(NPOL,BBS,REL,ANT,AB)
      WRITE(6,44)
      DO 45 I=1,4
      IM=(I-1)*2
45    WRITE(6,46) IM,AA(I),AB(I)
      WRITE(6,47)
      DO 48 I=1,3
      ITT=2*I
      ITM=ITT-1
48    WRITE(6,49) ANB(ITT),ANB(ITM),ANT(ITT),ANT(ITM)
      DO 301 I=1,3
      II=2*I
301   OMS(I)=-ANT(II)
      MAXI=3
      DO 70 I=1,2
      IF(OMS(I).GT.OMS(MAXI)) MAXI=I
70    CONTINUE
      GO TO (71,72,73),MAXI
```

```
 71   I1=2                                                              SUPPL217
      I2=3                                                              SUPPL218
      GO TO 74                                                          SUPPL219
 72   I1=1                                                              SUPPL220
      I2=3                                                              SUPPL221
      GO TO 74                                                          SUPPL222
 73   I1=1                                                              SUPPL223
      I2=2                                                              SUPPL224
 74   IF(OMS(I1).GT.OMS(I2)) GO TO 75                                   SUPPL225
      MINI=I1                                                           SUPPL226
      MIDI=I2                                                           SUPPL227
      GO TO 76                                                          SUPPL228
 75   MINI=I2                                                           SUPPL229
      MIDI=I1                                                           SUPPL230
 76   SORT(1)=OMS(MINI)                                                 SUPPL231
      SORT(2)=OMS(MIDI)                                                 SUPPL232
      SORT(3)=OMS(MAXI)                                                 SUPPL233
      DO 77 I=1,3                                                       SUPPL234
      OMS(I)=SORT(I)                                                    SUPPL235
 77   OMEGA(I)=DSQRT(OMS(I))                                            SUPPL236
      DO 302 I=1,3                                                      SUPPL237
 302  ALPHA(I,I)=1.                                                     SUPPL238
      DENB=BETA(2,1)*BETA(3,2)-BETA(3,1)*(BETA(2,2)-OMS(1))             SUPPL239
      ALPHA(1,2)=(BETA(1,2)*BETA(3,1)-BETA(3,2)*(BETA(1,1)-OMS(1)))/DENBSUPPL240
      ALPHA(1,3)=((BETA(2,2)-OMS(1))*(BETA(1,1)-OMS(1))-BETA(1,2)*BETA(2SUPPL241
     1,1))/DENB                                                         SUPPL242
      CHK(1)=BETA(1,3)*ALPHA(1,1)+BETA(2,3)*ALPHA(1,2)+(BETA(3,3)-OMS(1)SUPPL243
     1)*ALPHA(1,3)                                                      SUPPL244
      DENB=BETA(3,2)*(BETA(1,1)-OMS(2))-BETA(3,1)*BETA(1,2)             SUPPL245
      ALPHA(2,1)=(BETA(3,1)*(BETA(2,2)-OMS(2))-BETA(2,1)*BETA(3,2))/DENBSUPPL246
      ALPHA(2,3)=(BETA(2,1)*BETA(1,2)-(BETA(1,1)-OMS(2))*(BETA(2,2)-OMS(SUPPL247
     12)))/DENB                                                         SUPPL248
      CHK(2)=BETA(1,3)*ALPHA(2,1)+BETA(2,3)*ALPHA(2,2)+(BETA(3,3)-OMS(2)SUPPL249
     1)*ALPHA(2,3)                                                      SUPPL250
      DENB=BETA(2,3)*(BETA(1,1)-OMS(3))-BETA(1,3)*BETA(2,1)             SUPPL251
      ALPHA(3,1)=(BETA(2,1)*(BETA(3,3)-OMS(3))-BETA(3,1)*BETA(2,3))/DENBSUPPL252
      ALPHA(3,2)=(BETA(3,1)*BETA(1,3)-(BETA(1,1)-OMS(3))*(BETA(3,3)-OMS(SUPPL253
     13)))/DENB                                                         SUPPL254
      CHK(3)=BETA(1,2)*ALPHA(3,1)+(BETA(2,2)-OMS(3))*ALPHA(3,2)+BETA(3,2SUPPL255
     1)*ALPHA(3,3)                                                      SUPPL256
      WRITE(6,488)                                                      SUPPL257
      WRITE(6,489) (I,OMEGA(I),BETA(I,1),BETA(I,2),BETA(I,3),ALPHA(I,1),SUPPL258
     1ALPHA(I,2),ALPHA(I,3),CHK(I),I=1,3)                               SUPPL259
      SORT(1)=1.                                                        SUPPL260
      SORT(2)=0.                                                        SUPPL261
      SORT(3)=0.                                                        SUPPL262
      DO 432 J=1,3                                                      SUPPL263
      GO TO (381,382,383), J                                            SUPPL264
 382  SORT(1)=0.                                                        SUPPL265
      SORT(2)=1.                                                        SUPPL266
      SORT(3)=0.                                                        SUPPL267
      GO TO 381                                                         SUPPL268
 383  SORT(1)=0.                                                        SUPPL269
      SORT(2)=0.                                                        SUPPL270
      SORT(3)=1.                                                        SUPPL271
```

```
381  DO 384 I=1,3                                                        SUPPL272
     DO 384 K=1,3                                                        SUPPL273
384  DELTA(I,K)=ALPHA(I,K)                                               SUPPL274
     CALL ALSOL(3,DELTA,SORT,3)                                          SUPPL275
     DO 431 I=1,3                                                        SUPPL276
431  GAMMA(I,J)=SORT(I)                                                  SUPPL277
432  CONTINUE                                                            SUPPL278
     WRITE(6,11)                                                         SUPPL279
     WRITE(6,12) (I,GAMMA(I,1),GAMMA(I,2),GAMMA(I,3),I=1,3)              SUPPL280
     AMPLU = XMUAVB * (1. - ROVB**3) / (1. - ROVB**4) * 1.33333333333D0SUPPL284
     SA = SMALS * S1LB + RM * S2LB * HIGHS                               SUPPL285
     SB = SMALS * S1LB**2 + RM * S2LB**2 * HIGHS                         SUPPL286
     DEL(1,1) =  XMUVB * (1. - ROVB**4) / (4. * (1. - SLAMZ * XBAB**2) SUPPL287
    A       *  RRDBR * FM11 )                                            SUPPL288
     DEL(1,2) =  2. * SLAMZ * XBAB * DEL(1,1)                            SUPPL289
     DEL(1,3) = A1 * (SLAMZ * XBAB * SB - SA ) / (1. - SLAMZ * XBAB**2)SUPPL290
    A      +   B * HIGHS * S2LB                                          SUPPL291
     DEL(2,1) =  A2 / A1 * DEL(1,1)                                      SUPPL292
     DEL(2,2) =  A2 / A1 * DEL(1,2)                                      SUPPL293
     DEL(2,3) = A2 * (SLAMZ * XBAB  * SB - SA) / (1. - SLAMZ * XBAB**2)SUPPL294
    A      -  B * SMALS * S2LB                                           SUPPL295
     DEL(3,1) = - SLAMZ * XBAB * DEL(1,1) / A1                           SUPPL296
     DEL(3,2) = -2. * SLAMZ * DEL(1,1) / A1                              SUPPL297
     DEL(3,3) = (BDBR / RRDBR)**2 +  SLAMZ * (XBAB * SA - SB) /          SUPPL298
    A       (1. - SLAMZ * XBAB**2)                                       SUPPL299
     CMPA(2) = CMPAVB                                                    MAIN
     CL(2) = CLVB                                                        MAIN
          NDIMC= 60
        COSPSI=  1.
        SINPSI= 0.
     TO = ATOVB + ATCVB  * COS PSI  + ATSVB * SIN PSI
        TOT(1)= TO - ATOVB
       DO 50 I=1,3
 50     SMALLG(I)= DEL(I,1) * CLVB + DEL( I,2) * CMPAVB
        DO 51  I=1,3
        GCAP(I,1)=0.
      DO 52  J=1,3
          YPR(I,J)=0.
 52       GCAP(I,1)     =  GCAP(I,1) + ALPHA(I,J) * SMALLG(J)
        GCAP(I,2)= GCAP(I,1)
        Y(I,1)= GCAP(I,1) / CMS(I)
 51       Y(I,2) = Y(I,1)
        IF ( PLOTOP .LT. 0.)
    1 WRITE( 6,9000) TO, Z, TOPR, ZPR, Y, YPR, DEL, SMALLG
 9000   FORMAT(//'   TO=', 1P1E13.6, '  Z=', 1P3E13.6,' TOPR=', 1P1E13.6
    1 , ' ZPR=', 1P3E13.6 / '  Y=', 1P9E13.6/'  YPR=   ' , 1P9E13.6
    2 / '  DEL=   ', 1P9E13.6/ '  SMALLG=   ', 1P9E13.6//)
      RETURN                                                             SUPPL300
C                                                                        SUPPL301
C                                                                        SUPPL302
      ENTRY SUPPI (UINF)                                                 SUPPL303
C                                                                        SUPPL304
C                                                                        SUPPL305
      CMPA(3) = CMPA(2)                                                  SUPPL306
      CMPA(2) = CMPAVB                                                   MAIN
```

```
      CMPA(1) = 2. * CMPA(2) - CMPA(3)                                  SUPPL308
      CL(3) = CL(2)                                                     SUPPL309
      CL(2) = CLVB                                                      MAIN
      CL(1) = 2. * CL(2) - CL(3)                                        SUPPL311
      PSI =  (BDBR / RRDBR ) * NTIME * DXI                              SUPPL312
      SIN PSI = SIN(PSI)                                                SUPPL313
      COS PSI = COS(PSI)                                                SUPPL314
         TOT(2)= TOT(1)
      TO = ATOVB + ATCVB  * COS PSI  + ATSVB * SIN PSI                  SUPPL315
        TOT(1)= TO - ATOVB
      TO PR = (BDBR/ RRDBR) * (ATSVB * COS PSI - ATCVB * SIN PSI)       SUPPL316
      DO 60 K = 1, 2
      DO 64 I = 1, 3                                                    SUPPL317
64    SMALL G(I) = UINF **2 * (DEL(I,1) * CL(K) +  DEL(I,2) * CMPA(K))
     A    +   DEL(I,3) * TOT(K)                                         SUPPL319
      DO 65 I = 1, 3                                                    SUPPL320
      G CAP(I, K) = 0.
      DO 65 J = 1, 3                                                    SUPPL322
65    G CAP(I, K)  = GCAP(I, K) + ALPHA(I,J) * SMALLG(J)
60         CONTINUE
      DO 62 I = 1, 3                                                    SUPPL328
      Y(I,2) = Y(I,1)                                                   SUPPL329
      YPR(I,2) = YPR(I,1)                                               SUPPL330
      WDXI    = OMEGA(I) * DXI                                          SUPPL331
      SWDXI    = SIN(WDXI)                                              SUPPL332
      CWDXI    = COS(WDXI)                                              SUPPL333
      Y(I,1) = Y(I,2) * CWDXI  + YPR(I,2) * SWDXI /OMEGA(I)             SUPPL334
     A +((GCAP(I,2) - GCAP(I,1)) * (SWDXI - WDXI * CWDXI) / WDXI        SUPPL335
     B  + GCAP(I,1) * (1. - CWDXI)) / OMEGA(I)**2                       SUPPL336
62    YPR(I,1) = YPR(I,2) * CWDXI - OMEGA(I) * Y(I,2) * SWDXI           SUPPL337
     A  + ( (GCAP(I,2) - GCAP(I,1) ) * (WDXI * SWDXI + CWDXI - 1.)      SUPPL338
     B     / WDXI  + GCAP(I,1) * SWDXI) / OMEGA(I)                      SUPPL339
      DC 61 I = 1, 3                                                    SUPPL340
      Z(I) = 0.                                                         SUPPL341
      ZPR(I) = 0.                                                       SUPPL342
      DO 61 J = 1, 3                                                    SUPPL343
      Z(I) = Z(I) + GAMMA(I,J) * Y(J,1)                                 SUPPL344
61    ZPR(I) = ZPR(I) + GAMMA(I,J) * YPR(J,1)                           SUPPL345
      ALPH1 = TO + Z(3)                                                 SUPPL346
      ALPH2 = TO PR + ZPR(3)                                            SUPPL347
      HEAVE =  - ZPR(1)  - ZPR(2)
        IF ( PLOTOP .LT. 0.)
     1 WRITE( 6,9000) TO, Z, TOPR, ZPR, Y, YPR, DEL, SMALLG
     2 , TOT
      RETURN                                                            SUPPL351
1     FORMAT(5F10.4)                                                    SUPPL352
2     FORMAT(5F10.4)                                                    SUPPL353
3     FORMAT(1H1,10X,'ITERATION FOR XBAR DIVERGED' )                    SUPPL354
4     FORMAT(1H1,5X,4HF1 =E13.5,5X,4HF2 =E13.5,5X,4HF3 =E13.5//5X,4HRM =SUPPL355
     1E13.5////)                                                        SUPPL356
5     FORMAT(5X,5HM11 =E13.5,5X,5HM22 =E13.5,5X,5HM33 =E13.5,5X,5HM13 =ESUPPL357
     113.5,5X,5HM23 =E13.5/)                                            SUPPL358
6     FORMAT(5X,5HT11 =E13.5,5X,5HT22 =E13.5,5X,5HT33 =E13.5,5X,5HT13 =ESUPPL359
     113.5,5X,5HT23 =E13.5///)                                          SUPPL360
7     FORMAT(20X,6HXB/R =E13.5,10X,6HXB/B =E13.5/20X,6HL1/R =E13.5,10X,6SUPPL361
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```
     1HL1/B =E13.5/20X,6HL2/R =E13.5,10X,6HL2/B =E13.5/9X,7HK1/M1 =E13.5SUPPL362
     1/9X,7HK2/M2 =E13.5)                                               SUPPL363
41    FCRMAT(//10X,5HB/R =E13.5,20X,6HRR/R =E13.5///)                   SUPPL364
44    FORMAT(1H1,20X,'POLYNOMIAL COEFFICIENTS'///7X,5HPOWER,12X,5HBLADE,SUPPL365
     126X,3H2-D/)                                                       SUPPL366
46    FORMAT(I10,2D30.9)                                                SUPPL367
47    FORMAT(1H1,20X,'ROOTS OF POLYNOMIALS'///30X,'BLADE',60X,'2-D'/20X,SUPPL368
     14HREAL,21X,4HIMAG,31X,4HREAL,21X,4HIMAG/)                         SUPPL369
49    FORMAT(2D25.9,10X,2D25.9)                                         SUPPL370
11    FORMAT(/////9X,1HI,15X,10HGAMMA(I,1),15X,10HGAMMA(I,2),15X,10HGAMMSUPPL371
     1A(I,3)/)                                                          SUPPL372
12    FORMAT(I10,3E25.5)                                                SUPPL373
488   FORMAT(1H1,8X,1HI,7X,5HOMEGA,4X,9HBETA(I,1),4X,9HBETA(I,2),4X,9HBESUPPL374
     1TA(I,3),3X,10HALPHA(I,1),3X,10HALPHA(I,2),3X,10HALPHA(I,3),8X,3HCHSUPPL375
     1K//)                                                              SUPPL376
489   FORMAT(I10,8E13.5)                                                SUPPL377
721   FORMAT(///10X,5HFR1 =E13.5,10X,5HFR2 =E13.5,10X,5HFR3 =E13.5//10X,SUPPL378
     14HSA =E13.5,10X,4HSB =E13.5///)                                   SUPPL379
      END
```

```
      SUBROUTINE SETUPS                                                 SETUPS 1
C                                                                       SETUPS 2
      IMPLICIT REAL*8 (A-H,O-Z)                                         SETUPS 3
C                                                                       SETUPS 4
C                                                                       SETUPS 5
      REAL   FTVB,  FPVB,  FPPRVB,  DIDRVB,  XMVB,  DELVB,  XMUVB,      SETUPS 6
     A    FOVB,  XMUAVB,  ATOVB,  ATCVB,  ATSVB,  ROVB,  RVB,  MVB      SETUPS 7
      REAL   ELSIG,  DXI,  REB,  RDBB,  FRZ,  ARR,  AMPLU,  FREQU,      SETUPS 8
     A  ALPH1,  ALPH2,  HEAVE,  AROT,  FREQF,  PHIH,       RY1,  DRY,   SETUPS 9
     B  Y,  TEST,  UPRIM,  XU,  YU,  XL,  YL,  ER1,  ER2,  ER3,  BDBR,  SETUPS10
     C    RRDBR                                                         SETUPS11
     H,  CMPA, CMPAS, BARG, EM1, HVOR,       SSPA, SVOR, TORF, X1VOR
     I, PLOTOP, PSILOW, PSIUP
C                                                                       SETUPS12
      INTEGER  TABLE(7, 80) /560 * ' '/
C                                                                       SETUPS14
C                                                                       SETUPS15
      COMMON /BL1/       NTIME                                          SETUPS16
C                                                                       SETUPS17
      COMMON /INPTVB/   FTVB(64),   FPVB(64),   FPPRVB(64),  DIDRVB(64),SETUPS18
     A       XMVB(64),   DELVB,      XMUVB,      FOVB,    XMUAVB,       SETUPS19
     B       ATOVB,      ATCVB,       ATSVB,       ROVB,        RVB(64),SETUPS20
     C       MVB(64),    NVB                                            SETUPS21
C                                                                       SETUPS22
      COMMON /INPUTS/   NSBL,      NZ,        NOFF,      NGAM,      NSIG,   SETUPS23
     A         NCOI,      NCORD,     LOWER,     MSTOP,     MAXT,      MOTR,   SETUPS24
     B         NOTBL,     INDV,      ELSIG,     DXI,       REB,       RDBB,   SETUPS25
     C         FRZ,       ARR,       AMPLU,     FREQU,     ALPH1,     ALPH2,  SETUPS26
     D         HEAVE,     AROT,      FREQF,     PHIH,      NY,        RY1,    SETUPS27
     E         DRY,       Y(100),    TEST,      UPRIM,     XU(30),    YU(30), SETUPS28
     F         XL(30),    YL(30),    ER1,       ER2,       ER3,       BDBR,   SETUPS29
     G        RRDBR                                                        SETUPS30
     H,  CMPA, CMPAS, BARG, EM1, HVOR, NVOR, SSPA, SVOR, TORF, X1VOR
     I, PLOTOP, PSILOW, PSIUP
     J , NOUT
C                                                                       SETUPS31
C                                                                       SETUPS32
C                                                                       SETUPS33
      CALL WHERE(TABLE)                                                 SETUPS34
      CALL ZEROIN                                                       SETUPS35
C                                                                       SETUPS36
C                                                                       SETUPS37
      CALL SETUP('ALPH1     ',4, ALPH1                 )                SETUPS38
      CALL SETUP('ALPHA1    ',4, ALPH1                 )                SETUPS39
      CALL SETUP('ALPH2     ',4, ALPH2                 )                SETUPS40
      CALL SETUP('ALPHA2    ',4, ALPH2                 )                SETUPS41
      CALL SETUP('AMPLU     ',4, AMPLU                 )                SETUPS42
      CALL SETUP('ARR       ',4, ARR                   )                SETUPS43
      CALL SETUP('AROT      ',4, AROT                  )                SETUPS44
      CALL SETUP('ATOVB     ',4, ATOVB                 )                SETUPS45
      CALL SETUP('ATCVB     ',4, ATCVB                 )                SETUPS46
      CALL SETUP('ATSVB     ',4, ATSVB                 )                SETUPS47
      CALL SETUP('BARG      ',4,  BARG                 )
      CALL SETUP('BDBR      ',4,  BDBR                 )                SETUPS48
      CALL SETUP('CMPA      ',4,  CMPA                 )
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      CALL SETUP('CMPAS   ',4,  CMPAS               )
      CALL SETUP('DELVB   ',4,  DELVB               )                  SETUPS49
      CALL SETUP('DIDRVB  ',4,  DIDRVB, 64          )                  SETUPS50
      CALL SETUP('DRY     ',4, DRY                  )                  SETUPS51
      CALL SETUP('DXI     ',4, DXI                  )                  SETUPS52
      CALL SETUP('ELSIG   ',4, ELSIG                )                  SETUPS53
      CALL SETUP('EMI     ',4,  EMI                 )
      CALL SETUP('ER1     ',4,  ER1                 )                  SETUPS54
      CALL SETUP('ER2     ',4,  ER2                 )                  SETUPS55
      CALL SETUP('ER3     ',4,  ER3                 )                  SETUPS56
      CALL SETUP('FPVB    ',4,  FPVB,   64          )                  SETUPS57
      CALL SETUP('FPPRVB  ',4,  FPPRVB, 64          )                  SETUPS58
      CALL SETUP('FRZ     ',4, FRZ                  )                  SETUPS59
      CALL SETUP('FREQU   ',4, FREQU                )                  SETUPS60
      CALL SETUP('FREQF   ',4, FREQF                )                  SETUPS61
      CALL SETUP('FTVB    ',4,  FTVB,   64          )                  SETUPS62
      CALL SETUP('FOVB    ',4,  FOVB                )                  SETUPS63
      CALL SETUP('HEAVE   ',4, HEAVE                )                  SETUPS64
      CALL SETUP('HVOR    ',4,  HVOR                )
      CALL SETUP('INDV    ',4, INDV                 )                  SETUPS65
      CALL SETUP('LOWER   ',4, LOWER                )                  SETUPS66
      CALL SETUP('MAXT    ',4, MAXT                 )                  SETUPS67
      CALL SETUP('MOTR    ',4, MOTR                 )                  SETUPS68
      CALL SETUP('MSTOP   ',4, MSTOP                )                  SETUPS69
      CALL SETUP('MVB     ',4,  MVB ,   64          )                  SETUPS70
      CALL SETUP('NCOI    ',4, NCOI                 )                  SETUPS71
      CALL SETUP('NCORD   ',4, NCORD                )                  SETUPS72
      CALL SETUP('NGAM    ',4, NGAM                 )                  SETUPS73
      CALL SETUP('NOFF    ',4, NOFF                 )                  SETUPS74
      CALL SETUP('NOTBL   ',4, NOTBL                )                  SETUPS75
         CALL SETUP('NOUT    ', 4, NOUT)
      CALL SETUP('NSBL    ',4, NSBL                 )                  SETUPS76
      CALL SETUP('NSIG    ',4, NSIG                 )                  SETUPS77
      CALL SETUP('NVB     ', 4, NVB)                                   SETUPS78
      CALL SETUP('NVOR    ',4,  NVOR                )
      CALL SETUP('NY      ',4, NY                   )                  SETUPS79
      CALL SETUP('NZ      ',4, NZ                   )                  SETUPS80
      CALL SETUP('PHIH    ',4, PHIH                 )                  SETUPS81
      CALL SETUP('PLOTOP  ', 4, PLOTOP)
      CALL SETUP('PSILOW  ', 4,  PSILOW)
      CALL SETUP('PSIUP   ',  4, PSIUP)
      CALL SETUP('RVB     ', 4,  RVB, 64 )                             SETUPS82
      CALL SETUP('RDBB    ',4, RDBB                 )                  SETUPS83
      CALL SETUP('REB     ',4, REB                  )                  SETUPS84
      CALL SETUP('RRDBR   ',4,  RRDBR               )                  SETUPS85
      CALL SETUP('ROVB    ',4, ROVB                 )                  SETUPS86
      CALL SETUP('RYI     ',4, RYI                  )                  SETUPS87
      CALL SETUP('SSPA    ',4,  SSPA                )
      CALL SETUP('SVOR    ',4,  SVOR                )
      CALL SETUP('TEST    ',4, TEST                 )                  SETUPS88
      CALL SETUP('TORF    ',4,  TORF                )
      CALL SETUP('UPRIM   ',4, UPRIM                )                  SETUPS89
      CALL SETUP('XIVOR   ',4,  XIVOR               )
      CALL SETUP('XL      ',4,  XL,  30             )                  SETUPS90
      CALL SETUP('XMVB    ',4,  XMVB,   64          )                  SETUPS91
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      CALL SETUP('XMUVB    ',4,  XMUVB                )            SETUPS92
      CALL SFTUP('XMUAVB   ',4, XMJAVB                )            SETUPS93
      CALL SETUP('XU       ',4,  XU,  30              )            SETUPS94
      CALL SETUP('Y        ',4, Y,  100               )            SETUPS95
      CALL SETUP('YL       ',4,  YL,  30              )            SETUPS96
      CALL SETUP('YU       ',4,  YU,  30              )            SETUPS97
C                                                                  SETJPS98
C                                                                  SETUPS99
C                                                                  SETUP100
C                                                                  SFTUP101
C                                                                  SETUP102
C                                                                  SETUP103
C                                                                  SETUP104
         PSILOW= 1.E10
         PSIUP=      -1.E10
        PLOTOP = 1.
             NOUT= 0
      RETURN                                                       SETUP105
C                                                                  SETUP106
      END
```

```
      SUBROUTINE BLC(X,Y,MST.MEND,NY,RY,DRY,DXI,REB,UPRIM,FLAM,XFLAM,TESBLC       1
     1T,U,SCALE,UE,UC,V,XSEP,USEP,DISS,THETS,LOWER,LAMQ,MSEP,XC,USAV,SCABLC       2
     1LS,NITS,NTIME, NCTBL, XTEST, NZ, NOUT )
C                                                                       BLC       4
C   PROGRAM FOR ANALYZING LAMINAR AND TURBULENT BOUNDARY LAYERS         BLC       5
C   BY THE METHOD OF FINITE DIFFERENCES.  IF THE INTEGER LAMQ           BLC       6
C   IS GREATER THAN ZERO, THE BOUNDARY LAYER IS LAMINAR.                BLC       7
C                                                                       BLC       8
      COMMON /BL1/     NDUMMY, NDIMC , ISTD
      DIMENSION USAV(300,100),SCALS(300)                                BLC       9
      DIMENSION X(300),Y(100),UE(300,3),UC(100,3),V(100,2),XC(300)      BLC      10
      DIMENSION SD(100),SE(100),SF(100),VISC(100,2),GRAD(100)           BLC      11
      DIMENSION A(100),B(100),C(100),D(100),F(100)                      BLC      12
      DIMENSION ALPHA(100),BETA(100),GAMMA(100),DELTA(100)              BLC      13
      DIMENSION SCALE(300,2),VAR1(100),VAR2(100)                        BLC      14
      DIMENSION FLAM(10),XFLAM(10),YB1(100),YB2(100)                    BLC      15
      DIMENSION U(300,100,2)                                            BLC      16
      DIMENSION CAPG(100),CAPH(100),CAPJ(100),CAPK(100)                 BLC      17
      DOUBLE PRECISION AP(100),BP(100),CP(100),DP(100),FP(100),UP(100)  BLC      18
   10 FORMAT(1H1,41X,36H  ANALYSIS OF LAMINAR BOUNDARY LAYER///51X,12HTIBLC      19
     1ME STEP NOI3//51X,12HITERATION NOI3///4X,1HM,8X,1HX,13X,2HXC,12X,2BLC      20
     1HUE,10X,6H-DP/DX,9X,5HDELTA,9X,5HDISPL,9X,5HTHETA,9X,5HSHEAR/)    BLC      21
   11 FORMAT(1H1,41X,36HANALYSIS OF TURBULENT BOUNDARY LAYER///51X,12HTIBLC      22
     1ME STEP NOI3//51X,12HITERATION NOI3///4X,1HM,8X,1HX,13X,2HXC,12X,2BLC      23
     1HUE,10X,6H-DP/DX,9X,5HDELTA,9X,5HDISPL,9X,5HTHETA,9X,5HSHEAR,4X,  BLC      24
     3 'I'/)
   12 FORMAT(I5,8E14.4,I3)
   20 FORMAT(1H1,2X,3HM =I4//2X,3HX =E14.5//2X, 4HUE =E14.5,10X,17H-(1/RBLC      26
     1HO)(DP/DX) =E14.5,10X,5HREB =E14.5,10X,4HU* =E14.5///)            BLC      27
   24 FORMAT(2X,25HPHYSICAL            DELTA =E14.5,8X,12HDELTA STAR =E14.BLC      28
     15,8X,7HTHETA =E14.5//2X,25HTRANSFORMED         DELTA =E14.5,8X,12HDEBLC      29
     1LTA STAR =E14.5,8X,7HTHETA =E14.5///)                             BLC      30
   21 FORMAT(25X,1HY,19X,1HU,19X,1HV,16X,5HDU/DY,14X,6HNUE/NU/)         BLC      31
   22 FORMAT(10X,5E20.5)                                                BLC      32
   23 FORMAT(//30X,17HSEPARATION AT X =E13.5,6H, XC =E13.5)             BLC      33
   25 FORMAT(///40X,12HWALL SHEAR =E14.5//)                             BLC      34
   30 FORMAT(//50X,17HTRANSITION AT X =E14.5)                           BLC      35
   35 FORMAT(/20X,35HSCALE CHANGE - Y-MAX INCREASED FROME12.4,3H TOE12.4BLC      36
     1/)                                                                BLC      37
  810 FORMAT(10X,7HAT STEPI3,22H, THE WALL GRADIENT ISE12.4)            BLC      38
      BCON = 1.5/DXI                                                    BLC      51
      FCON = 1./(2.*DXI)                                                BLC      52
          IF( ISTD.NE. 1) GO TO 900
        DXI=1.E30
         BCON=0.
         FCON= 0.
  900     CONTINUE
      MOUT=6                                                            BLC      39
      MTRAN=-1                                                          BLC      40
      YSUB2=Y(2)                                                        BLC      41
      MST2 = MST - 2                                                    BLC      42
      MST1=MST-1                                                        BLC      43
           NOUT1= NOUT +1
         MST1MD=  MOD(  MST1, NOUT1)
        MAXIT=0
```

```
      GO TO (543,550),LOWER
  543 IF(LAMQ) 544,544,545
  544 WRITE(MOUT,11) NTIME,NITS
      GO TO 550
  545 WRITE(MOUT,10) NTIME,NITS
  550 CONTINUE
      YTR = SQRT(REB)
      UC(1,1) = 0.
      V(1,1) = 0.
      NV = NY - 2
      NVM1 = NV - 1
      NVP1 = NV + 1
      CALL YDIFF(NY,ALPHA,BETA,GAMMA,DELTA,SD,SE,SF,C2,C3,C4,Y)
      DO 41 N=1,NVP1
      VISC(N,1) = 1.
   41 VISC(N,2) = 1.
      DO 42 M=MST2,MST1
      L = MST1-M+2
      DO 50 N=1,NV
   50 GRAD(N+1) = SD(N+1)*UC(N+2,L)+SE(N+1)*UC(N+1,L)-SF(N+1)*UC(N,L)
      GRAD(1) = C2*UC(2,L)+C3*UC(3,L)+C4*UC(4,L)
      MM=M-1
      CALL PGRAD(MM,X,UE,DXI,PRESS,SA,SB,SC,SR,SS)
      DO 456 N=1,NY
  456 UC(N,1)=UC(N,L)
      CALL SETIT(LAMQ,M,NV,REB,X,Y,UC,PRESS,GRAD,DELT,DISP,THETA,VISC,MT
     1RAN)
   42 CONTINUE
      MEND1 = MEND - 1
      GRADS=GRAD(1)
      GRADSS=GRAD(1)
C
C  THE MAIN CALCULATION STARTS HERE.
C
      DO 99 M=MST1,MEND1
      ITER=0
      WALLG=0.
      MP1=M+1
      DELTP = DELT/YTR
      DISPT = DISP*YTR
      THETT = THETA*YTR
      SHEAR = GRAD(1)/YTR
            IF( MOD(M,  NOUT1).NE. MST1MD) GO TO 225
      GO TO (561,562),LOWER
  561 WRITE(MOUT,12) M,X(M),XC(M),UE(M,1),PRESS,DELTP,DISP,THETA, SHEAR
     1 , MAXIT
      GO TO 225
  562 WRITE(MOUT,20) M,X(M),UE(M,1),PRESS,REB,UPRIM
      WRITE(MOUT,24) DELTP,DISP,THETA,DELT,DISPT,THETT
      WRITE(MOUT,21)
      WRITE(MOUT,22) (Y(N),UC(N,2),V(N,1),GRAD(N),VISC(N,1),N=1,NVP1)
      WRITE(MOUT,25) SHEAR
  225 IF(GRADSS-GRADS-1.E-6) 229,229,408
  408 XSX=X(M-2)+(X(M-1)-X(M-2))*GRADSS/(GRADSS-GRADS)
      IF(XSX-X(M)) 409,409,229
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 409  WFS=(XSX-X(M-1))/(X(M)-X(M-1))                                   BLC   99
      GO TO 224                                                        BLC  100
 229       IF ( GRAD(1) )  227, 227, 273
 273    IF (DISP .GT. 0. .AND. THETA .GT. 0.) GO TO 223
 283     CONTINUE
        XSEP= XC(M-1)
         USEP=UE(M-1,1)
          XBL=X(M-1)
           WRITE(MOUT,23) XBL, XSEP
          RETURN
 227  WFS=GRADS/(GRADS-GRAD(1))                                        BLC  102
 224  WFS1=1.-WFS                                                      BLC  103
      XSEP=WFS1*XC(M-1)+WFS*XC(M)                                      BLC  104
      XBL=WFS1*X(M-1)+WFS*X(M)                                         BLC  105
      USEP=WFS1*UE(M-1,1)+WFS*UE(M,1)                                  BLC  106
      WFP=(XBL-X(M-2))/(X(M-1)-X(M-2))                                 BLC  107
      WFP1=1.-WFP                                                      BLC  108
      DISS=DISSS*WFP1+DISS*WFP                                         BLC  109
      THETS=THETSS*WFP1+THETS*WFP                                      BLC  110
      WRITE(MOUT,23) XBL,XSEP                                          BLC  111
      IF(LAMQ.EQ.0.AND.M.LT.MTRAN+5) LAMQ=1                            BLC  112
      GO TO 222                                                        BLC  113
 223  CONTINUE                                                         BLC  114
         IF(  NOTBL .EQ. 2 .AND. NITS  .GT. 1 .AND. M.GT. NZ .AND.
     1  XC(M) .GT. XTEST) GO TO 283
      IF(LAMQ) 801,801,802                                             BLC  115
 802        IF( NOTBL .EQ. 2) GO TO 801
      CALL TRANS(UPRIM,PRESS,THETA,REB,UC,NY,FLAM,XFLAM,LAMQ)          BLC  116
      IF(LAMQ) 805,805,801                                             BLC  117
 805  WRITE(MOUT,30) X(M)                                              BLC  118
      MTRAN = M+1                                                      BLC  119
 801  CONTINUE                                                         BLC  120
      IF(Y(NV)-DELT) 620,641,641                                       BLC  121
 620  RY=RY+DRY                                                        BLC  122
C                                                                      BLC  123
C  RESCALING CALCULATION STARTS HERE.                                  BLC  124
C                                                                      BLC  125
      DO 632  N=1,NY                                                   BLC  126
      YB1(N)  =  Y(N)                                                  BLC  127
      VAR1(N)  =  UC(N,2)                                              BLC  128
 632  VAR2(N)  =  UC(N,3)                                              BLC  129
      CALL  YSET(RY,YSUB2,NY,Y)                                        BLC  130
      WRITE(MOUT,35)  YB1(NY),Y(NY)                                    BLC  131
      DO 633  N=2,NVP1                                                 BLC  132
      YIN  =  Y(N)                                                     BLC  133
      CALL  TERP(YIN,YB1,VAR1,NY,UPAS1)                                BLC  134
      UC(N,2)  =  UPAS1                                                BLC  135
      CALL  TERP(YIN,YB1,VAR2,NY,UPAS2)                                BLC  136
 633  UC(N,3)  =  UPAS2                                                BLC  137
      CALL  YDIFF(NY,ALPHA,BETA,GAMMA,DELTA,SD,SE,SF,C2,C3,C4,Y)       BLC  138
      IF(LAMQ)  700,700,701                                            BLC  139
 700  DO 635  N=2,NVP1                                                 BLC  140
      VAR1(N)  =  VISC(N,1)                                            BLC  141
 635  VAR2(N)  =  VISC(N,2)                                            BLC  142
      DO 636  N=2,NVP1                                                 BLC  143
```

```
      YIN = Y(N)
      CALL TERP(YIN,YB1,VAR1,NVP1,UPAS1)
      VISC(N,1) = UPAS1
      CALL TERP(YIN,YB1,VAR2,NVP1,UPAS2)
  636 VISC(N,2) = UPAS2
  701 DO 637 N=2,NVP1
      VAR1(N) = V(N,1)
  637 VAR2(N) = V(N,2)
      DO 638 N=2,NVP1
      YIN = Y(N)
      CALL TERP(YIN,YB1,VAR1,NVP1,UPAS1)
      V(N,1) = UPAS1
      CALL TERP(YIN,YB1,VAR2,NVP1,UPAS2)
  638 V(N,2) = UPAS2
  641 CONTINUE
C
C  RESCALING CALCULATION ENDS HERE.
C
      CALL PGRAD(M,X,UE,DXI,PRESS,SA,SB,SC,SR,SS)
C
C   RECURSION RELATIONS ARE SET UP HERE.
C
        IF (ISTD.EQ. 1) GO TO 820
      IF(SCALE(M+1,1)-1.) 522,522,521
  521 IF(SCALE(M+1,2)-1.) 522,522,523
  522 LACKU=1
      FACU1=UE(M+1,2)/UE(M+1,1)
      FACU2=UE(M+1,3)/UE(M+1,1)
      GO TO 820
  523 LACKU=2
      DO 610 NN=1,NY
      VAR1(NN) = U(M+1,NN,1)
  610 VAR2(NN) = U(M+1,NN,2)
      CALL YSET(SCALE(M+1,1),YSUB2,NY,YB1)
      CALL YSET(SCALE(M+1,2),YSUB2,NY,YB2)
  820 DO 88 N=2,NV
      CALL CAPS(ITER,N,CAPG,CAPH,CAPJ,CAPK,SR,SS,SD,SE,SF,VISC,V,UC)
      A(N)=-SF(N)*CAPG(N)-DELTA(N)*CAPH(N)+SF(N)*CAPJ(N)
      B(N)=BCON+SA*CAPK(N)+SF(N)*CAPG(N)-GAMMA(N)*CAPH(N)-SE(N)*CAPJ(N)
      C(N)=SD(N)*CAPG(N)-BETA(N)*CAPH(N)-SD(N)*CAPJ(N)
      D(N) = -ALPHA(N)*CAPH(N)
        IF(ISTD .EQ. 1) GO TO 576
      GO TO (574,575),LACKU
  574 UPAS1=FACU1*UC(N,1)
      UPAS2=FACU2*UC(N,1)
      GO TO 576
  575 YIN = Y(N)
      CALL TERP(YIN,YB1,VAR1,NY,UPAS1)
      CALL TERP(YIN,YB2,VAR2,NY,UPAS2)
  576 F(N) = PRESS+FCON*(4.*UPAS1-UPAS2)+CAPK(N)*(SB*UC(N,2)-SC*UC(N,3))
   88 CONTINUE
C
C    SOLUTION FOR VELOCITY PROFILE STARTS HERE.
C
      DO 89 N=2,NV
```

```
      AP(N) = A(N)
      BP(N) = B(N)
      CP(N) = C(N)
      DP(N) = D(N)
   89 FP(N) = F(N)
      DO 77 N=2,NVM1
      CP(N) = CP(N)/BP(N)
      DP(N) = DP(N)/BP(N)
      FP(N) = FP(N)/BP(N)
      BP(N+1) = BP(N+1) - CP(N)*AP(N+1)
      CP(N+1) = CP(N+1) - DP(N)*AP(N+1)
   77 FP(N+1) = FP(N+1) - FP(N)*AP(N+1)
      UP(NY) = UE(M+1,1)
      UP(NVP1) = UP(NY)
      UP(NV) = (FP(NV)-UP(NY)*(DP(NV) + CP(NV)))/BP(NV)
      DO 66 N=3,NV
      NN=NV+2-N
   66 UP(NN) = FP(NN) - DP(NN)*UP(NN+2) - CP(NN)*UP(NN+1)
      DO 65 N=2,NY
   65 UC(N,1) = UP(N)
      IF(ITER) 843,841,843
  841 DO 842 N=2,NVP1
      V(N,2) = V(N,1)
  842 VISC(N,2)=VISC(N,1)
      DISSS=DISS
      DISS=DISP
      THETSS=THETS
      THETS=THETA
      GRADSS=GRADS
      GRADS=GRAD(1)
  843 DO 55 N=2,NVP1
   55 V(N,1) = V(N-1,1)-.5*(Y(N)-Y(N-1))*(SA*(UC(N,1)+UC(N-1,1))-SB*(UC(
     1N,2)+UC(N-1,2))+SC*(UC(N,3)+UC(N-1,3)))
      DO 56 N=1,NV
   56 GRAD(N+1) = SD(N+1)*UC(N+2,1)+SE(N+1)*UC(N+1,1)-SF(N+1)*UC(N,1)
      GRAD(1) = C2*UC(2,1)+C3*UC(3,1)+C4*UC(4,1)
      CALL SETIT(LAMQ,MP1,NV,REB,X,Y,UC,PRESS,GRAD,DELT,DISP,THETA,VISC,
     1MTRAN)
      ITER=ITER+1
      GO TO (830,809),LOWER
  809 WRITE(MOUT,810) ITER,GRAD(1)
  830 IF(ITER-9) 811,811,812
  811 EPW=ABS(GRAD(1)-WALLG)
      IF(WALLG-1.) 120,120,119
  119 EPW=EPW/WALLG
  120 IF(EPW-TEST) 812,814,814
  814 WALLG=GRAD(1)
      GO TO 820
  812 DO 44 N=1,NY
      UC(N,3) = UC(N,2)
      UC(N,2) = UC(N,1)
   44        CONTINUE
               MAXIT=ITER
         IF(ISTD .EQ. 1) GO TO  99
             DO 48  N= 1,NY
```

```
48    USAV(M+1,N)=UC(N,1)
      SCALS(M+1)=RY                                                   BLC  249
99    CONTINUE                                                        BLC  250
      XSEP=1.1                                                        BLC  251
      USEP=UE(MX,1)                                                   BLC  252
222   CONTINUE                                                        BLC  253
      RETURN                                                          BLC  254
      END
```

```
        SUBROUTINE PLOTSB( PLOTOP   ,   P , L   )
         REAL * 8  ORD(6)
           DIMENSION  P(200,7), TIT1(56)                    , NF(5,4)
     1 , NFP(6)
       DATA    N1 , N2 , NO   , N42
     1 / 1 ,  2   , 0   , 42  /
        DATA  ORD/       ' THETA-P' ,'  TORS   ', ' FLAP-H ', ' BEND-H ',
     1  '   CL    ',     '    CM-A ' /
        IF(PLOTOP .EQ.  0. ) RETURN
         IF(               L .LT. 2) RETURN
         IF ( PLOTOP .EQ. 2.) GO TO 2
        PLOTOP = 2.
           CALL IDFRMV ( 'CRIMI -PETE ', '30',   '5100'   )
2        CONTINUE
3       NL=1
          DO    1    J =  1,  6
       CALL  EZPLOT(9.   ,  N1     , N1,  P , P(1,J+1), L  , -N1 ,   N2
     1  ,  N42    ,  1 ,' ' , 12   , ' PSI-DEGREES'   , 8 ,  ORD(  J)
     2  ,  N1   ,    N1 ,  XL   ,  XU    ,  N1   ,  YL   ,  YU ,N1   , NO , NL)
   1      CONTINUE
         NFP(1)= -1
        NFP(2)= 66
         NFP(3)= 50
       NFP(4)= 50
         NFP(5)= 680
       CALL  EZPLOT(9.   ,  N1     , N1,  P , P(1,2   ), L  , -N1 ,   N2
     1  ,  N42    ,  1 ,' ' , 12   , ' PSI-DEGREES'   , 8 ,  ORD(  1)
     2  , NFP   ,    N1 ,  XL   ,  XU    ,  N1   ,  YL   ,  YU ,N1, NO, N1)
          NFP(1)= -2
          NFP(2)= 66
            NFP(4)= 350
            NFP(5)= 380
       CALL  EZPLOT(9.   ,  N1     , N1,  P , P(1,6   ), L  , -N1 ,   N2
     1  ,  N42    ,  1 ,' ' , 12   , '                '   , 8 ,  ORD(  5)
     2  , NFP   ,    N1 ,  XL   ,  XU    ,  N1   ,  YL   ,  YU ,N1, NO, N1)
         NFP(2)= 50
       NFP(4)= 690
         NFP(5)= 40
       CALL  EZPLOT(9.   ,  N1     , N1,  P , P(1,7   ), L  , -N1 ,   N2
     1  ,  N42    ,  1 ,' ' , 12   , '                '   , 8 ,  ORD(  6)
     2  , NFP   ,    N1 ,  XL   ,  XU    ,  N1   ,  YL   ,  YU ,N1, NO, N1)
         NFP(1)=-1
            NFP(2)= 50
            NFP(3)=50
           NFP(4)=50
            NFP(5)= 680
       CALL  EZPLOT(9.   ,  N1     , N1,  P , P(1,3   ), L  , -N1 ,   N2
     1  ,  N42    ,  1 ,' ' , 12   , ' PSI-DEGREES'   , 8 ,  ORD(  2)
     2  , NFP   ,    N1 ,  XL   ,  XU    ,  N1   ,  YL   ,  YU ,N1, NO, N1)
            NFP(1)=-2
         NFP(2)= 66
            NFP(4)= 350
        NFP(5)= 380
       CALL  EZPLOT(9.   ,  N1     , N1,  P , P(1,4   ), L  , -N1 ,   N2
     1  ,  N42    ,  1 ,' ' , 12   , '                '   , 8 ,  ORD(  3)
```

```
2   , NFP   ,    N1 ,   XL   ,   XU    ,   N1   ,   YL   ,   YU ,N1, NO, N1)
      NFP(2)= 50
       NFP(4)= 690
      NFP(5)= 40
    CALL  EZPLOT(9.   ,   N1     , N1,   P , P(1,5  ), L , -N1 ,  N2
1   ,  N42    ,  1 ,' ' , 12  , '                  ' , 8 ,  ORD(  4)
2   , NFP   ,    N1 ,   XL   ,   XU    ,   N1   ,   YL   ,   YU ,N1, NO, N1)
         RETURN
            END
```

```
      SUBROUTINE STAG(MX,NY,MSTOP,MST,DXI,RY,DRY,X,Y,UE,UC,V,USAV,SCALS,
     1ISEP)
C   PROGRAM FOR CALCULATING THE BOUNDARY LAYER PROFILE NEAR
C   THE STAGNATION POINT
C
      COMMON /BL1/       NTIME, NDIMC , ISTD
      DIMENSION USAV(300,100),SCALS(300)
      DIMENSION PHIZ(24),PHIP(24),ETAP(24)
      DIMENSION X(300),Y(100),UE(300,3),UC(100,3),V(100,2)
      DIMENSION EF(100),EFP(100)
      DATA ETAP /0.,.2,.4,.6,.8,1.,1.2,1.4,1.6,1.8,2.,2.2,2.4,2.6,2.8,3.
     1,3.2,3.4,3.6,3.8,4.,4.2,4.4,4.6/
      DATA PHIZ /0.,.0233,.0881,.1867,.3124,.4592,.622,.7967,.9793,1.168
     19,1.362,1.5578,1.7553,1.9538,2.153,2.3526,2.5523,2.7522,2.9521,3.1
     1521,3.3521,3.5521,3.7521,3.9521/
      DATA PHIP /0.,.2266,.4145,.5663,.6859,.7779,.8467,.8968,.9323,.956
     18,.9732,.9839,.9905,.9946,.997,.9984,.9992,.9996,.9998,.9999,1.,1.
     1,1.,1./
      BAG=.08
      IF(ISEP) 10,10,5
 5    BAG=.5
 10   EF(1) = 0.
      EFP(1) = 0.
      DO 20 M=1,MX
      IF(UE(M,1)) 20,20,19
 19   MSP = M
      GO TO 21
 20   CONTINUE
 21   ASTAG = (UE(MSP+2,1)-UE(MSP+1,1))/(X(MSP+2)-X(MSP+1))
      IF(ASTAG) 22,22,23
 22   ASTAG=(UE(MSP,1)-UE(MSP-1,1))/(X(MSP)-X(MSP-1))
 23   SQAS = SQRT(ASTAG)
      DELT = 2.6/SQAS
 309  IF(DELT-Y(NY-3)) 311,310,310
 310  RY=RY+DRY
      CALL YSET(RY,Y(2),NY,Y)
      GO TO 309
 311  CONTINUE
      DO 80 N=2,NY
      YET = Y(N)*SQAS
      DO 33 NN=1,24
      IF(YET-ETAP(NN)) 408,408,33
 408  MARK = NN
      GO TO 410
 33   CONTINUE
      EF(N) = YET-.6479
      EFP(N) = 1.
      GO TO 80
 410  FRACT = (YET-ETAP(MARK-1))/(ETAP(MARK)-ETAP(MARK-1))
      FRAC1 = 1.-FRACT
      EF(N) = PHIZ(MARK-1)*FRAC1+PHIZ(MARK)*FRACT
      EFP(N) = PHIP(MARK-1)*FRAC1+PHIP(MARK)*FRACT
 80   CONTINUE
      M1 = MSP-MSTOP
      M2 = MSP+MSTOP
```

```
      M=M1-1                                                            STAG  55
 50   M=M+1                                                             STAG  56
      MST=M+1                                                           STAG  57
      SCALS(M)=RY                                                       STAG  58
      DO 71 N=1,NY                                                      STAG  59
      UC(N,3) = UC(N,2)                                                 STAG  60
      UC(N,2) = UE(M,1)*EFP(N)                                          STAG  61
      V(N,2) = V(N,1)                                                   STAG  62
      V(N,1) = -SQAS*EF(N)                                              STAG  63
        IF(ISTD .EQ. 1) GO TO  71
      USAV(M,N)=UC(N,2)                                                 STAG  64
 71   CONTINUE                                                          STAG  65
      IF(M-M2) 50,55,55                                                 STAG  66
 55   IF(UF(M,1)-BAG) 50,50,81                                          STAG  67
 81   CONTINUE                                                          STAG  68
      RETURN                                                            STAG  69
      END                                                               STAG  70
```

```
      SUBROUTINE ATTPR(PREC,XSIG,NSIG,ASZ,AS,AR,CMAT,RMAT,NGAM,NF,ACAP,TATTPR   1
     1HICK,RDBB,GAMAW,UINF,UDOT,DXI,BCAP)                               ATTPR   2
      DIMENSION XSIG(100),ASZ(30),AS(30,30),AR(30),BCAP(100,3)          ATTPR   3
      DIMENSION ACAP(30,3),THICK(24),GAMAW(1000)                        ATTPR   4
      DOUBLE PRECISION CMAT(60,60   ),RMAT(130)                         ATTPR   5
      PI=3.14159                                                        ATTPR   6
      NGP1=NGAM+1                                                       ATTPR   7
      DO 50 M=1,NGP1                                                    ATTPR   8
      CMAT(M,1)=ASZ(M)                                                  ATTPR   9
      RMAT(M)=AR(M)                                                     ATTPR  10
      DO 25 N=1,NGAM                                                    ATTPR  11
   25 CMAT(M,N+1)=AS(M,N)                                               ATTPR  12
   50 CONTINUE                                                          ATTPR  13
      CALL ALSOL(NGP1,CMAT,RMAT)                                        ATTPR  14
      DO 75 M=1,NGP1                                                    ATTPR  15
   75 ACAP(M,1)=RMAT(M)                                                 ATTPR  16
      GAMAW(1)=GAM1(ACAP,DXI,PI)                                        ATTPR  17
      SAVE=XSIG(NSIG+1)                                                 ATTPR  18
      XSIG(NSIG+1)=2.                                                   ATTPR  19
      CALL CPC(0,NGAM,NF,XSIG,NSIG,XSIG,NSIG,XSIG,NSIG,ACAP,BCAP,THICK,RATTPR  20
     1DBB,GAMAW,UINF,UDOT,1.,SAVE,DXI,PREC)                             ATTPR  21
      XSIG(NSIG+1)=SAVE                                                 ATTPR  22
      RETURN                                                            ATTPR  23
      END                                                               ATTPR  24
```

```
      SUBROUTINE UNPOP(NGAM,AR,ALAM,AFACT,RMAT,CMAT,XGAM,AS,ACAP,MX,NZ,N
     1F,XSIG,BCAP,THICK,RDBB,UINF,XC,UE)
      DIMENSION AR(30),ALAM(30),XGAM(30),AS(30,30),ACAP(30,3),XSIG(100),
     1BCAP(100,3),THICK(24),XC(300),UE(300,3)
      DOUBLE PRECISION RMAT(130),CMAT(60,60   )
      NGP1=NGAM+1
      DO 5 M=1,NGP1
      SUB=AR(M)-ALAM(M)*AFACT/3.
      RMAT(M)=SUB
      CMAT(M,1)=1.
      CMAT(M,2)=XGAM(M)
      DO 5 N=2,NGAM
    5 CMAT(M,N+1)=AS(M,N)
      CALL ALSOL(NGP1,CMAT,RMAT)
      DO 10 N=1,NGP1
   10 ACAP(N,1)=RMAT(N)
      DO 15 M=1,MX
      SIGN=1.
      IF(M-NZ) 12,14,14
   12 SIGN=-SIGN
   14 CALL QECAL(0,NGAM,NGAM,NF,XSIG,ACAP,BCAP,THICK,RDBB,0.,UINF,XC(M),
     1UE(M,1),SIGN)
   15 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE ALSOL(NT, C, R)
      DOUBLE PRECISION C    NDIMC,NDIMC), R(130)                             ALSOL
      DOUBLE PRECISION CMAX,SAVE,SUM
      COMMON /BL1/      NTIME, NDIMC
      NT1 = NT-1
      DO 99 J=1,NT1
      CMAX = C(NT,J)
      L=NT
      DO 10 I=J,NT1
      IF (DABS(CMAX)-DABS(C(I,J))) 5,10,10
5     CMAX = C(I,J)
      L=I
10    CONTINUE
      DO 15 JJ=J,NT
      SAVE = C(L,JJ)
      C(L,JJ) = C(J,JJ)
15    C(J,JJ) = SAVE/CMAX
      SAVE = R(L)
      R(L) = R(J)
      R(J) = SAVE/CMAX
      JP1 = J+1
      DO 25 I=JP1,NT
      DO 20 JJ=JP1,NT
20    C(I,JJ) = C(I,JJ) - C(I,J)*C(J,JJ)
25    R(I) = R(I) - R(J)*C(I,J)
99    CONTINUE
      R(NT) = R(NT)/C(NT,NT)
      DO 150 K=1,NT1
      I=NT-K
      IP1 = I+1
      SUM = 0.
      DO 125 J=IP1,NT
125   SUM = SUM + R(J)*C(I,J)
150   R(I) = R(I) - SUM
      RETURN
      END
```

```
      SUBROUTINE CPC(ISEP,NGAM,NF,XSIG,NSIG,XSIGA,NSIGA,XSIGB,NSIGB,ACAP
     1,BCAP,THICK,RDBB,GAMAW,UINF,UDOT,SIGN,XC,DXI,CP)
      DIMENSION XSIG(100),XSIGA(100),XSIGB(100),ACAP(30,3),BCAP(100,3)
      DIMENSION GAMAW(1000),THICK(24)
      THETA=ARCT(XC)
      RECIP=1./(UINF*UINF)
      SUM=0.
      ANGLE=0.
      DO 5 N=1,NF
      ANGLE=ANGLE+THETA
    5 SUM=SUM+THICK(N)*COS(ANGLE)
      CP=UDOT*RECIP*(THICK(1)+2.*(1.-XC)*SUM)
      CALL QECAL(ISEP,NGAM,NSIG,NF,XSIG,ACAP,BCAP,THICK,RDBB,GAMAW(1),UI
     1NF,XC,U,SIGN)
      CP=CP+2.*(SIGN*U/UINF-1.)
      CALL EGAMI(1,NGAM,ACAP,BCAP(1,1),XSIG(1),XSIG(NSIG+1),GAMAW(1),XC,
     1VAL1)
      CALL EGAMI(2,NGAM,ACAP,BCAP(1,2),XSIGA(1),XSIGA(NSIGA+1),GAMAW(2),
     1XC,VAL2)
      CALL EGAMI(3,NGAM,ACAP,BCAP(1,3),XSIGB(1),XSIGB(NSIGB+1),GAMAW(3),
     1XC,VAL3)
      CP=CP+SIGN*RECIP*(1.5*VAL1-2.*VAL2+.5*VAL3)/DXI
      IF(ISEP) 20,20,10
   10 CALL ESIGI(1,NSIG,XSIG,BCAP,XC,VAL1)
      CALL ESIGI(2,NSIGA,XSIGA,BCAP,XC,VAL2)
      CALL ESIGI(3,NSIGB,XSIGB,BCAP,XC,VAL3)
      CP=CP+RECIP*(1.5*VAL1-2.*VAL2+.5*VAL3)/DXI
   20 CP=-CP
      RETURN
      END
```

```
      SUBROUTINE CLCM(NCOI,ISEP,NGAM,XSIG,NSIG,XSIGA,NSIGA,XSIGB,NSIGB,ACLCM
     1CAP,BCAP,THICK,RDBB,GAMAW,UINF,UDOT,DXI,AROT,CMPA)               CLCM
      COMMON /CLCMBL/   CLVB, CMVB,   CMPAVB                           MAIN
      DIMENSION ARGL(21),ARGM(21)                                      CLCM
      DIMENSION GAMAW(1000),THICK(24)                                  CLCM
      DIMENSION XSIG(100),XSIGA(100),XSIGB(100),ACAP(30,3),BCAP(100,3) CLCM
    4 FORMAT(//40X,4HCL =E13.5/40X,4HCM =E13.5,17H (ABOUT MIDCHORD))/40X,CLCM
     14HCM =E13.5,24H (ABOUT PITCH AXIS - A =F7.4,1H))                 CLCM
      MOUT=6                                                           CLCM
      SAVE=THICK(1)                                                    CLCM
      THICK(1)=0.                                                      CLCM
      DT=3.14159/FLOAT(NCOI)                                           CLCM
      CL=0.                                                            CLCM
      CM=0.                                                            CLCM
      XI=-1.                                                           CLCM
      ANGLE=0.                                                         CLCM
      FLI=0.                                                           CLCM
      FMI=0.                                                           CLCM
      IF(ISEP) 5,5,7                                                   CLCM
    7 XATT=XSIG(NSIG+1)                                                CLCM
      IF(XATT-.95) 8,5,5                                               CLCM
    8 XAQ=XATT+5.E-4                                                   CLCM
      XAP=XAQ+.025                                                     CLCM
      C1=-.5*(1.+XATT)                                                 CLCM
      C2=C1+XATT                                                       CLCM
      C1P=.5*(1.-XAP)                                                  CLCM
      C2P=C1P+XAP                                                      CLCM
      DO 10 I=1,NCOI                                                   CLCM
      ANGLE=ANGLE+DT                                                   CLCM
      XIP1=C1*COS(ANGLE)+C2                                            CLCM
      CALL CPC(ISEP,NGAM,1,XSIG,NSIG,XSIGA,NSIGA,XSIGB,NSIGB,ACAP,BCAP,TCLCM
     1HICK,RDBB,GAMAW,UINF,UDOT,1.0,XIP1,DXI,CPU)                      CLCM
      CALL CPC(ISEP,NGAM,1,XSIG,NSIG,XSIGA,NSIGA,XSIGB,NSIGB,ACAP,BCAP,TCLCM
     1HICK,RDBB,GAMAW,UINF,UDOT,-1.,XIP1,DXI,CPL)                      CLCM
      FLIP1=CPL-CPU                                                    CLCM
      FMIP1=XIP1*FLIP1                                                 CLCM
      CL=CL+(XIP1-XI)*(FLIP1+FLI)                                      CLCM
      CM=CM+(XIP1-XI)*(FMIP1+FMI)                                      CLCM
      XI=XIP1                                                          CLCM
      FLI=FLIP1                                                        CLCM
   10 FMI=FMIP1                                                        CLCM
      XI=1.                                                            CLCM
      FLI=0.                                                           CLCM
      FMI=0.                                                           CLCM
      ANGLE=0.                                                         CLCM
      DO 15 I=1,NCOI                                                   CLCM
      ANGLE=ANGLE+DT                                                   CLCM
      XIP1=C1P*COS(ANGLE)+C2P                                          CLCM
      CALL CPC(ISEP,NGAM,1,XSIG,NSIG,XSIGA,NSIGA,XSIGB,NSIGB,ACAP,BCAP,TCLCM
     1HICK,RDBB,GAMAW,UINF,UDOT,1.0,XIP1,DXI,CPU)                      CLCM
      CALL CPC(ISEP,NGAM,1,XSIG,NSIG,XSIGA,NSIGA,XSIGB,NSIGB,ACAP,BCAP,TCLCM
     1HICK,RDBB,GAMAW,UINF,UDOT,-1.,XIP1,DXI,CPL)                      CLCM
      FLIP1=CPL-CPU                                                    CLCM
      FMIP1=XIP1*FLIP1                                                 CLCM
      CL=CL-(XIP1-XI)*(FLIP1+FLI)                                      CLCM
```

```
      CM=CM-(XIP1-XI)*(FMIP1+FMI)                                       CLCM  55
      XI=XIP1                                                           CLCM  56
      FLI=FLIP1                                                         CLCM  57
   15 FMI=FMIP1                                                         CLCM  58
      XIP1=XAQ                                                          CLCM  59
      DO 16 I=1,21                                                      CLCM  60
      CALL CPC(ISEP,NGAM,1,XSIG,NSIG,XSIGA,NSIGA,XSIGB,NSIGB,ACAP,BCAP,TCLCM  61
     1HICK,RDBB,GAMAW,UINF,UDOT,1.0,XIP1,DXI,CPU)                       CLCM  62
      CALL CPC(ISEP,NGAM,1,XSIG,NSIG,XSIGA,NSIGA,XSIGB,NSIGB,ACAP,BCAP,TCLCM  63
     1HICK,RDBB,GAMAW,UINF,UDOT,-1.,XIP1,DXI,CPL)                       CLCM  64
      ARGL(I)=CPL-CPU                                                   CLCM  65
      ARGM(I)=XIP1*ARGL(I)                                              CLCM  66
   16 XIP1=XIP1+.00125                                                  CLCM  67
      SUML=0.                                                           CLCM  68
      SUMM=0.                                                           CLCM  69
      DO 17 I=1,19,2                                                    CLCM  70
      SUML=SUML+2.*ARGL(I)+4.*ARGL(I+1)                                 CLCM  71
   17 SUMM=SUMM+2.*ARGM(I)+4.*ARGM(I+1)                                 CLCM  72
      CL=CL+0.833333E-3*(SUML+ARGL(21)-ARGL(1))                         CLCM  73
      CM=CM+0.833333E-3*(SUMM+ARGM(21)-ARGM(1))                         CLCM  74
      BCON=16.*BCAP(1,1)*SQRT(5.E-4*(XATT-XSIG(1)))/UINF                CLCM  75
      CL=CL+BCON                                                        CLCM  76
      CM=CM+XATT*BCON                                                   CLCM  77
      GO TO 100                                                         CLCM  78
    5 DO 99 I=1,NCOI                                                    CLCM  79
      ANGLE=ANGLE+DT                                                    CLCM  80
      XIP1=-COS(ANGLE)                                                  CLCM  81
      CALL CPC(ISEP,NGAM,1,XSIG,NSIG,XSIGA,NSIGA,XSIGB,NSIGB,ACAP,BCAP,TCLCM  82
     1HICK,RDBB,GAMAW,UINF,UDOT,1.0,XIP1,DXI,CPU)                       CLCM  83
      CALL CPC(ISEP,NGAM,1,XSIG,NSIG,XSIGA,NSIGA,XSIGB,NSIGB,ACAP,BCAP,TCLCM  84
     1HICK,RDBB,GAMAW,UINF,UDOT,-1.,XIP1,DXI,CPL)                       CLCM  85
      FLIP1=CPL-CPU                                                     CLCM  86
      FMIP1=XIP1*FLIP1                                                  CLCM  87
      CL=CL+(XIP1-XI)*(FLIP1+FLI)                                       CLCM  88
      CM=CM+(XIP1-XI)*(FMIP1+FMI)                                       CLCM  89
      XI=XIP1                                                           CLCM  90
      FLI=FLIP1                                                         CLCM  91
   99 FMI=FMIP1                                                         CLCM  92
  100 CL=.25*CL                                                         CLCM  93
      CM=-.125*CM                                                       CLCM  94
      CMPA=CM+AROT*CL*.5                                                CLCM  95
      WRITE(MOUT,4) CL,CM,CMPA,AROT                                     CLCM  96
      THICK(1)=SAVE                                                     CLCM  97
      CLVB = CL                                                         MAIN
      CMVB = CM                                                         MAIN
      CMPAVB = CMPA                                                     MAIN
      RETURN                                                            CLCM  98
      END
```

```
      SUBROUTINE QECAL(ISEP,NGAM,NSIG,NF,XSIG,ACAP,BCAP,THICK,RDBB,GAMMA
     1,UINF,XC,U,SIGN)
      DIMENSION ACAP(30,3),BCAP(100,3),XSIG(100)
      DIMENSION THICK(24)
      EPS=1.E-6
      CORR=.707107/(1.-.63662*SQRT(RDBB)+.25*RDBB)
      SINT=SQRT(1.-XC*XC)
      THETA=ARCT(XC)
      COUNT=0.
      SUM=0.
      SINT2=SIN(.5*THETA)
      COST2=COS(.5*THETA)
      IF(SINT -EPS) 4,6,6
4     FACT=THETA*.5
      GO TO 8
6     FACT=(1.-XC)/SINT
8     DO 10 N=1,NF
      COUNT=COUNT+1.
      ANGLE=THETA*COUNT
      SUM=SUM+THICK(N)*(COUNT*FACT*SIN(ANGLE)-COS(ANGLE))
10    CONTINUE
      U=2.*SIGN*UINF*COST2*SUM+ACAP(1,1)*SINT2+.25*COST2*(1.+XC)*(3.*XC-
     11.)*GAMMA
      SUM=0.
      ANGLE=0.
      DO 12 N=1,NGAM
      ANGLE=ANGLE+THETA
12    SUM=SUM+ACAP(N+1,1)*SIN(ANGLE)
      U=U+COST2*SUM
      IF(ISEP) 25,99,25
25    SUM=0.
      XSEP=XSIG(1)
      XATT=XSIG(NSIG+1)
      DO 40 N=2,NSIG
40    SUM=SUM+BCAP(N,1)*FB(XSIG(N-1),XSIG(N),XSIG(N+1),XC)
      IF(XC-XATT-EPS) 45,45,46
46    FACT=(1.-XATT)**(-1.5)*SQRT((XATT-XSEP)*(1.-XC)/(XC-XATT))*(1.+3.*
     1XATT-4.*XC)-SIGN*(1.-SQRT((XSEP-XC)/(XATT-XC)))
      GO TO 55
45    IF(XSEP-XC) 49,49,48
48    FACT=-SIGN*(1.-SQRT((XSEP-XC)/(XATT-XC)))
      GO TO 55
49    FACT=-SIGN
55    U=U+COST2*(BCAP(1,1)*FACT+SIGN*SUM)
99    U=(SIGN*UINF*SQRT(1.+XC)+   CORR*U)/SQRT(1.+XC+.5*RDBB)
      RETURN
      END
```

```
      SUBROUTINE YVB(Y, I)
      REAL Y(10)
      REAL  MVB
      COMMON /INPTVB/   FTVB(64),  FPVB(64),  FPPRVB(64),  DIDRVB(64),
     A       XMVB(64),   DELVB,     XMUVB,     FOVB,   XMUAVB,
     B       ATOVB,      ATCVB,      ATSVB,      ROVB,        RVB(64),
     C       MVB(64),    NVB
      Y(1) =  (RVB(I) - DELVB)**2  *  MVB(I)
      Y(2) =  FPVB(I)**2 *  MVB(I)
      Y(3) =  FTVB(I)**2  * DIDRVB(I)
      Y(4) = (DELVB - RVB(I)) * FTVB(I) * XMVB(I) * MVB(I)
      Y(5) = FPVB(I) * FTVB(I) * XMVB(I) * MVB(I)
      Y(6) =  RVB(I) * (DELVB - RVB(I))* MVB(I)
      Y(8) =  (RVB(I) - DELVB) * FPPRVB(I) * FTVB(I) * XMVB(I) * MVB(I)
      IP1 = I+1
      IF(IP1 .GE. NVB) GO TO 12
      SUM = 0.
      DO 10  J = IP1, NVB
10    SUM = SUM -(RVB(4+1) - RVB(4)) * (RVB(4+1) * MVB(J+1)
     A        +  RVB(J) * MVB(J))
12    Y(7) = FPPRVB(I) ** 2  * SUM / 2.
      RETURN
      END
```

```
      SUBROUTINE POLLY(N,BBS,REL,AN,AA)
      IMPLICIT REAL*8 (A-H,O-Z)
C     COMPLEX ROOTS OF A POLYNOMIAL BAIRSTOWS METHOD
      DIMENSION A(30),AN(60),C(26),ABAR(26),B(30),AA(30)
      III=1
    7 NP1=N+1
      NPP1=N+2
      DO 601 I=1,NP1
      LLL=NPP1-I
  601 A(I)=AA(LLL)
   13 DO 14 K=1,NP1
   14 ABAR(K)=A(K)
      ABSSQ=BBS*BBS
      RELSQ=REL*REL
      NBAR=N
      B(1)=A(1)
      C(1)=A(1)
   15 IF(NBAR-2)200,210,17
   17 P1=.2
      Q1=.1
   18 ITER=0
   19 P1=P1*5.
      Q1=Q1*10.
   33 P=P1
      Q=Q1
      NBP1=NBAR+1
   34 L=1
      LAST=NBAR
      DTEST=9.99D36
C     BAIRSTOW ITERATION
   37 B(2)=ABAR(2)-P*B(1)
      DO 40 K=3,NBP1
   40 B(K)=ABAR(K)-P*B(K-1)-Q*B(K-2)
   45 C(2)=B(2)-P*C(1)
      DO 50 K=3,LAST
   50 C(K)=B(K)-P*C(K-1)-Q*C(K-2)
      C(LAST)=C(LAST)-B(LAST)
      D=C(LAST-1)*C(LAST-1)-C(LAST)*C(LAST-2)
      DSQR=D*D
      IF(DSQR-1.D-36)19,19,60
   60 DELP=(B(LAST)*C(LAST-1)-B(LAST+1)*C(LAST-2))/D
      DELQ=(B(LAST+1)*C(LAST-1)-B(LAST)*C(LAST))/D
C     TEST FOR CONVERGENCE
      RELP=DELP/P
      RELQ=DELQ/Q
      RELPS=RELP*RELP
      RELQS=RELQ*RELQ
      DELSQ=RELPS+RELQS
      P=P+DELP
      Q=Q+DELQ
      IF(RELPS-RELSQ)70,70,65
   65 IF(DELP*DELP-ABSSQ)70,70,80
   70 IF(RELQS-RELSQ)120,120,75
   75 IF(DELQ*DELQ-ABSSQ)120,120,80
   80 GO TO (90,100),L
```

```
   90 ITER=ITER+1
      IF(250-ITER)310,37,37
  100 IF(DTEST-DELSQ)34,34,110
  110 DTEST=DELSQ
      B(2)=A(2)-P*B(1)
      DO 115 K=3,NP1
  115 B(K)=A(K)-P*B(K-1)-Q*B(K-2)
      GO TO 45
C     ITERATION HAS CONVERGED
  120 GO TO (130,140),L
  130 L=2
      LAST=N
      GO TO 110
C     FACTOR OUT QUADRATIC
  140 NBAR=NBAR-2
      NBP1=NBAR+1
      ABAR(2)=ABAR(2)-P*ABAR(1)
      DO 150 K=3,NBP1
  150 ABAR(K)=ABAR(K)-P*ABAR(K-1)-Q*ABAR(K-2)
      GO TO  250
C     SOLVE LINEAR EQUATION
  200 NBAR=NBAR-1
      R1=-ABAR(2)/ABAR(1)
      R2=0.
      GO TO 262
C     NORMALIZE QUADRATIC
  210 P=ABAR(2)/ABAR(1)
      Q=ABAR(3)/ABAR(1)
      NBAR=NBAR-2
C     SOLVE NORMALIZED QUADRATIC
  250 R1=-P/2.
      C1=R1*R1-Q
      IF(C1)270,280,260
  260 C1=DSQRT(C1)
      R2=R1-C1
      R1=R1+C1
  262 C1=0.
      GO TO 290
  270 C1=-C1
      C1=DSQRT(C1)
  280 R2=R1
  290 C2=-C1
      AN(III)=C1
      AN(III+1)=R1
      AN(III+2)=C2
      AN(III+3)=R2
      III=III+4
      IF(NBAR-1)4,200,15
C     SPECIAL CONDITIONS
  310 WRITE (6,600)
  600 FORMAT(1X,50HNO CONVERGENCE IN 250 ITERATIONS ,POLLY HAS SPOKEN)
    4 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE SETUP(LGO,M,NV,REB,X,Y,UC,PRESS,GRAD,DELT,DISP,THETA,VI
     1SC,MTRAN)
C
C   SUBROUTINE FOR CALCULATION OF BOUNDARY LAYER THICKNESS,
C   DISPLACEMENT THICKNESS, MOMENTUM THICKNESS AND EDDY VISCOSITY.
C
      DIMENSION X(300),Y(100),UC(100,3),VISC(100,2),GRAD(100)
      RTR=SQRT(REB)
      NY = NV + 2
      UEDGE = .995*UC(NY,1)
      DO 10 N=1,NV
      IF(UEDGE-UC(N+1,1)) 41,41,10
   41 NDELT = N
      GO TO 20
   10 CONTINUE
   20 DELT = Y(NDELT)+(UEDGE-UC(NDELT,1))*(Y(NDELT+1)-Y(NDELT))/(UC(NDEL
     1T+1,1)-UC(NDELT,1))
      SUM = 0.
      DO 50 N=2,NY
   50 SUM = SUM+(Y(N)-Y(N-1))*(UC(N,1)+UC(N-1,1))
      DISP = (Y(NY)-.5*SUM/UC(NY,1))/RTR
      SUM = 0.
      UEDGE = UC(NY,1)
      DO 60 N=2,NY
   60 SUM = SUM+(Y(N)-Y(N-1))*((UEDGE-UC(N,1))*UC(N,1)+(UEDGE-UC(N-1,1))
     1*UC(N-1,1))
      THETA = .5*SUM/(RTR*UEDGE**2)
      IF(LGO) 53,53,56
   53 NVPI=NV+1
      EASE = 1.
      IF(M-MTRAN) 31,32,32
   32 IF(MTRAN+5-M) 31,31,33
   33 EASE = (X(M)-X(MTRAN))/(X(MTRAN+5)-X(MTRAN))
   31 CONTINUE
      INNER=0
      FAC1 = .16*RTR*EASE
      FAC2 = .0168*UEDGE*DISP*REB*EASE
      EFAC1 = -RTR/26.
      EFAC2 = PRESS/RTR
      TAUW = GRAD(1)/RTR
      DO 160 N=2,NVPI
      ALTER = 1.+FAC2/(1.+5.5*(Y(N)/DELT)**6)
      IF(INNER) 402,401,402
  402 VISC(N,1)=ALTER
      GO TO 160
  401 CONTINUE
      TAUMY=TAUW-Y(N)*EFAC2
      IF(TAUMY) 701,701,702
  701 VISC(N,1)=1.
      GO TO 703
  702 EX=Y(N)*EFAC1*SQRT(TAUMY)
      VISC(N,1) = 1.+FAC1*Y(N)*Y(N)*ABS(GRAD(N))*(1.-EXP(EX))**2
  703 IF(VISC(N,1)-ALTER) 160,160,521
  521 VISC(N,1)=ALTER
      INNER=1
```

```
 160  CONTINUE                                                        SETUP 56
      SAVE=1.                                                         SETUP 57
      DO 162 N=2,NV                                                   SETUP 58
      RAVE=VISC(N,1)                                                  SETUP 59
      VISC(N,1)=(VISC(N+1,1)+RAVE+SAVE)/3.                            SETUP 60
 162  SAVE=RAVE                                                       SETUP 61
 56   CONTINUE                                                        SETUP 62
      RETURN                                                          SETUP 63
      END
```

```
      SUBROUTINE MIXER(FPRES,PREC,UINF,UDOT,THICK,NF,XBSIG,NSIG,INDT,DEL
     11,THET1,REB,USEP,X4,CP4)
      DIMENSION FPRES(100),THICK(24),XBSIG(100)
      FCAP(X)=-19.556*X+107.535*X*X-336.33*X**3+508.1*X**4-295.96*X**5
      UI1(X)=-.46532*X+.68425*X*X-.45293*X**3+.6592*X**4
      UI2(X)=-.045929*X-1.91615*X*X+2.91843*X**3-5.42125*X**4
      DIST=.5*(XBSIG(2)-XBSIG(1))
      XSEP=XBSIG(1)-DIST
      XATT=XBSIG(NSIG)+DIST
C
C   IF INDT IS NONZERO, THE BOUNDARY LAYER IS TURBULENT
C   AT SEPARATION.
C
      CALL H4X4(INDT,XSEP,DEL1,THET1,XATT,REB,USEP,X3,H3,X4,H4)
      IF(XSEP-1.) 24,25,25
 25   CP4=0.
      GO TO 27
 24   URAT=EXP(-.08712-UI1(H4)-.24723*(.3255+UI2(H4)))
      CP4=1.-(1.-PREC)/URAT**2
      DEADL=XATT-XSEP
      IF(DEADL-2.) 5,6,6
 5    G=(.5*DEADL)**2
      GO TO 7
 6    G=1.
 7    CP4=PREC+(CP4-PREC)*(1.-G*XSEP)
 27   CONTINUE
      COEF=(PREC-CP4)/(XATT-X4)
      CZ=2.*UDOT/UINF
      C2 = -2.*UINF
      DO 20 M=1,NSIG
      SUM=0.
      COUNT=0.
      X=XBSIG(M)
      IF(X-1.) 2,2,3
 2    THETA = ARCT(X)
      TANT = SIN(.5*THETA)/COS(.5*THETA)
      C1 = -CZ*(1.-COS(THETA))
      DO 10 N=1,NF
      COUNT=COUNT+1.
      ANGLE=COUNT*THETA
 10   SUM=SUM+THICK(N)*(C1*COS(ANGLE)+C2*(COUNT*TANT*SIN(ANGLE)-COS(ANGL
     1E)))
      SUM=SUM-.5*CZ*THICK(1)
      GO TO 35
 3    C1=CZ*(1.-X)
      XRAD=1./(X+SQRT(X*X-1.))
      C1=CZ*(X-1.)
      RF=SQRT((X-1.)/(X+1.))
      SUM=THICK(1)*XRAD*(C2*(RF-1.)-CZ*(1.-.5*XRAD))
      FRAD=XRAD
      COUNT=1.
      DO 30 N=2,NF
      COUNT=COUNT+1.
      FRAD=FRAD*XRAD
 30   SUM=SUM+THICK(N)*FRAD*(C2*(COUNT*RF-1.)+C1)
```

```
35    CP=CP4                                                            MIXER 56
      IF(X-X4) 55,50,50                                                 MIXER 57
50    CP=CP+(X-X4)*COEF                                                 MIXER 58
55    CONTINUE                                                          MIXER 59
      FPRES(M)=-UINF*CP+SUM                                             MIXER 60
20    CONTINUE                                                          MIXER 61
      RETURN                                                            MIXER 62
      END                                                               MIXER 63
```

```
      SUBROUTINE BUBB(DEL1,THET1,REB,XC1,U1,XC5,DCP,DEL5,X,XC,MX,NZ,X5,UB
     15,UE,ALTC,RENEL,USTOP)
      DIMENSION X(300),XC(300),UE(300,3)
      FCAP(X)=-19.556*X+107.535*X*X-336.33*X**3+508.1*X**4-295.96*X**5
      UI1(X)=-.46532*X+.68425*X*X-.45293*X**3+.6592*X**4
      UI2(X)=-.045929*X-1.91615*X*X+2.91843*X**3-5.42125*X**4
      FDELT(X)=EXP(2.5773-.34252*X-.4379*X*X-.076511*X**3-.0039707*X**4)
      FAICH(X)=EXP(-3.7481+.038772*X+.41967*X*X+.071046*X**3+.0032162*X*
     1*4)
      DELI(X)=-.045929*ALOG(X)-3.9242*X+.54535*X*X-1.39147*X**3-10.8425*
     1X**4
25    FORMAT(1H1,44X,31HANALYSIS OF LEADING-EDGE BUBBLE////34X,1HX,19X,1
     1HU,19X,1HH,18X,4HDISP/)
30    FORMAT(20X,4E20.5)
      MOUT=6
      H1=.25
      H5=.429
      DO 5 M=NZ,MX
      IF(XC1-XC(M)) 4,4,5
4     M1=M
      GO TO 6
5     CONTINUE
6     X1=X(M1-1)+(X(M1)-X(M1-1))*(XC1-XC(M1-1))/(XC(M1)-XC(M1-1))
      X4=X1+RENEL/(U1*REB)
      ARG=ALOG((X4-X1)/(REB*DEL1*DEL1*U1))
      H4=  .25*FAICH(ARG)
      DEL4=.58*FDELT(ARG)*DEL1
      X5=X4+10.5*DEL4*(1.-(H4/.429)**2)
      IF(U1-USTOP) 41,41,40
40    ALTL=ALTC*DEL1
      IF(X5-X1.LT.ALTL) X5=X1+ALTL
41    URAT=EXP(-.08712-UI1(H4)-.24723*(.3255+UI2(H4)))
      DCP=U1*U1*(1.-URAT**2)
      DRAT=EXP(-2.24374-FCAP(H4)+.24723*(2.0214+DELI(H4)))
      DEL5=DRAT*DEL4
      DO 7 M=NZ,MX
      IF(X5-X(M)) 16,16,7
16    M5=M
      GO TO 8
7     CONTINUE
8     FACT=(X5-X(M5-1))/(X(M5)-X(M5-1))
      FACT1=1.-FACT
      XC5=XC(M5-1)*FACT1+XC(M5)*FACT
      U5=UE(M5-1,1)*FACT1+UE(M5,1)*FACT
      WRITE(MOUT,25)
      WRITE(MOUT,30) X1,U1,H1,DEL1
      WRITE(MOUT,30) X4,U1,H4,DEL4
      WRITE(MOUT,30) X5,U5,H5,DEL5
      RETURN
      END
```

```
      SUBROUTINE YSET(R,A,NY,Y)
      DIMENSION Y(100)
      RP1=1.+R
      Y(1)=0.
      Y(2)=A
      DO 10 N=3,NY
10    Y(N)=RP1*Y(N-1)-R*Y(N-2)
      RETURN
      END
```

```
      SUBROUTINE H4X4(INDT,X1,DEL1,THET1,X5,REB,U1,X3,H3,X4,H4)
      CURLF(H)=26.703/H+305.03*ALOG(H)-2111.3*H+3327.8*H*H-2403.9*H**3
      FDELT(X)=EXP(2.5773-.34252*X-.4379*X*X-.076511*X**3-.0039707*X**4)
      FAICH(X)=EXP(-3.7481+.038772*X+.41967*X*X+.071046*X**3+.0032162*X*
     1*4)
   10 FORMAT(//20X,54HA SOLUTION FOR X4 COULD NOT BE OBTAINED IN 1000 TR
     1IALS)
      MOUT=6
C
C  IF INDT IS NONZERO, THE BOUNDARY LAYER IS TURBULENT
C  AT SEPARATION.
C
      IF(INDT) 2,5,2
    2 H3=THET1/DEL1
      X3=X1
      DEL3=DEL1
      GO TO 20
    5 X3=X1+5.E4/(U1*REB)
      ARG=ALOG((X3-X1)/(REB*DEL1*DEL1))
      H3=THET1*FAICH(ARG)/DEL1
      DEL3=.58*FDELT(ARG)*DEL1
      IF(X3-X5) 20,15,15
   15 H4=.429
      X4=X5
      GO TO 50
   20 CONTINUE
      IGO=0
      DIST=X5-X1
      UNDER=0.
      H4=H3+H3
      COEF1=DEL3*H3
      COEF2=10.5*DEL3*H3
      SUB=X3-COEF1*CURLF(H3)
   95 OVER=H4
      H4=.5*(H4+UNDER)
      X4=CURLF(H4)*COEF1+SUB
      ALTER=X5-COEF2*(1.-(H4/.429)**2)/H4
      IGO=IGO+1
      IF(X4-ALTER) 41,50,42
   41 IF(IGO-1000) 95,61,61
   42 IF(ABS(X4-ALTER)/DIST-.001) 50,50,43
   43 UNDER=H4
      H4=.5*(OVER+H4)
      X4=CURLF(H4)*COEF1+SUB
      ALTER=X5-COEF2*(1.-(H4/.429)**2)/H4
      IGO=IGO+1
      IF(X4-ALTER) 52,50,51
   51 IF(IGO-1000) 43,61,61
   52 IF(ABS(X4-ALTER)/DIST-.001) 50,50,95
   61 H4=.429
      X4=X5
      WRITE(MOUT,10)
   50 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE SETSX(NSP1,XSEP,XATT,XSIG,ANGLE)                       SETSX  1
      DIMENSION XSIG(100)                                               SETSX  2
      A=.5*(XSEP+XATT)                                                  SETSX  3
      B=.5*(XATT-XSEP)                                                  SETSX  4
      ARG=0.                                                            SETSX  5
      DO 5 N=1,NSP1                                                     SETSX  6
      XSIG(N)=A-B*COS(ARG)                                              SETSX  7
    5 ARG=ARG+ANGLE                                                     SETSX  8
      RETURN                                                            SETSX  9
      END                                                               SETSX 10
```

```
      FUNCTION ARCT(X)                                                  ARCT   1
      PI=3.14159                                                        ARCT   2
      IF(ABS(X)-1.E-6) 1,2,2                                            ARCT   3
    1 ARCT=.5*PI                                                        ARCT   4
      GO TO 6                                                           ARCT   5
    2 IF(X+.99999) 3,4,4                                                ARCT   6
    3 ARCT=PI                                                           ARCT   7
      GO TO 6                                                           ARCT   8
    4 ARCT=ATAN(SQRT(1.-X*X)/X)                                         ARCT   9
      IF(ARCT) 5,6,6                                                    ARCT  10
    5 ARCT=ARCT+PI                                                      ARCT  11
    6 CONTINUE                                                          ARCT  12
      RETURN                                                            ARCT  13
      END                                                               ARCT  14
```

```
      FUNCTION GAM1(ACAP,DXI,PI)
      DIMENSION ACAP(30,3)
      GAM1=PI*(-1.5*ACAP(1,1)-.75*ACAP(2,1)+2.*ACAP(1,2)+ACAP(2,2)-.5*AC
     1AP(1,3)-.25*ACAP(2,3))/DXI
      RETURN
      END
```

```
      FUNCTION FB(X1,X2,X3,Y)
      D1=1./(X2-X1)
      D2=1./(X3-X2)
      T1=ABS(Y-X1)
      T2=ABS(Y-X2)
      T3=ABS(Y-X3)
      EPS=1.E-6
      IF(T1-EPS) 2,3,3
    2 F1=0.
      F2=ALOG(T2)
      F3=ALOG(T3)
      GO TO 10
    3 F1=ALOG(T1)
      IF(T2-EPS) 4,5,5
    4 F2=0.
      F3=ALOG(T3)
      GO TO 10
    5 F2=ALOG(T2)
      IF(T3-EPS) 6,7,7
    6 F3=0.
      GO TO 10
    7 F3=ALOG(T3)
   10 FB=((Y-X1)*F1*D1+(D1+D2)*(X2-Y)*F2+(Y-X3)*F3*D2)/3.14159
      RETURN
      END
```

```
      SUBROUTINE EGAMI(NU,NG,A,B,XSEP,XATT,GAMMA,Y,GI)                  EGAMI   1
      DIMENSION A(30,3)                                                 EGAMI   2
      SINT=SQRT(1.-Y*Y)                                                 EGAMI   3
      THETA=ARCT(Y)                                                     EGAMI   4
      SUM=0.                                                            EGAMI   5
      COUNT=1.                                                          EGAMI   6
      DO 6 N=2,NG                                                       EGAMI   7
      COUNT=COUNT+1.                                                    EGAMI   8
6     SUM=SUM+A(N+1,NU)*(SIN((COUNT+1.)*THETA)/(COUNT+1.)-SIN((COUNT-1.)EGAMI   9
     1*THETA)/(COUNT-1.))                                               EGAMI  10
      GI=(3.14159-THETA+SINT)*(A(1,NU)+.5*A(2,NU))+.5*SUM-.25*GAMMA*(1.+EGAMI  11
     1Y)*SINT*SINT                                                      EGAMI  12
      IF(Y-XATT) 8,8,7                                                  EGAMI  13
7     DIFF=1.-XATT                                                      EGAMI  14
      IF(DIFF-1.E-6) 8,8,9                                              EGAMI  15
9     GI=GI+2.*B*DIFF**(-1.5)*SQRT((XATT-XSEP)*(1.-Y)*(Y-XATT))         EGAMI  16
8     CONTINUE                                                          EGAMI  17
      RETURN                                                            EGAMI  18
      END                                                               EGAMI  19
```

```
      SUBROUTINE ESIGI(NU,NX,XS,B,Y,SI)                                 ESIGI   1
      DIMENSION XS(100),B(100,3)                                        ESIGI   2
      SUM=0.                                                            ESIGI   3
      DO 10 I=2,NX                                                      ESIGI   4
   10 SUM=SUM+B(I,NU)*GB(XS(I-1),XS(I),XS(I+1),Y)                       ESIGI   5
      SI=B(1,NU)*BINT(XS(1),XS(NX+1),Y)+SUM                             ESIGI   6
      RETURN                                                            ESIGI   7
      END                                                               ESIGI   8
```

```
      FUNCTION GB(X1,X2,X3,X)                                           GB       1
      GB=ABINT(X1,X2,X)-ABINT(X3,X2,X)                                  GB       2
      GB=GB/3.14159                                                     GB       3
      RETURN                                                            GB       4
      END                                                               GB       5
```

```
      FUNCTION ABINT(A,B,X)
      ARGA=ABS(X-A)
      ARGB=ABS(X-B)
      COEF=2.*(B-A)
      AP1=A+1.
      BP1=B+1.
      IF(ARGA-1.E-6) 2,3,3
    2 CA=0.
      GO TO 5
    3 CA=ALOG(ARGA)
      IF(ARGB-1.E-6) 4,5,5
    4 CB=0.
      GO TO 6
    5 CB=ALOG(ARGB)
    6 ABINT=(CA-.5)*ARGA**2-(CB-.5)*ARGB**2-(ALOG(AP1)-.5)*AP1**2+(ALOG(
     1BP1)-.5)*BP1**2-COEF*((X-B)*(CB-1.)+BP1*(ALOG(BP1)-1.))
      ABINT=ABINT/COEF
      RETURN
      END
```

```
      FUNCTION BINT(XS,XZ,X)
      RTS=SQRT(1.+XS)
      RTZ=SQRT(1.+XZ)
      BINT=-1.-X+RTS*RTZ
      IF(XZ-X) 2,3,3
2     RTSX=SQRT(X-XS)
      RTZX=SQRT(X-XZ)
      BINT=BINT+(XZ-XS)*ALOG((RTSX+RTZX)/(RTS+RTZ))+RTSX*RTZX
      GO TO 50
3     IF(X-XS) 5,5,4
4     BINT=BINT+(XZ-XS)*ALOG(SQRT(XZ-XS)/(RTS+RTZ))
      GO TO 50
5     RTSX=SQRT(XS-X)
      RTZX=SQRT(XZ-X)
      BINT=BINT+(XZ-XS)*ALOG((RTSX+RTZX)/(RTS+RTZ))-RTSX*RTZX
50    CONTINUE
      RETURN
      END
```

```
      SUBROUTINE SCAL(SBL,NSBL,FRZ,ARR,RDBB)
      DIMENSION SBL(300)
      DELZ=FRZ*RDBB
      EN=ARR/FRZ
      DO 5 N=1,300
      IF(EN-N) 4,4,5
    4 NE=N
      GO TO 6
    5 CONTINUE
    6 NG=NSBL-NE
      EN=FLOAT(NG)
      NGM1=NG-1
      SBL(1)=0.
      DO 7 N=2,NE
    7 SBL(N)=SBL(N-1)+DELZ
      FRACT=2.2/DELZ
      FRAC1=FRACT-1.
      R=FRACT**(1./FLOAT(NGM1))
    8 SAVE=R
      R=R-(R**NG-FRACT*R+FRAC1)/(EN*R**NGM1-FRACT)
      IF(ABS(SAVE-R)-1.E-6) 9,9,8
    9 RP1=R+1.
      DO 10 N=NE,NSBL
   10 SBL(N+1)=RP1*SBL(N)-R*SBL(N-1)
      RETURN
      END
```

```
      SUBROUTINE TERPF(XI,J,TAB1,TAB2,TAB3,TAB4,XITAB,FP)
      DIMENSION TAB1(24),TAB2(24),TAB3(24),TAB4(24),XITAB(24)
      IF(XI-.0001) 2,2,10
 2    GO TO (3,4,5,6),J
 3    FP=2.53-2.439*ALOG(XI)
      GO TO 99
 4    FP=3.54-1.725*ALOG(.7071*XI)
      GO TO 99
 5    FP=4.58-1.2195*ALOG(.5*XI)
      GO TO 99
 6    FP=10.12
      GO TO 99
 10   DO 12 N=1,24
      IF(XI-XITAB(N)) 11,11,12
 11   NX=N
      GO TO 13
 12   CONTINUE
 13   TX=(XI-XITAB(NX-1))/(XITAB(NX)-XITAB(NX-1))
      TX1=1.-TX
      GO TO (14,15,16,17),J
 14   FP=TX1*TAB1(NX-1)+TX*TAB1(NX)
      GO TO 99
 15   FP=TX1*TAB2(NX-1)+TX*TAB2(NX)
      GO TO 99
 16   FP=TX1*TAB3(NX-1)+TX*TAB3(NX)
      GO TO 99
 17   FP=TX1*TAB4(NX-1)+TX*TAB4(NX)
 99   CONTINUE
      RETURN
      END
```

```
      SUBROUTINE EVAL(NNF,XX,SSC,SST,CCB,TTB,CCM,TTM)
      DIMENSION SSC(50),SST(50)
      COST = 2.*XX - 1.
      COSTS = COST**2
      IF(COSTS-1.E-8) 303,304,304
  304 TANT = SQRT(1./COSTS - 1.)
      THE = ATAN(TANT)
      GO TO 305
  303 THE = 1.5708
  305 IF(COST) 403,404,404
  403 THE = 3.14159 - THE
  404 ARG = 0.
      SUM1 = 0.
      SUM2 = 0.
      DO 551 N=1,NNF
      ARG = ARG + THE
      SUM1 = SUM1 + SSC(N)*SIN(ARG)
  551 SUM2 = SUM2 + SST(N)*SIN(ARG)
      CCB = SUM1*SIN(THE)*CCM
      TTB = (1. - COS(THE))*SUM2*TTM
      RETURN
      END
```

```
      SUBROUTINE SIMP(NS,DX,ORD,FIND)
      DIMENSION ORD(50)
C         INTEGRATION OF NS + 1 EQUALLY SPACED ORDINATE VALUES
C         BY SIMPSON'S RULE.  NS MUST BE EVEN
      SUM = 0.
      DO 88 I=2,NS,2
   88 SUM = SUM + 2.*ORD(I-1) + 4.*ORD(I)
      FIND = DX*(SUM - ORD(1) + ORD(NS+1))/3.
      RETURN
      END
```

```
      SUBROUTINE SECT(XU,YU,XL,YL,NOFF,NF,RCDBC,TMAX,CMAX,ST,SC)
C  PROGRAM TO COMPUTE COEFFICIENTS TN AND CN OF THE FOURIER SERIES
C  REPRESENTATION OF SECTION THICKNESS AND CAMBER DISTRIBUTIONS
      DIMENSION XU(30),YU(30),XL(30),YL(30),YUC(30),YLC(30),ST(24),SC(24
     1),DUM(50),TBAR(50),CBAR(50)
 12   FORMAT(////47X,26HINPUT AND COMPUTED OFFSETS/)
  13  FORMAT(19X,4HX1/C,12X,4HYU/C,11X,5HYUC/C,20X,4HX1/C,12X,4HYL/C,11X
     1,5HYLC/C/)
  14  FORMAT(8X,3F16.5,8X,3F16.5)
      NA=6
      RNA=6.
      RNF=FLOAT(NF)
      MOUT=6
      PI = 3.14159
      DELT = PI/(2.*RNF)
      NTC = 2*NF - 1
      NINT = NTC + 2
      NSIMP = NTC + 1
      RDBC=.5*RCDBC
      VARY = 0.
      CB = 0.
      TB = 0.
      THETA = 0.
      DO 89 K=1,NTC
      THETA = THETA + DELT
      X1 = .5*(1. + COS(THETA))
      DO 90 LAM=2,NOFF
      IF(X1-XU(LAM)) 110,90,90
  110 YUINT = YU(LAM-1) + (X1 - XU(LAM-1))*(YU(LAM) - YU(LAM-1))/(XU(LAM
     1) - XU(LAM-1))
      GO TO 111
  90  CONTINUE
  111 DO 80 LAM=2,NOFF
      IF(X1-XL(LAM)) 210,80,80
  210 YLINT = YL(LAM-1) + (X1 - XL(LAM-1))*(YL(LAM) - YL(LAM-1))/(XL(LAM
     1) - XL(LAM-1))
      GO TO 112
  80  CONTINUE
  112 TBAR(K+1) = .5*(YUINT - YLINT)
  89  CBAR(K+1) = .5*(YUINT + YLINT)
      TMAX = 0.
      CMAX = 0.
      DO 79 K = 2,NSIMP
      IF(TBAR(K)-TMAX) 801,802,802
  802 TMAX = TBAR(K)
  801 IF(CBAR(K)-CMAX) 79,702,702
  702 CMAX = CBAR(K)
  79  CONTINUE
      IF(CMAX-1.E-5) 1201,1202,1202
 1201 CMAX=1.
 1202 CONTINUE
      IF(TMAX-1.E-5) 1140,1141,1141
 1140 TMAX=1.
 1141 DO 69 K=2,NSIMP
      TBAR(K) = TBAR(K)/TMAX
```

```
   69 CBAR(K) = CBAR(K)/CMAX
      TBAR(1) = 0.
      CBAR(1) = 0.
      TBAR(NINT) = 0.
      CBAR(NINT) = 0.
      TTA = TBAR(NA)
      TTB = TBAR(NA+1)
      TTC = TBAR(NA+2)
      TAA = DELT*(RNA-1.)
      TBB = TAA + DELT
      TCC = TBB + DELT
      XA = .5*COS(TAA)
      XB = .5*COS(TBB)
      XC = .5*COS(TCC)
      SLOPE = ((TTC-TTB)*(XB-XA)/(XC-XB) + (TTB-TTA)*(XC-XB)/(XB-XA))/(X
     1C-XA)
      THETA = 0.
      COSB = COS(TBB)
      DO 456 I=2,NA
      THETA = THETA + DELT
      COST = COS(THETA)
  456 TBAR(I) = (SQRT(1.-COST)/(1.-COSB)**1.5)*(TTB*(1.+COST-2.*COSB)/(1
     1.-COSB) + .5*SLOPE*(COST-COSB))
      NLE = 2*NF + 1 - NA
      COSR1 = 1. + COS(PI-RNA*DELT)
      THETA = PI
      SINAS=SIN(RNA*DELT)**2
      COSAS=COS(RNA*DELT)
      ANG=0.
      DO 457 I=2,NA
      IND = 2*NF + 2 - I
      THETA = THETA - DELT
      COST1 = 1. + COS(THETA)
      ANG=ANG+DELT
      COEF=(SINAS-SIN(ANG)**2)/(COSR1*(COS(ANG)+COSAS))
  457 TBAR(IND) = (SQRT(RDBC*COST1)*COEF/TMAX+TBAR(NLE)*(COST1/COSR1)**1
     1.5)/(2.-COST1)
      THETA = TAA
      NAP1 = NA + 1
      DO 458 I = NAP1,NLE
      THETA = THETA + DELT
  458 TBAR(I) = TBAR(I)/(1.-COS(THETA))
      THETA = 0.
      DO 459 I=2,NSIMP
      THETA = THETA + DELT
  459 CBAR(I) = CBAR(I)/SIN(THETA)
      RKK = 0.
      DO 59 K=1,NF
      RKK = RKK + 1.
      THETA = 0.
      DO 777 I=1,NINT
      DUM(I) = TBAR(I)*SIN(THETA*RKK)
  777 THETA = THETA + DELT
      CALL SIMP(NSIMP,DELT,DUM,VARY)
      ST(K) = 2.*VARY/PI
```

```
      THETA = 0.                                                        SECT 111
      DO 888 I=1,NINT                                                   SECT 112
      DUM(I) = CBAR(I)*SIN(THETA*RKK)                                   SECT 113
  888 THETA = THETA + DELT                                              SECT 114
      CALL SIMP(NSIMP,DELT,DUM,VARY)                                    SECT 115
   59 SC(K) = 2.*VARY/PI                                                SECT 116
      DO 969 I=1,NOFF                                                   SECT 117
      X = XU(I)                                                         SECT 118
      CALL EVAL(NF,X,SC,ST,CB,TB,CMAX,TMAX)                             SECT 119
  969 YUC(I) = CB + TB                                                  SECT 120
      DO 869 I=1,NOFF                                                   SECT 121
      X = XL(I)                                                         SECT 122
      CALL EVAL(NF,X,SC,ST,CB,TB,CMAX,TMAX)                             SECT 123
  869 YLC(I) = CB - TB                                                  SECT 124
      SUM1 = 0.                                                         SECT 125
      COUNT = 0.                                                        SECT 126
      DO 699 I=1,NF                                                     SECT 127
      COUNT = COUNT + 1.                                                SECT 128
  699 SUM1 = SUM1 - ST(I)*COUNT*(-1.)**I                                SECT 129
      RCDBC = 8.*(TMAX*SUM1)**2                                         SECT 130
      RCDBC=2.*RCDBC                                                    SECT 131
      TMAX=2.*TMAX                                                      SECT 132
      CMAX=2.*CMAX                                                      SECT 133
      WRITE(MOUT,12)                                                    SECT 134
      WRITE(MOUT,13)                                                    SECT 135
      WRITE(MOUT,14) (XU(I),YU(I),YUC(I),XL(I),YL(I),YLC(I),I=1,NOFF)   SECT 136
      RETURN                                                            SECT 137
      END                                                               SECT 138
```

```
      SUBROUTINE CORDX(NSBL,NZ,RDBB,SBL,X,XC)
C
C  BOUNDARY LAYER COORDINATES AND CORRESPONDING CHORDAL
C  COORDINATES ARE COMPUTED HERE.
C
      DIMENSION SBL(300),X(300),XC(300)
  336 FORMAT(//10X,31HITERATION TO COMPUTE XC FOR M =I5,32H DID NOT CONV
     1ERGE IN 1000 STEPS.)
  337 FORMAT(1H1,25X,1HM,20X,1HS,25X,1HX,24X,2HXC//)
  338 FORMAT(22X,I5,3E25.5)
      MOUT=6
      MX = NSBL + NZ - 1
      RZERO = RDBB/2.
      XC(NZ) = -1.
      DO 255 M=1,NZ
      MM = NZ + 1 - M
  255 X(M) = SBL(NZ) - SBL(MM)
      DO 256 M=NZ,MX
      MM = M + 1 - NZ
  256 X(M) = SBL(NZ) + SBL(MM)
      DO 257 M=1,MX
      IF(NZ-M) 333,257,335
  333 K = M + 1 - NZ
      GO TO 334
  335 K = NZ - M + 1
  334 XC(M) = -1. + SBL(K)
      IF(SBL(K)-RZERO) 341,341,342
  341 XC(M) = -1. + SBL(K)**2/(4.*RZERO)
  342 CONTINUE
      DO 258 L=1,1000
      SAVE = XC(M)
      CALC1 = SQRT((1.+XC(M))/RZERO)
      CALC2 = SQRT(1.+(1.+XC(M))/RZERO)
      XC(M)=XC(M)+CALC1*(SBL(K) -   RZERO*(CALC1*CALC2+ALOG(CALC1+CALC2))
     1)/CALC2
      IF(ABS(SAVE-XC(M))-1.E-6) 257,257,258
  258 CONTINUE
      WRITE(MOUT,336) M
  257 CONTINUE
      WRITE(MOUT,337)
      DO 264 M=1,MX
      IF(NZ-M) 261,261,262
  262 K=NZ-M+1
      GO TO 263
  261 K=M+1-NZ
  263 WRITE(MOUT,338) M,SBL(K),X(M),XC(M)
  264 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE PGRAD(M,X,UE,DXI,PRESS,SA,SB,SC,SR,SS)
C
C   SUBROUTINE FOR CALCULATION OF PRESSURE GRADIENT AND
C   DERIVATIVE COEFFICIENTS.
C
      DIMENSION X(300),UE(300,3)
      D1Z=X(M+1)-X(M)
      D2Z=X(M+2)-X(M)
      D21=X(M+2)-X(M+1)
      D1M1=X(M+1)-X(M-1)
      DZM1=X(M)-X(M-1)
      XIM=D1Z/(D2Z*D21)
      ETAM=1./D1Z-1./D21
      ZETAM=D21/(D1Z*D2Z)
      PRESS = (3.*UE(M+1,1)-4.*UE(M+1,2)+UE(M+1,3))/( 2.*DXI)+UE(M+1,1)*(
     1XIM*UE(M+2,1)+ETAM*UE(M+1,1)-ZETAM*UE(M,1))
      SA=1./D1Z+1./D1M1
      SB=D1M1/(D1Z*DZM1)
      SC=D1Z/(D1M1*DZM1)
      SR=D1M1/DZM1
      SS=D1Z/DZM1
      RETURN
      END
```

```
      SUBROUTINE TRANS(UPRIM,PRESS,THETA,REB,UC,NY,FLAM,XFLAM,LAMQ)     TRANS  1
C                                                                       TRANS  2
C  SUBROUTINE TO TEST FOR TRANSITION IN A LAMINAR BOUNDARY LAYER.       TRANS  3
C                                                                       TRANS  4
      DIMENSION UC(100,3),FLAM(10),XFLAM(10)                            TRANS  5
      F(X) = .11746 - 1.0582E-3*X - 1.1023E-4*X*X                       TRANS  6
      TKAY = PRESS*REB*THETA**2/UC(NY,2)                                TRANS  7
      IF(TKAY-.077) 2,2,99                                              TRANS  8
 2    IF(ABS(TKAY)-.0001) 3,3,4                                         TRANS  9
 3    ARG = TKAY*72.48                                                  TRANS 10
      GO TO 5                                                           TRANS 11
 4    ARG = 0.                                                          TRANS 12
      DO 6 N=1,1000                                                     TRANS 13
      SAVE = ARG                                                        TRANS 14
      ARG = ARG - (ARG*F(ARG)**2-TKAY)/(F(ARG)*(.11746-ARG*3.1746E-3 - ATRANS 15
     1RG*ARG*5.5115E-4))                                                TRANS 16
      IF(ABS(1.-SAVE/ARG)-1.E-6) 7,7,6                                  TRANS 17
 6    CONTINUE                                                          TRANS 18
 7    IF(ARG+11.) 8,8,5                                                 TRANS 19
 8    EF = 1.75                                                         TRANS 20
      GO TO 10                                                          TRANS 21
 5    DO 15 N=1,10                                                      TRANS 22
      IF(ARG-XFLAM(N)) 24,24,15                                         TRANS 23
 24   NBAR = N                                                          TRANS 24
      GO TO 16                                                          TRANS 25
 15   CONTINUE                                                          TRANS 26
 16   EF = FLAM(NBAR-1)+(ARG-XFLAM(NBAR-1))*(FLAM(NBAR)-FLAM(NBAR-1))/(XTRANS 27
     1FLAM(NBAR)-XFLAM(NBAR-1))                                         TRANS 28
 10   B = .5*EF                                                         TRANS 29
      A = 3.36*(UPRIM/UC(NY,2))**2                                      TRANS 30
      RTH = F(ARG)*(SQRT(B*B+9860.*A)-B)/A                              TRANS 31
      IF(REB*THETA-RTH) 99,50,50                                        TRANS 32
 50   LAMQ = 0                                                          TRANS 33
 99   CONTINUE                                                          TRANS 34
      RETURN                                                            TRANS 35
      END                                                               TRANS 36
```

```
      SUBROUTINE CAPS(ITER,N,CAPG,CAPH,CAPJ,CAPK,SR,SS,SD,SE,SF,VISC,V,UCAPS    1
     1C)                                                                 CAPS    2
      DIMENSION CAPG(100),CAPH(100),CAPJ(100),CAPK(100)                  CAPS    3
      DIMENSION VISC(100,2),V(100,2),UC(100,3),SD(100),SE(100),SF(100)   CAPS    4
      IF(ITER) 4,2,4                                                     CAPS    5
    2 CAPG(N)= SR*V(N,1) - SS*V(N,2)                                     CAPS    6
      CAPH(N)=SR*VISC(N,1)-SS*VISC(N,2)                                  CAPS    7
      CAPJ(N)=SR*(SD(N)*VISC(N+1,1)+SE(N)*VISC(N,1)-SF(N)*VISC(N-1,1))-SCAPS    8
     1S*(SD(N)*VISC(N+1,2)+SE(N)*VISC(N,2)-SF(N)*VISC(N-1,2))            CAPS    9
      CAPK(N)= SR*UC(N,2)-SS*UC(N,3)                                     CAPS   10
      GO TO 6                                                            CAPS   11
    4 CAPG(N)=.5*(CAPG(N)+V(N,1))                                        CAPS   12
      CAPH(N)=.5*(CAPH(N)+VISC(N,1))                                     CAPS   13
      CAPJ(N)=.5*(CAPJ(N)+SD(N)*VISC(N+1,1)+SE(N)*VISC(N,1)-SF(N)*VISC(NCAPS   14
     1-1,1))                                                             CAPS   15
      CAPK(N)=.5*(CAPK(N)+UC(N,1))                                       CAPS   16
    6 CONTINUE                                                           CAPS   17
      RETURN                                                             CAPS   18
      END                                                                CAPS   19
```

```
      SUBROUTINE TERP(YIN,YBASE,VARY,NY,VALUE)
C
C   SUBROUTINE FOR DETERMINING INTERPOLATED VALUE OF THE
C    FUNCTION VARY AT Y = YIN.
C
      DIMENSION YBASE(100),VARY(100)
      IF(YIN-YBASE(NY-1)) 2,3,3
  3   VALUE = VARY(NY)
      GO TO 10
  2   DO 15 N=1,NY
      IF(YIN-YBASE(N)) 24,24,15
   24 NBAR=N
      GO TO 16
  15  CONTINUE
  16  D21=YBASE(NBAR)-YBASE(NBAR-1)
      D31=YBASE(NBAR+1)-YBASE(NBAR-1)
      D32=D31-D21
      D3A=YBASE(NBAR+1)-YIN
      D2A=YBASE(NBAR)-YIN
      DA1=YIN-YBASE(NBAR-1)
      VALUE=D3A*D2A*VARY(NBAR-1)/(D21*D31)+D3A*DA1*VARY(NBAR)/(D21*D32)-
     1D2A*DA1*VARY(NBAR+1)/(D31*D32)
  10  CONTINUE
      RETURN
      END
```

```
      SUBROUTINE YDIFF(NY,ALPHA,BETA,GAMMA,DELTA,SD,SE,SF,C2,C3,C4,Y)    YDIFF  1
      DIMENSION ALPHA(100),BETA(100),GAMMA(100),DELTA(100)               YDIFF  2
      DIMENSION SD(100),SE(100),SF(100),Y(100)                           YDIFF  3
      NV=NY-2                                                            YDIFF  4
      NVP1=NV+1                                                          YDIFF  5
      DO 40 N=2,NV                                                       YDIFF  6
      ALPHA(N) = 2.*(2.*Y(N)-Y(N-1)-Y(N+1))/((Y(N+2)-Y(N-1))*(Y(N+2)-Y(NYDIFF  7
     1+1))*(Y(N+2)-Y(N)))                                                YDIFF  8
      DELTA(N) = 2.*(Y(N+2)+Y(N+1)-2.*Y(N))/((Y(N+2)-Y(N-1))*(Y(N+1)-Y(NYDIFF  9
     1-1))*(Y(N)-Y(N-1)))                                                YDIFF 10
      BETA(N) = (DELTA(N)*(Y(N)-Y(N-1))**3-ALPHA(N)*(Y(N+2)-Y(N))**3)/(YYDIFF 11
     1(N+1)-Y(N))**3                                                     YDIFF 12
      GAMMA(N) = -ALPHA(N)-BETA(N)-DELTA(N)                              YDIFF 13
   40 CONTINUE                                                           YDIFF 14
      DO 39 N=2,NVP1                                                     YDIFF 15
      SD(N) = (Y(N)-Y(N-1))/((Y(N+1)-Y(N-1))*(Y(N+1)-Y(N)))              YDIFF 16
      SE(N) = 1./(Y(N)-Y(N-1))-1./(Y(N+1)-Y(N))                          YDIFF 17
      SF(N) = (Y(N+1)-Y(N))/((Y(N)-Y(N-1))*(Y(N+1)-Y(N-1)))              YDIFF 18
   39 CONTINUE                                                           YDIFF 19
      C2 = Y(3)*Y(4)/(Y(2)*(Y(3)-Y(2))*(Y(4)-Y(2)))                      YDIFF 20
      C3 = -Y(2)*Y(4)/(Y(3)*(Y(4)-Y(3))*(Y(3)-Y(2)))                     YDIFF 21
      C4 = Y(2)*Y(3)/(Y(4)*(Y(4)-Y(3))*(Y(4)-Y(2)))                      YDIFF 22
      RETURN                                                             YDIFF 23
      END                                                                YDIFF 24
```

```
      SUBROUTINE ELDER(BCAP,XSIG,NSIG,UINF,ELD,Y,YMAX)                  ELDER   1
      DIMENSION BCAP(100,3),XSIG(100)                                   ELDER   2
      BCAP(NSIG+1,1)=0.                                                 ELDER   3
      XS=XSIG(1)                                                        ELDER   4
      XZ=XSIG(NSIG+1)                                                   ELDER   5
      IF(XZ-1.) 16,16,1                                                 ELDER   6
    1 DEADL=XZ-XS                                                       ELDER   7
      YMAX=1.E-10                                                       ELDER   8
      SUM=.5*(XSIG(2)-XS)*BCAP(2,1)                                     ELDER   9
      DO 10 N=2,NSIG                                                    ELDER  10
      X=XSIG(N+1)                                                       ELDER  11
      SUM=SUM+.5*(X-XSIG(N))*(BCAP(N+1,1)+BCAP(N,1))                    ELDER  12
      IF(N-NSIG) 4,2,4                                                  ELDER  13
    2 ANGLE=1.5708                                                      ELDER  14
      GO TO 6                                                           ELDER  15
    4 ANGLE=ATAN(SQRT((X-XS)/(XZ-X)))                                   ELDER  16
    6 Y=SUM+BCAP(1,1)*(DEADL*ANGLE-SQRT((X-XS)*(XZ-X)))                 ELDER  17
      IF(Y-YMAX) 10,10,8                                                ELDER  18
    8 YMAX=Y                                                            ELDER  19
   10 CONTINUE                                                          ELDER  20
      ELD=Y/YMAX                                                        ELDER  21
      IF(ABS(ELD)-UINF) 20,20,12                                        ELDER  22
   12 IF(ELD) 14,16,16                                                  ELDER  23
   14 ELD=-UINF                                                         ELDER  24
      GO TO 20                                                          ELDER  25
   16 ELD=UINF                                                          ELDER  26
   20 CONTINUE                                                          ELDER  27
      RETURN                                                            ELDER  28
      END                                                               ELDER  29
```

```
      SUBROUTINE REATT(UC,V,X,Y,MX,NY,RY,DRY,UE,X5,DEL5,MST,REB)
      DIMENSION UC(100,3),V(100,2),Y(100)
      DIMENSION X(300),UE(300,3)
      DIMENSION TAB1(24),TAB2(24),TAB3(24),TAB4(24),XITAB(24)
      DATA TAB1 /24.98,23.29,21.04,19.33,17.61,15.29,13.46,11.54,10.36,9
     1.38,8.35,7.32,6.29,5.31,4.4,3.57,2.22,1.26,.66,.31,.14,.01,0.,0./
      DATA TAB2 /20.05,18.85,17.25,16.04,14.8,13.12,11.77,10.3,9.36,8.65
     1,7.95,7.2,6.43,5.66,4.9,4.18,2.89,1.86,1.11,.62,.32,.04,0.,0./
      DATA TAB3 /16.65,15.8,14.67,13.8,12.91,11.66,10.65,9.48,8.71,8.11,
     17.59,7.01,6.41,5.77,5.13,4.5,3.31,2.28,1.48,.9,.51,.09,.01,0./
      DATA TAB4 /10.12,10.05,9.93,9.78,9.58,9.17,8.72,8.08,7.6,7.2,6.85,
     16.53,6.18,5.79,5.36,4.91,3.98,3.05,2.21,1.5,.95,.22,.03,0./
      DATA XITAB /.0001,.0002,.0005,.001,.002,.005,.01,.02,.03,.04,.05,.
     106,.07,.08,.09,.1,.12,.14,.16,.18,.2,.25,.3,.35/
    3 FORMAT(///40X,23HAT REATTACHMENT, BETA =E13.5)
      MOUT=6
      RTR=SQRT(REB)
      UC(1,2)=0.
      UC(1,3)=0.
      V(1,1)=0.
      V(1,2)=0.
      DO 5 M=1,MX
      IF(X5-X(M)) 4,4,5
    4 MST=M+2
      GO TO 6
    5 CONTINUE
    6 XA=X(MST-2)
      XB=X(MST-1)
      UA=UE(MST-2,1)
      UB=UE(MST-1,1)
      ZA=ALOG(UA*DEL5*REB)
      PGRAD=2.*(UA-UB)/((UA+UB)*(XB-XA))
      BETM2=(.0974-SQRT(DEL5*PGRAD))/(.0249+.004565*ZA)
      IF(BETM2-1.) 8,7,7
    7 BETM2=1.
      GO TO 10
    8 IF(BETM2-.3) 9,9,10
    9 BETM2=.3
   10 BETA=1./(BETM2*BETM2)
      WRITE(MOUT,3) BETA
      AGAM=.0974*BETM2-.0249/BETA
      BGAM=.004565/BETA
      AH=1.-(5.3+3.9*BETM2)*(.0974-.0249*BETM2)
      BH=BETM2*(5.3+3.9*BETM2)*.004565
      GAMA=AGAM-BGAM*ZA
      DERIV=UA*REB*EXP(-ZA)*GAMA*GAMA*(1.+BETA*(1.+AH+BH*ZA))/(AH+BH+BH*
     1ZA)
      ZB=ZA+DERIV*(XB-XA)
      DELB=EXP(ZB)/(UB*REB)
      GAMB=AGAM-BGAM*ZB
      DELL=.35*DELB*RTR*BETM2/GAMB
   11 IF(DELL-Y(NY-3)) 14,12,12
   12 RY=RY+DRY
      CALL YSET(RY,Y(2),NY,Y)
      GO TO 11
```

```
14    IF(BETA-4.) 102,101,101                                          REATT 56
101   TERPB=1.-4./BETA                                                 REATT 57
      INDEX=3                                                          REATT 58
      GO TO 110                                                        REATT 59
102   IF(BETA-2.) 104,103,103                                          REATT 60
103   TERPB=.5*BETA-1.                                                 REATT 61
      INDEX=2                                                          REATT 62
      GO TO 110                                                        REATT 63
104   TERPB=BETA-1.                                                    REATT 64
      INDEX=1                                                          REATT 65
110   K=0                                                              REATT 66
      TERP1=1.-TERPB                                                   REATT 67
50    K=K+1                                                            REATT 68
      GO TO (16,17,99),K                                               REATT 69
16    G=GAMA                                                           REATT 70
      DELTA=DELS                                                       REATT 71
      UEDGE=UA                                                         REATT 72
      L=3                                                              REATT 73
      GO TO 18                                                         REATT 74
17    G=GAMB                                                           REATT 75
      DELTA=DELB                                                       REATT 76
      UEDGE=UB                                                         REATT 77
      L=2                                                              REATT 78
18    XICO=G/(DELTA*RTR*BETM2)                                         REATT 79
      UCOW=RTR*(UEDGE*G)**2                                            REATT 80
      EFCO=G/BETM2                                                     REATT 81
      NLAM=NY                                                          REATT 82
      DO 75 N=2,NY                                                     REATT 83
      XI=Y(N)*XICO                                                     REATT 84
      IF(XI-.35) 20,19,19                                              REATT 85
19    UC(N,L)=UEDGE                                                    REATT 86
      GO TO 75                                                         REATT 87
20    CALL TERPF(XI,INDEX,TAB1,TAB2,TAB3,TAB4,XITAB,FP1)               REATT 88
      INDP1=INDEX+1                                                    REATT 89
      CALL TERPF(XI,INDP1,TAB1,TAB2,TAB3,TAB4,XITAB,FP2)               REATT 90
      FP=TERP1*FP1+TERPB*FP2                                           REATT 91
      UC(N,L)=UEDGE*(1.-EFCO*FP)                                       REATT 92
      IF(N-NLAM) 21,75,75                                              REATT 93
21    ALTER=UCOW*Y(N)                                                  REATT 94
      IF(ALTER-UC(N,L)) 33,33,32                                       REATT 95
32    UC(N,L)=ALTER                                                    REATT 96
      GO TO 75                                                         REATT 97
33    NLAM=N                                                           REATT 98
75    CONTINUE                                                         REATT 99
      GO TO 50                                                         REATT100
99    DO 60 K=2,3                                                      REATT101
      SAVE2=0.                                                         REATT102
      DO 60 N=3,NY                                                     REATT103
      SAVE1=UC(N-1,K)                                                  REATT104
      UC(N-1,K)=(SAVE2+SAVE1+UC(N,K))/3.                               REATT105
60    SAVE2=SAVE1                                                      REATT106
      DUDX=0.                                                          REATT107
      COD=.5/(XB-XA)                                                   REATT108
      DO 65 N=2,NY                                                     REATT109
      DUDXP=COD*(UC(N,2)-UC(N,3))                                      REATT110
```

```
      V(N,1)=V(N-1,1)-(Y(N)-Y(N-1))*(DUDXP+DUDX)                        REATT111
      V(N,2)=V(N,1)                                                     REATT112
 65   DUDX=DUDXP                                                        REATT113
      RETURN                                                            REATT114
      END                                                               REATT115
```

```
      SUBROUTINE ELPIT(ALPH1,ALPH2,EMI,TORF,THETZ,UINF,DXI,CMPA,CMPAS)
      SAVET=ALPH1
      STEP=TORF*DXI
      SINS=SIN(STEP)
      COSS=COS(STEP)
      CONST=2.*EMI*(UINF/TORF)**2
      ALPH1=THETZ+(ALPH1-THETZ)*COSS+ALPH2*SINS/TORF+CONST*(2.*CMPA-CMPA
     1S)*(1.-COSS)+CONST*(CMPAS-CMPA)*(SINS-STEP*COSS)/(TORF*DXI)
      ALPH2=ALPH2*COSS-TORF*SINS*(SAVET-THETZ)+CONST*(CMPA-CMPAS)*(1.-CO
     1SS)/DXI+CONST*CMPA*TORF*SINS
      RETURN
      END
```

```
      SUBROUTINE VWASH(BARG,H,S,NVOR,X1,UINF,VZIP,XGAM,NGP1,DXI)
      DIMENSION VZIP(30),XGAM(30)
      DO 10 N=1,NGP1
      DIFF=XGAM(N)-X1
      SUM=0.
      DO 5 K=1,NVOR
      SUM=SUM+DIFF/(DIFF*DIFF+H)
    5 DIFF=DIFF-S
   10 VZIP(N)=VZIP(N)+SUM*BARG
      RETURN
      END
```

```
      SUBROUTINE WASH(XGAM,NGAM,TIME,ALPH1,ALPH2,HEAVE,AROT,FREQF,PHIH,U
     1INF,CAMBR,NF,VZIP,MOTR,INDV)
      DIMENSION XGAM(30),VZIP(30),CAMBR(24)
      NGP1 = NGAM+1
      ANGLE = FREQF*TIME
      GO TO (108,120), INDV
  108 GO TO (110,120),MOTR
  110 CONST =-ALPH2*COS(ANGLE)*UINF+HEAVE*COS(ANGLE+PHIH)+ALPH1*UINF
      FACT =-ALPH2*FREQF*SIN(ANGLE)*UINF
      GO TO 130
  120 CONST=UINF*ALPH1+HEAVE
      FACT=-UINF*ALPH2
  130 DO 10 M=1,NGP1
      X=XGAM(M)
      THETA = ARCT(X)
      SUM=0.
      COUNT=0.
      DO 20 N=1,NF
      COUNT=COUNT+1.
   20 SUM=SUM+COUNT*CAMBR(N)*COS(COUNT*THETA)
      IF(M-1) 2,4,2
    2 IF(NGP1-M) 3,4,3
    4 SUM = SUM + SUM
      GO TO 50
    3 COUNT = 0.
      COTT = X/SIN(THETA)
      DO 30 N=1,NF
      COUNT = COUNT+THETA
   30 SUM=SUM+COTT*CAMBR(N)*SIN(COUNT)
   50 VZIP(M) = UINF*SUM+CONST+FACT*(AROT-X)
   10 CONTINUE
      RETURN
      END
```

APPENDIX B

DETERMINATION OF COUPLING PARAMETERS

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DETERMINATION OF COUPLING PARAMETERS

The characteristic equation for the rotor blade is

$$\sum_{k=0}^{3} B_{2k} \lambda^{2k} = 0$$

where

$$B_0 = f_0 - \frac{\bar{\omega}_{\phi}^2 T_{\beta\theta}^2}{M_{\beta\beta} M_{\theta\theta}} - \frac{\bar{\omega}_{\beta}^2 T_{\phi\theta}^2}{M_{\phi\phi} M_{\theta\theta}}$$

$$B_2 = f_2 + 2 \frac{\bar{\omega}_{\phi}^2 M_{\beta\theta} T_{\beta\theta}}{M_{\beta\beta} M_{\theta\theta}} + 2 \frac{\bar{\omega}_{\beta}^2 M_{\phi\theta} T_{\phi\theta}}{M_{\phi\phi} M_{\theta\theta}}$$

$$- \frac{T_{\beta\theta}^2}{M_{\beta\beta} M_{\theta\theta}} - \frac{T_{\phi\theta}^2}{M_{\phi\phi} M_{\theta\theta}}$$

$$B_4 = f_4 - \frac{\bar{\omega}_{\phi}^2 M_{\beta\theta}}{M_{\beta\beta} M_{\theta\theta}} - \frac{\bar{\omega}_{\beta}^2 M_{\phi\theta}^2}{M_{\phi\phi} M_{\theta\theta}}$$

$$+ 2 \frac{M_{\beta\theta} T_{\beta\theta}}{M_{\beta\beta} M_{\theta\theta}} + 2 \frac{M_{\phi\theta} T_{\phi\theta}}{M_{\phi\phi} M_{\theta\theta}}$$

$$B_6 = 1 - \frac{M_{\beta\theta}^2}{M_{\beta\beta} M_{\theta\theta}} - \frac{M_{\phi\theta}^2}{M_{\phi\phi} M_{\theta\theta}}$$

in which

$$f_0 = \bar{\omega}_\beta^{\,2} \; \bar{\omega}_\phi^{\,2} \; \bar{\omega}_\theta^{\,2}$$

$$f_2 = \bar{\omega}_\beta^{\,2} \; \bar{\omega}_\phi^{\,2} + \bar{\omega}_\beta^{\,2} \; \bar{\omega}_\theta^{\,2} + \bar{\omega}_\phi^{\,2} \; \bar{\omega}_\theta^{\,2}$$

$$f_4 = \bar{\omega}_\beta^{\,2} + \bar{\omega}_\phi^{\,2} + \bar{\omega}_\theta^{\,2}$$

The characteristic equation for the two-dimensional system is found to be

$$\sum_{k=0}^{3} D_{2k} \; \lambda^{2k} = 0$$

where

$$D_0 = f_0 - \bar{\omega}_\phi^{\,2} \, h_a \, a_1^{\,2} - \bar{\omega}_\beta^{\,2} \, h_b \, b_1^{\,2}$$

$$D_2 = f_2 - \bar{\omega}_\phi^{\,2} \, g_a \, \bar{x} \, a_1 - \bar{\omega}_\beta^{\,2} \, g_b \, \bar{x} \, b_1$$

$$- h_a \, a_1^{\,2} - h_b \, b^{\,2}$$

$$D_4 = f_4 - C_4 \, \bar{x}^2 - g_a \, \bar{x} \, a_1 - g_b \, \bar{x} \, b_1$$

$$D_6 = 1 - C_6 \, \bar{x}^2$$

in which

$$h_a = \frac{M_{\beta\beta}}{R^2 \, M_{\theta\theta}} \qquad h_b = \frac{M_{\phi\phi}}{M_{\theta\theta}}$$

$$g_a = 2\,h_a\,A_1 \qquad\qquad g_b = 2\,h_b\,A_2$$

$$C_4 = \bar{\omega}_{\phi}^{\,2}\,h_a\,A_1^{\,2} + \bar{\omega}_{\beta}^{\,2}\,h_b\,A_2^{\,2}$$

$$C_6 = h_a\,A_1^{\,2} + h_b\,A_2^{\,2}$$

$$a_1 = A_1\left(\bar{\omega}_{\beta}^{\,2}\,l_{s_1} + r_m\,\bar{\omega}_{\phi}^{\,2}\,l_{s_2}\right) - B\,\bar{\omega}_{\phi}^{\,2}\,l_{s_2}$$

$$b_1 = A_2\left(\bar{\omega}_{\beta}^{\,2}\,l_{s_1} + r_m\,\bar{\omega}_{\phi}^{\,2}\,l_{s_2}\right) + B\,\bar{\omega}_{\phi}^{\,2}\,l_{s_2}$$

Equating D_0/D_6 to B_0/B_6, D_2/D_6 to B_2/B_6 and D_4/D_6 to B_4/B_6 provides three relations in the three unknowns $\bar{x}$, l_{s1}, and l_{s2}. If a_1 and b_1 are eliminated, the following equation for $\bar{x}$ is obtained:

$$(r_1\,t_2 - r_2\,t_1)^2 + (r_1\,s_2 - r_2\,s_1)(t_2\,s_1 - t_1\,s_2) = 0$$

where

$$r_1 = -\left[h_a + \frac{h_b\,g_a^{\,2}}{g_b^{\,2}}\right] \qquad\qquad r_2 = \left[\frac{\bar{\omega}_{\phi}^{\,2}}{\bar{\omega}_{\beta}^{\,2}} - 1\right] h_a$$

$$s_2 = \left(\bar{\omega}_{\beta}^{\,2} - \bar{\omega}_{\phi}^{\,2}\right) g_a\,\bar{x}\,, \qquad s_1 = s_2 + \frac{2\,h_b\,g_a\,F}{g_b^{\,2}\,\bar{x}}$$

$$t_1 = (1 - C_6\,\bar{x}^2)\,B_2/B_6 - f_2 + \bar{\omega}_{\beta}^{\,2}\,F + \frac{h_b\,F^2}{g_b^{\,2}\,\bar{x}^2}$$

$$t_2 = (1 - C_6 \bar{x}^2)(B_2 - B_0/\bar{\omega}_\beta{}^2)/B_6 - f_2 + \bar{\omega}_\beta{}^2 F + f_0/\bar{\omega}_\beta{}^2$$

in which

$$F = f_4 - B_4/B_6 + (B_4 C_6/B_6 - C_4) \bar{x}^2$$

With some algebraic manipulation, a polynomial of fourth degree in $\bar{x}^2$ can be extracted from that equation. The value of $\bar{x}$ is taken to be the square root of the smallest positive root of that polynomial. The original equations are then used to solve for a_1 and b_1, from which l_{s1} and l_{s2} are readily obtained.

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