FINITE ELEMENT ANALYSIS OF A COMPOSITE MATERIAL INTERFACE

A Technical Report
by
Kenneth H. Murray

Prepared for the

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
LANGLEY RESEARCH CENTER
HAMPTON, VIRGINIA 23665

Under
Master Contract Agreement NAS1-9434
Task Order No. 45

SCHOOL OF ENGINEERING
OLD DOMINION UNIVERSITY
NORFOLK, VIRGINIA

August 1973

Technical Report 73–C1

NASA CR-132341

FINITE ELEMENT ANALYSIS OF A COMPOSITE MATERIAL INTERFACE

A FINAL TECHNICAL REPORT

By

Kenneth H. Murray

Prepared for the
NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
Langley Research Center
Hampton, Virginia 23665

Under
Master Contract Agreement No. NAS1-9434
Task Order No. 45

School of Engineering
Old Dominion University
Technical Report 73-C1

Submitted by the
Old Dominion University Research Foundation
P.O. Box 6173
Norfolk, Virginia 23508

August 1973

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

FINITE ELEMENT ANALYSIS OF A COMPOSITE MATERIAL INTERFACE

By

Kenneth H. Murray*

SUMMARY

A finite element model of a composite material interface is developed to study the influence of the interface on the thermal strain in the composite. A plane stress model is used with an axisymmetric model as a check. The interface thickness, thermal coefficient, modulus, Poisson's ratio and the percent of mineral in the composite are variables in the study. The results confirmed the usability of the finite element model in studying the polymer-mineral interface.

INTRODUCTION

Composite materials can be considered to be a combination of two separate phases or materials with an interphase or interface between them. The interface transmits the forces and displacements between the phases and thus makes possible the combined action of the phases[[1]. These forces and displacements have been analyzed by means of finite elements to estimate the matrix stresses [2-4] as well as the effect of the interface [5-7]. In all of these analyses, perfect bonding and zero thickness interface between the phases was assumed, although such an assumption is physically unrealistic.

Consequently, a finite element analysis was made of a composite material

---

* Associate Professor of Engineering, School of Engineering, Old Dominion University, Norfolk, Virginia 23508.

consisting of a continuous polymer phase (the matrix) containing a discontinuous mineral phase (the filler). The phases were separated by a finite thickness interface to which a range of properties could be assigned. The effects of the interface properties on the thermal strain of the composite were determined and are reported in the next section.

MODELS AND ANALYSIS

Composite Model

The model polymer-mineral composite material had properties which approximated those of an epoxy-glass system in which the glass is in the form of small, uniformly sized, beads. The properties were:

	Polymer	Mineral
Elastic modulus, GN/m^2	3.047	68.95
Thermal expansion coef., K^{-1}	88.6×10^{-6}	8.5×10^{-6}
Poisson's ratio	0.44	0.22
Specific gravity	1.15	2.45
Bead diameter, μm	---	125

A composite with these properties was being investigated concurrently in an experimental program at NASA Langley Research Center.

Finite Element Model

The finite element model of the composite was basically a plane stress model of a unit cell of polymer surrounding a single glass bead, separated by a finite thickness interface (Figure 1). The bead was modeled as a hexagon because only straight line boundaries were permitted with the plane stress elements available in the computer program which was used in the analysis. The interface, which can vary in thickness and material properties, is essentially a geometrical extension of the hexagon. The bead diameter was taken as the distance between opposite sides of the hexagon. Since the diameter was fixed (125 μm), different volume percentages of mineral were obtained by changing the size of the unit cell. Thus a large mineral

volume percentage (e.g. 60%) required a smaller unit cell than did a small percentage (e.g. 5%).

Using the basic unit cell, four finite element models were constructed. The first model (Figure 2) had a flexible link element as the interface. This link has been used successfully to model the bonding between concrete and reinforcing steel [8]. The material properties of this interface element were independent of the polymer and mineral because of the linking arrangement, so these properties could be varied over an extremely wide range. The second model used the same geometry except for the interface element (Figure 3). The interface element was a normal element geometrically but had its material properties bounded by the material properties of the polymer and mineral; e.g. it is unreasonable to expect the interface modulus to be larger than the mineral or smaller than the polymer modulus. The third model was made up of two of the second models stacked on top of each other (Figure 4). This was done to determine if the strain for two unit cells would differ from that for one cell. The fourth model was an axisymmetric idealization of the bead and polymer (Figure 5). This model used the same type of interface element as the second model. The idealization shown in Figure 5 was rotated 360° about the z-axis to develop a three-dimensional solid.

Analysis

The four models were analyzed using the finite element method by way of NASTRAN. NASTRAN, an acronym for NASA Structural Analysis, is a general purpose finite element computer program for structural analysis. A discussion of the finite element method and of NASTRAN is given in Appendix A.

The loading system for the models was a temperature change from 300 K (approximately room temperature) to 400 K. In the finite element method the thermal displacements are found by defining a set of nodal loads which approximate the thermal load on an element. The nodal loads from each element are summed to obtain the total load vector. This approximation along with the basic idealization of polymer-mineral composite are necessary to analyze the system using finite elements.

The boundary conditions for the first three models were the same

(Figures 3 and 4). The x-axis nodes (16, 17, 21, and 22) were restrained in the y-direction and the y-axis nodes (18, 22, and 23) were restrained in the x-direction. The boundary conditions approximated those of a piece of the composite two unit cells wide, resting on a plate, with the composite axis of symmetry coincident with the y-axis. As a result of the symmetry only half of the piece (one unit cell) had to be analyzed. The boundary conditions for the fourth (axisymmetric) model were zero displacement along the r-axis in the y-direction (see Figure 5).

The variables used in the analysis are listed in Table I. One composite variable, the volume percent of mineral, was included to confirm the results obtained with the other variables; if the mineral volume is increased, the thermal strain should decrease [9]. Four interface variables were used, the interface thickness plus the interface properties of elastic modulus E, coefficient of thermal expansion α, and Poisson's ratio μ. The interface properties were assumed to change little over the 300-400 K temperature range investigated, and indeed, the large drop in the epoxy modulus does not occur until a temperature of 440 K is reached [10]. The interface properties were assigned values equal to the polymer, the mineral, or the average of the two values. The particular value is indicated in the text and the tables by a lower case suffix p (polymer), m (mineral), or a (average). Thus the designation αa, Em means that the average value of the coefficient of thermal expansion (48.55×10^{-6}/K) and the mineral value of the elastic modulus (68.95 GN/m^2) were assigned to the interface. Only the interface properties were varied; the polymer and mineral always were assigned their respective values listed in Table I.

The geometry of the models, the variable mineral volume, and the variable interface properties created a major data preparation problem. To reduce the effort and the possible errors associated with the input data, a computer program was written to punch the NASTRAN input deck. This program is given in Appendix B along with an outline of its operation.

RESULTS AND DISCUSSION

The results of the finite element program (NASTRAN) are displacements

at the nodes and stresses within the element. The displacements were divided by the length (height or width) of the unit cell to obtain strain values. These strain values are listed in Tables II through VII. An analysis of variance (ANOVA) of selected data is given in Tables VIII through X.

Preliminary Results

Model One: No results are presented in this report because the model never gave reasonable results. The material properties of the linking element were varied systematically from zero to 10^{20} and no combination of values gave reasonable results.

Poisson's ratio: Table II shows some of the results when the Poisson's ratio was varied in Model Two. It is obvious from the table that this quantity does not affect the thermal strain values, therefore it was eliminated from further study and an average value was used ($\mu = 0.33$) for the interface.

Interface thickness: Most researchers agree that the interface is a thin layer between the mineral and the polymer. Using this, the larger value of F did not appear valid. The remainder of the study uses only F = 1 and 10 μm.

Model Three: The two cell model gave strain values identical to the one cell model (Model Two). The mineral content was 34% and the interface was 10 μm with all combinations of αm, αp, Ep, and Em.

Variables examined: The final results are based on the following variables with their discrete values listed in parentheses. All combinations were studied for:

Volume Percent of Mineral - PER (5, 34, and 60%)
Interace thickness - F (1 and 10 μm)
Interface thermal coefficient - $\underline{\alpha}$ (αp, αm, and αa)
Interface modulus - E (Ep, Em, and Ea)
Models two and four

The reference model was Model Two with 34% mineral and an interface thickness of 1 μm.

Final Results

Mineral content: Comparison of data in Tables III, IV, and V and the ANOVA in Table VIII indicates that the mineral content is a very significant variable. The thermal strain in the model decreases with increasing mineral content. This can be predicted because the mineral is less responsive to temperature change than the polymer, thus the more mineral the less thermal strain.

Interface thickness: Comparison of data in Tables IV and VI and the ANOVA in Table IX indicates that the interface thickness is a significant variable.

Model: Comparison of data in Tables IV and VII and the ANOVA in Table X for Models Two and Four indicates no apparent significance for the model. This, of course, is good because now only the plane stress model needs to be studied in detail.

Strain location: Comparison of Tables III through VII and the ANOVA in Tables VIII and IX indicates that the strain location is a significant variable. This is somewhat disturbing because it should not be significant. The variation in strain along the boundary is plotted in Figure 6. The low strain values in the middle region near the mineral bead are caused by the mineral's presence.

Table XI gives the results of the basic model with the boundary conditions varied at the corner node (22). Standard conditions fix the node in both the x- and y-directions. To help explain the nonuniform strain across the top and side of the model, this node (22) was released first in the x- and then in the y-direction. Only the two outside nodes and node 22 are presented, to observe the difference between them. It is quite obvious that the boundary conditions affect the final strains. The only way to eliminate this effect is to increase the number of elements near the boundaries to isolate the effect.

Interface material properties: Observation of Table IV for Model Two, F = 1 μm and 34% mineral, indicates that as α and E increase the thermal strain increases. The thermal coefficient α has the most effect as can be seen from ANOVA Table XII. It should be noted that the total effect of both variables is less than 1% in general. This presents critical questions as to the real effect of the interface on the final thermal strains.

CONCLUDING REMARKS

The basic procedure used in this study was to model the polymer-mineral composite as a plane stress elasticity problem modeled with finite elements. A unit cell with one bead and a variable interface was idealized as plane stress triangles and quadrilaterals. The interface was varied in thickness and material properties through a realistic range to obtain data on its effect on the thermal strain of the unit cell during a temperature change.

For the standard model used in this study (34% mineral, interface thickness approximately 1% of the bead diameters, 1 μm, and interface material properties equal to the polymer, mineral and their average) the following conclusions can be drawn:

1. The major factors influencing the thermal strain of the composite are:
   a) Mineral content
   b) Interface thickness
   c) Interface thermal coefficient
2. The thermal strain decreases with increasing mineral content.
3. The thermal strain decreases with increasing interface stiffness.
4. More elements and nodes are needed to eliminate the variation in displacement if the plane stress elements are used.
5. Experimental results are needed to establish the correct interface properties.
6. The plane stress finite element idealization does a reasonable job representing the polymer-mineral composite, but needs some more refinement.
7. A three-dimensional representation needs to be made to evaluate the usefulness of the plane stress model beyond what was done here.

ACKNOWLEDGEMENTS

The author wishes to express his appreciation to Mr. Howard Price for his constant help throughout this study.

REFERENCES

1. Holliday, L.: "Geometrical Considerations and Phase Relationships" Holliday, L., (Ed.): Composite Materials, Elsevier Publishing Co., 1966, pp. 44.

2. Chen, P.E. and T.B. Lewis: "Stress Analysis of Ribbon Reinforced Composites", Polymer Engineering and Science, Vol. 10, No. 1, Jan. 1970, p. 43.

3. Iqbal, M.A. and E.M. Krokosky: "Interaction Stresses in Composite Systems", ASCE Journal Engn. Mechanics Division, Vol. 96, No. EM6, Dec. 1970, p. 825.

4. MacLaughlin, T.F. and R.M. Barker: "Effect of Modulus on Stress Near a Discontinuous Fiber", Society for Experimental Stress Analysis, Spring Meeting May 1971, Paper 1826A.

5. Carrora, A.S. and F.J. McGarry: "Matrix and Interface Stresses in a Discontinuous Fiber Composite Model", J. Composite Materials, Vol. 2, No. 2, April 1968, p. 222.

6. Sahu, S. and L.J. Broutman: "Mechanical Properties of Particulate Composites", Polymer Engineering and Science, Vol. 12, No. 2, March 1972, pp. 91-100.

7. Broutman, L.J. and B.D. Agarwal: "A Theoretical Study of the Effect of the Interface on Composite Toughness", 28th Annual Technical Conf., 1973, Reinforced Plastics/Composites Institute.

8. Murray, K.H.: "Theoretical and Experimental Analysis of Crack Formation in Reinforced Concrete", Ph.D. Thesis, Virginia Polytechnic Institute and State University, Oct. 1968.

9. Nielsen, L.E.: "Mechanical Properties of Particulate Filled Systems", J. Composite Materials, Vol. 1, 1967, p. 100.

10. Woodward, A.E. and J.A. Sauer: Physics and Chemistry of the Organic Solid State, Vol. II. Interscience Publishers Inc., (1965), pp. 710.

Appendix A

Finite Element Analysis - NASTRAN

I. Basic Procedure (Reference A.1)

A general continuous body can be idealized using the finite element method by dividing it into a discrete number of small elements. These elements are interconnected only at a finite number of points called nodes or joints. At each node a finite number of degrees of freedom, usually displacements and rotations, are prescribed. Thus the real system with displacements continuous throughout the body is idealized by a discrete system with displacements specified only at nodes. This approach to structural analysis has been used on framed structures for quite some time, but in the last twenty years it has been extended to include two- and three-dimensional continua. Because it is a systematic procedure it is very well suited for implementation on digital computers. It is this fact plus the versatility of the method that has led to its great popularity.

The first step in the analysis procedure is to idealize the continuum as a discrete number of finite elements. For each element a functional relationship between the internal displacements and the nodal displacements is written. Next the stress-strain-displacement relationships are developed and used to write the potential energy of the element. From the potential energy equation the stiffness matrix for the element is derived. The stiffness matrices of all elements are

then summed to determine the equations of equilibrim for the continuum. These equilibrium equations are then solved for the nodal displacements using modern numerical techniques. Once the nodal displacements have been computed, the forces, stresses and strains in each element are found.

II. Thermal Stress Analysis

The temperature load is input as a nodal force which is equivalent to the thermal strain associated with the temperature change. The thermal stress analysis by the finite element method can be best summarized by the following steps:

1) Idealization

The continuum is subdivided into a system of discrete or finite elements, interconnected at a discrete number of nodes. The unknown parameters of the problem are normally the displacements or rotations at each node.

2) Functional

A displacement function is selected which expresses the internal displacement of the element in terms of the unknown nodal displacements:

$$\{f\} = [N] \{\delta\}_e \qquad (1)$$

3) Strain

The state of strain within an element is expressed in terms of the nodal displacements:

$$\{\varepsilon\} = [B] \{\delta\}_e \quad . \qquad (2)$$

4) Stress

The state of stress within an element is expressed in terms of the nodal displacements using the element strains, initial strains and constitutive properties of the material. In equation form:

$$\{\sigma\} = [D]\ (\{\varepsilon\} - \{\varepsilon_o\}) \quad , \tag{3a}$$

or

$$\{\sigma\} = [D]\ ([B]\ \{\delta\}_e - \{\varepsilon_o\}) \ . \tag{3b}$$

5) Total Potential Energy

The total potential energy is expressed in terms of the nodal displacements using the stress and strain functions:

$$\Pi = U + V \tag{4}$$

where U is the internal strain energy

$$U = 1/2 \int_{V_e} \{\varepsilon\}^T \{\sigma\}\ dV_e$$

or after substituting equations (2) and (3b)

$$U = 1/2 \int_{V_e} \{\delta\}_e^T\ [B]^T[D][B]\{\delta\}_e dV_e - 1/2 \int_{V_e} \{\delta\}_e^T\ [B]^T[D]\{\varepsilon_o\} dV_e . \tag{5}$$

The potential energy of the applied forces can be written as

$$V = \int_{V_e} - \{f\}^T\{p\}\ dV_e - \int_S \{f\}^T\{g\}\ dS - \{\delta\}_e^T\ F_n \tag{6}$$

where $\{p\}$ = body force per unit volume

$\{g\}$ = surface tractions per unit area

S = surface (area) over which $\{g\}$ acts

$\{F_n\}$ = applied nodal forces

For the case of thermal strain V = 0 and is omitted from the following. The total potential energy then is U,

$$\Pi = 1/2 \int_e \{\delta\}_e^T [B]^T [D] [B] \{\delta\}_e dV_e - 1/2 \int_{V_e} \{\delta\}_e^T [B]^T [D] \{\varepsilon_o\} dV_e \quad (7)$$

6) Minimize Total Potential Energy

The equilibrium of the system can be established by minimizing the total potential energy of the system. In equation form this can be stated:

$$\frac{\partial \Pi}{\partial \{\delta\}_e} = 0$$

Performing the minimization, after using equation (1), yields

$$\int_{V_e} [B]^T [D] [B] \{\delta\}_e dV_e - \int_{V_e} [B]^T [D] \{\varepsilon_o\} dV_e = 0. \quad (8)$$

7) Nodal Forces

In the minimization above four load types are usually present, but here only one is present. Each must be converted into equivalent nodal forces. The second integral in equation (8) represents nodal loads resulting from initial strains or

$$\{F_o\} = \int_{V_e} [B]^T [D] \{\varepsilon_o\} dV_e \; . \quad (9)$$

The equivalent nodal force then is

$$\{F\}_e = \{F_o\} \tag{10}$$

8) Stiffness Matrix

Another outgrowth of the minimization is the basic element relationship between load and displacement or the stiffness matrix [k]. The basic equilibrium equation in discrete cooredinates can be written as

$$\{F\}_e = [k]\{\delta\}_e$$

and comparing it to equation (8) defines

$$[k] = \int_{V_e} [B]^T[D][B]dV_e \quad . \tag{11}$$

9) Assemblage

Each element has now been reduced to a set of equilibrium equations. The continuum can now be represented by the sum of the equilibrium equations. If N denotes the total number of elements, then

$$\{F\} = [K]\{\delta\} \tag{12}$$

where

$$\{F\} = \sum_{e=1}^{N} \{F\}_e \tag{13a}$$

$$[K] = \sum_{e=1}^{N} [k]_e \tag{13b}$$

and

$$\{\delta\} = \sum_{e=1}^{N} \{\delta\}_e \quad . \tag{13c}$$

10) Boundary Conditions

The external constraints are applied using modern numerical technique procedures.

11) Solution

The unknown displacements of equation (12) with constraints are obtained using appropriate numerical procedures to solve the equations.

12) Stress Recovery

The displacements are used to determine the stress and strains in the continuum using equations (2) and (3a).

This entire procedure has been programed many times. The program used in this study is NASA's Structural Analysis Program - NASTRAN. The input for NASTRAN must contain the following information:

1) Type of analysis (static)
2) Type of element (plane stress)
3) Nodal point locations
4) Element connections
5) Material properties
6) Loads (temperature change)
7) Boundary conditions
8) Output requirements

Most of the input is quite easy, but the nodal locations and element connection cards can be difficult. For this reason a FORTRAN program was written to generate the geometry of the

polymer-mineral composite and then convert this into NASTRAN input format for punching. Appendix B gives a detailed description of this procedure.

REFERENCES

A.1 Thornton, E.A. and K.H. Murray, Fundamentals of the Finite Element Method, Old Dominion University, Norfolk, Va., July 1973.

Appendix B

NASTRAN Input Generation Program

A FORTRAN program was written to generate the geometry of the mineral polymer composite and punch it in NASTRAN format. The Executive Control and Case Control decks are read into this program as data for the final NASTRAN deck. Also read into this program are the GRDSET, PROPERTY, MATERIAL and TEMP cards from the Bulk Data deck. All other cards are generated and punched. The mineral bead diameter, mineral content in percent volume, interface thickness and the number of basic blocks in the model are also read into this program. This allows great flexibility in defining the variables of this polymer-mineral composite study.

The program is listed on the following pages with a typical data deck.

```
      DIMENSION DECK (30,18),GRIDX(35),GRIDY(35),IE(22),JE(22),KE(22)
      DIMENSION LE(22)
C     B = BEAD RADIUS IN MICRONS
C     F = INTERFACE THICKNESS
C     PER = PERCENT BY VOLUME OF BEADS
C     A = INSCRIBED CIRCLE OF LARGE HEX
C     R = RADIUS TO NODE OF LARGE HEX
C     IN = LOGICAL UNIT FOR INPUT
C     IOO = LOGICAL UNIT FOR PUNCHED OUTPUT
C     IO = LOGICAL UNIT FOR PRINTED OUTPUT
C     NO = NUMBER OF HEXES
C        MAXIMUM OF 2 FOR MODEL ONE AND 1 FOR MODEL TWO
C     NDEC = NUMBER OF LEADING CARDS
C     BHEX = RADIUS OF BEAD HEX
C     ITYPE = TYPE OF INTERFACE
C        0 FOR MODEL ONE AND 1 FOR MODEL TWO
C
C      SET UP INPUT-OUTPUT UNITS
C
      IN=2
      IO=3
      IOO=7
      READ (IN,1) B,F,PER,NO,NDEC ,ITYPE
    1 FORMAT (3F10.0,3I2)
      NDEC1=NDEC+1
C
C      READ IN NASTRAN EXECUTIVE CONTROL DECK,
C                      CASE CONTROL DECK AND
C                      THE FOLLOWING BULK DATA CARDS
C                         GRDSET
C                         PROPERTY
C                         MATERIAL
C                         TEMP.
C
      WRITE (IO,600)
  600 FORMAT(1H1)
      DO 15 I=1,NDEC1
      READ (IN,11)(DECK(I,J),J=1,18)
      WRITE(IO,11)(DECK(I,J),J=1,18)
   15 CONTINUE
   11 FORMAT (18A4)
      BHEX = 1.15469*B
      R = SQRT(75.0/PER)*B/.86603
      GRIDX(1) = 0.
      GRIDY(1) = 0.
      IF (ITYPE) 300,19,300
C
C      LINKAGE ELEMENT  MODEL ONE
C
   19 DO 20 I=2,13
   20 GRIDX(I)=BHEX
      DO 21 I=14,25
   21 GRIDX(I)=BHEX+F
      DO 22 I=26,31
   22 GRIDX(I)=R
      JJ=1
```

```
      I=3
      GRIDY(2)=0
   23 GRIDY(I)=GRIDY(I-1)+60.
      IF (I-31) 24,30,30
   24 IF (  JJ-5        ) 25,26,27
   25 I=I+1
      JJ=JJ+1
      GO TO 23
   26 I=I+1
       JJ=1
      GRIDY(I)=0.
      I=I+1
      GO TO 23
   27 WRITE(IO,3)
    3 FORMAT('PROBLEMS')
      STOP
   30 DO 40 I=1,31
      RR=GRIDX(I)
      TH=(-GRIDY(I)+60.)*0.01745329
      GRIDX(I)=RR*COS(TH)+R
   40 GRIDY(I)=RR*SIN(TH)+0.86603*R
      GRIDX(32)=2.*R
      GRIDX(33)=2.*R
      GRIDX(34)=0.
      GRIDX(35)=0.
      GRIDY(32)=1.73206*R
      GRIDY(33)=0.
      GRIDY(34)=0.
      GRIDY(35)=1.73206*R
      DO 50 I=1,6
      IE(I)=1
      JE(I)=3+I-1
      LE(I)=0
   50 KE(I)=2+I-1
      JE(6)=2
      DO 60 I=7,12
      IE(I)=2+I-7
      JE(I)=9+I-7
      KE(I)=21+I-7
   60 LE(I)= 14+I-7
      JE(12)=8
      KE(12)=20
      DO 70 I=13,18
      IE(I)=20+I-13
      JE(I)=21+I-13
      KE(I)=27+I-13
   70 LE(I)=26+I-13
      JE(18)=20
      KE(18)=26
      DO 80 I=19,22
      IE(I)=27+I-19
      JE(I)=32+I-19
      KE(I)=26+I-19
   80 LE(I)=0
      JE(21)=30
      JE(22)=31
      KE(21)=34
      KE(22)=35
      DO 16 I=1,NDEC
   16  WRITE(100,11) (DECK(I,J),J=1,18)
      DO 90 I=1,35
```

```
   90 WRITE (IOO,4) I,GRIDX(I),GRIDY(I)
    4 FORMAT ('GRID',4X,I8,8X,2F8.3)
  316 DO 100 I=1,6
  100 WRITE (IOO,5) I,IE(I),JE(I),KE(I)
    5 FORMAT ('CTRMEM'2X,I8,7X,'1',3I8)
      DO 110 I=19,22
  110 WRITE (IOO,6) I,IE(I),JE(I),KE(I)
    6 FORMAT ('CTRMEM',2X,I8,7X,'2',3I8)
      DO 120 I=7,12
  120 WRITE (IOO,7) I, IE(I),JE(I),KE(I),LE(I)
    7 FORMAT ('CQDMEM',2X,I8,7X,'2',4I8)
      DO 130 I=13,18
  130 WRITE (IOO,8) I,IE(I),JE(I),KE(I),LE(I)
    8 FORMAT ('CQDMEM',2X,I8,7X,'1',4I8)
      IF (ITYPE)380,139,380
C
C      SPC CARDS
C
  139 I1=1
      I2=34
      I3=12
      I4=30
      I5=1
      WRITE(IOO,12)I1,I2,I3,I4,I5
   12 FORMAT('SPC'5X,3I8,8X,2I8)
      I1=2
      I2=35
      I3=1
      I4=29
      I5=2
      WRITE(IOO,12)I1,I2,I3,I4,I5
      I1=3
      I2=28
      I3=2
      I4=33
      WRITE(IOO,12)I1,I2,I3,I4,I5
  149 IF(NO-1)27,240,140
  140 DO 141 I=1,35
  141 GRIDY(I)=GRIDY(I)+1.73206*R
      DO 150 I=1,27
      II=I+35
  150 WRITE(IOO,4)II,GRIDX(I),GRIDY(I)
      II=63
      WRITE(IOO,4)II,GRIDX(30),GRIDY(30)
      II=64
      WRITE(IOO,4)II,GRIDX(31),GRIDY(31)
      II=65
      WRITE(IOO,4)II,GRIDX(32),GRIDY(32)
      II=66
      WRITE(IOO,4)II,GRIDX(35),GRIDY(35)
      DO 151 I=1,6
      IE(I)=36
      JE(I)=38+I-1
      KE(I)=37+I-1
  151 LE(I)=0
      JE(6)=37
      DO 160 I=7,12
      IE(I)=37+I-7
      JE(I)=44+I-7
      KE(I)=56+I-7
```

```
  160 LE(I)=49+I-7
      JE(12)=43
      KE(12)=55
      DO 170 I=13,18
      IE(I)=56+I-13
  170 LE(I)=55+I-13
      IE(18)=55
      JE(13)=62
      JE(14)=26
      JE(15)=31
      JE(16)=63
      JE(17)=64
      JE(18)=61
      KE(13)=61
      KE(14)=62
      KE(15)=26
      KE(16)=31
      KE(17)=63
      KE(18)=64
      DO 180 I=19,22
  180 LE(I)=0
      IE(19)=62
      IE(20)=26
      IE(21)=31
      IE(22)=63
      JE(19)=65
      JE(20)=32
      JE(21)=63
      JE(22)=64
      KE(19)=61
      KE(20)=62
      KE(21)=35
      KE(22)=66
      DO 200 I=1,6
      II=I+22
  200 WRITE(100,5)II,IE(I),JE(I),KE(I)
      DO 210 I=19,22
      II=I+22
  210 WRITE(100,6)II,IE(I),JE(I),KE(I)
      DO 220 I=7,12
      II=I+22
  220 WRITE(100,7)II,IE(I),JE(I),KE(I),LE(I)
      DO 230 I=13,18
      II=I+22
  230 WRITE(100,8)II,IE(I),JE(I),KE(I),LE(I)
      I1=4
      I2=63
      I3=1
      I4=66
      I5=1
C
C       SPC CARDS
C
      WRITE(100,12)I1,I2,I3,I4,I5
      IF(NO-1)27,240,241
  240 WRITE(100,13)
   13 FORMAT('SPCADD'7X,'100',7X,'1',7X,'2',7X,'3')
      GO TO 131
  241 WRITE(100,14)
   14 FORMAT('SPCADD'7X'100'7X'1'7X'2'7X'3'7X'4')
  131 WRITE (100,9)
```

```
    9 FORMAT ('ENDDATA')
      STOP
C
C       REGULAR ELEMENT  MODEL TWO
C
  300 DO 310 I=2,7
  310 GRIDX(I)=BHEX
      DO 311 I=8,13
  311 GRIDX(I)=BHEX+F
       DO 312 I=14,19
  312 GRIDX(I)=R
       JJ=1
      I=3
      GRIDY(2)=0
  323 GRIDY(I)=GRIDY(I-1)+60.
      IF (I-19)324,330,330
  324 IF (JJ-5) 325,326,327
  325 I=I+1
      JJ=JJ+1
      GO TO 323
  326 I=I+1
       JJ=1
      GRIDY(I)=0.
      I=I+1
      GO TO 323
  327 WRITE(IO,3)
      STOP
C
C       DEVELOPE AND PUNCH GRID AND CONNECTION CARDS
C
  330 DO340 I=1,19
      RR=GRIDX(I)
      TH=(-GRIDY(I)+60.)*0.01745329
      GRIDX(I)=RR*COS(TH)+R
  340 GRIDY(I)=RR*SIN(TH)+0.86603*R
      GRIDX(20)=2.*R
      GRIDX(21)=2.*R
      GRIDX(22)=0.
      GRIDX(23)=0.
      GRIDY(20)=1.73206*R
      GRIDY(21)=0.
      GRIDY(22)=0.
      GRIDY(23)=1.73206*R
      DO350 I=1,6
      IE(I)=1
      JE(I)=3+I-1
      LE(I)=0
  350 KE(I)=2+I-1
      JE(6)=2
       DO 360 I=7,18
      IE(I)=2+I-7
      JE(I)=3+I-7
      KE(I)=9+I-7
  360 LE(I)=8+I-7
      JE(12)=2
      JE(18)=8
      KE(12)=8
      KE(18)=14
      DO 370 I=19,22
      KE(I)=20+I-19
  370 LE(I)=0.
```

```
      IE(19)=14
      IE(20)=15
      IE(21)=17
      IE(22)=18
      JE(19)=15
      JE(20)=16
      JE(21)=18
      JE(22)=19
      DO416 I=1,NDEC
  416  WRITE(IOO,11) (DECK(I,J),J=1,18)
      DO390 I=1,23
  390 WRITE (IOO,4) I,GRIDX(I),GRIDY(I)
      GO TO 316
C
C      DEVELOPE AND PUNCH SPC CARDS
C
  380 I1=1
      I2=22
      I3=12
      I4=18
      I5=1
      WRITE(IOO,12)I1,I2,I3,I4,I5
      I1=2
      I2=23
      I3=1
      I4=17
      I5=2
      WRITE(IOO,12)I1,I2,I3,I4,I5
      I1=3
      I2=16
      I3=2
      I4=21
      WRITE(IOO,12)I1,I2,I3,I4,I5
      GO TO 149
      END
C
C      EXAMPLE DATA DECK FOLLOWS
C
/*
 125.     1.0         5.         126 1
ID POLYMER,MINERAL
APP DISP
SOL 1,0
TIME 10
CEND
TITLE = ONE HEX - 5 PERCENT MINERAL
SUBTITLE = INTERFACE = 1 , VARYING BOND REGULAR ELEMENT
ECHO = BOTH
SPCFORCES = ALL
OLOAD = ALL
ELSTRESS = ALL
DISPLACEMENT = ALL
SUBCASE 1
LABEL = ALPHA = MIN , E = MIN AND MU = 0.33
TEMPERATURE(LOAD)=1
SPC = 100
BEGIN BULK
GRDSET                                                    3456
PTRMEM          1       1       1.
PTRMEM          2       2       1.
```

```
PQDMEM          1         2       1.
PQDMEM          2         3       1.
MAT1            1 68.95                   .22     1.      8.5E-6
MAT1            2 3.047                   .44     1.      88.6E-6
MAT1            2 68.95                   .33     1.      8.5E-6
TEMPD           1 100.
```

TABLE I

Variables and their Values

Mineral Content by Volume	PER	5, 34 and 60%
Interface Elastic Modulus	E	Ep (3.047 GN/m^2), Ea (35.9985) and Em (68.95)
Interface Thermal Coefficient	α	αp ($88.6 \times 10^{-6}/°K$) αa (48.55×10^{-6} and αm (8.5×10^{-6})
Interface Thickness	F	1, 10, and 20μm
Interface Poisson's Ratio	μ	μp (0.44), μa (0.33) and μm (0.22)
Unit Cell Length X-Direction	L_x	(1118.0, 428.7, 322.7)μm
Unit Cell Length Y-Direction	L_y	(968.2, 371.3, 279.5)μm

TABLE II

Effect of Poisson's Ratio

Average % Strain For F = 10μm

		μm	μa	μp
αm	Em	0.5136	0.5135	0.5134
αp	Em	0.5603	0.5621	0.5603
αm	Ep	0.5249	0.5244	0.5234
αp	Ep	0.5518	0.5538	0.5556

Average % Strain For F = 1μm

		μm	μa	μp
αm	Em	0.5491	0.5491	0.5491
αp	Em	0.5537	0.5540	0.5543
αm	Ep	0.5502	0.5502	0.5501
αp	Ep	0.5528	0.5531	0.5532

Notes:

All values are % strain averaged over all exterior nodes

Mineral content is 34% in model two

TABLE III Model Two

Interface Thickness = 1μm 5% Mineral

X - Strain (%)

		Node 21		Node 15		Node 20	
		Disp.	Strain	Disp.	Strain	Disp.	Strain
αm	Em	9.180648	.8211	8.849059	.7914	9.074740	.8116
αm	Ea	9.181101	.8211	8.849708	.7915	9.075237	.8117
αm	Ep	9.188110	.8218	8.859923	.7924	9.083009	.8124
αa	Em	9.185943	.8216	8.856776	.7921	9.080808	.8122
αa	Ea	9.185296	.8215	8.855821	.7920	9.080044	.8121
αa	Ep	9.191171	.8220	8.864385	.7928	9.086519	.8127
αp	Em	9.191225	.8220	8.864473	.7928	9.086861	.8127
αp	Ea	9.189481	.8219	8.861919	.7926	9.084840	.8125
αp	Ep	9.194225	.8223	8.868835	.7932	9.090019	.8130

L_x = 1118.0μmm L_y = 968.2μm

Y - Strain (%)

		Node 23		Node 19		Node 14		Node 20	
		Disp.	Strain	Disp.	Strain	Disp.	Strain	Disp.	Strain
αm	Em	8.010276	.8272	7.727471	.7980	7.733756	.7987	7.922758	.8182
αm	Ea	8.010647	.8273	7.728028	.7981	7.734310	.7987	7.923195	.8183
αm	Ep	8.016359	.8279	7.736614	.7990	7.742869	.7996	7.929959	.8190
αa	Em	8.014427	.8277	7.733687	.7987	7.739926	.7993	7.927548	.8187
αa	Ea	8.013935	.8276	7.732952	.7986	7.739197	.7993	7.92699	.8186
αa	Ep	8.018758	.8281	7.740206	.7994	7.746434	.8000	7.932726	.8192
αp	Em	8.018567	.8281	7.739887	.7993	7.746080	.8000	7.932326	.8192
αp	Ea	8.017215	.8280	7.737863	.7991	7.744072	.7998	7.930775	.8190
αp	Ep	8.021150	.8284	7.743789	.7997	7.749990	.8004	7.935486	.8195

TABLE IV Model Two

Interface Thickness = 1μm 34% Mineral

X - Strain (%)

		Node 21		Node 15		Node 20	
		Disp.	Strain	Disp.	Strain	Disp.	Strain
αm	Em	2.581950	.6022	2.084823	.4863	2.470469	.5762
αm	Ea	2.582105	.6022	2.085121	.4863	2.470584	.5762
αm	Ep	2.584319	.6028	2.090296	.4875	2.472322	.5766
αa	Em	2.590835	.6043	2.097338	.4892	2.480168	.5785
αa	Ea	2.589148	.6039	2.095041	.4886	2.478272	.5780
αa	Ep	2.589499	.6040	2.097584	.4892	2.477980	.5780
αp	Em	2.599698	.6064	2.109822	.4921	2.489843	.5807
αp	Ea	2.596173	.6055	2.104936	.4910	2.485941	.5798
αp	Ep	2.594666	.6052	2.104853	.4909	2.483624	.5793

L_x = 428.7μm L_y = 371.3μm

Y - Strain (%)

		Node 23		Node 19		Node 14		Node 20	
		Disp.	Strain	Disp.	Strain	Disp.	Strain	Disp.	Strain
αm	Em	2.322191	.6254	1.827982	.4923	1.811206	.4878	2.128779	.5733
αm	Ea	2.322403	.6255	1.828354	.4924	1.811596	.4879	2.129142	.5734
αm	Ep	2.325521	.6263	1.833783	.4939	1.817498	.4895	2.134732	.5749
αa	Em	2.329257	.6273	1.838657	.4925	1.822003	.4907	2.137257	.5756
αa	Ea	2.328002	.6270	1.836814	.4947	1.820152	.4902	2.135860	.5752
αa	Ep	2.329634	.6274	1.839994	.4955	1.823778	.4912	2.139659	.5763
αp	Em	2.336305	.6292	1.8349305	.4981	1.832773	.4936	2.145714	.5779
αp	Ea	2.333588	.6285	1.8452530	.4970	1.828688	.4925	2.142562	.5770
αp	Ep	2.333737	.6285	1.846189	.4972	1.830043	.4929	2.144574	.5776

TABLE V Model Two

Interface Thickness = 1μm 60% Mineral

X - Strain (%)

		Node 21		Node 15		Node 20	
		Disp.	Strain	Disp.	Strain	Disp.	Strain
αm	Em	1.455489	.4509	.7777187	.2408	1.342661	.4160
αm	Ea	1.455686	.4510	.7780778	.2410	1.342902	.4160
αm	Ep	1.458024	.4517	.7842753	.2430	1.345512	.4168
αa	Em	1.465742	.4541	.7929214	.2456	1.353738	.4194
αa	Ea	1.463813	.4535	.7901271	.2448	1.351682	.4188
αa	Ep	1.464003	.4536	.7931280	.2457	1.351971	.4188
αp	Em	1.475970	.4573	.8080860	.2503	1.364787	.4228
αp	Ea	1.471919	.4560	.8021463	.2485	1.360439	.4215
αp	Ep	1.469966	.4554	.8019585	.2484	1.358413	.4208

L_x = 322.7μm L_y = 279.5μm

Y - Strain (%)

		Node 23		Node 19		Node 14		Node 20	
		Disp.	Strain	Disp.	Strain	Disp.	Strain	Disp.	Strain
αm	Em	1.313377	.4698	.6586718	.2356	.6295804	.2252	.9786822	.3501
αm	Ea	1.313512	.4699	.6590877	.2358	.6299332	.2253	.9790183	.3502
αm	Ep	1.315411	.4706	.6646017	.2377	.6354205	.2273	.9846786	.3522
αa	Em	1.321870	.4729	.6719461	.2404	.6430672	.2300	.9896197	.3540
αa	Ea	1.320244	.4723	.6696079	.2395	.6406222	.2291	.9876866	.3533
αa	Ep	1.320363	.4723	.6723304	.2405	.6432737	.2301	.9910420	.3545
αp	Em	1.330343	.4759	.6851872	.2451	.6565203	.2348	1.0005300	.3579
αp	Ea	1.326959	.4747	.6801019	.2433	.6512846	.2330	.9963333	.3564
αp	Ep	1.325304	.4741	.6800399	.2432	.6511073	.2329	.9973896	.3568

TABLE VI Model Two

Interface Thickness = 10μm 34% Mineral

X - Strain (%)

		Node 21		Node 15		Node 20	
		Disp.	Strain	Disp.	Strain	Disp.	Strain
αm	Em	2.465179	.5750	1.909221	.4453	2.349107	.5479
αm	Ea	2.466478	.5753	1.912151	.4460	2.350022	.5481
αm	Ep	2.486261	.5799	1.961386	.4575	2.364397	.5515
αa	Em	2.553462	.5956	2.034311	.4745	2.445074	.5703
αa	Ea	2.546541	.5940	2.0351985	.4747	2.4338925	.5677
αa	Ep	2.539620	.5923	2.036086	.4749	2.422711	.5651
αp	Em	2.641965	.6162	2.159714	.5037	2.541281	.5927
αp	Ea	2.609993	.6088	2.115385	.4934	2.506083	.5845
αp	Ep	2.593113	.6048	2.110973	.4921	2.481170	.5787

L_x = 428.7μm L_y = 371.3μm

Y - Strain (%)

		Node 23		Node 19		Node 14		Node 20	
		Disp.	Strain	Disp.	Strain	Disp.	Strain	Disp.	Strain
αm	Em	2.219723	.5978	1.667474	.4491	1.645357	.4431	1.992020	.5365
αm	Ea	2.221704	.5984	1.671203	.4501	1.649213	.4442	1.995726	.5375
αm	Ep	2.250633	.6061	1.722984	.4640	1.705969	.4595	2.050503	.5522
αa	Em	2.290563	.6169	1.774876	.4780	1.754223	.4724	2.077936	.5596
αa	Ea	2.291722	.6172	1.780780	.4796	1.762292	.4746	2.089412	.5627
αa	Ep	2.292881	.6175	1.786684	.4812	1.770361	.4768	2.100888	.5658
αp	Em	2.361581	.6360	1.882546	.5018	1.863361	.5018	2.164067	.5828
αp	Ea	2.336764	.6294	1.845567	.4970	1.825945	.4918	2.135131	.5750
αp	Ep	2.335234	.6289	1.850543	.4984	1.834913	.4942	2.151399	.5794

TABLE VII Model Four

Interface Thickness = 1μm 34% Mineral

X - Strain (%)

	Node 4		Node 8		Node 15		Node 22		Node 26	
	Disp.	Strain	Disp.	Strain	Disp.	Strain	Disp.	Strain	Disp.	Strain
αm Em	1.866125	.4353	1.863651	.4347	1.865150	.4350	1.864674	.4349	1.881719	.4389
αm Ea	1.877305	.4379	1.873785	.4370	1.873817	.4370	1.875932	.4375	1.897558	.4426
αm Ep	1.891039	.4411	1.885982	.4399	1.882969	.4392	1.893998	.4418	1.919856	.4478
αa Em	.961881	.2244	.953011	.2223	.854874	.1994	.949353	.2214	.935087	.2181
αa Ea	.948323	.2212	.936498	.2184	.802615	.1872	.929207	.2167	.912071	.2127
αa Ep	.940095	.2193	.922438	.2152	.700744	.1634	.901921	.2104	.884370	.2063
αp Em	1.737150	.4052	1.719656	.4011	1.523232	.3553	1.712238	.3994	1.682002	.3923
αp Ea	1.708915	.3986	1.685618	.3932	1.417848	.3307	1.670821	.3897	1.634386	.3812
αp Ep	1.691086	.3944	1.656277	.3863	1.213191	.2830	1.614441	.3766	1.576755	.3678

L_x = 428.7μm L_y = 371.3μm

Y - Strain (%)

	Node 23		Node 24		Node 25		Node 26	
	Disp.	Strain	Disp	Strain	Disp.	Strain	Disp.	Strain
αm Em	3.235837	.8715	3.269108	.8804	3.244813	.8739	3.215430	.8660
αm Ea	3.263461	.8789	3.299375	.8886	3.268303	.8802	3.231418	.8703
αm Ep	3.346422	.9013	3.360812	.9051	3.311613	.8919	3.269374	.8805
αa Em	1.578804	.4252	1.529106	.4418	1.594495	.4294	1.633722	.4400
αa Ea	1.492672	.4020	1.441762	.3883	1.530823	.4123	1.579264	.4253
αa Ep	1.308284	.3523	1.259974	.3393	1.397209	.3763	1.457008	.3924
αp Em	2.834025	.7633	2.731302	.7356	2.864509	.7715	2.945901	.7934
αp Ea	2.658998	.7161	2.553586	.6877	2.734815	.7365	2.835386	.7636
αp Ep	2.281927	.6146	2.183868	.5882	2.463257	.6634	2.587078	.6968

TABLE VIII Analysis of Variance

Factor	Sum of Squares	Degrees of Freedom	Mean Squares	Computed F
Per cent Mineral (PER)	679.53556	2	339.76778	2472.8
Location of Strain	55.002587	6	9.1670972	66.7
Interface Thermal Coef.(α)	0.0109865	2	0.00549324	0.04
Interface Modulus (E)	0.0169027	2	0.0084513	0.06
Inter Error	24.182964	176	0.1374032	1.0
	all x 10^{-6}		all x 10^{-6}	

Notes:

Both Y- and X- Strains are included

Interface thickness is 1μm

Model 2

Data from Tables III, IV, & V

TABLE IX Analysis of Variance

Factor	Sum of Squares	Degrees of Freedom	Mean Squares	Computed F
Interface Thickness (F)	0.59049875	1	0.59049875	27.4
Strain Location	37.240708	6	6.2067838	288.2
Interface Thermal Coef. (α)	0.9876678	2	0.49383391	22.9
Interface Modulus (E)	0.00863101	2	0.00431550	.2
Inter Error	2.455409	114	0.0215387	1.0
	all x 10^{-6}		all x 10^{-6}	

Notes:

Per cent mineral is 34%

Model 2

Data from Tables IV & VI

TABLE X Analysis of Variance

Factor	Sum of Squares	Degrees of Freedom	Mean Squares	Computed F
Model	2.3122947	1	2.31229	1.4
Location of Strain	89.936526	6	14.98942	8.8
Interface Thermal Coef.(α)	76.152995	2	38.62117	22.8
Interface Modulus (E)	0.7774556	2	0.388727	0.2
Inter Error	193.208262	114	1.694809	1.0
	all x 10^{-6}		all x 10^{-6}	

Notes:

Models two and four are compared

Interface thickness is 1μm

Per cent mineral is 34%

Data from Tables IV & VII

TABLE XI Model Two

Effect of Boundary Conditions

Interface Thickness = 1μm 34% Mineral

X - Strain (%)

				Node 21		Node 20		Node 22	
				Disp.	Strain	Disp.	Strain	Disp.	Strain
αp	Ep	μp	Y	2.612068	.6092	2.439823	.5691	0	0
αp	Ep	μp	X	2.487895	.5803	2.466908	.5754	-.4721402	-.1101
αp	Ep	μp	O	2.598213	.6053	2.484349	.5794	0	0
αm	Em	μm	Y	2.598778	.6061	2.425600	.5657	0	0
αm	Em	μm	X	2.473490	.5769	2.453071	.5722	-.4798611	-.1119
αm	Em	μm	O	2.582011	.6022	2.470542	.5762	0	0

Lx = 428.7 μm Ly = 371.3 μm

Y - Strain (%)

				Node 23		Node 20		Node 22	
				Disp.	Strain	Disp.	Strain	Disp.	Strain
αp	Ep	μp	Y	2.249300	.6058	2.157036	.5809	-.4625057	-.1246
αp	Ep	μp	X	2.359073	.6353	2.057258	.5541	0	0
αp	Ep	μp	O	2.334413	.6287	2.145183	.5777	0	0
αm	Em	μm	Y	2.236905	.6024	2.140730	.5765	-.4693512	-.1264
αm	Em	μm	X	2.347228	.6322	2.039707	.5493	0	0
αm	Em	μm	O	2.322228	.6254	2.128804	.5733	0	0

X - Node 22 x displacement released
Y - Node 22 y displacement released
O - Node 22 fixed

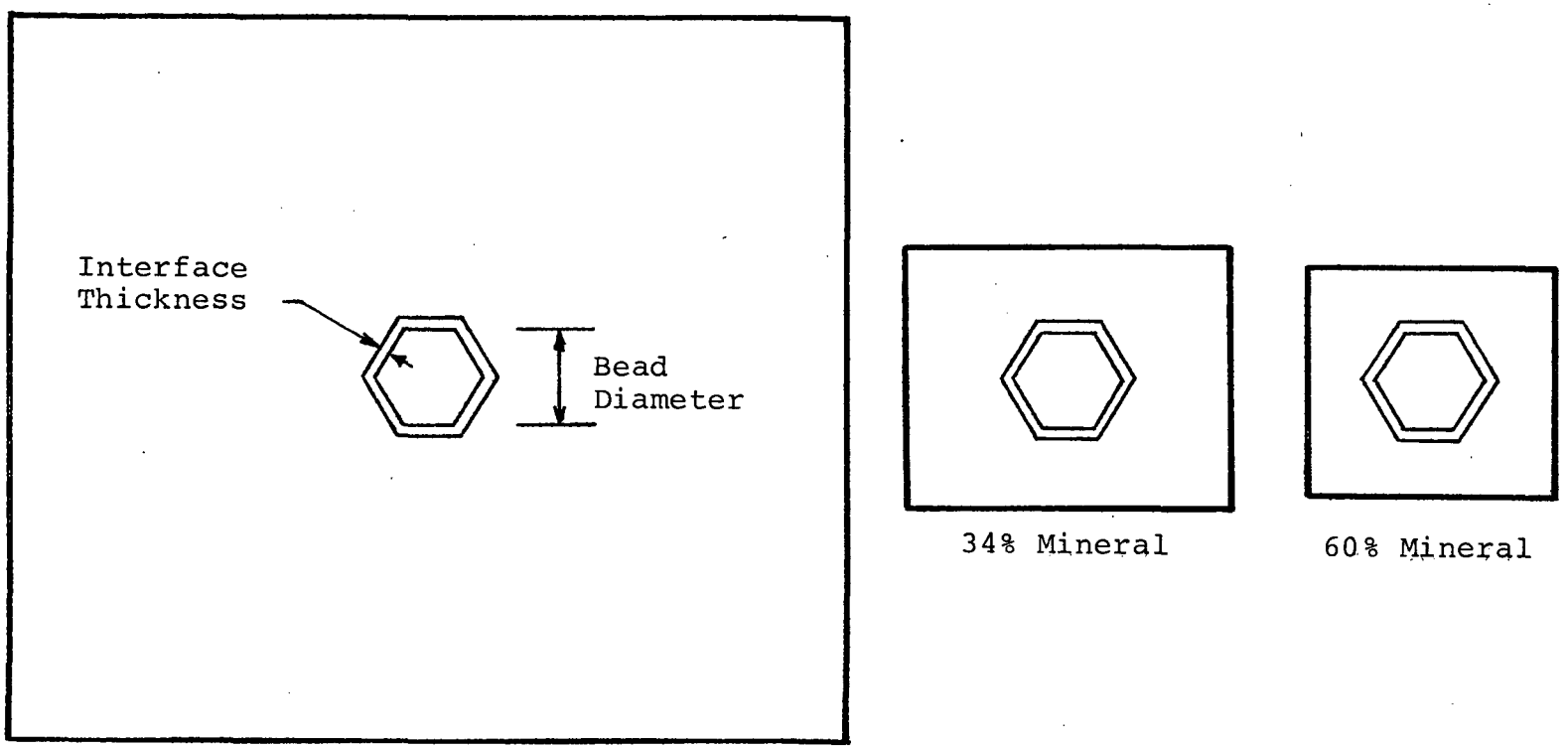

Figure 1 Basic Unit Cell Model of Polymer - Mineral Composite

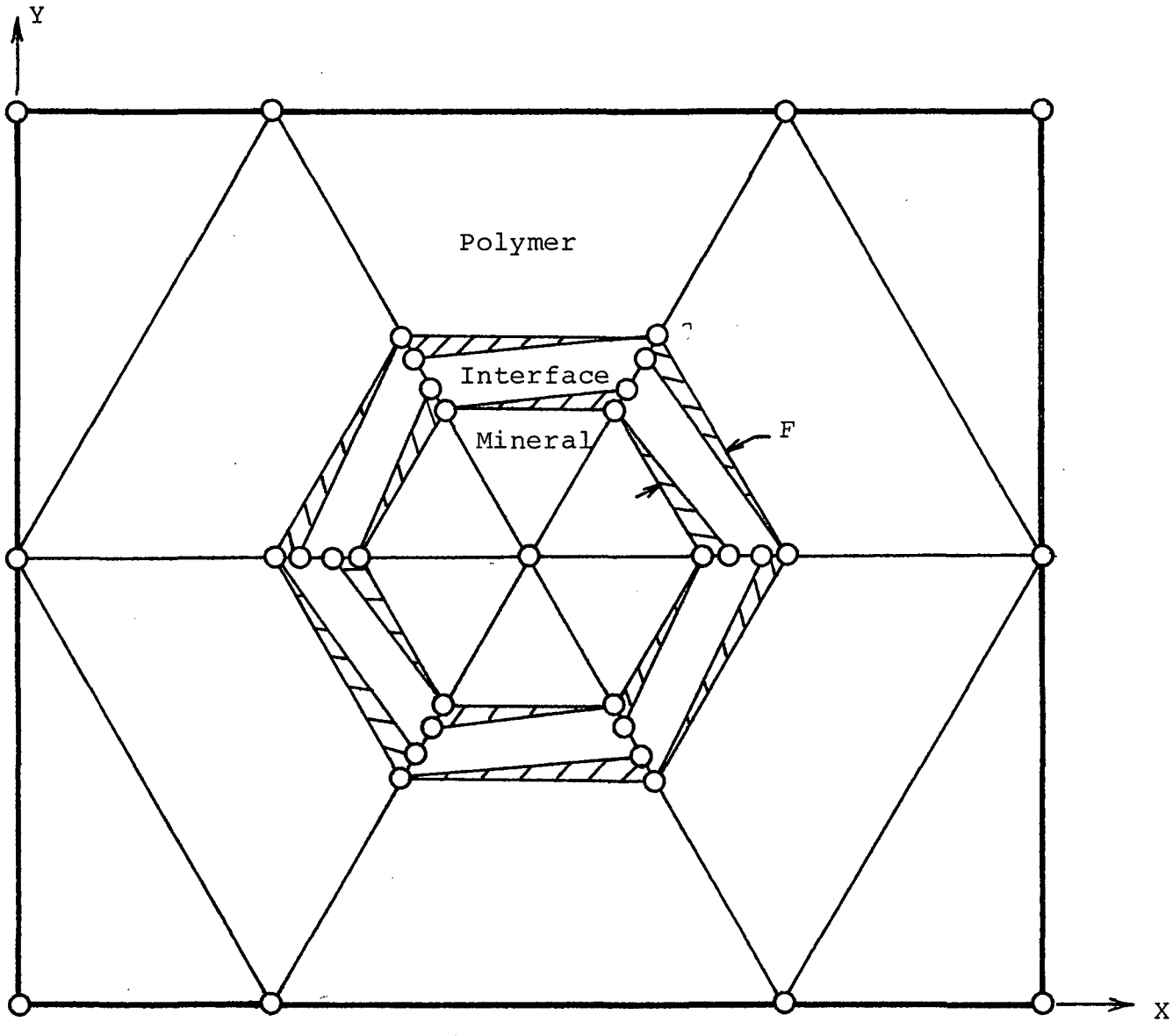

Figure 2 Model One

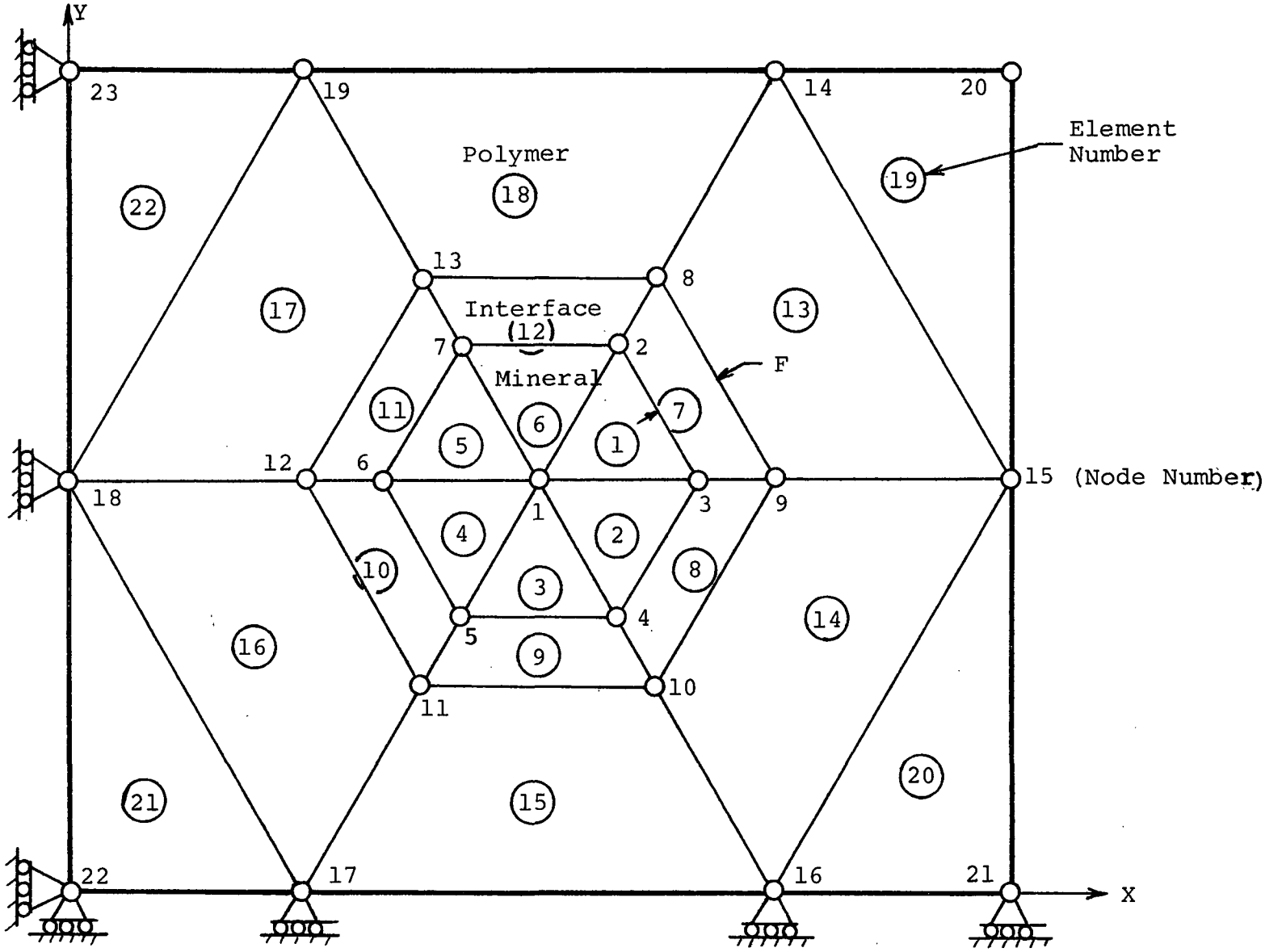

Figure 3 Model Two

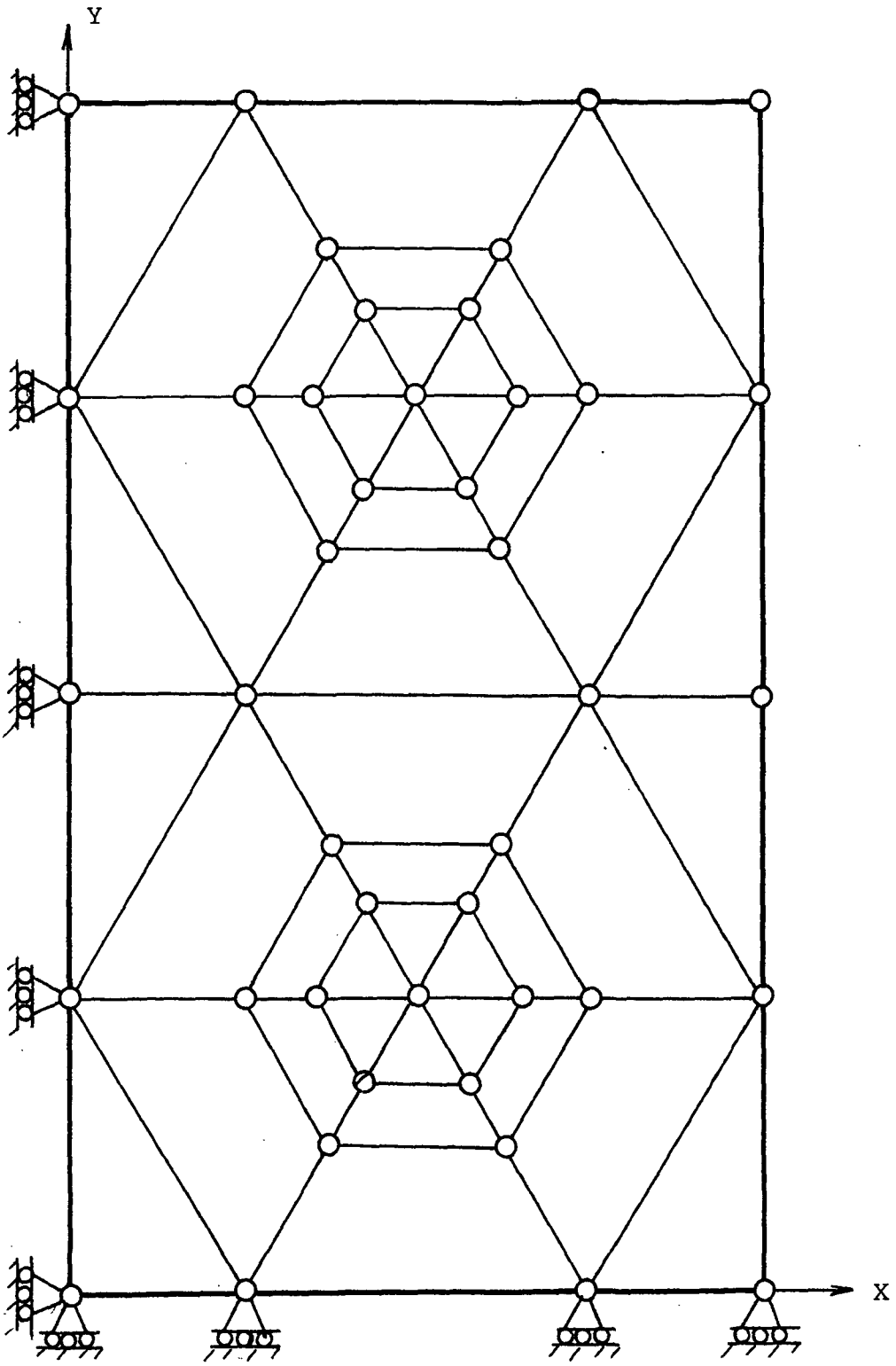

Figure 4 Model Three

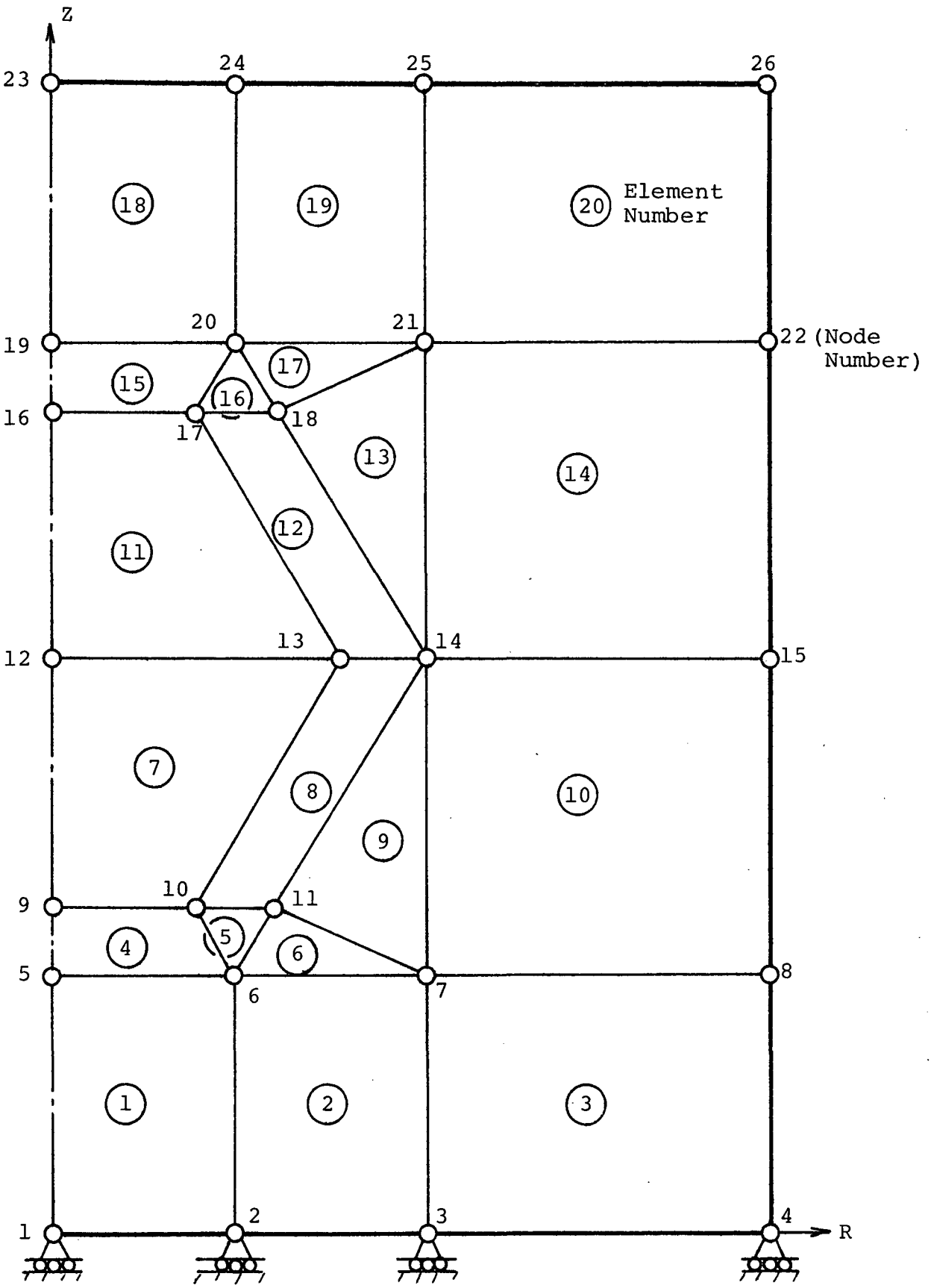

Figure 5 Model Four (Axisymmetric)

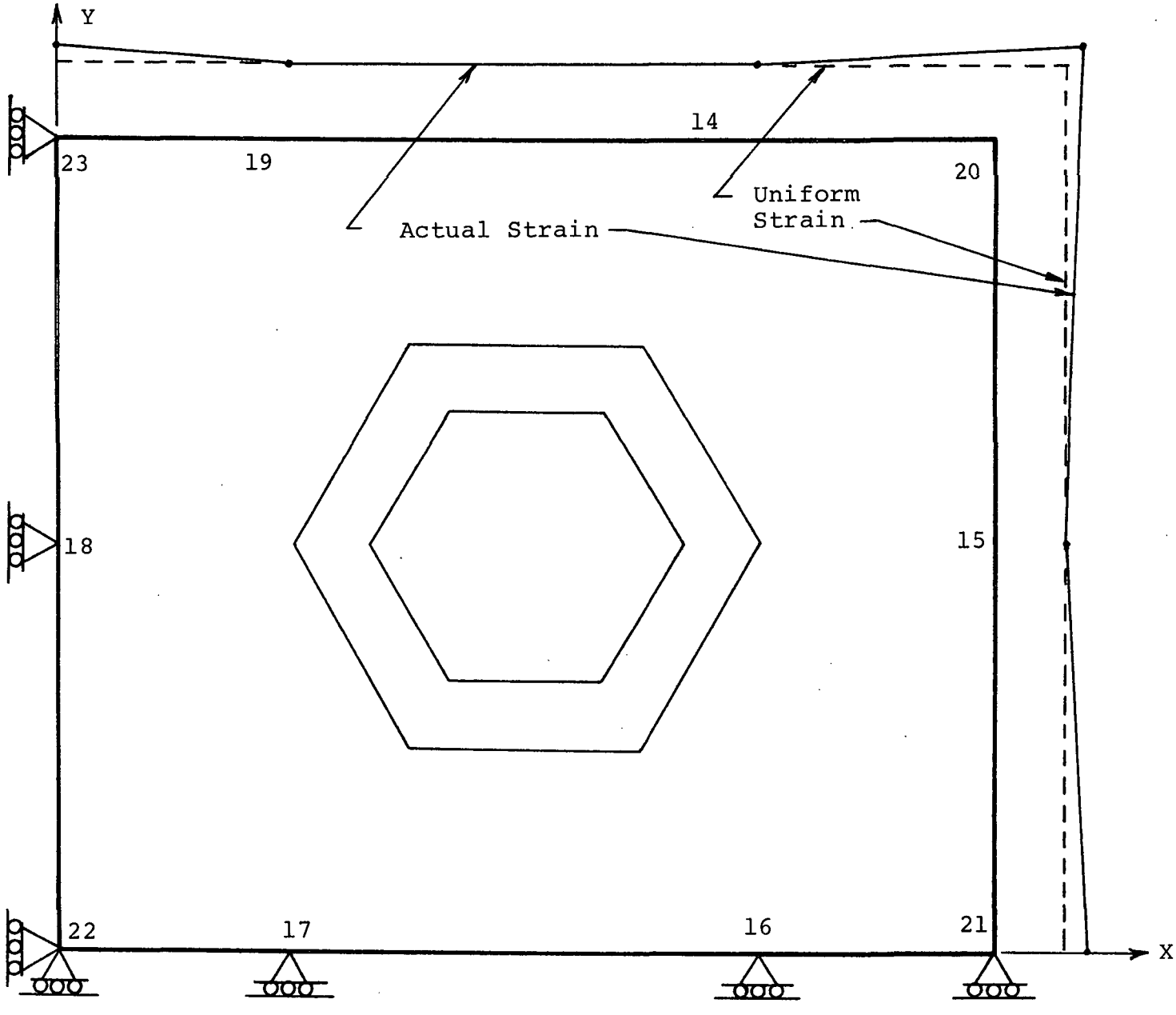

Figure 6 Non Uniform Strain