# FREQUENCY MODULATION TELEVISION ANALYSIS DISTORTION ANALYSIS 

Prepared by<br>William H. Hodge<br>and<br>Wing H . Wong

# Prepared for GODDARD SPACE FLIGHT CENTER 

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# COMPUTER SCIENCES CORPORATION 

6565 Arlington Boulevard
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## PREFACE

This study under NASA Contract NAS5-21872 was initiated to perform a parametric analysis using computer simulation and analysis techniques of the threshold and signal distortion effects in FM TV systems. This report is a study of the signal distortion effects. The FM threshold study is presented in a separate report.

Computer simulation is used to model an FM TV transmission system. A T-pulse-andbar test signal is passed through the simulated system and a distorted output signal approximating the input test signal is the result of the simulation. The output signal can be compared with the input signal for various specified systems. Thus, the distortion quality of various systems can be evaluated.

As a result of the analysis, the following conclusions can be drawn:

1. The T-pulse-and-bar test signal is distorted more in a system with preemphasis than in one without preemphasis.
2. In the case of worst distortion a preemphasized system with a half-power RF bandwidth equal to twice the peak deviation produces a T-bar overshoot of only $9.9 \%$ less than the $13 \%$ limit specified by C.C.I.R. This occurs for a four-pole Chebyshev filter having a $0.1-\mathrm{dB}$ ripple bandwidth of 10.0 MHz and a half-power bandwidth of 12.13 MHz .
3. An increase in the peak deviation of a system increases signal distortion.
4. The signal distortion increases as the number of poles of the predetection filter increases.
5. A four-pole Chebyshev predetection filter causes slightly more signal distortion than a four-pole Butterworth filter with the same half-power bandwidth.

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## SECTION 1 - INTRODUCTION

Reception of a frequency-modulated (FM) television signal requires predetection filtering to remove noise outside the FM frequency band prior to demodulation. To reject as much noise as possible the predetection filter should be as narrow as possible without significant distortion of the desired signal.

The primary objective of this study is to simulate an FM television transmission system by computer programming and to evaluate the signal distortion due to predetection filtering. The system is modeled mathematically, and a $T(T=100$ nanoseconds) pulse-and-bar test signal is used as the input video signal. The output wave resulting from the test signal is calculated and plotted. Then the distortion of the signal can be seen and used as a measure of the distortion quality of the system. No audio subcarriers are considered in this analysis. The simulation is implemented for 64 cases specified by NASA, and the results of those cases are presented herein.

A typical video luminance signal has a format as shown in Figure 1-1, and the test signal is shown in Figure 2-3. Portions of the test signal are shown in more detail in Figures 4-1 and 4-2. Note that the T-pulse has a width of 100 nanoseconds, corresponding to a $5.0-\mathrm{MHz}$ video bandwidth, rather than 125 nanoseconds, corresponding to a $4.0-\mathrm{MHz}$ video bandwidth. Therefore, the distortion results will be somewhat conservative for application to a $4.0-\mathrm{MHz}$ system.

The system being modeled is shown in Figure 1-3. The video test signal enters the preemphasis filter which is a 525 -line filter specified by C.C.I.R. (Reference 3 ). The model also includes the case for a flat, or unpreemphasized, system in which the preemphasis and deemphasis filters are not used.

The signal goes next to the RF frequency modulator from which the video signal is transmitted in an RF form through a noisy RF channel; however, RF additive noise is not included in this model.


Figure 1-1. Typical Video Luminance Signal


Figure 1-2. Block Diagram of the FM Video Transmission System Being Modeled

In passing through the predetection filter the signal is distorted somewhat depending on the filter characteristics. Narrowing the predetection filter bandwidth cuts out noise. On the other hand, narrowing the filter causes signal distortion. Thus, the filter is desired to be narrow but not so narrow that significant distortion occurs. The predetection filter characteristics are inputs to the computer program for system evaluation.

The RF demodulator recovers an estimate of the modulating signal. It is only an estimate because the signal includes distortion. In a flat system, this is the estimate of the video test signal. In a preemphasized system the demodulated signal must pass through a deemphasis filter to recover the estimate of the video test signal.

The final output signal is compared with the input test signal for various specifications of three- and four-pole Butterworth and Chebyshev filters and for various peak deviations of the RF carrier. The distortion of the final test signal is used as a measure of system quality.

## SECTION 2 - APPROACH

### 2.1 INTRODUCTION

The mathematical theory required to reclaim a modulating wave in a flat or preemphasized FM television system is presented first with block diagrams to illustrate the sequences of mathematical operations. Then, after the standard test signal is described, special treatment of the test signal to facilitate analysis is explained. Finally, the numerical methods used in the analysis are indicated.

Computer implementation of the mathematics involved in the analysis demands careful consideration particularly because convolutions are involved. Convolutions performed by numerical methods require lengthy computations in either the frequency domain with the use of Fourier transforms or directly in the time domain. Much is written about the economy of the fast Fourier transform (FFT) technique, and at first it appears that use of the FFT would be more economical than direct implementation of the convolution in the time domain. This is true when operations involve the same number of points in both the frequency and the time domains. If the number of points required for an operation in the time domain is less than that required for the equivalent operation in the frequency domain, then the time domain is preferred. To illustrate this point, consider the time impulse function, $\delta(t)$, whose Fourier transform is unity for all values of frequency, while $\delta(t)$ is nonzero at only one point in the time domain. Some other difficulties with the FFT method are integration with respect to time and division of two time functions. For these various reasons the time convolution is preferred over the FFT method for this application.

### 2.2 A MATHEMATICAL ANALYSIS OF THE VIDEO TEST SIGNAL DISTORTION

This mathematical analysis involves a method to reclaim the modulating wave of a flat or preemphasized FM television system. The case of a preemphasized FM television system is first considered, while the case of a flat FM television system can be regarded as a special case of the former.

Let $f(t)$ denote a test signal which enters the preemphasis filter having impulse response, $c_{1} h_{1}(t)$, where $h_{1}(t)$ is the response of a passive filter and $c_{1}$ is the voltage gain required to satisfy the preemphasis specification. The modulating wave $m(t)$ after preemphasis is given by

$$
\begin{equation*}
\mathrm{m}(\mathrm{t})=\mathrm{c}_{1} \int_{-\infty}^{\mathrm{t}} \mathrm{f}(\mathrm{x}) \mathrm{h}_{1}(\mathrm{t}-\mathrm{x}) \mathrm{dx} \tag{2-1}
\end{equation*}
$$

The modulated carrier which is filtered in the predetection filter is

$$
\begin{equation*}
\mathrm{s}_{1}(\mathrm{t})=\cos \left[\omega_{\mathrm{o}} \mathrm{t}+\phi(\mathrm{t})\right] \tag{2-2}
\end{equation*}
$$

where $\omega_{0}$ is the angular frequency of the carrier and

$$
\begin{equation*}
\phi(\mathrm{t})=\mathrm{k} \int_{-\infty}^{\mathrm{t}} \mathrm{~m}(\mathrm{x}) \mathrm{dx} \tag{2-3}
\end{equation*}
$$

A constant of proportionality, $k$, relates the instantaneous frequency deviation to the voltage of the modulating wave, $m(t)$.

The modulated carrier is then passed through the predetection bandpass filter. The impulse response, $h_{2}(t)$, of the bandpass filter can be found from the specification of the filter and can be written for a filter symmetric about its center frequency, $\omega_{0}$, as

$$
\begin{equation*}
h_{2}(t)=2 h_{l}(t) \cos \omega_{0} t \tag{2-4}
\end{equation*}
$$

where $h_{\boldsymbol{l}}(\mathrm{t})$ is the impulse response of the equivalent low-pass filter corresponding to the bandpass filter. The theory of converting a bandpass to an equivalent lowpass filter for analysis purposes is developed in Reference 1. The output, $s_{2}(t)$, of the filter is then found by convolving the RF signal, $s_{1}(t)$, with the impulse response, $h_{2}(t)$, as follows:

$$
\begin{equation*}
s_{2}(t)=\int_{-\infty}^{\mathrm{t}} \cos \left[\omega_{0} \mathrm{x}+\phi(\mathrm{x})\right] \mathrm{h}_{2}(\mathrm{t}-\mathrm{x}) \mathrm{dx} \tag{2-5}
\end{equation*}
$$

On substituting for $h_{2}(t-x)$ the output becomes

$$
\begin{equation*}
s_{2}(t)=\int_{-\infty}^{\mathrm{t}} \cos \left[\omega_{\mathrm{o}} \mathrm{x}+\phi(\mathrm{x})\right] 2 \mathrm{~h}_{\rho}(\mathrm{t}-\mathrm{x}) \cos \left[\omega_{\mathrm{o}}(\mathrm{t}-\mathrm{x})\right] \mathrm{dx} \tag{2-6}
\end{equation*}
$$

On expanding the last cosine term above and multiplying one gets

$$
\begin{align*}
s_{2}(t)= & \cos \omega_{0} t \int_{-\infty}^{t} 2 \cos \omega_{0} x \cos \left[\omega_{0} x+\phi(x)\right] h_{\ell}(t-x) d x \\
& +\sin \omega_{0} t \int_{-\infty}^{t} 2 \sin \omega_{0} x \cos \left[\omega_{0} x+\phi(x)\right] h_{\ell}(t-x) d x \tag{2-7}
\end{align*}
$$

A further expansion of the terms of this equation gives

$$
\begin{align*}
s_{2}(t)= & \cos \omega_{0} t \int_{-\infty}^{t} \cos \left[2 \omega_{0} x+\phi(x)\right] h_{\ell}(t-x) d x \\
& +\cos \omega_{0} t \int_{-\infty}^{t} \cos \phi(x) h_{\ell}(t-x) d x \\
& +\sin \omega_{0} t \int_{-\infty}^{t} \sin \left[2 \omega_{0} x+\phi(x)\right] h_{\ell}(t-x) d x  \tag{2-8}\\
& -\sin \omega_{0} t \int_{-\infty}^{t} \sin \phi(x) h_{\ell}(t-x) d x
\end{align*}
$$

Note that the first and third integrals are convolutions of an RF signal with the impulse response of a lowpass filter. Thus, those terms are zero and $s_{2}(t)$ can be written as

$$
\begin{equation*}
s_{2}(t)=A(t) \cos \omega_{0} t+B(t) \sin \omega_{0} t \tag{2-9}
\end{equation*}
$$

where

$$
\begin{equation*}
A(t)=\int_{-\infty}^{t} \cos \phi(x) h_{\ell}(t-x) d x \tag{2-10}
\end{equation*}
$$

and

$$
\begin{equation*}
B(t)=\int_{-\infty}^{t} \sin \phi(x) h_{\ell}(t-x) d x \tag{2-11}
\end{equation*}
$$

Finally the band-filtered RF carrier can be written as

$$
\begin{equation*}
\mathrm{s}_{2}(\mathrm{t})=\sqrt{\mathrm{A}^{2}(\mathrm{t})+\mathrm{B}^{2}(\mathrm{t})} \cos \left[\omega_{\mathrm{o}} \mathrm{t}+\psi(\mathrm{t})\right] \tag{2-12}
\end{equation*}
$$

where

$$
\begin{equation*}
\psi(t)=\tan ^{-1}\left[\frac{B(t)}{A(t)}\right] \tag{2-13}
\end{equation*}
$$

This is a convenient form since the demodulator output is proportional to the derivative of $\psi(t)$, and

$$
\begin{equation*}
\psi^{\prime}(t)=\frac{d}{d t} \tan ^{-1}\left[\frac{\mathrm{~B}(\mathrm{t})}{\mathrm{A}(\mathrm{t})}\right]=\frac{1}{1+\left[\frac{\mathrm{B}(\mathrm{t})}{\mathrm{A}(\mathrm{t})}\right]^{2}} \frac{\mathrm{~d}}{\mathrm{dt}}\left[\frac{\mathrm{~B}(\mathrm{t})}{\mathrm{A}(\mathrm{t})}\right] \tag{2-14}
\end{equation*}
$$

The modulating signal, $m(t)$, can be estimated as $\hat{m}(t)$ by replacing $\phi(t)$ by $\psi(t)$ and $m(t)$ by $\hat{m}(t)$ in Equation (2-3). Thus,

$$
\begin{equation*}
\psi(\mathrm{t})=\mathrm{k} \int_{-\infty}^{\mathrm{t}} \hat{\mathrm{~m}}(\mathrm{x}) \mathrm{dx} \tag{2-15}
\end{equation*}
$$

Differentiation of both sides of Equation (2-15) gives $\psi^{\prime}(t)=k \hat{m}(t)$ and the estimate of the modulating signal is given by

$$
\begin{equation*}
\hat{\mathrm{m}}(\mathrm{t})=\frac{\psi^{\prime}(\mathrm{t})}{\mathrm{k}} \tag{2-16}
\end{equation*}
$$

However, in the preemphasized FM television system $\hat{m}(t)$ is not yet the final output, which is obtainable by passing $\hat{\mathrm{m}}(\mathrm{t})$ through the deemphasis filter having an impulse response $c_{3} h_{3}(t)$, where $h_{3}(t)$ is the response of a passive filter and $c_{3}$ is the voltage gain required to satisfy the deemphasis specification. The final output $\hat{f}(t)$ is, therefore,

$$
\begin{equation*}
\hat{f}(\mathrm{t})=\mathrm{c}_{3} \int_{-\infty}^{\mathrm{t}} \hat{\mathrm{~m}}(\mathrm{t}) \mathrm{h}_{3}(\mathrm{t}-\mathrm{x}) \mathrm{dx} \tag{2-17}
\end{equation*}
$$

The circumflex accent in $\hat{f}(t)$ is used to indicate an estimate of $f(t)$. On substituting for $\hat{\mathrm{m}}(\mathrm{t})$ the final output becomes

$$
\begin{equation*}
\hat{\mathrm{f}}(\mathrm{t})=\frac{\mathrm{c}_{3}}{\mathrm{k}} \int_{-\infty}^{\mathrm{t}} \psi^{\prime}(\mathrm{x}) \mathrm{h}_{3}(\mathrm{t}-\mathrm{x}) \mathrm{dx} \tag{2-18}
\end{equation*}
$$

A block diagram to summarize the sequence of mathematical operations involved in the above analysis is presented in Figure 2-1.

For the case of a flat FM television system, the modulating wave $m(t)$ is simply the test signal, $f(t)$; i. e., the function, $\phi(t)$, in Equation (2-3) is given by

$$
\begin{equation*}
\phi(\mathrm{t})=\mathrm{k} \int_{-\infty}^{\mathrm{t}} \mathrm{f}(\mathrm{x}) \mathrm{dx} \tag{2-19}
\end{equation*}
$$

and the final output, $\hat{f}(t)$, is given in accordance with Equation (2-16) as

$$
\begin{equation*}
\hat{\mathrm{f}}(\mathrm{t})=\frac{\psi^{\prime}(\mathrm{t})}{\mathrm{k}} \tag{2-20}
\end{equation*}
$$

with k in Equations (2-19) and (2-20) having the same meaning as before. A block diagram summarizing the sequence of mathematical operations involved in this flat FM television system is presented in Figure 2-2.

The determinations of the impulse responses of the preemphasis and deemphasis filters are described in Appendix A, and those of the equivalent lowpass filters of the two types of predetection filters involved are described in Appendix B.

The determination of the value of $k$ in Equation (2-3) is shown below. To have equal maximum frequency deviations on both sides of the center frequency of the predetection filter, the zero frequency deviation is assigned at a level 0.2 volt above the baseline of the test signal shown in Figure 2-3. The instantaneous frequency deviation, $\phi^{\prime}(t)$, is given by differentiating the expression in Equation (2-3); i. e.,

$$
\begin{equation*}
\phi^{\prime}(t)=k m(t) \tag{2-21}
\end{equation*}
$$

Let $v_{1}$ be the minimum voltage of the input signal at an instant, $t_{1}$, and let $v_{2}$ be the maximum voltage of the input signal at another instant, $\mathrm{t}_{2}$. Then, from Equation (2-21) the following relations are obtained:

$$
\begin{align*}
& \phi^{\prime}\left(\mathrm{t}_{1}\right)=\mathrm{k} \mathrm{v}_{1}  \tag{2-22}\\
& \phi^{\prime}\left(\mathrm{t}_{2}\right)=\mathrm{k} \mathrm{v}_{2} \tag{2-23}
\end{align*}
$$


note: ' Indicates the convolution operation

Figure 2-1. Block Diagram of the Sequence of Mathematical Operations for a Preemphasized FM Television System

nole. -indicates the convolution operation

Figure 2-2. Block Diagram of the Sequence of Mathematical Operations for a Flat FM Television System


NOTES:
A: TPULSE
B: TBAR
$\mathrm{T}=100 \mathrm{~ns}$

Figure 2-3. T-Pulse-and-Bar Test Signal

If $D_{p}$ denotes the peak frequency deviation, then

$$
\begin{align*}
2 \pi \mathrm{D}_{\mathrm{p}} & =\frac{\phi^{\prime}\left(\mathrm{t}_{2}\right)-\phi^{\prime}\left(\mathrm{t}_{1}\right)}{2}  \tag{2-24}\\
& =\frac{\mathrm{k}\left(\mathrm{v}_{2}-\mathrm{v}_{1}\right)}{2}
\end{align*}
$$

for equal maximum frequency deviations from the center frequency of the predetection filter. Since the voltage difference, $v_{2}-v_{1}$, is 1 volt,

$$
\begin{equation*}
\mathrm{k}=4 \pi \mathrm{D}_{\mathrm{p}} \tag{2-25}
\end{equation*}
$$

### 2.3 THE TEST SIGNAL AND ITS TREATMENT

The test signal used in the distortion analysis contains a T-pulse and a T-bar, as specified in Reference 2. A T-pulse is one cycle of a raised sinusoid having a frequency of $\frac{1}{2 \mathrm{~T}}$. The T -pulse is often referred to as a sine-squared pulse because it can be generated by squaring a sinewave of half the frequency just mentioned. A T-bar is a rectangular pulse with half a T-pulse used for the rise and the other half used for the fall. The test signal is presented in Figure 2-3. A more detailed view of the pulse and bar are shown in Figures 4-1 and 4-2. In any practical situation the T-pulse is sufficiently separated from the synchronizing pulse that one can assume settling of any transient effect by the beginning of the T-pulse. Similarly one can assume that any transient effect preceding the T-bar has settled by the time it starts. The T-pulse is treated as though the synchronizing pulse and the T-bar did not exist, and the T-bar is treated as though the T-pulse and synchronizing pulse did not exist. Thus, the T-pulse and T-bar are considered separately. The methods of decomposing each of these signals to simplify the computer simulation are discussed.

### 2.3.1 T-Pulse Implementation

The T-pulse with its dc bias can be represented mathematically as

$$
\begin{align*}
f(t) & =0.7 \sin ^{2}\left[\omega\left(\mathrm{t}-\mathrm{t}_{1}\right)\right]-0.2, \mathrm{t}_{1} \leq \mathrm{t} \leq \mathrm{t}_{2}  \tag{2-26}\\
& =-0.2 \text { elsewhere }
\end{align*}
$$

where $\omega=2 \pi\left(\frac{1}{4 \mathrm{~T}}\right)$
$\mathrm{T}=100$ nanoseconds
The duration of the pulse interval $\left(t_{1}, t_{2}\right)$ is 200 nanoseconds.
For the case of a flat FM television system, the T-pulse applied to the system is given by Equation (2-26) with intervals, $\left(0, t_{1}\right)$ and $\left(t_{2}, t_{3}\right)$, each of 600 -nanosecond duration. The signal is considered over the time interval from 0 to $t_{3}$. This provides an interval of observation both before and after the transient pulse. The interval preceding the pulse allows time for settling of the system after the signal is applied to the predetection filter and before the pulse occurs. The period following the pulse is required to observe the delay and distortion on the output signal. The leading interval is required for settling of the predetection filter transient which occurs upon application of the signal at time zero. The preemphasis and deemphasis filters are tested with a dc voltage level applied for a long time prior to the T-pulse. Thus, the resultant dc output signal from these filters can be calculated simply as a constant times the input de voltage without actually using the convolution technique. The response to the T-pulse can be superimposed on the calculated de response to arrive at the output waveform. Thus, for the case of a preemphasized FM television system, the principle of superposition is applied in order to facilitate the analysis by computer.

The input time function, $f(t)$, is divided up into two components.

$$
\begin{equation*}
\mathrm{f}(\mathrm{t})=\mathrm{f}_{1}(\mathrm{t})+\mathrm{f}_{2}(\mathrm{t}) \tag{2-28}
\end{equation*}
$$

where

$$
\begin{align*}
\mathrm{f}_{1}(\mathrm{t}) & =0.7 \sin ^{2}\left[\omega\left(\mathrm{t}-\mathrm{t}_{1}\right)\right], \quad \mathrm{t}_{1} \leq \mathrm{t} \leq \mathrm{t}_{2}  \tag{2-29}\\
& =0, \text { elsewhere; }
\end{align*}
$$

and

$$
\begin{equation*}
\mathrm{f}_{2}(\mathrm{t})=-0.2, \text { all } \mathrm{t} \tag{2-30}
\end{equation*}
$$

When $f(t)$ enters the preemphasis filter having an impulse response, $c_{1} h_{1}(t)$, the output modulating signal, $m(t)$, is given by a convolution operation.

$$
\begin{align*}
m(t) & =c_{1} \int_{-\infty}^{\infty} f(x) h_{1}(t-x) d x \\
& =c_{1} \int_{t_{1}}^{t} f_{1}(x) h_{1}(t-x) d x+c_{1} \int_{-\infty}^{t} f_{2}(x) h_{1}(t-x) d x  \tag{2-31}\\
& \left.=c_{1} \int_{t_{1}}^{t} 0.7 \sin ^{2}\left[\omega_{(x-t}\right)\right] u\left(-x+t_{2}\right) h_{1}(t-x) d x+c_{1} \int_{-\infty}^{t}(-0.2) h_{1}(t-x) d x
\end{align*}
$$

where $u$ is the Heaviside unit function. The upper limit of the integrals is changed to $t$ because the filter is causal, and the lower limit of the first integral is changed to $t_{1}$ because the signal, $f_{1}(t)$, is zero for smaller values of time. The second term on the right side of Equation (2-31) is the response of the filter to a dc voltage. If the transient effect of applying a dc voltage is assumed to have subsided before the beginning of the sine-squared pulse, then the effect of the filter on the dc voltage can be considered as merely a constant attenuation. This constant attenuation is specified as a value of 10 dB in Reference 3. Then, Equation (2-31) can be written as

$$
\begin{align*}
\mathrm{m}(\mathrm{t}) & =-\frac{0.2}{3.162278}, \quad \mathrm{t} \leq \mathrm{t}_{1} \\
& =1.479 \int_{\mathrm{t}_{1}}^{\mathrm{t}} 0.7 \sin ^{2}\left[\omega\left(\mathrm{x}-\mathrm{t}_{1}\right)\right] \mathrm{u}\left(-\mathrm{x}+\mathrm{t}_{2}\right) \mathrm{h}_{1}(\mathrm{t}-\mathrm{x}) \mathrm{dx}-\frac{0.2}{3.162278}, \mathrm{t} \geq \mathrm{t}_{1} \tag{2-32}
\end{align*}
$$

where the factor, $\frac{1}{3.162278}$, corresponds to a dc attenuation of 10 dB and the factor, $\mathrm{c}_{1}=$ 1. 479 , corresponds to a gain of 3.39 dB .

Thus, the modulating signal can be written as the superposition of two signals as follows:

$$
\begin{equation*}
\mathrm{m}(\mathrm{t})=\mathrm{m}_{1}(\mathrm{t})+\mathrm{m}_{2}(\mathrm{t}) \tag{2-33}
\end{equation*}
$$

where

$$
\begin{align*}
m_{1}(t) & =-0.0632456 \text { for all } t \\
m_{2}(t) & =1.479 \int_{t_{1}}^{t} 0.7 \sin ^{2}\left[\omega\left(x-t_{1}\right)\right] u\left(-x+t_{2}\right) h_{1}(t-x) d x, t \geq t_{1}  \tag{2-34}\\
& =0 \text { elsewhere }
\end{align*}
$$

The first of these signals is the response of the preemphasis filter to the dc portions of the input signal, while the second is the response to the sine-squared component.

Since the frequency modulation and demodulation involved in this analysis are nonlinear processes, the principle of superposition cannot be applied as it was with the preemphasis filter. The modulating wave $m(t)$ must be applied as it is given in Equation (2-32), but the duration of the time intervals can be adjusted appropriately to reduce computation time. It was found that a value of 600 nanoseconds for $t_{1}$ and a value of 800 nanoseconds for the length of the time interval from $t_{1}$ to $t_{3}$ were large enough to evaluate the applied signal (see Figure 4-131).

The steps outlined mathematically in Paragraph 2.2 are implemented for finding the demodulated output, $\psi^{\prime}(\mathrm{t})$, as an estimate of $\mathrm{m}(\mathrm{t})$. The details of this implementation are not discussed since no special simplifying technique is used for that portion of the calculations.

Superposition is again applied at the deemphasis filter to simplify the calculations. The demodulated output $\psi^{\prime}(\mathrm{t})$ can be considered to consist of two parts, $\psi_{1}^{\prime}$ and $\psi_{2}^{\prime}$, where $\psi_{1}^{\prime}$ is the response to the dc portion of the modulating wave, $m_{1}(\mathrm{t})$, and $\psi_{2}^{\prime}$ is the response to $\mathrm{m}_{2}(\mathrm{t})$. Both $\mathrm{m}_{1}{ }^{(\mathrm{t})}$ and $\mathrm{m}_{2}{ }^{(\mathrm{t})}$ are defined by Equation (2-34). The demodulated output, $\psi^{\prime}(t)$, can be written as

$$
\begin{equation*}
\psi^{\prime}(\mathrm{t})=\psi_{1}^{\prime}(\mathrm{t})+\psi_{2}^{\prime}(\mathrm{t}) \tag{2-35}
\end{equation*}
$$

consisting of the de component, $\psi_{1}^{\prime}(t)$, and the transient component, $\psi_{2}^{\prime}(t)$. The input wave can be written as

$$
\begin{equation*}
\psi^{\prime}(t)=\psi_{1}^{\prime}(\infty)+\psi_{2}^{\prime}(t) \tag{2-36}
\end{equation*}
$$

where $\psi_{1}^{\prime}(\infty)$ is the steady-state dc response having a value of -0.0632456 .
If $\mathrm{c}_{3} \mathrm{~h}_{3}(\mathrm{t})$ denotes the impulse response of the deemphasis filter, the response of the deemphasis filter to the input wave, $\psi^{\prime}(t)$, is given by

$$
\begin{align*}
\hat{\mathrm{f}}(\mathrm{t}) & =\mathrm{c}_{3} \int_{-\infty}^{\infty}\left[\psi_{1}^{\prime}(\infty)+\psi_{2}^{\prime}(\mathrm{x})\right] \mathrm{h}_{3}(\mathrm{t}-\mathrm{x}) \mathrm{dx} \\
& =\mathrm{c}_{3} \int_{-\infty}^{\mathrm{t}}-(0.0632456) \mathrm{h}_{3}(\mathrm{t}-\mathrm{x}) \mathrm{dx}+\mathrm{c}_{3} \int_{\mathrm{t}_{1}}^{\mathrm{t}} \psi_{2}^{\prime}(\mathrm{x}) \mathrm{h}_{3}(\mathrm{t}-\mathrm{x}) \mathrm{dx}  \tag{2-37}\\
& =\hat{\mathrm{f}}_{1}(\mathrm{t})+\hat{\mathrm{f}}_{2}(\mathrm{t})
\end{align*}
$$

where the upper limit of the integrals has been changed to $t$ because the deemphasis filter is causal, and the lower limit of the second integral has been changed to $t_{1}$ because the function, $\psi_{2}^{\prime}(\mathrm{t})$, is zero for smaller values of time.
If only the steady-state value, $\hat{\mathrm{f}}_{1}(t)$, is considered, it is given by

$$
\begin{equation*}
\hat{\mathbf{f}}_{1}(\mathrm{t})=\hat{\mathrm{f}}_{1}(\infty)=(-0.0632456)(3.162278) \tag{2-38}
\end{equation*}
$$

where 3.162278 is the value of the deemphasis amplification factor, $\mathrm{c}_{3}$. The final output, $\hat{\mathrm{f}}(\mathrm{t})$, of the system is given by

$$
\begin{align*}
\hat{\mathrm{f}}(\mathrm{t}) & =-0.0632456(3.162278)+3.162278 \int_{\mathrm{t}_{1}}^{\mathrm{t}} \psi_{2}^{\prime}(\mathrm{x}) \mathrm{h}_{3}(\mathrm{t}-\mathrm{x}) \mathrm{dx}  \tag{2-39}\\
& =-0.2+3.162278 \int_{\mathrm{t}_{1}}^{\mathrm{t}} \psi_{2}^{\prime}(\mathrm{x}) \mathrm{h}_{3}(\mathrm{t}-\mathrm{x}) \mathrm{dx}
\end{align*}
$$

From the previous derivations, it is seen that the response of the preemphasis and deemphasis filters given by convolution operations are broken up into two separate convolutions, one of which can be evaluated without performing the operation. In this way, the required computer time is greatly reduced.

### 2.3.2 T-Bar Implementation

The procedures described for the T-pulse implementation are also applicable to the T-bar test signal when it is resolved into the right component parts. The following shows how this can be done.

### 2.3.2.1 The Rise Portion of the T-Bar

Let $T_{R}(t)$ represent the rise-portion of the $T$-bar having a mathematical representation given by

$$
\begin{array}{rlrl}
\mathrm{T}_{\mathrm{R}}(\mathrm{t}) & =-0.2 & \mathrm{t} \leq \mathrm{t}_{4} \\
& =0.7 \sin ^{2}\left[\boldsymbol{\omega}\left(\mathrm{t}-\mathrm{t}_{1}\right)\right]-0.2, & \mathrm{t}_{4} \leq \mathrm{t} \leq \mathrm{t}_{5}  \tag{2-40}\\
& =0.5 & & \mathrm{t}_{5} \leq \mathrm{t} \leq \mathrm{t}_{6}
\end{array}
$$

where $t_{4}$ is the instant at which the half sine-squared pulse starts,
$\mathrm{t}_{5}$ is the instant at which the half sine-squared pulse ends, the duration of the interval $\left(t_{4}, t_{5}\right)$ is 100 nanoseconds.

The duration of the interval, $\left(\mathrm{t}_{5}, \mathrm{t}_{6}\right)$, is arbitrary as long as it is large enough for the transient response of the filters to subside.
To make $T_{R}{ }^{(t)}$ suitable for the previously developed procedures of analysis in the case of the $T$-pulse, $T_{R}\left({ }^{(t)}\right.$ is reconstructed in the following way.

$$
\begin{equation*}
T_{R}(\mathrm{t})=\mathrm{T}_{R 1}{ }^{(\mathrm{t})+\mathrm{T}_{R 2}}{ }^{(\mathrm{t})} \tag{2-41}
\end{equation*}
$$

where

$$
\begin{array}{rlrl}
\mathrm{T}_{\mathrm{R} 1}(\mathrm{t}) & =0 & \mathrm{t}<\mathrm{t}_{4} \\
& =0.7 \sin ^{2}\left[\omega\left(\mathrm{t}-\mathrm{t}_{4}\right)\right], & \mathrm{t}_{4} \leq \mathrm{t} \leq \mathrm{t}_{5}  \tag{2-42}\\
& =0.7 & & \mathrm{t}_{5} \leq \mathrm{t} \leq \mathrm{t}_{6}
\end{array}
$$

and

$$
\begin{equation*}
\mathrm{T}_{\mathrm{R} 2}(\mathrm{t})=-0.2 \text {, all } \mathrm{t} \tag{2-43}
\end{equation*}
$$

When comparison is made with Equation (2-28), it is clear that $T_{R 1}{ }^{(t)}$ corresponds to $\mathrm{f}_{1}(\mathrm{t})$ while $\mathrm{T}_{\mathrm{R} 2}{ }^{(\mathrm{t})}$ corresponds to $\mathrm{f}_{2}(\mathrm{t})$, and the same procedures of analysis can be applied.

### 2.3.2.2 The Fall Portion of the T-Bar

It is assumed that any transient filtering effects have subsided before reaching the fall portion of the T-bar signal. This is reasonable because the pulse is quite long. Therefore, the fall portion of the T-bar signal, $T_{F}(t)$, can be represented by

$$
\begin{array}{rlrl}
\mathrm{T}_{\mathrm{F}}(\mathrm{t}) & =0.5 & \mathrm{t}<\mathrm{t}_{7} \\
& =0.5-0.7 \sin ^{2}\left[\omega\left(\mathrm{t}-\mathrm{t}_{7}\right)\right], & \mathrm{t}_{7} \leq \mathrm{t} \leq \mathrm{t}_{8}  \tag{2-44}\\
& =-0.2 & & \mathrm{t}>\mathrm{t}_{8}
\end{array}
$$

where the duration of the interval $\left(t_{7}, t_{8}\right)$ is also 100 nanoseconds, and the value of $t_{7}$ is arbitrary.
$\mathrm{T}_{\mathrm{F}}{ }^{(\mathrm{t})}$ is also reconstructed in the following manner:

$$
\begin{equation*}
\mathrm{T}_{\mathrm{F}}{ }^{(\mathrm{t})}=\mathrm{T}_{\mathrm{F} 1}{ }^{(\mathrm{t})+\mathrm{T}} \mathrm{~F}_{2}{ }^{(\mathrm{t})} \tag{2-45}
\end{equation*}
$$

where

$$
\begin{array}{rlrl}
\mathrm{T}_{\mathrm{F} 1}(\mathrm{t}) & =0 & \mathrm{t}<\mathrm{t}_{7} \\
& =-0.7 \sin ^{2}\left[\omega\left(\mathrm{t}-\mathrm{t}_{7}\right)\right], & \mathrm{t}_{7} \leq \mathrm{t} \leq \mathrm{t}_{8}  \tag{2-46}\\
& =-0.7 & & \mathrm{t}>\mathrm{t}_{8}
\end{array}
$$

and

$$
\begin{equation*}
\mathrm{T}_{\mathrm{F} 2}(\mathrm{t})=0.5, \text { all } \mathrm{t} \tag{2~47}
\end{equation*}
$$

Again, with reference to Equation (2-28), it is seen that $T_{F 1}$ ( t$)$ corresponds to $\mathrm{f}_{1}(\mathrm{t})$ and $\mathrm{T}_{\mathrm{F} 2}{ }^{(\mathrm{t})}$ to $\mathrm{f}_{2}$ (t) so that the same procedures of analysis can be applied.
For the case of a flat FM television system, the constant voltage portion of the T-bar test signal was shortened to 600 nanoseconds and then the entire modified $T$-bar signal was applied directly to the system.

### 2.4 NUMERICAL METHODS USED IN THE COMPUTER IMPLEMENTATION

The numerical formula used in the analysis for evaluating a definite integral is simply the rectangular formula.

$$
\begin{equation*}
\int_{\mathbf{t}_{1}}^{\mathbf{t}_{2}} \mathrm{f}(\mathrm{x}) \mathrm{dx} \approx \sum_{\mathrm{n}=1}^{\mathrm{N}} \mathrm{f}\left(\mathrm{t}_{1}+\mathrm{n} \Delta \mathrm{t}\right) \Delta \mathrm{t} \tag{2-48}
\end{equation*}
$$

where $f(t)$ is a given continuous time function,
$t_{1}$ and $t_{2}$ are the limits of integration,
$N$ is the number of sections into which the interval of integration ( $t_{2}-t_{1}$ ) is partitioned,
$\Delta t=\frac{t_{2}-t_{1}}{N}$, the size of an increment of time.
The numerical formula used for differentiation is a three-point one (Reference 4) as follows:

$$
\begin{equation*}
f_{i}^{\prime}=\frac{1}{2 \Delta t}\left(-3 f_{i}+4 f_{i+1}-f_{i+2}\right) \tag{2-49}
\end{equation*}
$$

where $f_{i}, f_{i+1}$, and $f_{i+2}$ are the values of the given function $f(t)$ at the $i^{\text {th }},(i+1)^{\text {th }}$, and $(1+2)^{\text {th }}$ points of the t -axis,
$f_{i}^{\prime}$ is the approximate value of the derivative of $f(t)$ at the $i^{\text {th }}$ point of the $t$-axis, $\Delta t$ is the step-size into which the $t$-axis is divided.

The numerical formula for evaluating a given convolution of two causal time functions is given below, and its rigorous proof can be found in Reference 5.

Let

$$
\begin{equation*}
R(t)=\int_{0}^{t} f(x) g(t-x) d x \tag{2-50}
\end{equation*}
$$

where $f(t)$ and $g(t)$ are two causal functions.
If the range of integration is divided into $n$ intervals of $\Delta t$ second, then $R(n \Delta t)$ is given by

$$
\begin{equation*}
R(n \Delta t)=\Delta t \sum_{m=0}^{n} f_{m} g_{n-m} \tag{2-51}
\end{equation*}
$$

## SECTION 3 - DESCRIPTION OF COMPUTER PROGRAMS

### 3.1 INTRODUCTION

The computer programs, developed in FORTRAN for the FM distortion analysis, are discussed in this section and a listing of the programs is found in Appendix C. The programs are not combined into a single program, because some results are calculated only once and are stored in data files for repeated use as input data for some of the other programs. There are sixteen numbered programs each of which performs a particular function as shown in Table 3-1. The final results of each program are stored in data files which can be printed out if desired or used directly as input data for other programs. Some of the computer programs are very similar and will be described as a group.

The computer programs all use a 1-nanosecond time increment for sampling and mathematical operations such as integration and differentiation.

### 3.2 COMPUTER PROGRAM 1

This computer program is used to generate sample values of components of a T-bar test signal. Its flow hart is shown in Figure 3-1. The data generated by this program are used as input data for Program 7. The program calculates samples of the rise and fall components of the T-bar according to Equations $(2-42)$ and $(2-46)$, respectively.

It generates samples at one-nanosecond intervals from the leading edge of the sine-squared curve at time, T1, to some later time, TEND, following the end of the sine-squared curve. In this program, T 1 is always set to zero, and the end of the sine-squared curve is designated T 3 with a value of 100 nanoseconds. The sample values generated by this program are printed out and stored in two data files for later use - one for the rise and one for the fall.

### 3.3 COMPUTER PROGRAM 2

This computer program is used to generate sample values of an unbiased T-pulse. The flow chart, similar to that for the T-bar, is shown in Figure 3-2. The sample values are calculated, according to Equation(2-29), at 1-nanosecond intervals from the leading edge of the sine-squared curve at time, Tl , to some later time, TEND, following the end of the sine-squared pulse. The sample values for times later than T 3 are zero. In this program, T 1 is always set to zero, and the end of the sine-squared curve is designated T3 with a value of 200 nanoseconds. The sample values are printed out and stored in a data file for later use in Program 8.

### 3.4 COMPUTER PROGRAM 3

This program generates samples of the biased and shortened T-bar test signal. The flowchart for this program is shown in Figure 3-3. The data generated by this program are used as input data for Program 9. It generates samples at 1-nanosecond intervals from zero time to some later time, TEND (1800 nanoseconds). The samples are calculated for the T-bar signal starting at time $T 1$ ( 600 nanoseconds) and ending at T4 ( 1400 nanoseconds), and they include values of the -0.2 -volt de bias for 600 nanoseconds preceding the shortened T-bar and 400 nanoseconds following it.

The sample values generated by this program are printed out and stored in a data file for later use.

### 3.5 COMPUTER PROGRAM 4

This program generates samples of the biased T-pulse test signal. The flowchart for this program is shown in Figure 3-4. The data generated by this program are used as input data for Program 9. It generates samples at onenanosecond intervals from zero time to some later time, TEND (1400 nanoseconds). The samples are calculated for the T-pulse signal starting at time T2 (600 nanoseconds) and ending at T3 (800 nanoseconds), and they include
values of the -0.2 -volt dc bias for 600 nanoseconds preceding the $T$-pulse and 600 nanoseconds following it.

### 3.6 COMPUTER PROGRAMS 5 and 6

These two computer programs are used to generate sample values of the preemphasis and deemphasis filters, respectively. Their flowcharts are identical and therefore only one flowchart is shown for them in Figure 3-5.

The samples, calculated by Program 5 and used as input data for Program 7, are printed out and stored in a data file for later use. Those calculated by Program 6 and used as input data for Programs 15 and 16 are also printed out and stored in a separate data file.

### 3.7 COMPUTER PROGRAM 7

This computer program is used to generate sample values of the modulating wave for the case of a preemphasized FM television system with a T-bar test signal. Rise and fall sections of the T-bar are calculated separately and are stored in two separate files. The flowehart for this program is shown in Figure 3-6. The input data are generated by Programs 1 and 5. Program 1 supplies the samples of the rise and fall components of the T-bar test signal, while Program 5 supplies sample values of the preemphasis impulse response. This program performs the convolutions required and adds in the dc voltage levels to generate samples of the output waveforms of the preemphasis filter. These data are printed out and stored in files for later use in Program 9.

## 3. 8 COMPUTER PROGRAM 8

This computer program is used to generate sample values of the modulating wave for the case of a preemphasized FM television system with a T-pulse test signal. The flowchart for this program is shown in Figure 3-7. The input data are generated by Programs 2 and 5. Program 2 supplies samples of the T-pulse, while Program 5 supplies sample values of the preemphasis impulse response. This program performs the convolutions required and adds in the dc voltage levels to generate samples of the output waveform of the preemphasis filter. These data are printed out and stored in files for later use in Program 9.

### 3.9 COMPUTER PROGRAM 9

This program calculates sample values of $\cos \phi(t)$ and $\sin \phi(t)$ for both flat and preemphasized FM television systems. The fiowchart is shown in Figure 3-8. The input samples of the modulating wave generated by Program 3, 4, 7, or 8, are used to calculate samples of $\phi(t)$ in accordance with Equation (2-3) for the preemphasized case and in accordance with Equation (2-19) for the flat case. After integrating to obtain $\phi(\mathrm{t})$ the sample values of $\cos \phi(\mathrm{t})$ and $\sin \phi(\mathrm{t})$ are calculated and stored in two separate files for later use in Program 14, 15, or 16.

### 3.10 COMPUTER PROGRAMS 10 THROUGH 13

All these computer programs are used to generate the sample values of the impulse responses of three - and four-pole Butterworth lowpass filters, the three- and fourpole Chebyshev lowpass filters. The single flowchart for Programs 10 and 12 is shown in Figure 3-9, and the flowchart for Programs 11 and 13 is shown in Figure 3-10. The variable, FB, in these programs represents the bandwidth in Hertz and should be assigned a value accordingly. In Programs 10 and 11, FB is the halfpower bandwidth of a Butterworth filter, while in Programs 12 and 13 it represents the ripple bandwidth of a Chebyshev filter. The resulting sample values of the impulse response are stored in a file for later use in Program 14, 15, or 16.

### 3.11 COMPUTER PROGRAM 14

This computer program performs the final mathematical operations required to obtain sample values of the final output of a flat FM television system. The flowchart is shown in Figure 3-11. There are three input data files for this program, and they contain sample values of $\cos \phi(t), \sin \phi(t)$ and $h(t)$. Values of $\cos \phi^{(t)}$ and $\sin \phi(t)$ are generated by Computer Program 9 and values of $h_{\ell}(t)$ are generated by Programs $10,11,12$, or 13 , depending on the predetection filter specifications. The peak deviation, DP, must be specified within the program. The program functions as follows:

Two convolution operations produce the time functions, $\mathrm{A}(\mathrm{t})$ and $\mathrm{B}(\mathrm{t})$, defined by Equations (2-10) and (2-11). $A(t)$ and $B(t)$ are obtained by convolving $\cos \phi(t)$ and $\sin \phi(\mathrm{t})$ with the impulse response $\mathrm{h}_{\ell}(\mathrm{t})$ of an equivalent lowpass predetection filter. The sample values of $\cos \phi(t), \sin \phi(t)$, and $h_{\ell}(t)$ are read from files into the vectors FLCOS, FLSIN, and HBF, respectively. The resulting sample values of $\mathrm{A}(\mathrm{t})$ and $\mathrm{B}(\mathrm{t})$ are retained in the vectors A and B , respectively.

The time function $\psi(t)$, which is the arctangent of $B(t) / A(t)$, is then produced and the sample values of $\psi(t)$ are retained in the vector PHI. When $A(t)$ takes a value of zero, the corresponding calculation of the arctangent of $B(t) / A(t)$ is skipped. This fact is reported by printing out the message " $\mathrm{A}=0.0$."

Finally, the differentiation of $\psi(\mathrm{t})$ is performed and the sample values of the resulting function, $\psi^{\prime}(t)$, are then divided by $K$. Then the resulting sample values of the final output are retained in a file and are also printed out.

### 3.12 COMPUTER PROGRAMS 15 AND 16

Programs 15 and 16 are used to calculate sample values of the final output waveform for a preemphasized FM television system. Program 15 calculates the system response to the T-pulse or to the rise portion of the T-bar test signal. Program 16 calculates the system response to the fall portion of the T-bar. These two programs are identical except for the values of two constants; therefore, only one flowchart, shown in Figure 3-12, is used to describe the programs. There are four input data files for this program, and they contain sample values of $\cos \phi(t), \sin \phi(t), h_{\ell}(t)$, and $h_{3}(t)$. The values of $\cos \phi(t)$ and $\sin \phi(t)$ are generated by Computer Program 9 and values of $h_{\ell}(t)$ are generated by Program $10,11,12$, or 13 , depending on the predetection filter specification. The peak deviation must be specified within each program. The value $M$ in Program 15 is specified as 1200 for T-bar calculations and as 1400 for $T$-pulse calculations. The value of $M$ does not change from 1200 in Program 16. Each program functions as follows:

Two convolution operations produce the time function, $\mathrm{A}(\mathrm{t})$ and $\mathrm{B}(\mathrm{t})$, defined by Equations (2-10) and (2-11). $\mathrm{A}(\mathrm{t})$ and $\mathrm{B}(\mathrm{t})$ are obtained by convolving $\cos \phi(\mathrm{t})$ and $\sin \phi(t)$ with the impulse response, $\mathrm{h}_{\ell}(\mathrm{t})$, of an equivalent lowpass predetection filter. The sample values of $\cos \phi(t), \sin \phi(t)$, and $h_{\ell}(t)$ are read from files into the vectors FLOCOS, FLSIN, and HBF, respectively. The resulting sample values of $A(t)$ and $B(t)$ are retained in the vectors $A$ and $B$, respectively.

The time function, $\psi(\mathrm{t})$, which is the arctangent of $\mathrm{B}(\mathrm{t}) / \mathrm{A}(\mathrm{t})$ is then produced and the sample values of $\psi(t)$ are retained in the vector PHI. When $A(t)$ takes a value of zero, the corresponding calculation of arctangent of $B(t) / A(t)$ is skipped. This fact is reported by printing out the message " $\mathrm{A}=0.0$ ".

The differentiation of $\psi(t)$ is then performed and the sample values of the resulting function, $\psi^{\prime}(t)$, are divided by $K$ and retained in the vector PHID. Then all these sample values are examined for computational errors. The errors, occurring in pairs, are corrected by interpolation.

After corrections are made to the components of the vector PHID, this vector is convolved with the vector HDE, which contains the sample values of the impulse response of the deemphasis filter, to give the final output of the preemphasized FM television system.

Table 3-1. Functions of Computer Programs

PROGRAM
NUMBER

1

2

3

4

PROGRAM FUNCTION

Generates the sample values of a T-bar signal for a preemphasized FM television system.

Generates the sample values of a T-pulse signal for a preemphasized FM television system.

Generates the sample values of a T-bar sig nal for a flat FM television system.

Generates the sample values of a T-pulse signal for a flat FM television system.

Generates the sample values of the impulse response of the preemphasis filter.

Generates the sample values of the impulse response of the deemphasis filter.

Calculates the sample values of the modulating wave corresponding to a T-bar signal in a preemphasized FM television system.

Calculates the sample values of the modulating wave corresponding to a T-pulse test signal in a preemphasized FM television system.

Calculates the sample values of the time function $\cos \phi(\mathrm{t})$ and $\sin \phi(\mathrm{t})$.

Generates the sample values of the impulse response of a symmetrical 3-pole Butterworth filter.

Table 3-1. Functions of Computer Programs

## PROGRAM

NUMBER

PROGRAM
FUNCTION
Generates the sample values of the impulse response of a symmetrical 4-pole Butterworth filter.

Generates the sample values of the impulse response of a symmetrical 3-pole Chebyshev filter.

Generates the sample values of the impulse response of a symmetrical 4-pole Chebyshev filter.

Calculates the sample values of the final output of a flat FM television system.

Calculates the sample values of the final output of a preemphasized FM television system when the input is either a $T$-pulse or the rise portion of the T-bar signal.

Calculates the sample values of the final output of a preemphasized FM television system when the input is the fall portion of the T-bar signal.


Figure 3-1. Flowchart for Computer Program 1


Figure 3-2. Flowchart for Computer Program 2


Figure 3-3. Flowchart for Computer Program 3


Figure 3-4. Flowchart for Computer Program 4


Figure 3-5. Flowchart for Computer Programs 5 and 6


Figure 3-6. Flowchart for Computer Program 7


Figure 3-7. Flowchart for Computer Program 8


Figure 3-8. Flowchart for Computer Program 9


Figure 3-9. Flowchart for Computer Programs 10 and 12


Figure 3-10. Flowchart for Computer Programs 11 and 13


Figure 3-11. Flowchart for Computer Program 14


Figure 3-12. Flowchart for Computer Programs 15 and 16

## SECTION 4 - ANALYSIS OF COMPUTER RESULTS

### 4.1 INTRODUCTION

The computer programs described in Section 3 and listed in Appendix B are developed to calculate responses of FM television systems to a T-pulse-and-bar test signal. The responses are calculated for 64 different combinations of system parameters. These responses are used as a measure of the distortion quality of the systems being evaluated. The response of a system can be compared with the input test signal and with the responses of other systems.

Table 4-1 is convenient for finding the figure showing the calculated system response for particular system parameters.

### 4.2 INPUT TEST SIGNAL

The T-pulse-and-bar test signal is shown in Figure 2-3 in terms of the time relation of the pulse and bar to a single television line scan. Figure 4-1 shows the T-pulse in detail, while Figure 4-2 shows the T-bar in detail. The width of the pulse at half amplitude is $T$, having a value of 100 nanoseconds. The entire length of the pulse is 200 nanoseconds. The height of the pulse is 0.7 volt extending from -0.2 volt to 0.5 volt.

The T-bar has a width of 25.0 microseconds at half amplitude and a total length of 25.2 microseconds. Its height is also 0.7 volt extending from -0.2 volt to 0.5 volt. Note that a break in the graph of Figure 4-2 is used to indicate that the T-bar continues at the same level over a long period of time from one side of the break to the other. This technique is used to show detail of the T-bar on a single page.

### 4.3 PRESENTATION OF THE RESULTS

The calculated system responses to the T-pulse-and-bar are shown in Figures 4-3 through 4-130. Table 4-1 is an index for these figures. In all cases the T-bar response for a particular specified system is found in the figure immediately following the T-pulse response for that same system. This is clearly presented in Table 4-1. For example, the T-pulse
response of a preemphasized FM system with a three-pole Butterworth predetection filter having a half-power bandwidth of 10 MHz and with a $5-\mathrm{MHz}$ peak deviation is found in Figure 4-67. The T-bar response for the same system is found in Figure 4-68.

The RF bandwidths of the predetection filters are specified in two ways. Half-power bandwidths are specified for the Butterworth filters and ripple bandwidths are specified for the Chebyshev filters. Table 4-2 shows the half-power bandwidths corresponding to the ripple bandwidths for all the cases analyzed.

### 4.4 DISCUSSION OF THE RESULTS

The distortion of the test signal in passing through the system can be evaluated as a function of the system parameters. The effect of the RF predetection filter type, its number of poles, its bandwidth, the peak deviation, and preemphasis are discussed.

### 4.4.1 Effects of Predetection Filter Types

A comparison is made of the distortion effects resulting from the use of Butterworth and Chebyshev filters with the same number of poles and the same specified bandwidths. Comparison of the cases exhibiting a rather large distortion is first made. Consider the cases for the three-pole, $10.0-\mathrm{MHz}$ filters and $5.0-\mathrm{MHz}$ peak deviation. Table $4-1$ shows that Figures 4-3 and 4-4 give the test signal response for a flat system utilizing a Butterworth filter and that Figures 4-19 and 4-20 give the corresponding test signal response for a flat system utilizing a Chebyshev filter. The undershoot of the T-pulse in these cases is $3.1 \%(0.022$ volt) for the Butterworth and $2.3 \%$ ( 0.016 volt) for the Chebyshev cases. These undershoot values compare, respectively, with $6.9 \%$ ( 0.048 volt) and $2.1 \%$ ( 0.015 volt ) for a preemphasized system. These latter numbers are derived from Figures 4-67 and 4-83. The T-bar responses also show more distortion for the Butterworth than for the Chebyshev cases. If the same comparison is made for fourpole filters, the Chebyshev filter again looks better. On the basis of this comparison of the use of a Butterworth filter having a given half-power bandwidth with a Chebyshev filter having a ripple bandwidth of the same value, the Butterworth filter produces more distortion.

Since this comparison may not be fair to the Butterworth filter a similar comparison is made comparing Butterworth and Chebyshev filters having the same half-power bandwidths as well as the same number of poles. Table 4-2 shows that a four-pole Chebyshev filter having a $10.0-\mathrm{MHz}$ ripple bandwidth has a half-power bandwidth of 12.13 MHz ; therefore, the four-pole Chebyshev filter with a $10.0-\mathrm{MHz}$ ripple Bandwidth can be compared with the four-pole Butterworth filter with a $12.0-\mathrm{MHz}$ half-power bandwidth. The comparison for the T-pulse in a flat system can be made by comparing Figures 4-43 and 4-57. These cases utilize a peak deviation of 6.0 MHz . The undershoot is $3.9 \%$ ( 0.028 volt) for the Butterworth case compared with $4.3 \%$ ( 0.030 volt) for the Chebyshev. This difference is rather small. On the fall side of the T-pulse the overshoot is $2.9 \%$ ( 0.020 volt) for the Butterworth filter compared with $4.3 \%$ ( 0.030 volt) for the Chebyshev. Thus, it appears that the Chebyshev filter still produces slightly more distortion than the corresponding Butterworth filter. When the T-bar signals are compared in Figures 4-44 and 4-58, one sees that the Butterworth filter produces an overshoot of $2.7 \%$ ( 0.019 volt) on the fall compared with $2.3 \%$ ( 0.016 volt) for the Chebyshev. This would indicate that the Butterworth filter produces slightly more distortion than the Chebyshev. Thus, there appears to be no significant difference in the performance of the comparable Butterworth and Chebyshev filters for a flat system.

A similar comparison for a preemphasized system shows a more significant difference in performance of the two types of filters. A comparison of T-pulse distortions is made by observing Figures $4-107$ and $4-121$. The undershoot is $5.6 \%$ ( 0.039 volt) for the Butterworth case compared with $6.0 \%$ ( 0.042 volt) for the Chebyshev, and on the fall side of the $T$-pulse the overshoot is $4.4 \%$ ( 0.031 volt) for the Butterworth compared with $9.9 \%$ ( 0.069 volt), the largest of all overshoots, for the Chebyshev. That the Chebyshev filter produces more distortion than the Butterworth in this case is strengthened by comparing the overshoots at the fall ends of the T-bar output signals. Figures 4-108 and $4-122$ show overshoots of $1.4 \%(0.010$ volt) and $3.6 \%(0.025$ volt), respectively, for the Butterworth and Chebyshev filters. Thus, the Chebyshev overshoot at the end of each signal is approximately twice as large as for the corresponding Butterworth overshoot.

For broadband cases the distortion from one filter to another is small enough that any differences in the distortions are also small. In such cases the differences may be within the accuracy of the computer estimation of the output signal.

In conclusion, if preemphasis is used, it appears that the Chebyshev filter produces more distortion than a Butterworth filter with the same number of poles and the same half-power bandwidth and that without preemphasis there is no significant difference. It also appears that the Butterworth filter with a certain half-power bandwidth produces significantly more distortion than a Chebyshev filter with a ripple bandwidth numerically equal to the half-power bandwidth of the Butterworth filter.

### 4.4.2 Effects of the Number of Poles of the Predetection Filter

If all other parameters of the FM system remain fixed, the number of poles of the RF predetection filter affect the distortion of the system. In fact, distortion increases with the number of poles, and some examples are chosen to illustrate this point.

First, an example of Butterworth filters is chosen. Figures 4-3 and 4-9 are compared for T-pulse distortion. The peak deviation is 5.0 MHz and the half-power bandwidth is 10.0 MHz for this case. The undershoot at the top of the pulse is only $3.1 \%$ ( 0.022 volt) for the three-pole filter compared with $3.7 \%(0.026$ volt) for the four-pole filter. The overshoot at the end of the pulse is $4.3 \%$ ( 0.030 volt) compared with $5.7 \%$ ( 0.040 volt) with the larger number of poles causing the larger distortion. A similar effect is found in a preemphasized system. It also shows up in the T-bar responses, which can be compared in Figures 4-4 and 4-10. An example chosen for the Chebyshev case is the comparison of Figures 4-99 and 4-105 for a preemphasized system with a peak deviation of 6.0 MHz and a ripple bandwidth of 10.0 MHz . The T-pulse exhibits an undershoot of $7.4 \%(0.052$ volt) at the peak for the threepole filter compared with $8.6 \%$ ( 0.060 volt) for the four-pole case. Again a four-pole filter is seen to cause a larger distortion than a three-pole filter.

### 4.4.3 Effects of the Bandwidth of the Predetection Filter

The bandwidth of the predetection filter definitely affects the distortion of the test
signal and this effect is clearly seen in the output waveforms. If all other system parameters are unchanged, an increase in bandwidth results in a decrease in distortion. As the bandwidth of the three-pole Butterworth predetection filter increases from 10.0 MHz to 12.0 MHz and then to 15.0 MHz , the T -pulse distortion shown in Figures $4-3$, 4-5, and 4-7 decreases. The T-pulse undershoot at its peak is $3.1 \%$ ( 0.022 volt) for a $10.0-$ MHz bandwidth, $2.6 \%$ ( 0.018 volt) for a $12.0-\mathrm{MHz}$ bandwidth, and $1.4 \%$ ( 0.010 volt) for a $15.0-$ MHz bandwidth. The overshoot at the end of the pulse decreases from $4.3 \%(0.030$ volt) to $2.1 \%$ ( 0.015 volt) and finally to $1.1 \%$ ( 0.008 volt) as the bandwidth increases. Similar decreases in distortion occur in other examples as the bandwidth of the predetection filter increases. As it increases without other changes, more of the $R F$ spectrum is passed by the filter and this results in less distortion of the modulated waveform, which, in the end, results in a less distorted output signal.

### 4.4.4 Effects of Peak Deviation

A change in the peak deviation affects the distortion if all other system parameters remain unchanged. The figures listed at the top of Table 4-1 can be compared on a one-to-one basis with those at the bottom of the table. Those at the top are for a 5.0MHz peak deviation, and those at the bottom are for a $6.0-\mathrm{MHz}$ peak deviation. In comparing Figure $4-3$ with Figure $4-35$ one sees that the T-pulse undershoots by $3.1 \%(0.022$ volt) at the peak in the first figure and by $4.3 \%(0.030$ volt $)$ in the latter. However, the overshoot at the end of the pulse is $4.3 \%$ in both figures. In general, such comparisons as this indicate more distortion with more peak deviation. This is to be expected, since more of the FM spectrum is rejected by the predetection filter as the peak deviation increases.

### 4.4.5 Effects of Preemphasis

If all other system parameters are unchanged, the distortion of the test signal depends on whether or not preemphasis is used. It is quite clear from the data that preemphasis increases distortion of the test signal. As an example, compare the distorted T-pulses of Figures 4-3 (flat) and 4-67 (preemphasized). The undershoot of the pulse peak in the flat system is $3.1 \%(0.022$ volt) compared with $6.9 \%(0.048$ volt) in the preemphasized
system, and the overshoot at the end of the pulse is $4.3 \%(0.030$ volt ) in the flat system compared with $4.6 \%$ ( 0.032 volt) in the preemphasized system.

This effect can be explained if the modulating signals are compared for the two cases. In the flat case the modulating signal is just the T-pulse-and-bar as shown in Figures 4-1 and $4-2$. The modulating voltage ranges from $\mathbf{- 0 . 2}$ volt to 0.5 volt and the peak deviations correspond to $\pm 0.5$ volt. If a preemphasis filter is added, the modulating waveform takes the form shown in Figures 4-131 through 4-133.

Figure 4-131 shows the response of the preemphasis filter to the T-pulse. Note that the modulating voltage reaches a peak of 0.79 volt corresponding to a peak deviation that is $58 \%$ larger than that which occurs without preemphasis. Naturally the predetection filter distorts this peak, since the RF frequency corresponding to the peak is farther from the center of the predetection filter. At the end of the pulse the preemphasis filter causes an overshoot to -0.33 volt compared with -0.20 volt for the flat case. Thus, one could possibly expect more distortion at the end of the pulse in a narrow-band system, although the peak deviation is not excessive on that side of the carrier frequency.

Figure 4-132 shows the response of the preemphasis filter to the rise portion of the T -bar. Again, as in the case of the T -pulse, the voltage rises to 0.79 volt which provides a $58 \%$ increase in peak deviation over that without preemphasis. Then the voltage slowly falls to a lower constant de level. This results in an increased deviation on one side of the carrier frequency and distortion by the predetection filter.

Figure 4-133 shows the response of the preemphasis filter to the fall portion of the T-bar. Here the voltage falls to approximately -0.7 volt, causing an increase of $40 \%$ in peak deviation over that achieved at the peak of the synchronizing pulse in the absence of preemphasis. Preemphasis increases the deviation of the T-bar test signal on the low side of center frequency by $350 \%$, since the $T$-bar test signal goes only as low as -0.2 volt in the absence of preemphasis.

Clearly preemphasis causes increased distortion of a T-pulse-and-bar test signal over that of a flat system, and that increased distortion results from the increased deviation of the RF carrier and resultant distortion of the RF carrier.

Table 4-1. List of Figures Showing Pulse and Bar Responses of Specified FM Systems

| SYSTEM PARAMETERS |  |  |  | FIGURE NUMBERS |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Filter Type* | Number of Poles | $\begin{aligned} & \text { Bandwidth } \\ & (\mathrm{MHz}) \end{aligned}$ | Peak Deviation (MHz) | Flat |  | Preemphasized |  |
|  |  |  |  | Pulse | Bar | Pulse | Bar |
| B | 3 | 10 | 5 | 4-3 | 4-4 | 4-67 | 4-68 |
| B | 3 | 12 | 5 | 4-5 | 4-6 | 4-69 | 4-70 |
| B | 3 | 15 | 5 | 4-7 | 4-8 | 4-71 | 4-72 |
| B | 4 | 10 | 5 | 4-9 | 4-10 | 4-73 | 4-74 |
| B | 4 | 12 | 5 | 4-11 | 4-12 | 4-75 | 4-76 |
| B | 4 | 15 | 5 | 4-13 | 4-14 | 4-77 | 4-78 |
| B | 4 | 20 | 5 | 4-15 | 4-16 | 4-79 | 4-80 |
| B | 4 | 30 | 5 | 4-17 | 4-18 | 4-81 | 4-82 |
| C | 3 | 10 | 5 | 4-19 | 4-20 | 4-83 | 4-84 |
| C | 3 | 12 | 5 | 4-21 | 4-22 | 4-85 | 4-86 |
| C | 3 | 15 | 5 | 4-23 | 4-24 | 4-87 | 4-88 |
| C | 4 | 10 | 5 | 4-25 | 4-26 | 4-89 | 4-90 |
| C | 4 | 12 | 5 | 4-27 | 4-28 | 4-91 | 4-92 |
| C | 4 | 15 | 5 | 4-29 | 4-30 | 4-93 | 4-94 |
| C | 4 | 20 | 5 | 4-31 | 4-32 | 4-95 | 4-96 |
| C | 4 | 30 | 5 | 4-33 | 4-34 | 4-97 | 4-98 |
| B | 3 | 10 | 6 | 4-35 | 4-36 | 4-99 | 4-100 |
| B | 3 | 12 | 6 | 4-37 | 4-38 | 4-101 | 4-102 |
| B | 3 | 15 | 6 | 4-39 | 4-40 | 4-103 | 4-104 |
| B | 4 | 10 | 6 | 4-41 | 4-42 | 4-105 | 4-106 |
| B | 4 | 12 | 6 | 4-43 | 4-44 | 4-107 | 4-108 |
| B | 4 | 15 | 6 | 4-45 | 4-46 | 4-109 | 4-110 |
| B | 4 | 20 | 6 | 4-47 | 4-48 | 4-111 | 4-112 |
| B | 4 | 30 | 6 | 4-49 | 4-50 | 4-113 | 4-114 |
| C | 3 | 10 | 6 | 4-51 | 4-52 | 4-115 | 4-116 |
| C | 3 | 12 | 6 | 4-53 | 4-54 | 4-117 | 4-118 |
| C | 3 | 15 | 6 | 4-55 | 4-56 | 4-119 | 4-120 |
| C | 4 | 10 | 6 | 4-57 | 4-58 | 4-121 | 4-122 |
| C | 4 | 12 | 6 | 4-59 | 4-60 | 4-123 | 4-124 |
| C | 4 | 15 | 6 | 4-61 | 4-62 | 4-125 | 4-126 |
| C | 4 | 20 | 6 | 4-63 | 4-64 | 4-127 | 4-128 |
| C | 4 | 30 | 6 | 4-65 | 4-66 | 4-129 | 4-130 |

*Note: B for Buttersworth; C for Chebyshev

Table 4-2. Half-Power Bandwidths Corresponding to the 0.1-dB Ripple Bandwidths for Three- and Four-Pole Chebyshev Filters

| Ripple <br> Bandwidth <br> (MHz) | 3 Poles | 4 Poles |
| :---: | :---: | :---: |
| 10 | 13.89 | 12.13 |
| 12 | 16.67 | 14.55 |
| 15 | 20.84 | 18.20 |
| 20 | -- | 24.26 |
| 30 | -- | 36.39 |



Figure 4-1. T-Pulse Test Signal


Figure 4-2. T-Bar Test Signal


Figure 4-3. Response of a Flat FM Television System Having a 3-Pole Butterworth Filter to a T-Pulse Test Signal ( $\mathrm{B}=10 \mathrm{MHz}, \mathrm{D}_{\mathrm{p}}=5 \mathrm{MHz}$ )


Figure 4-4. Response of a Flat FM Television System Having a 3-Pole Butterworth Filter to a T-Bar Test Signal ( $\mathrm{B}=10 \mathrm{MHz}, \mathrm{D}_{\mathrm{p}}=5 \mathrm{MHz}$ )


Figure 4-5. Response of a Flat FM Television System Having a 3-Pole Butterworth Filter to a T-Pulse Test Signal ( $\mathrm{B}=12 \mathrm{MHz}, \mathrm{D}_{\mathrm{p}}=5 \mathrm{MHz}$ )


Figure 4-6. Response of a Flat FM Television System Having a 3-Pole Butterworth Filter to a T-Bar Test Signal ( $\mathrm{B}=12 \mathrm{MHz}, \mathrm{D}_{\mathrm{p}}=5 \mathrm{MHz}$ )


Figure 4-7. Response of a Flat FM Television System Having a 3-Pole Butterworth Filter to a T-Pulse Test Signal ( $\mathrm{B}=15 \mathrm{MHz}, \mathrm{D}_{\mathrm{p}}=5 \mathrm{MHz}$ )


Figure 4-8. Response of a Flat FM Television System Having a 3-Pole Butterworth Filter to a T-Bar Test Signal

$$
\left(\mathrm{B}=15 \mathrm{MHz}, \mathrm{D}_{\mathrm{p}}=5 \mathrm{MHz}\right)
$$



Figure 4-9. Response of a Flat FM Television System Having a 4-Pole Butterworth Filter to a T-Pulse Test Signal
( $\mathrm{B}=10 \mathrm{MHz}, \mathrm{D}_{\mathrm{p}}=5 \mathrm{MHz}$ )


Figure 4-10. Response of a Flat FM Television System Having a 4-Pole Butterworth Filter to a T-Bar Test Signal
( $\mathrm{B}=10 \mathrm{MHz}, \mathrm{D}_{\mathrm{p}}=5 \mathrm{MHz}$ )


Figure 4-11. Response of a Flat FM Television System Having a 4-Pole Butterworth Filter to a T-Pulse Test Signal ( $\mathrm{B}=12 \mathrm{MHz}, \mathrm{D}_{\mathbf{p}}=5 \mathrm{MHz}$ )


Figure 4-12. Response of a Flat FM Television System Having a 4-Pole Butterworth Filter to a T-Bar Test Signal ( $\mathrm{B}=12 \mathrm{MHz}, \mathrm{D}_{\mathrm{p}}=5 \mathrm{MHz}$ )


Figure 4-13. Response of a Flat FM Television System Having a 4-Pole Butterworth Filter to a T-Pulse Test Signal ( $\mathrm{B}=15 \mathrm{MHz}, \mathrm{D}_{\mathrm{p}}=5 \mathrm{MHz}$ )


Figure 4-14. Response of a Flat FM Television System Having a 4-Pole Butterworth Filter to a T-Bar Test Signal ( $\mathrm{B}=15 \mathrm{MHz}, \mathrm{D}_{\mathrm{p}}=5 \mathrm{MHz}$ )


Figure 4-15. Response of a Flat FM Television System Having a 4-Pole Butterworth Filter to a T-Pulse Test Signal ( $\mathrm{B}=20 \mathrm{MHz}, \mathrm{D}_{\mathrm{p}}=5 \mathrm{MHz}$ )


Figure 4-16. Response of a Flat FM Television System
Having a 4-Pole Butterworth Filter to a T-Bar Test Signal

$$
\left(\mathrm{B}=20 \mathrm{MHz}, \mathrm{D}_{\mathrm{p}}=5 \mathrm{MHz}\right)
$$



Figure 4-17. Response of a Flat FM Television System Having a 4-Pole Butterworth Filter to a T-Pulse Test Signal ( $\mathrm{B}=30 \mathrm{MHz}, \mathrm{D}_{\mathrm{p}}=5 \mathrm{MHz}$ )


Figure 4-18. Response of a Flat FM Television System Having a 4-Pole Butterworth Filter to a T-Bar Test Signal
( $\mathrm{B}=30 \mathrm{MHz}, \mathrm{D}_{\mathrm{p}}=5 \mathrm{MHz}$ )


Figure 4-19. Response of a Flat FM Television System Having a 3-Pole Chebyshev Filter to a T-Pulse Test Signal $\left(\mathrm{B}=10 \mathrm{MHz}, \mathrm{D}_{\mathrm{p}}=5 \mathrm{MHz}\right.$, Ripple $\left.=0.1 \mathrm{~dB}\right)$


Figure 4-20. Response of a Flat FM Television System Having a 3-Pole Chebyshev Filter to a T-Bar Test Signal $\left(B=10 \mathrm{MHz}, \mathrm{D}_{\mathrm{p}}=5 \mathrm{MHz}\right.$, Ripple $\left.=0.1 \mathrm{~dB}\right)$


Figure 4-21. Response of a Flat FM Television System Having a 3-Pole Chebyshev Filter to a T-Pulse Test Signal $\left(B=12 \mathrm{MHz}, \mathrm{D}_{\mathrm{p}}=5 \mathrm{MHz}\right.$, Ripple $=0.1 \mathrm{~dB}$ )


Figure 4-22. Response of a Flat FM Television System Having a 3-Pole Chebyshev Filter to a T-Bar Test Signal $\left(\mathrm{B}=12 \mathrm{MHz}, \mathrm{D}_{\mathrm{p}}=5 \mathrm{MHz}\right.$, Ripple $\left.=0.1 \mathrm{~dB}\right)$


Figure 4-23. Response of a Flat FM Television System Having a 3-Pole Chebyshev Filter to a T-Pulse Test Signal $\left(\mathrm{B}=15 \mathrm{MHz}, \mathrm{D}_{\mathrm{p}}=5 \mathrm{MHz}\right.$, Ripple $\left.=0.1 \mathrm{~dB}\right)$


Figure 4-24. Response of a Flat FM Television System
Having a 3-Pole Chebyshev Filter to a T-Bar Test Signal $\left(\mathrm{B}=15 \mathrm{MHz}, \mathrm{D}_{\mathrm{p}}=5 \mathrm{MHz}\right.$, Ripple $\left.=0.1 \mathrm{~dB}\right)$


Figure 4-25. Response of a Flat FM Television System Having a 4-Pole Chebyshev Filter to a T-Pulse Test Signal $\left(\mathrm{B}=10 \mathrm{MHz}, \mathrm{D}_{\mathrm{p}}=5 \mathrm{MHz}\right.$, Ripple $\left.=0.1 \mathrm{~dB}\right)$


Figure 4-26. Response of a Flat FM Television System
Having a 4-Pole Chebyshev Filter to a T-Bar Test Signal
$\left(\mathrm{B}=10 \mathrm{MHz}, \mathrm{D}_{\mathrm{p}}=5 \mathrm{MHz}\right.$, Ripple $\left.=0.1 \mathrm{~dB}\right)$


Figure 4-27. Response of a Flat FM Television System Having a 4-Pole Chebyshev Filter to a T-Pulse Test Signal
$\left(\mathrm{B}=12 \mathrm{MHz}, \mathrm{D}_{\mathrm{p}}=5 \mathrm{MHz}\right.$, Ripple $\left.=0.1 \mathrm{~dB}\right)$


Figure 4-28. Response of a Flat FM Television System Having a 4-Pole Chebyshev Filter to a T-Bar Test Signal $\left(\mathrm{B}=12 \mathrm{MHz}, \mathrm{D}_{\mathrm{p}}=5 \mathrm{MHz}\right.$, Ripple $\left.=0.1 \mathrm{~dB}\right)$


Figure 4-29. Response of a Flat FM Television System Having a 4-Pole Chebyshev Filter to a T-Pulse Test Signal
$\left(\mathrm{B}=15 \mathrm{MHz}, \mathrm{D}_{\mathrm{p}}=5 \mathrm{MHz}\right.$, Ripple $\left.=0.1 \mathrm{~dB}\right)$


Figure 4-30. Response of a Flat FM Television System Having a 4-Pole Chebyshev Filter to a T-Bar Test Signal
$\left(\mathrm{B}=15 \mathrm{MHz}, \mathrm{D}_{\mathrm{p}}=5 \mathrm{MHz}\right.$, Ripple $\left.=0.1 \mathrm{~dB}\right)$


Figure 4-31. Response of a Flat FM Television System Having a 4-Pole Chebyshev Filter to a T-Pulse Test Signal
$\left(\mathrm{B}=20 \mathrm{MHz}, \mathrm{D}_{\mathrm{p}}=5 \mathrm{MHz}\right.$, Ripple $\left.=0.1 \mathrm{~dB}\right)$


Figure 4-32. Response of a Flat FM Television System
Having a 4-Pole Chebyshev Filter to a T-Bar Test Signal
$\left(\mathrm{B}=20 \mathrm{MHz}, \mathrm{D}_{\mathrm{p}}=5 \mathrm{MHz}\right.$, Ripple $\left.=0.1 \mathrm{~dB}\right)$


Figure 4-33. Response of a Flat FM Television System Having a 4-Pole Chebyshev Filter to a T-Pulse Test Signal $\left(\mathrm{B}=30 \mathrm{MHz}, \mathrm{D}_{\mathrm{p}}=5 \mathrm{MHz}\right.$, Ripple $\left.=0.1 \mathrm{~dB}\right)$


Figure 4-34. Response of a Flat FM Television System Having a 4-Pole Chebyshev Filter to a T-Bar Test Signal
$\left(\mathrm{B}=30 \mathrm{MHz}, \mathrm{D}_{\mathrm{p}}=5 \mathrm{MHz}\right.$, Ripple $\left.=0.1 \mathrm{~dB}\right)$


Figure 4-35. Response of a Flat FM Television System
Having a 3-Pole Butterworth Filter to a T-Pulse
Test Signal (B $\left.=10 \mathrm{MHz}, \mathrm{D}_{\mathrm{p}}=6 \mathrm{MHz}\right)$


Figure 4-36. Response of a Flat FM Television System Having a 3-Pole Butterworth Filter to a T-Bar Test Signal
( $\mathrm{B}=10 \mathrm{MHz}, \mathrm{D}_{\mathrm{p}}=6 \mathrm{MHz}$ )


Figure 4-37. Response of a Flat FM Television System Having a 3-Pole Butterworth Filter to a T-Pulse

Test Signal $\left(B=12 \mathrm{MHz}, \mathrm{D}_{\mathrm{p}}=6 \mathrm{MHz}\right)$


Figure 4-38. Response of a Flat FM Television System Having a 3-Pole Butterworth Filter to a T-Bar Test Signal

$$
\left(\mathrm{B}=12 \mathrm{MHz}, \mathrm{D}_{\mathrm{p}}=6 \mathrm{MHz}\right)
$$



Figure 4-39. Response of a Flat FM Television System Having a 3-Pole Butterworth Filter to a T-Pulse

Test Signal $\left(B=15 \mathrm{MHz}, D_{p}=6 \mathrm{MHz}\right)$


Figure 4-40. Response of a Flat FM Television System Having a 3-Pole Butterworth Filter to a T-Bar Test Signal ( $\mathrm{B}=15 \mathrm{MHz}, \mathrm{D}_{\mathrm{p}}=6 \mathrm{MHz}$ )


Figure 4-41. Response of a Flat FM Television System Having a 4-Pole Butterworth Filter to a T-Pulse Test Signal ( $B=10 \mathrm{MHz}, \mathrm{D}_{\mathrm{p}}=6 \mathrm{MHz}$ )


Figure 4-42. Response of a Flat FM Television System Having a 4-Pole Butterworth Filter to a T-Bar Test Signal ( $\mathrm{B}=10 \mathrm{MHz}, \mathrm{D}_{\mathrm{p}}=6 \mathrm{MHz}$ )


Figure 4-43. Response of a Flat FM Television System
Having a 4-Pole Butterworth Filter to a T-Pulse Test Signal (B $=12 \mathrm{MHz}, \mathrm{D}_{\mathrm{p}}=6 \mathrm{MHz}$ )


Figure 4-44. Response of a Flat FM Television System Having a 4-Pole Butterworth Filter to a T-Bar Test Signal
( $\mathrm{B}=12 \mathrm{MHz}, \mathrm{D}_{\mathrm{p}}=6 \mathrm{MHz}$ )


Figure 4-45. Response of a Flat FM Television System
Having a 4-Pole Butterworth Filter to a T-Pulse
Test Signal ( $B=15 \mathrm{MHz}, D_{p}=6 \mathrm{MHz}$ )


Figure 4-46. Response of a Flat FM Television System
Having a 4-Pole Butterworth Filter to a T-Bar Test Signal

$$
\left(\mathrm{B}=15 \mathrm{MHz}, \mathrm{D}_{\mathrm{p}}=6 \mathrm{MHz}\right)
$$



Figure 4-47. Response of a Flat FM Television System
Having a 4-Pole Butterworth Filter to a T-Pulse
Test Signal ( $B=20 \mathrm{MHz}, \mathrm{D}_{\mathrm{p}}=6 \mathrm{MHz}$ )


Figure 4-48. Response of a Flat FM Television System Having a 4-Pole Butterworth Filter to a T-Bar Test Signal ( $\mathrm{B}=20 \mathrm{MHz}, \mathrm{D}_{\mathrm{p}}=6 \mathrm{MHz}$ )


Figure 4-49. Response of a Flat FM Television System Having a 4-Pole Butterworth Filter to a T-Pulse Test Signal ( $B=30 \mathrm{MHz}, \mathrm{D}_{\mathrm{p}}=6 \mathrm{MHz}$ )


Figure 4-50. Response of a Flat FM Television System Having a 4-Pole Butterworth Filter to a T-Bar Test Signal ( $\mathrm{B}=30 \mathrm{MHz}, \mathrm{D}_{\mathrm{p}}=6 \mathrm{MHz}$ )


Figure 4-51. Response of a Flat FM Television System Having a 3-Pole Chebyshev Filter to a T-Pulse Test Signal
$\left(\mathrm{B}=10 \mathrm{MHz}, \mathrm{D}_{\mathrm{p}}=6 \mathrm{MHz}\right.$, Ripple $\left.=0.1 \mathrm{~dB}\right)$


Figure 4-52. Response of a Flat FM Television System
Having a 3-Pole Chebyshev Filter to a T-Bar Test Signal $\left(\mathrm{B}=10 \mathrm{MHz}, \mathrm{D}_{\mathrm{p}}=6 \mathrm{MHz}\right.$, Ripple $\left.=0.1 \mathrm{~dB}\right)$


Figure 4-53. Response of a Flat FM Television System Having , a 3-Pole Chebyshev Filter to a T-Pulse Test Signal
$\left(\mathrm{B}=12 \mathrm{MHz}, \mathrm{D}_{\mathrm{p}}=6 \mathrm{MHz}\right.$, Ripple $\left.=0.1 \mathrm{~dB}\right)$


Figure 4-54. Response of a Flat FM Television System
Having a 3-Pole Chebyshev Filter to a T-Bar Test Signal $\left(B=12 \mathrm{MHz}, \mathrm{D}_{\mathrm{p}}=6 \mathrm{MHz}\right.$, Ripple $\left.=0.1 \mathrm{~dB}\right)$


Figure 4-55. Response of a Flat FM Television System Having a 3-Pole Chebyshev Filter to a T-Pulse Test Signal $\left(\mathrm{B}=15 \mathrm{MHz}, \mathrm{D}_{\mathrm{p}}=6 \mathrm{MHz}\right.$, Ripple $\left.=0.1 \mathrm{~dB}\right)$


Figure 5-56. Response of a Flat FM Television System Having a 3-Pole Chebyshev Filter to a T-Bar Test Signal $\left(B=15 \mathrm{MHz}, \mathrm{D}_{\mathrm{p}}=6 \mathrm{MHz}\right.$, Ripple $\left.=0.1 \mathrm{~dB}\right)$


Figure 4-57. Response of a Flat FM Television System Having a 4-Pole Chebyshev Filter to a T-Pulse Test Signal
$\left(B=10 \mathrm{MHz}, D_{p}=6 \mathrm{MHz}\right.$, Ripple $\left.=0.1 \mathrm{~dB}\right)$


Figure 4-58. Response of a Flat FM Television System Having a 4-Pole Chebyshev Filter to a T-Bar Test Signal
$\left(\mathrm{B}=10 \mathrm{MHz}, \mathrm{D}_{\mathrm{p}}=6 \mathrm{MHz}\right.$, Ripple $\left.=0.1 \mathrm{~dB}\right)$


Figure 4-59. Response of a Flat FM Television System Having a 4-Pole Chebyshev Filter to a T-Pulse Test Signal
$\left(\mathrm{B}=12 \mathrm{MHz}, \mathrm{D}_{\mathrm{p}}=6 \mathrm{MHz}\right.$, Ripple $\left.=0.1 \mathrm{~dB}\right)$


Figure 4-60. Response of a Flat FM Television System Having a 4-Pole Chebyshev Filter to a T-Bar Test Signal $\left(B=12 \mathrm{MHz}, \mathrm{D}_{\mathrm{p}}=6 \mathrm{MHz}\right.$, Ripple $\left.=0.1 \mathrm{~dB}\right)$


Figure 4-61. Response of a Flat FM Television System Having a 4-Pole Chebyshev Filter to a T-Pulse Test Signal
$\left(\mathrm{B}=15 \mathrm{MHz}, \mathrm{D}_{\mathrm{p}}=6 \mathrm{MHz}\right.$, Ripple $\left.=0.1 \mathrm{~dB}\right)$


Figure 4-62. Response of a Flat FM Television System

## Having a 4-Pole Chebyshev Filter to a T-Bar Test Signal

 $\left(\mathrm{B}=15 \mathrm{MHz}, \mathrm{D}_{\mathrm{p}}=6 \mathrm{MHz}\right.$, Ripple $\left.=0.1 \mathrm{~dB}\right)$

Figure 4-63. Response of a Flat FM Television System Having a 4-Pole Chebyshev Filter to a T-Pulse Test Signal $\left(\mathrm{B}=20 \mathrm{MHz}, \mathrm{D}_{\mathrm{p}}=6 \mathrm{MHz}\right.$, Ripple $\left.=0.1 \mathrm{~dB}\right)$


Figure 4-64. Response of a Flat FM Television System
Having a 4-Pole Chebyshev Filter to a T-Bar Test Signal
$\left(B=20 \mathrm{MHz}, \mathrm{D}_{\mathrm{p}}=6 \mathrm{MHz}\right.$, Ripple $\left.=0.1 \mathrm{~dB}\right)$


Figure 4-65. Response of a Flat FM Television System Having a 4-Pole Chebyshev Filter to a T-Pulse Test Signal
$\left(\mathrm{B}=30 \mathrm{MHz}, \mathrm{D}_{\mathrm{p}}=6 \mathrm{MHz}\right.$, Ripple $\left.=0.1 \mathrm{~dB}\right)$


Figure 4-66. Response of a Flat FM Television System
Having a 4-Pole Chebyshev Filter to a T-Bar Test Signal
$\left(\mathrm{B}=30 \mathrm{MHz}, \mathrm{D}_{\mathrm{p}}=6 \mathrm{MHz}\right.$, Ripple $\left.=0.1 \mathrm{~dB}\right)$


Figure 4-67. Response of a Preemphasized FM Television System Having a 3-Pole Butterworth Filter to a T-Pulse Test Signal ( $\mathrm{B}=10 \mathrm{MHz}, \mathrm{D}_{\mathrm{p}}=5 \mathrm{MHz}$ )


Figure 4-68. Response of a Preemphasized FM Television System
Having a 3-Pole Butterworth Filter to a T-Bar Test Signal
( $\mathrm{B}=10 \mathrm{MHz}, \mathrm{D}_{\mathrm{p}}=5 \mathrm{MHz}$ )


Figure 4-69. Response of a Preemphasized FM Television System
Having a 3-Pole Butterworth Filter to a T-Pulse Test Signal

$$
\left(\mathrm{B}=12 \mathrm{MHz}, \mathrm{D}_{\mathrm{p}}=5 \mathrm{MHz}\right)
$$



Figure 4-70. Response of a Preemphasized FM Television System
Having a 3-Pole Butterworth Filter to a T-Bar Test Signal
( $\mathrm{B}=12 \mathrm{MHz}, \mathrm{D}_{\mathrm{p}}=5 \mathrm{MHz}$ )


Figure 4-71. Response of a Preemphasized FM Television System Having a 3-Pole Butterworth Filter to a T-Pulse Test Signal ( $\mathrm{B}=15 \mathrm{MHz}, \mathrm{D}_{\mathrm{p}}=5 \mathrm{MHz}$ )


Figure 4-72. Response of a Preemphasized FM Television System
Having a 3-Pole Butterworth Filter to a T-Bar Test Signal

$$
\left(\mathrm{B}=15 \mathrm{MHz}, \mathrm{D}_{\mathrm{p}}=5 \mathrm{MHz}\right)
$$



Figure 4-73. Response of a Preemphasized FM Television System
Having a 4-Pole Butterworth Filter to a T-Pulse Test Signal
$\left(\mathrm{B}=10 \mathrm{MHz}, \mathrm{D}_{\mathrm{p}}=5 \mathrm{MHz}\right)$


Figure 4-74. Response of a Preemphasized FM Television System
Having a 4-Pole Butterworth Filter to a T-Bar Test Signal
( $\mathrm{B}=10 \mathrm{MHz}, \mathrm{D}_{\mathrm{p}}=5 \mathrm{MHz}$ )


Figure 4-75. Response of a Preemphasized FM Television System Having a 4-Pole Butterworth Filter to a T-Pulse Test Signal ( $\mathrm{B}=12 \mathrm{MHz}, \mathrm{D}_{\mathrm{p}}=5 \mathrm{MHz}$ )


Figure 4-76. Response of a Preeamphasized FM Television System
Having a 4-Pole Butterworth Filter to a T-Bar Test Signal
( $\mathrm{B}=12 \mathrm{MHz}, \mathrm{D}_{\mathrm{p}}=5 \mathrm{MHz}$ )


Figure 4-77. Response of a Preemphasized FM Television System
Having a 4-Pole Butterworth Filter to a T-Pulse Test Signal
(B=15 MHz, $\left.D_{p}=5 \mathrm{MHz}\right)$


Figure 4-78. Response of a Preemphasized FM Television System
Having a 4-Pole Butterworth Filter to a T-Bar Test Signal
( $\mathrm{B}=15 \mathrm{MHz}, \mathrm{D}_{\mathrm{p}}=5 \mathrm{MHz}$ )


Figure 4-79. Response of a Preemphasized FM Television System Having a 4-Pole Butterworth Filter to a T-Pulse Test Signal
( $\mathrm{B}=20 \mathrm{MHz}, \mathrm{D}_{\mathrm{p}}=5 \mathrm{MHz}$ )


Figure 4-80. Response of a Preemphasized FM Television Sys tem
Having a 4-Pole Butterworth Filter to a T-Bar Test Signal
( $\mathrm{B}=20 \mathrm{MHz}, \mathrm{D}_{\mathrm{p}}=5 \mathrm{MHz}$ )


Figure 4-81. Response of a Preemphasized FM Television System Having a 4-Pole Butterworth Filter to a T-Pulse Test Signal
( $\mathrm{B}=30 \mathrm{MHz}, \mathrm{D}_{\mathrm{p}}=5 \mathrm{MHz}$ )


Figure 4-82. Response of a Preemphasized FM Television System Having a 4-Pole Butterworth Filter to a T-Bar Test Signal

$$
\left(\mathrm{B}=30 \mathrm{MHz}, \mathrm{D}_{\mathrm{p}}=5 \mathrm{MHz}\right)
$$



Figure 4-83. Response of a Preemphasized FM Television System Having a 3-Pole Chebyshev Filter to a T-Pulse Test Signal
$\left(\mathrm{B}=10 \mathrm{MHz}, \mathrm{D}_{\mathrm{p}}=5 \mathrm{MHz}\right.$, Ripple $\left.=0.1 \mathrm{~dB}\right)$


Figure 4-84. Response of a Preemphasized FM Television System
Having a 3-Pole Chebyshev Filter to a T-Bar Test Signal
$\left(B=10 \mathrm{MHz}, D_{p}=5 \mathrm{MHz}\right.$, Ripple $\left.=0.1 \mathrm{~dB}\right)$


Figure 4-85. Response of a Preemphasized FM Television System Having a 3-Pole Chebyshev Filter to a T-Pulse Test Signal $\left(\mathrm{B}=12 \mathrm{MHz}, \mathrm{D}_{\mathrm{p}}=5 \mathrm{MHz}\right.$, Ripple $\left.=0.1 \mathrm{~dB}\right)$


Figure 4-86. Response of a Preemphasized FM Television System
Having a 3-Pole Chebyshev Filter to a T-Bar Test Signal
$\left(\mathrm{B}=12 \mathrm{MHz}, \mathrm{D}_{\mathrm{p}}=5 \mathrm{MHz}\right.$, Ripple $\left.=0.1 \mathrm{~dB}\right)$


Figure 4-87. Response of a Preemphasized FM Television System Having a 3-Pole Chebyshev Filter to a T-Pulse Test Signal $\left(B=15 \mathrm{MHz}, D_{p}=5 \mathrm{MHz}\right.$, Ripple $\left.=0.1 \mathrm{~dB}\right)$


Figure 4-88. Response of a Preemphasized FM Television System
Having a 3-Pole Chebyshev Filter to a T-Bar Test Signal
$\left(B=15 \mathrm{MHz}, D_{p}=5 \mathrm{MHz}\right.$, Ripple $\left.=0.1 \mathrm{~dB}\right)$


Figure 4-89. Response of a Preemphasized FM Television System
Having a 4-Pole Chebyshev Filter to a T-Pulse Test Signal $\left(\mathrm{B}=10 \mathrm{MHz}, \mathrm{D}_{\mathrm{p}}=5 \mathrm{MHz}\right.$, Ripple $\left.=0.1 \mathrm{~dB}\right)$


Figure 4-90. Response of a Preemphasized FM Television System
Having a 4-Pole Chebyshev Filter to a T-Bar Test Signal

$$
\left(\mathrm{B}=10 \mathrm{MHz}, \mathrm{D}_{\mathrm{p}}=5 \mathrm{MHz}, \text { Ripple }=0.1 \mathrm{~dB}\right)
$$



Figure 4-91. Response of a Preemphasized FM Television System Having a 4-Pole Chebyshev Filter to a T-Pulse Test Signal $\left(\mathrm{B}=12 \mathrm{MHz}, \mathrm{D}_{\mathrm{p}}=5 \mathrm{MHz}\right.$, Ripple $\left.=0.1 \mathrm{~dB}\right)$


Figure 4-92. Response of a Preemphasized FM Television System
Having a 4-Pole Chebyshev Filter to a T-Bar Test Signal
$\left(\mathrm{B}=12 \mathrm{MHz}, \mathrm{D}_{\mathrm{p}}=5 \mathrm{MHz}\right.$, Ripple $\left.=0.1 \mathrm{~dB}\right)$


Figure 4-93. Response of a Preemphasized FM Television System Having a 4-Pole Chebyshev Filter to a T-Pulse Test Signal $\left(\mathrm{B}=15 \mathrm{MHz}, \mathrm{D}_{\mathrm{p}}=5 \mathrm{MHz}\right.$, Ripple $\left.=0.1 \mathrm{~dB}\right)$


Figure 4-94. Response of a Preemphasized FM Television System
Having a 4-Pole Chebyshev Filter to a T-Bar Test Signal
$\left(B=15 \mathrm{MHz}, \mathrm{D}_{\mathrm{p}}=5 \mathrm{MHz}\right.$, Ripple $\left.=0.1 \mathrm{~dB}\right)$


Figure 4-95. Response of a Preemphasized FM Television System Having a 4-Pole Chebyshev Filter to a T-Pulse Test Signal $\left(B=20 \mathrm{MHz}, \mathrm{D}_{\mathrm{p}}=5 \mathrm{MHz}\right.$, Ripple $\left.=0.1 \mathrm{~dB}\right)$


Figure 4-96. Response of a Preemphasized FM Television System
Having a 4-Pole Chebyshev Filter to a T-Bar Test Signal
$\left(\mathrm{B}=20 \mathrm{MHz}, \mathrm{D}_{\mathrm{p}}=5 \mathrm{MHz}\right.$, Ripple $\left.=0.1 \mathrm{~dB}\right)$


Figure 4-97. Response of a Preemphasized FM Television System Having a 4-Pole Chebyshev Filter to a T-Pulse Test Signal $\left(\mathrm{B}=30 \mathrm{MHz}, \mathrm{D}_{\mathrm{p}}=5 \mathrm{MHz}\right.$, Ripple $\left.=0.1 \mathrm{~dB}\right)$


Figure 4-98. Response of a Preemphasized FM Television System
Having a 4-Pole Chebyshev Filter to a T-Bar Test Signal
$\left(\mathrm{B}=30 \mathrm{MHz}, \mathrm{D}_{\mathrm{p}}=5 \mathrm{MHz}\right.$, Ripple $\left.=0.1 \mathrm{~dB}\right)$


Figure 4-99. Response of a Preemphasized FM Television System Having a 3-Pole Butterworth Filter to a T-Pulse Test Signal

$$
\left(\mathrm{B}=10 \mathrm{MHz}, \mathrm{D}_{\mathrm{p}}=6 \mathrm{MHz}\right)
$$



Figure 4-100. Response of a Preemphasized FM Television System
Having a 3-Pole Butterworth Filter to a T-Bar Test Signal
$\left(\mathrm{B}=10 \mathrm{MHz}, \mathrm{D}_{\mathrm{p}}=6 \mathrm{MHz}\right)$


Figure 4-101. Response of a preemphasized FM Television System Having a 3-Pole Butterworth Filter to a T-Pulse Test Signal ( $\mathrm{B}=12 \mathrm{MHz}, \mathrm{D}_{\mathrm{p}}=6 \mathrm{MHz}$ )


Figure 4-102. Response of a Preemphasized FM Television System
Having a 3-Pole Butterworth Filter to a T-Bar Test Signal
( $\mathrm{B}=12 \mathrm{MHz}, \mathrm{D}_{\mathrm{p}}=6 \mathrm{MHz}$ )


Figure 103. Response of a Preemphasized FM Television System
Having a 3-Pole Butterworth Filter to a T-Pulse Test Signal

$$
\left(\mathrm{B}=15 \mathrm{MHz}, \mathrm{D}_{\mathrm{p}}=6 \mathrm{MHz}\right)
$$



Figure 4-104. Response of a Preemphasized FM Television System
Having a 3-Pole Butterworth Filter to a T-Bar Test Signal
( $\mathrm{B}=15 \mathrm{MHz}, \mathrm{D}_{\mathrm{p}}=6 \mathrm{MHz}$ )


Figure 4-105. Response of a Preemphasized FM Television System Having a 4-Pole Butterworth Filter to a T-Pulse Test Signal
$\left(B=10 \mathrm{MHz}, D_{p}=6 \mathrm{MHz}\right)$


Figure 4-106. Response of a Preemphasized FM Television System

## Having a 4-Pole Butterworth Filter to a T-Bar Test Signal

$$
\left(B=10 \mathrm{MHz}, \mathrm{D}_{\mathrm{p}}=6 \mathrm{MHz}\right)
$$



Figure 4-107. Response of a Preemphasized FM Television System Having a 4-Pole Butterworth Filter to a T-Pulse Test Signal
( $\mathrm{B}=12 \mathrm{MHz}, \mathrm{D}_{\mathrm{p}}=6 \mathrm{MHz}$ )


Figure 4-108. Response of a Preemphasized FM Television System
Having a 4-Pole Butterworth Filter to a T-Bar Test Signal
( $\mathrm{B}=12 \mathrm{MHz}, \mathrm{D}_{\mathrm{p}}=6 \mathrm{MHz}$ )


Figure 4-109. Response of a Preemphasized FM Television System
Having a 4-Pole Butterworth Filter to a T-Pulse Test Signal
( $\mathrm{B}=15 \mathrm{MHz}, \mathrm{D}_{\mathrm{p}}=6 \mathrm{MHz}$ )


Figure 4-110. Response of a Preemphasized FM Television System
Having a 4-Pole Butterworth Filter to a T-Bar Test Signal
( $\mathrm{B}=15 \mathrm{MHz}, \mathrm{D}_{\mathrm{p}}=6 \mathrm{MHz}$ )


Figure 4-111. Response of a Preemphasized FM Television System Having a 4-Pole Butterworth Filter to a T-Pulse Test Signal

$$
\left(B=20 \mathrm{MHz}, D_{p}=6 \mathrm{MHz}\right)
$$



Figure 4-112. Response of a Preemphasized FM Television System
Having a 4-Pole Butterworth Filter to a T-Bar Test Signal
( $\mathrm{B}=20 \mathrm{MHz}, \mathrm{D}_{\mathrm{p}}=6 \mathrm{MHz}$ )


Figure 4-113. Response of a Preemphasized FM Television System
Having a 4-Pole Butterworth Filter to a T-Pulse Test Signal
( $\mathrm{B}=30 \mathrm{MHz}, \mathrm{D}_{\mathrm{p}}=6 \mathrm{MHz}$ )


Figure 4-114. Response of a Preemphasized FM Television System
Having a 4-Pole Butterworth Filter to a T-Bar Test Signal

$$
\left(\mathrm{B}=30 \mathrm{MHz}, \mathrm{D}_{\mathrm{p}}=6 \mathrm{MHz}\right)
$$



Figure 4-115. Response of a Preemphasized FM Television System
Having a 3-Pole Chebyshev Filter to a T-Pulse Test Signal
$\left(\mathrm{B}=10 \mathrm{MHz}, \mathrm{D}_{\mathrm{p}}=6 \mathrm{MHz}\right.$, Ripple $\left.=0.1 \mathrm{~dB}\right)$


Figure 4-116. Response of a Preemphasized FM Television System
Having a 3-Pole Chebyshev Filter to a T-Bar Test Signal
$\left(\mathrm{B}=10 \mathrm{MHz}, \mathrm{D}_{\mathrm{p}}=6 \mathrm{MHz}\right.$, Ripple $\left.=0.1 \mathrm{~dB}\right)$


Figure 4-117. Response of a Preemphasized FM Television System
Having a 3-Pole Chebyshev Filter to a T-Pulse Test Signal
$\left(\mathrm{B}=12 \mathrm{MHz}, \mathrm{D}_{\mathrm{p}}=6 \mathrm{MHz}\right.$, Ripple $\left.=0.1 \mathrm{~dB}\right)$


Figure 4-118. Response of a Preemphasized FM Television System
Having a 3-Pole Chebyshev Filter to a T-Bar Test Signal
$\left(\mathrm{B}=12 \mathrm{MHz}, \mathrm{D}_{\mathrm{p}}=6 \mathrm{MHz}\right.$, Ripple $\left.=0.1 \mathrm{~dB}\right)$


Figure 4-119. Response of a Preemphasized FM Television System
Having a 3-Pole Chebyshev Filter to a T-Pulse Test Signal
$\left(\mathrm{B}=15 \mathrm{MHz}, \mathrm{D}_{\mathrm{p}}=6 \mathrm{MHz}\right.$, Ripple $\left.=0.1 \mathrm{~dB}\right)$


Figure 4-120. Response of a Preemphasized FM Television System
Having a 3-Pole Chebyshev Filter to a T-Bar Test Signal
$\left(\mathrm{B}=15 \mathrm{MHz}, \mathrm{D}_{\mathrm{p}}=6 \mathrm{MHz}\right.$, Ripple $\left.=0.1 \mathrm{~dB}\right)$


Figure 4-121. Response of a Preemphasized FM Television System Having a 4-Pole Chebyshev Filter to a T-Pulse Test Signal
$\left(B=10 \mathrm{MHz}, \mathrm{D}_{\mathrm{p}}=6 \mathrm{MHz}\right.$, Ripple $\left.=0.1 \mathrm{~dB}\right)$


Figure 4-122. Response of a Preemphasized FM Television System
Having a 4-Pole Chebyshev Filter to a T-Bar Test Signal
$\left(\mathrm{B}=10 \mathrm{MHz}, \mathrm{D}_{\mathrm{p}}=6 \mathrm{MHz}\right.$, Ripple $\left.=0.1 \mathrm{~dB}\right)$


Figure 4-123. Response of a Preemphasized FM Television System Having a 4-Pole Chebyshev Filter to a T-Pulse Test Signal
$\left(\mathrm{B}=12 \mathrm{MHz}, \mathrm{D}_{\mathrm{p}}=6 \mathrm{MHz}\right.$, Ripple $\left.=0.1 \mathrm{~dB}\right)$


Figure 4-124. Response of a Preemphasized FM Television System
Having a 4-Pole Chebyshev Filter to a T-Bar Test Signal
$\left(\mathrm{B}=12 \mathrm{MHz}, \mathrm{D}_{\mathrm{p}}=6 \mathrm{MHz}\right.$, Ripple $\left.=0.1 \mathrm{~dB}\right)$


Figure 4-125. Response of a Preemphasized FM Television System Having a 4-Pole Chebyshev Filter to a T-Pulse Test Signal $\left(B=15 \mathrm{MHz}, D_{p}=6 \mathrm{MHz}\right.$, Ripple $\left.=0.1 \mathrm{~dB}\right)$


Figure 4-126. Response of a Preemphasized FM Television System
Having a 4-Pole Chebyshev Filter to a T-Bar Test Signal
( $\mathrm{B}=15 \mathrm{MHz}, \mathrm{D}_{\mathrm{p}}=6 \mathrm{MHz}$, Ripple $=0.1 \mathrm{~dB}$ )


Figure 4-127. Response of a Preemphasized FM Television System
Having a 4-Pole Chebyshev Filter to a T-Pulse Test Signal $\left(\mathrm{B}=20 \mathrm{MHz}, \mathrm{D}_{\mathrm{p}}=6 \mathrm{MHz}\right.$, Ripple $\left.=0.1 \mathrm{~dB}\right)$


Figure 4-128. Response of a Preemphasized FM Television System Having a 4-Pole Chebyshev Filter to a T-Bar Test Signal $\left(B=20 \mathrm{MHz}, D_{p}=6 \mathrm{MHz}\right.$, Ripple $\left.=0.1 \mathrm{~dB}\right)$


Figure 4-129. Response of a Preemphasized FM Television System Having a 4-Pole Chebyshev Filter to a T-Pulse Test Signal $\left(\mathrm{B}=30 \mathrm{MHz}, \mathrm{D}_{\mathrm{p}}=6 \mathrm{MHz}\right.$, Ripple $\left.=0.1 \mathrm{~dB}\right)$


Figure 4-130. Response of a Preemphasized FM Television System
Having a 4-Pole Chebyshev Filter to a T-Bar Test Signal
$\left(\mathrm{B}=30 \mathrm{MHz}, \mathrm{D}_{\mathrm{p}}=6 \mathrm{MHz}\right.$, Ripple $\left.=0.1 \mathrm{~dB}\right)$


Figure 4-131. Response of a 525-Line Preemphasis Filter to a $T$-Pulse ( $T=100 \mathrm{~ns}$ )


Figure 4-132. Response of a 525-Line Preemphasis Filter
to the Rise Portion of a $T-\operatorname{Bar}(\mathrm{T}=100 \mathrm{~ns}$ )


Figure 4-133. Response of a 525-Line Preemphasis Filter
to the Fall Portion of a $T-\operatorname{Bar}(T=100 \mathrm{~ns}$ )

## SECTION 5 - CONC LUSIONS

Several conclusions can be drawn from this frequency modulation distortion analysis for television transmission. The conclusions are based on the computed distortions of a T-pulse-and-bar test signal which passes through the computer-simulated system. The conclusions are:

An FM television transmission system can be modeled for computer simulation of a well defined signal in the time domain from the video input terminals to the video output terminals.

If preemphasis is employed, a Chebyshev predetection filter produces more signal distortion than a Butterworth filter having four poles and the same half-power bandwidth. Without preemphasis no significant difference between Butterworth and Chebyshev filters was observed.

Whether a Butterworth or Chebyshev filter is used, a four-pole filter produces more distortion than a three-pole filter.

An increase in the RF predetection bandwidth reduces the system distortion if all other parameters remain fixed.

An increase in the peak deviation of a system increases the system distortion if all other parameters remain fixed.

The T-pulse-and-bar test signal is distorted more in a system utilizing preemphasis than in a flat system, which does not employ preemphasis.

The case producing the worst distortion (Figure 4-121) is a preemphasized system utilizing a four-pole Chebyshev predetection filter, having a ripple bandwidth of 10.0 MHz , corresponding to a half-power bandwidth of 12.13 MHz . This bandwidth, which is approximately twice the peak deviation of 6.0 MHz and significantly less than Carson's bandwidth, produces an overshoot distortion of only $9.9 \%$-less than the C.C.I.R. limit of $13 \%$ (Reference 6). This is achieved without any equalization in the system.

## REFERENCES

1. Papoulis, A., The Fourier Integral and Its Applications. McGraw-Hill Book Company, Inc., N. Y., 1962.
2. Annex to Part 2, Linear Waveform Distortion, Luminance Channel, C.C.I.R. XIIth Plenary Assembly, New Delhi, 1970, Vol. V, Part 2.
3. Recommendation 405-1, Radio Relay Systems for Television (Pre-emphasis Characteristics for Frequency - Modulation Systems) C.C.I.R. XIIth Plenary Assembly, New Delhi, 1970, Vol. IV, Part 1.
4. Moursund, D. G., and Duris, C. S., Elementary Theory and Application of Numerical Analysis, McGraw-Hill Book Company, Inc., N. Y., 1967.
5. Lathi, B. P., Signals, Systems and Communication. John Wiley and Sons, Inc., N. Y., 1965.
6. Recommendation 421-2, Requirements for the Transmission of Television Signals over Long Distances (System I Excepted), C.C. I. R. XIIth Plenary Assembly, New Delhi, 1970, Vol. V, Part 2.

## APPENDIX A - PREEMPHASIS AND DEEMPHASIS FILTER ANALYSIS

## A. 1 INTRODUCTION

The impulse responses of preemphasis and deemphasis filters for 525-line FM television transmission are derived from the circuits recommended by C.C.I.R. (Reference 1). The transfer functions are first derived from the specified circuits. Then the impulse responses are calculated from the transfer functions.

In Reference 1 the configurations and component values of preemphasis and deemphasis filters are specified for different television systems. For the case of the 525-line television system, the filter configurations are shown in Figure A-1 and A-2, while their component values are tabulated in Tables A-1 and A-2, respectively.

Figure A-3 shows the preemphasis magnitude and phase characteristics for a 525 -line television system which, besides the intrinsic frequency response of a passive filter, includes a constant amplification of 3.39000 dB . This is equivalent to a voltage multiplication factor of 1.479. Similarly for the deemphasis characteristic, a voltage amplification factor of 3.162278 is included.

## A. 2 DETERMINA TION OF PREEMPHASIS AND DEEMPHASIS TRANSFER FUNCTIONS

Consider a circuit which consists of a generator at the input terminals and a black-box network coupled to a load at the output terminals, as shown in Figure A-4. If the blackbox network is a T-section, as shown in the same figure, the elements of the impedance matrix can be written in terms of the branch impedances of the T-section as follows:

$$
\left.\begin{array}{l}
\mathrm{z}_{11}=\mathrm{Z}_{\mathrm{a}}+\mathrm{Z}_{\mathrm{c}}  \tag{A-1}\\
\mathrm{z}_{12}=\mathrm{Z}_{\mathrm{c}} \\
\mathrm{z}_{22}=\mathrm{Z}_{\mathrm{b}}+\mathrm{z}_{\mathrm{c}}
\end{array}\right\}
$$

If $V_{1}$ and $V_{2}$ denote the input and output voltages respectively, if $\Delta z$ is defined by $\Delta \mathrm{z}=\mathrm{z}_{11} \mathrm{z}_{22}{ }^{-\mathrm{z}_{12}^{2}}$, and if $\mathrm{Z}_{\mathrm{L}}$ denotes the load at the output terminals, then the transfer function, $\mathrm{H}_{12}(\mathrm{~s})$, defined as the ratio of $\mathrm{V}_{2}$ to $\mathrm{V}_{1}$, is given by

$$
\begin{equation*}
\mathrm{H}_{12}(\mathrm{~s})=\frac{\mathrm{V}_{2}}{\mathrm{~V}_{1}}=\frac{\mathrm{z}_{12} \mathrm{Z}_{\mathrm{L}}}{\mathrm{~s}_{\mathrm{z}}+\mathrm{z}_{11} \mathrm{Z}_{\mathrm{L}}} \tag{A-2}
\end{equation*}
$$

If $Z_{a}, Z_{b}, Z_{c}$ and $Z_{L}$ are known, the transfer function can be determined by Equation (A-2). Thus, the transfer function of both the preemphasis and deemphasis filters can be obtained if the filter networks are first transformed to T-sections without the parallel branch across the upper pair of terminals. To achieve this, the delta-star transformation can be applied to a portion of each of the two filters. The configurations of delta and star networks are shown in Figure A-5 and the necessary expressions of the transformation are given by Equation । (A-3).

$$
\left.\begin{array}{c}
\alpha=\frac{\mathrm{Z}_{1} \mathrm{Z}_{2}}{\mathrm{Z}_{1}+\mathrm{Z}_{2}+\mathrm{Z}_{3}}  \tag{A-3}\\
\beta=\frac{\mathrm{Z}_{2} \mathrm{Z}_{3}}{\mathrm{Z}_{1}+\mathrm{Z}_{2}+\mathrm{Z}_{3}} \\
\gamma=\frac{\mathrm{Z}_{1} \mathrm{Z}_{3}}{\mathrm{Z}_{1}+\mathrm{Z}_{2}+\mathrm{Z}_{3}}
\end{array}\right\}
$$

where $\alpha, \beta$, and $\gamma$ are impedances of the arms of the star section.
In the case of the preemphasis filter, the portion of the filter to be transformed contains the elements of $C, R_{1}, R_{2}$ on the left, and $R_{2}$ on the right, which form a delta-section. Then

$$
\left.\begin{array}{l}
\mathrm{z}_{1}=\frac{\mathrm{R}_{1}}{1+\mathrm{CR}_{1} \mathrm{~s}}  \tag{A-4}\\
\mathrm{z}_{2}=\mathrm{Z}_{3}=\mathrm{R}_{2}
\end{array}\right\}
$$

For the deemphasis filter, the portion of it involved in the transformation contains the elements $L, R_{1}, R_{2}$ on the left, and $R_{2}$ on the right. Similarly,

$$
\begin{align*}
& Z_{1}=\frac{L R_{1} s}{R_{1}+L s}  \tag{A-5}\\
& Z_{2}=Z_{3}=R_{2}
\end{align*}
$$

From Equations (A-3) and (A-4), for the preemphasis filter,

$$
\begin{align*}
& \alpha=\frac{\mathrm{R}_{1} \mathrm{R}_{2}}{\mathrm{R}_{1}+2 \mathrm{R}_{2}+2 \mathrm{R}_{1} \mathrm{R}_{2} \mathrm{Cs}} \\
& \beta=\frac{\mathrm{R}_{2}^{2}+\mathrm{R}_{1} \mathrm{R}_{2}^{2} \mathrm{Cs}}{\mathrm{R}_{1}+2 \mathrm{R}_{2}+2 \mathrm{R}_{1} \mathrm{R}_{2} \mathrm{Cs}}  \tag{A-6}\\
& \gamma=\alpha
\end{align*}
$$

and from Equations (A-3) and (A-5), for the deemphasis filter

$$
\begin{align*}
& \alpha=\frac{\mathrm{LR}_{1} \mathrm{R}_{2} \mathrm{~s}}{2 \mathrm{R}_{1} \mathrm{R}_{2}+\left(2 \mathrm{R}_{2}+\mathrm{R}_{1}\right) \mathrm{Ls}} \\
& \beta=\frac{\mathrm{R}_{1} \mathrm{R}_{2}^{2}+\mathrm{LR}_{2}^{2} \mathrm{~s}}{2 \mathrm{R}_{1} \mathrm{R}_{2}+\left(2 \mathrm{R}_{2}+\mathrm{R}_{1}\right) \mathrm{Ls}}  \tag{A-7}\\
& \gamma=\alpha
\end{align*}
$$

After $\alpha, \beta$, and $\gamma$ are determined, the preemphasis and deemphasis filters can be converted to T-sections, as shown in Figure A-6, from which the values of $Z_{a}, Z_{b}$, and $Z_{c}$ and then the values of $z_{11}, z_{12}$, and $z_{22}$ can be determined. Since $Z_{L}$ is 75 ohms as specified, the transfer function $\mathrm{H}_{12}(\mathrm{~s})$ can be determined by Equation (A-2).

If $H_{p}(s)$ and $H_{d}(s)$ denote the transfer functions of the preemphasis and the deemphasis filters respectively, they can be represented by

$$
\begin{align*}
\mathrm{H}_{\mathrm{p}}(\mathrm{~s}) & =\frac{4.57032(10)^{8}+608.897 \mathrm{~s}+2.12161(10)^{-4} \mathrm{~s}^{2}+2.11953(10)^{-11} \mathrm{~s}^{3}}{2.13754(10)^{9}+1418.22 \mathrm{~s}+3.03767(10)^{-4} \mathrm{~s}^{2}+2.11953(10)^{-11} \mathrm{~s}^{3}}  \tag{A-8}\\
\mathrm{H}_{\mathrm{d}}(\mathrm{~s}) & =\frac{1.28361(10)^{11}+1.9881(10)^{5} \mathrm{~s}+8.82881(10)^{-2} \mathrm{~s}^{2}+1.02537(10)^{-8} \mathrm{~s}^{3}}{1.28361(10)^{11}+2.84657(10)^{5} \mathrm{~s}+0.205638 \mathrm{~s}^{2}+4.79565(10)^{-8} \mathrm{~s}^{3}} \tag{A-9}
\end{align*}
$$

## A. 3 DETERMINATION OF IMPULSE RESPONSES

To obtain the impulse responses of the preemphasis and deemphasis filters, the inverse Laplace transforms of the expressions on the right side of Equations (A-8) and (A-9) must be obtained. These expressions are first expanded into partial fractions having the following form:

$$
\begin{equation*}
\mathrm{H}(\mathrm{~s})=\mathrm{N}_{1}+\mathrm{N}_{2}\left[\frac{\mathrm{As}+\mathrm{B}}{\left(\mathrm{~s}-\alpha_{1}\right)^{2}+\beta_{1}}{ }^{2}+\frac{\mathrm{C}}{\mathrm{~s}-\alpha_{2}}\right] \tag{A-10}
\end{equation*}
$$

where $N_{1}$ is a numerical constant term,
$\mathrm{N}_{2}$ is the reciprocal of the coefficient of the highest-power term of the polynomial in the denominators of the transfer functions in Equations (A-8) and (A-9),
$A, B, C$ are real constants to be determined,
$\left(\alpha_{1} \pm \mathrm{j} \beta_{1}\right)$ is the complex root of the polynomial,
$\alpha_{2}$ is the real root of the polynomial.
The roots of the polynomials are determined by a computer subroutine available in the CSC INFONET library. Equation (A-10) is rearranged to the form shown in Equation (A-11) before taking the inverse Laplace transform.

$$
\begin{equation*}
H(s)=N_{1}+N_{2}\left[\frac{A\left(s-\alpha_{1}\right)}{\left(s-\alpha_{1}\right)^{2}+\beta_{1}^{2}}+\frac{A \alpha_{1}+B}{\left(s-\alpha_{1}\right)^{2}+\beta_{1}^{2}}+\frac{C}{s-\alpha_{2}}\right] \tag{A-11}
\end{equation*}
$$

The four terms on the right side are recognizable Laplace transforms of some elementary functions the transform pairs of which are listed below.

## Laplace Transforms <br> Inverse Laplace Transforms

(1)


$$
N_{1} \delta(t)
$$

(2)

(3)

$$
\frac{\left|\beta_{1}\right|}{\left(s-\alpha_{1}\right)^{2}+\beta_{1}^{2}}
$$


(4)

$$
\frac{1}{s-\alpha_{2}}
$$


$e^{\alpha_{2}}{ }_{u(t)}$

Furthermore, the second and third transforms can be combined into the form, $K \mathrm{e}^{\alpha_{1} \mathrm{t}} \cos \left(\left|\beta_{1}\right| \mathrm{t}+\phi\right) \mathrm{u}(\mathrm{t})$, in which K and $\phi$ are constants to be determined.
The partial fraction expansions of (A-8) and (A-9) are given by

$$
\begin{equation*}
H_{p}(s)=1-N_{p}\left[\frac{A_{p}\left(s-\alpha_{1}\right)}{\left(s-\alpha_{1}^{2}+\beta_{1}^{2}\right.}+\frac{A_{p} \alpha_{1}+\beta_{p}}{\left|\beta_{1}\right|} \frac{\left|\beta_{1}\right|}{\left(s-\alpha_{1}\right)^{2}+\beta_{1}{ }^{2}}+\frac{C_{p}}{s-\alpha_{2}}\right] \tag{A-12}
\end{equation*}
$$

$$
\text { where } \begin{aligned}
\mathrm{N}_{\mathrm{p}} & =\frac{1}{2.11953(10)^{-11}} \\
\mathrm{~A}_{\mathrm{p}} & =9.16115(10)^{-5} \\
\beta_{\mathrm{p}} & =503.718 \\
\mathrm{C}_{\mathrm{p}} & =-5.45646(10)^{-9}
\end{aligned}
$$

$$
\begin{align*}
\alpha_{1} & =-5.4976376(10)^{6} \\
\beta_{1} & = \pm 4.3298779(10)^{4} \\
\alpha_{2} & =-3.3365331(10)^{6} \\
H_{d}(\mathrm{~s})= & 0.213813+\mathrm{N}_{\mathrm{d}}\left[\frac{\mathrm{~A}_{\mathrm{d}}\left(\mathrm{~s}-\alpha_{3}\right)}{\left(\mathrm{s}-\alpha_{3}\right)^{2}+\beta_{3}^{2}}+\frac{\mathrm{A}_{\mathrm{d}} \alpha_{3}+\beta_{\mathrm{d}}}{\left|\beta_{3}\right|} \frac{\left|\beta_{3}\right|}{\left(\mathrm{s}-\alpha_{3}\right)^{2}+\beta_{3}^{2}}+\frac{\mathrm{C}_{\mathrm{d}}}{\mathrm{~s}-\alpha_{4}}\right]\left(\mathrm{m}_{4}\right]  \tag{A-13}\\
\text { where } \mathrm{N}_{\mathrm{d}} & =\frac{1}{4.79565(10)^{-8}} \\
\mathrm{~A}_{\mathrm{d}} & =4.43178(10)^{-2} \\
\beta_{\mathrm{d}} & =5.20966(10)^{4} \\
\mathrm{C}_{\mathrm{d}} & =2.31291(10)^{-6} \\
\alpha_{3} & =-1.1754901(10)^{6} \\
\beta_{3} & = \pm 5.950369(10)^{3} \\
\alpha_{4} & =-1.9370308(10)^{6}
\end{align*}
$$

From the listed Laplace transform pairs one can easily find the time func tions for the terms involved in (A-12) and (A-13). Let $h_{p}(t)$ and $h_{d}(t)$ denote the respective impulse responses of the preemphasis and the deemphasis filters. Then
$h_{p}(t)=\delta(t)-\frac{1}{2.11953(10)^{-11}}\left[e^{\alpha_{1} t} K_{1} \cos \left(\left|\beta_{1}\right| t-\phi_{1}\right)+K_{2} e^{\alpha_{2} t}\right] u(t)$
where $\alpha_{1}=-5.4976376(10)^{6}$

$$
\begin{aligned}
& \left|\beta_{1}\right|=4.3298779(10)^{4} \\
& \alpha_{2}=-3.3365331(10)^{6} \\
& \mathrm{~K}_{1}=9.16262(10)^{-5} \\
& \phi_{1}=1.79422(10)^{-2} \\
& \mathrm{~K}_{2}=-5.45646(10)^{-9}
\end{aligned}
$$

and
$h_{d}(t)=0.213813 \delta(t)+\frac{1}{4.79565(10)^{-8}}\left[e^{\alpha_{3} t} K_{3} \cos \left(\left|\beta_{3}\right| t-\phi_{2}\right)+K_{4} e^{\alpha_{4} t^{t}}\right] u(t) \quad(A-15)$
where $\alpha_{3}=-1.1754901(10)^{6}$

$$
\begin{aligned}
& \left|\beta_{3}\right|=5.950369(10)^{3} \\
& \alpha_{4}=-1.9370308(10)^{6} \\
& \mathrm{~K}_{3}=0.0443185 \\
& \mathrm{~K}_{4}=2.31291(10)^{-6} \\
& \phi_{2}=-0.55566(10)^{-2}
\end{aligned}
$$

## A. 4 REFERENCE

Recommendation 405-1, Radio Relay Systems for Television (Pre-emphasis Characteristics for Frequency-Modulation Systems) C.C.I.R. XIIth Plenary Assembly, New Delhi, 1970, Vol. IV, Part 1.
A-7


Figure A-1. Preemphasis Filter for 525-Line Television

Table A-1. Component Values of Preemphasis Filter

|  |  |
| :---: | :---: |
| $L(\mu \mathrm{H})$ | 17.35 |
| $\mathrm{C}(\mathrm{p} \mathrm{F})$ | 3085 |
| $R_{1}(\Omega)$ | 275.8 |
| $R_{2}(\Omega)$ | 75 |
| $R_{3}(\Omega)$ | 20.4 |
|  |  |



Figure A-2. Deemphasis Filter for 525-Line Television

Table A-2. Component Values of Deemphasis Filter

|  |  |
| :---: | :---: |
| $L(\mu \mathrm{H})$ | 50.16 |
| $\mathrm{C}(\mathrm{p})$ | 8917 |
| $\mathrm{R}_{1}(\Omega)$ | 275.8 |
| $R_{2}(\Omega)$ | 75 |
| $R_{3}(\Omega)$ | 20.4 |



Figure A-3. Preemphasis Characteristics for Television 525-Line System


Figure A-4. A General Circuit and T-Section


DELTA - SECTION


Figure A-5. Delta-Section and Star-Section


Figure A-6. A Converted T-Section

# APPENDIX B - DERIVATION OF THE IMPULSE RESPONSE FOR <br> BUTTERWORTH AND CHEBYSHEV LOW-PASS FILTERS 

## B. 1 INTRODUCTION

The impulse responses are derived from the transfer functions for two-, three-, and four-pole Butterworth low-pass filters and for three- and four-pole Chebyshev low-pass . filters. The filters are assumed to have an arbitrary bandwidth of $\omega_{\mathrm{B}}$ radians per second, where $\omega_{B}$ is the half-power bandwidth for the Butterworth filters and is the ripple bandwidth for the Chebyshev filters. The general transfer functions are usually presented in a form such that the radian bandwidth is unity; however, replacement of s by $\frac{\mathrm{S}}{\omega_{\mathrm{B}}}$ in those expressions results in transfer functions with a radian-frequency bandwidth of $\omega_{B}$. The constants required for the Chebyshev filters are evaluated for a $0.1-\mathrm{dB}$ ripple.

## B. 2 IMPULSE RESPONSES OF BUTTERWORTH FILTERS

In Reference 1, the general transfer function of a low-pass Butterworth filter is given as

$$
\begin{equation*}
\mathrm{H}_{\mathrm{nB}}(\mathrm{~s})=\frac{1}{\mathrm{~B}_{\mathrm{n}}(\mathrm{~s})} \tag{B-1}
\end{equation*}
$$

where $B_{n}(s)$ is the Butterworth polynomial of order $n$. The coefficients of $B_{n}(s)$, up to the tenth order are listed in the mentioned reference. The second, third, and fourth order Butterworth polynomials are given as follows:

$$
\begin{align*}
& \mathrm{B}_{2}(\mathrm{~s})=\mathrm{s}^{2}+\sqrt{2} \mathrm{~s}+1  \tag{B-2}\\
& \mathrm{~B}_{3}(\mathrm{~s})=\mathrm{s}^{3}+2 \mathrm{~s}^{2}+2 \mathrm{~s}+1  \tag{B-3}\\
& \mathrm{~B}_{4}(\mathrm{~s})=\mathrm{s}^{4}+2.6131259 \mathrm{~s}^{3}+3.4142136 \mathrm{~s}^{2}+2.6131259 \mathrm{~s}+1 \tag{B-4}
\end{align*}
$$

To obtain the transfer function of a low-pass filter with bandwidth, $\omega_{B}$ radians per second, the complex frequency, $s$, representing $j \omega$, in Equations ( $B-2$ ), ( $B-3$ ), and (B-4) is replaced by $\frac{\mathrm{s}}{\omega_{\mathrm{B}}}$. Let $\mathrm{H}_{2 \mathrm{~B}}(\mathrm{~s}), \mathrm{H}_{3 \mathrm{~B}}(\mathrm{~s})$, and $\mathrm{H}_{4 \mathrm{~B}}$ (s) be respectively the transfer functions of the low-pass Butterworth filters of two, three, and four poles. Then

$$
\begin{align*}
& \mathrm{H}_{2 \mathrm{~B}}(\mathrm{~s})=\frac{\omega_{\mathrm{B}}^{2}}{\mathrm{~s}^{2}+\sqrt{2} \omega_{\mathrm{B}} \mathrm{~s}+\omega_{\mathrm{B}}^{2}}  \tag{B-5}\\
& \mathrm{H}_{3 \mathrm{~B}}(\mathrm{~s})=\frac{\omega_{\mathrm{B}}^{3}}{\mathrm{~s}^{3}+2 \omega_{\mathrm{B}} \mathrm{~s}^{2}+2 \omega_{\mathrm{B}}^{2} \mathrm{~s}+\omega_{\mathrm{B}}^{3}}  \tag{B-6}\\
& \mathrm{H}_{4 \mathrm{~B}}(\mathrm{~s})=\frac{(\mathrm{B}-5)}{\mathrm{s}^{4}+2.6131259 \omega_{\mathrm{B}} \mathrm{~s}^{3}+3.4142136 \omega_{\mathrm{B}}^{2} \mathrm{~s}^{2}+2.6131259 \omega_{\mathrm{B}}^{3} \mathrm{~s}+\omega_{\mathrm{B}}^{4}} \tag{B-7}
\end{align*}
$$

To obtain the impulse responses, the inverse Laplace transforms of the expressions in Equations (B-5), (B-6), and (B-7) must be determined. The expression for $\mathrm{H}_{2 \mathrm{~B}}{ }^{(\mathrm{s})}$ in Equation ( $\mathrm{B}-5$ ) can be rearranged easily into a recognizable transform given by

$$
\begin{equation*}
H_{2 B}(s)=\frac{\omega_{B}^{2}}{\left(s+\frac{\omega_{B}}{\sqrt{2}}\right)^{2}+\frac{\omega_{B}^{2}}{2}} \tag{B-8}
\end{equation*}
$$

but the rearrangement of the other two require the use of partial fractions. The inverse Laplace transform of $\mathrm{H}_{2 \mathrm{~B}}{ }^{(\mathrm{s})}$, given by

$$
\begin{equation*}
h_{2 B}(t)=\sqrt{2} \omega_{B} e^{-\frac{\omega_{B}}{\sqrt{2}} t} \sin \frac{\omega_{B}}{\sqrt{2}} t u(t) \tag{B-9}
\end{equation*}
$$

is the impulse response of the two-pole filter.

The expression in Equation (B-6) is expanded into partial fractions as shown in

$$
\begin{equation*}
\mathrm{H}_{3 \mathrm{~B}}(\mathrm{~s})=\frac{\mathrm{A}_{1}\left(\mathrm{~s}-\alpha_{1}\right)}{\left(\mathrm{s}-\alpha_{1}\right)^{2}+\beta_{1}^{2}}+\frac{\mathrm{A}_{1} \alpha_{1}+\mathrm{B}_{1}}{\left(\mathrm{~s}-\alpha_{1}\right)^{2}+\beta_{1}^{2}}+\frac{\mathrm{C}_{1}}{\mathrm{~s}-\alpha_{2}} \tag{B-10}
\end{equation*}
$$

where $\left(\alpha_{1} \pm j \beta_{1}\right)$ is the complex root
$\alpha_{2}$ is the real root of the denominator of ( $\mathrm{B}-6$ )
$A_{1}, B_{1}$, and $C_{1}$ are coefficients of the partial fractions.
The roots are determined by a computer subroutine in the CSC INFONET Library.
The values of the roots and coefficients are:

$$
\begin{aligned}
& \alpha_{1}=-0.5 \omega_{\mathrm{B}} \\
& \beta_{1}= \pm 0.8660254 \omega_{\mathrm{B}} \\
& \alpha_{2}=-\omega_{\mathrm{B}} \\
& \mathrm{~A}_{1}=-\omega_{\mathrm{B}} \\
& \mathrm{~B}_{1}=0 \\
& C_{1}=\omega_{\mathrm{B}}
\end{aligned}
$$

The corresponding impulse response, $\mathrm{h}_{3 \mathrm{~B}}{ }^{(\mathrm{t})}$, of a low-pass three-pole Butterworth filter is given by

$$
\begin{equation*}
\mathrm{h}_{3 \mathrm{~B}}(\mathrm{t})=\left[\mathrm{Ce}^{\alpha_{2}^{\alpha^{t}}}-\mathrm{K}_{1} \mathrm{e}^{\alpha_{1}^{\mathrm{t}}} \cos \left(\left|\beta_{1}\right| \mathrm{t}-\phi_{1}\right)\right] \mathrm{u}(\mathrm{t}) \tag{B-11}
\end{equation*}
$$

where $C={ }^{\omega} B$

$$
\begin{aligned}
& \left.\mathrm{K}_{1}=\sqrt{\mathrm{A}_{1}^{2}+\left(\frac{\left|\mathrm{A}_{1}\right| \alpha_{1}}{\left|\beta_{1}\right|}\right.}\right)^{2} \\
& \phi_{1}=\tan ^{-1}\left(\frac{\alpha_{1}}{\left|\beta_{1}\right|}\right)
\end{aligned}
$$

Similarly for the case of the four-pole Butterworth filter.

$$
\begin{equation*}
\mathrm{H}_{4 \mathrm{~B}}(\mathrm{~s})=\frac{\mathrm{A}_{2} \mathrm{~s}+\mathrm{B}_{2}}{\mathrm{~s}^{2}-2 \alpha_{3} \mathrm{~s}+\alpha_{3}^{2}+\beta_{3}^{2}}+\frac{\mathrm{C}_{2} \mathrm{~s}+\mathrm{D}_{2}}{\mathrm{~s}^{2}-2 \alpha_{4} \mathrm{~s}+\alpha_{4}^{2}+\beta_{4}^{2}} \tag{B-12}
\end{equation*}
$$

where $\alpha_{3}=-0.92387948 \omega_{B}$

$$
\begin{aligned}
& \beta_{3}= \pm 0.38268356 \omega_{\mathrm{B}} \\
& \alpha_{4}=-0.38268345 \omega_{\mathrm{B}} \\
& \beta_{4}= \pm 0.92387951 \omega_{\mathrm{B}} \\
& \mathrm{~A}_{2}=-\mathrm{C}_{2} \\
& \mathrm{~B}_{2}=2\left(\alpha_{3}-\alpha_{4}\right) \mathrm{C}_{2}-\mathrm{D}_{2}
\end{aligned}
$$

$$
\mathrm{C}_{2}=\frac{2\left(\alpha_{3}-\alpha_{4}\right) \omega_{\mathrm{B}}^{4}}{\left[\left(\beta_{3}^{2}-\beta_{4}^{2}\right)+\left(\alpha_{3}-\alpha_{4}\right)\left(\alpha_{3}-3 \alpha_{4}\right)\right]\left[\left(\alpha_{3}^{2}+\beta_{3}^{2}\right)-\left(\alpha_{4}^{2}+\beta_{4}^{2}\right)\right]+4\left(\alpha_{3}-\alpha_{4}\right)^{2}\left(\alpha_{4}^{2}+\beta_{4}^{2}\right)}
$$

$$
\mathrm{D}_{2}=\frac{\omega_{\mathrm{B}}^{4}-2\left(\alpha_{3}-\alpha_{4}\right)\left(\alpha_{4}^{2}+\beta_{4}^{2}\right) \mathrm{C}_{2}}{\left(\alpha_{3}^{2}-\beta_{3}^{2}\right)-\left(\alpha_{4}^{2}+\beta_{4}^{2}\right)}
$$

The two complex roots of the denominator of ( $B-7$ ) are ( $\alpha_{3} \pm j \beta_{3}$ ) and ( $\alpha_{4} \pm j \beta_{4}$ ) and the coefficients of the partial fractions are $A_{2}, B_{2}, C_{2}$, and $D_{2}$. Since $C_{2}$ and $D_{2}$ are found negative, $\mathrm{H}_{4 \mathrm{~B}}{ }^{(\mathrm{s})}$ can be rearranged as

$$
\mathrm{H}_{4 \mathrm{~B}}(\mathrm{~s})=\frac{\mathrm{A}_{2}\left(\mathrm{~s}-\alpha_{3}\right)}{\left(\mathrm{s}-\alpha_{3}\right)^{2}+\beta_{3}^{2}}+\frac{\mathrm{A}_{2} \alpha_{3}+\mathrm{B}_{2}}{\left(\mathrm{~s}-\alpha_{3}\right)^{2}+\beta_{3}^{2}}-\left[\frac{\left|\mathrm{C}_{2}\right|\left(\mathrm{s}-\alpha_{4}\right)}{\left(\mathrm{s}-\alpha_{4}\right)^{2}+\beta_{4}^{2}}+\frac{\left|\mathrm{C}_{2}\right| \alpha_{4}+\left|\mathrm{D}_{2}\right|}{\left(\mathrm{s}-\alpha_{4}\right)^{2}+\beta_{4}^{2}}\right](\mathrm{B}-13)
$$

The corresponding impulse response, $\mathrm{h}_{4 \mathrm{~B}}{ }^{(t)}$, is given by
$h_{4 B}(t)=\left[e^{\alpha_{3} t} K_{2} \cos \left(\left|\beta_{3}\right| t-\phi_{2}\right)-e^{\alpha_{4} t} K_{3} \cos \left(\left|\beta_{4}\right| t-\phi_{3}\right)\right] u(t)$
where $K_{2}=\sqrt{A_{2}^{2}+\left(\frac{A_{2} \alpha_{1}+B_{2}}{\left|\beta_{3}\right|}\right)^{2}}$

$$
\begin{aligned}
& \phi_{2}=\tan ^{-1}\left(\frac{\mathrm{~A}_{2} \alpha_{3}+\mathrm{B}_{2}}{\mathrm{~A}_{2}\left|\beta_{3}\right|}\right) \\
& \mathrm{K}_{3}=\sqrt{\mathrm{C}_{2}^{2}+\left(\frac{\left|\mathrm{C}_{2}\right| \alpha_{4}+\left|\mathrm{D}_{2}\right|}{\left|\beta_{4}\right|}\right)^{2}} \\
& \delta_{3}=\tan ^{-1}\left(\frac{\left|\mathrm{C}_{2}\right| \alpha_{2}+\left|\mathrm{D}_{2}\right|}{\left|\mathrm{C}_{2}\right| \cdot\left|\beta_{4}\right|}\right)
\end{aligned}
$$

Thus, the impulse response of the Butterworth filters can be determined if the halfpower bandwidth is specified.

## B. 3 IMPULSE RESPONSE OF CHEBYSHEV FILTERS

The impulse responses of three- and four-pole Chebyshev filters are derived in this paragraph, but first some general properties of Chebyshev filters are presented. The frequency response of the n-pole Chebyshev filter is given by

$$
\begin{equation*}
\left|\mathrm{H}_{\mathrm{nC}}(\omega)\right|^{2}=\frac{1}{1+\epsilon^{2} \mathrm{~T}_{\mathrm{n}}^{2}(\omega)} \tag{B-15}
\end{equation*}
$$

where $T_{n}(\omega)$ is the Chebyshev polynomial of order $n$ and $\epsilon$ is the ripple factor.

Some properties of this response are given below

1. In the passband, the maximum value of $\left|\mathrm{H}_{\mathrm{n}^{\prime}} \mathrm{C}^{(\omega)}\right|$ is unity which occurs when $T_{n}(\omega)=0$, and the minimum value is $\frac{1}{\sqrt{1+\epsilon^{2}}}$ which occurs when $\left|T_{n}(\omega)\right|=1$.
2. $\left|H_{n C^{(0)}}\right|=1$ for all odd $n$, corresponding to the maximum values in the pass band.
3. $\left|\mathrm{H}_{\mathrm{nC}}{ }^{0}\right|=\frac{1}{\sqrt{1+\epsilon^{2}}}$ for all even $n$, corresponding to the minimum values in the pass band.
4. $\left|\mathrm{H}_{\mathrm{n}} \mathrm{C}^{(1)}\right|=\frac{1}{\sqrt{1+\epsilon^{2}}}$ for all $n$, corresponding to the minimum values in the pass band.

From these properties it is clear that the ratio of the maximum to the minimum power response in the pass band is given by $1+\epsilon^{2}$. Therefore, when a ripple is specified by $R$ in decibels, the ripple factor can be calculated by

$$
\begin{equation*}
\epsilon=\sqrt{10^{\mathrm{R} / 10}-1} \tag{B-16}
\end{equation*}
$$

For example a ripple of 0.1 dB , i.e. $R=0.1$, yields a ripple factor of 0.152302 .
The transfer function of a given Chebyshev filter has the form:

$$
\begin{equation*}
\mathrm{H}_{\mathrm{nC}}(\mathrm{~s})=\frac{\mathrm{G}}{\mathrm{~s}^{\mathrm{n}}+\mathrm{b}_{\mathrm{n}-1} \mathrm{~s}^{\mathrm{n}-1}+\ldots+b_{1} s+b_{0}} \tag{B-17}
\end{equation*}
$$

where $b_{n-1}, \ldots, b_{0}$ are positive real coefficients of the polynomial in the denominator, and $G$ is the real constant to be determined to satisfy the properties of $\left|H_{n C}{ }^{(\omega)}\right|$ mentioned previously. The roots, $s_{v}$, of the polynomial are given in Reference 1 as follows:

$$
\begin{equation*}
s_{v}=-\sinh \phi \sin \frac{(2 v+1) \pi}{2 n}+j \cosh \phi \cos \frac{(2 v+1) \pi}{2 n} ; v=0,1,2, \ldots, n-1 \tag{B-18}
\end{equation*}
$$

where $\sinh \phi \quad \frac{\left(\sqrt{\frac{1}{\epsilon^{2}}+1}+\frac{1}{\epsilon}\right)^{1 / n}-\left(\sqrt{\frac{1}{\epsilon^{2}}+1}+\frac{1}{\epsilon}\right)^{-1 / n}}{2}$ and $\quad \cosh \phi=\frac{\left(\sqrt{\frac{1}{\epsilon^{2}}+1}+\frac{1}{\epsilon}\right)^{1 / n}+\left(\sqrt{\frac{1}{\epsilon^{2}}+1}+\frac{1}{\epsilon}\right)^{-1 / n}}{2}$

The constant should be determined by setting $s=0$ in Equation (B-17) and noting that

$$
\begin{align*}
& \mathrm{H}_{\mathrm{nC}} \mathrm{C}^{(0)}=\frac{\mathrm{G}}{\mathrm{~b}_{0}}=1, \text { for odd } \mathrm{n}  \tag{B-19}\\
& \mathrm{H}_{\mathrm{nC}}(0)=\frac{\mathrm{G}}{\mathrm{~b}_{0}}=\frac{1}{\sqrt{1+\epsilon^{2}}}, \text { for even } \mathrm{n} \tag{B-20}
\end{align*}
$$

as required by the properties of $H_{n C}(\omega)$ mentioned previously.
For the case $\mathrm{n}=3$, the transfer function $\mathrm{H}_{3 \mathrm{C}}(\mathrm{s})$ of a low pass three-pole Chebyshev filter has the form:

$$
\begin{equation*}
\mathrm{H}_{3 \mathrm{C}}(\mathrm{~s})=\frac{\mathrm{G}_{1}}{\left(\mathrm{~s}^{2}-2 \alpha_{1} \mathrm{~s}+\alpha_{1}^{2}+\beta_{1}^{2}\right)\left(\mathrm{s}-\alpha_{2}\right)} \tag{B-21}
\end{equation*}
$$

where $\alpha_{1} \pm{ }_{j} \beta_{1}$ is the complex root of the polynomial,
$\alpha_{2}$ is the real root of the polynomial, and
$G_{1}$ is a constant to be evaluated.
Similarly for $n=4$, the transfer function $H_{4 C}(s)$ of a low-pass four-pole Chebyshev filter has the form:

$$
\begin{equation*}
\mathrm{H}_{4 \mathrm{C}}(\mathrm{~s})=\frac{\mathrm{G}_{2}}{\left(\mathrm{~s}^{2}-2 \alpha_{3} \mathrm{~s}+\alpha_{3}^{2}+\beta_{3}^{2}\right)\left(\mathrm{s}^{2}-2 \alpha_{4} \mathrm{~s}+\alpha_{4}^{2}+\beta_{4}^{2}\right)} \tag{B-22}
\end{equation*}
$$

where $\left(\alpha_{3} \pm \mathrm{j} \beta_{3}\right)$ and ( $\alpha_{4} \pm \mathrm{j} \beta_{4}$ ) are the two complex roots of the polynomial, and $\mathrm{G}_{2}$ is a constant to be determined.
To obtain the transfer function of a low-pass filter with ripple bandwidth $\omega_{B}$, replace $s$ in Equations $(B-21)$ and $(B-22)$ by $\frac{S}{\omega_{B}}$. Then

$$
\begin{equation*}
\mathrm{H}_{3 \mathrm{C}}\left(\frac{\mathrm{~s}}{\omega_{\mathrm{B}}}\right)=\frac{\mathrm{G}_{2}{ }^{\omega}{ }_{\mathrm{B}}^{3}}{\left(\mathrm{~s}^{2}-2 \alpha_{5} \mathrm{~s}+\alpha_{5}^{2}+\beta_{5}^{2}\right)\left(\mathrm{s}-\alpha_{6}\right)} \tag{B-23}
\end{equation*}
$$

where $\alpha_{5}=\alpha_{1}{ }^{\omega}{ }_{\mathrm{B}}$

$$
\begin{aligned}
& \beta_{5}=\beta_{1}{ }^{\omega} \mathrm{B} \\
& \alpha_{6}=\alpha_{2}{ }^{\omega} \mathrm{B}
\end{aligned}
$$

and

$$
\begin{equation*}
\mathrm{H}_{4 \mathrm{C}}\left(\frac{\mathrm{~s}}{\omega_{\mathrm{B}}}\right)=\frac{\mathrm{G}_{2} \omega_{\mathrm{B}}^{4}}{\left(\mathrm{~s}^{2}-2 \alpha_{7} \mathrm{~s}+\alpha_{7}^{2}+\beta_{7}^{2}\right)\left(\mathrm{s}^{2}-2 \alpha_{8} \mathrm{~s}+\alpha_{8}^{2}+\beta_{8}^{2}\right)} \tag{B-24}
\end{equation*}
$$

where $\alpha_{7}=\alpha_{3} \omega_{B}$

$$
\begin{aligned}
& \beta_{7}=\beta_{3} \omega_{\mathrm{B}} \\
& \alpha_{8}=\alpha_{4} \omega_{\mathrm{B}} \\
& \beta_{8}=\beta_{4} \omega_{\mathrm{B}}
\end{aligned}
$$

To obtain the impulse responses, the right sides of Equations (B-23) and (B-24) are expanded into partial fractions and then the inverse Laplace transforms are taken. This procedure has been described previously and is not repeated here, but the results of calculations involved are given below.

The value of $R$ is specified to be 0.1 dB ; therefore, $\epsilon=0.152302$.
By Equation ( $\mathrm{B}-18$ ), the values of the following roots are obtained, and by Equation (B-19) $G_{1}$ and $G_{2}$ are determined:

$$
\begin{align*}
\alpha_{1} & =-0.484703 \\
\beta_{1} & = \pm 1.20616 \\
\alpha_{2} & =-0.969406  \tag{B-25}\\
G_{1} & =1.63806
\end{align*}
$$

$\alpha_{3}=-0.264156$
$\beta_{3}= \pm 1.12261$
$\alpha_{4}=-0.637730$
$\beta_{4}= \pm 0.4650$
$\mathrm{G}_{2}=0.819025$
$H_{3 C}\left(\frac{s}{\omega_{B}}\right)$ and $H_{4 C}\left(\frac{s}{\omega_{B}}\right)$ in partial fractions are given below.
$H_{3 C}\left(\frac{\mathrm{~s}}{\omega_{B}}\right)=\frac{\mathrm{A}_{1} \mathrm{~s}+\mathrm{B}_{1}}{\left(\mathrm{~s}-\alpha_{5}\right)^{2}+\beta_{5}^{2}}+\frac{\mathrm{C}_{1}}{\mathrm{~s}-\alpha_{6}}$
where $A_{1}=-C_{1}$

$$
\begin{aligned}
& \mathrm{B}_{1}=0 \\
& \mathrm{C}_{1}=\frac{\mathrm{G}_{1} \omega_{B}^{3}}{\left(\alpha_{5}-\alpha_{6}\right)^{2}+\beta_{5}^{2}}
\end{aligned}
$$

and

$$
\begin{equation*}
\mathrm{H}_{4 \mathrm{C}}\left(\frac{\mathrm{~s}}{\omega_{\mathrm{B}}}\right)=\frac{\mathrm{A}_{2} \mathrm{~s}+\mathrm{B}_{2}}{\left(\mathrm{~s}-\alpha_{7}\right)^{2}+\beta_{7}^{2}}+\frac{\mathrm{C}_{2} \mathrm{~s}+\mathrm{D}_{2}}{\left(\mathrm{~s}-\alpha_{8}\right)^{2}+\beta_{8}^{2}} \tag{B-28}
\end{equation*}
$$

where $A_{2}=-C 2$

$$
\begin{gathered}
\mathrm{B}_{2}=2\left(\alpha_{7}-\alpha_{8}\right) \mathrm{C}_{2}-\mathrm{D}_{2} \\
\mathrm{C}_{2}=\frac{2\left(\alpha_{7}-\alpha_{8}\right) \mathrm{D}_{2}}{\left(\beta_{7}^{2}-\beta_{8}^{2}\right)+\left(\alpha_{7}-\alpha_{8}\right)\left(\alpha_{7}-3 \alpha_{8)}\right.} \\
\mathrm{D}_{2}=\frac{\left[\left(\beta_{7}^{2}-\beta_{8}^{2}\right)+\left(\alpha_{7}-\alpha_{4}\right)\left(\alpha_{7}-3 \alpha_{8}\right)\right] \mathrm{G}_{2} \omega_{\mathrm{B}}^{4}}{4\left(\alpha_{7}-\alpha_{8}\right)^{2}\left(\alpha_{8}^{2}+\beta_{4}^{2}\right)+\left[\left(\alpha_{7}^{2}+\beta_{7}^{2}\right)-\left(\alpha_{8}^{2}+\beta_{8}^{2}\right)\right]\left[\left(\beta_{7}^{2}-\beta_{8}^{2}\right)+\left(\alpha_{7}-3 \alpha_{8}\right)\right]}
\end{gathered}
$$

Let $\mathrm{h}_{3 \mathrm{C}}{ }^{(t)}$ and $\mathrm{h}_{4 \mathrm{C}}{ }^{(t)}$ denote the required impulse responses of low-pass three-pole and four-pole Chebyshev filters with $\omega_{B}$ specified. Then

$$
\begin{equation*}
h_{3 C}(t)=\left[C_{1} e^{\alpha_{6} t}-e^{\alpha_{5} t} K_{1} \cos \left(\left|\beta_{5}\right| t-\phi_{1}\right)\right] u(t) \tag{B-29}
\end{equation*}
$$

where $\mathrm{K}_{1}=\mathrm{C}_{1} \sqrt{1+\left(\frac{\alpha_{5}}{\left|\beta_{5}\right|}\right)^{2}}$

$$
\phi_{1}=\tan ^{-1}\left(\frac{\alpha_{5}}{\left|\beta_{5}\right|}\right)
$$

and

$$
\begin{equation*}
\mathrm{h}_{4 \mathrm{C}}(\mathrm{t})=\left[\mathrm{e}^{\alpha_{7} \mathrm{t}} \mathrm{~K}_{2} \cos \left(\left|\beta_{7}\right| \mathrm{t}-\phi_{2}\right)-\mathrm{e}^{\alpha_{8} \mathrm{t}} \mathrm{~K}_{3} \cos \left(\left|\beta_{8}\right| \mathrm{t}-\phi_{3}\right)\right] \mathrm{u}(\mathrm{t}) \tag{B-30}
\end{equation*}
$$

where $K_{2}=\sqrt{A_{2}^{2}+\left(\frac{A_{2} \alpha_{7}+B_{2}}{\left|\beta_{7}\right|}\right)^{2}}$

$$
\phi_{2}=\tan ^{-1}\left(\frac{\mathrm{~A}_{2} \alpha_{7}+\mathrm{B}_{2}}{\mathrm{~A}_{2}\left|\beta_{7}\right|}\right)
$$

$$
\begin{aligned}
& \mathrm{K}_{3}=\sqrt{\mathrm{C}_{2}^{2}+\left(\frac{\left.\left|\mathrm{C}_{2}\right| \frac{\alpha_{8}+\left|\mathrm{D}_{2}\right|}{\left|\beta_{8}\right|}\right)^{2}}{\phi_{3}}=\tan ^{-1}\left(\frac{\left|\mathrm{C}_{2}\right| \alpha_{8}+\left|\mathrm{D}_{2}\right|}{\mathrm{C}_{2}\left|\beta_{8}\right|}\right)\right.}
\end{aligned}
$$

Thus, the impulse responses can be determined for three- and four-pole Chebyshev filters if the ripple bandwidth is specified.

## B. 4 REFERENCES

1. Louis Weinberg, Network Analysis and Synthesis, McGraw-Hill Book Company, Inc., 1962

## APPENDIX C - LISTING OF COMPUTER PROGRAMS

C. 1 COMPUTER PROGRAM 1

```
* tpulse: to genepate t-pulse
* DT=STEP SIZE
* -------------------------------------------
DIMENSION X(1800),Y(1800)
* --------------------------------------------
* TIME-TNTERVALS:
* -----------------------------------------------
T1=0.0
T3=0.1E-00
TEND=1.8E-06
q ----------------------------------------------
* STARTING TIME:
q --------------------------------------------
DT=1.0E-09
W=3.14159265*5.0E06
T=T1
N=1
* ---------------------------------------------
500 IF(T .GT. T3)GOTO 20
A=W*(T)
x(N)=0.70*SIN(A)*#?
Y(N)=-X(N)
GOTO 50
20 X(N)=0.7
Y(N)=-0.7
50 T=T+DT
IF(T .GT. TEND)GOTO 1000
N=N+1
GOTO 500
* ------------------------------------------------
1000 WRITE(11,*)X
WRITE(12,*)Y
WRITE (6,100)(X(I),I=1,1800)
WRITE(6,101)
101 FORMAT(/IX,'Y(I):'/)
WRITE(6,100)(Y(I),I=1,1800)
100 FORMAT(1X,9E14.6)
* --------------------------------------------------
STOP
END
```


## C. 2 COMPUTER PROGRAM 2

```
4n TPULSE: TO GENE-ATE T-HULSE
4 DT=STEP SI\angleE
y------------------------------------------------
O)IMENSIOM x(&10)
%-----------------------------------------------------
% TIMF-INTEKVALS:
% --------------------------------------------------
T1=0.11
T 3=0.2゙E- \6
TEND=0.BE-06
% ----------------------------------------------------
% STAKTING TIME:
& --------------------------------------------------
nT=1.0t-04
w=3.14159265*5.OFO%.
T=Tl
N=1
* -------------------------------------------------
500 IF(T .GT. T3)GOTO 20
A=w*T
x(iv) =0.70%SIN(A)*%.
GOTU SO
20 x (N) =0.0
5 0 T = T + D T
IF(T .GY. TEND)GOT` 1000
N=N+1
GOTO SU0
*-*----------------------------------------------------
1000 WHITE(11**)X
WRITF(12,100)(X(I),I=1,800)
100 FORMAT(1X,9El4.6)
4---------------------------------------------------------
STOP
ENO
```


## C. 3 COMPUTER PROGRAM <br> 3

```
* TPuLSE: to geng_ate t-pulse
* DT=STEN SIZE
* --------------
IIMENSION x(1900)
& -----------------
* TIMF-INTEHVALS:
T1=0.AE-OAS
TZ=0.7E-OG
```

```
T 3=1.3E-1)n
T4=1.4E-06
TFND=1.8L-00
%-----------------------------------------------------
U. STARTING TIME:
y-m----------------------------------------------------
i\T=1.0E-00
y=3.1415926545.UE0h
T=0.0
r=1
%-----------------------------------------------------
500 IF(T .GT. Tl)G(:TO) 20
x(N)=-0.<0
GOTO 50
2n IF(T .GT. TZ)GO10 30
O=w*(T-Tl)
x(NI)=0.7*SIN(A)**2-0.2
GOTO לO
3(IF(T .GT. T3)GOTO 40
x(N)}=0.5
GOTO 50
40 IF(T .GT. T4)GOTO bu
A=W%(T-T.j)
x(N)=0.71j*\operatorname{Cos}(A)**2-0.C0
fir)TO bO
A0 x(m)=-0.20
50 T=T+UT
TF(T .GT. TENU)GOTII 1000
N=N+1
GOTU 500
4-----------------------------------------------------
1000 CONTINUE
w!TE(11,*)x
WHITE(6,101)W,OT,N
101 FORMAT(1X,'T-AMK:FRE(HAD)=',E12.D,1X,'DT=',E12.6,1X,*
1HOINTS=1,I4/)
WHITE(0,102)
IUZ FOHMAT(IX.'UATA STORED) IN FILE: FBAKI'//)
WHTTE(10,100)(X(I),I=1,N)
100 FONNAT(1X, ЧE 14.6)
&-------------------------------------------------------
STOP
ENO
```


## C. 4 COMPUTER PROGRAM 4

```
* TpulSE: to genfaate t-hulse
* DT=STEP SIZE
& --------------------------------------------
OIMENSIUN X(1410)
& -------------------------------------------
    TIME-INTEHVALS:
% --------------------------------------------
Tl=0.0
TR=0.6E-J6
Ts=0.HE-i)h
TEND=1.4E-00
幺 ----------------------------------------------
* STARTING TIME:
4 ---------------------------------------------
TT=1.0F-0,
l:=3.14154265*5.0EDAK
T=Tl
N=1
* -------------------------------------------
500 IF(T .GT. TZ)GOTO 40
X(N)=-0.20
riatu bo
40 IF(T .GT. T3)GOTO 20
A=W*(T-TC)
x(N)=0.70%SIN(A)**2-0.20
goto ho
20 x(iv)=-0.20
50) T=T+[ T
TF(T.GT. TENO)GOTS 1000
N=N+1
GOTO 500
% --------------------------------------------
1000 WHITE(11,*)x
viritE.(12,100)(X(I),I=1,1400)
100 FOHMAT(1X,9E14.6)
%.---------------------------------------------
STOP
FND
```


## C. 5 COMPUTER PROGRAM 5

```
* PRDIMP: IMPULSE RESPONSE OF PREEMPHASIS FILTER
DIMENSION X(2000)
% ------------------------------------------------------
* CONSTANTS:
* ----------------------------------------------------
A=9.16115E-05
B=503.718
```

```
C= -5.45646E-09
ALPHAl= -5.4976376E06
ALPHA2=-3.3365331E06
日ETA=4.3298779E04
Al=(A*ALPHAl+B)/BETA
AR=A1/A
AK=SQRT(A*A+Al*Al)
PHI=ATAN(A2)
C2=1.0/2.11953E-11
N=2000
T=0.0
DT=1.0E-09
```



```
OO 10 I=1,N
Cl=EXP(ALPHAI*T)*AK*COS(BETA*T-PHI) + C*EXP(ALPHAZ*T)
X(I)=CI*C?
T=T+DT
10 CONTINUE
% --ー-ー-ー-ー-
WRITE(11,*)X
WRITE(6,102)
102 FORMAT(IX, 'PREIMP:PREEMPHASIS RESPONSE', 2X.'DATA STORED *
IN FILE:FPRIMT (//)
WRITE(6,101)(X(I),I=1,N)
101 FORMAT(1X,9E14.6)
*
STOP
END
```

C． 6 COMPUTER PROGRAM 6

```
* DEIMP: IMPULSE RESPONSE OF DEEMPHASIS FILTER
DIMENSION X(1800)
q --------------------------------------------------
% CONSTANTS:
& ---------------------------------------------------
A=4.43178F-02
B=52096.6
C=2.31291E-06
ALPHAI=-1.1754901E06
ALPHAL=-1.9370308E06
BETA=5.950369E03
Al=(A*ALPHAl + R)/BETA
AL=Al/A
AK=SQRT(A#A+Al#Al)
PHI=ATAN(AZ)
N=1800
T=0.0
DT=1.0E-09
Cl=1.0/4.79565E-08
```



```
00 10 I=1,N
C2=EXP(ALPHAI*T)*AK*COS(BETA*T-PHI) + C*EXP(ALPHA2*T)
X(I)=Cl*C2
T=T+DT
10 CONTINUE
% -------------------------------------------------------------
WFITE(6,101)
101 FORMAT (1X, DEIMP:DEEMPHASIS RESPONSE,/)
WRITE(6.102)
102 FORMAT(IX,'DATA STORED IN FILE:FDEIMT'/)
WRITE(6,103)AK,PHI,Cl
103 FORMAT(/1X,3(E14.6.1X)/)
WRITE(11.*)X
WPITE(6,100)(X(I),I=1,N)
100 FORMAT(1X,9E14.6)
% ---*--------------------------------------------------------
STOP
END
```


## C. 7 COMPUTER PROGRAM 7

```
* SINPUL: TO FORM COMPOSITE SIGNAL
DIMENSION SIG (1800), SIGI (1800), PRE (1800), Z (1800), Y(1800), \%
X(1800), W (1800)
READ (11,*)SIG
PEAD (12,*)SIG1
PEAD (13,*)PRE
```



```
\(M=1200\)
OT=1.0E-09
```



```
\% CONVOLUTION \& *1.479
```



```
OO \(10 \quad \mathrm{I}=1, \mathrm{M}\)
\(S=0.0\)
\(\mathrm{Sl}=0.0\)
\(L=I\)
กo \(20 \mathrm{~J}=1\).I
\(S=D T * S I G(L) * \operatorname{PRE}(J)+S\)
\(S I=D T * S I G I(L) * \operatorname{PRE}(J)+S I\)
\(L=L-1\)
20 CONTINUE
\(Z(I)=(S I G(I)-S) * 1.479\)
\(x(I)=(S I G 1(I)-S I) * 1.479\)
10 CONTINUE
```



```
\% FORMING THE COMPOSITE SIGNAL
```



```
\(\Delta=0.2 / 3.162277\)
```

```
Al=0.5/3.162277
00 199 I=1,M
Z(I)=Z(I)-A
X(I) =X(I)+AI
199 CONTINUE
DO 200 I=1,600
Y(I) =-A
W(I) =Al
200 CONTINUE
DO 601 I=601,1800
Y(I)=Z(I-600)
W(I) =X(I-600)
601 CONTINUE
\00 FORMAT(1X,9E14.6)
WRITE(14,*)Y
WRITE(15,*)W
WRITE(6,10.3)
103 FORMAT (1X,'COMPOSITE SIGNAL-1,STORED IN FCOMT1'/)
WRITE (6,100)(Y(I),I=1,1800)
WPITE(6,104)
104 FORMAT (/1X,'COMPOSITE SIGNAL-2,STORED IN FCOMT2'/)
WRITE (6,100)(W(I),I=1,1800)
STOP
END
```

C. 8 COMPUTER PROGRAM 8

```
* SINPUL: TO FORM COMPOSITE SIGNAL
GIMENSIUN SIG(EO1), PKE (1KO0),Z(800),Y(1400)
FEAD(11,*)SIG
READ(12,*)PRE
心--------------------------------------------
M=400
DT=1.0E-UY
& ------------------------------------------------
* CINVOLUTION & *1.479
#%---------
S=0.0
L=I
[0] 20 J=1, I
S=DT*SIO(L)*NKE(J) +S
L=L-1
?O CONTINUE
L(I)=(SIG(I)-S)*1.474
10 CONTINUE
*-----------------------------------------------
u=0.2/3.1ヵ2ट77
OO 14y I=1,M
L(I)= Z.(I)-A
```

```
199 cuntinue
103 FOんMAT(/1X,'CO.APOSITE SIGNAL,STO!EED IN FCOMFI'//)
nO 200 I=1,600
Y(I) =-A
200 conttivue
OO 201 I=n01,1400
Y(I)=Z(I-n00)
?01 COMTINUE
WHITE (13,*)Y
WHITE(0,10.3)
YHITE(G,100)(Y(I),I=1,1400)
100 FOHMAT(IX,9E14.6)
STOP
FNig
```


## C． 9 COMPUTER PROGRAM 9

```
& SINCOS: TO GET SIN & COS OF PHI(T)
*------------------------------------------------------------------
* Y(I): RESULTS OF INTEGRATION
* FLCOS: COS OF PHI(T)
* FLSIN: SIN OF NHI(T)
* DH: HEAK ULVIATION
\geqslant----------------------------------------------------------------------
1)IMENSION Z(1400),Y(1400),FLCOS(1400),FLSIN(1400)
FEAD(11,*)Z
N}=120
IT T=1.UE-09
\GammaP=5.0EU6
AK=4.0*3.14159265*1:P
# -----------------------------------------------
* INTEGRATION TO SET PHI(T)
4---------------------------------------------------------------------
Y(1)=Z(1)*0T
1:0 30 I= C,M
Y(I)=Y(I-l)+Z(I)*OT
30 CONTINUE
% -----------------------------------------------------------------------
* TAKE COS & SIN
4.---------------------------------------------------------------------
in(1)}40\quadI=1.
\triangleA=AK*Y(I)
FLCOS(I)=COS(AA)
FLSIN(I) =SIN(AA)
40 CONIINUE
4---------------------------------------------------------------------
WHITE(12,*)FLCOS
WHITE(13.*)FLSIN
WRITE(6,1ちい)
150 FOHMAT(IX,'PROGHAM TO GENEHATE COS(PHI) & SIN(PHI) *
```

C-8

```
-- STUHE゙\ IN FPCOSन,FPSINb!//)
i:MITE(6,15I)DP.DT...
151 FOGMAT(1X.'PEAK-DEVIATION=',E゙14.0, 2X.'STEH-SIZE=',EIZ.6, in
?X, POINTS=',I5//)
153 FOFMAT(IX,GE.14.6)
WHITE(6.155)
155 FOHMAT(/ 1x,'(:SS(I):'/)
wHITE(o,l53)(FLCOS(I),I=I\bulletM)
\QITE(6,156)
156 FOHMAT(/ 1X,'SIN(I):'//)
4NITE(6,153)(FLSIN(I),I=1,M)
STOP
FN(
```


## C. 10 COMPUTER PROGRAM 10

```
* SN3BF: SYMMETRICAL 3-POLE BUTTERWORTH FILTER RESPONSE
% FB: BANDWIDTH HZ
* DT: STEP SIZE
% T: TIME
DIMENSION x(1800)
% ------------------*--------------------------------------------------------
FB=15.0E06
WR=2.0*3.14159265*FE
N=1800
OT=1.0E-09
T=0.0
```



```
ALPHAl=-0.5*WR
\triangleLPHAZ=-1.0*WB
BETAl=0.8660254*WB
C=WB**3/(BETAl**2+(ALPHAl-ALPHAZ)**2)
B=(2.0*ALPHAI-ALPHAZ)*C
A=-C
* --------------------------------------------------------------------
AK=SQRT(A*A*(1.0 + ALPHAl*ALPHAL/BETAl/BETAl))
PHI=ATAN(ALPHAl/BETAl)
```



```
DO 10 I=1.N
X(I) =-AK*EXP(ALPHAI*T)*COS(BETA1*T-PHI) +C*EXP(ALPHAZ*T)
T=T+DT
10 CONTINUE
```



```
WKITE(6,100)
100 FORMAT(1X:3-POLE BUTTERWORTH,STORED IN FS3BT3'//)
WRITE(6,101)FB,DT,N
```

101 FORMAT（1X，＇BANDWIDTH：FB（HZ）＝1，E14．6．2X，＇STEP－SIZE＝1，\％ E14．6．2X．POOINTS＝＇．I5／／）
WRITE（6，10．3）C，B，AK，PHI

E14．6，2X． 1 PHI $=1, E 14 \cdot 6 / / 1$
WRITE（12，＊）X
WRITE（ 6,102 ）（X（I），I＝I，N）
102 FORMAT（IX．9E14．6）
STOP
END

C． 11 COMPUTER PROGRAM
11
＊SN4BF：SYMMETRICAL 4－POLE BUTTERWORTH FILTER RESPONSE
＊FB：BANDWIDTH HZ
＊DT：STEP－SIZE
$\%$ T：TIME

DIMENSION $\times(1800)$

$\mathrm{FB}=12.0 \mathrm{E} 06$
$W \mathrm{H}=2.0 * 3.14159265 * F B$
$\mathrm{N}=1800$
$D T=1.0 E-09$
$T=0.0$

ALPHAI $=-0.92387948 * W B$
ALPHAZ $=-0.38268345 * W B$
BETA1 $=0.38268356 \% W$ W
BETA2 $=0.92387951 * W A$

$\Delta A 1=(B E T A l * 2-B E T A C * 2)+(A L P H A 1-A L P H A C) *(A L P H A 1-3.0 * A L P H A C)$
$A A 2=(4.0 *(A L P H A 1-A L P H A 2) * せ 2) *(A L P H A C * * 2+B E T A 2 * * 2)$
$A A 3=(A L P H A 1 * * 2+B E T A l * * 2)-(A L P H A C * * 2+B E T A C * * 2)$

$D=(W B * * 4 /(A A Z+A A 3 * A A 1)) * A A 1$
$C=2.0 *(A L P H A I-A L P H A C) * D / A A I$
$\mathrm{E}=2.0 *(A L P H A 1-A L P H A 2) * C-D$
$A=-C$
＊－ースーーーーーーー
WRITE（6．100）
100 FORMAT（ $1 \times, 14-P O L E$ BUTTERWORTH，STORED IN FS4BT2•／／）
WRITE（6，101）FB，N，DT
101 FORMAT（ $/ 1 \mathrm{X}$, ＇BANDWIDTH：FB（HZ）$=1$, E $14.6,2 \mathrm{X}$, POINTS $=, \%$
I5，2X，STEP－SIZE（SEC）＝＇，E14．6／／）
WRITE $(6,102)$ ALPHAl．ALPHAZ，BETA1，BETAZ


WRITE（6，103）A，B，C，D

```
103 FORMAT(1X,'A=',E14.6,2X,'B=',E14.6,2X,'C=',E14.6.2X,%
1D=',E14.6//)
q-------------------------------------------------------------------------
ABC=ABS(C)
ARD=ABS(D)
AKl=SQRT (A*A ( ( A*ALPHAl +B)/BETAl)**2)
PHIl=ATAN((A*ALPHAl+B)/A/BETAl)
AK2 = SQRT (ABC*ABC + ( (ABC*ALPHAZ + ABD)/BETAZ)**2)
PHI2=ATAN((ABC*ALPHAC+ABD)/ABC/BETAC)
* ----------------------ッ---------------------------------------------------
OO 10 I= 1.N
X(I)=AK1*EXP(ALPHA1*T)*COS(BETAI*T-PHII)-AK2*EXP(ALPHAZ*T)*%
COS(BETA2*T-PHI2)
T=T+DT
10 CONT INUE
*-----------------------------------------------------------------------------
WRITE(6,104)AK1,PHIl,AK2,PHI?
104 FORMAT(1X,'AKI,PHIL,ETC:',4(El4.6.2X))
WRITE(11,*)X
WRITE (6,105)(X(I),I=1,N)
105 FORMAT(1X,9E14.6)
$ --
END
```


## C. 12 COMPUTER PROGRAM <br> 12

```
* SN3CF: SYMMETRICAL 3-POLE CHEBYSHEV FILTER RESPONSE
* FB: GANDWIDTH NHZ
* DT: STEP SIZE
% T: TIME
* ------------------------------------------------------------
OIMENSION X(1800)
& -------------------------------------------------------------
FA=15.0E06
WA=2.0*3.14159265#FB
N=1800
OT=1.0E-09
T=0.0
* ---------------------------------------------------------------
ALPHAL=-0.484703*WR
ALPHAZ =-0.969406*WB
BETAl=1.20616*WB
C=1.63806*WB**3/(HETA1**2+(ALPHA1-ALPHAC)*#2)
B=(2.0*ALPHA1-ALPHAL)*C
A=-C
* ---------------------------------------------------------------
AK=SQRT(A*A*(1,0+ALPHAl*ALPHAl/BETAl/BETAl))
PHI=ATAN(ALPHAl/BETAl)
```

```
*
0O 10 I=1,N
X(I)=-AK*EXP(ALPHAI*T)*COS(BETAI*T-PHI) +C*EXP(ALPHAZ*T)
T=T+DT
10 CONTINUE
```



```
WRITF (6.100)
100 FORMAT(1X, 3-POLE CHEBYSHEV,STORED IN FS3CT3://1
WRITE(G,101)FB,DT,N
101 FORMAT (IX,'BANOWIDTH:FB(HZ)=', E14.6,2X,'STEP-SILE=',%
F14.6.2X,.POINTS=',I5//)
WRITE(6,103)C,B,AK,PHI
103 FORMAT(1X, 'C=',E14.6,2X,'B=',El4.6,2X,'AK=',**
F14.6.2X.'PHI=',El4.6//1
WRITE(11,*)X
WRITE (6,102)(x(I),I=1,N)
102 FORMAT(1X,9E14.6)
STOP
END
```

C. 13 COMPUTER PROGRAM 13

```
* SN4CF: SYMMETRICAL 4-POLE CHEBYSHEV FILTER RESPONSE
* FB:BANDWIDTH HZ
* DT:STEP-SIZE
* T:TIME
```



```
DIMENSION X(1800)
```



```
FH=30.0E06
WB=2.0*3.14159265*FB
N=1800
DT=1.0E-09
T=0.0
```



```
ALPHAl = =0.637730#wP.
ALPHA2 = - 0. 264156%WB
BETAl=0.4650*WB
RETA2=1.12261*WB
```



```
AA1=(BETA1**2-BETAC**2) + (ALPHAl-ALPHA2)*(ALPHAl-3.0*ALPHAZ)
AA2 = (4.0*(ALPHA1-ALPHA2)**2)*(ALPHA2**2+BETA2**2)
AA 3 = (ALPHAl**2 + EETAl**2)-(ALPHAC**2 + BETAC**2)
```



```
D=(0.819025*WB**4/(AAZ+AA3*AA1))*AA1
C=2.0*(ALPHAl-ALPHA2)*D/AA1
A=2.0*(ALPHA1-ALPHA2)*C-D
A=-C
```



```
WRITE (6,100)
```

```
100 FORMAT(1X,14-POLE CHEHYSHEV,STORED IN FS4CT5://)
WRITE(6,101)FR,N,DT
101 FORMAT (/1X,'BANDWIDTH:FB(HZ)=1,E14.6,2X,'POINTS=, %
I5,2X,'STEP-SIZE(SEC)=',E14.6//)
WRITE(6,102)ALPHAl,ALPHAZ,BETAl,BETAZ
102 FORMAT(/1X,'ALPHAL=',E14.6,2X,'ALPHAL=',El4.6,2X,'BETAl=',%
E14.6.2X,PBETAZ=',F14.6//)
WKITE(6,103)A,B,C,D
103 FORMAT(IX,'A=',E14.6,2X,'A=',El4.6,2X,'C=',E14.6,2X,%
-D=',E14.6//)
* ------------------------------------------------------------------
ABC=ABS (C)
ABD=ABS(D)
AKL=SQRT(A*A+((A*ALPHAl+B)/BETAl)**2)
PHIl=ATAN((A*ALPHAl+P)/A/BETA1)
AK2=SQRT (ABC*ABC+((ABC*ALPHA2 + ABO)/BETAZ)**2)
PHI2=ATAN((ABC*ALPHAZ +ABD)/ABC/BETAL)
* -------------------------------------------------------------------
DO 10 I =1,N
x(I)=AKl*EXP(ALPHAl*T)*COS(BETAl*T-PHIl)-AK2#EXP(ALPHAZ*T)**
COS(BETAZ*T-PHI2)
T=T+DT
10 CONTINUE
% WRITE(6,104)AKI,PHIl,AKZ,PHI2
104 FORMAT(1X,'AKI,PHIL,ETC:',4(E14.6,2X)/)
WRITE(11,*)X
WRITE(6,105)(X(I),I=1,N)
105 FORMAT(1X,9E14.6)
* -
STOP
END
```


## C. 14 COMPUTER PROGRAM 14

```
* CONDPH: TO GET PHID(T)
* FLCOS(I): COS OF PHI(T)
* FLSIN(I): SIN OF PHI(T)
* HBF(I): PREDETECTION FILTER RESPONSE
* A(I):A(T)
% B(I):B(T)
% PHI(I):PHI(T)
* PHID(I): DERIVATIVE OF PHI(T)
% AM:MOD-INDEX
* FM:MAX MOD-FREQUENCY
& --------------------------------------------
OIMENSION FLCOS(1800),FLSIN(1800), HBF(1800),A(1800). %
B(1200),PHI(1200),PHID(1200)
READ(11,*)FLCOS
READ(12,*)FLSIN
```

```
READ(13,*)HBF
```



```
* CONSTANTS:
* -----
OT=1.0E-09
DP=5.0E06
AK=4.0*3.14159265*nP
& ------------------------------------------------------
* CONVOLUTION TO GET A(T) & R(T)
%
NO 10 I=1,M
S1=0.0
S2=0.0
L=I
00 20 J=1,I
Sl=DT*HBF(J)*FLCOS(L)+Sl
S2=DT*HBF(J)*FLSIN(L) + S2
L=L-1
20 CONT INUE
A(I)=SI
F(I)=S2
10 CONTINUE
* ----------------------------------------------------
* TO COMPUTE ATAN(B/A)
* -----------------------------------------------------
DO 30 I=1,M
IF(A(I) .FQ. 0.0)GOTO 40
C2=B(I)/A(I)
PHI(I)=ATAN(C2)
GOTO 30
40 PHI(I) =0.0
WRITE(6,101)I
101 FORMAT(1X,'A=0.0', 2X,'I=',I5/)
30 CONTINUE
q -----------------------------------------------
% DERIVATIVE OF PHI(T):
* - ----
OO 50 I=1,N1
PHID(I)=(-3.0*PHI(I)+4.0*PHI(I+1)-PHI(I+2))/(2.0*DT*AK)
50 CONTINUE
* -------------=-=-
151 FORMAT(1X,'PEAK-DEVIATION:',EI4.6,2X, POINTS=',I5, 2X,%
-STEP-SILE=',E14.6//)
WRITE(14,*)PHIO
WPITE(6,153)(PHID(I),I=1,M)
153 FORMAT(1X,9E14.6)
*
STOP
FNO
```


## C． 15 COMPUTER PROGRAM 15

```
& CONDPH: TO GET HHID(T)
&FLCOS(I): COS OF HHI(T)
* FLSIN(I): SIN i)F PHI(T)
* HEF(I): PREDETECTION FILTER RESPONSE
* A(I):A(T)
* H(I):H(T)
#PHI(I):PHI(T)
* PHID(I): DERIVATIVE OF PHI(T)
& AM:MOU-INDEX
% FM:MAX MOD-FRE\HENCY
<---------------------------------------------------
OIMENSION FLCOS(1400),FLSIN(1400),HBF(1800),A(1400), *
H(1400),WHI(1400), OHIO(1460),HOL(1800), Z(1400),Y(1400)
HEAD(11,*)FLCOS
HEAD(12,*)FLSIN
QFAD(13,*)HBF
HEAD(14**)HDE
y.--------------------------------------------------
C CONSTANTS:
y ----------------------------------------------------
M=1200
nT=1.0E-UG
\capP=5.UE゙06
AK=4.0%3.14154265*にp
y---------------------------------------------------
* CONVULUTION TO (EET A(T) & G(T)
* -----------------------------------------------------
OO 10 I=I,M
S1=0.0
S2=0.0
L=I
00 20 J=1.I
Sl=DT*hBF(J)*FLCOS(L)+Sl
S2=DT*h*F(J)*FLSIN(L)+SZ
L=L-1
20 CONTINUE
A(I)}=S
F(I)=SC
10 CONTINUE
4----------------------------------------------------
* TO COMPUTE ATAN(H/A)
幺-------------------------------------------------------
UO 3U I=1,M
IF(A(I) .FW. U.0)GHTO 40
CX=H(I)/A(I)
DHI(I)=ATAN(C2)
GOTO 30
40 PHI(I) =0.0
wKITE(6,101)I
```

```
101 FORMAT(IX,'A=0.0.,2X,'I=.,I5/)
30 CONTINUE.
y ---------------------
NOL=M-2
NO 50 I=1,N1
HMID(I)=(-3.0*PHI(I)+4.0*PHI (I+1)-PHI(I+C))/(2.0*OT*AK)
GO CONTINUE
WRITE(6.151)DP,M,DT
151 FORMAT(1X,'PLAK-DEVIATION:',E14.6,2X,'POINTS=1,I5.2X.**
!STEP-SILE=',E14.G//)
153 FORMAT(1X,YEl4.6)
& COKFECTION OF FOINTS
*
MPITF(6,112)
112 FOFMAT(IX,'COLHECTED PHID'/)
I=1
F0 AH=ABS(PHID(I))
IF(AHS.GT. 10.0)GOTO 72
GOTO 71
72 WRITE(0,75)I,PHID(I-1),PHIU(I+2)
75 FORMAT(/1X,'ERROR POINT:.,I5, 己X, 2E14.6/)
Al=(PHIO(I-1)-PHIO(I+2))/5.0
PHIO(I)=PHIO(I-I)-\DeltaI
HHID(I+1)=PHIU(I-1)-2.U*AI
I=I + I
7l CONTINUE
I=I + I
IF(I.,GT. M)GOTO m.]
(G) TO &O
HI WKITE(6,153)(PHID(I),I=1,M)
* convOluTION OF CORRECTED PHID & HDE
Cl=-0.0032562*3.162C7%
N2=M-600
\O 201 I=601,M
Z(I-600)=OHID(I)+0.0G325
Z0l CONTINUE
WO 60 I =I NC
S=0.0
L=I
\Gamma:O 70 J=1, I
S=DT*HDE(J)*Z(L)*S
L=L-l
70 CONTINUE
Y(I) =(Z(I)*0.213813+S)*3.162278+C1
```

```
GO CONTINJE
% ------------------------------------------------------
WITE(15,*)Y
NITE(6,102)
102 FÖHMAT(/ lX,'FINAL OUTPUT'//)
H/KITE(6,153)(Y(I),I=1,N2)
STOP
FMO
```

C. 16 COMPUTER PROGRAM 16

```
& CONOPH: TO GET PHIO(T)
* FLCOS(I): COS OF PHI(T)
c FLSIN(I): SIN OF HHI(T)
* HHF(I): PREDETECTION FILTER RESPONSE
% A(I):A(T)
* B(I):H(T)
* PHI(I):PHI(T)
%. PHIU(I): DERIVATIVE OF PHI(T)
* AM:MOU-INDEX
* FM:MAX MOD-FRENHENCY
--------------------------------
OIMENSION FLCOS(1H0U),FLSIN(1H00),HBF(1800),A(1800), &
H(1800),WHI(1800),NHID(1800),HCE(1800),Z(1800),Y(1800)
HEAD(11,*)FLCOS
HEAD(12,*)FLSIN
READ(13,*)HBF
HEAD(14,*)HDE
% ------N---
```



```
M=1200
\GammaT=1.0E-09
1)P=6.0EOD
AK=4.0*3.14154265%1:p
* CONVGLUTION TO GET A(T) & B(T)
%0-- 10 I=1.M
sl=0.0
S2=0.0
I= I
กO 20 J=1.I
Sl=OT*HBF(J)*FLCOS(L)*Sl
S2=DT*HBF(J)*FLSIN(L) +S2
L=L-1
?O CONTINUE
A(I)=Sl
F(I)=52
10 CONTI vUE
```

```
* TO COMPUTE ATAN(B/A)
*------------------------------------------------------
DO 30 I=1,M
IF(A(I) , EQ. 0.0)f(OTO 40
C2=B(I)/A(I)
PHI(I)=4TAN(CZ)
GOTO 30
41) PHI(I)=0.0
WHITE(6,lUI)I
101 FORMAT (1X,'A=0.0',2X,'I=',I5/)
30 CONTINUE
* DEKIVATIVE OF PrI(T):
y-----------------------------------------------------
M1 = M-2
0050 I=1,NI
PHIO(I)=(-3.0*PHI(I) +4.0%PHI(I+1)-PHI(I+C))/(2.0#0T*AK)
50 CONTINUE
* ---------------------*-*----------------------------
*WITE(6,1bl)DP*M,OI
    151 FOHMAT(1X.'PEAK-DEVIATION:',E14.0.2X,'POINTS=',IS,2X,*
    'STEP-SILE=',El4.'ゥ//)
    153 FOMMAT(1X.9E14.6)
* ---------------------------------------------------
* CORRECTION OF POINTS
WHITE(6,112)
112 FORMAT(IX.'CORMECTED HHID'/)
I= l
M0 AH=ABS(PHID(I))
IF(AB GGT. 10.0)GOTO 7C
GOTO 7l
72 WRITE(6.75)I,PHIU(I-1), PHID(I +2)
75 FOHMAT(/1X, 'ERROR POINT:',I5.2X.2E14.6/)
Al=(PHI\cup(I-1)-PHIO(I+2))/3.0
OHID(I) =HHIC(I-I)-Al
FHIO(I+1)=PHID(I-1)-2.0*A1
I = I +1
71 CONTINUE
I=I + I
IF(I .GT. iA)GOTO HI
GU TO 80
*1 WRITE(6.153)(PHIO(I),I=1,M)
乡---------------------------------------------------------
* CONVOLUTION OF CORRLCTED PHIU & HUE
y=-------------
Cl=0.b0
N2=M-000
4-------------------------------------------------------------
```

```
00 201 I=601.m
Z(I-600)=PHIO(I)-C?
201 CONTINUE
00 60 I=1,N2
S=0.0
L=I
\capO 70 J=I.I
&=OT*HUE (J)* Z(L) +S
L=L-1
70 CONT I NUE
Y(I) =(2(I)*0.213813+5)*3.162278+Cl
H0 CONTINUE
4. -----------------------------------------------------
*以ITE(15,*)Y
WKITE(6,102)
    lO2 FOHMAT(/ IX, FINAL OUTPUT'//)
    vNITE(6.153)(Y(I),I=1,NC)
    STOP
    EASB
```


## APPENDIX D - GLOSSARY OF VARIABLES

## USED IN THE COMPUTER PROGRAMS

## D. 1 VARIABLES USED IN COMPUTER PROGRAM 1

A A variable which is the product of angular frequency W and time T .
DT An increment of time T.
I
N
An integer index for a DO-Loop.
An integer variable to count the number of points into which the time interval is divided. It also acts as the index of two vectors.

T The current value of time.
T1 A specified time interval.
T3 A specified time interval.
TEND The largest value of time for which calculations are made.
W Angular frequency of a sinusoidal function.
$\mathrm{X} \quad$ A vector used to retain a set of calculated values.
Y A vector used to retain a set of calculated values.
D. 2 VARIABLES USED IN COMPUTER PROGRAM 2

A A variable which is the product of angular frequency $W$ and time $T$.
DT An increment of time $T$.
I An integer index for a DO-Loop.
$\mathrm{N} \quad$ An integer variable to count th number of points into which the time interval is divided. It also acts as the index of two vectors.
$T \quad$ The current value of time.
T1 A specified time interval.
T3 A specified time interval.
TEND The largest value of time for which calculations are made.
W Angular frequency of a sinusoidal function.
$\mathrm{X} \quad$ A vector used to retain a set of calculated values.
D. 3 VARIABLES USED IN COMPUTER PROGRAM 3

A A variable which is the product of angular frequency W and time T .
DT An increment of time $T$.
An integer index for a DO-Loop.
N
An integer variable to count the number of points into which the time interval is divided. It also acts as the index of two vectors.
$\mathrm{T} \quad$ The current value of time.
T1 A specified time interval.
T2 A specified time interval.
T3 A specified time interval.
T4 A specified time interval.
TEND The largest value of time for which calculations are made.
W Angular frequency of a sinusoidal function.
$\mathrm{X} \quad \mathrm{A}$ vector used to retain a set of calculated values.
D. 4 VARIABLES USED IN COMPUTER PROGRAM 4

A A variable which is the product of angular frequency W and time T .
DT An increment of time T.
I An integer index for a DO-Loop.
$\mathrm{N} \quad$ An integer variable to count the number of points into which the time interval is divided. It also acts as the index of two vectors.

T The current value of time.
T1 A specified time interval.
T2 A specified time interval.
T3 A specified time interval.
TEND The largest value of time for which calculations are made.
W Angular frequency of a sinusoidal function.
$\mathrm{X} \quad$ A vector used to retain a set of calculated values.

## D. 5 VARIABLES USED IN COMPUTER PROGRAM 5

A A coefficient in the partial fractions of the transfer function of the preemphasis filter.
AK One of the two real constants of the impulse response of the preemphasis filter.
ALPHA1 The real part of the complex root of the polynomial associated with the transfer function of the preemphasis filter.
ALPHA2 The real root of the polynomial associated with the transfer function of the preemphasis filter.

A1 The value of an expression used repeatedly.
A2
B A coefficient in the partial fractions of the transfer function of the preemphasis filter.
BETA The imaginary part of the complex root of the polynomial associated with the transfer function of the preemphasis filter.
C A coefficient in the partial fractions of the transfer function of the preemphasis filter.

C1

C2 The second real constant in the impulse response function of the preemphasis filter.

DT An increment of time T.
I An integer index used in the DO-Loops.
N
PHI The phase angle in the argument of the cosine term of the impulse response of the preemphasis filter.
$T \quad$ The current value of time.
$\mathrm{X} \quad$ A vector to retain the values of the impulse response of the preemphasis filter.

## D. 6 VARIABLES USED IN COMPUTER PROGRAM 6

A A coefficient of the partial fractions of the transfer function of the deemphasis filter.

AK One in the two real constants of the impulse response functions of the deemphasis filter.
ALPHA1 The real part of the complex root of the polynomial associated with the transfer function of the deemphasis filter.
ALPHA2 The real root of the polynomial associated with the transfer function of the deemphasis filter.

A1 The value of an expression used repeatedly.
A2 The value of an expression used repeatedly.
B A coefficient of the partial fractions of the transfer function of the deemphasis filter.
BETA The imaginary part of the complex root of the polynomial associated with the transfer function of the deemphasis filter.
C A coefficient in the partial fractions of the transfer function of the deemphasis filter.
C1 The second real constant in the impulse response function of the deemphasis filter.
C2 The value of a portion of the expression for the impulse response function of the deemphasis filter.

DT An increment of time $T$.
I An integer index used in DO-Loops.
$\mathrm{N} \quad$ An integer specifying the number of calculations.
PHI The phase angle in the argument of the cosine term of the impulse response of the deemphasis filter.
$\mathrm{T} \quad$ The current value of time.

X
A vector used to retain the sample values of the impulse response function.

## D. 7 VARIABLES USED IN COMPUTER PROGRAM 7

A A real constant which is used repeatedly.
A1 A real constant used repeatedly.
DT The value of the increment of time.

I

J
L
M
PRE The impulse response of the preemphasis filter.
S
S1
SIG A vector to retain a test signal.
SIG1 A vector to retain a test signal.
$\mathrm{X} \quad \mathrm{A}$ vector to retain the results of a convolution.
W A vector to retain the final result formed by adding two component parts of a test signal by the principle of superposition.
Y A vector to retain the final result formed by adding two component parts of a test signal by the principle of superposition.

Z
A vector to retain the result of a convolution.
D. 8 VARIABLES USED IN COMPUTER PROGRAM 8

A A real constant which is used repeatedly.
DT The value of the increment of time.
I An index for DO-Loops.
$J$ An index for DO-Loops.
L An index used to furnish a special order of vector components.
M An integer to specify the number of calculations.
PRE The impulse response of the preemphasis filter.
S A variable used to form a summation in a convolution operation.

A vector to retain a test signal.
A vector to retain the results of a convolution.
A vector to retain the final result formed by adding two component parts of a test signal by the principle of superposition.

A vector to retain the result of a convolution.

## D. 9 VARIABLES USED IN COMPUTER PROGRAM 9

AA A product of two variables used repeatedly.
$\mathrm{AK} \quad$ The produce of $4 \pi$ times peak deviation.
DP Peak frequency deviation.
DT Step size of time interval used in integration.
FLCOS A vector used to retain values of the function $\cos \phi$.
FLSIN A vector used to retain values of the function $\sin \phi$.
I An index used in DO-Loops.
M An integer specifying the number of calculations.
Y A vector used to retain results of integration.
$\mathrm{Z} \quad$ A vector used to retain sample values of the modulating wave.
D. 10 VARIABLES USED IN COMPUTER PROGRAM 10

A A coefficient of the partial fractions of the transfer function of a symmetrical 3 -pole Butterworth filter.
$\mathrm{AK} \quad \mathrm{A}$ constant in the impulse response function of the symmetrical 3-pole Butterworth filter.
ALPHA1 The real part of the complex root of the Butterworth polynomial associated with the transfer function of the filter.
ALPHA2 The real root of the Butterworth polynomial associated with the transfer function of the filter.
B A coefficient of the partial fractions of the transfer function of a symmetrical 3 -pole Butterworth filter.
BETA1 The imaginary part of the complex root of the Butterworth polynomial associated with the transfer function of the filter.

DT Increment of time $T$.
FB Frequency bandwidth of the filter in Hz.

I An integer index used in DO-Loops.
$\mathrm{N} \quad$ An integer specifying the number of calculations.
PHI The phase angle in the argument of the cosine term of the impulse response function of the filter.
$\mathrm{T} \quad$ The current value of time.
WB Angular frequency bandwidth of the filter.

X
A vector to retain sample value of the impulse response.
D. 11 VARIABLES USED IN COMPUTER PROGRAM 11

A A coefficient of the partial fractions of the transfer function of a symmetrical four-pole Butterworth filter.

AA1 A real variable denoting an expression used repeatedly.
AA2 A real variable denoting an expression used repeatedly.
AA3 A real variable denoting an expression used repeatedly.
$A B C \quad$ The absolute value of $C$.
ABD The absolute value of $D$.
AK1 A constant in the impulse response function of the four-pole Butterworth filter.
AK2 A constant in the impulse response function of the four-pole Butterworth filter.
ALPHA1 The real part of the first complex root of the Butterworth polynomial of the four-pole filter.
ALPHA2 The real part of the second complex root of the Butterworth polynomial of the four-pole filter.
B A coefficient in the partial fractions of the transfer function of the four-pole Butterworth filter.

BETA1 The imaginary part of the first root of the Butterworth polynomial for the four-pole filter.
BETA2 The imaginary part of the second root of the Butterworth polynomial for the four-pole filter.
C A coefficient in the partial fractions of the transfer function of the four-pole filter.
D A coefficient in the partial fractions of the transfer function of the four-pole filter.

## D. 12 VARIABLES USED IN COMPUTER PROGRAM.

A A coefficient in the partial fractions of the transfer function of a symmetrical three-pole Chebyshev filter.

AK A constant in the impulse response function of the three-pole Chebyshev filter.
ALPHA1 The real part of the complex root of the Chebyshev polynomial associated with the transfer function of the three-pole filter.

ALPHA2 The real root of the Chebyshev polynomial associated with the transfer function of the three-pole filter.

B A coefficient in the partial fractions of the transfer function of the filter.
BETA1 The imaginary part of the complex root of the Chebyshev polynomial associated with the transfer function of the three-pole filter.

C A coefficient of the partial fractions of the transfer function of the three-pole Chebyshev filter.

DT Increment of time T.
FB The frequency bandwidth of the filter.
I
N
PHI The phase angle of the argument of the cosine term of the impulse response of the three-pole Chebyshev filter.

T The current value of time.
WB The angular frequency bandwidth of the three-pole Chebyshev filter.
X
A vector to retain the values of the impulse response of the three-pole Chebyshev filter.

## D. 13 VARIABLES USED IN COMPUTER PROGRAM 13

A A coefficient in the partial fractions of the transfer function of a symmetrical four-pole Chebyshev filter.

AA1 A variable representing an expression used repeatedly.
AA2 A variable representing an expression used repeatedly.
AA3 A variable representing an expression used repeatedly.
$\mathrm{ABC} \quad$ The absolute value of C .
$A B D \quad$ The absolute value of $D$.

AK1 One of the two constants in the impulse response function of the four-pole Chebyshev filter.

AK2 One of the two constants in the impulse response function of the four-pole Chebyshev filter.

ALPHA1 The real part of the first complex root of the Chebyshev polynomial associated with the transfer function of the four-pole filter.

ALPHA2 The real part of the second complex root of the Chebyshev polynomial associated with the transfer function of the four-pole filter.

B A coefficient in the partial fractions of the transfer function of the four-pole Chebyshev filter.

BETA1 The imaginary part of the first complex root of the Chebyshev polynomial associated with the transfer function of the four-pole filter.

BETA2 The imaginary part of the second complex root of the Chebyshev polynomial associated with the transfer function of the four-pole filter.
C A coefficient in the partial fractions of the transfer function of the four-pole Chebyshev filter.

D A coefficient in the partial fractions of the transfer function of the four-pole Chebyshev filter.

DT An increment of time $T$.
FB The frequency bandwidth of the four-pole Chebyshev filter.
I An integer index used in DO-Loops.
$\mathrm{N} \quad$ An integer specifying the number of calculations.
PHI1 The phase angle in the argument of the first cosine term of the impulse response function of the four-pole Chebyshev filter.

PHI2 The phase angle of the second cosine term of the impulse response function of the the four-pole Chebyshev filter.

T The current value of time.
WB The angular frequency bandwidth of the four-pole Chebyshev filter.
$\mathrm{X} \quad$ A vector to retain the values of the impulse response of the four-pole Chebyshev filter.

## D. 14 VARIABLES USED IN COMPUTER PROGRAM 14

A A vector representing the convolution of $\cos \phi(t)$ with $H_{7}(t)$, the impulse response of the equivalent lowpass filter of a predetection filter.
AK The constant relating peak frequency deviation to the voltage of the modulating wave.
B A vector representing the convolution of $\sin \phi(t)$ with $h_{1}(t)$.
C2

DP The value of a specified peak frequency deviation.
DT An increment of time T.
FLCOS A vector to retain sample values of $\cos \phi(t)$.
FLSIN A vector to retain sample values of $\sin \phi(t)$.
HBF A vector to retain sample values of the impulse response of a prediction filter.

I

J

L
M
N1
PHI
A vector, the components of which are generated by the arctangent of C 2 .
PHID A vector of sample values of the derivative of PHI.
S1
S2
The summation of terms involved in a convolution operation.

## D. 15 VARIABLES USED IN COMPUTER PROGRAM 15

A A vector representing the convolution of $\cos \phi(t)$ with $h_{1}(t)$, the impulse response of the equivalent lowpass filter of a predetection filter.
$\mathrm{AB} \quad$ The absolute value of the derivative of PHI .
$\mathrm{AK} \quad$ The constant relating peak frequency deviation to the voltage of the modulating wave.

A1 The value of an expression used repeatedly.
B A vector representing the convolution of $\sin \phi(t)$ with $h_{1}(t)$.

C1 A numerical constant which is the steady response of the deemphasis filter to a dc input voltage.
C2 The quotient of a component of the vector $B$ and a corresponding component of the vector $A$.

DP The peak frequency deviation.
DT An increment of time T.
FLCOS A vector used to retain the sample values of $\cos \phi(t)$.
FLSIN A vector used to retain the sample values of $\sin \phi(t)$.
HBF A vector used to retain sample values of impulse response function of the predetection filter.
HDE A vector used to retain sample values of the impulse response function of the deemphasis filter.
I An integer index used in DO-Loops.
$J \quad$ An integer index used in DO-Loops.
L An integer index used to furnish a special ordering for vector components.
$\mathrm{M} \quad$ An integer specifying the number of calculations.
N1 An integer specifying the number of calculations.
N2 An integer specifying the number of calculations to be printed out.
PHI A vector, the components of which are generated by the arctangent of C2.
PHID A vector of sample values of the derivative of PHI.
S A summation of terms involved in a convolution operation.
S1 A summation of terms involved in a convolution operation.
S2 A summation of terms involved in a convolution operation.

Z
A vector of sample values of the final output response function of the system. A vector of sample values of the partial final system output to which a single constant should be added to each component to give the final output vector.

## D. 16 VARIABLES USED IN COMPUTER PROGRAM 16

A A vector representing the convolution of $\cos \phi(t)$ with $H_{7}(t)$, the impulse response of the equivalent lowpass filter of a predetection filter.
$\mathrm{AB} \quad$ The absolute value of the derivative of PHI.
AK The constant relating peak frequency deviation to the voltage of the modulating wave.

A1 The value of an expression used repeatedly.
B A vector representing the convolution of $\sin \phi(t)$ and $h_{1}(t)$.
C1 A numerical constant which is the steady response of the deemphasis filter to a dc input voltage.

C2 The quotient of a component of the vector $B$ and a corresponding component of the vector A .

DP The peak frequency deviation.
DT An increment of time $T$.
FLCOS A vector used to retain the sample values of $\cos \phi(t)$.
FLSIN A vector used to retain the sample values of $\sin \phi(\mathrm{t})$.
HBF A vector used to retain sample values of the impulse response function of the predetection filter.

HDE A vector used to retain sample values of the impulse response function of the deemphasis filter.

I An integer index used in DO-Loops.
J An integer index used in DO-Loops.
L An integer index used to furnish a special ordering for vector components.
M An integer specifying the number of calculations to be performed.
N1 An integer specifying the number of calculations to be performed.
An integer specifying the number of calculations to be printed out.
PHI A vector, the components of which are generated by the arctangent of C2.
PHID A vector of sample values of the derivative of PHI.
S A summation of terms involved in a convolution operation.
S1 A summation of terms involved in a convolution operation.
S2 A summation of terms involved in a convolution operation.

Z A vector of sample values of the partial final system output to which a single constant should be added to each component to give the final output vector.

