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J. E. MAGGS

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J. E. Maggs
NAS-NRC Postdoctoral Resident Research Associate
Goddard Space Flight Center
Greenbelt, Md. 20771

Abstract

A purely growing electrostatic drift instability driven by the electron temperature gradient at the inner edge of the plasma sheet can grow for large enough values of the temperature gradient. The parallel electric field associated with the instability is localized near the magnetic equator. The growth of the drift instability leads to enhanced whistler noise and increased electron pitch angle diffusion. If the current limit is exceeded in the ionosphere while the parallel electric field of the drift instability exists along the field line, rapid electron precipitation (the auroral breakup) can result.

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AN AURORAL BREAKUP MECHANISM

This paper evaluates the possibility the auroral breakup is related to a purely growing drift instability at the inner edge of the plasma sheet. The auroral breakup here refers to the initial stage of an auroral substorm ($T = 0$), i.e. the sudden brightening at or near a pre-existing active auroral arc prior to the poleward expansion of auroral activity (see Akasofu (1968) for a discussion of the phases of a substorm). The inner edge of the plasma sheet refers to the near earth region of the electron plasma sheet where an exponential decrease in the electron energy density is observed (Vasyliunas, 1968; Schield and Frank, 1970).

Drift instabilities associated with the ring current particle population have been specifically discussed as causes of the auroral breakup by Swift (1967) and Liu (1970). The motivation for looking at a drift instability associated with the earthward termination of the plasma sheet as a cause of the auroral breakup is (1) its location on auroral field lines and its earthward movement with increasing magnetic activity (Frank, 1971), (2) the similarity between the energy spectra and energy densities of plasma sheet electrons and electrons precipitated in the auroral zone (Frank and Ackerson, 1971) and (3) the existence of a pressure gradient in the electrons at the inner edge of the plasma sheet. The purely growing drift instability is of interest because it becomes unstable at moderate pressure gradients and has an electric field component along the magnetic field.

Theory of the Drift Instability

In discussing the time scales of the instability the following simplifying assumptions are made: (1) the geomagnetic field is axially symmetric, (2) the perturbation fields are electrostatic, (3) the ionospheres are perfectly conducting, (4) the temporal and spatial variations of the perturbation fields are large enough to preserve the first and second adiabatic invariants μ and J. The limitations these assumptions place on the analysis will be discussed later. A method for studying drift instabilities in a general magnetic field configuration has been developed by Rutherford and Frieman (1968a) but with the assumption made above it is convenient to use a variational principle technique as used by Rosenbluth (1968) or Liu (1969a). The analysis of the instability will closely parallel that of Liu (1969b).

Euler coordinates α and β will be used to describe the magnetic field

$$\vec{B}(r, \theta, \phi) = \vec{\nabla}\alpha x\vec{\nabla}\beta \tag{1}$$

with an axially symmetric field it is judicious to choose β to be the azimuthal angle, ϕ, so that $\alpha = \alpha(r,\theta)$. The third coordinate of the system, denoted by the parameter s, is the distance along a field line from a fixed reference surface through which all field lines pass.

In discussing the time scales of the drift instability the pertinent frequencies are the gyrofrequency, the bounce frequency, the azimuthal drift frequency and the azimuthal diamagnetic drift frequency. The gyrofrequency is

$$\Omega_j = \frac{eB}{m_j c}$$

where B is the magnetic field strength, e is the particle charge, c the speed of light, and m_j the particle mass. The bounce frequency of a particle between mirror points in the magnetic field is

$$\nu_b = \frac{\partial H(\alpha, \beta, \mu, J)}{\partial J} = \int \frac{ds}{v_{\|}(s)}$$

where

$$H = K + e\Phi + \frac{\alpha}{c}\frac{\partial \beta}{\partial t} \qquad (2)$$

and K is the particle kinetic energy and Φ the electric potential. H does not depend on the distance along the field line and for bounced averaged guiding center motion cH/e plays the role of the Hamiltonian (Northrop, 1963)

$$\langle \dot{\alpha} \rangle = -\frac{c}{e}\frac{\partial H}{\partial \beta}; \qquad \langle \dot{\beta} \rangle = -\frac{c}{e}\frac{\partial H}{\partial \alpha}$$

The brackets $\langle\ \rangle$ denote a time average over the bounce period of the particle

$$\langle x \rangle \equiv \nu_b \int_{S_{m_1}}^{S_{m_2}} \frac{ds}{v_{\|}(s)}\, x = \nu_b \oint \frac{ds}{v_{\|}}\, x \qquad (3)$$

The particle velocity along the field line is denoted by $v_{\|}(s)$. The limits of the integration over s are the mirror points s_m where $v_{\|}(s_m) = 0$ and the magnetic field has the value $B(s_m) = B_m$. It is convenient to define $1/v_{\|}(s)$ to be zero for those values of s along the field line that the particle does not traverse in a bounce period, that is, where $B(s) > B_m$. Then the integration in equation 3 can be considered to be over the total length of the field line. The azimuthal drift frequency is

$$\langle \dot{\beta} \rangle = \omega_d = \frac{c}{e} \frac{\partial H}{\partial \alpha}$$

and the azimuthal diamagnetic drift frequency is

$$\omega_x = \frac{c}{e} kT \frac{\partial \ln p}{\partial \alpha} \tag{4}$$

where p is the plasma pressure, T the plasma temperature and k denotes Boltzmann's constant.

In the magnetosphere near the inner edge of the plasma sheet the ion gyrofrequency in the equatorial plane is on the order of 1 Hz. The bounce frequency for a 5 Kev proton mirroring near the ionospheres is on the order of 10^{-2} Hz. For a 5 Kev particle at 6 R_e the azimuthal drift frequency is on the order of 10^{-5} Hz. The diamagnetic drift frequency depends upon the pressure gradient and is here considered to be on the order of the magnitude of the azimuthal drift frequency.

The instability associated with the inner edge of the plasma sheet is analyzed under the assumptions it is electrostatic and preserves μ and J. The wave length of the perturbation field perpendicular to the geomagnetic field, $L_\perp/2\pi$, and parallel to the geomagnetic field, $L_\parallel/2\pi$, as well as the scale height of the pressure, L_p, are taken to be of the same order of magnitude

$$L_p \sim L_\parallel \sim L_\perp = L$$

The fields and frequencies associated with the perturbation are ordered with respect to the parameter $\epsilon = \rho/L$, ρ being the ion gyroradius. The frequencies are taken to be ordered as

$$\omega \sim \epsilon\nu_{bi} \sim \epsilon^2\Omega_i$$

where ω is the wave frequency, ν_{bi} the ion bounce frequency and Ω_i the ion gyrofrequency. The fields are ordered as

$$E_{\parallel} \sim E_{\perp} \sim \epsilon \frac{vB}{c} \tag{5}$$

where $E_{\parallel}$ and $E_{\perp}$ are, respectively, the components of the perturbation fields parallel and perpendicular to the geomagnetic fields. The requirement on the perpendicular electric field states that the electric force is of higher order than the Lorentz force. The requirement on the parallel electric field means $\Delta v_{\parallel}$, the change in $v_{\parallel}$ due to $E_{\parallel}$ in one gyroperiod, is of higher order than $v_{\parallel}$ ($\Delta v_{\parallel} = (e/m)\, E_{\parallel}/\Omega = cE_{\parallel}/B = \epsilon v_{\parallel}$). Note the parallel and perpendicular fields are assumed to be of the same order so the geometry appropriate to these waves is not the long thin geometry of the finite Lamor radius ordering in which

$$E_{\parallel} \sim \epsilon E_{\perp} \sim \epsilon^2 \frac{v}{c} B; \qquad L_{\perp} \sim \epsilon L_{\parallel}; \qquad \omega \sim \epsilon^2 \Omega_i$$

The existence of an electrostatic instability can be determined by investigating solutions to the coupled Poisson and Vlasov equations. In the ordering presented above, Poisson's equation is equivalent to the charge neutrality condition to second order in ϵ. Poisson's equation is

$$\vec{\nabla} \cdot (\vec{E}_{\parallel} + \vec{E}_{\perp}) = 4\pi e (n_i - n_e)$$

where n_i is the ion number density and n_e is the electron number density. With the fields ordered as in equation 5

$$\frac{c}{B\Omega}\vec{\nabla}\cdot\vec{E} \sim \frac{c}{B\Omega}\left[\frac{E_{\parallel}}{L_{\parallel}} + \frac{E_{\perp}}{L_{\perp}}\right] \sim \epsilon^2$$

and

$$\frac{c}{B\Omega}4\pi e(n_i - n_e) \sim \left[\frac{\rho}{\lambda_D}\right]^2 \frac{(n_i - n_e)}{n_i} \sim \frac{n_i - n_e}{n_i}\left[\frac{L}{\lambda_D}\right]^2 \epsilon^2$$

where $\lambda_D = [kT/4\pi n_i e^2]^{1/2}$ is the Debye length. Then Poisson's equation requires

$$\frac{n_i - n_e}{n_i}\left(\frac{L}{\lambda_D}\right)^2 \sim 1 \quad \text{or} \quad n_i - n_e \sim n_i \lambda_D^2/L^2$$

For temperatures, densities and field strengths near the inner edge of the plasma sheet

$$\lambda_D^2 << \rho^2 \qquad \text{and} \qquad n_i - n_e << \epsilon^2 n_i$$

Thus to order ϵ^2 Poisson's equation can be replaced by the charge neutrality condition

$$e(n_i - n_e) = 0$$

When the first and second adiabatic invariants are conserved Vlasov's equation can be written (Northrop, 1963)

$$\frac{\partial F}{\partial t} + \frac{c}{e}\left[\frac{\partial F}{\partial \beta}\frac{\partial H}{\partial \alpha} - \frac{\partial F}{\partial \alpha}\frac{\partial H}{\partial \beta}\right] = 0 \qquad (6)$$

where $F = F(\mu, J, \alpha, \beta, t)$ is the phase space distribution density. Denoting the equilibrium quantities with a subscript "O" and perturbation quantities with a subscript "1" F can be written

$$F(\mu, J, \alpha, \beta, t) = F_0(\mu, J, H) + F_1(\mu, J, \alpha, \beta, t)$$

The quantity F_0 (μ,J,H) can easily be seen to be an equilibrium solution ($\partial F/\partial t = 0$) to equation 6. With axial symmetry it is convenient to write the perturbation quantities Φ_1, H_1, F_1 in component form

$$\Phi_1 = \sum_{\ell} \Phi_{\ell}(\alpha, s) \exp i(\ell\beta - \omega t)$$

$$H_1 = \sum_{\ell} H_{\ell}(\alpha, s) \exp i(\ell\beta - \omega t)$$

$$F_1 = \sum_{\ell} F_{\ell}(\alpha, s) \exp i(\ell\beta - \omega t) \tag{7}$$

where a time dependence exp $-i\omega t$ has been assumed for the perturbation quantities. The condition J is conserved is interpreted in the sense $\langle \dot{J} \rangle = 0$. Using J conservation and noting that with $\partial\beta/\partial t = 0$

$$J \equiv \oint ds v_{\|}(s) = \oint ds \left[\frac{2}{m}(H - \mu B - e\Phi)\right]^{1/2}$$

the components of H_1 can be related to the components of Φ_1

$$H_{\ell} = e\langle\Phi_{\ell}\rangle = e\nu_b \oint \frac{ds}{v_{\|}} \Phi_{\ell}(\alpha, s) \tag{8}$$

The spatial number density of particles $n(\vec{r},t)$ at the point $\vec{r} = (\alpha', \beta', s')$ is related to the phase space distribution density as (Northrop and Teller, 1960)

$$n(\vec{r}, t) = 2B\int dJ \int d\mu \frac{\partial H}{\partial J} \frac{F}{v_{\|}(s')} (\mu, J, \alpha', \beta', t)$$

$$= 2B\int d\mu \int dJ v_{\|}^{-1}(\mu, J, s') F(\mu, J, \alpha', \beta', t)\nu_b \tag{9}$$

The integration over μ, J in equation 9 is restricted to those values of μ, J for which the bounce motion of a particle carries it through the point $(\alpha', \beta', s') = \vec{r}$. However, with the extended definition of $v_\parallel^{-1}(s)$ discussed above the integration will be taken over all μ, J since $v_\parallel^{-1}(\mu, J, s) = 0$ for particles with μ, J values such that they do not pass through s′.

Using equations 6 and 7 the components of the perturbed distribution density, F_1, can be expressed in terms of the components of the electric potential, Φ_ℓ, and the equilibrium distribution density $F_0(\mu, J, H)$

$$F_\ell = c \left\langle \Phi_\ell \right\rangle \frac{\partial F_0}{\partial \alpha}\bigg|_{\mu, J} \left\{ \omega_d - \frac{\omega}{\ell} \right\}^{-1}$$

or writing $\omega = \omega' + i\gamma$ with ω', γ real

$$F_\ell = c \left\langle \Phi_\ell \right\rangle \frac{\left[\left(\omega_d - \frac{\omega'}{\ell} \right) + i \frac{\gamma}{\ell} \right]}{\left(\omega_d - \frac{\omega'}{\ell} \right)^2 + \frac{\gamma^2}{\ell^2}} \frac{\partial F_0}{\partial \alpha}\bigg|_{\mu, J} \tag{10}$$

Using equation 9 and

$$\nu_b^{-1} = \oint \frac{ds}{v_\parallel} \qquad v_\parallel = \left[\frac{2}{m} (H - \mu B - e\Phi) \right]^{1/2}$$

the components of the perturbed number density can be written

$$n_\ell = 2B \int d\mu \int dJ \left\{ F_\ell - \frac{F_0}{m} \frac{H_\ell - e\Phi_\ell}{v_\parallel^2} + \frac{F_0}{m} \nu_b \oint \frac{ds}{v_\parallel^3} (H_\ell - e\Phi_\ell) \right\} \tag{11}$$

Since $H_\ell = e \left\langle \Phi_\ell \right\rangle$ and H is independent of s, $\left\langle \Phi_\ell \right\rangle$ is a function of μ, H, α only, and (after Rutherford and Frieman, 1968b)

$$\nu_b \frac{\partial}{\partial H}\left\{\oint \frac{ds}{v_{\|}}(\langle\Phi_\ell\rangle - \Phi_\ell)\right\}\Bigg|_{\mu,\alpha} = \nu_b \frac{\partial}{\partial H}\left\{\nu_b^{-1}\langle\Phi_\ell\rangle - \nu_b^{-1}\langle\Phi_\ell\rangle\right\}\Big|_{\mu,\alpha} = 0$$

or performing the indicated differentiation and using

$$\frac{\partial\Phi}{\partial H}\Big|_{\mu,\alpha} = 0 \qquad \text{and} \quad - v_{\|}^{-3} = m\frac{\partial}{\partial H}(v_{\|}^{-1})\Big|_{\mu,\alpha}$$

$$= \nu_b\left[\oint \frac{ds}{mv_{\|}^3}\langle\Phi_\ell\rangle + \nu_b^{-1}\frac{\partial}{\partial H}\langle\Phi_\ell\rangle\Big|_{\mu,\alpha} + \oint \frac{ds}{mv_{\|}^3}\Phi_\ell\right] = 0$$

or

$$\nu_b \oint \frac{ds}{mv_{\|}^3}(\langle\Phi_\ell\rangle - \Phi_\ell) = \frac{\partial}{\partial H}\langle\Phi_\ell\rangle\Big|_{\mu,\alpha}$$

Using the above equation and $\nu_b = \partial J/\partial H$ equation 11 is

$$n_\ell = 2B\int d\mu\int dJ\left\{\frac{\nu_b}{v_{\|}}F_\ell - e\left[F_0\frac{\partial v_{\|}^{-1}}{\partial J}(\langle\Phi_\ell\rangle - \Phi_\ell) + \frac{F_0}{v_{\|}}\frac{\partial}{\partial J}\langle\Phi_\ell\rangle\right]\right\}$$

or integrating by parts

$$n_\ell = 2B\int d\mu\int dJ\frac{\nu_b}{v_{\|}}\left[F_\ell - e(\langle\Phi_\ell\rangle - \Phi_\ell)\frac{\partial F_0}{\partial H}\Big|_{\mu,\alpha}\right]$$

The charge neutrality condition

$$\sum_j e_j n_{j\ell} = 0$$

(where $\sum_j$ is used to denote a sum of the electron and ion terms) along with equation 10 gives

$$\sum_j e_j \int d\mu \int dJ \; \frac{\nu_b}{v_\parallel(s')} \left\{ \left[\frac{c \langle \Phi_\ell \rangle \left(\left(\omega_d^j - \frac{\omega'}{\ell} \right) + i \frac{\gamma}{\ell} \right)}{\left(\omega_d^j - \frac{\omega'}{\ell} \right)^2 + \frac{\gamma^2}{\ell^2}} \right] \frac{\partial F_0^j}{\partial \alpha} \bigg|_{\mu, J} \right.$$

$$\left. - e_j(\langle \Phi_\ell \rangle - \Phi_\ell) \frac{\partial F_0^j}{\partial H} \bigg|_{\mu, \alpha} \right\} = 0 \tag{12}$$

The ionospheres are assumed to be perfectly conducting so that the boundary conditions associated with equation 12 are Φ_ℓ and $d\Phi_\ell/ds$ both equal zero at the ionospheres. If Φ_ℓ is constant at the ionosphere, $d\Phi_\ell/ds$ must be zero since the field lines are not normal to the ionospheres.

The ordering used to analyze the Vlasov equation has resulted in an eigenvalue equation for Φ_ℓ localized in α (eq. 12). A variational principle can be formed from equation 12 and used to obtain information concerning the eigenvalues and eigenfunctions by multiplying equation 12 by the adjoint solution, $\tilde{\Phi}_\ell$, and integrating over the length of the field line (see Morse and Feshbach, 1953). The adjoint solution to equation 12 can be found by investigating the nature of the kernel of the integral operator $\mathfrak{L}$, where eq. 12 is written

$$\mathfrak{L}\Phi_\ell = \mathfrak{M}\Phi_\ell \quad \text{and} \quad \mathfrak{L}\Phi_\ell = \int ds\, L(s'/s)\, \Phi_\ell(s)$$

The limits of the integration over s are now fixed at the ionospheric values of s, and the kernel L is

$$L(s'/s) = \int d\mu \int dJ\, g(\mu, J) \frac{\nu_b(\mu, J)}{v_\parallel(s')} \frac{\nu_b(\mu, J)}{v_\parallel(s)}$$

where $g(\mu, J)$ does not depend on s and

$$g(\mu, J) = \sum_j e_j \left\{ \frac{\left[\left(\omega_d^j - \frac{\omega'}{\ell}\right) + i\frac{\gamma}{\ell}\right]}{\left(\omega_d^j - \frac{\omega'}{\ell}\right)^2 + \frac{\gamma^2}{\ell^2}} \left.\frac{\partial F_0^j}{\partial \alpha}\right|_{\mu, J} - e_j \left.\frac{\partial F_0}{\partial H}\right|_{\mu, \alpha} \right\}$$

The operator $\mathcal{L}$ is self-adjoint since $L(s|s') = L(s'|s)$ but it is not Hermetian since $L^*(s|s') \neq L(s'|s)$ where "*" denotes the operation of complex conjugation. Since the boundary conditions are adjoint and $L(s|s') = L(s'|s)$ the adjoint equation is the same as equation 12 and $\tilde{\Phi}_\ell = \Phi_\ell$.

The variational principle which yields equation 12 is $\delta D/\delta\tilde{\Phi}_\ell = 0$ where

$$D = \sum_j e_j \int d\mu \int dJ \left\{ \langle\Phi_\ell\rangle^2 \frac{\left[\left(\omega_d^j - \frac{\omega'}{\ell}\right) + i\frac{\gamma}{\ell}\right]}{\left(\omega_d^j - \frac{\omega'}{\ell}\right)^2 + \frac{\gamma^2}{\ell^2}} \left.\frac{\partial F_0}{\partial \alpha}\right|_{\mu, J} - e_j\left(\langle\Phi_\ell\rangle^2 - \langle\Phi_\ell^2\rangle\right) \left.\frac{\partial F_0}{\partial H}\right|_{\mu, \alpha} \right\} \tag{13}$$

The eigenfunctions, Φ_ℓ', extremize D, i.e. $\delta D/\delta\Phi_\ell |_{\Phi_\ell = \Phi_\ell'} = 0$ and to be consistent with equation 12 the frequency must be such that $D(\omega, \Phi_\ell') = 0$. It will be shown below that under some conditions there exists an ω with a positive imaginary part ($\gamma > 0$) and a Φ_ℓ' such that ($D(\omega, \Phi_\ell') = 0$ and D is stationary at Φ_ℓ'. Thus Φ_ℓ is an eigenfunction of equation 12 with a corresponding ω such that it is purely growing in time.

Conditions for the Existence of the Purely Growing Drift Instability

For the purpose of illustrating the physical processes contributing to the growth of the instability and simplifying calculations the magnetospheric plasma is assumed to have an isotropic pitch angle distribution and a Maxwellian distribution in energy. A more realistic evaluation of the stability criteria using experimentally determined differential electron energy spectra will be made later. To express the equatorial number density it is convenient to make a change of variables from μ, J to λ, K where $\lambda = \mu/K$ is the inverse of the mirror point magnetic field strength. The equilibrium equatorial number density $n_j(\alpha)$ can be expressed as an integral over λ, K

$$n_j(\alpha) = 2B_E \int d\mu \int dJ \frac{\nu_b^j}{v_{\|}^j(s_E)} \bar{F}_0^j(\mu, J, \alpha) = 2B_E \int d\lambda \int dK\, K^{1/2} \sqrt{\frac{m_j}{2}}$$

$$\times \{(1 - \lambda B_E)^{-1/2} \bar{F}_0^j(\mu(\lambda, K), J(\lambda, K), \alpha)\} = 2B_E \sqrt{\frac{m_j}{2}} \iint \frac{d\lambda\, dK\, K^{1/2}}{(1 - \lambda B_E)^{1/2}} F_0^j(\lambda, K, \alpha)$$

where with the above assumptions and noting that the charge neutrality condition implies $n_e(\alpha) = n_i(\alpha) = n_0(\alpha)$

$$F_0^j(\lambda, K, \alpha) = \frac{n_0(\alpha) \exp -[K/kT_j(\alpha)]}{(2\pi m_j k^3 T_j^3(\alpha))^{1/2} (1 - B_E/B_I)^{1/2}}; \qquad B_E^{-1} \leq \lambda \leq B_I^{-1}$$

k is Boltzmann's constant, $T_j(\alpha)$ is the temperature of the jth species, B_E/B_I is the ratio of the equatorial to ionospheric magnetic field strength which is taken to be small enough to write

$$F_0^j(K, \alpha) = \frac{n_0(\alpha)}{(2\pi m_j k^3 T_j^3(\alpha))^{1/2}} \exp - [K/kT_j(\alpha)]$$

Also, using the variables (λ, K), the bounce frequency and the azimuthal drift frequency are factorable in terms of the particle kinetic energy, K. If the equilibrium electric field is zero

$$\nu_b^j = \nu_0(\lambda, \alpha)\left[\frac{2K}{m_j}\right]^{1/2}; \qquad \omega_d^j = \omega_0^j(\lambda, \alpha)\frac{K}{kT_j(\alpha)}$$

ω_0^j is the azimuthal drift frequency of a particle of the jth species with energy kT_j and

$$\nu_0 = \left[\int \frac{ds}{(1 - \lambda B_E)^{1/2}}\right]^{-1}$$

is independent of the particle species.

Equation 12 will be examined for eigenfunctions corresponding to purely growing modes (that is $\omega' = 0$). It will be shown that a class of functions exist that extremize D where the extreme values of D can satisfy D = 0 under some conditions with $\gamma > 0 (\omega = i\gamma)$. The azimuthal drift frequency ω_d^j will be taken to be independent of λ as is the case for a dipole field. Equation 12 for the purely growing modes and a Maxwellian energy distribution ($F_0 = F_M$) is

$$D = \int d\lambda \nu_0^{-1}\left\{\langle\Phi_\ell\rangle^2\left(g_1(\alpha, \gamma^2) + i\frac{\gamma}{\ell} g_2(\alpha, \gamma^2) - g_3(\alpha)\right) + g_3\langle\Phi_\ell^2\rangle\right\}$$

where

$$g_1(\alpha, \gamma^2) = \sum_j e_j\sqrt{\frac{m_j}{2}}\int K^{1/2}dK \frac{\omega_d^j}{\omega_d^{j2} + \frac{\gamma^2}{\ell^2}}\left[\frac{e_j}{c}\frac{\partial F_M^j}{\partial K}\bigg|_{\mu,\alpha}\omega_d^j + \frac{\partial F_M^j}{\partial \alpha}\bigg|_{\mu,K}\right]$$

$$g_2(\alpha, \gamma^2) = \sum_j e_j \sqrt{\frac{m_j}{2}} \int K^{1/2} \frac{c\left[\frac{e_j}{c} \left.\frac{\partial F_M^j}{\partial K}\right|_{\mu, a} \omega_d^j + \left.\frac{\partial F_M^j}{\partial \alpha}\right|_{\mu, K}\right]}{\omega_d^{j\,2} + \frac{\gamma^2}{\ell^2}}$$

$$g_3(\alpha) = \sum_j e_j \sqrt{\frac{m_j}{2}} \int K^{1/2} dK e_j \left.\frac{\partial F_M^j}{\partial K}\right|_{\mu, a}$$

The functions g_i are all real.

Now let $\Phi_\ell = R + i\, I$ where R and I are real functions of s, so that

$$\langle \Phi_\ell \rangle^2 = \langle R \rangle^2 - \langle I \rangle^2 + 2i \langle R \rangle \langle I \rangle$$

$$\langle \Phi_\ell^2 \rangle = \langle R^2 \rangle - \langle I^2 \rangle + 2i \langle RI \rangle$$

and with $D = D_r + i\, D_i$ (D_r, D_i real)

$$D_r = \int d\lambda \nu_0^{-1}(\lambda) \left[(\langle R \rangle^2 - \langle I \rangle^2)(g_1 - g_3) - 2\frac{\gamma}{\ell} g_2 \langle R \rangle \langle I \rangle + g_3 (\langle R^2 \rangle - \langle I^2 \rangle) \right]$$

and

$$D_i = \int d\lambda \nu_0^{-1}(\lambda) \left[(\langle R \rangle^2 - \langle I \rangle^2) \frac{\gamma}{\ell} g_2 + 2 \langle R \rangle \langle I \rangle (g_1 - g_3) + 2 g_3 \langle RI \rangle \right]$$

It can now be shown that there exists a class of functions Φ_ℓ that simultaneously extremizes D_r and D_i. The functions which accomplish this are those with $R = aI$ where a is a real constant, since then

$$D_r = \int d\lambda \nu_0^{-1} \left\{ \langle R \rangle^2 \left[(1 - a^2)(g_1 - g_3) - 2a \frac{\gamma}{\ell} g_2 \right] + g_3 \langle R^2 \rangle \right\}$$

and

$$D_i = \int d\lambda \nu_0^{-1} \left\{ \langle R \rangle^2 (1 - a^2) \frac{\gamma}{\ell} g_2 + 2a \langle R \rangle^2 (g_1 - g_3) + 2a g_3 \langle R^2 \rangle \right\}$$

Then the functions which extremize both D_r and D_i simultaneously are those which extremize the quantity

$$b = \frac{\int d\lambda\, \nu_0^{-1} \langle R^2 \rangle}{\int d\lambda\, \nu_0^{-1} \langle R \rangle^2}$$

Let the extreme value of b equal M then the conditions $D_r = D_i = 0$ for D stationary in Φ_ℓ are

$$D_r = 0 = (1 - a^2)\, g_3(M - 1) + (1 - a^2)\, g_1 - 2\frac{\gamma}{\ell} g_2 a \tag{14a}$$

$$D_i = 0 = (1 - a^2)\frac{\gamma}{\ell} g_2 + 2ag_1 + 2ag_3(M - 1) \tag{14b}$$

which are two equations in the real variables a and γ.

Solutions to equation 14 can be easily found for marginal stability ($\gamma = 0$) provided that the plasma pressure gradient satisfies a certain condition. Choosing a = 0 (i.e. Φ_ℓ real) gives $D_i = 0$ and $D_r = 0$ if

$$g_3(\alpha)\ (M - 1) + g_1(\alpha,\ \gamma^2 = 0) = 0$$

or with the definitions of g_1 and g_3 above the condition for marginal stability is

$$\frac{1}{\sum_j kT_j} \left\{ \sum_j \frac{c}{e_j} (\omega_0^j)^{-1} \left[\frac{d\, \ln n_0(\alpha)}{d\alpha} - \frac{d\, \ln T_j}{d\alpha} \right] \right\} = \frac{M}{2} \tag{15}$$

It can be shown that solutions exist for $\gamma \neq 0$. Suppose γ is small and approximate g_1 by

$$g_1(\gamma^2) \approx \frac{g_1(0)}{1 + \frac{\gamma^2}{\ell^2}} \approx g_1(0) - 2\frac{\gamma^2}{\ell^2} g_1(0)$$

and note g_3 is independent of γ. Equation 14a and 14b are

$$(1 - a^2)\, g_3(M - 1) + (1 - a^2)\, g_1(0) - 2\frac{\gamma^2}{\ell^2}(1 - a^2)\, g_1(0) + 2\frac{\gamma}{\ell}\, g_2(\gamma)\, a = 0$$

$$(1 - a^2)\frac{\gamma}{\ell}\, g_2(\gamma) + 2a\,[g_1(0) + g_3(M - 1)] - 4a\frac{\gamma^2}{\ell^2}\, g_1(0) = 0$$

the first equation then gives

$$\frac{\gamma}{\ell}\, g_2(\gamma^2) = \left[(1 - a^2)\,(g_3(M - 1) + g_1(0)) - 2\frac{\gamma^2}{\ell^2}\, g_1(0)\,(1 - a^2)\right] \Big/ 2a$$

an expression that can be satisfied by either sign of γ simply by changing the sign of ℓ. Using this result in the second equation gives

$$(1 + a^2)^2\, \frac{[g_3(M - 1) + g_1(0)]}{g_1(0)} - \left(2(1 - a^2)^2\frac{\gamma^2}{\ell^2} + 8a^2\gamma^2\right) = 0$$

so that if $g_1(0) > 0$ solutions exist if $g_3(M-1) + g_1(0) > 0$ or

$$\frac{1}{\sum_j kT_j}\left\{\sum_j \frac{c}{e_j}\left[\frac{d\ln n_0(\alpha)}{d\alpha} - \frac{d\ln T_j}{d\alpha}\right](\omega_0^j)^{-1}\right\} > \frac{M}{2}$$

It has been shown that the class of functions $f = \Phi_\ell(1 + ia)$ (Φ_ℓ a real function, a real) can simultaneously extremize the real and imaginary parts of D and for certain values of the plasma pressure gradient the condition D = 0 can be met and is an extremum for the function $f' = \Phi'_\ell\,(1 + ia')$ which extremizes

$$b = \frac{\int d\lambda\, \nu_0^{-1}\, \langle \Phi_\ell^2 \rangle}{\int d\lambda\, \nu_0^{-1}\, \langle \Phi_\ell \rangle^2}$$

where Φ_ℓ satisfies the boundary conditions noted above $\Phi_\ell = d\Phi_\ell/ds = 0$. According to the Schwartz inequality (with Φ_ℓ real, $\Phi_\ell^2 = \Phi_\ell^* \Phi_\ell = |\Phi_\ell^2|$) the minimum value of b occurs for $\langle \Phi_\ell^2 \rangle = \langle \Phi_\ell \rangle^2$, a value achieved if Φ_ℓ is a constant along the field line. However a non-zero constant potential is ruled out by the boundary conditions. So the minimum value of b will be somewhat greater than unity.

The fundamental or fastest growing eigenfunction for each value of ℓ is a symmetric function of s (i.e. centered about the magnetic equator $s = s_E$) and zero at the ionosphereic values of s. The dependence on s of the magnitude of the fundamental eigenfunctions is determined by the requirement b is minimum. Consider a potential function Φ' with scale size L'. The average value of Φ' can be approximated for a given λ by

$$\langle \Phi' \rangle \sim f(\lambda)\, \Phi_T' = \nu_b \frac{L'}{v_\parallel(s_T)} \Phi'(s_T)$$

where s_T is a value of s near the turning point so that $f(\lambda)$ can be interpreted as the fraction of the bounce period spent within a scale size, L', of the turning point. The factor $1/v_\parallel$ in the integrand of the averaging operation causes the potential in the vicinity of the turning point to make the largest contribution to the average value. From the Schwartz inequality $\langle |\Phi_\ell|^2 \rangle \geq |\langle \Phi_\ell \rangle|^2$ it follows $f(\lambda) \leq 1$, for all λ, and then

$$b \sim \int d\lambda\, \nu_0^{-1} f(\lambda)\, \Phi_T^2 \Big/ \left(\int d\lambda\, \nu_0^{-1} f^2(\lambda)\, \Phi_T^2 \right) \tag{16}$$

is smallest when $f(\lambda)$ is unity. $f(\lambda)$ is only unity for particles mirroring at the magnetic equator. b will be minimum for potentials with magnitudes maximum at the equator and falling rapidly to zero since then $f(\lambda)$ will be near unity when the potential is non-zero in the integration in equation 16. So the fastest growing potential eigenfunctions will be concentrated near the magnetic equator falling to zero in a distance on the order of the equilibrium pressure scale height.

Using a dipole field to approximate the geomagnetic field $\omega_0^j = (-3/\alpha)(c/e_j)kT_j$ and assuming the temperatures of the species are equal the condition for the existence of a purely growing instability becomes

$$-\left[\frac{d \ln n_0(\alpha)}{d \ln \alpha} - \frac{d \ln T(\alpha)}{d \ln \alpha}\right] > \frac{3}{2} M \sim \frac{3}{2} \qquad (17)$$

Gradients in the temperature and number density determine the stability of the equilibrium. At the inner edge of the plasma sheet the temperature gradient will be found to be the most important factor in determining the stability of the plasma.

The electric fields associated with a purely growing instability will mostly affect the orbits of low energy particles ($\omega_d^j < \omega_0^j$). The instability derives its energy from particles which are moved about in the geomagnetic field by the perturbation electric fields of the instability. If the net energy of the particles interchanged by the wave fields is less than before the interchange, energy is available for the instability to grow. In the familiar hydromagnetic interchange, instability onsets if a sufficiently large gradient in the number density exists in the same direction as the magnetic field gradient. For the purely growing instability equation 15 indicates the temperature gradient must be opposite the field gradient for instability to onset. This behavior of the temperature gradient arises from the term

$$\int \frac{dK K^{1/2}}{\omega_0 K/kT} \left[\frac{K}{kT} - \frac{3}{2} \right] \frac{d \ln T}{d\alpha}$$

in the evaluation of D at marginal stability. It might be thought that the sign of the gradient in the temperature indicated in equation 15 as leading to instability should actually be stable due to the decrease in the average energy of the particles with decreasing temperature. Actually the temperature gradient leads to instability in a manner quite similar to the familiar hydromagnetic interchange in that the number of interchanged particles (in this case the low energy particles) increases for increasing B. The low energy particles increase due to the change in the normalization factor $1/T^{3/2}$ with α (this change is expressed by the term

$$\int \frac{dK}{K^{1/2}} \left[-\frac{3}{2} \frac{d \ln T}{d\alpha} \right] \exp - \frac{K}{kT}$$

in the evaluation of D). The change in the normalization factor more than compensates for the decrease in the average energy of the particles with decreasing α (the decrease in average particle energy is represented by the term

$$\frac{d \ln T}{d\alpha} \int \frac{dK}{K^{1/2}} \frac{K}{kT} \exp - \frac{K}{kT}$$

in the evaluation of D). As a result the energy change favors the growth of the instability for

$$\frac{d \ln T}{d \ln \alpha} > 0$$

(in a dipole field) and the low energy particles in the plasma supply the free energy for the growth of the instability. It might be expected, then, that non-Maxwellian distributions in the energy of the plasma which have a similar behavior (i.e an increase in low energy particles with decreasing α) may also be subject to the purely growing instability.

The requirements for the onset of the instability can be expressed in terms of the directional differential intensity, I (a quantity commonly used to present satellite data). The directional differential intensity is related to the phase space distribution density, F_0 by $F_0 \propto I/K$. Using this expression in equation 13 leads to a requirement for marginal stability in a dipole field of

$$\sum_j (\omega_0^j)^{-1} \frac{\int dKK^{-3/2}\left[\frac{\partial I}{\partial \alpha} - \omega_d^j \frac{I}{K}\right]}{\int dK\,K^{1/2}\left[\frac{\partial I}{\partial K} - \frac{I}{K^2}\right]} = M \sim 1 \tag{18}$$

The expressions involving I in equation 18 can be approximated by the directional intensity measured for several energy bandpasses as a function of radial distance. Such measurements for electrons at the inner edge of the plasma sheet are available from satellite experiments (see e.g. Schield and Frank, 1970; Frank, 1971).

The growth rate of the instability with F Maxwellian can be found by solving equation 13 for γ. Unfortunately the integration over the energy is not trivial. The growth rate can be estimated by approximating the integrals in equation 13 by, for example,

$$\int \frac{dK\,\omega_0^2 K^2}{\omega_0^2 K^2 + \frac{\gamma^2}{\ell^2}} \exp - \frac{K}{kT} \equiv \mathcal{J}(\gamma) \sim \frac{\mathcal{J}(0)}{1 + \frac{\gamma^2}{\ell^2}} = \frac{1}{1 + \frac{\gamma^2}{\ell^2}} \int dK K^{1/2} \exp - \frac{K}{kT}$$

In equation 13 this approximation leads to a growth rate of

$$\gamma = b^{1/2} \ell [\omega_0 (\omega_x - \omega_0)]^{1/2} \approx \ell \omega_0 = - 3 \frac{\ell c k T}{e \alpha}$$

the last estimate following since the diamagnetic drift frequency is on the order of the azimuthal drift frequency. Note the growth rate does not maximize because particle losses arising from violation of μ, J have not been accounted for. The assumption of μ, J conservation requires that the distance a particle drifts azimuthally in a bounce period is small compared to the perpendicular wave length of the perturbation fields

$$r \omega_0 \nu_b^{-1} << K_\perp^{-1} \approx \frac{L}{\ell}$$

so that $\ell \omega_0 << \nu_b$. But $\gamma \approx \ell \omega_0$ so that even for short azimuthal wave lengths the fastest growth rate to be expected is on the order of 10^{-3} $\sec^{-1}$. The growth rate of the instability in relation to the auroral breakup will be discussed later.

In evaluating the proposed instability as a cause of the auroral breakup it is important to consider the factors which tend to stabilize the plasma. Objections have been raised to ascribing auroral breakup to electrostatic interchange instabilities (e.g. see Hasegawa, 1971) since they are easily stabilized by a small amount of homogeneous cold plasma. If it is assumed that such a plasma is present at the inner edge of the plasma sheet with number density n_c and temperature T_c then the requirement for instability with F Maxwellian becomes

$$\frac{d \ln T}{d \ln \alpha} > \frac{3}{2}\left(1 + \frac{T}{n_0}\frac{n_c}{T_c}\right)$$

If a cold plasma is to stabilize the interchange it should be noted it is necessary that the cold plasma be homogeneous as illustrated by the following example. Suppose the inner edge of the plasma sheet is formed when plasma sheet electrons are precipitated into the atmosphere and replaced by cold ionospheric electrons (somewhat heated during the replacement process) in such a way that the inner edge is a two component electron plasma with Maxwellian velocity distributions and with number densities and temperatures n_c, T_c and n, T where $T_c << T$. The electron number density at the inner edge is constant, $n_c + n =$ constant. The apparent decrease in temperature at the inner edge is now due to an increase in n_c. In this case

$$\frac{dn_c}{d\alpha} = -\frac{dn}{d\alpha}$$

and the stability criteria is

$$\left(\frac{T_c}{T} - 1\right)\frac{d \ln n_c}{d \ln \alpha} > \frac{3}{2}$$

At the inner edge of the plasma sheet

$$\frac{d \ln n_c}{d \ln \alpha} < 0$$

and the two component plasma could be unstable. The addition of a cold electron component to the plasma from the ionosphere might take place in such a way as to lead to a situation in which the added cold plasma eventually becomes the source of free energy for a purely growing instability. The existence of a cold plasma component in the vicinity of the inner edge of the plasma sheet has not been established by direct satellite measurement due to the experimental difficulties involved. In the following a homogeneous cold plasma component is assumed absent at the inner edge of the electron plasma sheet.

An anisoptropic pitch angle distribution also alters the stability criteria. For a fixed temperature gradient a flat pitch angle distribution (i.e. a distribution having more particles with velocities perpendicular to the magnetic field than an isotropic distribution) is more stable than a field aligned pitch angle distribution. The alteration of stability criteria is due to the energy during an interchange of particles at a fixed temperature gradient being larger for average larger J. The effect is small for pitch angle distributions not too far from isotropy and is not considered here.

The effect of an equilibrium electric field with potential $P(\alpha)$ depending only on α can be established from equation 12. Such a field is representative of the corotational field of the earth or convection-like fields in the dawn and dusk sectors. The effect of such a field is to make the equilibrium overstable. That is, the fastest growing mode corresponds to an eigenvalue with the real part of the frequency

$$\omega' = c\,\frac{dP}{d\alpha} = \frac{c}{RB}\,\frac{dP}{dR},$$

where R is the equatorial radial distance corresponding to α. A non-local analysis in α would introduce terms dependent upon $d^2 P/d^2\alpha$ and are not considered here. The physical process leading to the growth of the instability can be learned by investigating the zero equilibrium electric field case.

Stability of the Inner Edge of the Plasma Sheet

The electron plasma sheet is a population of electrons in the magnetosphere of average energy 1-5 keV forming a layer some 4-6 R_e in thickness centered about the neutral sheet (Vasyliunas, 1968; Schield and Frank, 1970). It extends near the earth into the dawn and dusk sectors where its thickness increases. The inner edge of the plasma sheet is in the portion of the magnetosphere where the magnetic field is nearly dipolar (Frank, 1971). The termination of the plasma sheet is identified by a sharp drop in the energy density of the electrons. The drop in the energy density is due to a decrease in the number of higher energy particles not a decrease in the number density. The plasma sheet terminates in the vicinity of 6-10 R_e from the earth and the location of the inner edge depends on Dst. The inner edge moves earthward as the level of magnetic activity increases. The plasma sheet electrons are characterized by their energy spectrum which shows a peak intensity for energies between 1-5 keV. The energy spectrum characteristic of the inner edge of the plasma sheet is similar but has a break in the slope of the higher energy portion of the spectrum. The change in the shape of the plasma sheet energy spectrum at the inner edge is shown in figure 1 (Figs. 4 and 7 of Schield and Frank, 1970). Nearer the earth the number density of particles, both protons and electrons, in the energy range 1-50 keV drops off at what is usually identified as the inner edge of the ring current. Usually the decrease in electron energy density occurs in the region where the number density is changing slowly. Thus the drop in energy density can be thought of as a decrease in the electron temperature. The important contribution to instability can be considered to come from the temperature gradient in the electrons. The scale length associated with the drop in electron temperature is .6-.4 R_e (Frank, 1971; Vasyliunas, 1968) and it occurs between the trapping boundary and the plasmapause. Some examples of the location of the decrease in plasma sheet electron energy density relative to the plasmapause, trapping boundary and proton ring current are shown in figure 2 (Figs. 1 and 5 of Frank, 1971; note also his Figs. 2-4,6).

The directional differential intensity of electrons for various energy bandpasses as functions of radial distance near the equator is shown in figure 3 (figure 2 of Schield and Frank, 1970). Note the increase of low energy particles with decreasing L value (decreasing α) as well as a notable lack of particles with energies below 500 eV compared with a Maxwellian distribution (see figure 4). The use of equation 17 with Maxwellian energy distributions to represent the data of figure 3 as shown in figure 4 yields a value of approximately 10 for the left hand side of equation 17. This method would predict the inner edge was unstable but the Maxwellian approximation is not a good choice due to the absence of low energy particles in the measured spectra. The departure of the distribution from Maxwellian makes it desirable to use equation 18 in evaluating the stability of the inner edge. A numerical calculation of the left hand side of equation 18 using the detailed data shown in figure 3 yields a maximum value of approximately 1 at 6.7 R_e. The data in figure 3 corresponds to figure 2a. and for this particular case the electron temperature gradient is near the inner edge of the ring current and there is probably a stabilizing contribution from the protons. The temperature drop in electrons does not usually occur in the region where the proton number density decreases so figure 2a. is not a "typical" example. The important thing here is to note that a realistic estimate of the stability criteria shows the inner edge is near marginal stability. It is not unreasonable to expect that the increase in low energy particles generally observed at the inner edge of the plasma sheet can be large enough that the purely growing drift instability described above can be expected to grow and eventually result in the relaxation of the temperature gradient.

Precipitation of Particles

The growth of the instability alters the distribution of particles in phase space. A distribution of charge (less than $\epsilon^2 n_0$ at any point along the field line) sets up a parallel electric field. As discussed above the potential associated with the fastest growing mode will be non-zero only in the region of the magnetic equator. The change in the equilibrium phase space distribution density resulting from the growth of the instability depends upon the values of λ, K and s. The equation giving the change in the spatial number density at any point along the field line is

$$n_{\ell}(s) = 2B(s)\sqrt{\frac{m_j}{2}}\iint \frac{d\lambda dK K^{1/2}}{(1-\lambda B(s))^{1/2}} \left\{ \frac{c\langle\Phi_{\ell}\rangle}{\omega_d - i\frac{\gamma}{\ell}} \left.\frac{dF_0}{d\alpha}\right|_{\mu, J} - e(\langle\Phi_{\ell}\rangle - \Phi_{\ell}(s)) \left.\frac{dF_0}{dK}\right|_{\mu, \alpha} \right\} \tag{19}$$

Note that replacing the term contained in braces in the integrand of equation 19 with the equilibrium distribution yields $n_0(\alpha)$ (a constant along the field line). Thus to compare the relative magnitudes of the changes in the equilibrium number density along the field line it is sufficient to compare the magnitude of the terms in braces in equation 19. The largest changes occur in the vicinity of the magnetic equator where $\langle\Phi_{\ell}\rangle$ and Φ_{ℓ} are large. Changes in the velocity space distribution for values of s near the ionosphere are a factor $\langle\Phi\rangle/\Phi$ ($\sim\epsilon$) less than the changes in the equilibrium distribution near the equator. The reason for the small change in the near ionospheric velocity space distribution is that the parallel electric field of the instability grows slowly enough that the first and second adiabatic invariants are conserved. As the instability grows the energy of the particle changes so that its J is conserved. Denoting equilibrium ($\Phi_{\ell} = 0$) quantities with the subscript "0," J at $\Phi_{\ell} \neq 0$ is

$$J = \int v_{\parallel} \, ds = \sqrt{\frac{2}{m}} H^{1/2} \int \left(1 - \frac{\mu}{H} B + \frac{e\Phi_{\ell}}{H}\right)^{1/2} ds \text{ and with } \lambda \sim \frac{\mu}{H}$$

$$\approx H_0 \left(1 + \frac{\Delta H}{H_0}\right)^{1/2} \left[\int (1 - \lambda B)^{1/2} ds + \frac{e}{2H_0} \int \frac{\Phi_{\ell}}{(1 - \lambda B)^{1/2}} ds\right]$$

$$\approx J_0 + \frac{1}{2} \frac{\Delta H}{H_0} J_0 + \frac{2}{m} \frac{e}{2} \langle \Phi_{\ell} \rangle \nu_b^{-1}$$

Since J does not change it is required that

$$\Delta H \approx \frac{2}{m} H_0 \frac{\nu_b^{-1}}{J_0} e \langle \Phi_{\ell} \rangle \approx e \langle \Phi_{\ell} \rangle$$

The change in the pitch angle, $\theta \equiv \tan^{-1} (V_{\perp} / V_{\parallel})$, of a particle mirroring near the ionosphere with energy kT will then be

$$\left[\frac{e \langle \Phi_{\ell} \rangle}{kT}\right]^{1/2}.$$

Thus even if the change in the potential near the equator is on the order of kT few particles are dumped into the loss cone, since the pitch angle of particles mirroring at the ionosphere changes from $\pi/2$ to $\pi/2 + \epsilon^{1/2} [(\Phi_E)/k]^{1/2}$. Actually the pitch angle distribution of electrons and protons in the region along the field line where the parallel electric field is zero must be the same. If it were not the pitch angle distribution would then cause a non-zero electric field along the field line in that region. So any particles dumped into the loss cone consist of equal numbers of protons and electrons. The number of particles dumped, however, is small. The instability can be considered to grow, setting up a parallel electric field near the magnetic equator, without precipitating particles.

An upper limit to the magnitude of the parallel electric field associated with the instability can be found from the ordering requirement $E_{\|} \sim \epsilon\,(vB/c)$. This requirement assures that the change in the parallel kinetic energy due to the electric field applied during one gyroperiod is of order ϵkT. An electric field consistent with this ordering could cause a change in the parallel kinetic energy of a particle

$$\Delta W_{\|} = e\Delta\Phi \approx eE_{\|} L \approx \epsilon \frac{evB}{c} L \approx e\Omega \frac{L}{v} mv^2 \approx \epsilon\Omega \frac{L}{v} kT$$

but $L/v \sim \nu_b^{-1}$ and $\epsilon\Omega \sim \nu_b$ so that $\Delta W_{\|} \sim kT$. The electric field of the instability can be large enough to change the parallel kinetic energy of particles a substantial amount as they move along the field line in the region of the magnetic equator. However, as discussed above, a potential change even of order kT does not cause rapid precipitation of particles.

Although the instability does not immediately precipitate particles the interaction of electrons with whistler-mode noise can be expected to diffuse electrons into the loss cone (Kennel and Petschek, 1966). The growth of the purely growing drift instability modifies the equilibrium in such a way that whistler-mode noise should be generated. The velocity space distribution will be unstable to waves of frequency ω' and wave number k' if the anisotropy in the electron distribution, $A^- > \omega'/(\Omega_E - \omega')$ (Kennel and Petschek, 1966, where

$$A^- = \left. \frac{\displaystyle\int_0^\infty v_\perp dv_\perp \tan\theta \frac{\partial F^-}{\partial \theta}}{\displaystyle 2\int_0^\infty v_\perp dv_\perp F^-} \right|_{v_\parallel = \omega' - \Omega_e / k'}$$

F^- is the electron velocity space distribution and θ is the pitch angle of the particle ($\theta = \tan^{-1} (v_\perp / v_\parallel)$). The change in the equilibrium equatorial number density is given approximately by (equation 19 with $\gamma^2/\ell^2 << \omega_d^2$)

$$n_{e\ell} = \int \frac{d\lambda}{(1 - \lambda B_E)^{1/2}} \int K^{1/2} dK \left[\frac{e\langle\Phi\rangle}{2K} \frac{d \ln T}{d \ln \alpha} - \frac{e}{kT} \left(\frac{\langle\Phi\rangle}{3} \frac{d \ln T}{d \ln \alpha} + \Phi \right) \right] \qquad (20)$$

The electron velocity space distribution is related to the term in brackets in equation 20. The distribution in velocity space after the instability has started to grow will depend on λ (since $\langle\Phi\rangle$ depends on λ) and thus θ since $B_E \lambda = \sin^2\theta$. Since the effect of the growth of the interchange is to relax the temperature gradient and decrease the number of low energy particles when

$$\frac{d \ln T}{d \ln \alpha} > 0,$$

choose $n_{e\ell} > 0$ for low energies ($K \lesssim kT$). For this to be the case it is required that $\langle\Phi\rangle > 0$. The magnitude of $\langle\Phi\rangle$ for the fastest growing mode decreases monotonically from large λ (particles mirroring near the equator) to small λ (particles mirroring near the ionosphere). Since $\langle\Phi\rangle > 0$, $\langle\Phi\rangle$ is monotonically decreasing as λ decreases (and thus θ decreases) so that

$$\frac{d\langle\Phi\rangle}{d\theta} > 0$$

Thus for parallel energies greater than 3/2 kT, $\partial F/\partial\theta$ is positive for all K and $A^- > 0$. For high enough resonant energies, E_R, the condition for instability becomes approximately, $A^- > 0$, and so some whistler noise should be generated at frequencies

$$\omega' = \frac{E_c}{E_{\|}}\,\Omega_e\left(1 - \frac{\omega'}{\Omega_e}\right)^3 \quad (E_{\|} > E_R)$$

where E_R is large enough that $A^- > \omega'/(\omega' - \Omega_e)$ where the frequency satisfies the above expression and $E_c = B^2/8\pi n$ is the magnetic energy per particle. A^-, for a given $E_{\|}$, depends on the magnitude of the potential associated with the parallel electric field of the drift instability. The larger the potential the wider the range of parallel energies which satisfy the criteria for instability and the larger the growth rate. Thus whistler noise should be generated over a wider band width and interact with more particles as Φ increases. For the high energy portion of the measured plasma sheet spectrum the Maxwellian is a good approximation and the above arguments can be expected to hold.

If whistler noise is present in the vicinity of the equator it can be expected to diffuse some electrons into the loss cone. If a parallel electric field is present the loss of these electrons will change the potential in the vicinity of the ionosphere. The reaction of ionospheric electrons to the changing potential near the ionosphere will be important to continued maintenance of an electric field along the field line. The loss of some electrons to the atmosphere will cause a temporary excess of positive charge along the field line near the ionosphere. This charge will be neutralized by the highly mobile ionospheric electrons thus maintaining zero electric field in the vicinity of the ionosphere. It will be assumed here that as long as ionospheric electrons maintain their mobility a small amount of plasma sheet electrons can be lost by pitch angle diffusion without altering the condition of zero electric field near the ionosphere. As long as this particle loss is small the purely growing drift instability should continue to grow with the same potential shape, i.e. non-zero only near the magnetic equator, described above.

If the diffusion of particles into the loss cone is large and the instability has grown to a point where the parallel electric field is large, rapid precipitation of electrons may occur. If the number of electrons diffused into the loss cone by whistler noise is large enough that the current of neutralizing ionospheric electrons exceeds a certain limit, a two stream instability will onset in the ionosphere (Swift, 1965; Kindel and Kennel, (1971). The dominant modes growing as a result of the current limit being exceeded depends upon ionospheric composition and the magnitude of the current as has been shown by Kindel and Kennel (1971). When the current is exceeded and instability onsets the resulting wave turbulence along the field line greatly impedes the ionospheric electron mobility. Under these conditions the ionospheric electrons cannot completely neutralize the excess positive charge near the ionosphere which results when plasma sheet electrons are precipitated into the atmosphere. As a result electrons coming down the field line will "see" more positive charge near the ionosphere than would exist if no electrons had been precipitated. As a result their parallel velocity increases and some are dumped into the loss cone. Since electrons move much faster than ions along the field line the ions can be considered fixed during this process. Dumping of electrons into the loss cone sustains the current and wave turbulence as well as increasing the excess of positive charge in the vicinity of the ionosphere which in turn leads to more electron precipitation and so forth. A large number of electrons will be rapidly precipitated until the ions redistribute themselves along the field line. This redistribution can be expected to take at least a few ion bounce periods (on the order of 5 minutes). The rapid precipitation which occurs if the critical current limit is exceeded when the parallel electric field of the purely growing drift instability exists along the field line is proposed here to be the auroral breakup.

Discussion

The auroral substorm is a part of the magnetospheric substorm which occurs in the polar ionosphere during magnetic storms and lasts for 1-3 hours (Akasofu, 1968). The magnetic perturbation associated with the auroral substorm and the latitudinal and longitudinal extent of the auroral substorm can vary greatly. The magnetospheric substorm has been proposed to consist of three phases: a growth phase (McPherron, 1970); an expansion phase; and a recovery phase (Akasofu, 1964). The growth phase has been cited as a period of enhanced magnetospheric convection during which (among other phenomena) the inner edge of the plasma sheet moves earthward, the plasma sheet thins, and a current system similar to the 3-dimensional electrojet current system (Akasofu and Meng, 1969; Bonnevier, et. al., 1970; Kisabeth 1972) is established (McPherron, 1972). The evidence presented for the existence of a growth phase has been disputed by Akasofu and Snyder (1972). The expansion phase marks the beginning of the auroral substorm as described by Akasofu (1964). Three main events are assigned to the expansion phase: (1) the sudden brightening near midnight at or near a pre-existing auroral arc (duration 0-5 minutes); (2) the auroral arcs become distorted and move poleward (5-10 mins.); (3) the westward traveling surge and auroral bulge is formed with the northern-most arc the most intense (10-30 mins.). The recovery phase begins when the arcs reach their most poleward position and start to recede equatorward and is marked by pulsating aurora and eastward drifting patches (1-2 hours). The rapid precipitation of electrons discussed in this paper deals with event 1 of the expansion phase. The growth of the purely growing drift instability would be an essential part of the proposed growth phase of the magnetospheric substorm.

Some observational evidence supporting the auroral breakup mechanism proposed here is available. First, as noted above, a temperature gradient large enough to cause the growth of the purely growing electrostatic drift instability is likely to exist at times at the inner edge of the electron plasma sheet. The inner edge of the plasma sheet does respond to magnetic activity in a manner similar to the behavior of the auroral oval and the position of the inner edge is certainly closely related to the auroral zone. The location of the breakup of a small isolated substorm has been placed at the inner edge of the plasma sheet by Johnstone, et. al. (1973). The existence of an electric field along the magnetic field lines has been cited by some investigators as consistent with observations of precipitated electron fluxes measured in breakup aurora (Johnstone, et. al., 1973; Whalen and McDiarmid, 1972; Pongratz, 1972). However, the precipitated fluxes expected from the existence of a parallel electric field requires a detailed analysis and greatly depends on the shape and location of the electric potential along the field line and the manner in which the field is established. The precipitation of particles by suddenly turning on a parallel electric field has been investigated analytically by Chamberlain (1969). Although the mathematical device of suddenly turning on a parallel electric field does not exactly correspond to the mechanism discussed here it is somewhat similar. A numerical investigation of the precipitated fluxes resulting from suddenly turning on a parallel electric field (after Chamberlain) has been performed by Matthews and Pongratz (1973). They use an initial energy spectrum similar to the plasma sheet spectrum and a parallel electric field confined to the region near the magnetic equator. They are able to reproduce the precipitated fluxes measured using a sounding rocket in a breakup aurora. It might be expected that the mechanism proposed here should lead to similar precipitated spectra, but a detailed numerical calculation is necessary to support this assertion.

The contention that the current limit is exceeded in the intense breakup arc may also have some observational support. Observations of precipitated particle fluxes in a moderate substorm after the onset of breakup have been reported by Whalen and McDiarmid (1972). They report that the field-aligned current in the northern-most arc (the breakup arc after the onset of poleward expansion) exceeds the critical current density given by Swift (1965) which is higher than the critical limit given by Kindel and Kennel (1971). They further observe that there is no correlation between proton and electron precipitation in the northern-most arc, a result which might be expected from the mechanism proposed here. It is also interesting to note that Johnstone, et. al. (1973) claim the precipitated electron fluxes observed spatially outside of the breakup arc are apparently due to "relatively strong" pitch angle diffusion. As pointed out above the growth of the drift instability should lead to enhanced pitch angle diffusion.

It is appropriate to review some of the approximations used to analyze the growth of the electrostatic drift instability. The electric potential associated with the instability has been assumed to vary azimuthally as $\exp i \ell \beta$ and to be slowly varying in α. The inner edge of the plasma sheet is known not to be azimuthally symmetric (Frank, 1971). The fastest growing mode, however, does have a large ℓ value and thus a short azimuthal wavelength. If the scale size of the azimuthal variations is much larger than the wavelength of the fastest growing mode then the assumption of azimuthal symmetry may be appropriate. Since the temperature gradient at the inner edge of the plasma sheet occurs over a narrow region (1-3 R_e) variations in the potential with α will probably be important. The solution for the electric potential in the curved field line geometry is a formidable problem and would only merit consideration if the breakup mechanism proposed here appears to be correct. The approximation of electrostatic perturbation fields is not accurate near the magnetic equatorial plane. Before the growth of the drift instability a diamagnetic current exists at the inner edge of the plasma sheet due to the temperature gradient. The growth of the instability diminishes the diamagnetic current as exp-γt, which implies the existence of a magnetic field B' with an associated electric field

$$E' \left(\vec{\nabla} \times E' = -\frac{1}{c} \frac{\partial B'}{\partial t} \right)$$

which the electrostatic approximation ignores. The magnitude of B' is related to the diamagnetic current,

$$\vec{\nabla} \times B' = 4\pi \frac{B \times \vec{\nabla} p}{B^2}$$

and thus $B'/B = 4\pi p_e / B^2 \equiv \beta_e^*$ where p_e is the electron pressure so that

$$\frac{cE'}{B} \approx \frac{\gamma}{L^{-1}} \frac{B'}{B} = \frac{\gamma}{L^{-1}} \beta_e^* \approx \frac{\gamma L}{v} \beta_e^* v \approx \frac{\gamma}{\nu_b} \beta_e^* v \sim \epsilon \beta_e^* v$$

but $cE/B \sim \epsilon\nu$ so $E' \sim \beta_e^* E$. Since β_e^*, the ratio of electron pressure to magnetic pressure is on the order of unity only near the magnetic equator, the magnetic field associated with the relaxation of the diamagnetic current will be important only near the equator. The growth of the electrostatic drift instability also has a large effect on the motion of particles mirroring near the equator. A thorough analysis of the purely growing drift should take into account the effects of the magnetic field associated with the relaxation of the diamagnetic current on particle motions. Hagege, et. al. (1973) have found in studying the "ion trapped mode" at higher frequencies ($\ell \omega_0 \sim \epsilon \omega$) that finite β_e^* effects are stabilizing. However, β_e^* appears in integrals over the field line which tends to mitigate the problem since β_e^* decreases rapidly away from the magnetic equator.

An important question concerns the ordering used to analyze the Vlasov equation. The ordering used here assumes a scale length of the same order of magnitude for the perpendicular electric field, the parallel electric field, and the temperature gradient. Frank (1971) asserts the pressure scale height is .4 R_e. The fastest growing mode has a short azimuthal wavelength (~ 1 R_e) and the parallel electric field will have as small a scale height as the ordering allows. Thus the ordering of scale heights used appears to be appropriate. The finite Lamor radius ordering should be used if the parallel electric field has a scale height much longer than the perpendicular scale heights. An investigation of the stability of the temperature gradient at the inner edge of the plasma sheet using the finite Lamor radius ordering has been carried out by Coroniti and Kennel (1970, see Chance et. al. (1973) for finite β_e^* corrections) in a straight field line geometry. They find an electromagnetic drift instability with a frequency on the order of 10^{-1} sec^{-1} and a growth rate on the order of 10^{-2} sec^{-1} having perpendicular wavelengths on the order of the ion gyroradius. This faster growing electromagnetic instability would, of course, relax the temperature gradient at the inner edge before the purely growing electrostatic drift instability could grow. The electromagnetic drift instability may not occur in the midnight-evening sector of the magnetosphere, however. In fact, Coroniti and Kennel suggest the instability exists only in the post midnight sector (0200-1000 LT). The electromagnetic instability can be expected to grow when the temperature scale height is .4 R_e only if the electron temperature is about 4 times the ion temperature. Satellite measurements (Frank, 1967), at least for the evening sector, indicate proton temperatures of about 10-20 keV for the ring current and electron temperatures near 1-5 keV (Vasyliunas, 1968; Schield and Frank, 1970). If the ion to electron temperature ratio is on the order of four to one, the scale height of the temperature gradient would have to be about 10^{-2} R_e for the electromagnetic instability to grow. If the ion temperature is larger than the electron temperature and the scale height of the temperature gradient is near .5 R_e the electromagnetic drift instability investigated by Coroniti and Kennel cannot be expected to grow whereas the purely growing electrostatic drift instability can grow.

The growth rate of the purely growing drift instability has been estimated as $\gamma \sim \ell\omega_0$, where ℓ is small enough that $\ell\omega_0 \gg \nu_b$. Since magnetospheric ions have a higher temperature than electrons ℓ can be on the order of 10^2 without violating the second adiabatic invariant, J, of the ions and $\gamma \sim 10^{-3}$ sec^{-1}. The loss of electrons to the atmosphere modifies the growth rate but this effect is ignored here. Kennel (1969) estimates the electron life-time is less than 10^3 seconds at the inner edge of the plasma sheet during strong diffusion. Strong diffusion must be assumed absent if the purely growing drift instability is to grow although as has been previously noted the growth of the instability should subsequently lead to strong diffusion. The drift instability can have a growth rate on the order of (10 minutes)$^{-1}$. This growth rate may be fast enough for the purely growing drift instability to near saturation during the growth phase of the magnetospheric substorm. McPherron (1972) gives the duration of the growth phase as about 1 hour. If the parallel electric field of the instability at saturation, $E_{\parallel S} = \Phi_S/L$, is estimated by

$$m \frac{d}{dt} v_{\parallel} = 0 = eE_{\parallel S} - \mu \frac{dB}{ds}; \qquad e\Phi_S \sim \frac{kTL}{L_B} \sim kT$$

the potential amplitude along the field line near the equator before breakup may be on the order of kT/e, a few kilovolts.

In summary, the mechanism for auroral breakup proposed here depends on a sequence of related events. The temperature gradient which exists at the inner edge of the electron plasma sheet is the source of energy for a purely growing drift instability (the pressure gradient appears to be strong enough to meet the requirements for instability). Initially, precipitation of electrons by whistler noise in the vicinity of the inner edge is assumed weak. The drift instability grows (sometime during the growth phase of the magnetospheric substorm) with little particle precipitation, alters the equatorial velocity space distribution creating a parallel electric field which is concentrated near the magnetic equator. The alteration of the equatorial velocity space distribution is an anisotropy in pitch angle which leads to generation of whistler noise and pitch angle diffusion of electrons. The growth rates and band width of the whistler noise increases as the parallel electric field of the drift instability gets stronger. The drift instability continues to grow until the electron loss by whistler generated pitch angle diffusion is large. If the current in the ionosphere due to electron loss is strong enough, the current limit is exceeded, and turbulent resistivity set up in the ionosphere leading to rapid and intense electron precipitation. This rapid electron precipitation when the current limit is exceeded is proposed to be the auroral breakup. If the current limit is not exceeded the parallel electric field is relaxed by strong pitch angle diffusion, and no breakup occurs.

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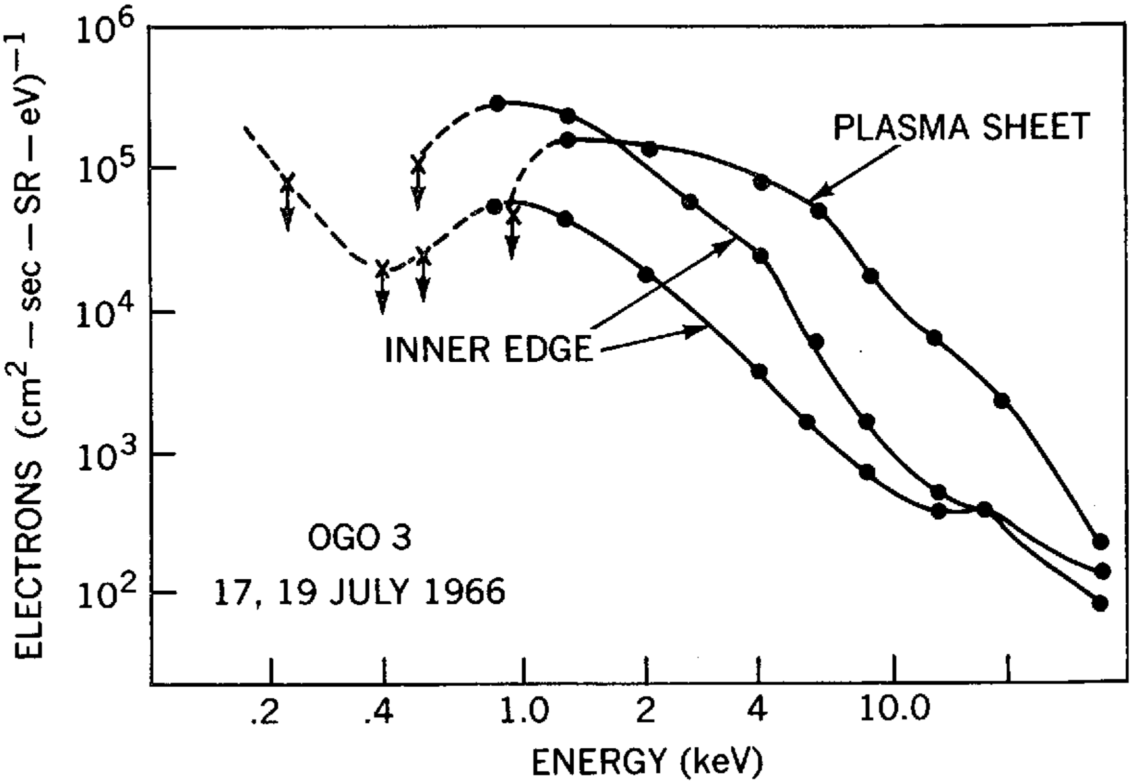
Figure Captions

Figure 1. Directional differential energy spectrums of electron intensities measured by OGO 3 in the midnight sector of the magnetosphere near the equatorial plane (Schield and Frank (1970) figures 4 and 7). The figures show the changes in the spectrum observed on satellite passes across the inner edge of the plasma sheet.

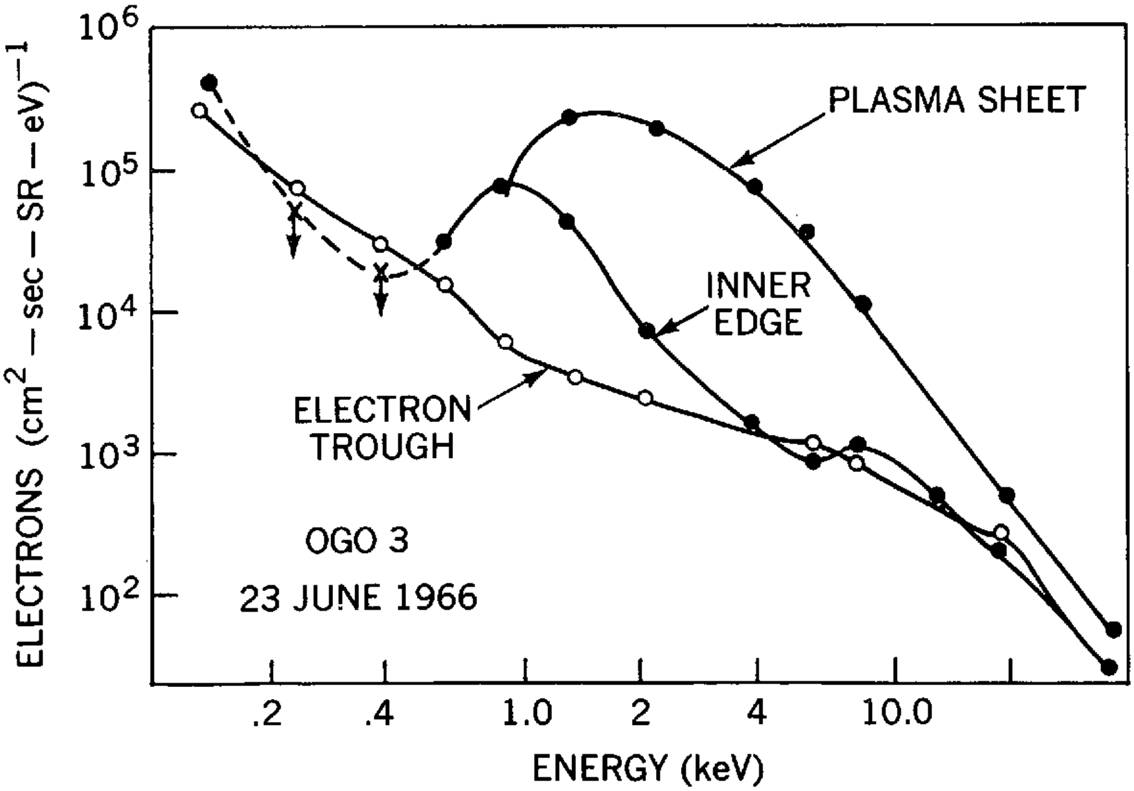
Figure 2. Energy densities of both protons and electrons for the energy range 1-50 keV as measured by OGO 3 (Frank (1971) figures 1 and 5), showing the location of the trapping boundary and plasmapause. Figure 2a corresponds to the detailed data shown in figure 3.

Figure 3. Directional differential electron intensities for various energy bandpasses as functions of L value. The center energies of the bandpasses are noted in the figure. Note the increase in low energy particles for decreasing L value as the inner edge of the plasma sheet is crossed. The data for this figure is taken from Shield and Frank (1970) figure 2.

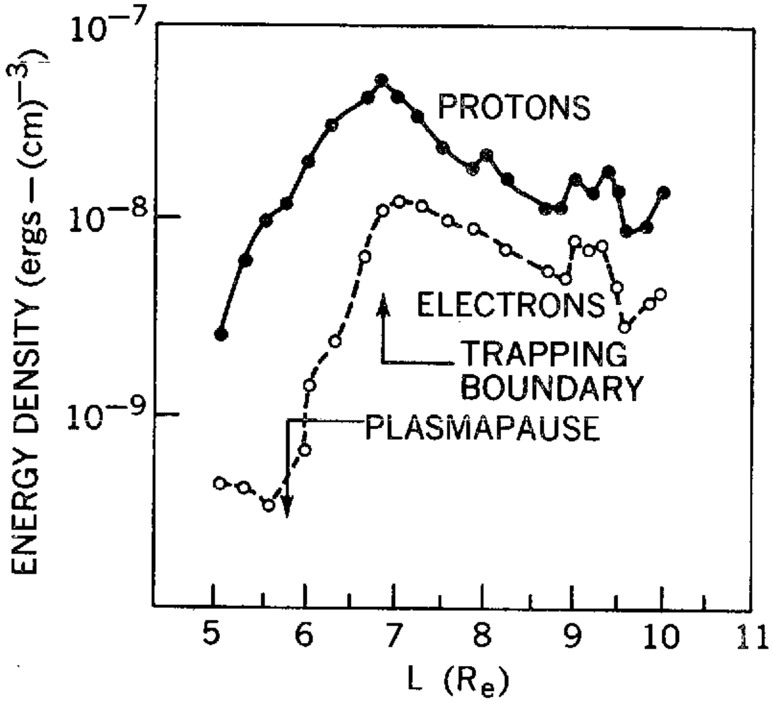
Figure 4. Directional differential energy spectrums of electron intensities corresponding to figure 3 at 6.6 and 6.8 R_e. The two curves are the intensities for a Maxwellian energy distribution at the temperatures shown (1.0 and 1.55 keV) for a constant number density. Note the lack of low energy particles for the measured plasma sheet intensities compared to the intensity for the corresponding Maxwellian.



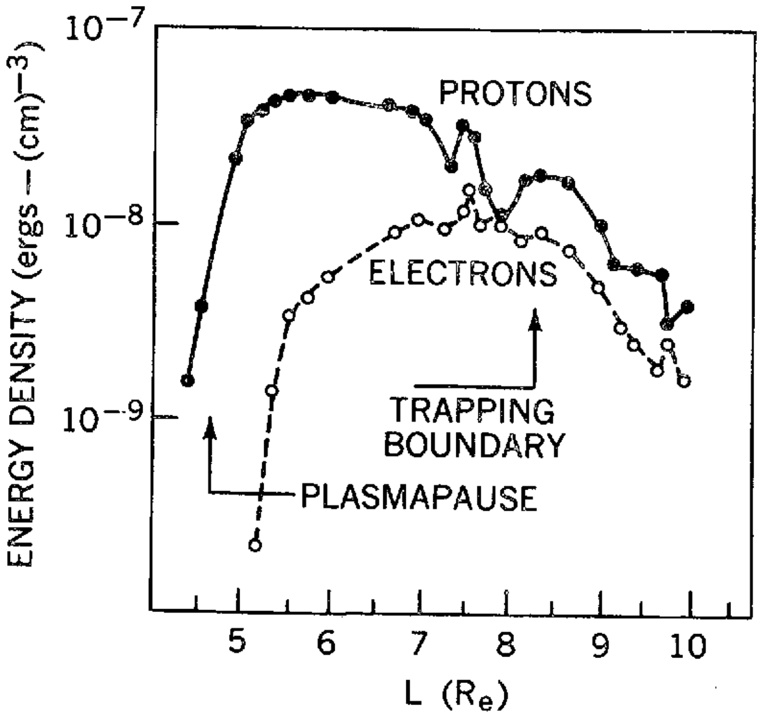
ELECTRONS (cm2 — sec — SR — eV)−1
10^6
10^5
10^4
10^3
10^2
PLASMA SHEET
INNER EDGE
OGO 3
17, 19 JULY 1966
.2
.4
1.0
2
4
10.0
ENERGY (keV)



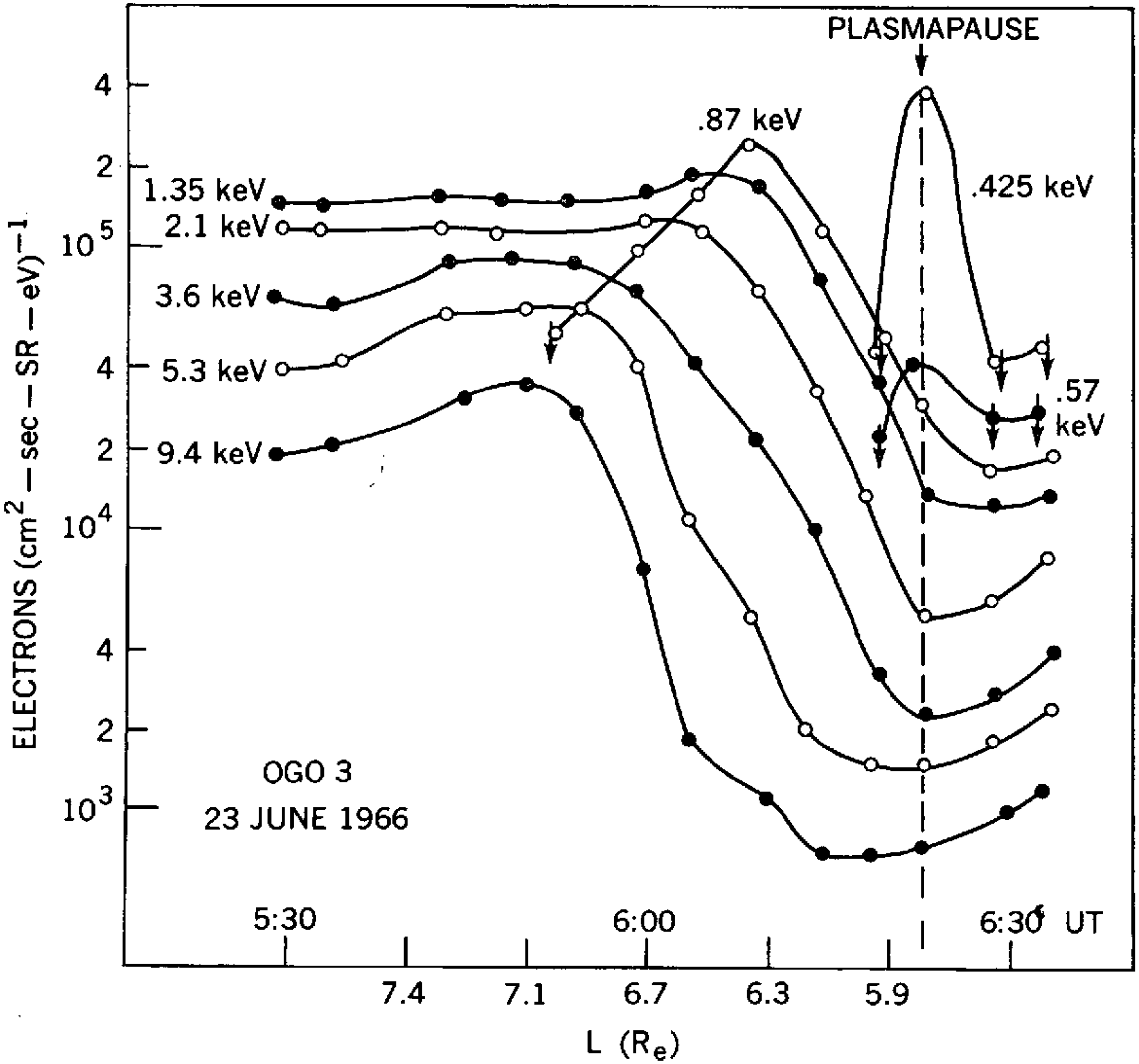
ELECTRONS (cm2 — sec — SR — eV)−1
10^6
10^5
10^4
10^3
10^2
PLASMA SHEET
INNER EDGE
ELECTRON TROUGH
OGO 3
23 JUNE 1966
.2
.4
1.0
2
4
10.0
ENERGY (keV)



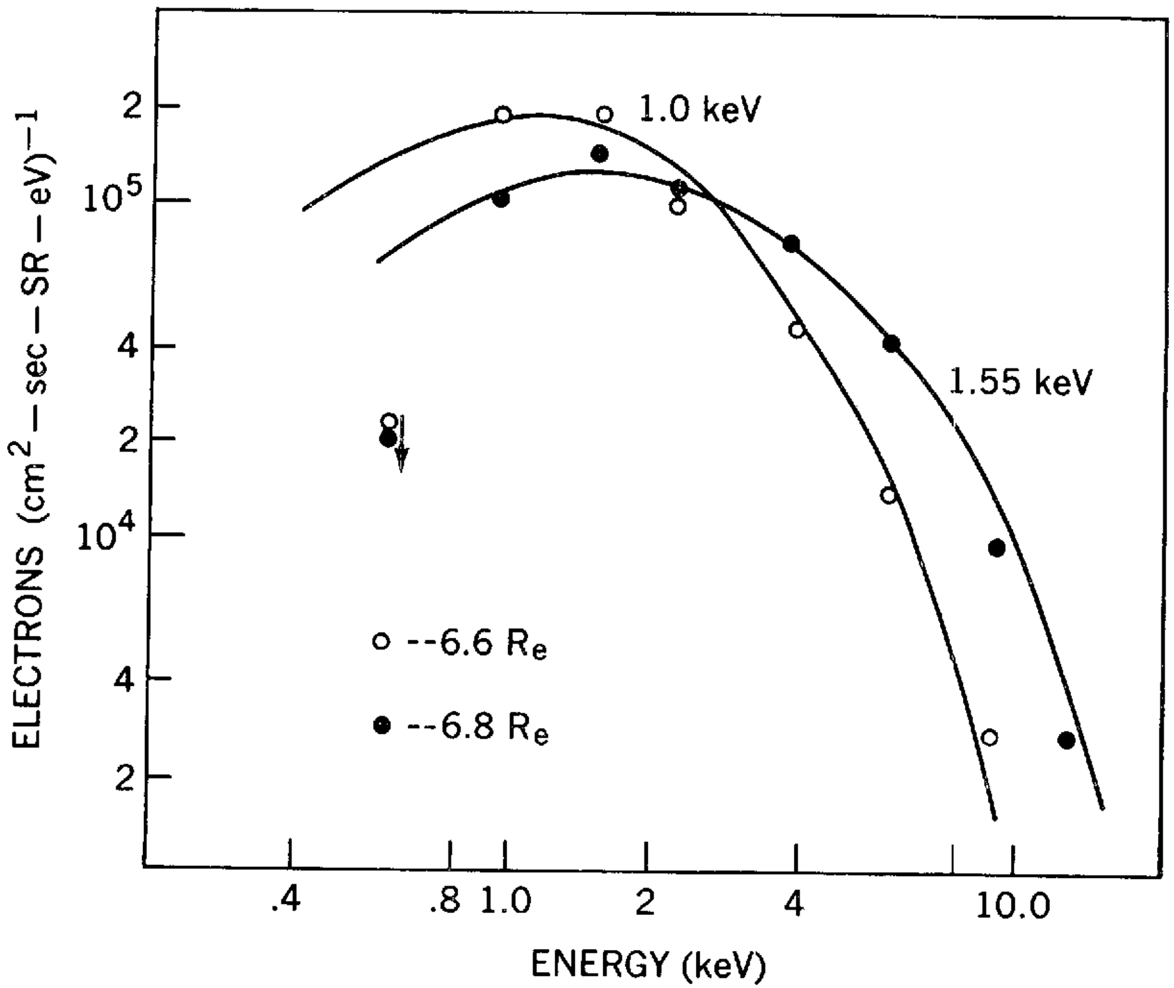
10^{-7}
10^{-8}
10^{-9}
ENERGY DENSITY (ergs — (cm)$^{-3}$)
PROTONS
ELECTRONS
TRAPPING BOUNDARY
PLASMAPAUSE
5
6
7
8
9
10
11
L (R_e)



10^{-7}
10^{-8}
10^{-9}
ENERGY DENSITY (ergs — (cm)$^{-3}$)
PROTONS
ELECTRONS
TRAPPING BOUNDARY
PLASMAPAUSE
5
6
7
8
9
10
L (R_e)



PLASMAPAUSE
.87 keV
.425 keV
1.35 keV
2.1 keV
3.6 keV
5.3 keV
9.4 keV
.57 keV
4
2
10^5
4
2
10^4
4
2
10^3
ELECTRONS $(cm^2 - sec - SR - eV)^{-1}$
OGO 3
23 JUNE 1966
5:30
6:00
6:30 UT
7.4
7.1
6.7
6.3
5.9
L (R_e)


ELECTRONS (cm^2 — sec — SR — eV)$^{-1}$
2
10^5
4
2
10^4
4
2
1.0 keV
1.55 keV
○ --6.6 R_e
● --6.8 R_e
.4
.8
1.0
2
4
10.0
ENERGY (keV)