CORRELATION OF HEAT TRANSFER FOR THE ZERO PRESSURE GRADIENT HYPERSONIC LAMINAR BOUNDARY LAYER FOR SEVERAL GASES

A TECHNICAL REPORT

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REPORT OF RESEARCH ACTIVITIES - SUMMER, 1973

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Introduction

This report covers the author's research activities at the NASA Langley Research Center during the ten-week period June 11 to August 17, 1973. This work was supported under NASA Contract No. NAS1-11707-21, and was carried out in the Hypervelocity Impulse Facilities Section of the Advanced Entry Analysis Branch, Space Systems Division. Mr. Jim Jones served as immediate supervisor. The effort involved work in the three areas described briefly below.

Correlation of Aerodynamic Heating Rates

This work involved a theoretical study of heat transfer for zero pressure gradient hypersonic laminar boundary layers for various gases with particular application to the flows produced in Langley expansion tube facility. A simple correlation based on results obtained from solutions to the governing equations for five gases was formulated. See the attached first draft of a paper entitled, "Correlation of Heat Transfer for the Zero Pressure Gradient Hypersonic Laminar Boundary Layer for Several Gases," which will be submitted to the AIAA Journal.

Heat Transfer Measurement in Short-Duration Flows

The author served as a consultant on data reduction techniques in relation to the measurement of heat transfer rates in the expansion tube facility. Techniques in use by the Langley scientists which were previously developed by the author for reduction of data obtained from thin-film resistance thermometer heat-flux gages were extended for use in conjunction with thin-film thermocouple gages.

Shock Tube Boundary Layer Analysis

The author served as a consultant on the subject of high temperature laminar boundary layer flows in shock tubes. Particular attention was directed toward the laminar boundary layer on shock tube splitter plates in carbon dioxide flows generated by high speed shock waves. Computer analysis of the splitter plate boundary layer flow provided information that is useful in interpreting experimental data obtained in shock tube gas radiation studies.

FIRST DRAFT

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NOMENCLATURE

C	Chapman-Rubesin coefficinet, $\rho\mu/\rho_e\mu_e$
h	Static enthalpy, J/kg
h_r	Recovery enthalpy, $h_e + ru_e^2/2$
H	Dimensionless enthalpy, $(h - h_e)/h_e$
k	Thermal conductivity, W/(m K)
p	Pressure, atm
q	Heat transfer rate per unit area, (heat flux), W/m^2
r	Recovery factor
Re	Reynolds number, $xu_e\rho_e/\mu_e$
M	Mach number
T	Temperature, K
s	Exponent: cone, $s = 1$; wedge, plate, $s = 0$
St	Stanton number, $-q_w/\rho_e u_e(h_r - h_w)$
u	x component of velocity, m/s
v	y component of velocity, m/s
x	Space coordinate parallel to wall, m
y	Space coordinate perpendicular to wall, m
β	u/u_e

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γ Specific heat ratio, c_p/c_v

τ Dimensionless shear stress, $\mu(\partial u/\partial y)/\rho_e u_e^2$

μ Viscosity coefficient, kg/(m s)

ρ Density, kg/m^3

ϕ Shear stress variable, Eq. (3)

σ Effective Prandtl number

Subscripts

e Boundary layer edge

w Wall

INTRODUCTION

Current research in the area of convective heating in high speed flows includes studies using gases and gas mixtures to simulate aerodynamic heating in planetary atmospheres. Test facilities such as the shock tunnel and the expansion tube are useful for generating the required high-energy flows for investigating aerodynamic heating over a range of high Mach numbers. Simple configurations such as the sharp flat plate, the wedge, and the sharp cone that avoid boundary layer pressure gradients are frequently used as basic body shapes in such studies. This note is concerned with correlation of hypersonic aerodynamic heating parameters for these body shapes for several gases under test conditions encountered in these facilities and for conditions under which the boundary layer is laminar.

Testing in the above-mentioned facilities characteristically involves models with cold surfaces (relative to the recovery temperature) since, due to short testing times involved, little change from the initial room temperature of the model surface occurs. Hence, considerations here will be directed mainly toward flows with the cold wall condition. Further, only the boundary layer flow downstream of the region of interaction of any leading edge shock wave and the boundary layer that occurs near the leading edge will be treated. To a good approximation the self-similar laminar boundary layer equations for flows with zero pressure gradient are then applicable.

In order to assess the influence of the characteristics of various real gases, hypersonic heating rates in air, argon, carbon dioxide, helium, and hydrogen have been theoretically studied. These gases as ideal gases range in molecular structure from monatomic to triatomic, the molecular weights range from 2 to 44 and values of γ vary from 1.28 to 1.67. In high Mach number flows real gas effects can play an important role since, because of pronounced dissipation effects, the temperature in the boundary layer can reach large values. Real gas effects in hypersonic flows may be present to varying degrees depending on the gas considered, e.g., carbon dioxide dissociates at relatively low temperatures while helium exhibits ideal gas behavior over a wide range of temperature. In view of these facts and the range of gas characteristics considered, there is no assurance that a precise universal correlation for heat transfer exists for a wide range of flow conditions. However, insight for the present problem can be gained by examining the boundary layer equations and solutions thereto for various gases.

ANALYSIS

The method employed here to solve the governing boundary layer equations is that of Crocco (e.g., see Ref. 1), in which the boundary layer equations in terms of x and y as the independent variables are transformed to equations that have x and u as the independent variables. For real gas flows in thermochemical equilibrium, the laminar boundary layer equations with $\beta = u/u_e$ and x as the independent variables and $\tau = \mu(\partial u/\partial y)/\rho_e u_e^2$ and $H = (h - h_e)/h_e$ as dependent variables are:

Momentum,

$$\tau_{\beta\beta} + \beta \left[\frac{\rho\mu}{\rho_e^2 u_e \tau}\right]_x + \left[\frac{\rho\mu\beta}{x\rho_e^2 u_e \tau}\right]^s = 0 \qquad (1)$$

Energy,

$$\tau_\beta H_\beta = \frac{u_e^2}{h_e}\tau + \left[\frac{\tau}{\sigma} H_\beta\right]_\beta - \frac{\rho\mu\beta}{\rho_e^2 u_e \tau} H_x \qquad (2)$$

where $s = 1$ for the case of the cone and $s = 0$ for the plate and the wedge. In Eq. (2) $\sigma = \sigma(h,p)$ is the effective Prandtl number that serves to incorporate real gas effects with regard to energy transport. Continuity is implicitly satisfied in Eqs. (1) and (2). The boundary conditions for these equations are,

$$\partial\tau/\partial\beta = 0, \quad H = H_w, \quad \text{at } \beta = 0$$

$$\tau = 0, \quad H = 0, \quad \text{at } \beta = 1$$

The boundary condition on τ at $\beta = 0$ results from $u = v = 0$ at the wall. By employing a separation of variables solution of the form $\tau(x,\beta) = \emptyset(\beta)/X(x)$, the shear stress becomes,

$$\tau(x,\beta) = \emptyset(\beta)/(Re)^{1/2} \tag{3}$$

After Crocco, the term H_x in Eq. (2) is taken as zero. Substituting Eq. (3) into Eqs. (1) and (2),

$$\emptyset\emptyset_{\beta\beta} + 3^s C\beta/2 = 0 \tag{4}$$

$$\emptyset\emptyset_\beta H_\beta = (u_e^2/h_e)\emptyset^2 + \emptyset\left(\frac{\emptyset}{\sigma} H_\beta\right)_\beta \tag{5}$$

where $C = \rho\mu/\rho_e\mu_e$. The boundary conditions become

$$\emptyset_\beta = 0, \quad H = H_w, \quad \text{at } \beta = 0$$

$$\emptyset = 0, \quad H = 0, \quad \text{at } \beta = 1$$

In integrated form Eqs. (4) and (5) can be written as,

$$\phi(\beta) = \sqrt{3^s}\left[\int_\beta^1 \phi(P)\int_0^P \frac{C(\lambda)}{\phi(\lambda)}\,\lambda d\lambda\right]^{1/2} \tag{6}$$

and

$$H(\beta) = -H_\beta(0) \frac{\Phi(0)}{\sigma(0)} \int_\beta^1 \frac{\sigma(P)}{\Phi(P)} dP$$

$$+ \frac{U^2e}{he} \int_\beta^1 \frac{\sigma(P)}{\Phi(P)} \int_0^P \Phi(\lambda) d\lambda dP \tag{7}$$

$$+ \int_\beta^1 \frac{\sigma(P)}{\Phi(P)} \int_0^P \frac{\Phi(\lambda)}{\sigma(\lambda)} H_\lambda \ \ell n \ [\phi(\lambda)] \ \sigma_\lambda \ d_\lambda \ dP$$

where $\Phi(\beta) = [\emptyset(\beta)]^{1-\sigma(\beta)}$. Equations (6) and (7) are coupled since $C = C(h,p)$. The heat flux $q_w = -k(\partial T/\partial y)_w$ for the gas becomes

$$q_w = -\emptyset_w (H_\beta)_w \rho_e u_e h_e / \sigma_w (Re)^{1/2} \tag{8}$$

where $\emptyset_w$ and $(H_\beta)_w$ are obtained from simultaneous solution of Eqs. (6) and (7). Substitution of Eq. (8) into the Stanton number expression yields,

$$St(Re)^{1/2} = \frac{\emptyset_w (H_\beta)_w}{\sigma_w \left[(\sigma_w)^{1/2} \frac{u_e^2}{2h_e} - H_w \right]} \tag{9}$$

where the recovery factor r has been taken as $(\sigma_w)^{1/2}$.

It is useful to examine the equations that lead to the terms in Eq. (9) in order to determine how the various flow quantities and gas properties influence $St(Re)^{1/2}$. To solve Eqs. (6) and (7) for $\emptyset_w$ and $(H_\beta)_w$ certain quantities must be specified. For a given gas the variations of the properties σ, T, and $\rho\mu$ with h at the specified pressure are required. Specification of T_w and T_e then establish h_w and h_e and therefore H_w. In addition, $C = \rho\mu/\rho_e\mu_e$ and σ can then be determined as functions of H.

Finally, with specification of u_e, the term u_e^2/h_e in Eq. (7) is established. Thus, if T_w, T_e, u_e, and the gas properties as a function of h are specified, sufficient information is available to solve Eqs. (6) and (7) for ϕ_w and $(H_\beta)_w$. Therefore, if T_w, T_e, u_e, and the pressure p are specified, and various gases are considered, any variation that occurs in ϕ_w and $(H_\beta)_w$ will arise from differences in the relations for T, $\rho\mu$, and σ vs h for the various gases. It should be noted however that fixed values for T_w and T_e do not yield the equal values of H_w for all gases except for the case $T_w = T_e$. Therefore, if comparisons are to be made between the solutions to Eqs. (6) and (7) for various gases at conditions other than $T_w = T_e$, they should be made at the same values of T_w and H_w rather than at the same T_w and T_e. On this basis, specification of fixed values of T_w, p, u_e^2/h_e, and H_w would permit the variations of $\rho\mu$ and σ vs h on ϕ_w and $(H_\beta)_w$ for various gases to be assessed. The terms u_e^2/h_e and H_w also appear in the denominator of Eq. (9). The term u_e^2/h_e is related to the Mach number. For ideal gases $u_e^2/h_e = M^2(\gamma - 1)$. Thus for large Mach numbers $u_e^2/h_e \gg H_w$. The Prandtl number σ_w in the denominator of Eq. (9) for the cold walls considered here is related to γ. (According to the theory of Eucken[2] the Prandtl number for gases can be closely approximated by $4\gamma/(9\gamma - 5)$.) Therefore, for high Mach numbers the denominator of Eq. (9) is related to the Mach number and γ. Computations performed to examine the influence of variations of gas properties on Eq. (9) and the terms therein are presented in the next section.

RESULTS

Figure 1a shows solutions to Eqs. (6) and (7) for a plate $(s = o)$ in terms of $\phi(\beta)$ and $H(\beta)$ obtained for carbon dioxide and helium for the values of T_w, p, u_e^2/h_e, and H_w noted in the figure. The numerical method employed to solve Eqs. (6) and (7) is outlined in ref. 3. Figure 1b shows $C(\beta)$ and $\sigma(\beta)$ for each of the gases. Property values for carbon dioxide were obtained from refs. 4 and 5. Thermodynamic properties for helium were computed from ideal gas equations. Also the relation $\mu \propto T^{0.647}$ and a constant Prandtl number $(\sigma = 0.688)$ were employed for helium.

It is noted that the influence of the difference between the curves for

$C(\beta)$ and $\sigma(\beta)$ on the profiles for $\phi(\beta)$ and $H(\beta)$ is small, but that the values for $\phi_w(H_\beta)_w$ noted in the figure differ by about 20 percent. However, the values of $St(Re)^{1/2}$ computed by Eq. (9) for the two gases differ by less than one percent. This suggests that $St(Re)^{1/2}$ and u_e^2/h_e are the correlating variables. Figure 2 presents results in terms of $St(R_e)^{1/2}/\sqrt{3^s}$ vs u_e^2/h_e for air, argon, carbon dioxide, helium, and hydrogen. The term $\sqrt{3^s}$ permits the results to be applied to the cone or the plate and wedge. The properties for argon and hydrogen were obtained from ref. 4. Those for air were taken from ref. 6. The Mach number and u_e^2/h_e ranges applicable for each gas are shown in the figure. Results obtained for $H_w = 0$ and $H_w = -0.68$ are indicated by the open and filled symbols respectively. These values of H_w correspond to $T_e/T_w = 0$ and $T_e/T_w \doteq 3$, and cover the range of T_e/T_w commonly encountered in shock tunnel and expansion tube flows. With the exception of carbon dioxide the results are relatively insensitive to the value of H_w. The results in Fig. 2 were obtained for $p = 0.01$ atm. However, computations at $p = 1$ atm for carbon dioxide (for which variation in pressure was expected to have the most influence) yielded results that are within one percent of those shown in Fig. 2, indicating that there is no significant influence of pressure for the conditions considered here.

For simplicity a straight line has been fitted through the points in Fig. 2. The computed results deviate by at most five percent from this curve. Thus for most engineering purposes this simple relation should be sufficiently accurate for the gases considered for Fig. 2. In view of the wide range of gas characteristics considered here it is reasonable to expect this relation would be applicable with the same accuracy to other gases and gas mixtures provided the cold wall condition is maintained and H_w is within the range noted in Fig. 2.

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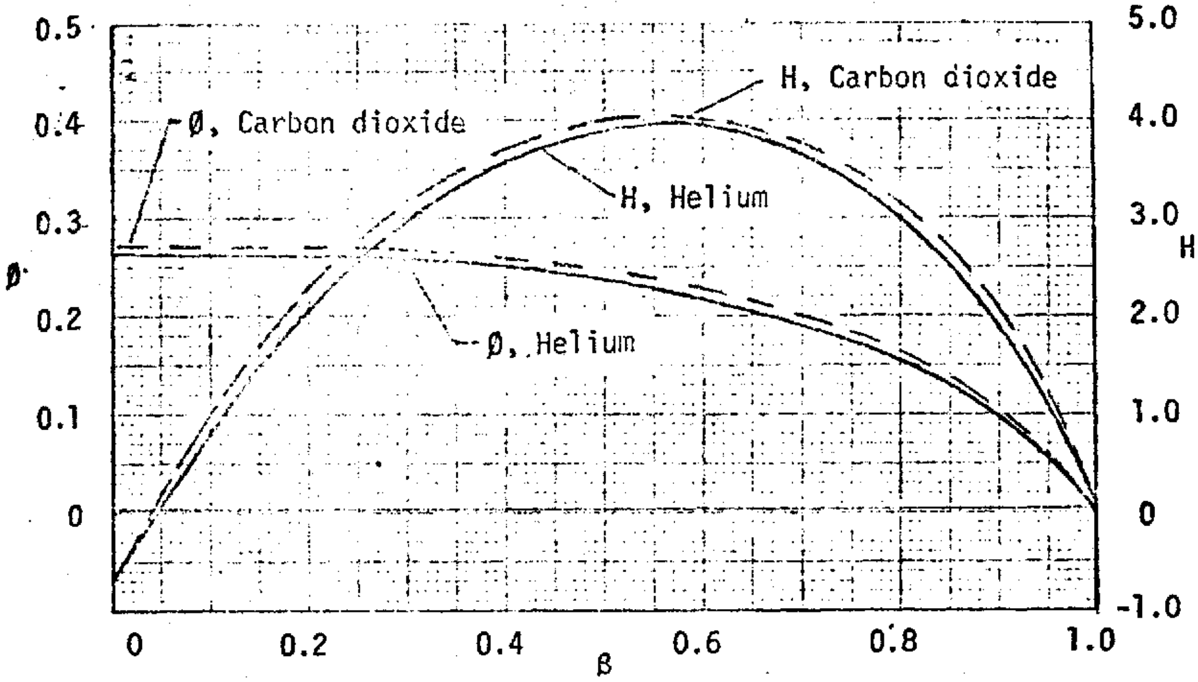
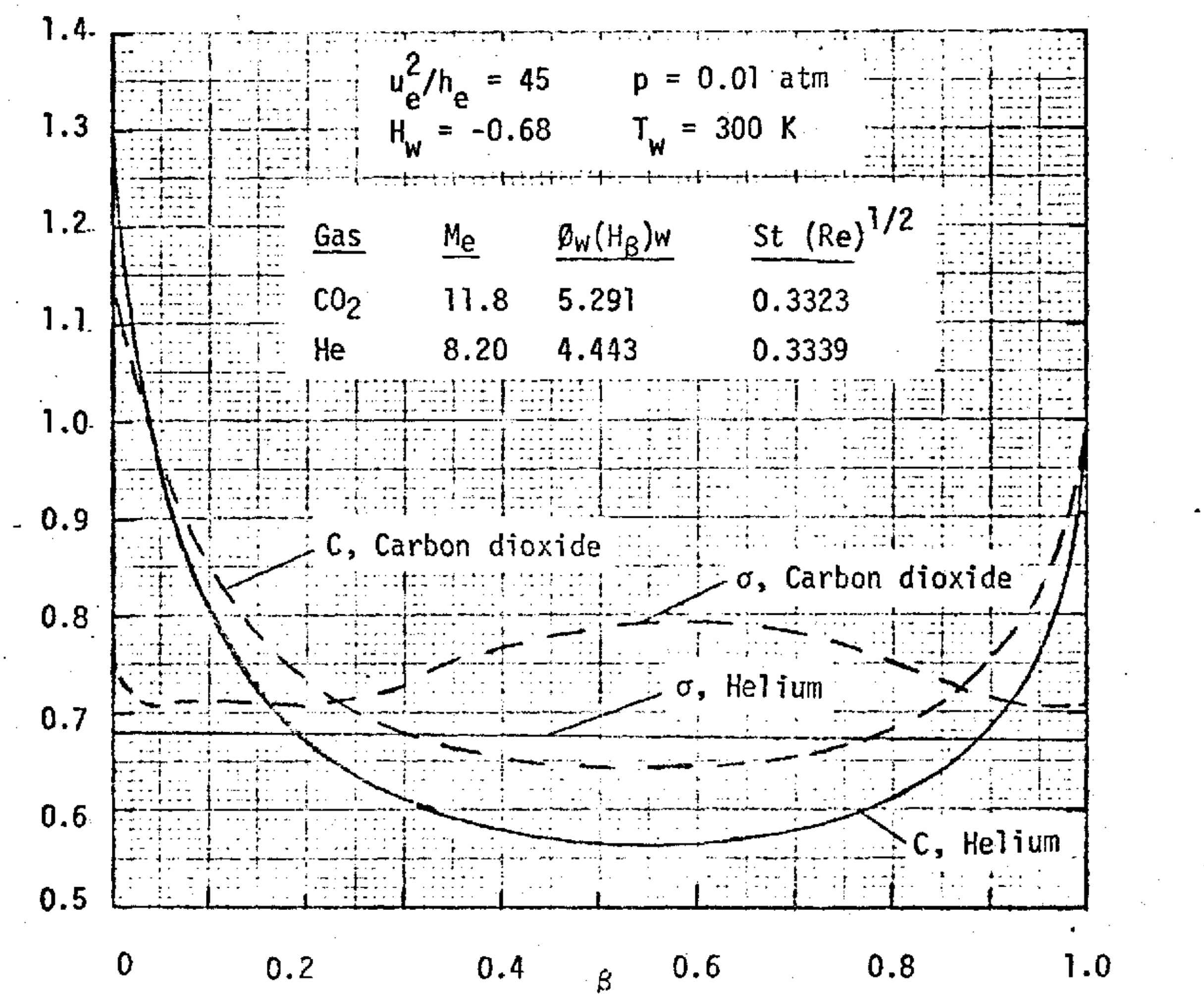


a. Ø and H vs β



b. C and σ vs β

Fig. 1 Comparison of results for carbon dioxide and helium.

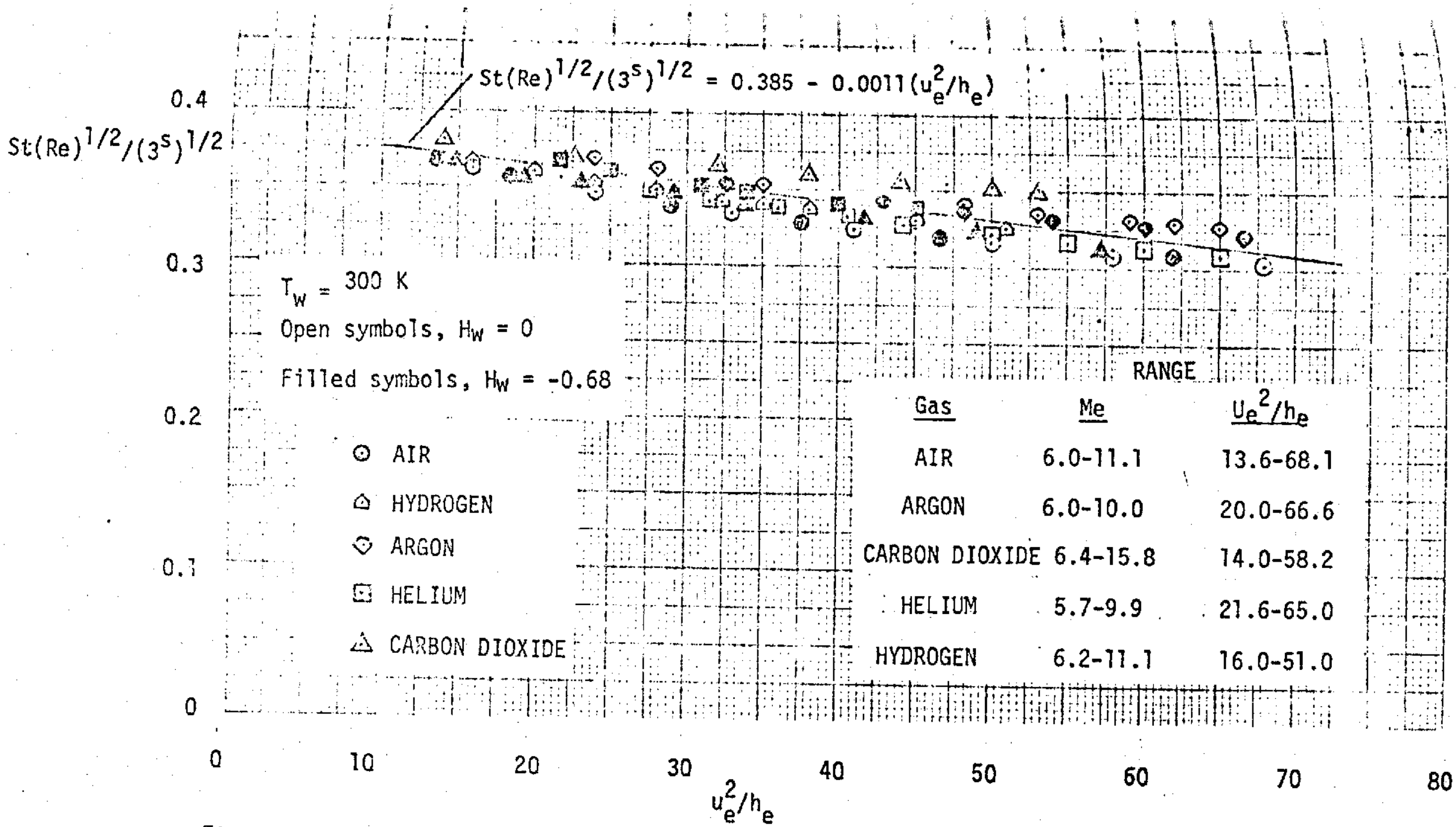


Fig. 2 Correlation of heat transfer for several gases.