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Reports of the Department of Geodetic Science

Report No. 194

THE COMPUTATION OF 15° AND 10° EQUAL AREA BLOCK TERRESTRIAL FREE AIR GRAVITY ANOMALIES

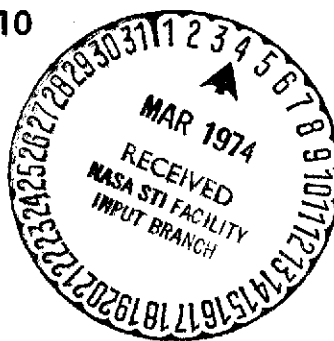
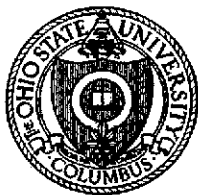
by

D. P. Hajela

Prepared for

National Aeronautics and Space Administration
Goddard Space Flight Center
Greenbelt, Maryland

Grant No. NGR 36-008-161
OSURF Project No. 3210



The Ohio State University
Research Foundation
Columbus, Ohio 43212

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Abstract

Starting with the set of 23,355 $1^\circ \times 1^\circ$ mean free air gravity anomalies used in Rapp (1972) to form a 5° equal area block terrestrial gravity field, this report describes the computation of 15° equal area block mean free air gravity anomalies along with estimates of their standard deviations. A new scheme of an integral division of a 15° block into 9 component 300 n. m. blocks, and each 300 n. m. block being subdivided into 25 60 n. m. blocks, is used. This insures that there is no loss in accuracy, which would have resulted if proportional values according to area were taken of the 5° equal area anomalies to form the 15° block anomalies.

A similar scheme is used for the computation of 10° equal area block mean free air gravity anomalies with estimates of their standard deviations. The scheme is general enough to be used for a 30° equal area block terrestrial gravity field, but this has not been computed in this report.

Foreword

This report was prepared by Mr. D. P. Hajela, Graduate Research Associate, Department of Geodetic Science, under NASA Grant NGR 36-008-161, The Ohio State University Research Foundation Project No. 3210 which is under the direction of Professor Richard H. Rapp. The contract covering this research is administered through the Goddard Space Flight Center, Mr. L. H. Carpenter, Technical Officer.

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1. Introduction

Rapp (1972) describes the formation of a global set of 1654 5° equal area mean free air gravity anomalies, with estimates of their standard deviations, using a set of 23,355 $1^\circ \times 1^\circ$ mean free air anomalies,¹ supplemented by model anomalies for those 5° blocks, in which no $1^\circ \times 1^\circ$ mean anomaly was available.

This report describes the formation of a global set of 15° and 10° equal area mean free air gravity anomalies with estimates of their standard deviations. The latitudinal and longitudinal extent of these blocks are described later. If these larger blocks would have contained an exact number of 5° equal area blocks, as described in Rapp (1972), the mean gravity anomalies could be obtained from the values in the 5° blocks. This was not so in the actual case. However, as the latitudinal extent of the $15^\circ/10^\circ$ blocks was an exact multiple of the 5° block, a larger $15^\circ/10^\circ$ block could be considered in $3/2$ 5° latitudinal belts. The mean anomaly in each of these belts could then be computed as these would have in general, consisted of one or two full 5° equal area blocks and balance, part 5° blocks, for which a proportional value could be computed, and thus finally, a mean value for the $15^\circ/10^\circ$ block. This procedure would have, however, not been accurate, as the mean anomaly for a part 5° block cannot be obtained accurately by taking a proportional value according to area from the mean anomaly of the whole 5° block.

A scheme, described in detail in Section 2, was accordingly devised to divide the larger $15^\circ/10^\circ$ block into an exact number of 5° blocks, i.e. 9 or 4 blocks. These component 5° blocks, which will nowonwards be referred to as 300 n. m. blocks in this report, have a slightly larger deviation in area from the 5° equal area blocks, as described in Rapp (1972). However, as an exact number of these 300 n. m. blocks could be fitted in a $15^\circ/10^\circ$ block, there would be no loss in accuracy while obtaining the mean anomaly of the $15^\circ/10^\circ$ block from the mean anomalies of the 300 n. m. integral blocks, as there would now be no part 300 n. m. block in a $15^\circ/10^\circ$ block.

In the same way as the 300 n. m. blocks were obtained as an exact number of subdivisions, (9 or 4), of a $15^\circ/10^\circ$ block, each 300 n. m. block was then subdivided into exactly 25 nominally 60 n. m. blocks, i.e. 60 n. m. in latitude and 60 ± 30 n. m. in longitude. These blocks will nowonwards be

¹ These anomalies (in the Canadian area) were corrected by a -2mgal base station correction which was earlier neglected.

referred to as 60 n. m. blocks in this report. The mean gravity anomalies in the 60 n. m. blocks were obtained by taking the mean of the available $1^\circ \times 1^\circ$ anomalies inside the 60 n. m. block. Estimates of the standard deviation of the 60 n. m. block anomalies was obtained by usual propagation of variance. If no $1^\circ \times 1^\circ$ anomaly was available in any 60 n. m. block, the mean gravity anomaly of the 60 n. m. block was predicted within the 300 n. m. block, (obtained, as already described, as integral components of the larger $15^\circ/10^\circ$ blocks), by linear regression as described in Rapp (1972) based on original programs written by Kaula in 1966.

Having thus obtained the mean anomalies, and variances, of all the 25 60 n. m. blocks comprising the 300 n. m. block, which in turn was an integral component of the larger $15^\circ/10^\circ$ block, the mean anomaly of the 300 n. m. block and the larger blocks was obtained, along with estimates of their standard deviations after allowing for the 60 n. m. and 300 n. m. blocks not being exactly equal area blocks. The detailed procedure is described later, but the main point is the formation of a fresh set of 300 n. m. block gravity anomalies, the limits of these blocks being set to give an exact number of 300 n. m. blocks inside a $15^\circ/10^\circ$ equal area block. The set of 300 n. m. anomalies, including the 60 n. m. values used in its computation along with predicted values of 60 n. m. blocks, was computed afresh from the $1^\circ \times 1^\circ$ data, and is different for the 15° and the 10° blocks, as the longitudinal limits of the sub blocks are worked out for the specific 15° or 10° block. The procedure for both cases is however similar, and would also apply to a 30° equal area block.

2. 15° and 10° Equal Area Block Limits and Their Subdivisions

The limits of $15^\circ/10^\circ$ equal area blocks were chosen according to Section 2 of Rapp (1971), except for the modification that the longitudinal limits were also kept as integral degrees, the latitudinal limits already being integral degrees. In all further subdivisions of these larger blocks, the longitudinal limits were again always kept as integral degrees, while the latitudinal limits of course continued to remain integral degrees. This scheme insured an exact number of $1^\circ \times 1^\circ$ blocks to fall in the 60 n. m. blocks. The word "integral" has been used in this report to indicate an exact number, i. e. a full integer value.

Following the notation of Rapp (1971), the area A_B , of the basic block bounded by the equator and the parallel of latitude, $\varphi = \pm\theta$, and of longitudinal extent, $\Delta\lambda = \theta$, where $\theta = 15^\circ$ (or 10°) is the size of the basic block, is given on a unit sphere by

$$A_B = \theta \sin \theta \quad (1)$$

which is easily verified from the expression for the area, A , of any θ° block in general, given by

$$A = \int_{\varphi_i}^{\varphi_{i+1}} \int_{\lambda_i}^{\lambda_{i+1}} \cos \varphi \, d\varphi \, d\lambda = \Delta\lambda (\sin \varphi_{i+1} - \sin \varphi_i) \quad (2)$$

Keeping the latitudinal extent of the blocks everywhere as θ , i. e.

$$\Delta\varphi = \varphi_{i+1} - \varphi_i = \theta \quad (3)$$

the longitudinal extent, $\Delta\lambda$, of the blocks away from the equator, required to make $A = A_B$, is given by

$$\Delta\lambda = \theta \sin \theta / (\sin \varphi_{i+1} - \sin \varphi_i) \quad (4)$$

Using this value of $\Delta\lambda$ as a guide, we first work out an integral number, n , of blocks in the specified latitudinal belt. We then recompute the value of $\Delta\lambda = 360^\circ/n$, and starting from the Greenwich meridian, round off the resulting longitudinal limits of each block to an integral number of degrees.

To describe the 15° equal area blocks first, the latitudinal extent of each block was 15° , starting from the equator and going North or South. The longitudinal extent, starting for convenience from the Greenwich meridian, was also kept as 15° for the equatorial blocks, i. e. with latitudinal extent from 0° to

15° N, and 0° to 15° S. The longitudinal extent of the blocks in other 15° latitude belts was then determined on the basis described above. Each of these 15° blocks was then divided into nine 300 n. m. blocks, with latitudinal extent kept everywhere as 5°, and longitudinal extent kept in integral degrees. Each of the resulting 300 n. m. blocks was then divided into 25 60 n. m. blocks, with latitudinal extent kept everywhere as 1°, and longitudinal extent kept in integral degrees.

The limits of the 10° equal area blocks, and its four component 300 n. m. blocks, in turn divided into 25 60 n. m. blocks, were obtained in a similar manner. The latitudinal and longitudinal extent of the 15° and 10° blocks, and the component 300 n. m. blocks, are shown in Tables 1 and 2 for the Northern/Southern hemisphere. The extent and area of the 60 n. m. blocks has not been given, but is obtained by working on a similar basis.

It is apparent that the areas of the component 300 n. m. /60 n. m. blocks are not equal, the main variation being due to the fact that the longitudinal extent has been kept everywhere in integral number of degrees. Instead of taking straight means of the anomalies of a smaller block to get the value for a larger block, we should then obtain a weighted mean, keeping the weights proportional to the areas of each of the smaller blocks.

The total number of 15° equal area blocks for the whole globe was 184, and the corresponding number of 300 n. m. blocks were 1656. The total number of 10° equal area blocks for the whole globe was 416, and the corresponding number of 300 n. m. blocks was 1664.

TABLE ONE
 Latitudinal and Longitudinal Extent and Area of 15°/10° Equal Area Blocks

15° Equal Area Blocks				
Latitudinal Extent in Degrees	Total No. of Blocks in 15° Lat. Belt	Longitudinal Extent in Degrees	Partial No. of Blocks of Differing Longitude Extent as shown in Column (3)	Approx. Area of Block on Unit Sphere with Long. Extent in Column (3)
(1)	(2)	(3)	(4)	(5)
0 - 15	24	15	24	.0677
15 - 30	22	16 or 17	14, 8	.0673, .0715
30 - 45	19	19 or 18	18, 1	.0686, .0650
45 - 60	15	24	15	.0665
60 - 75	9	40	9	.0697
75 - 90	3	120	3	.0713
10° Equal Area Blocks				
Latitudinal Extent in Degrees	Total No. of Blocks in 10° Lat. Belt	Longitudinal Extent in Degrees	Partial No. of Blocks of Differing Longitude Extent as shown in Column (8)	Approx. Area of Block on Unit Sphere with Long. Extent in Column (8)
(6)	(7)	(8)	(9)	(10)
0 - 10	36	10	36	.0303
10 - 20	35	10 or 11	25, 10	.0293, .0323
20 - 30	33	11 or 10	30, 3	.0303, .0275
30 - 40	30	12	30	.0299
40 - 50	26	14 or 13	22, 4	.0301, .0279
50 - 60	21	17 or 18	18, 3	.0296, .0314
60 - 70	15	24	15	.0308
70 - 80	12	30	12	.0314
80 - 90	3	120	3	.0318

TABLE TWO
Latitudinal and Longitudinal Extent and Area of 300 n. m. Blocks
As Integral Subdivisions of a 15°/10° Equal Area Block

Table 2A - Subdivision of a 15° Equal Area Block into nine 300 n. m. Blocks

Longitudinal Extent of a 15° E. A. Block in Degrees	Longitudinal Extent of 300 n. m. Blocks in Degrees	Latitudinal Extent of a 300 n. m. Block in Degrees	Approx. Area of 300 n. m. Blocks on Unit Sphere depending on Long. extent in Column (2)
(1) 15	(2) 5, 5, 5	(3) 0 - 5	(4) .0076
		5 - 10	.0075
		10 - 15	.0074
16 or 17	5, 6, 5 or 6, 5, 6	15 - 20	.0072, .0087
		20 - 25	.0070, .0084
19 or 18	6, 7, 6 or 6, 6, 6	25 - 30	.0067, .0081
		30 - 35	.0077, .0089
24	8, 8, 8	35 - 40	.0072, .0084
		40 - 45	.0067, .0078
40	13, 14, 13	45 - 50	.0082
		50 - 55	.0074
120	40, 40, 40	55 - 60	.0065
		60 - 65	.0091, .0098
		65 - 70	.0075, .0081
		70 - 75	.0059, .0064
		75 - 80	.0131
		80 - 85	.0079
		85 - 90	.0026

TABLE TWO
 Latitudinal and Longitudinal Extent and Area of 300 n. m. Blocks
 As Integral Subdivisions of a 15°/10° Equal Area Block

Table 2B - Subdivision of a 10° Equal Area Block into four 300 n. m. Blocks

Longitudinal Extent of a 10° E. A. Block in Degrees	Longitudinal Extent of 300 n. m. Blocks in Degrees	Latitudinal Extent of a 300 n. m. Block in Degrees	Approx. Area of 300 n. m. Blocks on Unit Sphere depending on Long. extent in Column (6)
(5)	(6)	(7)	(8)
10	5, 5	0 - 5	.0076
10 or 11	5, 5 or	5 - 10	.0075
11 or 10	6, 5	10 - 15	.0089, .0074
11 or 10	6, 5 or	15 - 20	.0087, .0072
12	5, 5	20 - 25	.0084, .0070
14 or 13	6, 6	25 - 30	.0081, .0067
17 or 18	7, 7 or	30 - 35	.0077
24	7, 6	35 - 40	.0072
30	9, 8 or	40 - 45	.0078, .0067
120	9, 9	45 - 50	.0072, .0061
	12, 12	50 - 55	.0083, .0074
	15, 15	55 - 60	.0073, .0065
	60, 60	60 - 65	.0084
		65 - 70	.0069
		70 - 75	.0091
		75 - 80	.0065
		80 - 85	.0119
		85 - 90	.0039

3. Utilization of 1° x 1° Data for 60 n. m. Block Values

The starting data was the 23,355 free air gravity anomalies¹ in 1° x 1° blocks, along with their assigned standard deviations as described in Rapp (1972). As described by Rapp, the anomalies were then corrected by the relation:

$$\Delta g_i = \Delta g_{i1} + (1.49 - 13.71 \sin^2 \varphi) \text{ mgals} \quad (5)$$

where φ is the mean latitude of the 1° x 1° anomaly block, Δg_{i1} is the original anomaly with respect to the International Gravity Formula referred to the Potsdam system, and the corrected anomaly Δg_i used in this report refers to the absolute system (IAG, 1971) with reference to the following gravity formula:

$$\gamma = \gamma_E (1 + .005,302,43 \sin^2 \varphi - .000,005,87 \sin^2 2\varphi) \quad (6)$$

with the value of equatorial gravity, γ_E , as 978,033.51 mgals.

As the initial anomalies were in nearest mgals, the corrections were also applied to the nearest mgals. A straight mean was then taken of all the 1° x 1° corrected anomalies falling in any particular 60 n. m. block with its latitudinal and longitudinal limits in integral degrees, to give the mean gravity anomaly of the 60 n. m. block. The variance of this 60 n. m. mean anomaly was then obtained by dividing the sum of variances of the 1° x 1° anomalies falling in it, by the square of the total number of 1° x 1° blocks. The 1° x 1° anomalies were thus considered as uncorrelated. All 1° x 1° variances in a 60 n. m. block were considered to belong to the same population, and the population variance could then be estimated as the average of their pooled variances.

The limits of the 60 n. m. blocks were in full integer values of degrees, thereby insuring that each 60 n. m. block encompassed an exact number of 1° x 1° blocks. As described earlier, the 60 n. m. blocks were obtained separately for a 15°/10° equal area block, through the specified subdivisions in its component 9/4 300 n. m. blocks, and each of these 300 n. m. blocks into its component 25 60 n. m. blocks.

¹ These anomalies (in the Canadian area) were corrected by a -2mgal base station correction which was earlier neglected.

If none of the 23,355 $1^\circ \times 1^\circ$ anomaly fell in any particular 60 n. m. block, the mean gravity anomaly was predicted as described below.

4. Computation of 300 n. m. Block Values

The 300 n. m. block values were based on the 25 component 60 n. m. block values. Hence, when any 60 n. m. block values could not be computed as in Section 3, these were predicted by linear regression, based on other 60 n. m. block values inside the particular 300 n. m. block by the well-known relation, using the notation of Rapp (1972):

$$g_p^* = \underline{C}_p (\underline{C} + \underline{D})^{-1} \underline{g} \quad (7)$$

where g_p^* is the predicted mean anomaly in the pth 60 n. m. block, based on the computed mean anomalies of the 60 n. m. blocks from the $1^\circ \times 1^\circ$ data as in Section 3, and denoted by the column vector \underline{g} . The variances of the computed 60 n. m. block anomalies are denoted by the diagonal matrix \underline{D} , while the covariances between the computed 60 n. m. block anomalies are denoted by the matrix \underline{C} . \underline{C}_p denotes a column vector of the covariances of 60 n. m. block, p, anomaly being predicted, with the computed 60 n. m. block anomalies.

The 60 n. m. block anomaly covariances required to be used in \underline{C}_p and \underline{C} were obtained from the 25 x 25 60 n. m. block anomaly variance covariance matrix. The matrix was formed on the basis of each 60 n. m. block anomaly referring to the center of the 60 n. m. block, and separated from its adjacent 60 n. m. block anomalies within a 300 n. m. block, by 60 n. m. i. e. 1° , in latitude and longitude. These covariances were obtained by straight line interpolation from the data of short-range autocovariances of 60 ± 30 n. m. free air gravity anomalies, using all $1^\circ \times 1^\circ$ data, given in Table 2 on page 6 of Rapp (1972). As described by Rapp, the 60 n. m. block anomaly variance covariance matrix was then scaled by a factor of 1.345 so that the variance of the 300 n. m. block anomalies conformed to 245 mgals²; and this resulted in 60 n. m. block anomaly variances being raised to 928 mgals² from the original value of 690 mgals².

After obtaining the gravity anomalies for all the 25 60 n. m. blocks inside a 300 n. m. block, the mean gravity anomaly \bar{g} of the 300 n. m. block was computed from the relation:

$$\bar{g} = \frac{\sum_{i=1}^{25} A_i g_i}{\sum_{i=1}^{25} A_i} \quad (8)$$

where g_i is the computed/predicted anomaly in the i th 60 n. m. block of area A_i .

Taking the value of $A_i = \Delta\varphi_i \Delta\lambda_i \cos \varphi_{m_i}$, where φ_{m_i} is the mean latitude of the i th 60 n. m. block of latitudinal and longitudinal extent $\Delta\varphi_i$, $\Delta\lambda_i$ respectively, and as $\Delta\varphi_i = 1^\circ$ was the same for all blocks, with the 25 blocks being arranged in 5 latitudinal belts, equation (8) reduces to:

$$\bar{g} = \frac{\sum_{j=1}^5 \cos \varphi_{m_j} \sum_{i=1}^5 g_i \Delta\lambda_i}{\sum_{j=1}^5 \cos \varphi_{m_j}} \quad (9)$$

where $j = 1, 2, \dots, 5$ refers to the 5 1° latitudinal belts.

As the main variation in areas of the blocks was due to the change in longitudinal extent, as noted in the end of Section 2, equation (9) was considered satisfactory instead of using an equation similar to (2) for getting the area of each block.

The variance m^2 , of the 300 n. m. block anomaly was estimated from the relation, using Rapp (1972) notation:

$$m^2 = \bar{C} - \bar{C}_i (\bar{C} + \bar{D})^{-1} \bar{C}_i^T \quad (10)$$

where \bar{C} is the variance of the 300 n. m. block anomalies, taken to be 245 mgals² and \bar{C}_i is a column vector of covariances between the i th computed 60 n. m. block anomaly with the mean anomaly of the 300 n. m. block in which it lies, and \bar{C} and \bar{D} have the same meaning as in equation (7).

The standard deviation of the 300 n. m. block anomalies, as obtained in equation (10) thus represents the contribution of the variances of computed 60 n. m. block anomalies based on $1^\circ \times 1^\circ$ data, as well as the effect of insufficient $1^\circ \times 1^\circ$ data, when 60 n. m. block anomalies were predicted based on the computed 60 n. m. block anomalies. In general, the standard deviation of the 300 n. m. block anomaly

was larger when more 60 n. m. block anomalies had to be predicted. Its value ranges from 0.5 to 3.0 mgals when all 25 60 n. m. blocks had some $1^\circ \times 1^\circ$ gravity data, and between 12 to 15 mgals when 1 to 3 60 n. m. blocks had some observed $1^\circ \times 1^\circ$ gravity data and the anomalies in the remaining 60 n. m. blocks were predicted.

If there was no $1^\circ \times 1^\circ$ gravity anomaly data in any of the 60 n. m. blocks inside a 300 n. m. block, the standard deviation of the 300 n. m. block anomaly was taken uniformly as 20 mgals, as was done in Rapp (1972). A proportional value of gravity anomaly according to longitudinal limits was then taken from the 1654 global set of 5° equal area gravity anomalies in Rapp (1972). In the case of 15° equal area blocks, there were 369 such 300 n. m. blocks out of a total of 1656 blocks, of which 60 were in Northern hemisphere and 309 in Southern hemisphere. It may be recalled that there were about the same number of blocks, 371 out of 1654 5° equal area blocks in Rapp (1972).

5. Computation of 15° Equal Area Block Values

A 15° equal area block gravity anomaly was computed from the mean anomalies of the 9 component 300 n. m. blocks, whose values were obtained in Section 4. To allow for the 300 n. m. blocks being of different areas, a weighted mean was taken in proportion to the areas. Analogous to equation (9), the gravity anomaly \bar{g} of a 15° equal area block was given by:

$$\bar{g} = \frac{\sum_{j=1}^3 \cos \varphi_{n_j} \sum_{i=1}^3 \bar{g}_i \lambda_i}{\sum_{j=1}^3 \cos \varphi_{n_j}} \quad (11)$$

where \bar{g}_i is the gravity anomaly of the 300 n. m. block, of longitudinal extent λ_i , arranged in 3 latitudinal belts whose central values are given by φ_{n_j} , $j = 1, 2, 3$.

The covariances of the 300 n. m. block anomalies were obtained by straight line interpolation from the data of long range autocovariances of 300 n. m. anomalies given in Table 6 on page 17 of Rapp (1972). Each 300 n. m. block anomaly was taken to refer to the center of the 300 n. m. block, and separated from the adjacent 300 n. m. block anomalies within a 15° equal area block by 300 n. m., i. e. 5° , in latitude and longitude. The resulting 9×9 variance

covariance matrix of 300 n. m. block anomalies is given below in Table 3. The 300 n. m. block numbering was 1 to 3 in the first latitudinal belt, 4 to 6 in the next latitudinal belt and 7 to 9 in the last latitudinal belt. The last row in Table 3 below gives the covariance of each 300 n. m. block anomaly with the mean anomaly of the 15° equal area block in which it lies. The average of these values represents the variance of the 15° equal area block anomalies and comes to 125 mgals^2 .

Subsequently, the actual variance computed for the 15° equal area block gravity anomalies, after these were obtained from equation (11), came to 86 mgals^2 . As this was less than 125 mgals^2 , the implicit value of variance of 15° equal area block gravity anomalies, no scale factor was required to be applied to the covariances of 300 n. m. block anomalies in Table 3, following usual procedures in adjustment of working with larger estimates of variances.

The variance, m^2 , of a particular 15° equal area block anomaly was obtained from an analogous relation to equation (10), where \bar{C} is now the variance of 15° equal area block anomalies obtained as 125 mgals^2 from Table 3, \bar{C}_i is the column vector of covariances of the i th 300 n. m. block anomaly with the mean anomaly of the 15° equal area block in which it lies, and is given by the last row in Table 3. The variances of the component 300 n. m. block anomalies are now denoted by \underline{D} , while \underline{C} denotes these covariances as given in Table 3.

TABLE THREE

(Covariances (mgals²) of 300 n. m. Free Air
Anomalies in a 15° Equal Area Block

300 n. m. Block No.	1	2	3	4	5	6	7	8	9
1	250.0	139.0	92.2	139.0	119.8	80.7	92.2	80.7	57.5
2	139.0	250.0	139.0	119.8	139.0	119.8	80.7	92.2	80.7
3	92.2	139.0	250.0	80.7	119.8	139.0	57.5	80.7	92.2
4	139.0	119.8	80.7	250.0	139.0	92.2	139.0	119.8	80.7
5	119.8	139.0	119.8	139.0	250.0	139.0	119.8	139.0	119.8
6	80.7	119.8	139.0	92.2	139.0	250.0	80.7	119.8	139.0
7	92.2	80.7	57.5	139.0	119.8	80.7	250.0	139.0	92.2
8	80.7	92.2	80.7	119.8	139.0	119.8	139.0	250.0	139.0
9	57.5	80.7	92.2	80.7	119.8	139.0	92.2	139.0	250.0

Covariance of a 300 n. m. block anomaly with the mean anomaly of the 15° Equal Area
block in which it lies:

116.8	128.9	116.8	128.9	142.8	128.9	116.8	128.9	116.8
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The gravity anomalies of all 184 15° equal area blocks, along with estimates of their standard deviation, are given in the last two columns of Table 5. The latitudinal and longitudinal limits of the 15° block, its area, the latitude and longitude of its center are also given. The third column in Table 5 gives the number of 300 n. m. blocks, if any, in which anomalies were taken on a proportional basis from Rapp (1972), when no 1° x 1° gravity data was available in the 300 n. m. block.

Some information regarding the minimum, maximum, R. M. S. and mean values of 15° block gravity anomalies and standard deviations is given in Table 4. With reference to not applying any scale factor to the covariances in Table 3, it may be mentioned here that the 95% confidence interval for the variance of 15° block gravity anomalies was 91 to 124 mgals² for the Northern hemisphere, 69 to 94 mgals² for the Southern hemisphere, and 84 to 104 mgals² for the whole globe. The estimated variance of 15° block anomalies of 125 mgals² from Table 3 is larger than these values.

TABLE FOUR

Some Statistics for 15° Equal Area Block Free Air Gravity Anomalies & Standard Deviations

Northern Hemisphere				Southern Hemisphere				Whole Globe			
Min.	Max.	R. M. S.	Mean	Min.	Max.	R. M. S.	Mean	Min.	Max.	R. M. S.	Mean
Gravity Anomalies in Mgals											
-29	25	10.6	-2.2	-22	20	7.9	-0.6	-29	25	9.4	-1.4
Standard Deviations in Mgals											
0.5	5.7	2.6	2.3	0.5	5.7	4.0	3.9	0.5	5.7	3.4	3.1

6. Computation of 10° Equal Area Block Values

The procedure for obtaining 10° equal area block gravity anomalies, and their standard deviations, was similar to that described for 15° blocks in Section 5. The main difference was that the number of component 300 n. m. blocks was now 4, instead of 9. The limits of these 300 n. m. blocks, and in turn of their 25 component 60 n. m. blocks, was set, as before, to insure an integral division, as described in Section 2.

Analogous to equation (11), the gravity anomaly, \bar{g} , of a 10° equal area block was given by:

$$\bar{g} = \frac{\sum_{j=1}^2 \cos \varphi_{nj} \sum_{i=1}^2 \bar{g}_i \lambda_i}{\sum_{j=1}^2 \cos \varphi_{nj}} \quad (12)$$

where \bar{g}_i is the gravity anomaly of the i th component 300 n. m. block, of longitudinal extent λ_i , arranged in two latitudinal belts, with central values as φ_{nj} , $j = 1, 2$.

The covariances of the 300 n. m. block anomalies in a 10° equal area block were obtained in the same manner as described for the 15° equal area block, except that it now resulted in a 4 x 4 matrix in place of the earlier 9 x 9 matrix in Table 3. The covariances of the 300 n. m. block anomalies in a 10° equal area block are given in Table 6 below, and it can be seen to be an extract of corresponding entries in rows and columns 1, 2, 4, 5 of Table 3. The overall mean in Table 6 represents the variance of 10° equal area block anomalies and comes to 162 mgals². Subsequently, the actual variance computed for the 10° equal area block gravity anomalies, after these were obtained from equation (12), came to 127 mgals². Following the usual procedure in adjustment of utilizing larger variance estimates, no scale factor was thus required to be applied to the covariances in Table 6.

TABLE SIX

Covariances (mgals²) of 300 n. m. Free Air Gravity Anomalies
in a 10° Equal Area Block

300 n. m. Block No.	1	2	3	4
1	250.0	139.0	139.0	119.8
2	139.0	250.0	119.8	139.0
3	139.0	119.8	250.0	139.0
4	119.8	139.0	139.0	250.0
Covariance of a 300 n. m. block anomaly with the mean anomaly of the 10° equal area block in which it lies:				
	162.0	162.0	162.0	162.0

The variance, m^2 , of a particular 10° equal area block anomaly was again obtained from an analogous relation to equation (10), where \bar{C} is now the variance of 10° equal area block anomalies, i. e. 162 mgals², \underline{C}_i is the column vector of covariances of the i th 300 n. m. block anomaly with the mean anomaly of the 10° equal area block in which it lies, and is given in the last line of Table 6. The variances of the component 300 n. m. block anomalies are again denoted by \underline{D} , while \underline{C} denotes their covariances as given in Table 6.

The gravity anomalies of all the 416 10° equal area blocks, and estimates of their standard deviations are given in the last two columns of Table 8. The arrangement of other columns in Table 8 is the same as described for Table 5. In particular, column 3 of Table 8 again gives the number of 300 n. m. blocks, if any, in which anomalies are taken on a proportional basis from Rapp (1972) in absence of any 1° x 1° gravity data in that 300 n. m. block.

Relevant information analogous to Table 4 is given in Table 8. The 95% confidence interval for the variance of 10° block gravity anomalies was 127 to 187 mgals² for the northern hemisphere, 84 to 124 mgals² for the southern hemisphere, and 112 to 147 mgals² for the whole globe. The estimated variance of 10° block anomalies of 162 mgals² from Table 6 is larger, and has thus been used without any scaling.

TABLE SEVEN

Some Statistics for 10° Equal Area Block Free Air Gravity Anomalies & Standard Deviations

Northern Hemisphere				Southern Hemisphere				Whole Globe			
Min.	Max.	R. M. S.	Mean	Min.	Max.	R. M. S.	Mean	Min.	Max.	R. M. S.	Mean
<u>Gravity Anomalies in Mgals</u>											
-41	28	12.5	-2.0	-29	34	10.0	-0.5	-41	34	11.3	-1.4
<u>Standard Deviations in Mgals</u>											
0.5	7.9	3.8	3.3	0.7	7.9	5.8	5.4	0.5	7.9	4.9	4.4

7. Conclusion

15° and 10° equal area block mean free air gravity anomalies, with estimates of their standard deviations have been given in Tables 5 and 9 of this report. The block limits have also been given there, and follow the scheme of Section 2 of Rapp (1971), with the modification of longitudinal limits being integral degrees.

The values are in terms of the starting data of the set of 23,355 1°x1° mean free air anomalies used by Rapp (1972), which were referred to the absolute system (IAG 1971) with the gravity formula given in equation (6), and the value of equatorial gravity as 978,033.51 mgals.

Each 15° and 10° equal area block was separately divided into its 9 and 4 component 300 n.m. blocks respectively, which were in turn divided into 25 60 n.m. blocks. The anomalies and variances of these 60 n.m. blocks were obtained from the 1° x 1° data mentioned above. The 300 n.m. blocks were used as intermediate step, specifically devised, in which 60 n.m. block anomalies could be predicted using autocovariance data in Rapp (1972). The latitudinal and longitudinal limits of all 300 and 60 n.m. was kept as integral degrees throughout, which served as a safeguard against the need of taking any proportional values, which would have resulted in less accurate values. However, in those 300 n.m. blocks, in which no 1° x 1° gravity data is presently available, proportional values were taken from Rapp (1972). This was for about 1% of the 300 n.m. blocks in the northern hemisphere, and for about 37% blocks in the southern hemisphere, representing about 22% for the whole globe. These anomalies were accordingly given a higher variance.

It is felt that the 15° and 10° equal area block mean free air gravity anomalies reported in this paper are the best estimates in the weighted least squares sense with the available global coverage of 1° x 1° data, with realistic estimates of their standard deviations.

References

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