NASA TECHNICAL NOTE

NASA TN D-7590

NASA TN D-7590

(NASA-TN-D-7590) DYNAMICS AND COLUMN DENSITIES OF SMALL PARTICLES EJECTED FROM SPACECRAFT (NASA) ~~37~~ 35 p HC $3.00 CSCL 22C

N74-17539

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DYNAMICS AND COLUMN DENSITIES OF SMALL PARTICLES EJECTED FROM SPACECRAFT

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NATIONAL AERONAUTICS AND SPACE ADMINISTRATION • WASHINGTON, D. C. • FEBRUARY 1974

1. REPORT NO. NASA TN D-7590	2. GOVERNMENT ACCESSION NO.	3. RECIPIENT'S CATALOG NO.
4. TITLE AND SUBTITLE Dynamics and Column Densities of Small Particles Ejected from Spacecraft		5. REPORT DATE February 1974
		6. PERFORMING ORGANIZATION CODE
7. AUTHOR(S) Robert J. Naumann		8. PERFORMING ORGANIZATION REPORT # M460
9. PERFORMING ORGANIZATION NAME AND ADDRESS George C. Marshall Space Flight Center, Marshall Space Flight Center, Alabama 35812		10. WORK UNIT NO.
		11. CONTRACT OR GRANT NO.
		13. TYPE OF REPORT & PERIOD COVERED Technical Note
12. SPONSORING AGENCY NAME AND ADDRESS National Aeronautics and Space Administration, Washington, D.C. 20546		
		14. SPONSORING AGENCY CODE

15. SUPPLEMENTARY NOTES

Prepared by Space Sciences Laboratory, Science and Engineering

16. ABSTRACT

Trajectories and relative motions of small particles ejected from a spacecraft have been analyzed, and modifications to the clearing times and column densities because of orbital dynamics have been assessed. It was found that despite the fact that such particles are confined by orbital dynamics to move along similar trajectories with the spacecraft rather than to continue their free expansion, the effect is negligible for viewing angles away from the orbital path. Small particles are rapidly swept away by drag and will not contribute significantly to the column density when viewing along the velocity vector in a 420-km earth orbit. However, substantial increases in column density can result when viewing in a direction opposite to the velocity vector because of drag effects. In the absence of drag, significant column densities can build up both in front of and behind the spacecraft in earth orbit for particles released at a few meters per second. This effect is much less pronounced in lunar orbit because the same release velocity produces a larger orbital perturbation for the particle.

17. KEY WORDS	18. DISTRIBUTION STATEMENT Distribution Category: 30

19. SECURITY CLASSIF. (of this report)	20. SECURITY CLASSIF. (of this page)	21. NO. OF PAGES	22. PRICE
Unclassified	Unclassified	35	$3.00

MSFC - Form 3292 (Rev December 1972)

For sale by National Technical Information Service, Springfield, Virginia 22151

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INTRODUCTION

Small particles are often generated by a spacecraft in flight and can interfere with intended operations in a number of ways. One of the primary areas of concern is the light scattered from such particles. Of secondary concern is the molecular material sublimating from them. To calculate these effects it is necessary to know the column density.

The column density depends on the source rate and the velocity. To a first order approximation the velocity may be taken as the initial velocity of separation between the particle and the spacecraft. However, this velocity can be modified significantly by orbital dynamic and drag forces. These effects are computed in this study.

ANALYSIS

Consider the satellite in a circular orbit with velocity $v_{\odot}$. Let the particle be ejected with a velocity δv making angle γ with the velocity vector and Υ with the radius vector. Define the x-axis to be along the velocity vector, the y-axis to be along the radius vector, and the z-axis to be opposite the orbital angular momentum. The components of δv are

$$\vec{\delta v} = |\delta v| \begin{pmatrix} \cos\gamma \\ \cos\Upsilon \\ \sqrt{1-\cos^2\gamma-\cos^2\Upsilon} \end{pmatrix} . \tag{1}$$

The particle will have a change in orbital inclination given by $\delta i = \delta v_Z / v_{\odot}$ and will oscillate about the z direction according to

$$\delta z = r \sin\phi \; \delta i \tag{2}$$

where r is the radius vector and ϕ is the angle traveled from the release point.

By defining a quantity

$$\epsilon = v^2/v_{\odot}^2 - 1$$

where

$$v = |\vec{v}_{\odot} + \vec{\delta v}| \qquad ,$$

$$\epsilon = 2\frac{\delta v_x}{v_{\odot}} + \left(\frac{\delta v}{v_{\odot}}\right)^2 \qquad . \tag{3}$$

The semimajor axis a and eccentricity e of the particle orbit are

$$e^2 = 1 - \frac{v^2}{v_{\odot}^2}\left(2 - \frac{v^2}{v_{\odot}^2}\right) \cos^2\delta \tag{4}$$

and

$$a = \frac{r_{\odot}}{2 - v^2/v_{\odot}^2} \tag{5}$$

where δ is the dispersion angle or $\delta = \delta v_y/v_{\odot}$. In terms of ϵ,

$$e^2 = 1 - (1 - \epsilon^2)(1 - \delta v_y/v_{\odot}) \tag{6}$$

$$= \epsilon^2 + \left(\frac{\delta\, v_y}{v_{\odot}}\right)^2 + \ldots$$

and

$$a = \frac{r_\odot}{1 - \epsilon} \quad . \tag{7}$$

Hereafter the subscript $\odot$ refers to the spacecraft.

The mean anomaly M_0 at the point of release is obtained by requiring the radius vector r to equal $r_\odot$. Using the relation between radius vector and eccentric anomaly E and equation (7),

$$r_\odot/a = 1 - e \cos E_0 = 1 - \epsilon \quad ,$$

from which

$$E_0 = \pm \cos^{-1} (\epsilon/e) \tag{8}$$

where E_0 is the eccentric anomaly at the time of release.

The sign ambiguity introduced by the arc cos may be resolved by taking E_0 positive if the particle is projected in the direction of the radius vector ($0 < \gamma < 180$ deg) and negative otherwise.

The mean anomaly M_0 is found from the relation

$$M_0 = E_0 - e \sin E_0 \quad . \tag{9}$$

The mean anomaly may be expressed as a function of time by

$$M = M_0 + \frac{2 \pi t}{T} \tag{10}$$

where T is the period. This period is related to the spacecraft period by

$$T = T_\odot (a/r_\odot)^{3/2} = T_\odot \left(1 + \frac{3}{2} \epsilon + \ldots\right) \quad . \tag{11}$$

The mean anomaly of the spacecraft relative to the particle perigee is just the true anomaly ν_0 of the particle at the time of release. This is found by requiring

$$\frac{1 - e^2}{1 + e \cos \nu_0} = \frac{r_\odot}{a} = 1 - \epsilon \tag{12}$$

or

$$\nu_0 = \pm \cos^{-1} \left[\frac{\epsilon - e^2}{e(1 - e)} \right] \quad . \tag{13}$$

The same resolution of the sign ambiguity stated for E_0 applies here.

The mean anomaly for the spacecraft is given as a function of time as

$$M_\odot = \nu_0 + \frac{2 \pi t}{T_\odot} \quad . \tag{14}$$

The position of the particle relative to the spacecraft in a space-fixed coordinate system with x' in the direction of the particle perigee is given by

$$\frac{\delta x'}{r_\odot} = \frac{1}{1 - \epsilon} \left[\cos M + \frac{1}{2} e (\cos 2M - 3) + \frac{1}{8} e^2 (3 \cos 3M - 3 \cos M) + \ldots \right] - \cos M_\odot \tag{15}$$

$$\frac{\delta y'}{r_\odot} = \frac{1}{1 - \epsilon} \left[\sin M + \frac{1}{2} e \sin 2M + \frac{1}{24} e^2 (9 \sin 3M - 15 \sin M) + \ldots \right] - \sin M_\odot \quad .$$

These may be expressed in a rotating coordinate system such that x is in the direction of the spacecraft velocity and y is along the radius vector by rotating the system through angle $M_{\odot}$.

The drag on the particles is treated by allowing the semimajor axis to vary according to

$$\frac{\dot{a}}{a} = -\frac{\dot{E}}{E} \tag{16}$$

where E is the energy of the orbit. From the Virial Theorem,

$$2 \langle KE \rangle = -\langle PE \rangle \tag{17}$$

where the bracket terms are the time-averaged kinetic and potential energies. Therefore, the energy of the orbit is

$$E = -\frac{1}{2} m \langle v^2 \rangle \quad . \tag{18}$$

The rate of energy loss caused by drag is

$$\dot{E} = -Fv = -C_D A \rho_a v^3 \tag{19}$$

where C_D is the drag coefficient which is the order of unity, A is the frontal area, and ρ_a is the atmospheric density.

For nearly circular orbits the velocity is almost constant so that the difference between the average velocity and the rms velocity is negligible. Therefore,

$$\frac{\dot{a}}{a} = -\frac{2 C_D A \rho_a v}{m} \quad . \tag{20}$$

It is useful to express the change in a in terms of the fraction change per orbit, or

$$\frac{da}{dn} = \dot{a}\,T \tag{21}$$

where the period T is

$$T = \frac{2\pi a}{v} \quad . \tag{22}$$

Therefore,

$$\frac{da}{dn} = -\frac{4\pi C_D A \rho_a a^2}{m} \tag{23}$$

where n is the revolution number. This has the form ,

$$\frac{da}{dn} = -k\,a^2 \quad . \tag{24}$$

Integrating with the boundary condition $a = a_0$ at $n = 0$,

$$\frac{1}{a_1} - \frac{1}{a_0} = k\,n \tag{25}$$

or

$$a = \frac{a_0}{1 + a_0\,k\,n} \approx a_0\,(1 - a_0\,k\,n) \quad . \tag{26}$$

Assuming the particles are uniform ice spheres, $C_D = 1$, $a_0 = 6.798 \times 10^6$, and $\rho_a = 4.82 \times 10^{-12}$ kg/m^3 (at 420 km),

$$a_0 \; k = \frac{4 \pi C_D A \rho_a a_0}{m} = \frac{6.15 \times 10^{-7}}{d\ (m)} \tag{27}$$

where d(m) is the particle diameter in meters. Finally, equation (15) becomes

$$\frac{\delta x'}{r_\odot} = \frac{1 - a_0 k n}{1 - \epsilon_0} \left[\cos M + \frac{1}{2} e (\cos 2M - 3) + \frac{1}{8} e^2 (3 \cos 3M - 3 \cos M) + \ldots \right] - \cos M_\odot \tag{28}$$

$$\frac{\delta y'}{r_\odot} = \frac{1 - a_0 k n}{1 - \epsilon_0} \left[\sin M + \frac{1}{2} e \sin 2M + \frac{1}{24} e^2 (9 \sin 3M - 15 \sin M) + \ldots \right] - \sin M_\odot$$

where $a_0 k$ is given by equation (27), and n is the time in orbital periods.

To find the distance the particle has moved in one orbit, replace M by $M + 2\pi$ and $M_\odot$ by $M_\odot + 2 \pi T/T_\odot = M_\odot + 2\pi (1 + \frac{3}{2} \epsilon + \ldots)$ in equation (15) and subtract the original equation. This yields

$$\begin{aligned} \delta x' &= a[\cos M_\odot (1 - \cos 3 \pi \epsilon) - \sin M_\odot \sin 3 \pi \epsilon] \\ \delta x' &= a[\sin M_\odot (i - \cos 3 \pi \epsilon) - \cos M_\odot \sin 3 \pi \epsilon] \quad . \end{aligned} \tag{29}$$

Since $\epsilon << 1$, $(1 - \cos 3\pi\epsilon)$ involves only second order terms which may be neglected. The distance traveled in one orbit ΔS becomes

$$\frac{\Delta S}{\Delta n} = (\delta x'^2 + \delta y'^2) \approx | a \sin 3 \pi \epsilon | \approx | 3 \pi \epsilon a | \quad . \tag{30}$$

Under the influence of drag, a changes by $a = a_0 (1 - a_0 kn)$ and ϵ changes by $\epsilon = \epsilon_0 - a_0 kn$. Both ϵ_0 and $a_0 kn \ll 1$. Ignoring terms of higher than first order,

$$\frac{\Delta S}{\Delta n} \approx |3 \pi a_0 (\epsilon_0 - a_0 k n)| \quad . \tag{31}$$

Integrating, the average separation as a function of time is given by

$$\overline{S} = 3 \pi a_0 \left| \frac{\epsilon t}{T} - \frac{a_0 k}{2} \frac{t^2}{T^2} + \ldots \right| \tag{32}$$

where

$$a_0 = \frac{r_\odot}{1 - \epsilon_0} \quad ,$$

$$\epsilon_0 = 2 \frac{\delta v}{v_\odot} \cos \gamma + \left(\frac{\delta v}{v_\odot} \right)^2 \quad ,$$

and T is the orbital period of the particles. Actually, this changes also with a, but the difference is only in terms higher than first order.

RESULTS

Typical trajectories in the rotating coordinate system are shown in Figures 1 through 3. Figure 1 shows particle trajectories for an ejection velocity of $\delta v = 0.001 \, v_\odot$ at various angles and no drag. It is interesting to note that particles ejected along the velocity vector pass above the spacecraft and move to the rear, while the opposite is true for those ejected to the rear. This can be easily understood from the fact that addition of velocity raises the orbit and increases the period; therefore, the spacecraft will pass under and ahead of the particle.

Particles ejected at 90 deg and 270 deg have approximately the same period and will, therefore, almost reencounter the spacecraft one orbit later. Also, note the fact that trajectories have a mirror symmetry; i.e., the 225-deg trajectory is the mirror image of the 45-deg trajectory.

Figure 2 shows a family of trajectories for an ejection velocity of $\delta v = 0.0005\ v_{\odot}$ and no drag. Trajectories for angles $180\ \text{deg} < \gamma < 360\ \text{deg}$ were deleted for clarity since they are mirror images of those shown. The epicycles can be seen for the $\gamma = 60$ deg and 120 deg case. Figure 3 is the same case as shown in Figure 2 except that the scale is smaller. The tendency for the particles to spread out along the orbit is seen. In the direction of the velocity, the particles tend to stay below the flight path, while the opposite is true in the negative velocity direction.

Figure 4 is a plot of the radial distance between the spacecraft and the particle as a function of time. For comparison, the trajectory corresponding to initial ejection velocity is shown. It may be seen that particles leave with this velocity and that clearing rates may either be enhanced or retarded by the orbital dynamics. The 45- to 225-deg trajectory provides the fastest initial separation. A similar set of curves plotted over several orbits is shown in Figure 5.

The effect of drag is the decrease of the semimajor axis. This will ultimately cause the particle to move ahead of and below the spacecraft. Figure 6 shows the trajectories of particles ranging from 100 μm to 1 cm ejected at $\gamma = 60$ deg with a velocity of $\delta v = 0.001\ v_{\odot}$. The 100-$\mu$m particle turns and is rapidly swept away. The 1-mm particle first proceeds opposite to the velocity, makes several loops, and then rapidly moves in front of and away from the spacecraft. The 1-cm particle moves similarly to the drag-free case. It is being retarded and will ultimately reverse direction and pass below and in front of the spacecraft.

Figure 7 is identical to Figure 4 except that drag for a 100-μm particle is considered. Now the symmetry is destroyed and the 225-deg ejection angle results in very rapid clearing. The 135-deg case has a slower initial clearing rate and the 45- and 315-deg cases delay the rapid acceleration from the drag until almost one orbital period. Therefore, if overboard dumps are required, there is some advantage to projecting the material toward the rear and down.

Figure 8 shows the separation rates for 100-μm, 1-mm, and 1-cm particles for different values of γ. Note that clearing is always more rapid for particles thrown to the rear of the spacecraft.

COLUMN DENSITIES

The ultimate use of this study is to compute densities in a column along some line of sight. To do this properly, the contribution from a given source must be obtained by following the trajectory of each particle and the probability of the particle being found in the specified column obtained. This can be done by Monte Carlo techniques. However, for more general estimates simplified models will be used.

First assume a free expansion model in which particles are emitted uniformly from the surface of a sphere with radius R_0 and radial velocity δv_r. The number density is just

$$N(R) = \frac{\dot{N}_T}{4 \pi R_0^2 \delta v_r} \tag{33}$$

where $\dot{N}_T$ is the total emission rate (particles per unit time). The column density is found by integrating along a column,

$$n_c = \int_{R_0}^{\infty} N(R)\, dR = \frac{\dot{N}_T}{4 \pi R_0 \delta v_r} \quad . \tag{34}$$

This model applies if the δv_r is large enough so that the particles disperse to an undetectable concentration in less than one orbit. However, because of the orbital motion, the particles do not expand indefinitely but are confined to a tube about the orbit.

For an order of magnitude estimate, assume that all the particles leaving the spacecraft are uniformly distributed in cross sector in a circular tube with radius ξ and whose length is increasing with time at the rate $\dot{S}$ along the orbit.

The y-component of particle oscillation is given by

$$\frac{\xi_y}{r_\odot} = \frac{ae}{r} = \left(4 \frac{\delta v_x^2}{v_\odot^2} + \frac{\delta v_y^2}{v_\odot^2} + \ldots\right)^{1/2} \tag{35}$$

and the tangential amplitude is

$$\frac{\xi_z}{r_\odot} = \frac{\delta v_z}{v_\odot} \quad . \tag{36}$$

The resultant amplitude is

$$\xi = (\xi_y{}^2 + \xi_z{}^2)^{1/2} = \frac{r_\odot}{v_\odot} (4\ \delta v_x{}^2 + \delta v_y{}^2 + \delta v_z{}^2)^{1/2} \tag{37}$$

or

$$\xi = r_\odot \frac{\delta v}{v_\odot} (3 \cos \gamma + 1)^{1/2} \quad .$$

Note that ξ varies only by a factor of 2 over the entire range of γ. For the accuracy required here, it is sufficient to ignore the γ dependence and set

$$\xi \approx r_\odot \frac{\delta v}{v_\odot} \quad . \tag{38}$$

The number density contribution from particles inserted at angle γ anywhere in the tube is given by the number inserted during the interval Δt divided by the volume of a length of the tube equal to $\dot{S}\Delta t$, or

$$\delta \eta = \frac{\Delta N (\gamma)}{\pi \xi^2 \dot{S} \Delta t} = \frac{\dot{N} (\gamma)}{\pi \xi^2 \dot{S}} \tag{39}$$

where $\dot{S}$ is given by

$$\dot{S} = \frac{1}{T} \frac{\Delta S}{\Delta n} = \frac{3 \pi a_0}{T} \left| \epsilon_0 - a_0 \frac{k t}{T} + \dots \right| \quad . \tag{40}$$

In terms of S, this becomes

$$\dot{S} = \sqrt{| \dot{S}_0{}^2 \pm 2 \ddot{S}_0 S |} \tag{41}$$

where

$$\dot{S}_0 = \frac{3\,\pi\,a_0\,\epsilon_0}{T} \qquad \text{and} \qquad \ddot{S}_0 = \frac{3\,\pi\,a_0{}^2\,k}{T^2} \quad .$$

The positive sign which decreases the column density applies to particles moving ahead of the spacecraft, i.e., those ejected to the rear with $\gamma > \pi/2$ resulting in $\epsilon < 0$.

The column density may be calculated by integrating $\eta(S)$ along the sight vector. Let the sight vector make angle ψ with the orbital path. The contribution from particles released with angle γ to the column density is

$$\delta\, n_c = \int_0^{S_1} \eta(S)\,\frac{ds}{\cos\psi}$$

$$= \frac{\dot{N}(\gamma)}{\pi\,\xi^2\,\cos\psi} \int_0^{S_1} \frac{ds}{[\dot{S}_0{}^2 \pm 2\,\ddot{S}_0\,S]^{1/2}} \quad . \tag{42}$$

$\dot{S}_0$ may be expressed in terms of γ. From equations (30) and (3),

$$\dot{S}_0 = -\frac{3\,\pi\,a_0}{T}\left(2\cos\gamma + \frac{\delta\,v}{v_\odot}\right)\frac{\delta\,v}{v_\odot} \tag{43}$$

or, using the fact that $T = 2\pi r_\odot / v_\odot$,

$$\dot{S}_0 = -\frac{3}{2}\,\delta\,v\left(2\cos\gamma + \frac{\delta\,v}{v_\odot}\right) \quad . \tag{44}$$

The sign was lost in the derivation when extracting the root; however, it is clear that particles ejected with $\gamma < \pi/2$ will move behind the spacecraft, whereas $\gamma > \pi/2$

results in positive velocities. Some care is required to insure the proper sign in equation (42). For $\psi < \pi/2$, particles ejected with $\gamma > \pi/2$ will be seen directly, and they will be accelerated by drag. Therefore for $\psi < \pi/2$,

$$n_C = \int_0^{\pi/2} \frac{\dot{N}(\gamma)\, d\gamma}{\pi\, \xi^2 \cos\psi} \int_0^{S_1} \frac{ds}{[\dot{S}_0^2 + 2\, \ddot{S}_0\, S]^{1/2}} \tag{45}$$

where S_1 is the distance at which the line of sight intersects the tube containing the particles. This is

$$S_1 = \begin{cases} \xi \cot\psi \\ \sqrt{2\, r_\odot\, \xi} \end{cases} \quad \text{whichever less.} \tag{46}$$

Particles ejected with $\gamma < \pi/2$ will initially move opposite to the velocity vector. They will be retarded by drag and will ultimately turn and be accelerated along the velocity vector. When they pass the spacecraft, from equation (41), it may be seen that their separation velocity will be equal in magnitude to $\dot{S}_0$, but they will be moving in the opposite direction. From this point on they will behave just like particles ejected at $\pi/2 - \gamma$. Therefore, the total column density for $\psi < \pi/2$ will be twice the integral of equation (45) over $0 \leqslant \gamma \leqslant \pi/2$.

For $\psi > \pi/2$, the situation is somewhat more complicated. There will be no contribution from particles ejected with $\gamma \geqslant \pi/2$ since they will drift in front of the spacecraft.[1] For $\gamma \leqslant \pi/2$, the particles will drift to the rear. The effect of drag is initially to slow the particles, which requires that the terms in the denominator are subtractive. This will come about naturally since S will be negative. The limit S_1 will be that specified in equation (46) with opposite sign, or the turning point of the particle, $S_1 = -\dot{S}_0^{\,2}/2\ddot{S}_0$, whichever occurs first. The returning particles will give a contribution equal to those going away. Therefore, the total will be twice the amount found by integrating equation (45) from $0 \leqslant \gamma \leqslant \pi/2$.

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1. Particles ejected with $\gamma > \pi/2$ will, of course, initially pass through the field of view as they make their initial loop around the spacecraft, but the long-term motion is being considered here.

Integrating equation (45) once yields

$$n_c = \int_0^{\pi/2} \frac{\dot{N}(\gamma)\, d\gamma}{\pi\, \xi^2 \cos\psi} \frac{2}{\ddot{S}_0} \left[(\dot{S}_0^2 + \ddot{S}_0 S_1)^{1/2} - |\dot{S}_0| \right] . \qquad (47)$$

For $\psi < \pi/2$, the above holds for all γ. If the spacecraft is emitting particles isotropically at a rate $\dot{N}_T/4\pi$ particles/sec/steradian, the number density can be found by replacing $\dot{N}(\gamma)$ by $\dot{N}_T d\Omega/4\pi$ or $\frac{1}{2}\dot{N}_T \sin\gamma$, multiplying by 2 and integrating from $0 \leqslant \gamma \leqslant \pi/2$. This results in

$$n_c = \frac{3\, \dot{N}_T\, \delta v}{4\, \pi\, \xi^2 \cos\psi\, \ddot{S}_0} \left\{ (2 + \delta v/v_\odot)^2 (A_1 - 1) + \left(\frac{\delta v}{v_\odot} \right)^2 (A_2 - 1) \right.$$

$$\left. + \frac{4\, \ddot{S}_0\, S_1}{9\, \delta v^2} \ln \left[\frac{(2 + \delta v/v_\odot)(1 + A_1)}{\delta v/v_\odot\, (1 + A_2)} \right] \right\} \qquad (48)$$

where

$$A_1 = \left[1 + \frac{4\, \ddot{S}_0\, S_1}{9\, \delta v^2\, (2 + \delta v/v_\odot)^2} \right]^{1/2}$$

and

$$A_2 = \left[1 + \frac{4\, \ddot{S}_0\, S_1}{9\, \delta v^2\, \delta v^2/v_\odot^2} \right]^{1/2} .$$

For $\psi > \pi/2$, the above holds for particles that do not turn before $-S_1$, or for which

$$-\frac{\dot{S}_0^2}{2\, \ddot{S}_0} < \begin{cases} \xi \cos\psi \\ -\sqrt{2\, r_\odot\, \xi} \end{cases} \quad \text{whichever greater.} \qquad (49)$$

For particles that turn before they move out of the viewing column, S_1 must be replaced by $-\dot{S}_0^2/2\ddot{S}_0$ in equation (47). This applies to all particles for which

$$\dot{S}_0^2 < 2\,\ddot{S}_0\,\xi\,|\cot\psi| \tag{50}$$

or for which

$$r_\odot k > \frac{3\pi\,[2\cos\gamma\,\delta v/v_\odot + (\delta v/v_\odot)^2]^2}{2\,\delta v/v_\odot\,|\cot\psi|} \tag{51}$$

At Skylab heights (420 km) the drag term $r_\odot k$ for a spherical particle with density of 1 gm/cm³ is 0.615/d (μm) where d(μm) is the diameter in microns. This means that all particles smaller than

$$d(\mu m) < \frac{0.615 \quad 2 \quad \delta v/v_\odot \quad |\cot\psi|}{3\pi\,[2\,\delta v/v_\odot \cos\gamma + (\delta v/v_\odot)^2]} \tag{52}$$

will turn before they move out of the viewing column. For $\gamma = 0$ and $\delta v/v_\odot = 0.001$ (7.8 m/sec),

$$d(\mu m) < 32.6\,|\cot\psi| .$$

Therefore, integrating equation (47) with $S_1 = -\dot{S}_0^2/2\ddot{S}_0$ applies to all cases for which the viewing angle is greater than $\psi \geqslant \cot^{-1}[-d(\mu m)/32.6]$. For 100-$\mu$m particles, this restricts ψ_{min} to greater than 162 deg. For 10-μm particles, ψ_{min} is only restricted to greater than 107 deg. Integrating equation (47) for $\psi > \psi_{min}$ and doubling to account for the particles after they turn yields

$$n_c = -\frac{3\sqrt{2}\,\dot{N}_T\,\delta v}{2\pi\,\xi^2\cos\psi\,\ddot{S}_0}\left[1 + \frac{\delta v}{v_\odot}\right] . \tag{53}$$

The intermediate cases, $\pi/2 < \psi < \psi_{min}$, must be treated by integrating equation (47) from 0 to γ_1 with S_1 given by $\xi \cot\psi$ and from γ_1 to $\pi/2$ with $S_1 = -\dot{S}_0{}^2/2\ddot{S}_0$ where γ_1 is the value for which equation (52) is an equality.

The column densities for a spacecraft emitting particles isotropically are plotted as a function of velocity in Figures 9, 10, and 11 for look angles $\psi = 0, \pi/2$, and π. The column densities are normalized by the column density that would result in the absence of orbital and drag effects, equation (30). The drag-free case is obtained by taking the lim as $\ddot{S}_0 \rightarrow 0$ of equation (47). This yields

$$n_c = \frac{\dot{N}_T \quad S_1}{6 \pi \xi^2 \cos \psi \; \delta v} \ln \left(\frac{2 + \delta v / v_\odot}{\delta v / v_\odot} \right) \quad . \tag{54}$$

For the $\psi = 0$ case, it may be seen that orbital effects on the column density are not important unless $\delta v < 4$ m/sec. For small particles which produce the highest scattering efficiency, drag effects rapidly clear the particles even if their initial velocity is low. The fact that at high ejection velocities the column density from small particles exceeds the no-drag case is because of the factor of two resulting from the drag eventually causing all particles to move into the field of view.

For the $\psi = \pi/2$ case, the orbital effects are negligible for all particles with velocities more than a few tens of centimeters per second. With drag effects, the smaller particles are accelerated out of the field of view so rapidly that they do not contribute to the column density regardless of their initial velocity.

For the direction $\psi = \pi$ the effect of drag is to greatly increase the column density, particularly for the larger particles. This is because of the increased number density brought about by the particle's low velocity in the vicinity of the turning point. Equation (53) is restricted to values of δv small enough to cause all particles to turn before $\sqrt{2 r_\odot \xi}$. These velocities are indicated by the termination of the plotted lines. As δv is made large, these curves will approach a value of twice the drag-free case.

Finally, Figure 12 shows the relative column density for the Apollo in lunar orbit. It was assumed that the Apollo was an isotropic source with an effective radius of 2 m. Shown for comparison is the $\psi = 0$, 90 deg no-drag cases for Skylab in a 420-km earth orbit. Note that in lunar orbit, orbital effects are not significant for ejection velocities more than 0.56 m/sec.

SUMMARY

The effects of orbital dynamics and drag on small particles released from spacecraft in circular orbits have been calculated. It is shown that such particles will become distributed along the orbital path and that the net separation rate between the particle and the spacecraft is given by

$$\bar{\dot{S}} = \frac{3}{2} v_{\Theta} \left| 2 \frac{\delta v}{v_{\Theta}} \cos \gamma + \left(\frac{\delta v}{v_{\Theta}} \right)^2 - \frac{2 C_D A \rho_a v_{\Theta} t}{m} \right| .$$

where v_{Θ} is orbital velocity, δv is ejection velocity, γ is the angle of ejection relative to the orbital velocity vector, C_D is the drag coefficient, A/m is the area to mass ratio of the particle, ρ_a is the atmospheric density, and t is the time after ejection. Particles ejected randomly will be confined by orbital dynamics to a tube of cross sectional area given approximately by $\pi \xi^2$ where $\xi = r_{\Theta} \delta v / v_{\Theta}$ and r_{Θ} is the orbit radius vector.

The column densities resulting from a continuous isotropic release of particles in circular orbit from the Skylab were compared to a similar release in the absence of orbital and drag effects. It was found that substantial increases in column density along the antivelocity vector can result from orbital dynamics and drag forces for particles larger than 10 μm ejected at velocities less than 4 m/sec. Drag and orbital effects significantly reduce the column density when viewing at 90 deg to the velocity vector. Drag forces produce rapid clearing of particles smaller than 100 μm in front of the spacecraft and subsequently reduce the column density when viewing along the velocity vector. Larger particles ejected at velocities less than 1 m/sec can cause some increase in the column density because of their longer time in the field of view resulting from their confinement along the orbital path.

Finally, the case for an Apollo spacecraft orbiting the moon was considered. Despite the lack of drag, there is negligible accumulation of particles even in a column along the velocity vector for ejection velocities larger than 0.5 m/sec.

George C. Marshall Space Flight Center
National Aeronautics and Space Administration
Marshall Space Flight Center, Alabama, September 1973
502-21-28-0000

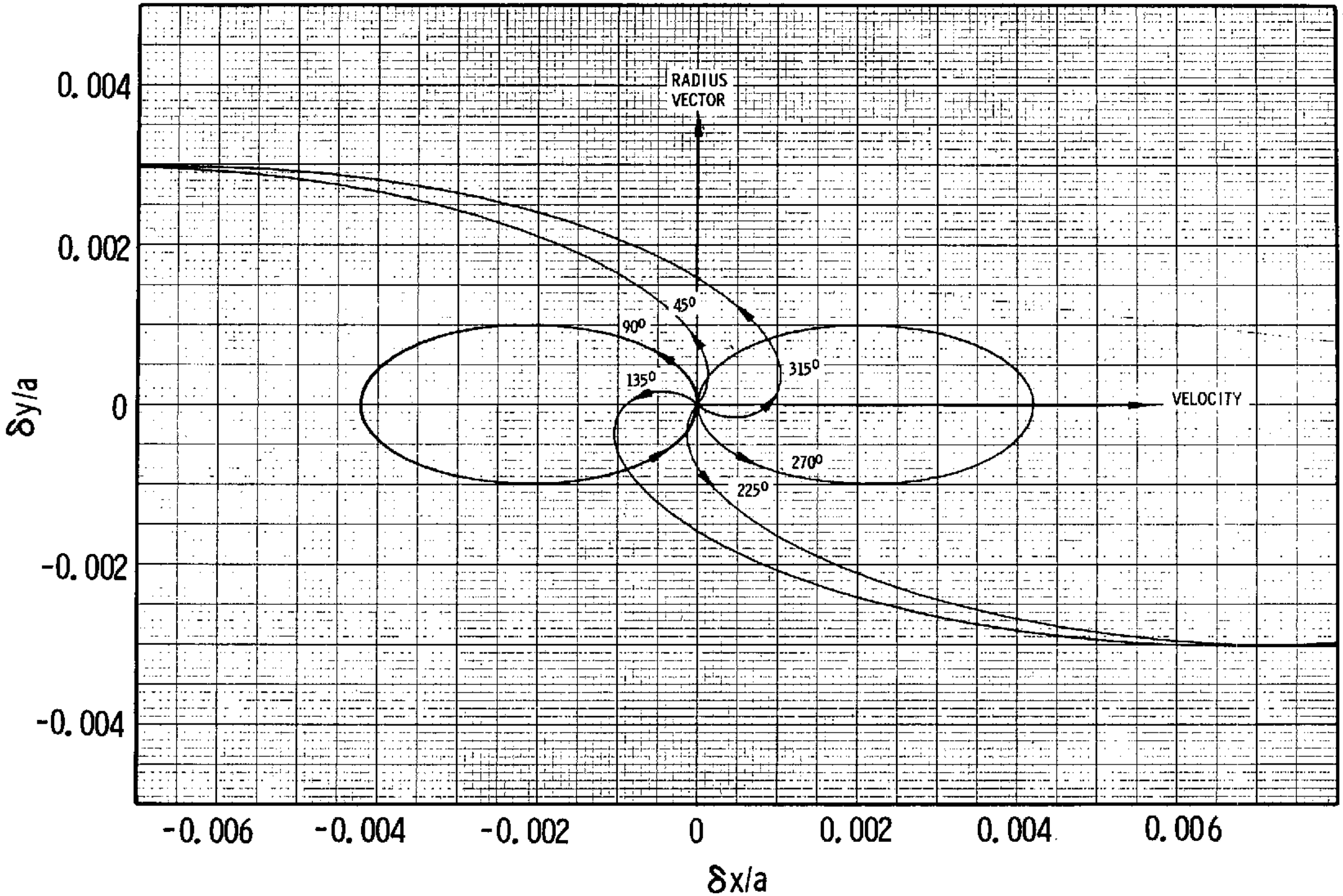

Figure 1. Particle trajectories for various ejection angles at $\delta v = 0.001\ v_\Theta$ for no drag.

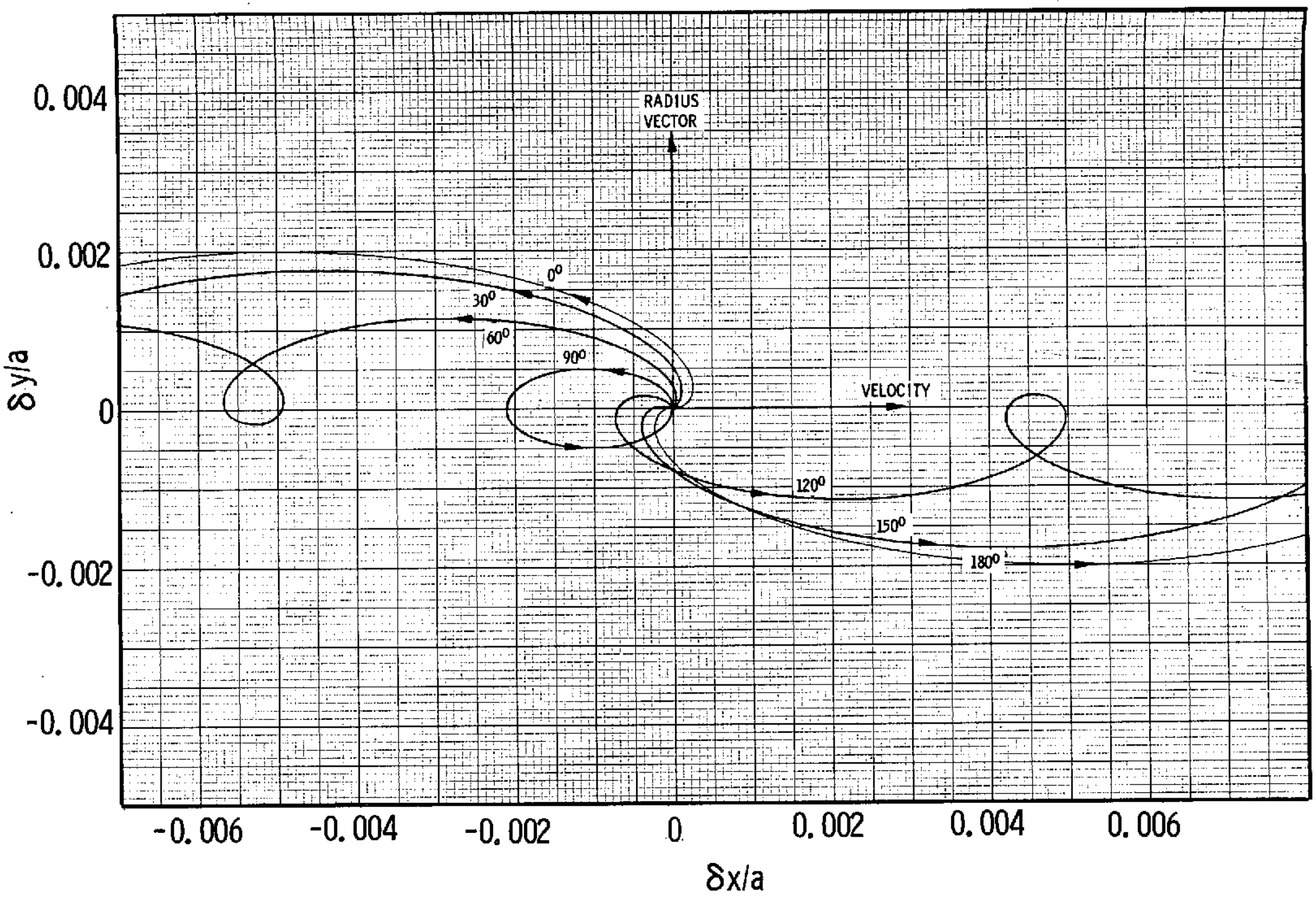

Figure 2. Particle trajectories for various ejection angles at $\delta v = 0.0005\ v_{\odot}$ for no drag.

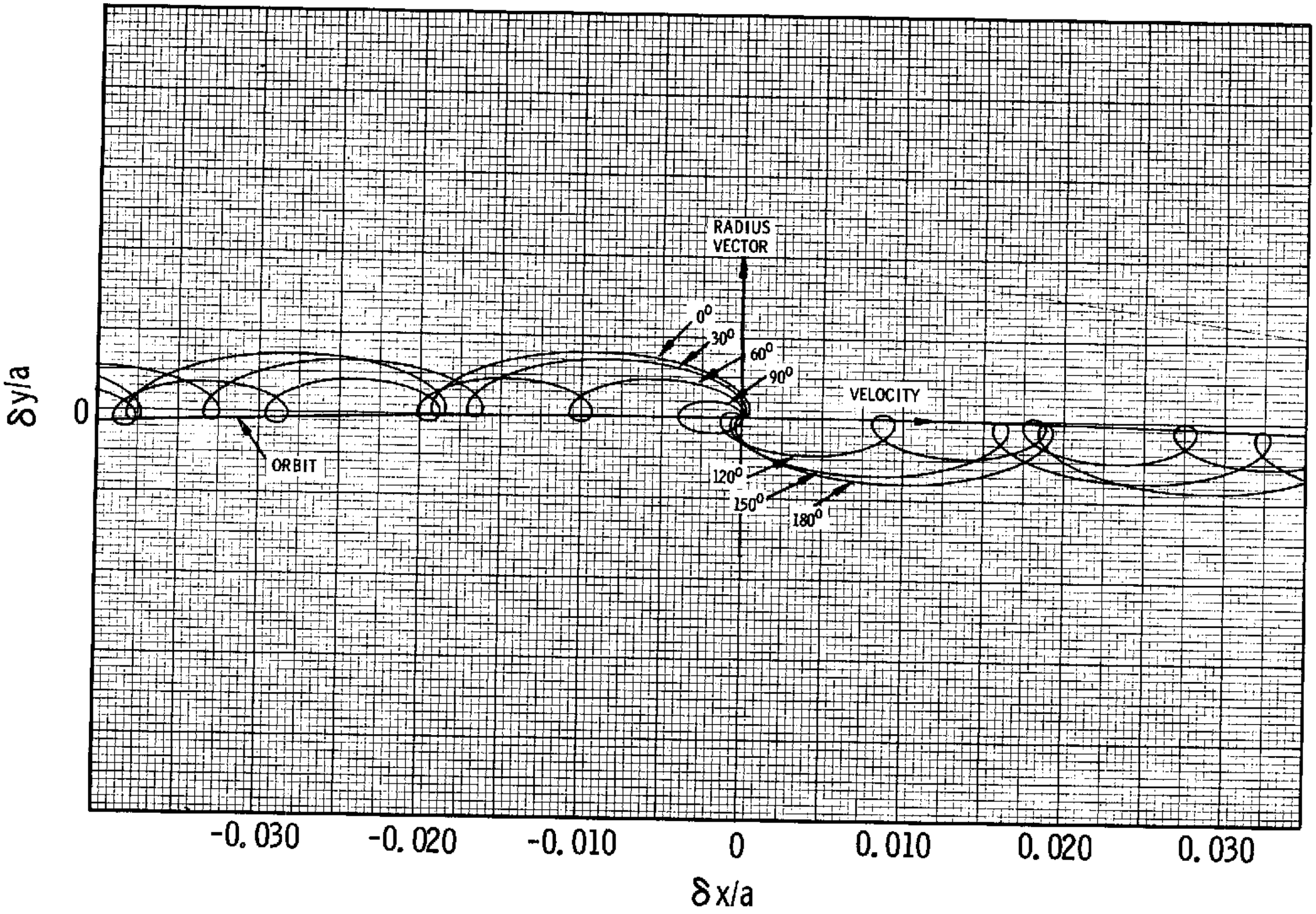

Figure 3. Particle trajectories of Figure 2 with reduced scale to show the relative motion of the particles along the orbit.

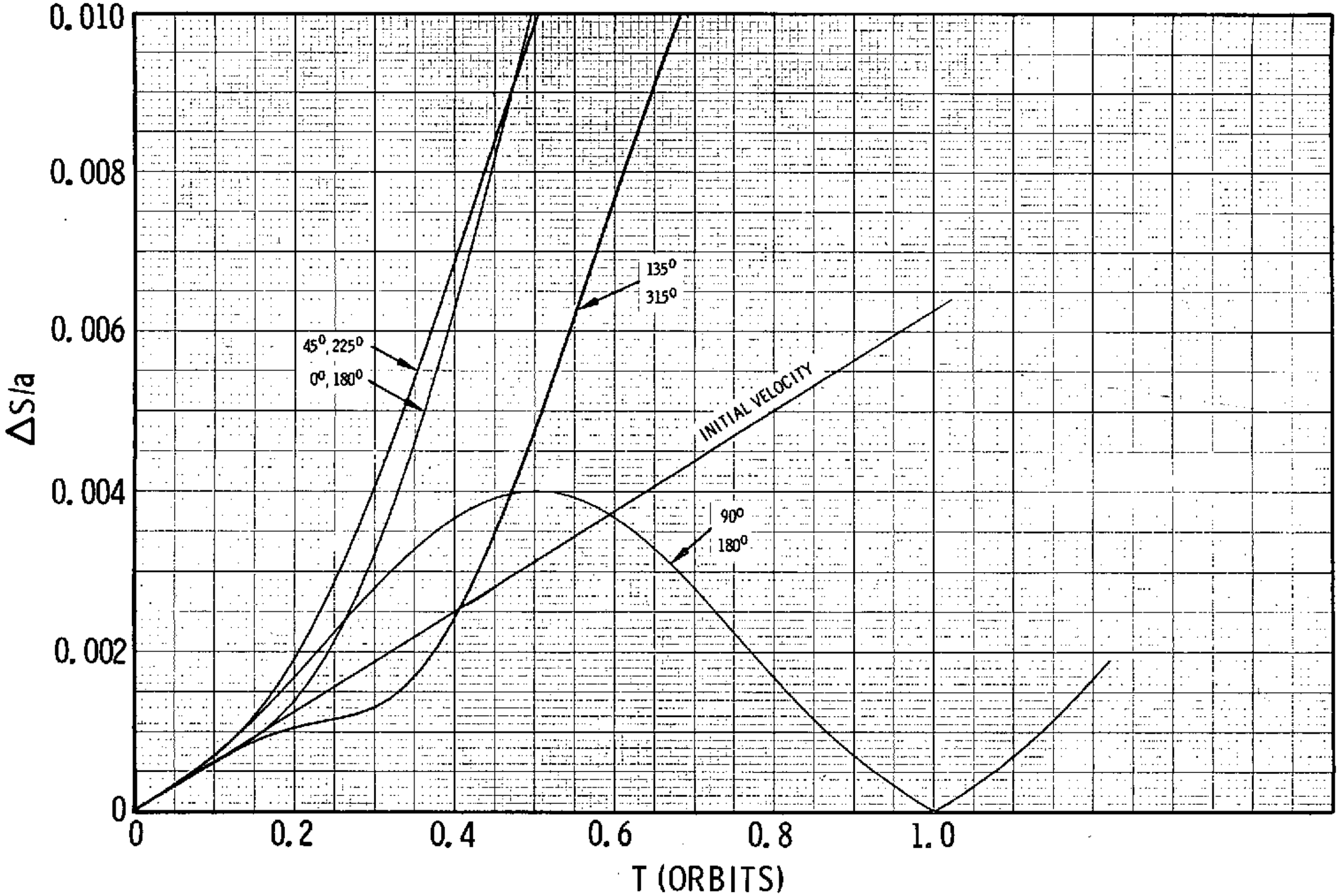

Figure 4. Separation distance between particle and spacecraft. (Initially the particles leave at their given separation velocity, but after ~ 0.1 orbit the orbital dynamics may either accelerate or retard the separation rate, depending on the ejection angle.)

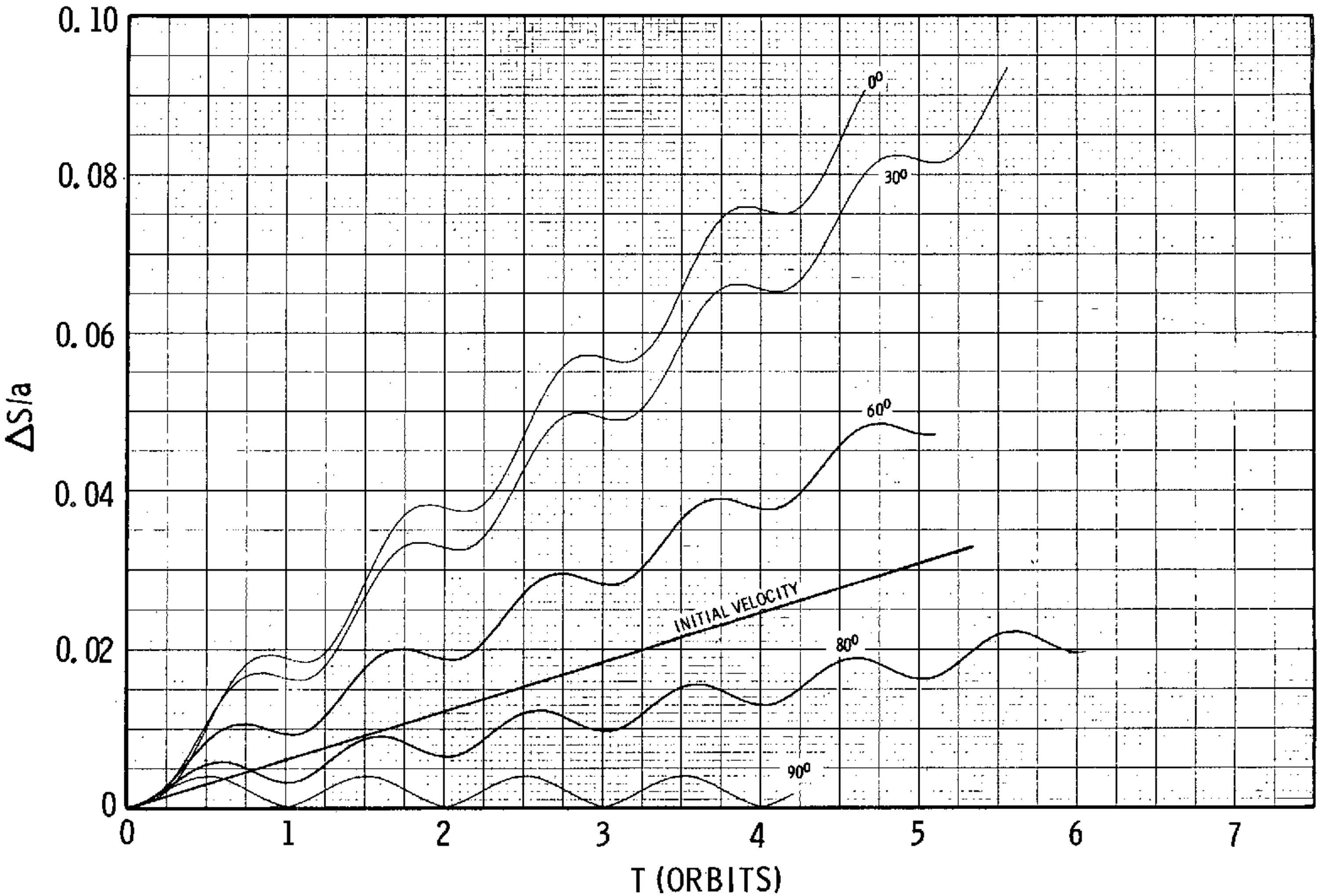

Figure 5. Separation distance between particle and spacecraft shown in Figure 4 but with reduced scale to show long-term behavior.

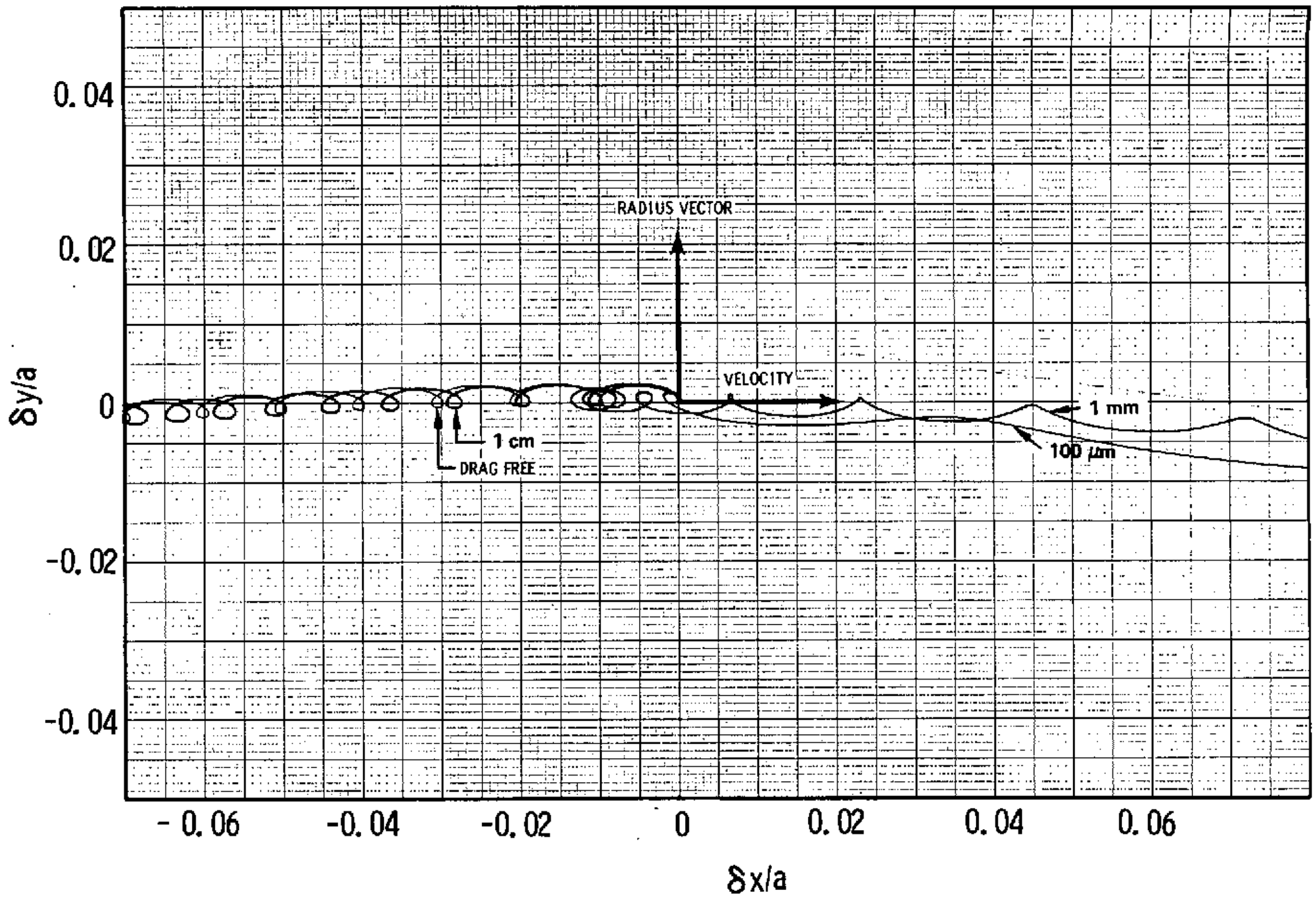

Figure 6. Particle trajectories for γ = 60 deg, $\delta v = 0.001\ v_{\odot}$, with drag at 420 km. (The 100-μm particle turns and is rapidly swept away. The 1-mm particle moves initially to the rear of the spacecraft, turns after several orbits, and then moves in front and separates rapidly. The 1-cm trajectory is similar to the drag-free case, but the effect of drag is seen to retard the separation. This particle will also eventually turn and move in front of the spacecraft.)

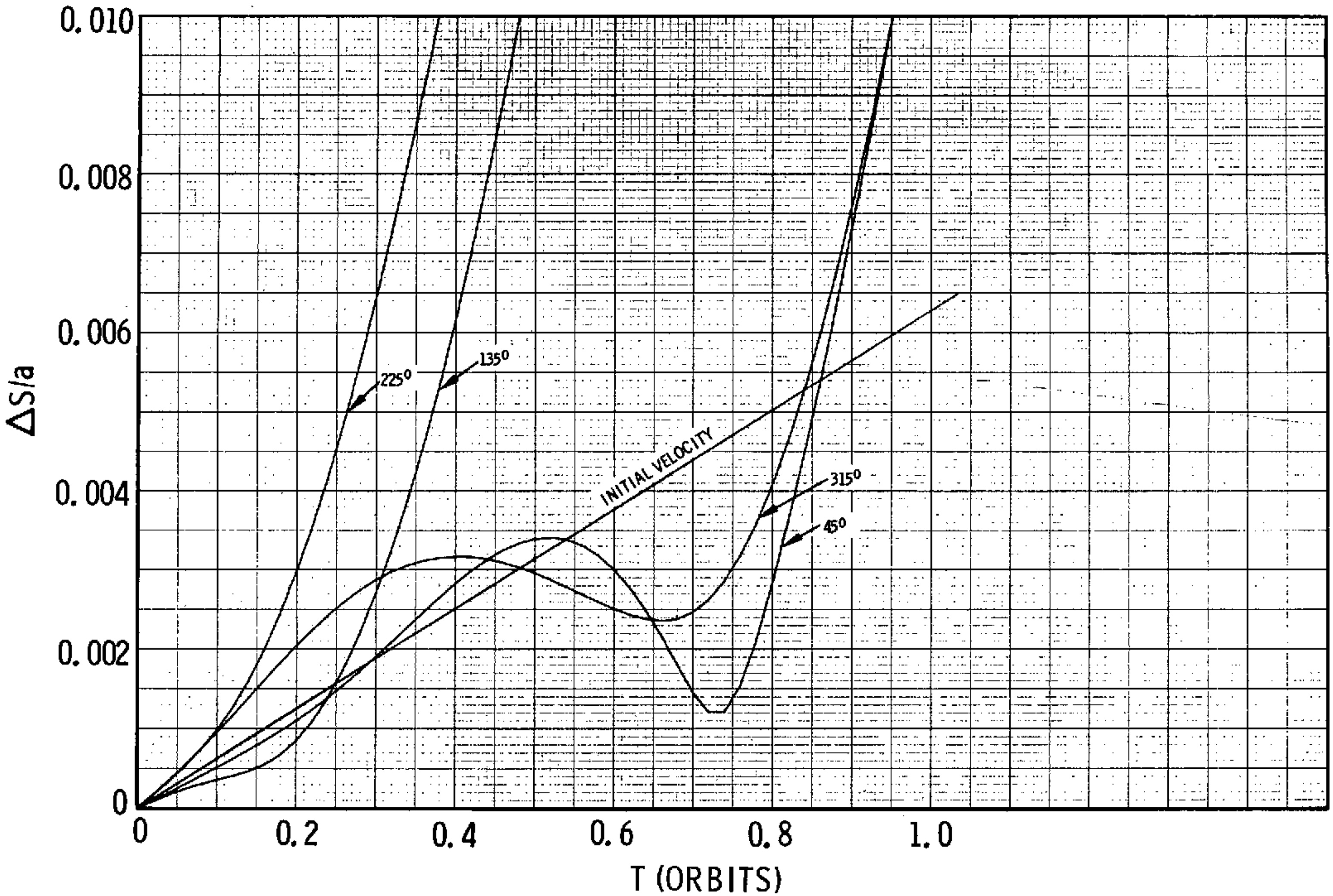

Figure 7. Separation distance between particle and spacecraft for the same conditions as in Figure 4 except that drag for a 100-μm particle is considered.

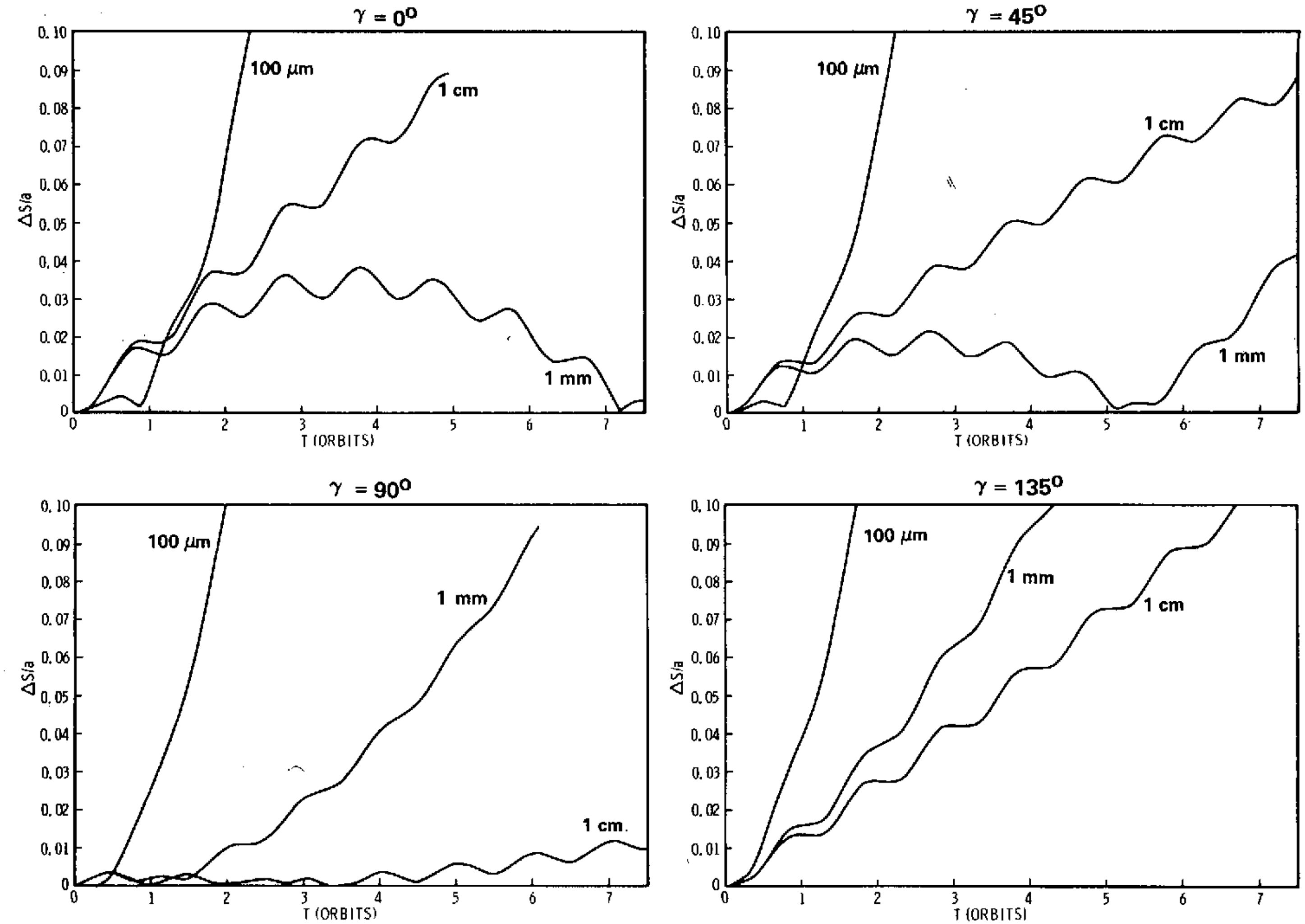

Figure 8. Separation distances for 1-μm, 1-mm, and 1-cm particles ejected at $\delta v = 0.001\ v_{\odot}$ at various γ. (The fact that clearing time is decreased by projecting particles rearward is clearly seen.)

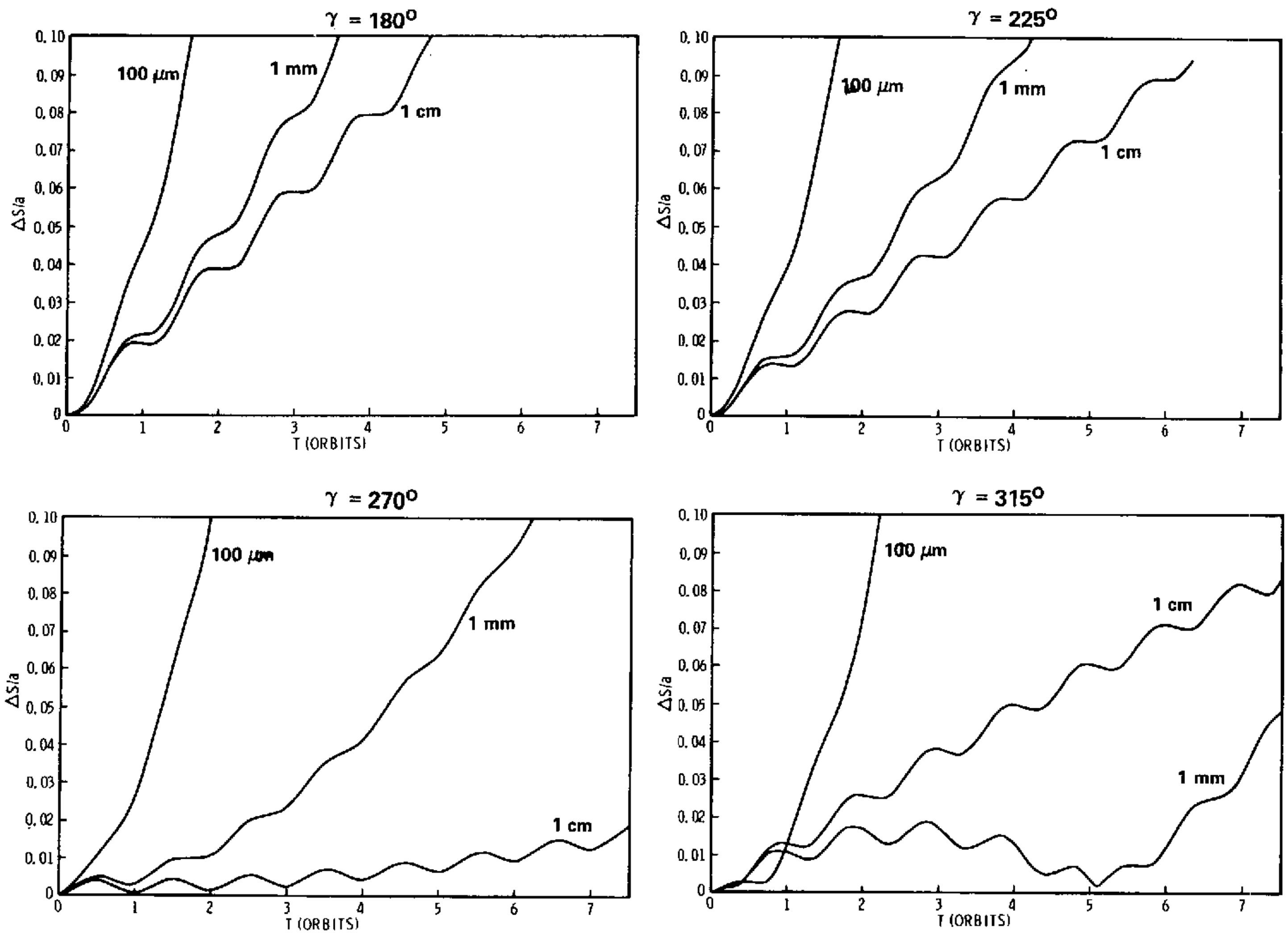

Figure 8. (Concluded)

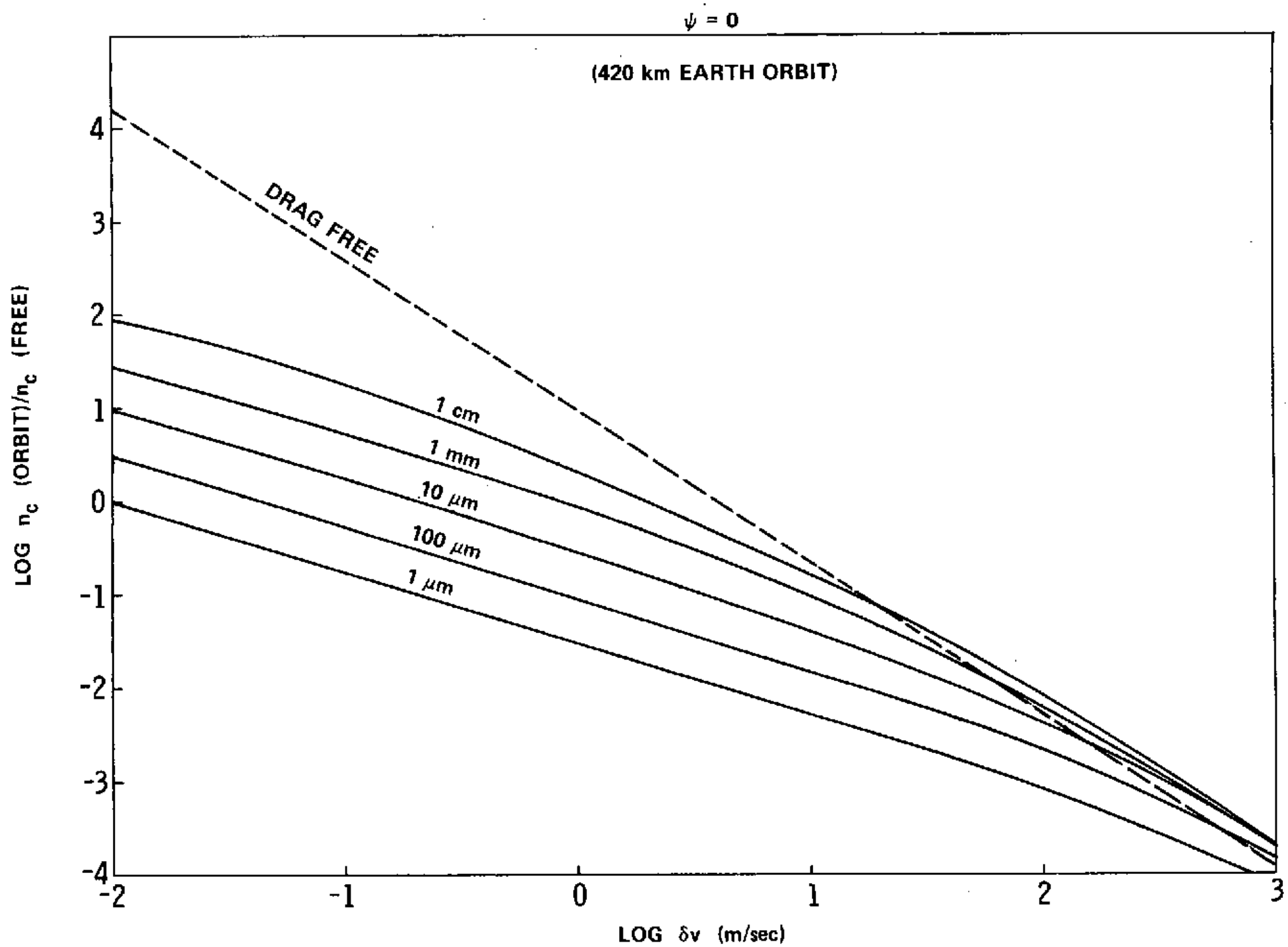

Figure 9. Column densities from particles released continuously and isotropically with velocity δv in a 420-km circular orbit looking along the velocity vector compared with column densities that would result from a free expansion of the same particles. (There is no significant accumulation from orbital effects for release velocities larger than 4 m/sec. Smaller particles are rapidly cleared by drag and particle sizes below 100 μm will not accumulate significantly for velocities above 18 cm/sec.)

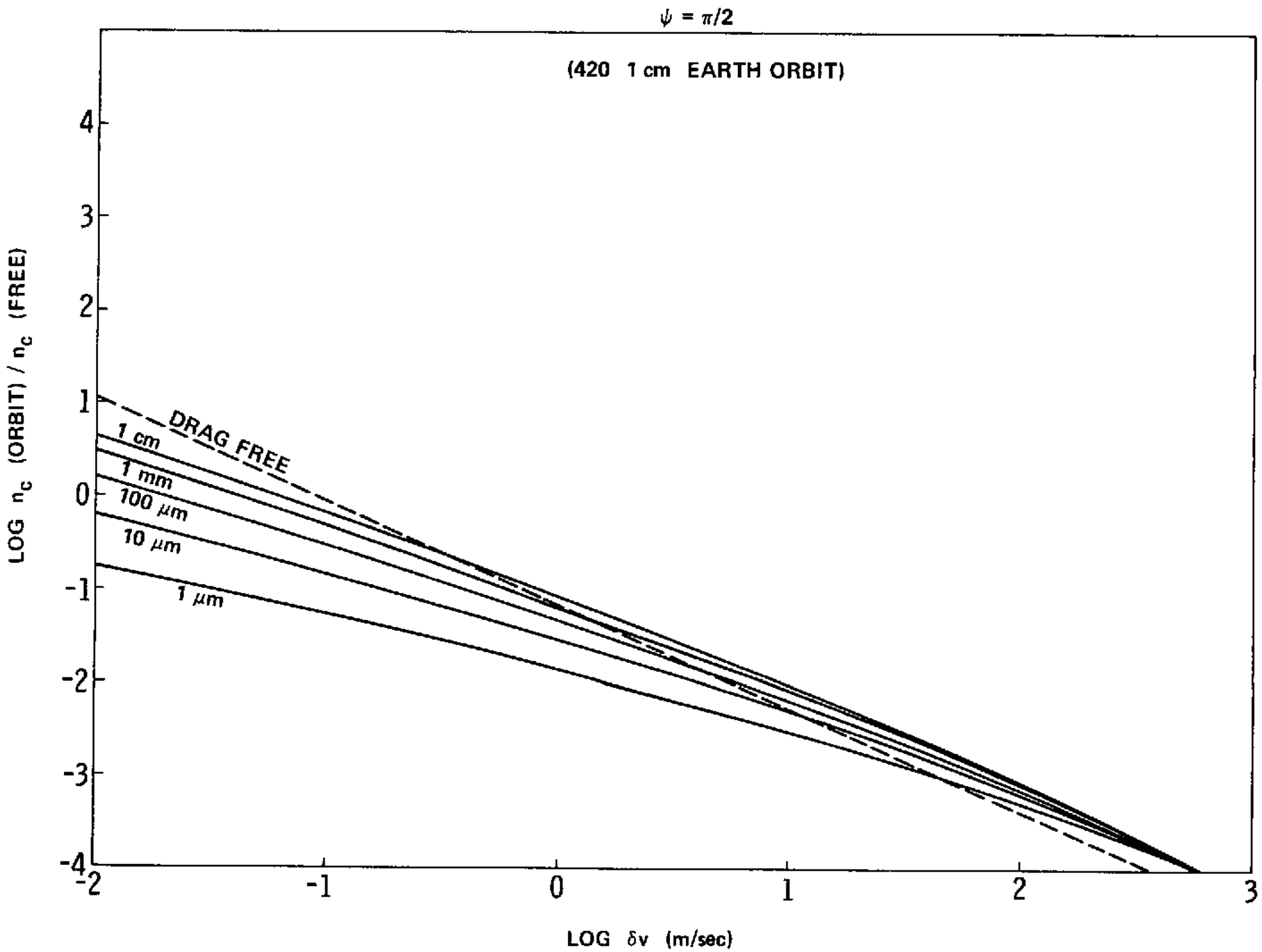

Figure 10. Similar to Figure 9 except that the viewing angle is 90 deg to the velocity vector. (The accumulation from orbital effects is negligible even in the absence of drag for release velocities in excess of 10 cm/sec.)

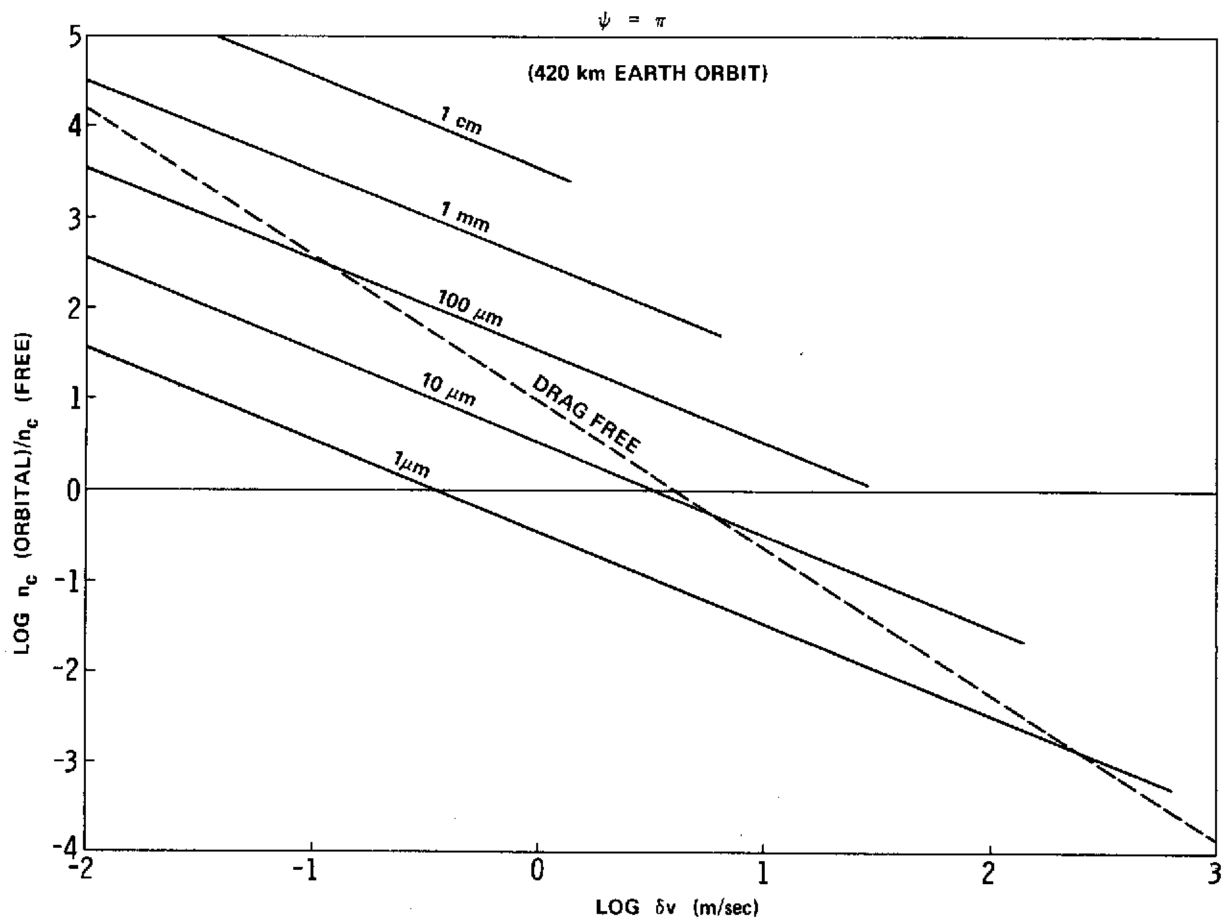

Figure 11. Similar to Figure 9 except that the viewing direction is opposite to the velocity vector. (In this case substantial accumulation can take place because the particles moving rearward are retarded by drag and eventually accelerated back toward the spacecraft. The lines are terminated where equation (53) no longer applies because particles with larger δv will move out of the field of view before they turn. The column densities for larger δv will eventually approach twice the drag-free value.)

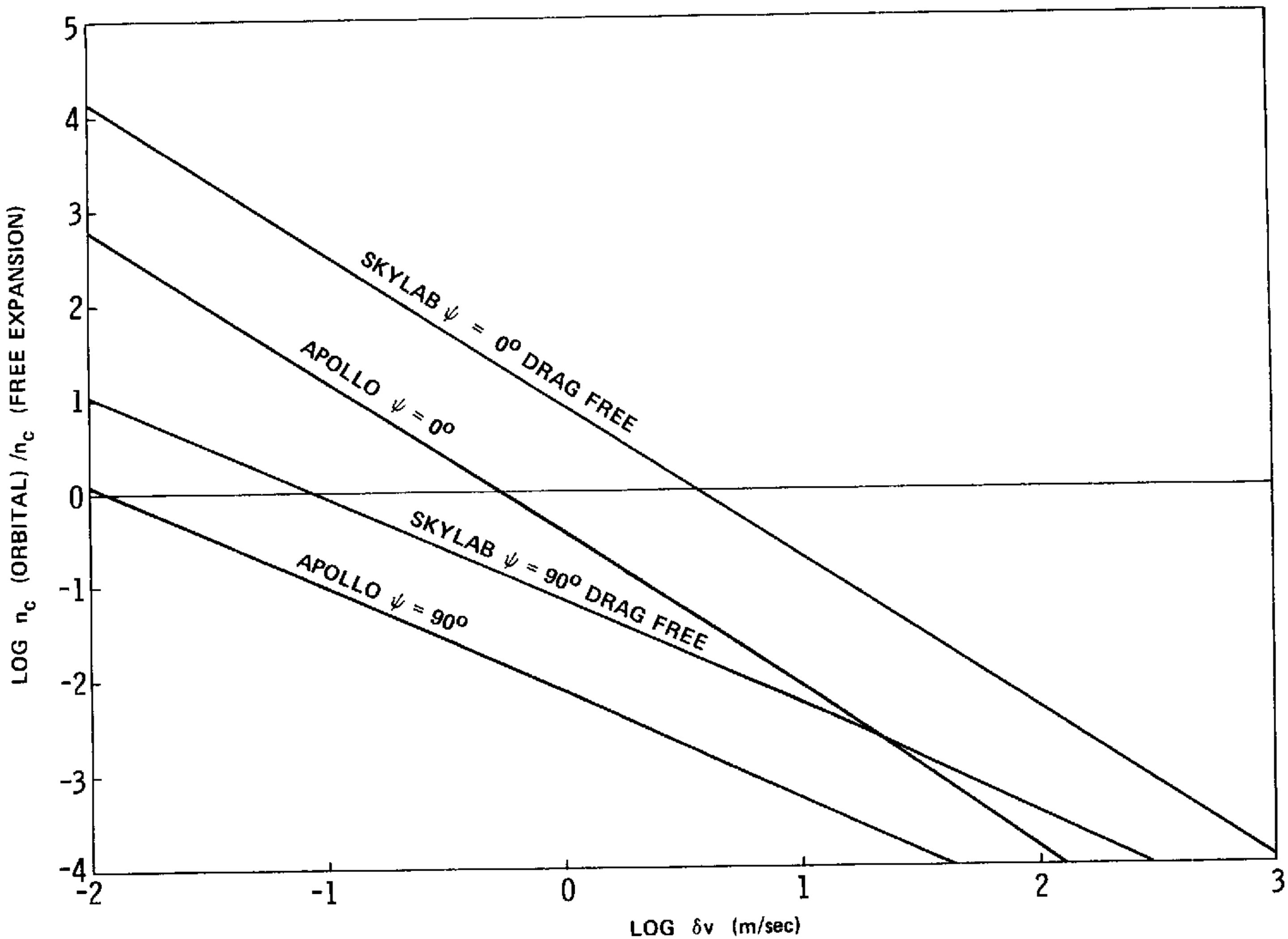

Figure 12. Column densities from orbital effects (no drag) in earth orbit (Skylab) compared with column densities from the same particle release in lunar orbit (Apollo).

NASA-Langley, 1974