# ANALYSIS OF MULTI-LAYERED FILMS

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## ABSTRACT

Dye densities of multi-layered films are determined by applying a regression analysis to the spectral response of the composite transparency. The amount of dye in each layer is determined by fitting the sum of the individual dye layer densities to the measured dye densities.  From this, dye content constants are calculated.

Methods of calculating "equivalent exposures" are discussed. "Equivalent exposures" are a constant amount of energy over a limited band-width that will give the same dye content constants as the real incident energy.  Methods of using these equivalent exposures for analysis of photographic data are presented.

## INTRODUCTION

All the information recorded at each point on multi-layered films can be characterized by one constant for each layer of the film.  The constant for each layer is the number a normalized dye density must be multiplied by in order to equal the dye in that layer for a particular scene response.

All multi-layered films have characteristic colors associated with each layer of the film.  The spectral properties of the dyes that make up these colors are well known[1], though not necessarily stable with time. An example of the spectral characteristics for Kodak Ektachrome MS Aerographic film type 2448 is given in Figure 1.  In this case the film
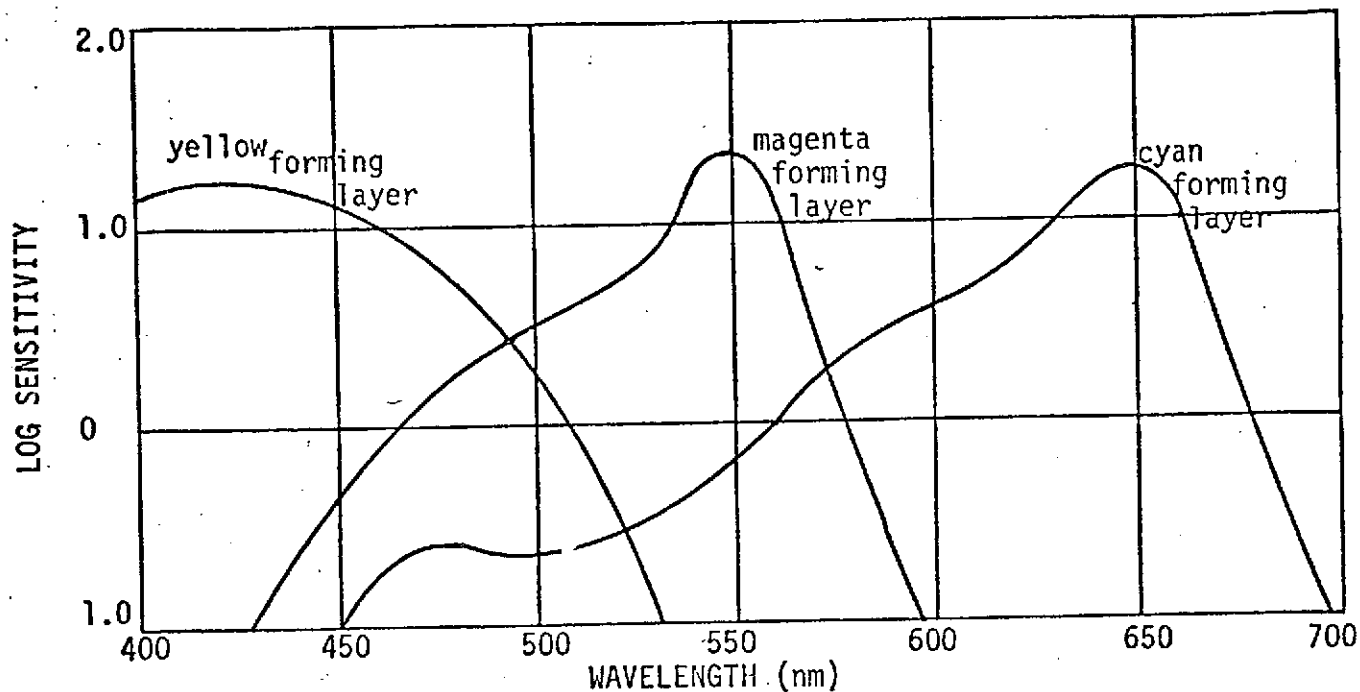
## SPECTRAL SENSITIVITY CURVES



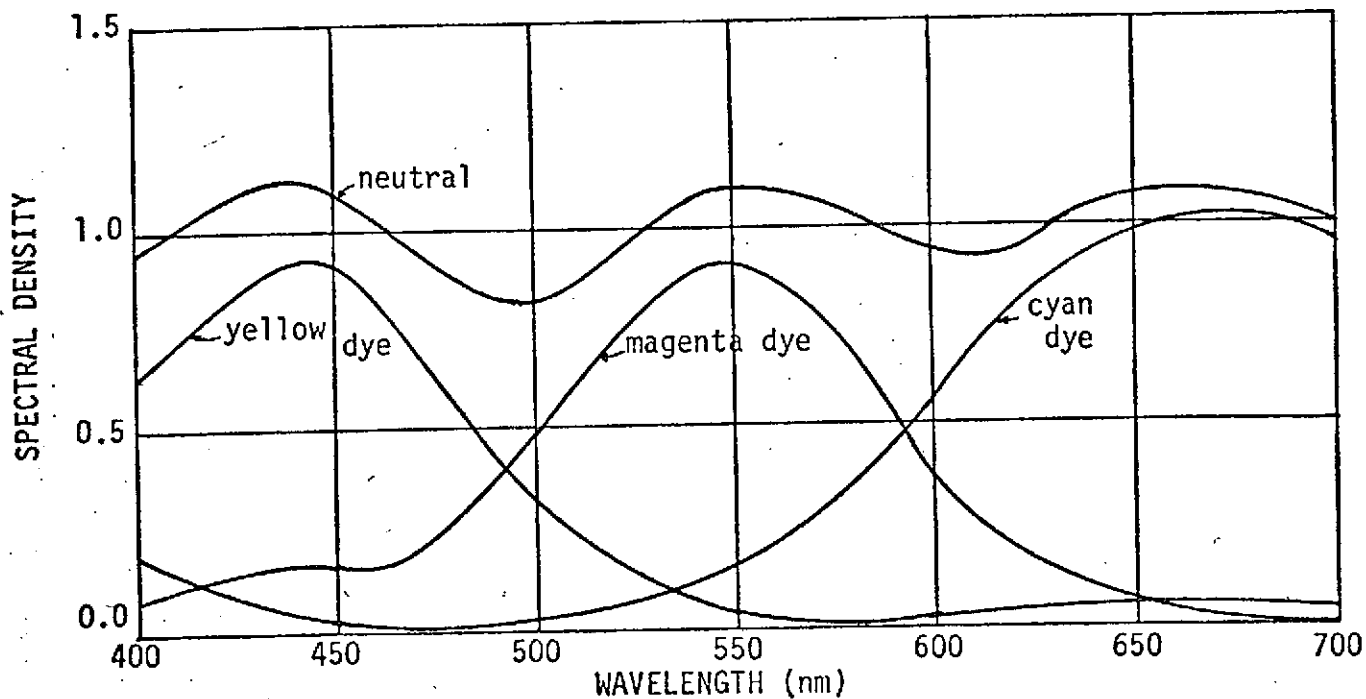## SPECTRAL DYE DENSITY CURVES



Figure 1. Spectral Characteristics of Kodak color film type 2448.

has three layers as do most multi-layered films. The colors formed in the three layers are yellow, magenta and cyan. The properties of this film are well known.[2] The method of analysis is developed using type 2448 as an example though the methods can be easily generalized to any multi-layered film.

The spectral response or color at each point on a transparency is due to the dyes in the three layers at that point. The spectral properties of the dyes in each of the layers at a point can be represented by a constant (one for each dye) times a normalized dye density at each wavelength. Figure 1 shows the three dye density curves for film type 2448. Thus the spectral properties of the transmitted light through the developed film transparency are determined by a linear superposition of constants times these dye density curves. Mathematically this can be written:

$$D(x,y,\lambda) = K_1(x,y)Y(\lambda) + K_2(x,y)M(\lambda) + K_3(x,y)C(\lambda) + B(\lambda) \qquad (1)$$

and
$$I(x,y,\lambda) = I_o(x,y,\lambda)10^{-D(x,y,\lambda)} \qquad (2)$$

where    $D(x,y,\lambda)$ is the density at any point as a function of x, y and of wavelength ($\lambda$)

Y($\lambda$) is the normalized dye density for the yellow layer as a function of wavelength

M($\lambda$) is the normalized dye density for the magenta layer as a function of wavelength

C($\lambda$) is the normalized dye density for the cyan layer as a function of wavelength

B($\lambda$) is the density of the film base as a function of wavelength

$K_1(x,y)$, $K_2(x,y)$, $K_3(x,y)$ are constants related to the amount of dye on the three respective film layers and only a function of position on the film format (dye content constants)

$I(x,y,\lambda)$ is the transmitted light through the transparency at

each point on the film and is a function of wavelength

$I_0(x,y,\lambda)$ is the incident light on the transparency

At any given point, the quantities experimentally determined are usually the total transmitted light and the incident light. From this the density can be calculated for that point. Because these quantities are a function of the wavelength of the light (color), they must be measured at a single wavelength. Actually, for this treatment, measurements must be acquired for at least three different wavelengths at the point of interest on the transparency. Three equations can be written and solved for the three dye content constants. It is then possible to determine an equivalent integral spectral density or analytical spectral density[4] at any wavelength.

The transmittance of a typical film base is represented in Figure 2. Predicted and observed values of transmittance for colored objects are shown in Figures 3, 4 and 5.

## Equivalent Energies

An equivalent energy is defined as a constant amount of energy throughout the spectrum that will give the same dye constant in a film layer as the real incoming radiation. There will be a separate equivalent energy for each layer of the film. These equivalent energies can be calculated by using the known sensitivities of the film layers.

Let: $S_i(\lambda)$ be the sensitivity of the $i^{th}$ layer of the film as

a function of wavelength

$\epsilon(\lambda)$ be the radiation falling at a point on the film (ergs/cm$^2$/ )

$E_i$ be the equivalent energy for the $i^{th}$ film layer

$i = 1$ for the yellow layer

# TRANSMITTANCE [4] OF FILM BASE
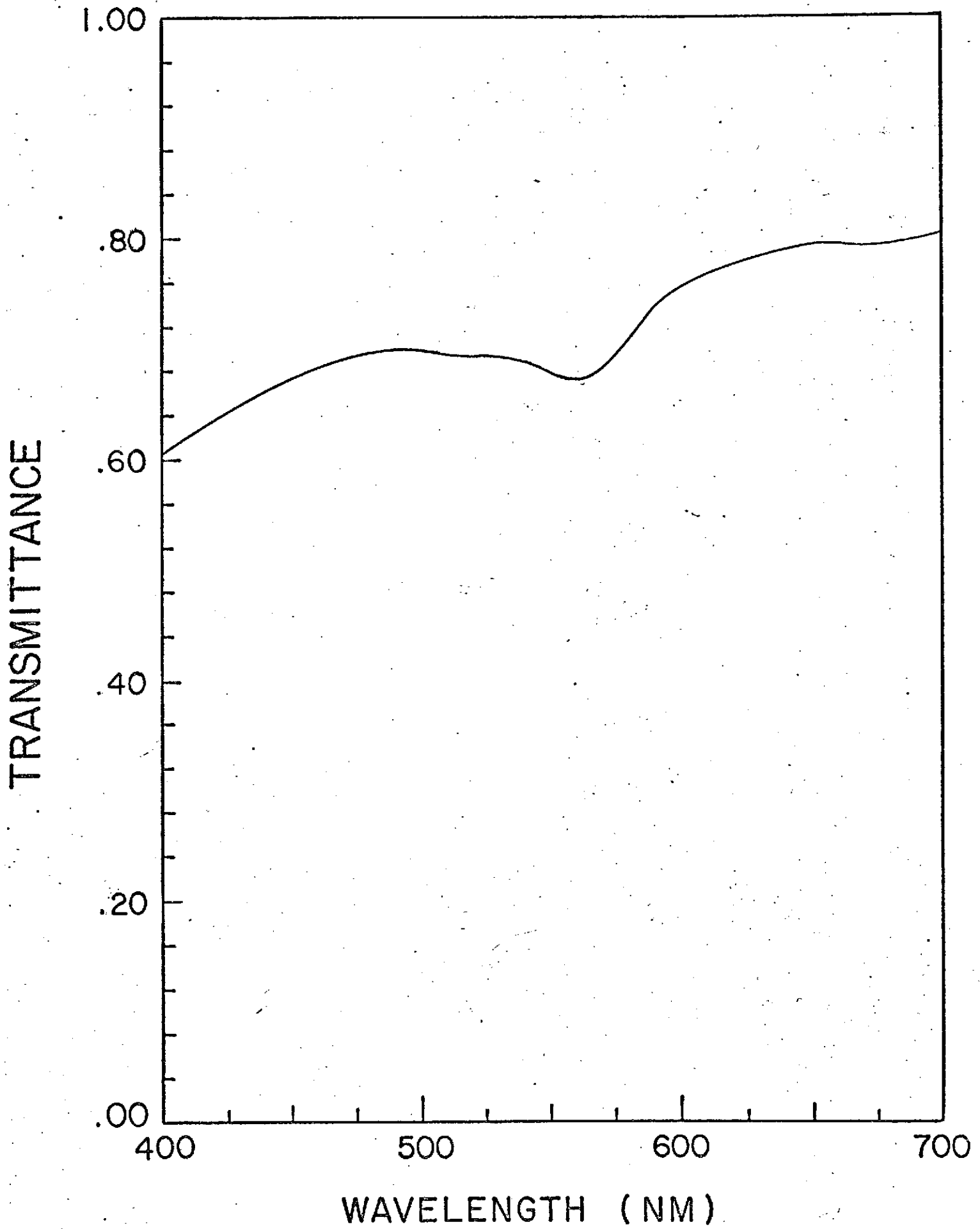


Figure 2

WAVELENGTH (NM)

# FRAME 3    BLUE OBJECT

Figure 3.    Transmittance vs. wavelength of a blue object.



— predicted values

● measured values used in prediction

⊙ measured values not used in prediction

Dye Content Constants

$K_1 = 1.14$

$K_2 = 1.40$

$K_3 = 2.29$
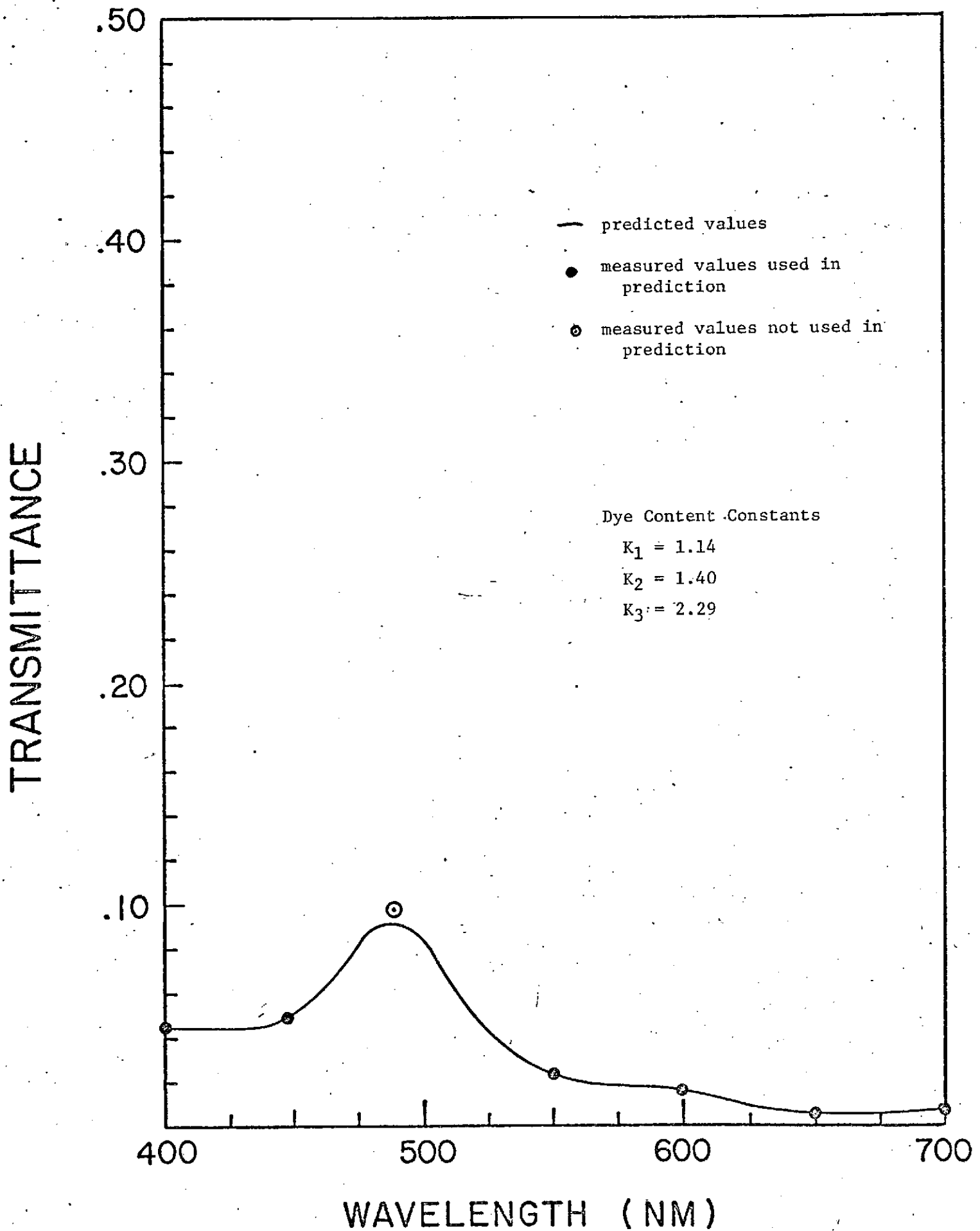
# FRAME 9 YELLOW OBJECT

Figure 4. Transmittance vs. wavelength of a yellow object.



Dye Content Constants
$K_1 = 1.939$
$K_2 = .426$
$K_3 = .289$

Legend:
— Predicted values using 7 measured values
--- Predicted values using 3 measured values
● Measured values
○ Measured values not used in solid line fit
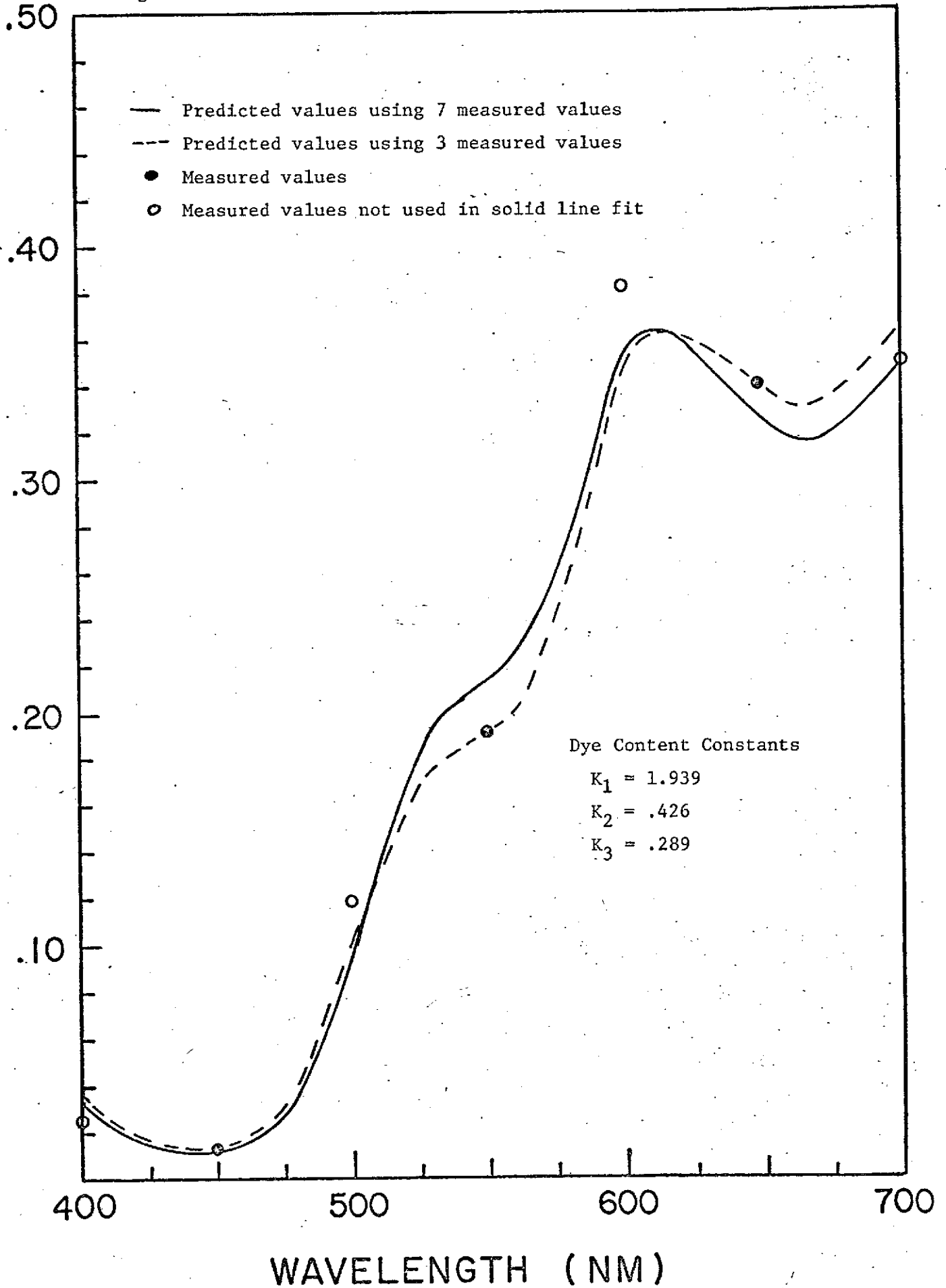
# FRAME 9 RED OBJECT

Figure 5. Transmittance vs. wavelength of a red object.



— Predicted values using 7 measured values

--- Predicted values using 3 measured values

● Measured values

○ Measured values not used in solid line fit

Dye Content Constants

$K_1 = 2.13$

$K_2 = 2.14$

$K_3 = .64$

TRANSMITTANCE

WAVELENGTH (NM)

7<
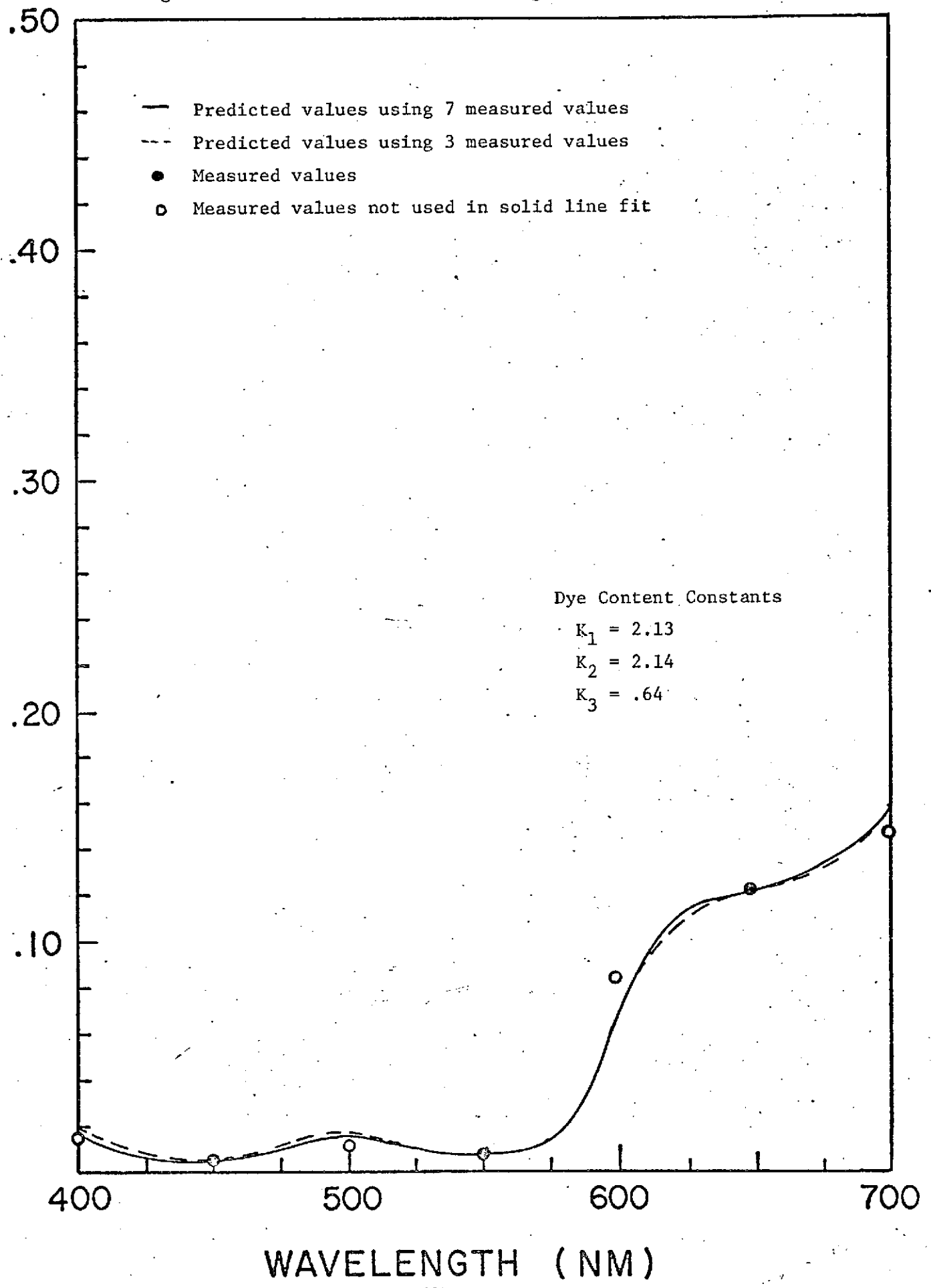
i = 2 for the magenta layer

i = 3 for the cyan layer

Mathematically we can write:

$$\int E_i S_i(\lambda)d\lambda = \int \epsilon(\lambda)S_i(\lambda)d\lambda \tag{3}$$

where the integral is over all wavelengths.

The units of $S_i(\lambda)$ are in $1/(ergs/cm^2)$ to give a density of 1 with standard processing. Thus the integral of radiation times the sensitivity will give the fraction of exposure required to give a final density of 1 with standard processing for that radiation.

Since $E_i$ on the left of (3) is a constant we can write (3) as:

$$E_i = \frac{\int \epsilon(\lambda)S_i(\lambda)d\lambda}{\int S_i(\lambda)d\lambda} \tag{4}$$

In essence, this $E_i$ is a weighted average of incoming radiation over the sensitivity of the layer.

In themselves these E's are not very useful. They represent three different constant energy spectrums. More useful would be one equivalent energy spectrum that can replace the incoming radiation spectrum and give the same dye densities. Using the same type of treatment, this equivalent energy spectrum can be found in terms of the E's and thus in terms of incident energy spectrum $\epsilon(\lambda)$ and the sensitivity of the film layer, $S_i(\lambda)$.

For color and color IR films these three equivalent energies are designated as EB, EG and ER and EG, ER and EI respectively. An example of these equivalent energies is graphically represented in Figure 3. EB* is a constant amount of energy between .4 and .5 microns, EG is a constant amount between .5 and .6 microns, ER is a constant amount between .6 and .7 microns, and EI is a constant amount between .7 and .9 microns.

The actual values of the three equivalent energies for each of the film types can be easily found by substituting the equivalent energy spectrum

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*Kodak filter 2B must be used for it if EB is to correspond to the range .4 to .5μ.

into equation (4) in the place of $\epsilon(\lambda)$, being careful to integrate over the proper limits. In the case of color film this reduces to:

$$E_i = \frac{\int_{.4}^{.5} EB\, S_i(\lambda)d\lambda + \int_{.5}^{.6} EG\, S_i(\lambda)d\lambda + \int_{.6}^{.7} ER\, S_i(\lambda)d\lambda}{\int_{.4}^{.7} S_i(\lambda)d\lambda} \qquad (5)$$

Equation (5) is really 3 equations that can be written explicitly as:

$$E_1 = A'_{11}EB + A'_{12}EG + A'_{13}ER \qquad (6)$$

$$E_2 = A'_{21}EB + A'_{22}EG + A'_{23}ER \qquad (7)$$

$$E_3 = A'_{31}EB + A'_{32}EG + A'_{33}ER \qquad (8)$$

where

$$A'_{11} = \frac{\int_{.4}^{.5} S_1(\lambda)d\lambda}{\int_{.4}^{.7} S_1(\lambda)d\lambda} \qquad A'_{12} = \frac{\int_{.5}^{.6} S_1(\lambda)d\lambda}{\int_{.4}^{.7} S_1(\lambda)d\lambda} \qquad A'_{13} = \frac{\int_{.6}^{.7} S_1(\lambda)d\lambda}{\int_{.4}^{.7} S_1(\lambda)d\lambda}$$

$$A'_{21} = \frac{\int_{.4}^{.5} S_2(\lambda)d\lambda}{\int_{.4}^{.7} S_2(\lambda)d\lambda} \qquad A'_{22} = \frac{\int_{.5}^{.6} S_2(\lambda)d\lambda}{\int_{.4}^{.7} S_2(\lambda)d\lambda} \qquad A'_{23} = \frac{\int_{.6}^{.7} S_2(\lambda)d\lambda}{\int_{.4}^{.7} S_2(\lambda)d\lambda}$$

$$A'_{31} = \frac{\int_{.4}^{.5} S_3(\lambda)d\lambda}{\int_{.4}^{.7} S_3(\lambda)d\lambda} \qquad A'_{32} = \frac{\int_{.5}^{.6} S_3(\lambda)d\lambda}{\int_{.4}^{.7} S_3(\lambda)d\lambda} \qquad A'_{33} = \frac{\int_{.6}^{.7} S_3(\lambda)d\lambda}{\int_{.4}^{.7} S_3(\lambda)d\lambda}$$

and   $S_1(\lambda)$ is the sensitivity of the yellow layer

   $S_2(\lambda)$ is the sensitivity of the magenta layer

   $S_3(\lambda)$ is the sensitivity of the cyan layer

The nine "A primes" (above) are just constants dependent only on the film sensitivity. The equations (6) through (8) can be solved for EB, EG and ER in terms of $E_1$, $E_2$ and $E_3$ by several methods.[5] The solutions to equations (6), (7) and (8) for EB, EG and ER in terms of $E_1$, $E_2$ and $E_3$ can be written as:
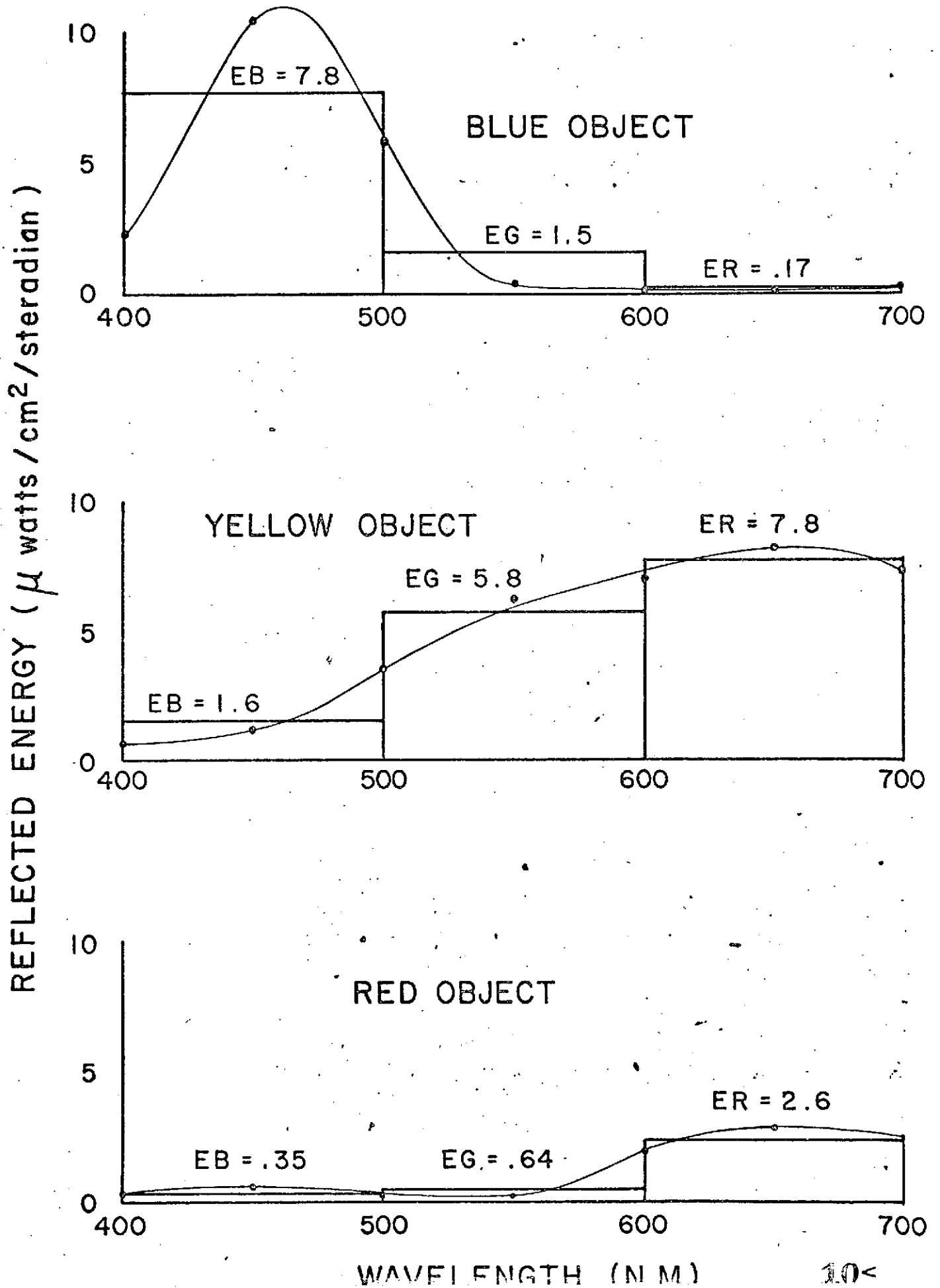
$$EB = A_{11}E_1 + A_{12}E_2 + A_{13}E_3 \qquad (9)$$

$$EG = A_{21}E_1 + A_{22}E_2 + A_{23}E_3 \qquad (10)$$

$$ER = A_{31}E_1 + A_{32}E_2 + A_{33}E_3 \qquad (11)$$

These A's are again only a function of the film sensitivity. The numerical values of these A's are given in Table 2 for two different types of film. An example of an equivalent energy spectrum is given in Figure 6 for a blue object.

# REAL AND EQUIVALENT REFLECTED ENERGY

Figure 6.  Typical values of real and equivalent reflected energy.

| FILM TYPE | $A_{11}$ | $A_{12}$ | $A_{13}$ | $A_{21}$ | $A_{22}$ | $A_{23}$ | $A_{31}$ | $A_{32}$ | $A_{33}$ |
|---|---|---|---|---|---|---|---|---|---|
| Kodak 2443 | 1.03 | −.0011 | −.02 | −.16 | 1.197 | −.032 | −.18 | −.55 | 1.7 |
| Kodak 2448 | 1.05 | −.0256 | −.0108 | −.134 | 1.176 | −.0155 | −.0008 | −.184 | 1.2 |

Table 2

In the case of color IR film, these equations can be written as:

$$EG = A_{11}E_1 + A_{12}E_2 + A_{13}E_3 \qquad (12)$$

$$ER = A_{21}E_1 + A_{22}E_2 + A_{23}E_3 \qquad (13)$$

$$EI = A_{31}E_1 + A_{32}E_2 + A_{33}E_3 \qquad (14)$$

Thus, when $E_1$, $E_2$ and $E_3$ have been found, EB, EG and ER can be calculated for color film or EG, ER and EI can be calculated for color IR film. The physical significance of these equivalent energies, EB, EG, ER and EI, is that they represent numbers proportional to average energies over a limited band width very similar to the data acquired by multi-spectral scanners. EG and ER correspond exactly to the ERTS satellite MSS bands 4 and 5. EI corresponds to a larger band pass than ERTS MSS band 6.

The calculation for $E_1$, $E_2$ and $E_3$ may be difficult if proper procedures are not followed. The critical step in the procedure is the exposure of a calibrated step wedge on the film before development. The important thing here is to be able to calculate $E_1$, $E_2$ and $E_3$ for each step of the wedge from equation (4). In this case the $\varepsilon(\lambda)$ is the radiation falling on the $n$th step of the film wedge, the film wedge being the exposed step wedge on the film of interest. These values are designated as $E_i^n$ ($i = 1,3$) for the $n$th step on the film wedge.

## Film Calibration

The dyes used in color and color IR film are not stable with time. On different rolls of film processing differences will cause different amounts of dye to be formed in each of the layers for the same exposures.

For these reasons the familiar D log E or characteristic curve must be used for calibration.

The two ingredients involved in successful calibration of a multi-layered film are (1) the step wedge densities as a function of wavelength must be known for each step, and (2) the spectral distribution of the light incident on the step wedge must be known.

Let $D_s^n(\lambda)$ = the density of the $n^{th}$ step of the step wedge as a function of $\lambda$

$E_o(\lambda)$ = the incident energy on the step wedge

$E_i^n$ = the exposure value for the $i^{th}$ layer from the $n^{th}$ step of the step wedge

$$E_i^n = \frac{\int \varepsilon_o(\lambda) 10^{-D_s^n(\lambda)} S_i(\lambda) d\lambda}{\int S_i(\lambda) d\lambda} \tag{15}$$

If $\varepsilon_o(\lambda)$, $D_s(\lambda)$, and $S_i(\lambda)$ are known, the exposures due to the light transmitted through each of the steps of the step wedge for each layer can be calculated.

One of the usual practices in sensitometric analysis is to plot the density vs (−) the step wedge densities as a characteristic curve. Let us look carefully at the assumption one must make for this to be valid.

If $\varepsilon_o(\lambda)$ is a constant, $\varepsilon_o$ and $D_s^n(\lambda)$ do not depend on $\lambda$, then equation (15) reduces to

$$E_i^n = \varepsilon_o 10^{-D_s^n} \tag{16}$$

Taking logs of both sides

$$\log E_i^n = -D_s^n + \log \varepsilon_o \tag{17}$$

If the assumptions of $D_n^s(\lambda)$ and $\varepsilon_o(\lambda)$ do not depend on wavelengths are true, the log of the exposure due to the step wedge can be represented by $-D_s^n$. The term $\log \varepsilon_o$ is only necessary if absolute calibration of the system is necessary. For most purposes, a relative characteristic curve is

sufficient, and only the values of the relative densities of the step wedge are necessary. It should be noted that in this approximation, the value of $\log \varepsilon_0$ is the same for each layer.

In many cases the density of the step wedge is a constant over the relevant wavelengths, but the incident energy on the step wedge has a distribution like a black body at 5800°K. This is a reasonable assumption since most sensitometers are color corrected to this temperature. In this approximation, equation (15) reduces to

$$E_i^n = 10^{-D_s^n} \frac{\int \frac{G_o S_i(\lambda)}{\lambda^5 (e\ hc/\lambda KT - 1)} d\lambda}{\int S_i(\lambda) d\lambda} \qquad (18)$$

or
$$E_i^n = 10^{-D_s^n} G_i \qquad (19)$$

where $G_i$ represents the three different values of ratio of the integrals in equation (18). The log of the exposure for each layer then reduces to

$$\log E_i^n = -D_s^n + \log G_i \qquad (20)$$

The important thing to note here is that the term $\log G_i$ is different for each of the layers. If the exposure for each layer is determined for a scene response and these exposures are used independently, only the $-D_s^n$ term need be used. But if the sums of these exposures are used, to calculate an equivalent energy spectrum for example, or ratios of the exposures are needed, the $\log G_i$ term must be included.

In the cases that neither $D_s^n(\lambda)$ or $\varepsilon_o(\lambda)$ are constant over the relevant wavelengths, equation (15) can be used throughout the film analysis. Measurements can be made on each step of the film wedge and dye contents constants can be calculated. Three characteristic curves can then be plotted; $K_i^n$'s are plotted against the log of the $E_i^n$'s as calculated in equation (15). Three different curves (one for each layer) will have been plotted. From these curves, for any selected point on the film, values

of $E_1$, $E_2$ and $E_3$ can be found for calculated values of $K_1$, $K_2$ and $K_3$. EB, EG and ER or EG, ER and EI can be calculated from $E_1$, $E_2$ and $E_3$ for color or color IR film, respectively from equations (9) to (14).

A necessary ingredient for the calculation of the dye constants $K_1$, $K_2$ and $K_3$ is the knowledge of the spectral densities of the dyes in the film. If the shapes of these curves are known and do not change with time, the procedures described so far are all that is necessary to calibrate multi-layered films. If the height of these curves change with time, the step wedge calibration will adequately deal with this problem. But if the relative shape of this curve changes with time, or if the spectral densities are not known, additional calibrations must be made.

The easiest way to calibrate the spectral density curves for a film is to expose three different step wedges. The best way to expose these step wedges is through narrow band interference filters. For color film the filters would be centered at 400 nm, 550 nm, and 650 nm; for color IR film the filters would be centered at 550 nm, 650 nm, and 750 nm.

In the case of color film, the film wedge exposed at 650 nm will give the spectral density of the cyan layer. The film wedge exposed at 550 nm will contain information on the spectral density of both the magenta and cyan layers. If the cyan spectral density has been determined then the magenta spectral density can be determined from combination density. The film wedge exposed at 400 nm will give the spectral density of the yellow dye. A similar argument can be made for color IR film.

If this technique is employed, the exposures used to plot the characteristic curves must be calculated by equation (15). For color film this reduces to:

$$E_1^n = \frac{\int \varepsilon_o(400) 10^{-D_s^n(400)} F_1 S_1(400) d\lambda}{\int S_1(\lambda) d\lambda} \tag{21}$$

$$E_1^n = \frac{\varepsilon_o(400)10^{-D_s^n(400)}S_1(400)}{\int S_1(\lambda)d\lambda} \qquad (22)$$

$$E_2^n = \frac{\varepsilon_o(550)10^{-D_s^n(550)}S_2(550)}{\int S_2(\lambda)d\lambda} \qquad (23)$$

$$E_3^n = \frac{\varepsilon_o(650)10^{-D_s^n(650)}S_3(650)}{\int S_3(\lambda)d\lambda} \qquad (24)$$

The characteristic curve for $K_1$ is determined from the film wedge exposed at 400 nm; the curve for $K_2$ is determined from wedge exposed at 550 nm; and the curve for $K_3$ is determined from the wedge exposed at 650 nm. The curve for $K_3$ could be determined from the wedge exposed at 550 nm also. In that case, $E_3^n$ would be:

$$E_3^n = \frac{\varepsilon_o(550)10^{-D_s^n(550)}S_3(550)}{\int S_3(\lambda)d\lambda} \qquad (25)$$

An alternate method of arriving at the same result is to expose one step wedge with a sensitometer and also one step of the wedge through the three different narrow band filters. Using the step exposed through the filters the spectral densities of the dyes can be determined. The characteristic curves are then plotted from the complete step wedge exposed by the sensitometer.

## Summary

We have presented a method of quantitatively analyzing multi-layered films. This type of analysis uses the physical characteristics of the film to relate the film's spectral response to the spectral distribution of the incoming radiation. A method has been presented to determine three dye content constants at each point on the film that can be easily related to the incoming radiation. The equivalent energies are simply a linear transformation of these dye content constants which are more closely related physically to the incoming radiation. These equivalent energies

can be thought of as the output from a three channel multi-spectral

scanner.  In this regard this method is well suited for comparison to

ERTS satellite data.

## REFERENCES

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