Asymptotic Solution of the Problem for a Thin Axisymmetric Cavern

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The boundary value problem which describes the axisymmetric separation of the flow around a body by a stationary infinite stream is considered. It is here understood that the cavitation number varies over the length of the cavern. Using the asymptotic expansions for the potential of a thin body, the orders of magnitude of terms in the equations of the problem are estimated. Neglecting small quantities on the order of $\delta^2 \ln \delta$, where δ is the cavern thinness parameter, a simplified boundary value problem is obtained which reduces to the solving of a second-order nonlinear integro-differential equation:

(1)
$$2\left(\frac{d\sqrt{\bar{R}^2}}{d\bar{x}}\right)^2 + \frac{d^2\bar{R}^2}{d\bar{x}^2}\ln\left|\frac{\bar{R}^2}{4(\bar{x}-1)(\bar{x}+1)}\right| - \int_{-1}^{1}\frac{\left.\frac{d^2\bar{R}^2}{d\bar{x}^2}\right|_{\bar{x}=\bar{x}_1} - \frac{d^2\bar{R}^2}{d\bar{x}^2}}{|\bar{x}_1-\bar{x}|}d\bar{x}_1 - \frac{\left.\frac{d\bar{R}^2}{d\bar{x}}\right|_{\bar{x}=-1}}{\bar{x}-1} - \frac{\left.\frac{d\bar{R}^2}{d\bar{x}}\right|_{\bar{x}=1}}{\bar{x}+1} = 2\sigma^* \quad (1)$$

with the boundary conditions

(2)
$$\bar{R}^2(\bar{x}) = 0\Big|_{\bar{x}=-1}\quad \bar{R}^2(\bar{x}) = 0\Big|_{\bar{x}=1}\quad \bar{R}^2(\bar{x}) = \delta^2\Big|_{\bar{x}=0}, \quad (2)$$

where $\bar{R} = R(\bar{x})$ is the equation of the cavern profile, and $\sigma^* = \sigma^*(\bar{x})$ is the variable cavitation number.

The solution of the problem (1),(2) is sought as the asymptotic series

(3)
$$\bar{R}^2 = \delta^2\left[\bar{R}_0^2 + \bar{R}_{-1}^2\left(\ln\frac{1}{\delta^2}\right)^{-1} + \bar{R}_{-2}^2\left(\ln\frac{1}{\delta^2}\right)^{-2} + \ldots\right]; \quad (3)$$

(4)
$$\sigma = \delta^2\left[\sigma_1\left(\ln\frac{1}{\delta^2}\right) + \sigma_0 + \sigma_{-1}\left(\ln\frac{1}{\delta^2}\right)^{-1} + \ldots\right] \quad (4)$$

and reduces to solving a sequence of linear boundary value problems /1/, of which the first two are

(5)
$$\frac{d^2\bar{R}_0^2}{d\bar{x}^2} + 2\sigma_1\tilde{\sigma}(\bar{x}) = 0,$$
$$\bar{R}_0^2 = 0\Big|_{\bar{x}=-1}\quad \bar{R}_0^2 = 0\Big|_{\bar{x}=1},\quad \bar{R}_0^2 = 1\Big|_{\bar{x}=0}, \quad (5)$$

(6)
$$2\left(\frac{d\sqrt{\bar{R}_0^2}}{d\bar{x}}\right)^2 - \frac{d^2\bar{R}_{-1}^2}{d\bar{x}^2} + \frac{d^2\bar{R}_0^2}{d\bar{x}^2}\ln\left|\frac{\bar{R}_0^2}{4(\bar{x}^2-1)}\right| - \int_{-1}^{1}\frac{\left.\frac{d^2\bar{R}_0^2}{d\bar{x}^2}\right|_{\bar{x}=\bar{x}_1} - \frac{d^2\bar{R}_0^2}{d\bar{x}^2}}{|\bar{x}_1-\bar{x}|}d\bar{x}_1 -$$
$$-\frac{\left.\frac{d\bar{R}_0^2}{d\bar{x}}\right|_{\bar{x}=1}}{(\bar{x}-1)} - \frac{\left.\frac{d\bar{R}_0^2}{d\bar{x}}\right|_{\bar{x}=-1}}{(\bar{x}+1)} = 2\sigma_0\tilde{\sigma}(\bar{x}), \quad (6)$$
$$\bar{R}_{-1}^2 = 0\Big|_{\bar{x}=-1},\quad \bar{R}_{-1}^2 = 0\Big|_{\bar{x}=1},\quad \bar{R}_{-1}^2 = 0\Big|_{\bar{x}=0}.$$

The problems (5), (6) are always solved in quadratures, and for the case of variable cavitation number given as a power series (horizontal and vertical caverns, etc.) are solved analytically in final form.

As an example, two terms of the asymptotic series are obtained for the usual cavern with constant cavitation number (the coordinate system is coupled to the middle of the cavern)*

$$R^2 = R_k^2\{(1-\bar{x}^2) + (\ln\lambda^2)^{-1}\left[(\ln 4)_{\bar{x}}^{-2} - \ln\left|(1-\bar{x})^{(1-\bar{x})}\cdot(1+\bar{x})^{(1+\bar{x})}\right|\right]\}, \qquad (7)$$

$$\sigma = 2\lambda^{-2}\ln\frac{\lambda}{\sqrt{e}},$$

where $\bar{x} = \frac{x}{l_k}$, $l_k = \lambda R_k$ is the half-length, and $\lambda = \frac{l}{\delta}$ the length of the cavern while e is the base of the natural logarithms and R_k is the cavern radius at the middle which remains undetermined in the solution. The known formula /2, 3/

$$R_k^2 = \frac{W_0}{k\pi(p_\infty - p_k)}, \qquad (8)$$

can be recommended for this, where W_0 is the cavitator drag.

The solutions obtained are in good agreement with theoretical and experimental results /2/ and afford a reason for recommending (7) for application in engineering computations.

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* An approximate solution of the problem for a thin axisymmetric cavern with constant cavitation number can also be found in /4, 5, 6/.

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