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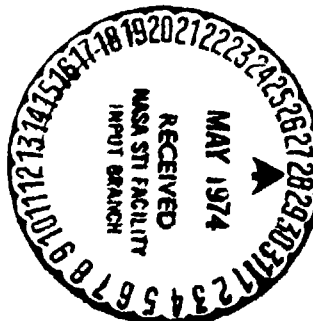
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## STOCHASTIC MODEL OF TEMPORAL CHANGES OF WIND SPECTRA IN THE FREE ATMOSPHERE

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16. ABSTRACT  Data for wind profile spectra changes with respect to time from Cape Kennedy, Florida for the time period from 28 November 1964 to 11 May 1967 have been analyzed. A "universal" statistical distribution of the spectral change which encompasses all vertical wave numbers, wind speed categories, and elapsed time has been developed for the standard deviation of the time changes of detailed wind profile spectra as a function of wave number.					
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## **FOREWORD**

The motivation for this work was the critical need to define the persistence of inflight gusts for the development of space vehicle wind biased control systems and the development of prelaunch wind monitoring techniques.

The work described in this report was conducted by Northrop Services, Inc., Huntsville, Alabama, for the National Aeronautics and Space Administration, George C. Marshall Space Flight Center, Aero-Astrodynamic Laboratory under Contract No. NAS8-21810, Appendix A, Schedule Order 13. Dr. George H. Fichtl was the Technical Coordinator for this task.

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## Section I

### INTRODUCTION

The inflight short wavelength ( $<2000$  m) gust environment is critical to the prelaunch simulation and flight evaluation of a space vehicle. It is desirable to have an accurate estimate of environmental wind at launch time. However, due to the limitation of existing operational wind prediction techniques and constraints imposed by detailed wind profile measurement and data processing procedures, it is impossible to state the inflight wind environment that will occur at launch or prior to launch with absolute certainty. Accordingly, the risk of exceeding critical inflight wind conditions and thus critical vehicle responses must be assessed with statistical procedures.

In this report the statistics of inflight gust spectra will be examined in order to define the changes that can occur in the short wavelength gust environment. Such statistics must be determined empirically and this is the philosophy used in the analysis described herein. The short wavelength gust spectra used in this study were derived from FPS-16 Radar/Jimsphere detailed wind profiles. The data consisted of approximately 2700 wind profiles measured at Cape Kennedy, Florida during the period 28 November 1964 to 11 May 1967. The smallest time period between the various profiles was approximately 3 hours. Thus, it was possible to develop wind spectra time change statistics for time periods as short as 3 hours and as long as desired. The largest time period used in this study was 72 hours. The wind gust profile spectra were calculated with standard correlation-Fourier transform techniques as described in reference 1. The inflight wind gust profiles were obtained by removing the long wavelength ( $> 2000$  m) Fourier components. This was accomplished using the high-pass digital filters described by DeMandel and Krivo (ref. 2) which were primarily developed to filter geophysical data of the kind under consideration.

The time change statistics of the inflight gust spectra developed give valuable information about the persistency of gust amplitude in time. In

general, the persistence of the various Fourier components of the wind field tends to be a monotonically increasing function of horizontal wavelength, i.e., the longer the wavelength of the wind feature the more persistent it is in time. For example, synoptic scale cyclones with horizontal wavelength on the order of 2000 km tend to have a typical persistence in a region of 3 to 5 days, while a small turbulent eddy, with a horizontal wavelength in the order of 100 meters, for example, has a characteristic time scale or persistency on the order of a minute. However, the variations or wavelengths considered in this report are along the vertical rather than the horizontal as in the particular examples noted above. The largest variation or wavelength along the vertical is on the order of the height of the tropopause (~10-15 km) which should be contrasted with the typical horizontal synoptic scale of 2000 km noted above. Nevertheless, the monotonically increasing relationship between wavelength and persistence as discussed in the context of horizontal variations or wavelengths is still valid in a general sense for the vertical wavelength Fourier components of the wind field. This results from the fact that as the horizontal scale of atmospheric flows increase so does the associated vertical scale. Thus, for example, the typical vertical extent of synoptic scale eddies (horizontal wavelength ~1000-3000 km) is typically on the order of 10-15 km, while the typical vertical extent or wavelength of the mesoscale (horizontal wavelength ~10-1000 km) motions of the atmosphere is on the order of 1-5 km and the associated microscale (horizontal wavelength  $\leq$  10 km) vertical wavelengths are on the order of 100-1000 meters.

## Section II

### THE DATA

The analysis herein is based on a set of statistics of time change of detailed wind profile spectra provided by the Aerospace Environment Division of the NASA-Marshall Space Flight Center. These spectra were calculated with detailed wind profile data acquired with the WPS-16 Radar/Jimsphere system at Cape Kennedy, Florida during the period of 28 November 1964 to 11 May 1967. A discussion of this wind sensing technique can be found in references 3 and 4. The empirical spectral time change statistics provided by NASA included distribution functions at various percentage points for the quantity

$$\xi(K;t,\tau) = \phi(K;t + \tau) - \phi(K;t) \quad (2-1)$$

where  $\phi(K;t)$  is the spectrum at vertical wave number  $K$  of the vertical wind profile of horizontal wind speed at time  $t$ , and  $\tau$  is the time interval between wind profiles. The elapsed time or time lag  $\tau$  took a value between 3 to 72 hours (3, 6, 7.5, 9, 12, 18, 24, 48, 72 hours) and the quantity  $K$  range between 0.00025 cycles  $m^{-1}$  to 0.0075 cycles  $m^{-1}$  (0.00025, 0.0005, 0.001, 0.0015, 0.002, 0.0025, 0.00375, 0.005, 0.00625, 0.0075 cycles  $m^{-1}$ ). In addition, the empirical statistics included associated unbiased values of the mean, variance, skewness and kurtosis calculated according to the following formulae

$$\text{mean: } \mu_1 = \frac{1}{n} \sum_{i=1}^n \xi_i \quad (2-2)$$

$$\text{standard deviation: } \sigma = \mu_2^{1/2} = \left\{ \frac{1}{n-1} \sum_{i=1}^n [\xi_i - \mu_1]^2 \right\}^{1/2} \quad (2-3)$$

$$\text{skewness: } S = \frac{1}{\sigma^3} \frac{n}{(n-1)(n-2)} \sum_{i=1}^n [\xi_i - \mu_1]^3 \quad (2-4)$$

$$\text{kurtosis: } K = \frac{1}{\sigma^4} \left\{ \frac{n^2 - 2n - 3}{(n-1)(n-2)(n-3)} \sum_{i=1}^n [\xi_i - \mu_1]^4 - \frac{3(2n-3)}{n(n-1)(n-2)(n-3)} \left( \sum_{i=1}^n [\xi_i - \mu_1]^2 \right)^2 \right\}$$

For details concerning these formulae see reference 5.

To compute the distribution function and the above moments it was assumed that the process  $\xi(K;t,\tau)$  is stationary and that the ergodic hypothesis is available. Thus, the empirical distributions and moments of  $\xi$  are functions of  $K$  and  $\tau$  only.

In an attempt to graduate the spectral statistics the data in the sample were categorized according to the largest mean flow wind speed in the 5-15 km region at time  $t$ , which will be denoted by  $V$ . The mean flow is defined to be the flow associated with Fourier component possessing vertical wavelengths  $>2000$  m. The categorization scheme consisted of the partition of the sample of values for  $\xi(K;t,\tau)$  into two subsamples, i.e., a sample of  $\xi(K;t,\tau)$  for  $V \leq 45 \text{ msec}^{-1}$  and one for  $V > 45 \text{ msec}^{-1}$ . Thus the distribution function and moments in this report are conditional ones, with the conditionalized parameter being the quantity  $V$ .

### Section III

## DATA ANALYSIS

In this section the data analysis methods that were used to construct a model of the time difference statistics of detailed inflight wind profile spectra will be discussed. Preliminary analysis revealed that it was possible to develop a "universal" statistical distribution of the spectral changes which encompassed all vertical wave numbers, wind speed categories, and more importantly, elapsed time. This, as will be shown later, is a fortuitous result which will be extremely useful in operational applications.

To develop a "universal" probability distribution of wind spectra time changes the distributions of  $\xi$  supplied by the NASA were transformed to distributions of the standardized variable

$$\rho(p_1, \tau_j, K_k) = \frac{\xi(p_1, \tau_j, K_k) - \mu_1(\tau_j, K_k)}{\sigma(\tau_j, K_k)} \quad (3-1)$$

where

$\rho$  is the standardized wind spectrum difference

$\xi$  is the original wind spectrum difference

$\mu_1$  is the mean of  $\xi$

$\sigma$  is the standard deviation of  $\xi$ .

The notation inside the parentheses serve to indicate percentage point ( $p_1 = 1, 5, 10, 25, 50, 75, 90, 95, 99, 99.9$  percent), elapsed time ( $\tau_j = 3, 6, 7.5, 9, 12, 18, 24, 48, 72$  hours), and wave numbers ( $K_k = 0.00025, 0.0005, 0.001, 0.0015, 0.002, 0.0025, 0.00375, 0.005, 0.00625, 0.0075$  cycles  $m^{-1}$ ).

After reviewing the plot of the standardized wind spectra difference  $\rho$  versus percentage point  $p_1$  for various elapsed time  $\tau_j$ , they showed no significant difference. Therefore, they can be averaged into one curve by the following formula,

$$\rho(p_i, K_k) = \frac{\sum_{j=1}^9 \rho(p_i, \tau_j, K_k) \times n(\tau_j, K_k)}{\sum_{j=1}^9 n(\tau_j, K_k)}, \quad (3-2)$$

where  $\rho$  is wind spectra difference averaged over elapsed time,  $n$  is the number of distributions for specified elapsed time and wave number. The notation inside the parentheses is previously defined. Also, plots of the wind spectra difference averaged over elapsed time for various wave numbers show that the probability distribution curves of the wind spectra difference averaged over elapsed time did not vary much among various wave numbers. A similar formula can be used to average these curves to obtain the "universal" probability distribution of wind spectra time changes.

$$\rho(p_i) = \frac{\sum_{k=1}^{10} \rho(p_i, K_k) \times m(K_k)}{\sum_{k=1}^{10} m(K_k)} \quad (3-3)$$

where

$\rho(p_i)$  is wind spectra difference averaged over wave number and elapsed time

$m(K_k)$  is the number of distributions for wave number  $K_k$  and

$$m(K_k) = \sum_{j=1}^9 n(\tau_j, K_k).$$

Thus, the "universal" probability distribution of wind spectrum change has been averaged in terms of the random variable  $\rho$ . To recover the  $\xi$  distribution, models are required for  $\mu_1$  and  $\sigma$ . The mean value,  $\mu_1$ , is identically equal zero (as will be shown later). To develop a model of the standard deviation as a function of wave number  $K$ , the following equation was used to average the standard deviation of wind spectrum differences over elapsed time. The standard deviation  $\sigma$  is an exponential function of wave number  $K$ , as will be shown later.

$$\sigma(K_k) = \frac{\sum_{j=1}^9 \sigma(\tau_j, K_k) \times n(\tau_j, K_k)}{\sum_{j=1}^9 n(\tau_j, K_k)} \quad (3-4)$$

where  $\sigma(K_k)$  is standard deviation of wind spectra average over elapsed time.  
All other notations have been previously defined.

Also, similar formula will be used to get  $\mu_1(K_k)$  the standardized mean of wind spectra difference averaged over elapsed time  $\tau$ .

## Section IV

### RESULTS

Figures 4-1 through 4-10 show the probability curves of the standardized gust spectra difference averaged over elapsed time  $\tau$  for the wave number range 0.00025 cycles  $m^{-1}$  through 0.0075 cycles  $m^{-1}$  for the wind speeds less than 45 msec $^{-1}$ . These curves match very well from the 10 percent level up to 95 percent level, but diverge on both ends. There are no definitive patterns for the divergence so that it is tentatively concluded that no trends exist in the tails. This is valid for engineering purposes. An inspection of Figures 4-1 through 4-10 shows that the transformation of the random variables  $\xi$  to  $\rho$  eliminates the dependence on wave number. To develop a "universal" distribution curve of  $\rho$ , the curves in Figures 4-1 through 4-10 were averaged to obtain a composite. Table 4-1 and Figure 4-11 shows this composite averaged probability curve of the standardized wind spectra difference ("universal" probability curve) for the wind speeds less than 45 msec $^{-1}$ . Figures 4-12 through 4-21 show the probability curves of the standardized gust spectra difference averaged over elapsed time  $T$  for the wave number range 0.00025 cycles  $m^{-1}$  through 0.0075 cycles  $m^{-1}$  for the wind speeds greater than 45 msec $^{-1}$ . Similar to those curves of the previous case these curves agree very well from the 20 percent level up to 95 percent level, but diverge on both ends. It is also tentatively concluded that no trends exist in the tails. The composite averaged probability curve of the standardized wind spectra difference ("universal" probability curve) for the wind speeds greater than 45 msec $^{-1}$  is shown in Table 4-2 and Figure 4-22. Comparison of Figures 4-11 and 4-22 shows that the two curves agree very well from the one percent level up to the 99 percent level. There is a slight difference at the 99.9 percent level, however, this is unimportant because most applications will be concerned within probability levels less than or equal to 99 percent.

Figures 4-23 and 4-24 show the standard deviation of the wind spectra difference versus elapsed times for the wave number range 0.00025 cycles  $m^{-1}$  through 0.0075 cycles  $m^{-1}$  for the wind speeds less than 45 msec $^{-1}$  and greater than 45 msec $^{-1}$  respectively. Similarly, Figures 4-25 and 4-26 show the standardized mean of the wind spectra difference versus elapsed times for the wave



Table 4-1. PROBABILITY DISTRIBUTION OF STANDARDIZED WIND SPECTRA DIFFERENCE AVERAGED OVER ELAPSED TIMES AND WAVE NUMBER FOR WIND SPEED LESS THAN 45 M SEC<sup>-1</sup> ("UNIVERSAL" PROBABILITY DISTRIBUTION)

PERCENT LEVEL	WIND SPECTRA DIFFERENCE
1	-1.334
5	-1.199
10	-1.050
25	-0.740
50	-0.202
75	0.522
90	1.331
95	1.925
99	3.459
99.9	4.919

Table 4-2. PROBABILITY DISTRIBUTION OF STANDARDIZED WIND SPECTRA DIFFERENCE AVERAGED OVER ELAPSED TIMES AND WAVE NUMBER FOR WIND SPEEDS GREATER THAN 45 M SEC<sup>-1</sup> ("UNIVERSAL" PROBABILITY DISTRIBUTION)

PERCENT LEVEL	WIND SPECTRA DIFFERENCE
1	-1.382
5	-1.231
10	-1.072
25	-0.701
50	-0.160
75	0.521
90	1.370
95	2.037
99	3.533
99.9	4.343

number range  $0.00025 \text{ cycles m}^{-1}$  through  $0.0075 \text{ cycles m}^{-1}$  for the wind speeds less than  $45 \text{ msec}^{-1}$  and greater than  $45 \text{ msec}^{-1}$ , respectively. There are no definite trends in Figures 4-23 and 4-24. But, they show that the standard deviation of the wind spectra difference changes rapidly with respect to elapsed time at elapsed time less than 24 hours for all wave numbers. Also, in Figures 4-25 and 4-26, the standardized mean of the spectral changes do not have patterns along various wave numbers. They do show rapid fluctuation with respect to elapsed times at elapsed time less than 24 hours for all wave numbers. This variability is due to statistical instability. The mean, by necessity, must be identically equal to zero. To prove this, note that  $\xi = \phi(t + \tau) - \phi(t)$  ensemble average, yields

$$\langle \xi \rangle = \langle \phi(t + \tau) \rangle - \langle \phi(t) \rangle$$

where  $\langle \rangle$  is the ensemble average operation. In a stationary process  $\langle \phi(t + \tau) \rangle = \langle \phi(t) \rangle$ , so that  $\langle \xi \rangle = 0$ . Furthermore, ergodicity demands that ensemble averages equal time averages. These are precisely the assumption used in the analysis, so by necessity  $\mu_1 = 0$ . Tables 4-3, 4-4, and Figure 4-27 show the standardized mean of the wind spectra difference averaged over elapsed times. There are similar patterns in both cases. As shown, theoretically, the averaged normalized mean of the wind spectra difference should equal to zero. Thus, the non-zero mean values of  $\xi$  in these figures represent a bias of the data. Tables 4-3, 4-4, and Figure 4-28 show the standard deviation of the wind spectra difference averaged over elapsed times. A power law of the form

$$\sigma(K) = \sigma(K_0) \left( \frac{K}{K_0} \right)^q \quad (4-1)$$

with  $q = -2.567$  fit the results quite well. This is shown in Figure 4-28 where

$\sigma(K)$  is the standard deviation of the wind spectra difference at wave number  $K$ .

$\sigma(K_0) = 0.178$ , is the standard deviation of the wind spectra difference at wave number  $k_0 = 0.001 \text{ cycles m}^{-1}$ .

Table 4-3. STANDARD DEVIATION AND STANDARDIZED MEAN OF THE WIND SPECTRA DIFFERENCE AVERAGED OVER ELAPSED TIMES (WIND SPEEDS LESS THAN 45 M SEC<sup>-1</sup>)

K	STANDARD DEVIATION	MEAN/STANDARD DEVIATION
0.00025	5.164204	0.035962
0.0005	1.006005	-0.396706
0.001	0.19115	-1.048616
0.0015	0.072866	-0.417769
0.002	0.019199	-0.065735
0.0025	0.009953	-0.541371
0.00375	0.005549	-0.591095
0.005	0.003138	0.355242
0.00625	0.001663	-0.096475
0.0075	0.001	-0.068066

Table 4-4. STANDARD DEVIATION AND STANDARDIZED MEAN OF THE WIND SPECTRA DIFFERENCE AVERAGED OVER ELAPSED TIMES (WIND SPEEDS GREATER THAN 45 M SEC<sup>-1</sup>)

K	STANDARD DEVIATION	MEAN/STANDARD DEVIATION
0.00025	5.908361	0.427843
0.0005	0.922322	-0.346052
0.001	0.146474	-1.503471
0.0015	0.055182	-0.783911
0.002	0.018193	0.008803
0.0025	0.009121	-0.455056
0.00375	0.003837	-1.302469
0.005	0.003306	0.645182
0.00625	0.002606	0.503403
0.0075	0.001302	0.339731

It should be noted that Figures 4-11 and 4-22 show negative values for 57 percent of the occasions. This means the wind spectra decreases 57 percent of the time, and thus it might be concluded that the Eulerian increases in gust spectra are faster than the decreases.

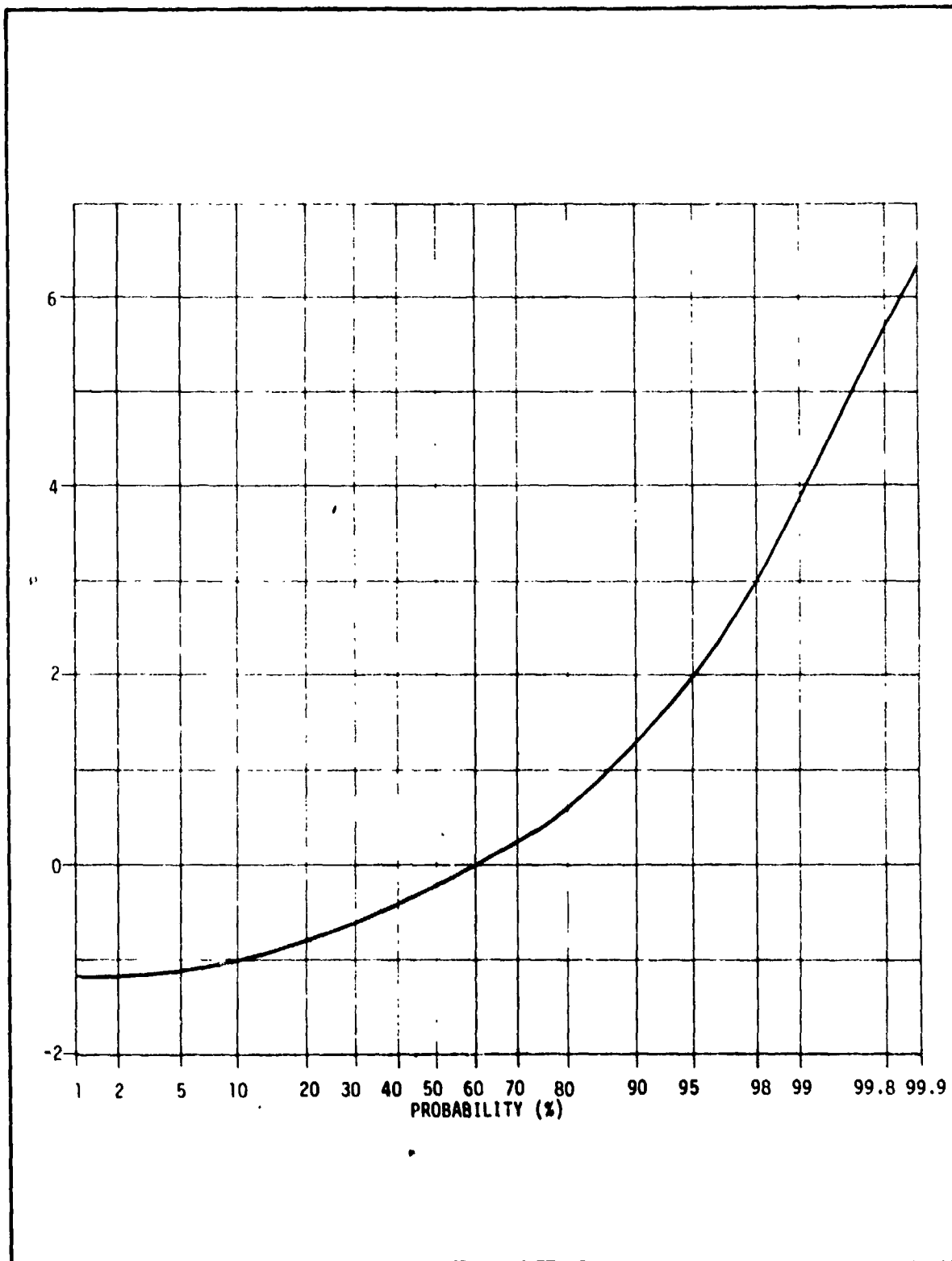


Figure 4-1. PROBABILITY DISTRIBUTION CURVE OF STANDARDIZED WIND SPECTRA DIFFERENCE AVERAGED OVER ELAPSED TIMES AT WAVE NUMBER 0.0025 CYCLES M<sup>-1</sup> FOR WIND SPEEDS LESS THAN 45 M SEC<sup>-1</sup>

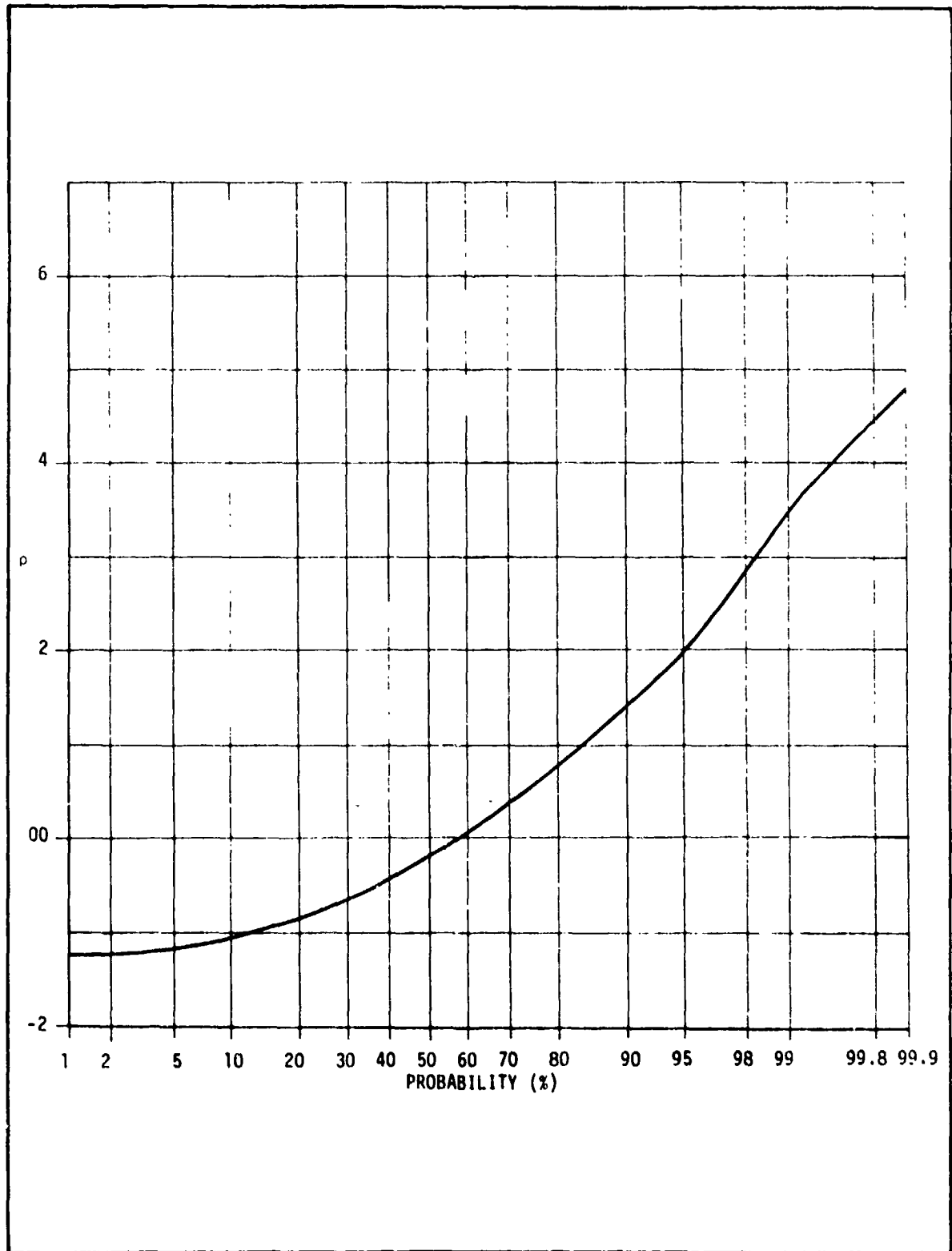


Figure 4-2. PROBABILITY DISTRIBUTION CURVE OF STANDARDIZED WIND SPECTRA DIFFERENCE AVERAGED OVER ELAPSED TIMES AT WAVE NUMBER 0.0005 CYCLES M<sup>-1</sup> FOR WIND SPEEDS LESS THAN 45 M SEC<sup>-1</sup>

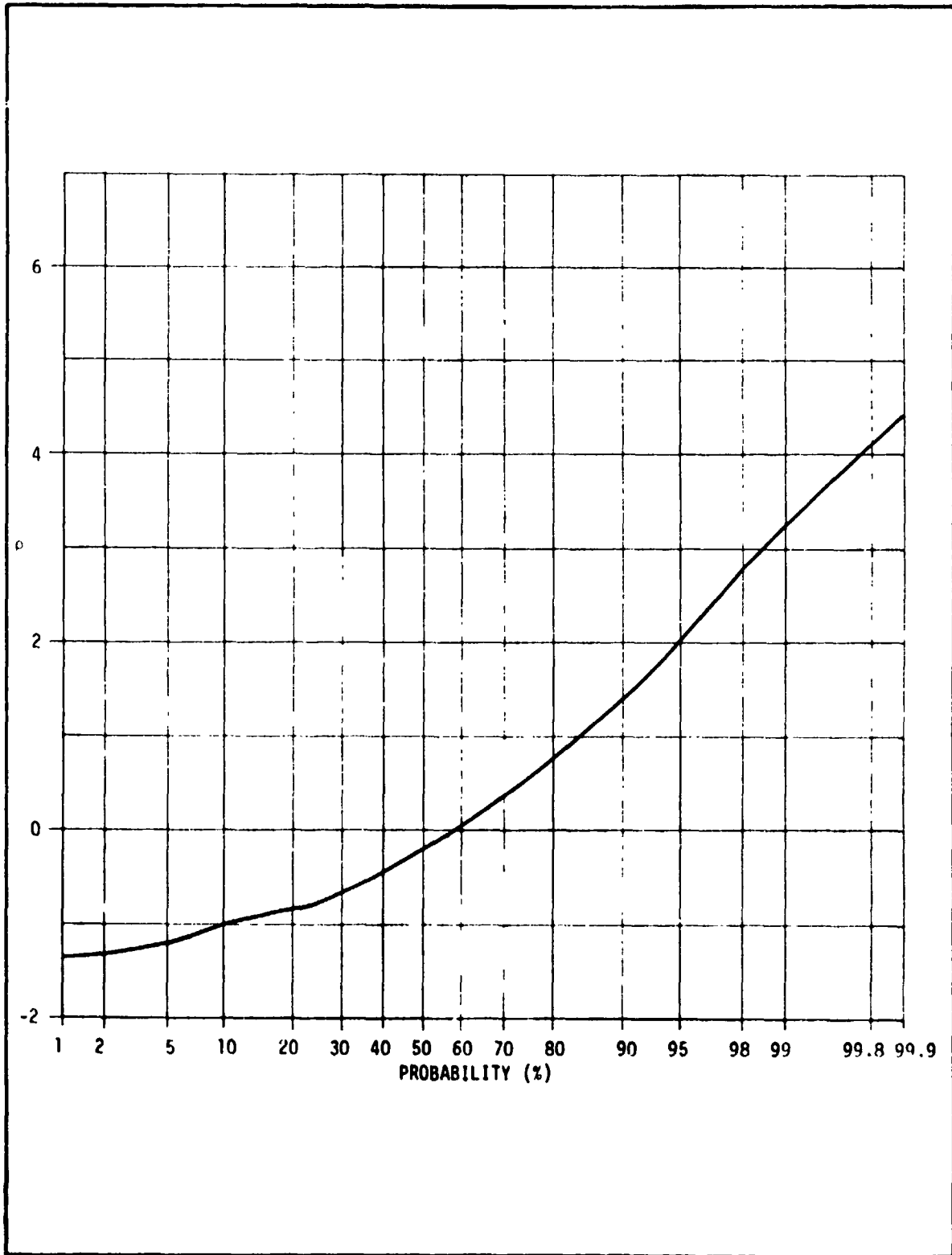


Figure 4-3. PROBABILITY DISTRIBUTION CURVE OF STANDARDIZED WIND SPECTRA DIFFERENCE AVERAGED OVER ELAPSED TIMES AT WAVE NUMBER 0.001 CYCLES M<sup>-1</sup> FOR WIND SPEEDS LESS THAN 45 M SEC<sup>-1</sup>

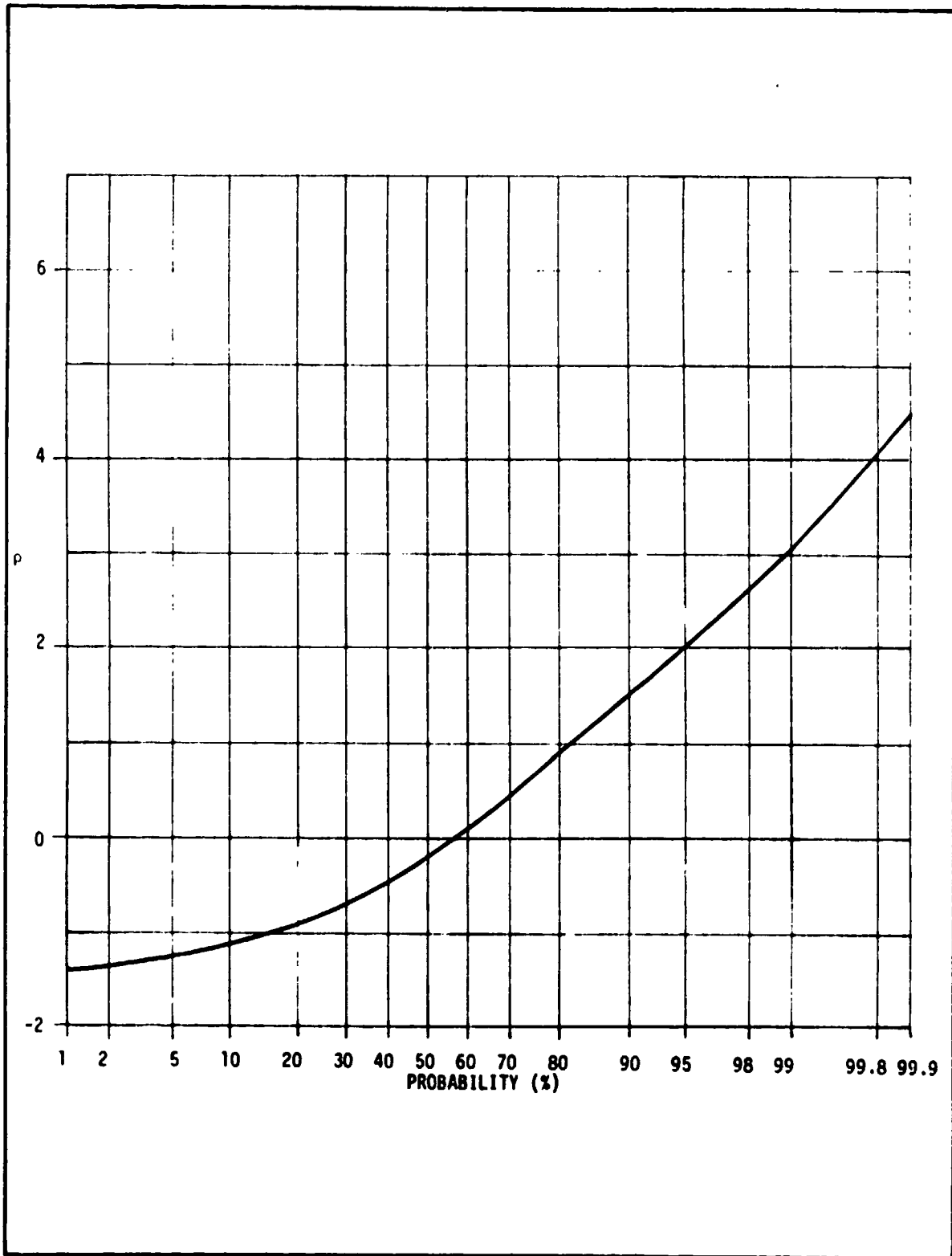


Figure 4-4. PROBABILITY DISTRIBUTION CURVE OF STANDARDIZED WIND SPECTRA DIFFERENCE AVERAGED OVER ELAPSED TIMES AT WAVE NUMBER 0.0015 CYCLES M<sup>-1</sup> FOR WIND SPEEDS LESS THAN 45 M SEC<sup>-1</sup>



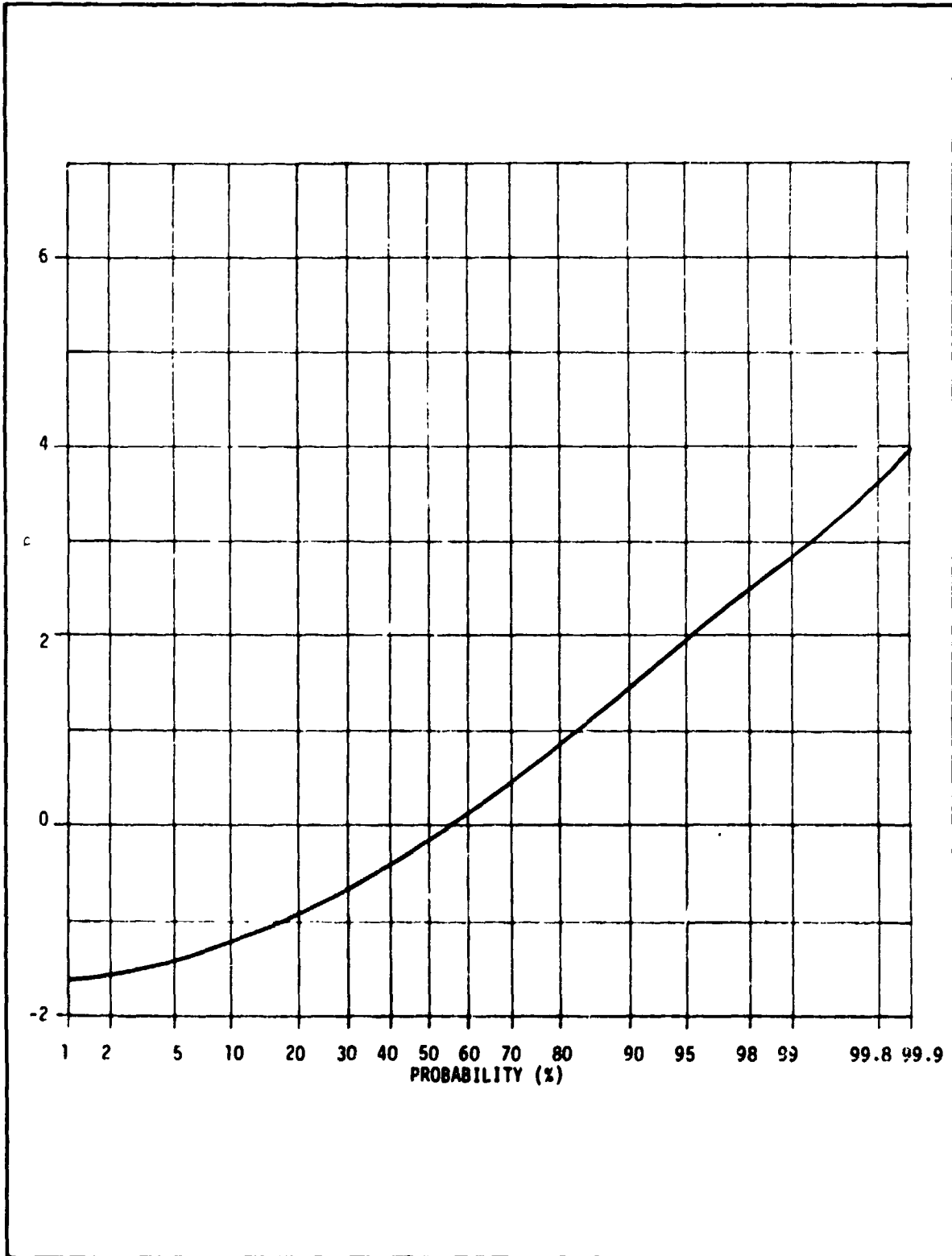


Figure 4-5. PROBABILITY DISTRIBUTION CURVE OF STANDARDIZED WIND SPECTRA DIFFERENCE AVERAGED OVER ELAPSED TIMES AT WAVE NUMBER 0.002 CYCLES M<sup>-1</sup> FOR WIND SPEEDS LESS THAN 45 M SEC<sup>-1</sup>

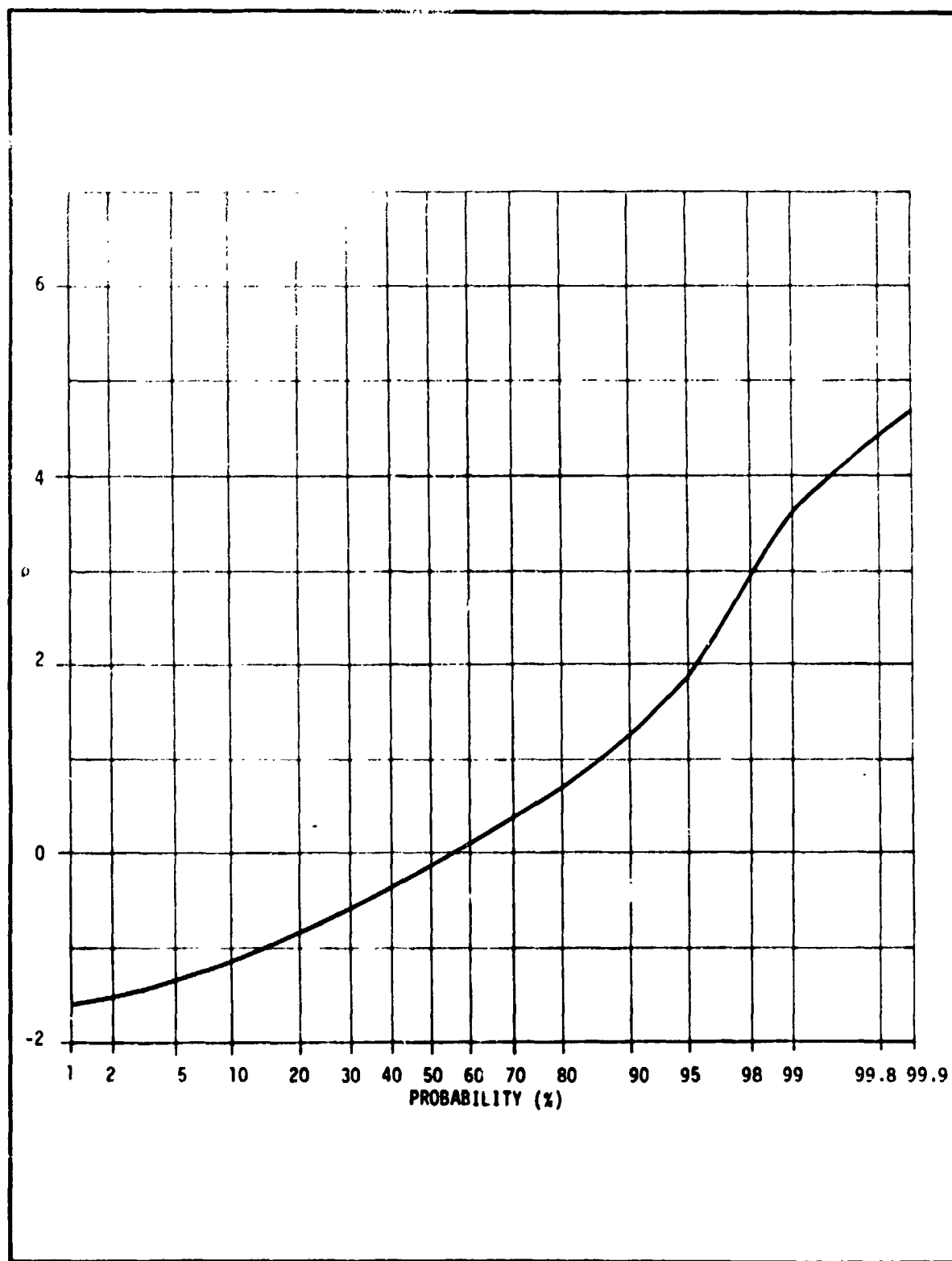


Figure 4-6. PROBABILITY DISTRIBUTION CURVE OF STANDARDIZED WIND SPECTRA DIFFERENCE AVERAGED OVER ELAPSED TIMES AT WAVE NUMBER 0.0025 CYCLES M<sup>-1</sup> FOR WIND SPEEDS LESS THAN 45 M SEC<sup>-1</sup>

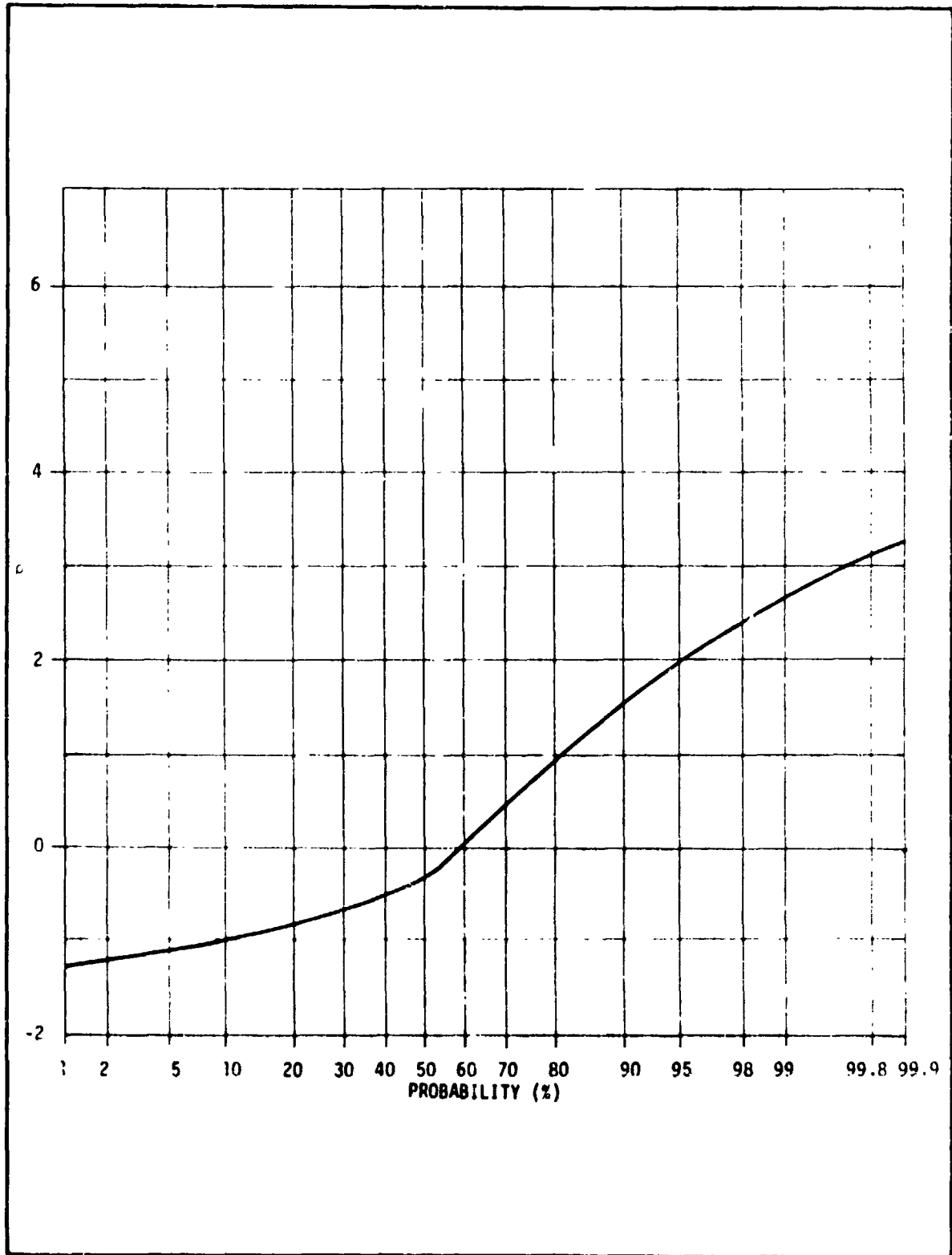


Figure 4-7. PROBABILITY DISTRIBUTION CURVE OF STANDARDIZED WIND SPECTRA DIFFERENCE AVERAGED OVER ELAPSED TIMES AT WAVE NUMBER 0.00375 CYCLES M<sup>-1</sup> FOR WIND SPEEDS LESS THAN 45 M SEC<sup>-1</sup>

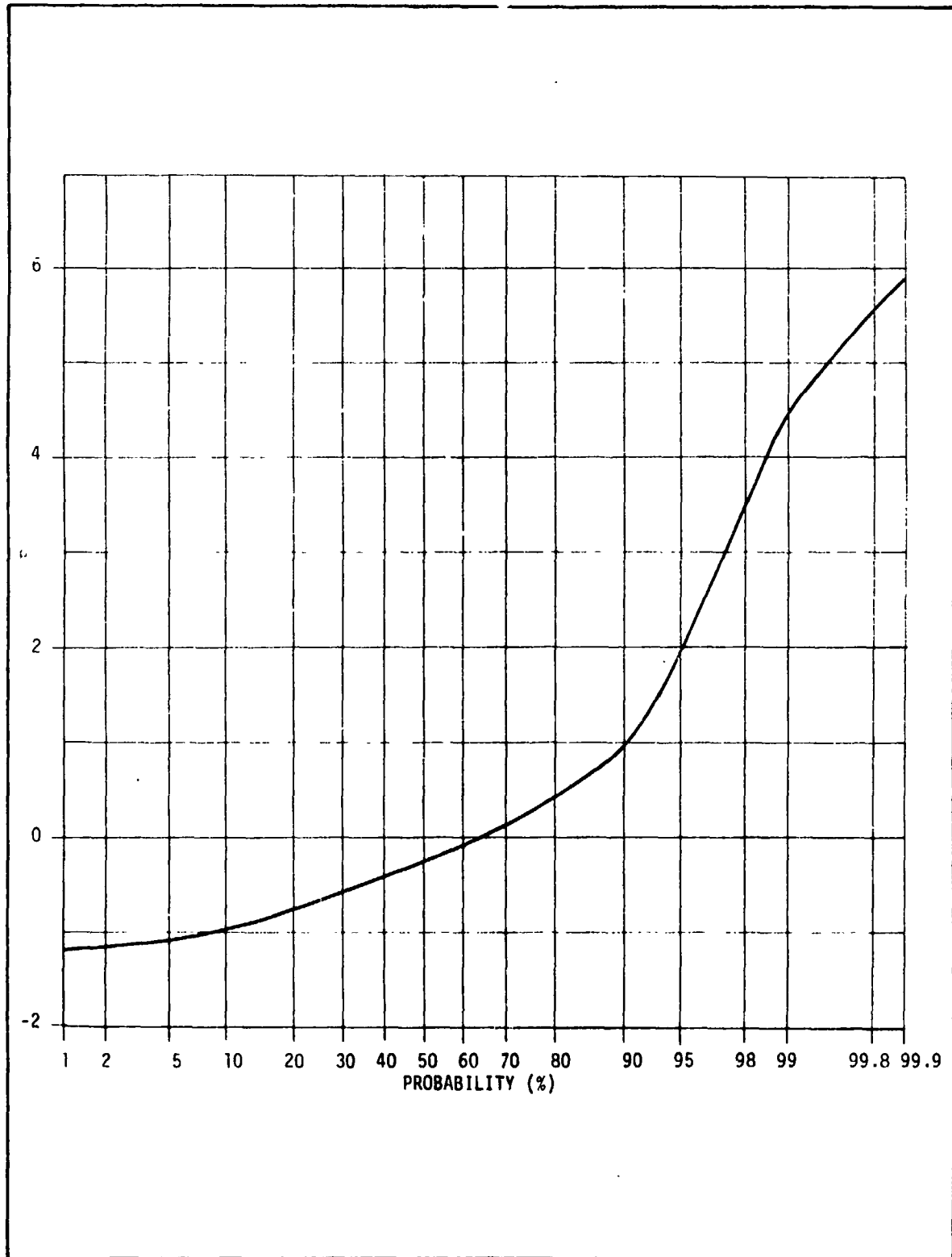


Figure 4-8. PROBABILITY DISTRIBUTION CURVE OF STANDARDIZED WIND SPECTRA DIFFERENCE AVERAGED OVER ELAPSED TIMES AT WAVE NUMBER 0.005 CYCLES M<sup>-1</sup> FOR WIND SPEEDS LESS THAN 45 M SEC<sup>-1</sup>

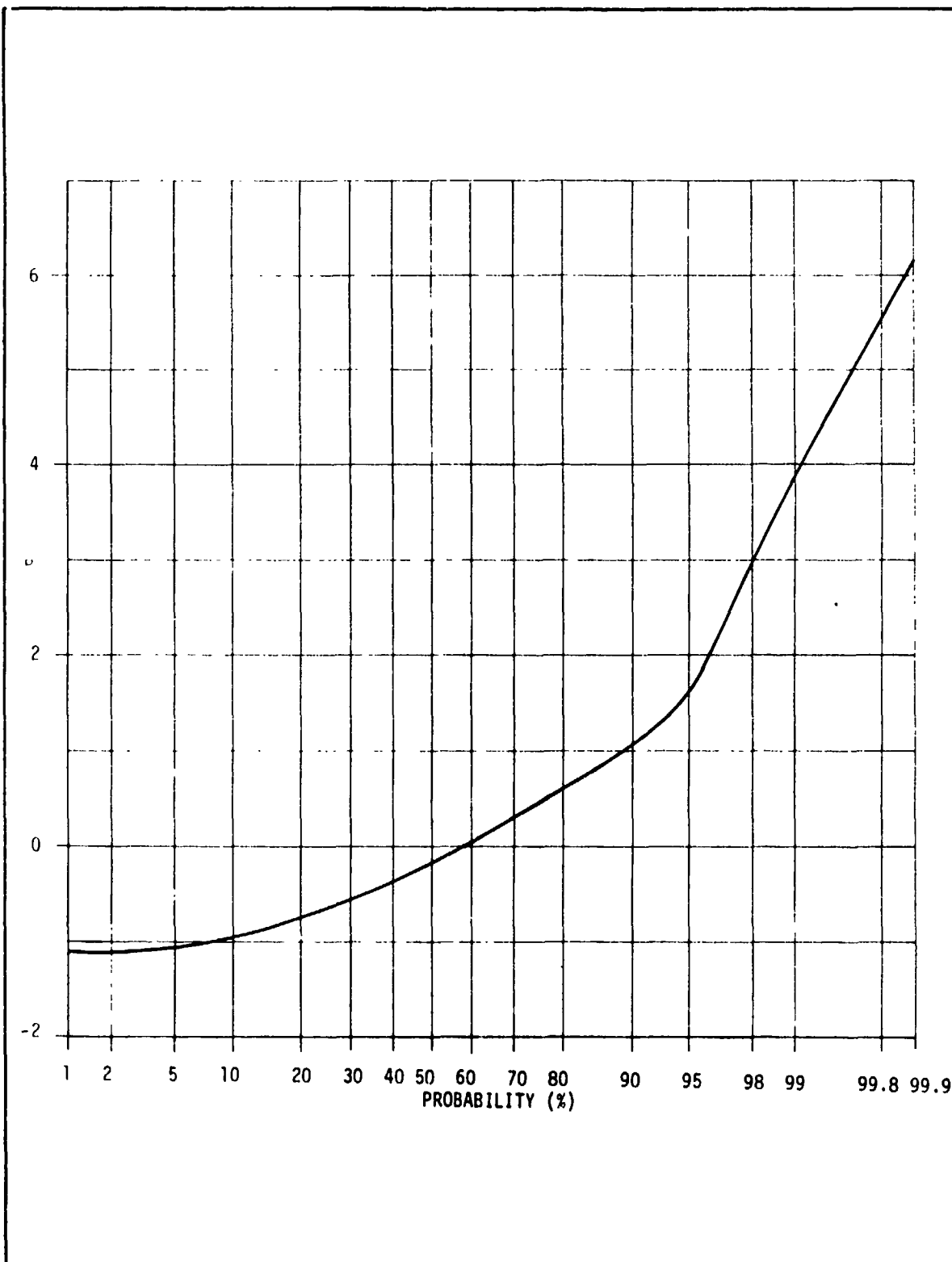


Figure 4-9. PROBABILITY DISTRIBUTION CURVE OF STANDARDIZED WIND SPECTRA DIFFERENCE AVERAGED OVER ELAPSED TIMES AT WAVE NUMBER 0.00625 CYCLES M<sup>-1</sup> FOR WIND SPEEDS LESS THAN 45 M SEC<sup>-1</sup>

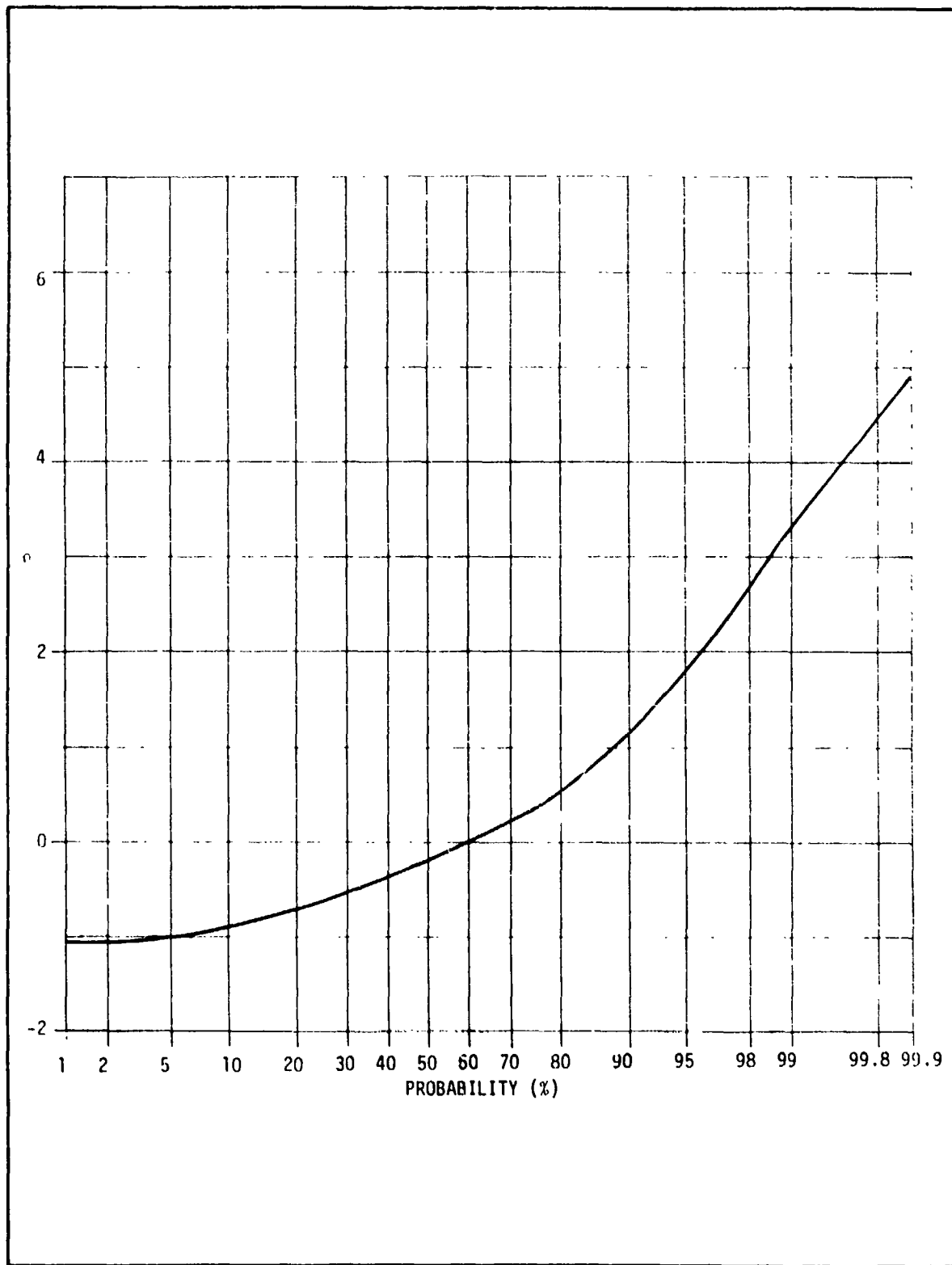


Figure 4-10. PROBABILITY DISTRIBUTION CURVE OF STANDARDIZED WIND SPECTRA DIFFERENCE AVERAGED OVER ELAPSED TIMES AT WAVE NUMBER 0.0075 CYCLES M<sup>-1</sup> FOR WIND SPEEDS LESS THAN 45 M SEC<sup>-1</sup>

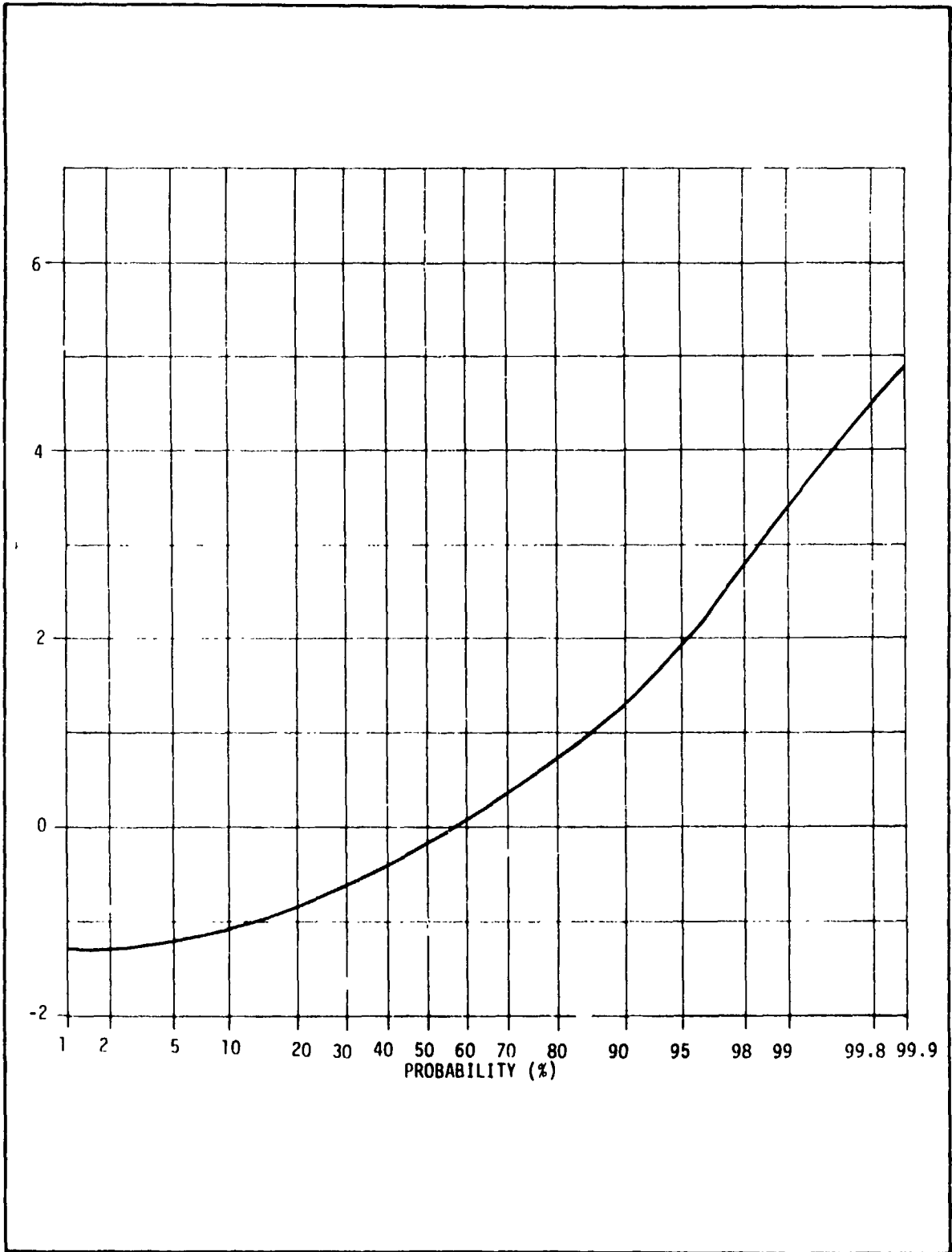


Figure 4-11. PROBABILITY DISTRIBUTION CURVE OF STANDARDIZED WIND SPECTRA DIFFERENCE AVERAGED OVER ELAPSED TIMES AND WAVE NUMBERS FOR WIND SPEEDS LESS THAN 45 M SEC<sup>-1</sup>

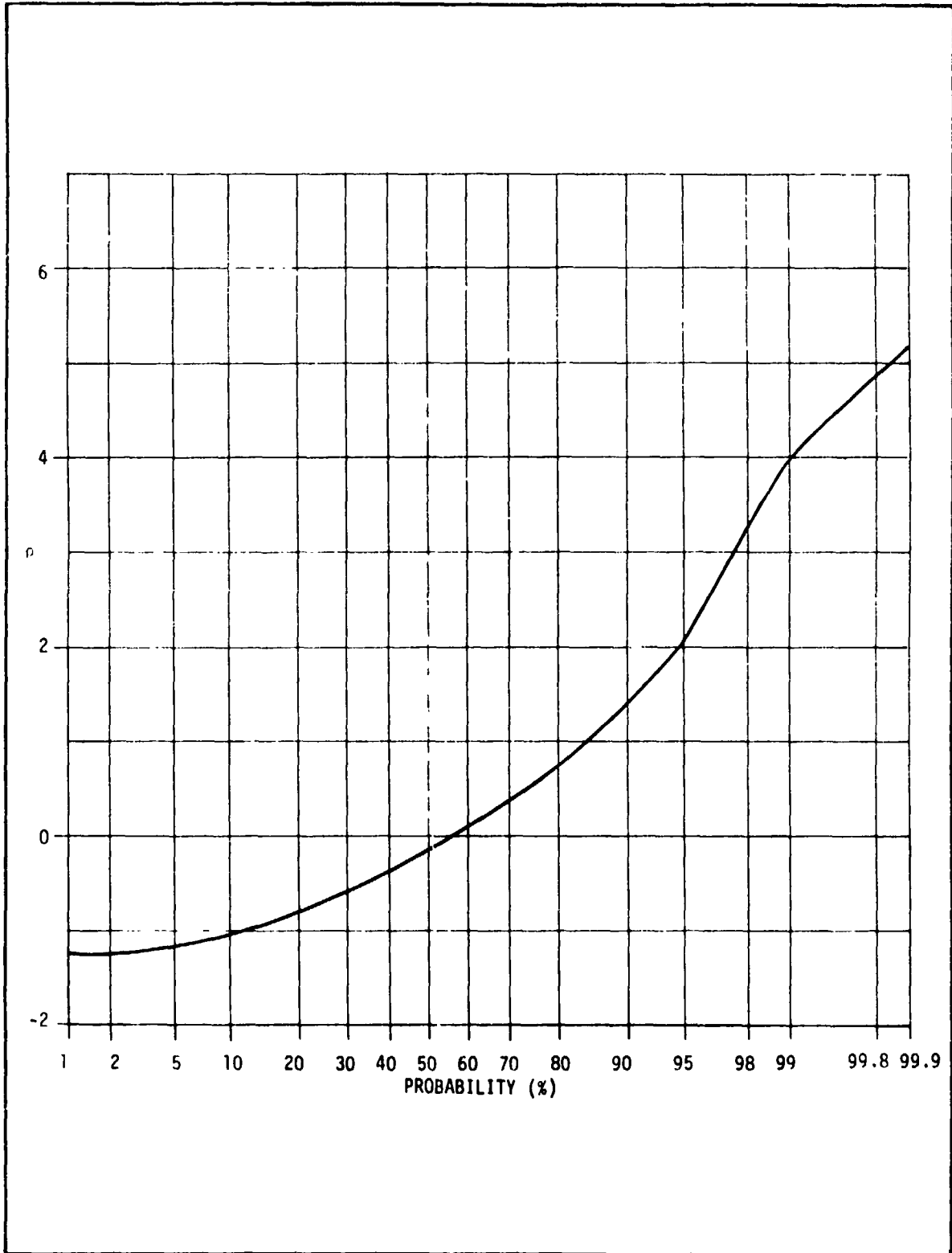


Figure 4-12. PROBABILITY DISTRIBUTION CURVE OF STANDARDIZED WIND SPECTRA DIFFERENCE AVERAGED OVER ELAPSED TIMES AT WAVE NUMBER 0.00025 CYCLES M<sup>-1</sup> FOR WIND SPEEDS GREATER THAN 45 M SEC<sup>-1</sup>



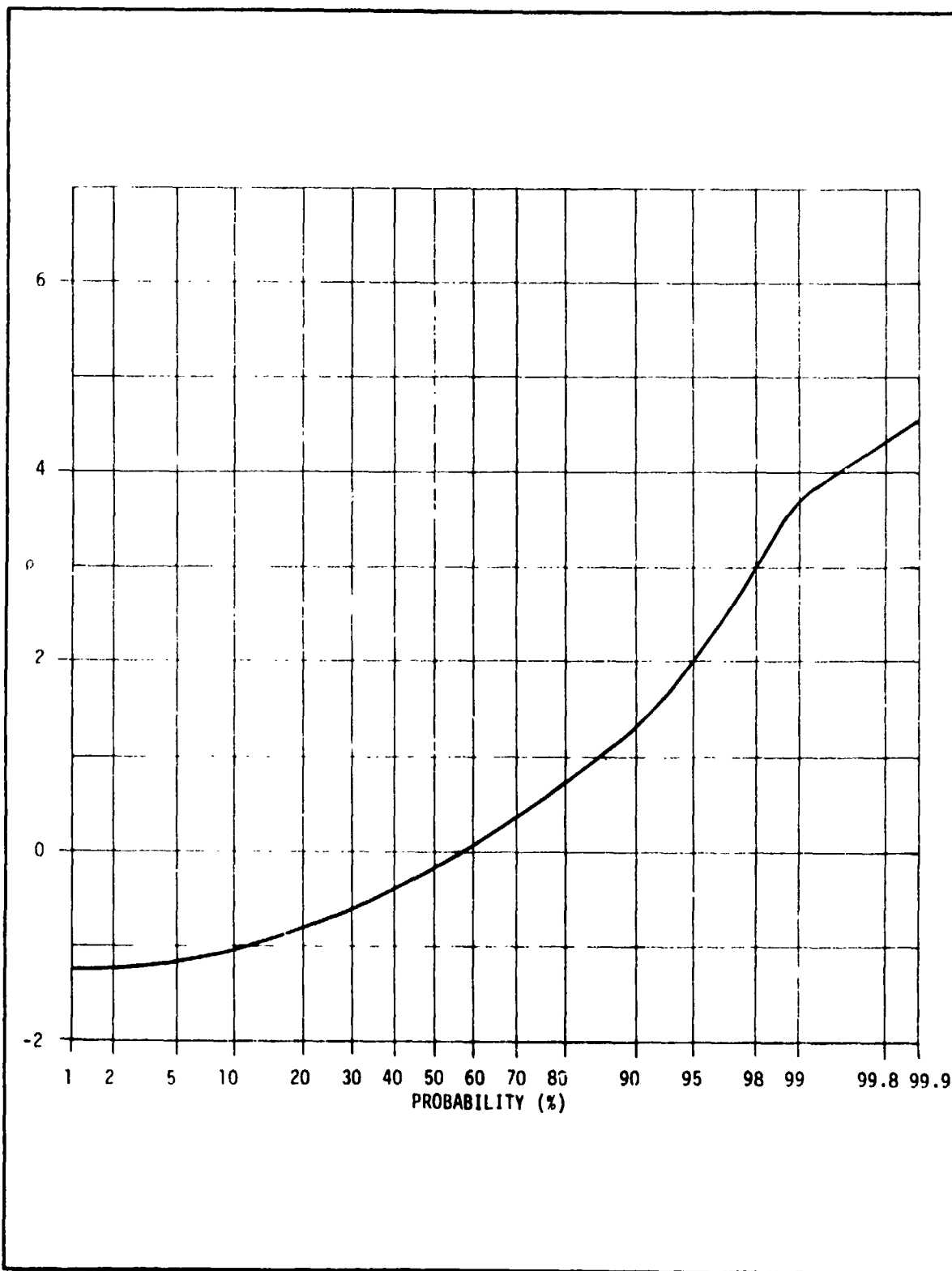


Figure 4-13. PROBABILITY DISTRIBUTION CURVE OF STANDARDIZED WIND SPECTRA DIFFERENCE AVERAGED OVER ELAPSED TIMES AT WAVE NUMBER 0.0005 CYCLES M<sup>-1</sup> FOR WIND SPEEDS GREATER THAN 45 M SEC<sup>-1</sup>

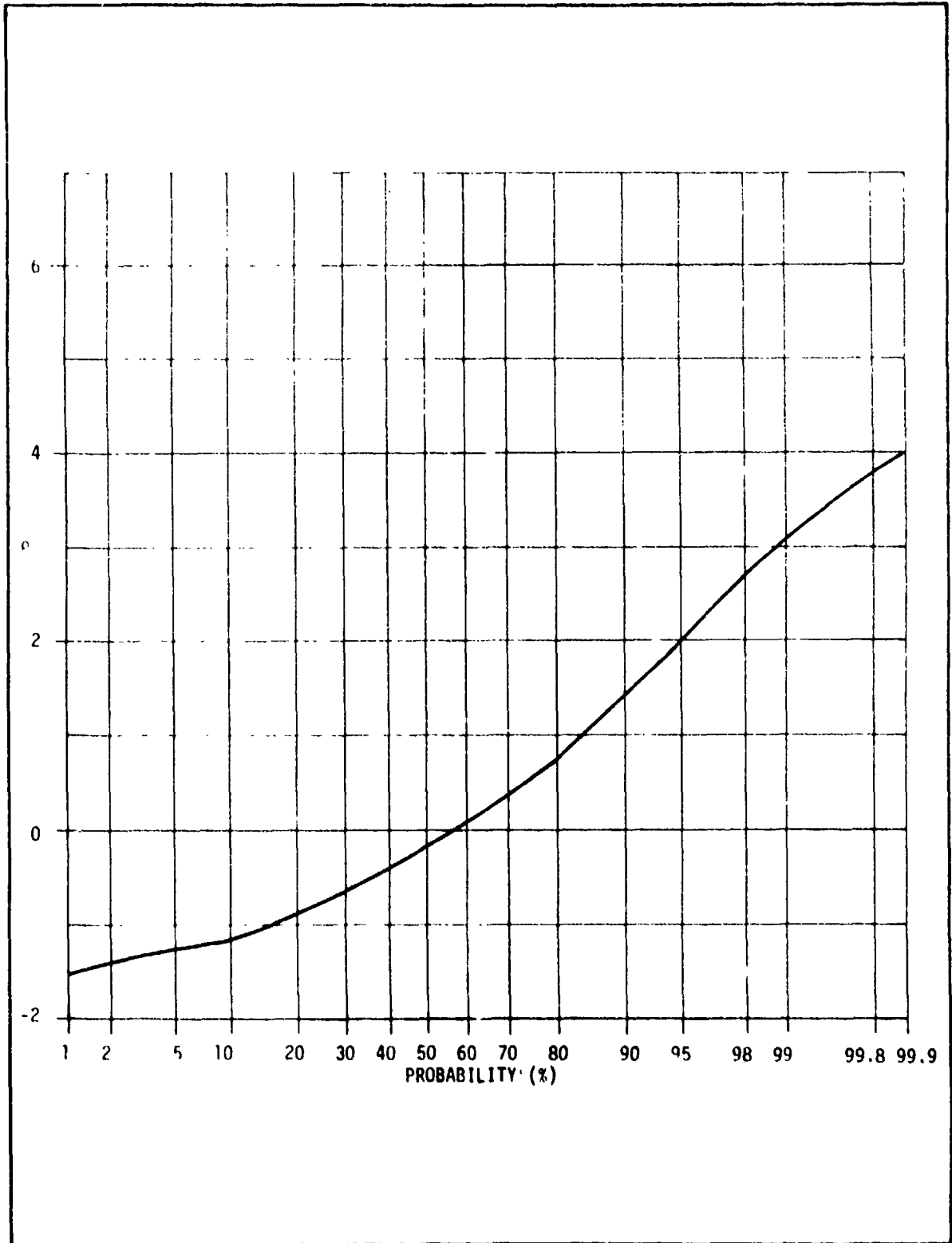


Figure 4-14. PROBABILITY DISTRIBUTION CURVE OF STANDARDIZED WIND SPECTRA DIFFERENCE AVERAGED OVER ELAPSED TIMES AT WAVE NUMBER 0.001 CYCLES M-1 FOR WIND SPEEDS GREATER THAN 45 M SEC-1

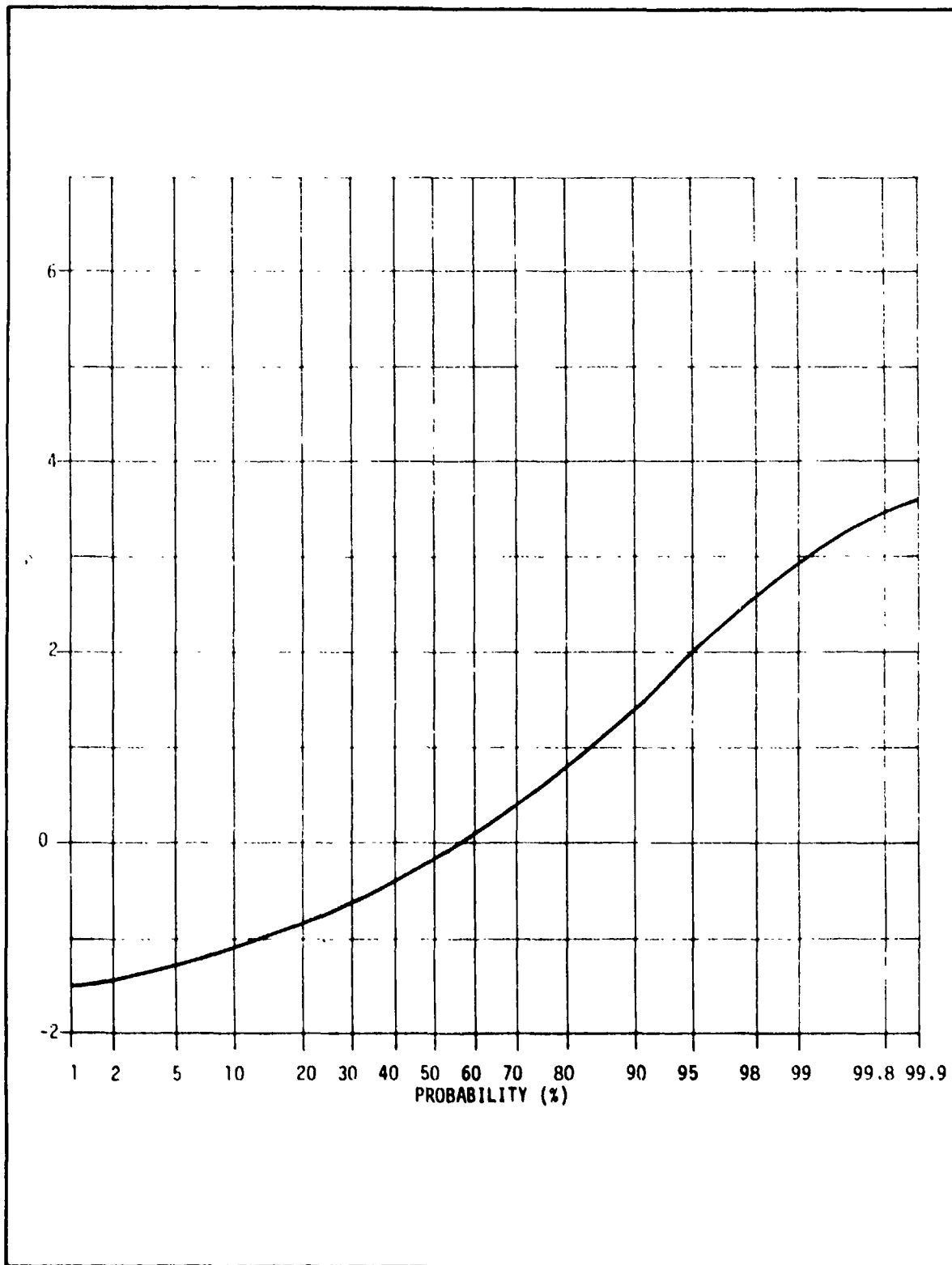


Figure 4-15. PROBABILITY DISTRIBUTION CURVE OF STANDARDIZED WIND SPECTRA DIFFERENCE AVERAGED OVER ELAPSED TIMES AT WAVE NUMBER 0.0015 CYCLES M<sup>-1</sup> FOR WIND SPEEDS GREATER THAN 45 M SEC<sup>-1</sup>

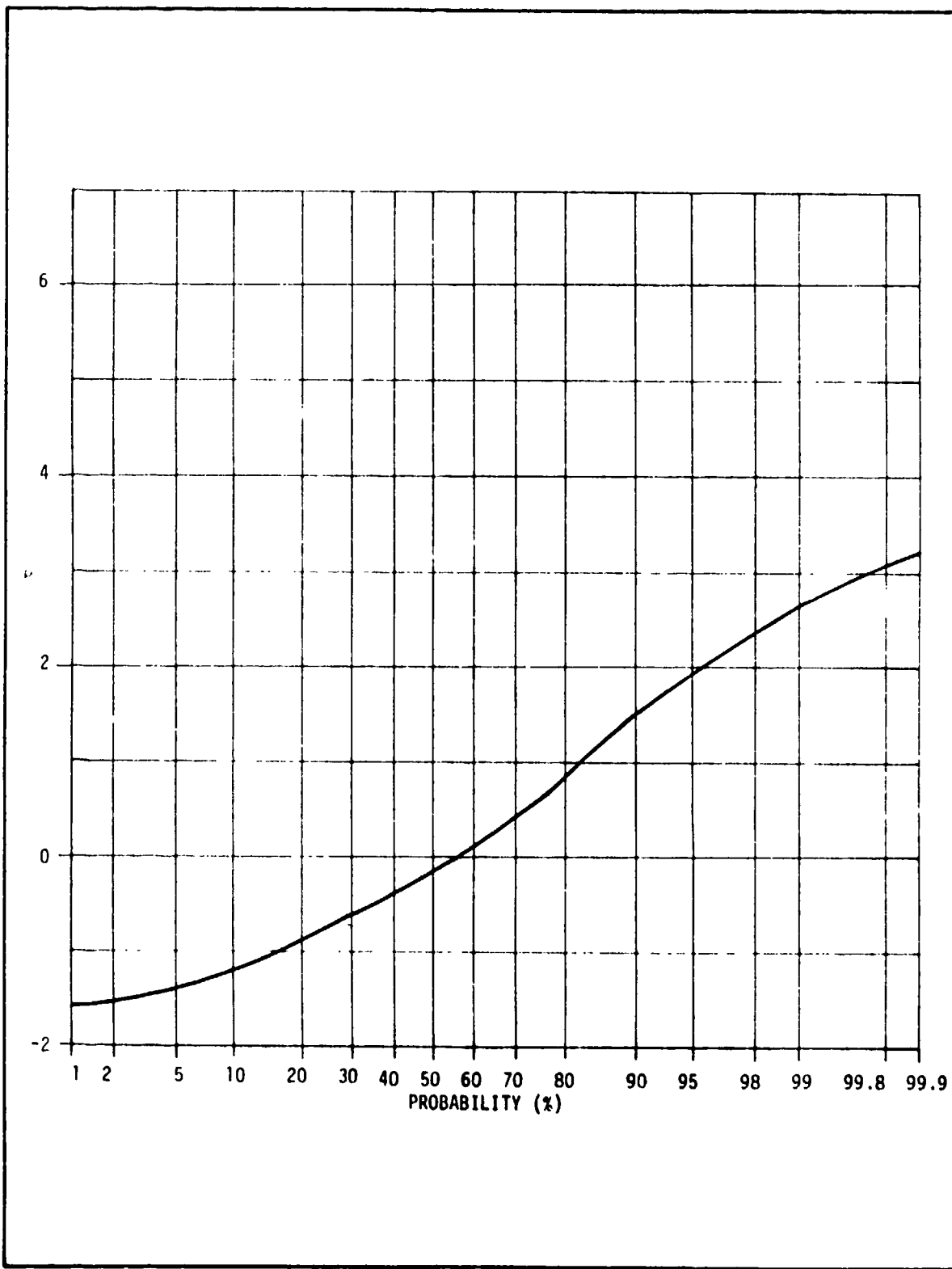


Figure 4-16. PROBABILITY DISTRIBUTION CURVE OF STANDARDIZED WIND SPECTRA DIFFERENCE AVERAGED OVER ELAPSED TIMES AT WAVE NUMBER 0.002 CYCLES M<sup>-1</sup> FOR WIND SPEEDS GREATER THAN 45 M SEC<sup>-1</sup>

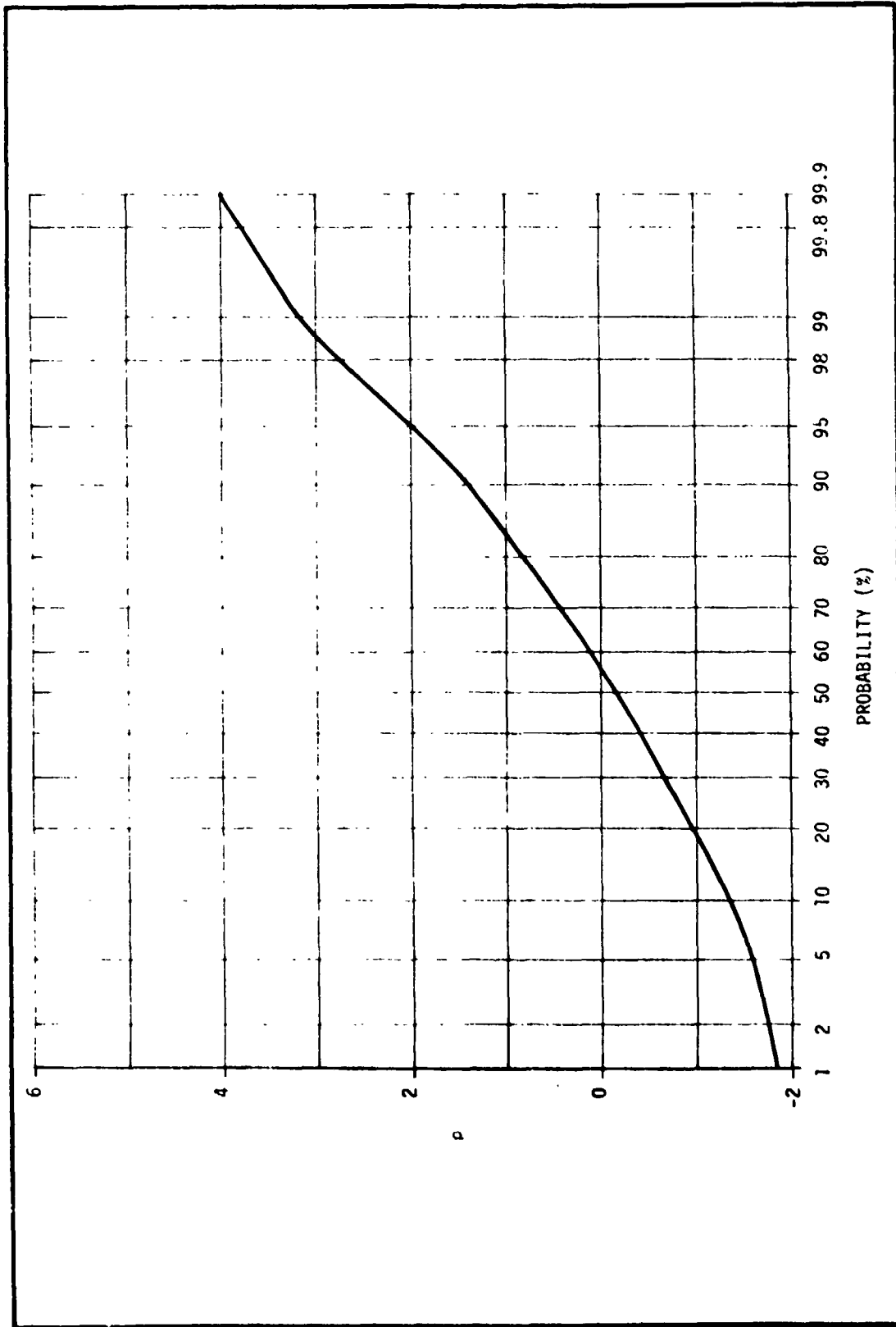


Figure 4-17. PROBABILITY DISTRIBUTION CURVE OF STANDARDIZED WIND SPECTRA DIFFERENCE AVERAGED OVER ELAPSED TIMES AT WAVE NUMBER 0.0025 CYCLES M<sup>-1</sup> FOR WIND SPEEDS GREATER THAN 45 M SEC<sup>-1</sup>

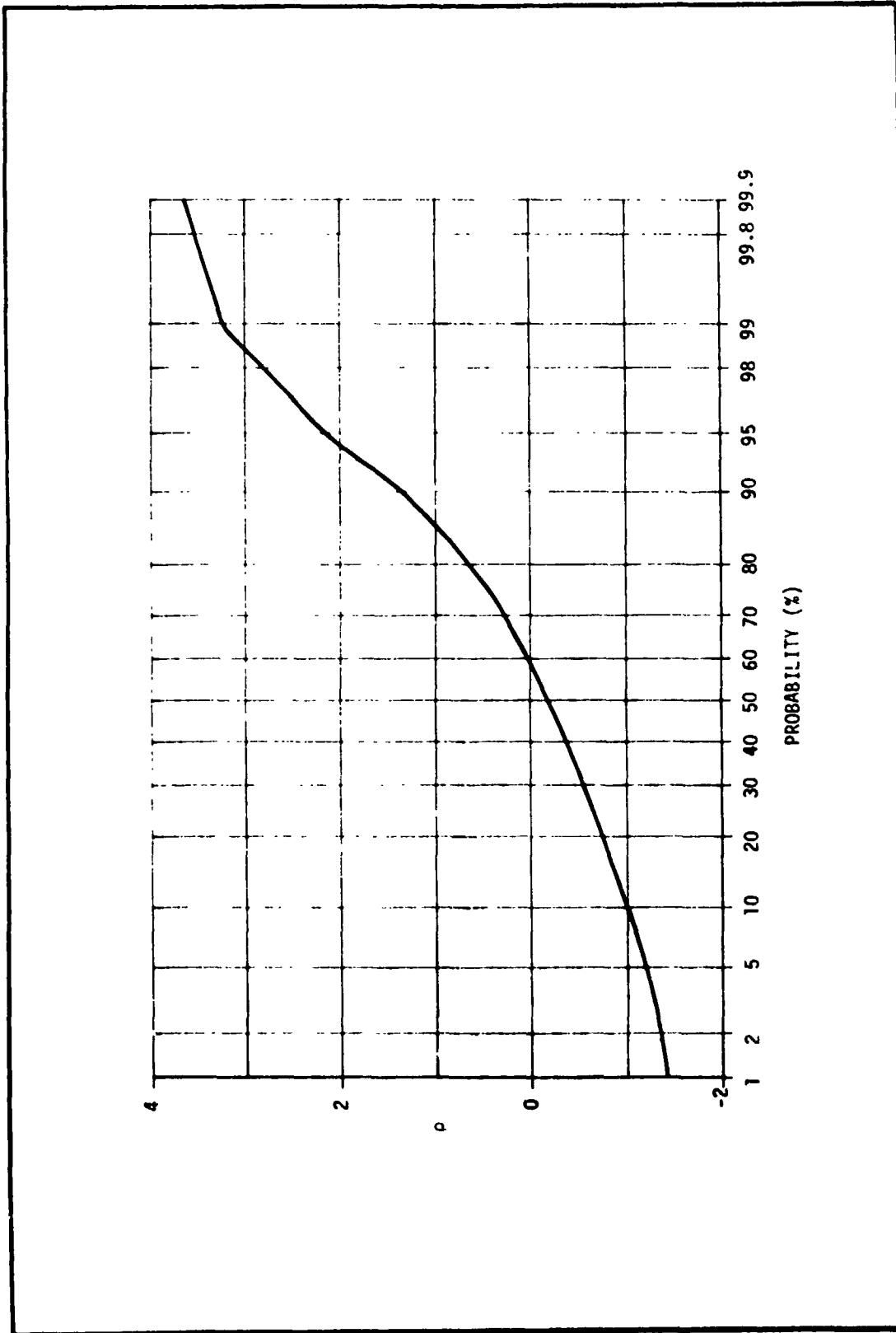


Figure 4-18. PROBABILITY DISTRIBUTION CURVE OF STANDARDIZED WIND SPECTRA DIFFERENCE AVERAGED OVER ELAPSED TIMES AT WAVE NUMBER 0.00375 CYCLES M-1 FOR WIND SPEEDS GREATER THAN 45 M SEC-1

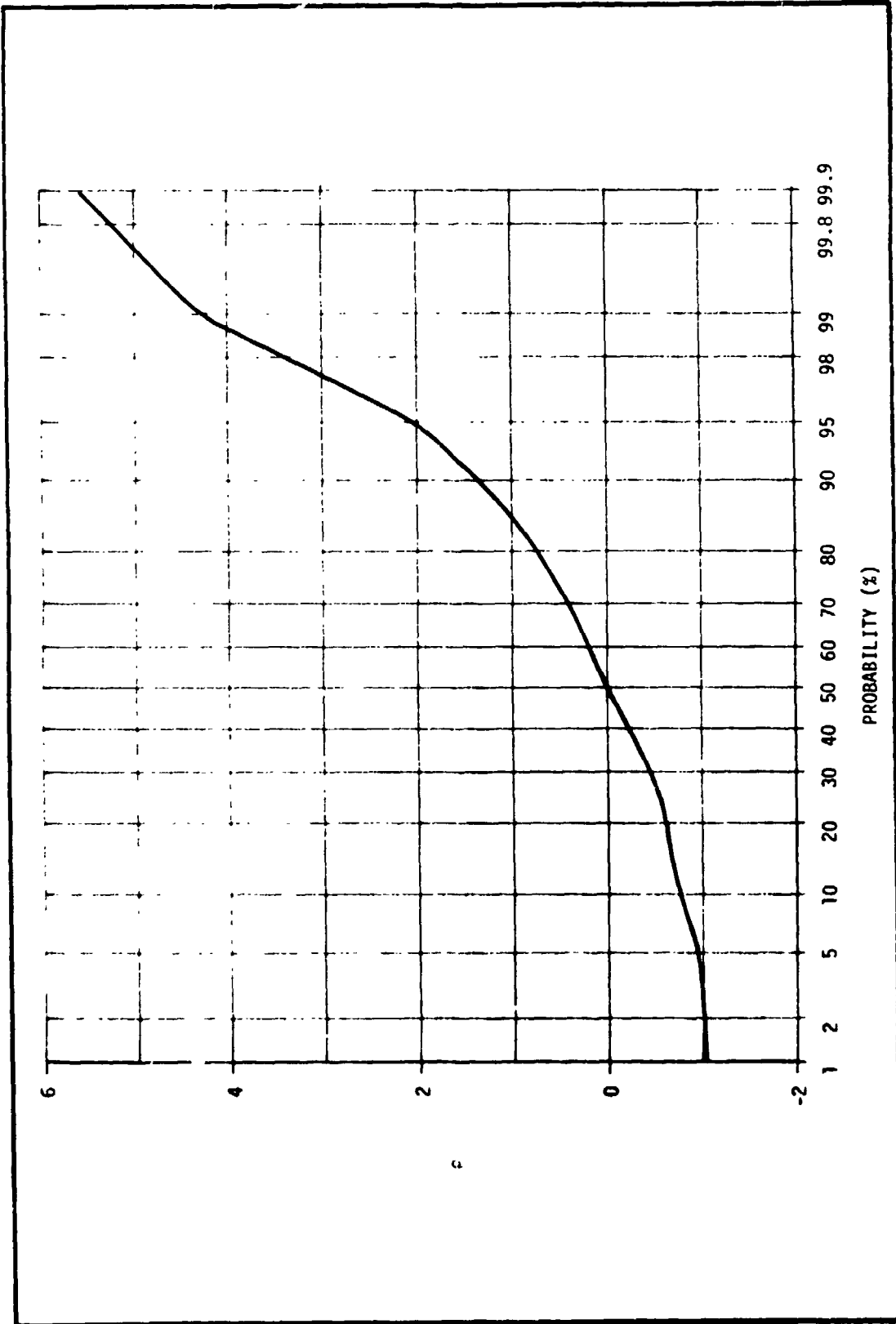


Figure 4-19. PROBABILITY DISTRIBUTION CURVE OF STANDARDIZED WIND SPECTRA DIFFERENCE AVERAGED OVER ELAPSED TIMES AT WAVE NUMBER 0.005 CYCLES M<sup>-1</sup> FOR WIND SPEEDS GREATER THAN 45 M SEC<sup>-1</sup>

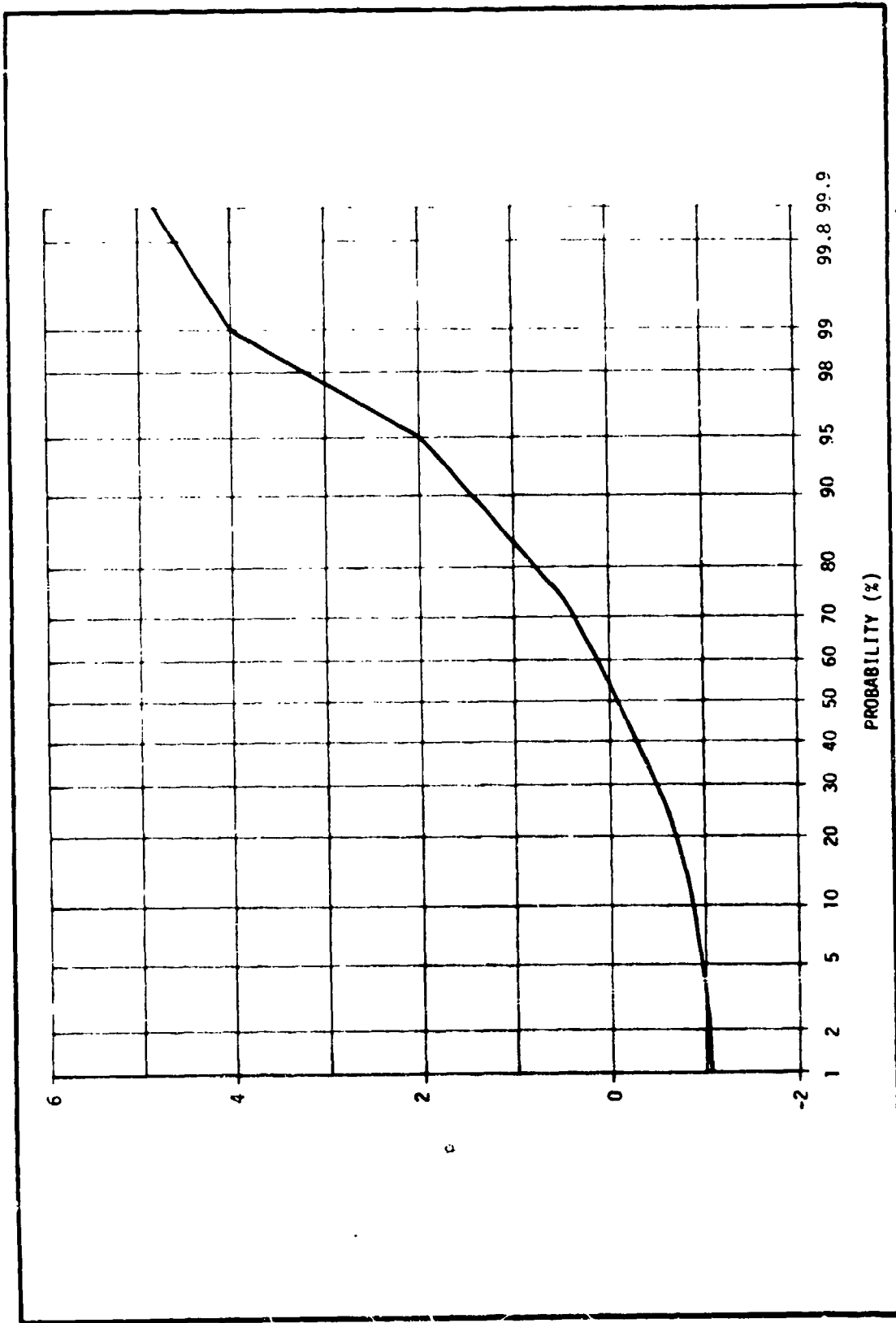


Figure 4-20. PROBABILITY DISTRIBUTION CURVE OF STANDARDIZED WIND SPECTRA DIFFERENCE AVERAGED OVER ELAPSED TIMES AT WAVE NUMBER 0.00625 CYCLES M-1 FOR WIND SPEEDS GREATER THAN 45 M SEC-1



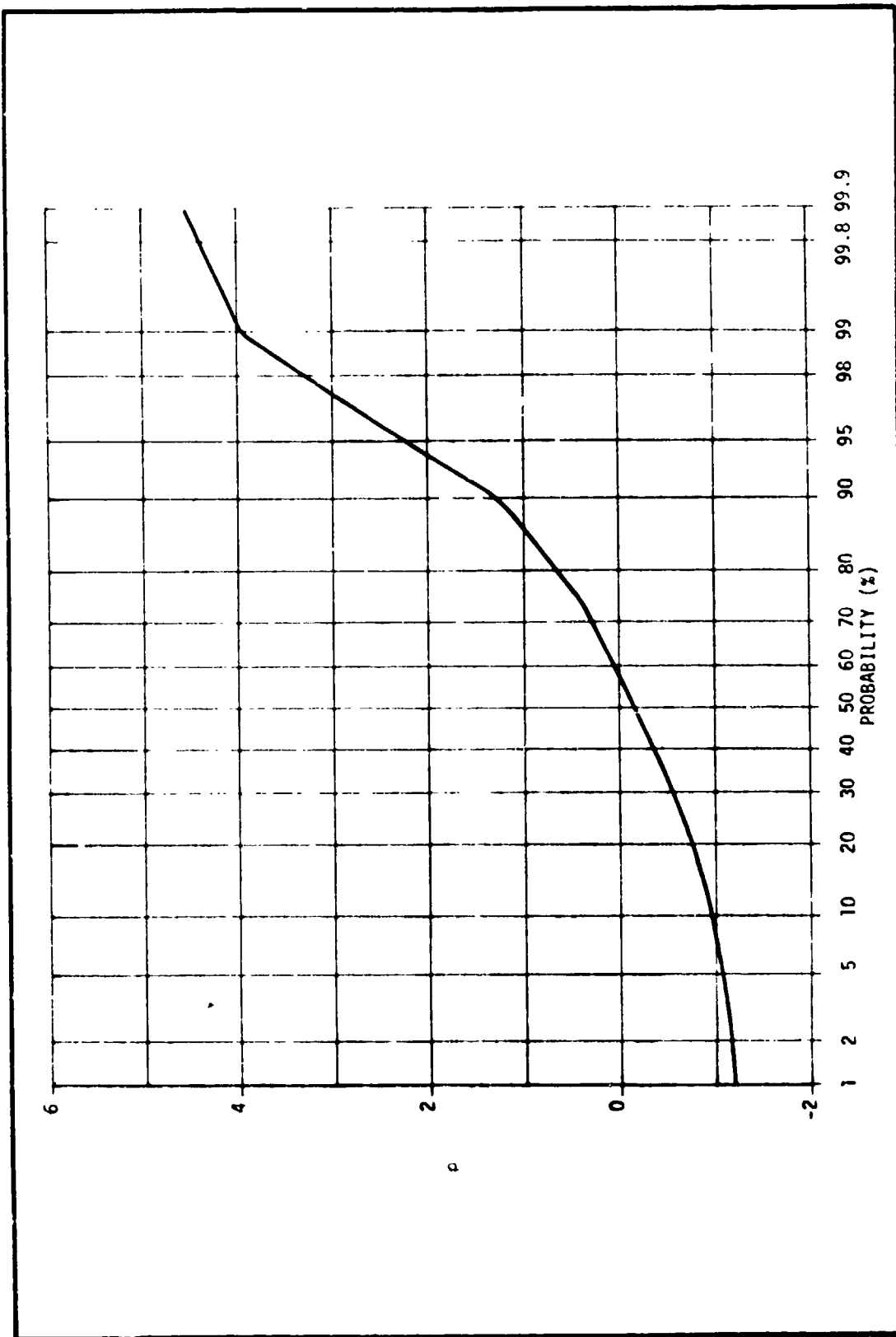


Figure 4-21. PROBABILITY DISTRIBUTION CURVE OF STANDARDIZED WIND SPECTRA DIFFERENCE AVERAGED OVER ELAPSED TIMES AT WAVE NUMBER 0.0075 CYCLES M<sup>-1</sup> FOR WIND SPEEDS GREATER THAN 45 M SEC<sup>-1</sup>

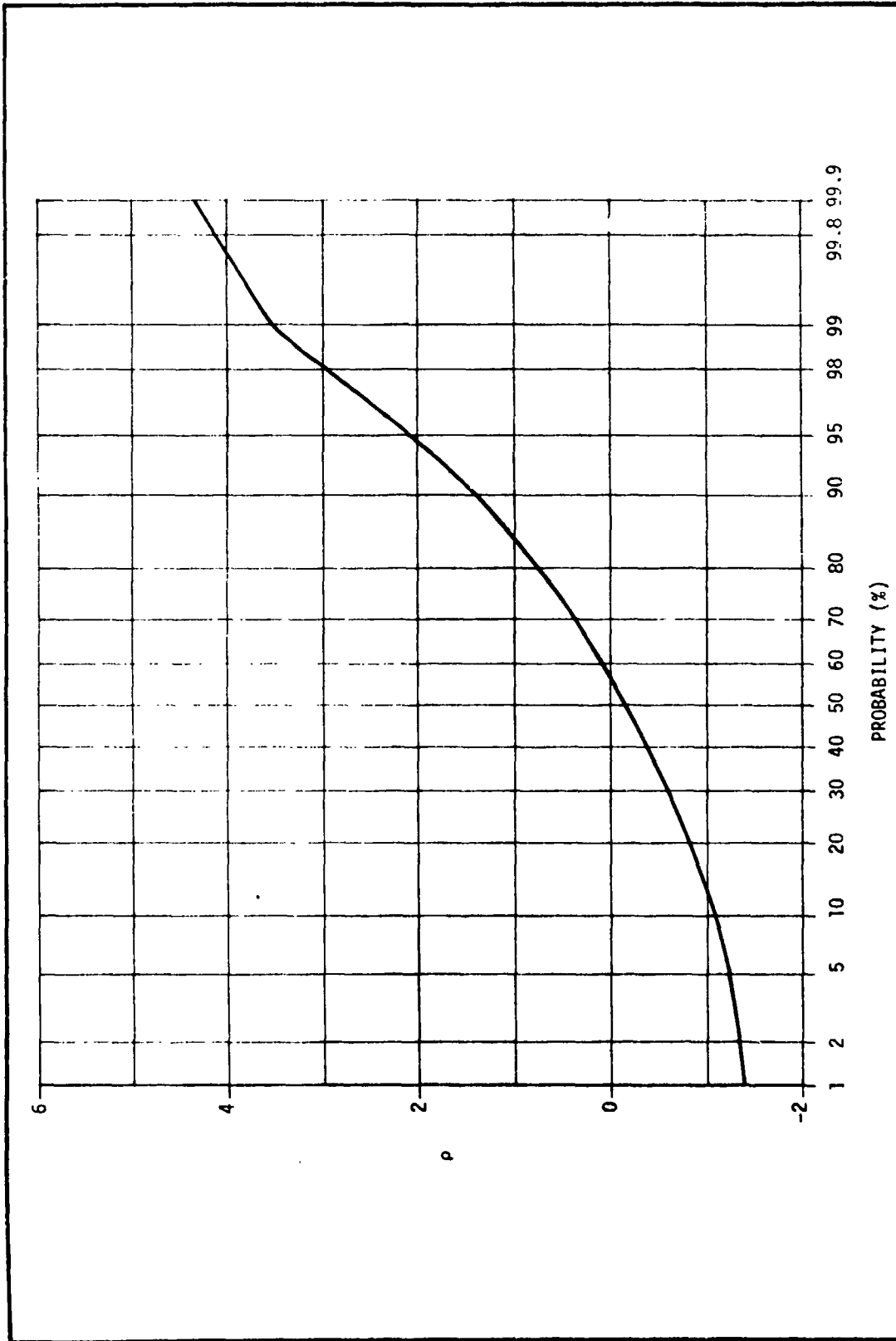


Figure 4-22. PROBABILITY DISTRIBUTION CURVE OF STANDARDIZED WIND SPECTRA DIFFERENCE AVERAGED OVER ELAPSED TIMES AND WAVE NUMBERS FOR WIND SPEEDS GREATER THAN 45 M SEC-1

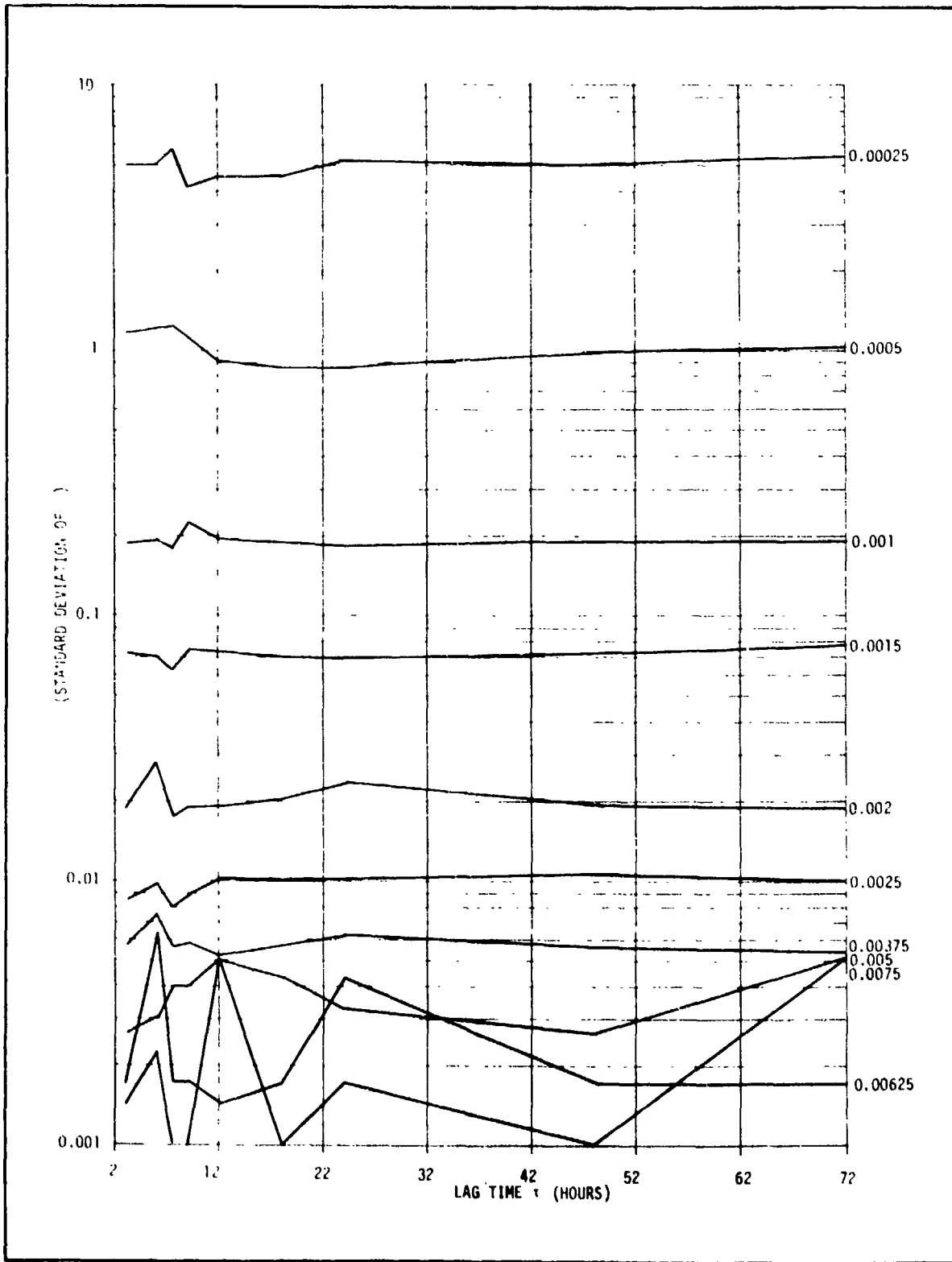


Figure 4-23. STANDARD DEVIATION OF WIND SPECTRA DIFFERENCE VERSUS ELAPSED TIMES AT VARIOUS WAVE NUMBERS FOR WIND SPEEDS LESS THAN 45 M SEC-1

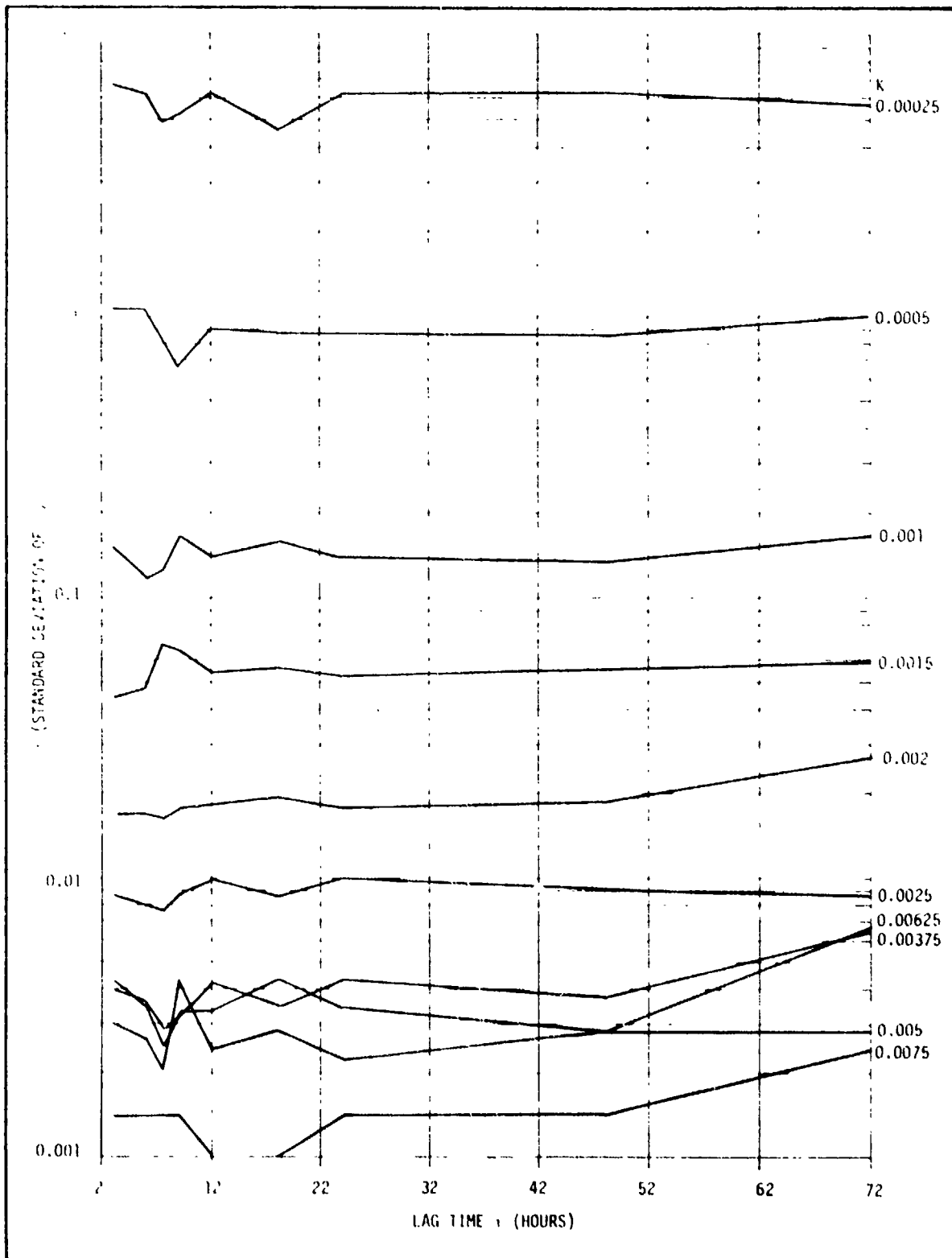


Figure 4-24. STANDARD DEVIATION OF WIND SPECTRA DIFFERENCE VERSUS ELAPSED TIMES AT VARIOUS WAVE NUMBERS FOR WIND SPEEDS GREATER THAN 45 M SEC<sup>-1</sup>

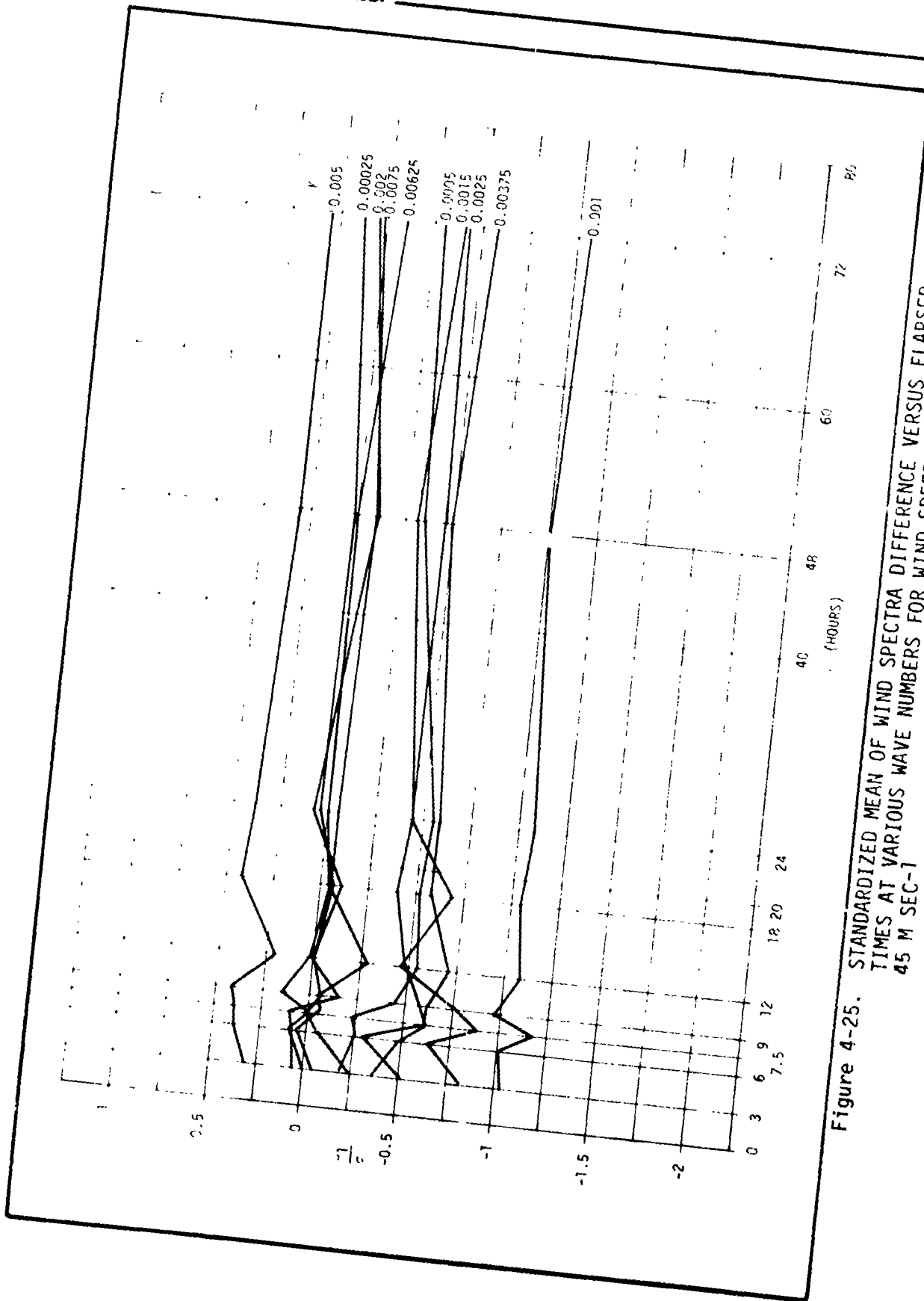


Figure 4-25. STANDARDIZED MEAN OF WIND SPECTRA DIFFERENCE VERSUS ELAPSED TIMES AT VARIOUS WAVE NUMBERS FOR WIND SPEEDS LESS THAN 45 M SEC-1

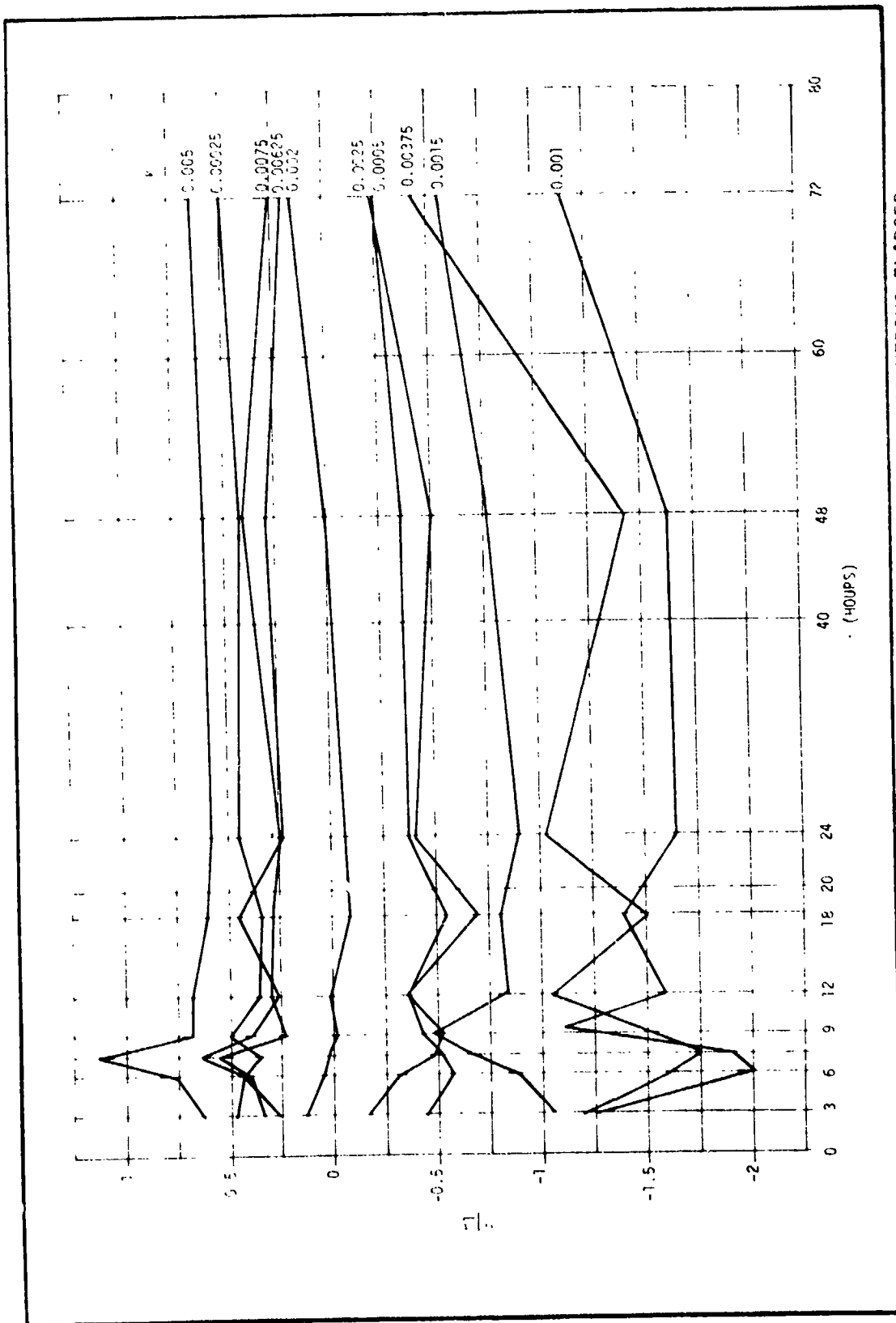


Figure 4-26. STANDARDIZED MEAN OF WIND SPECTRA DIFFERENCE VERSUS ELAPSED TIMES AT VARIOUS WAVE NUMBERS FOR WIND SPEEDS GREATER THAN 45 M SEC-1

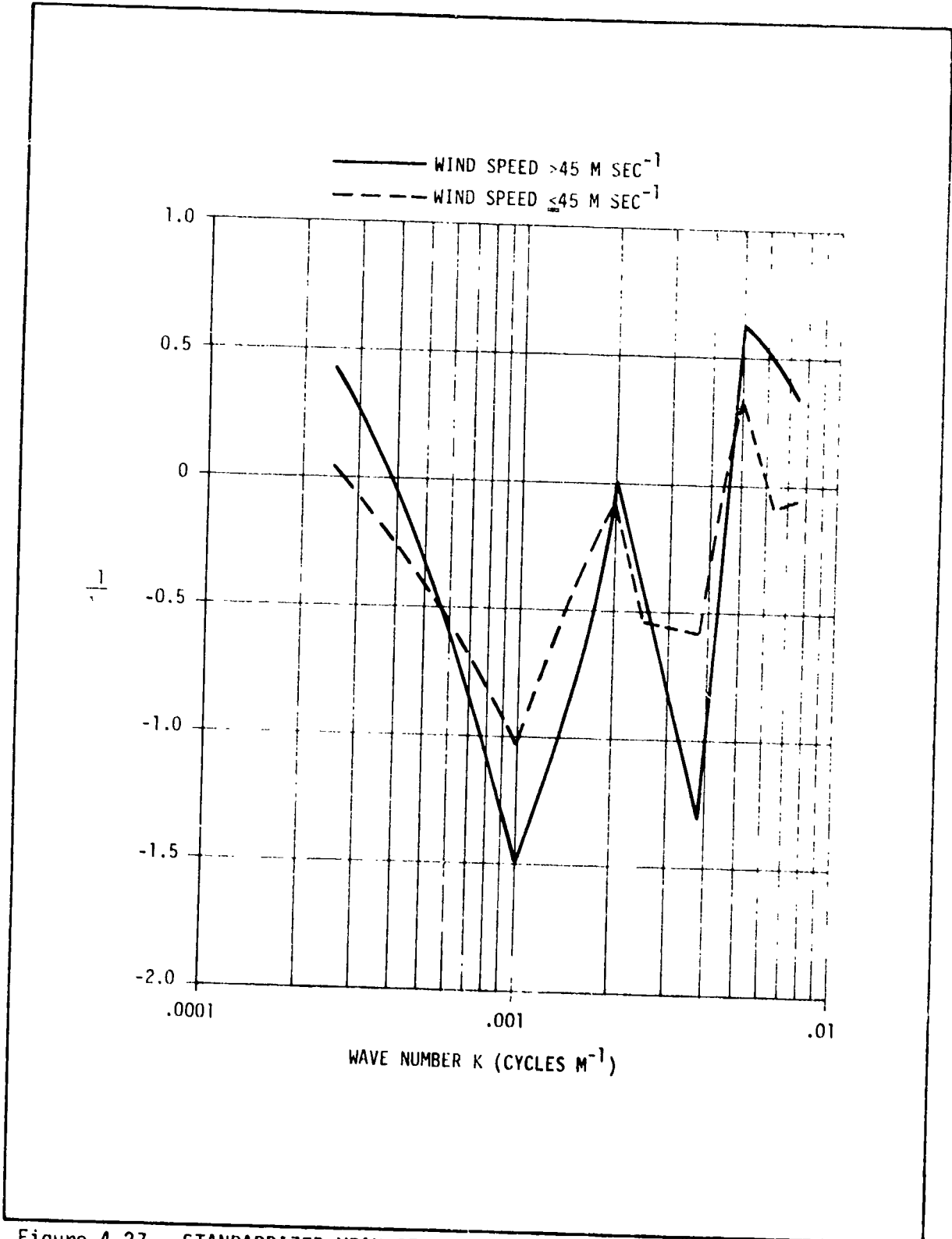


Figure 4-27. STANDARDIZED MEAN OF WIND SPECTRA DIFFERENCE VERSUS ELAPSED TIMES VERSUS WAVE NUMBERS FOR WIND SPEED LESS THAN 45 M SEC-1 AND GREATER THAN 45 M SEC-1

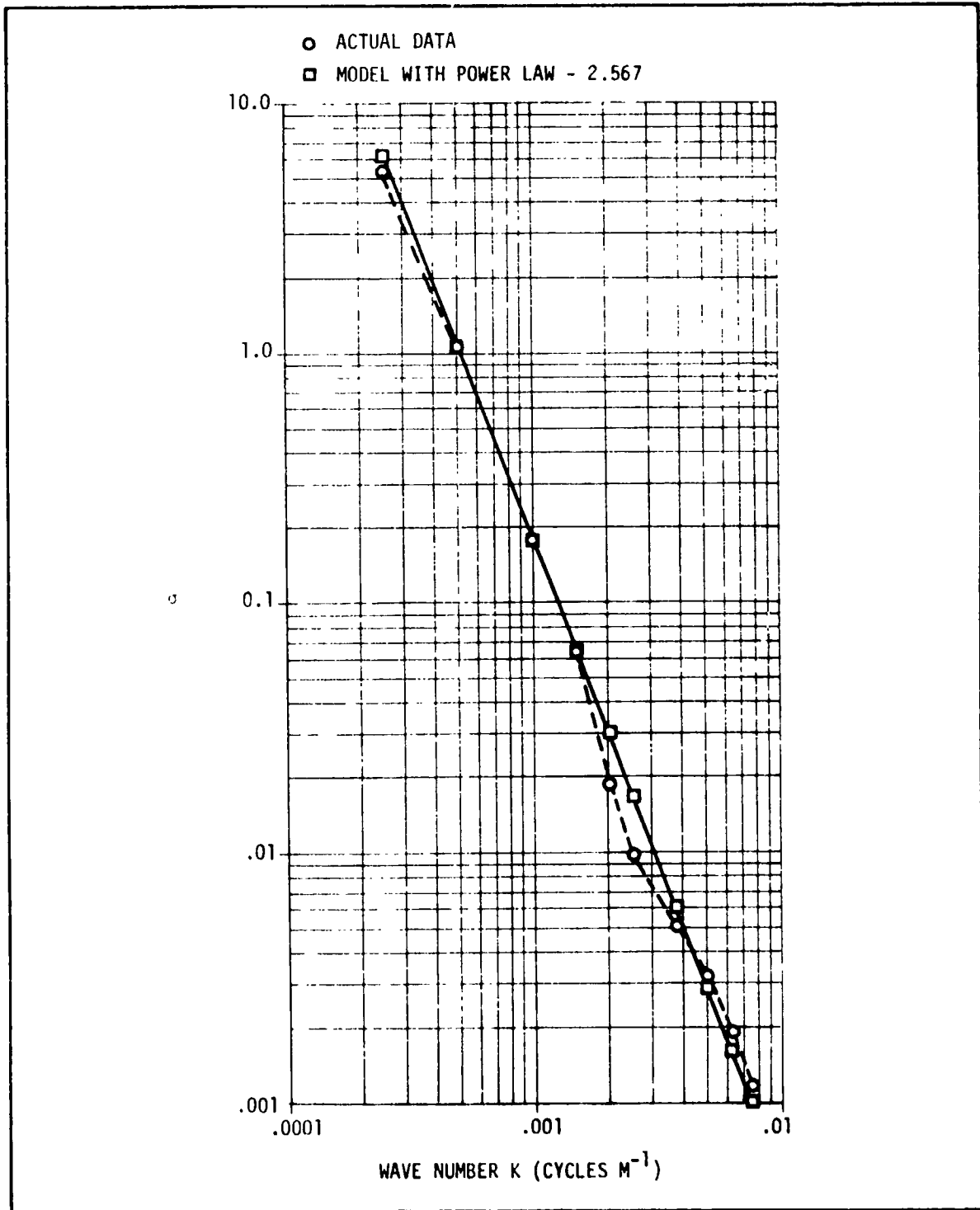


Figure 4-28. STANDARD DEVIATION OF WIND SPECTRA DIFFERENCE AVERAGED OVER ELAPSED TIMES AND WIND SPEEDS VERSUS WAVE NUMBERS AND THE POWER LAW  $\sigma (K) = \sigma(K_0) \left(\frac{K}{K_0}\right)^q$  WITH  $K_0 = 0.001$  CYCLES M<sup>-1</sup> AND  $q = -2.567$



## Section V

### CONCLUSIONS

Figures 4-11 and 4-22 are the "universal" curves for the probability of the wind spectra difference. These two curves agree very well between one and 99 percent level. The mean of the wind spectra difference is equal to zero. The standard deviation of the wind spectra difference can be found by using equation (3-1). By employing the universal curve, the wind spectra at later time can be estimated by giving present wind spectra.

The universal curves show 57 percent of the time the spectral changes are negative. This indicates most of the time the wind spectra decrease. So the detailed wind spectrum on the average build-up faster than it decays in an Eulerian context.

Here, for example, the wind spectra equals to  $10 \text{ m}^2 \text{ sec}^{-1} (\text{cycles m}^{-1})^{-1}$  at wave number  $0.001 \text{ cycles m}^{-1}$  for wind speeds less than  $45 \text{ msec}^{-1}$ . From Figure 4-11, the 95 percent value of standardized wind spectra difference is 1.9. From Figure 4-28, the standard deviation of wind spectra difference for wave number  $0.001 \text{ cycles m}^{-1}$  is 0.175. So the wind spectra difference at wave number  $0.001 \text{ cycles m}^{-1}$  is 0.3325, and the wind spectra at a later time will have 95 percent probability of not exceeding  $10.3325 \text{ m}^2 \text{ sec}^{-1} (\text{cycles m}^{-1})^{-1}$  at wave number  $0.001 \text{ cycles m}^{-1}$ .

**Section VI**  
**REFERENCES**

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2. DeMandel, R. E. and Krivo, S. J., "Selecting Digital Filters for Application to Detailed Wind Profiles", NASA CR-61325, The National Aeronautics and Space Administration, 1969, 136 pp.
3. Scoggins, J. R., "Sphere Behavior and Measurement of Wind Profiles", NASA TN D-3994, The National Aeronautics and Space Administration, 1967, 53 pp.
4. DeMandel, R. E. and Krivo, S. J., Radar/Balloon Measurement of Vertical Air Motions Between the Surface and 15 KM, J. Appl. Meteor., Vol. 10, April 1971, pp. 313-319.
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