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TRIANGULATION OF MULTISTATION CAMERA DATA TO LOCATE A CURVED LINE IN SPACE

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16. Abstract

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SUMMARY

A method is described for finding the location of a curved line in space from local azimuth as a function of elevation data obtained at several observation sites. A least-squares criterion is used to insure the best fit to the data. The method is applicable to the triangulation of an object having no identifiable structural features, provided its width is very small compared with its length so as to approximate a line in space. The method was implemented with a digital computer program and was successfully applied to data obtained from photographs of a barium ion cloud which traced out the Earth's magnetic field line at very high altitudes.

INTRODUCTION

A unique and powerful tool in magnetospheric studies involves the deposition of barium vapor at a point in the magnetosphere through the use of chemicals or explosives carried aloft by rockets. The barium atoms are rapidly ionized by sunlight and thus form a barium ion cloud which extends along the magnetic field line and becomes "frozen" to it. The barium ion cloud, on account of its resonant scattering of sunlight, is visible to ground sites when viewed against the night sky and hence serves to delineate the magnetic field line over a considerable arc length. This condition permits a determination of magnetic field line orientation and shape. From the motion of the cloud one may obtain the convective motion of magnetospheric plasma and hence the electric fields which drive such motions.

The accuracy of locating an object in the distant magnetosphere by triangulation methods is extremely limited by the relatively short baselines available. In the case of the barium ion cloud (ref. 1), even though the observation sites were widely dispersed on the Earth, the cloud altitude was about five times the baseline distance. Hence, it was necessary to use every means possible to improve the accuracy such as calibration for distortion in the cameras, orientation of cameras using stars in the photographs, and the use of more than two observation sites.

Existing triangulation methods were limited in one respect or another: methods using several observation sites triangulated only on points (refs. 2 and 3) and methods which triangulated on lines could use only two observation sites (refs. 4 to 6). Table I shows a comparison of the various methods.

In view of these limitations, it was desired to develop a triangulation method which allows for a cloud that is curved in space, extends over a large arc, and is to be photographed at several observation sites. The difficulty in obtaining a solution by triangulating on an extended object from several sites lies in the fact that the intersection of the several surfaces defined by the necessarily inaccurate data from several sites does not determine a unique line in space. Thus, some criterion is needed to determine the most probable solution when all the data are taken into account.

Methods already exist (refs. 7 to 9) for precise conversion of images on photographs to pointing directions from each site, such as azimuth as a function of elevation. This note describes a method and a computer program that find the most probable line solution from input data on azimuth as a function of elevation from several observation sites.

FORTRAN VARIABLES AND SYMBOLS

FORTRAN variables are the same as algebraic symbols, except that they are underlined when used in equations in the text.

A	equatorial radius, 6378.166 km; in subroutine SUMRES, angle in radians used to weight residuals
AZ(L,N)	azimuth angle from site L, data point N
a,b,c	coefficients in quadratic equation
B	polar radius, 6356.784 km
BC(I,L,N)	Ith coefficient in least-squares fit for site L, data point N
CLAT	geocentric latitude
CR	geocentric radius
C1,C2,C3	coefficients in least-squares fit to data for azimuth as a function of elevation (az-el)

D	residual (minimum angle between point P and az-el curve)
D(L)	residual from site L
DA	increment given to PLON
DC	discriminant of cubic equation
DDA	interpolation correction given to PLON to give minimum point
DEG	degrees per radian
DIST(L)	angle between point P and end data point on az-el curve, site L
DNA	altitude increment
DR	increment given to PLAT
DX,DY,DZ	axes in topocentric coordinate system; on horizontal plane in easterly direction, on horizontal plane in northerly direction, and in vertical direction perpendicular to horizontal plane, respectively
E	root-mean-square residual; in subroutine GGRGCN, ratio A^2/B^2
E(N)	minimum root-mean-square residual with respect to both latitude and longitude; thus, the solution at point N is defined
EL(L,N)	elevation angle from site L, data point N
EM	minimum root-mean-square residual (with variable longitude) for a given latitude
E1,E2,E3	in subroutine LONMIN, root-mean-square residual for three consecutive longitudes
F	flattening factor, 1/298.3; in subroutine SUMRES, weighting factor depending on DIST(L)
GLAT	geographic latitude

GLON	geographic or geocentric longitude
H	geographic altitude, km
HI	altitude of first estimated trial point
IN	number of iterations in subroutine MINISOL (usually 3)
IS	number of sites using data points on the end of az-el curve
L	observation site number
LP	index to designate line (LP=1) or point (LP=2) solution
N	in program LARC, data point number, or solution point number; in subroutine RESDUE, data point nearest the trial point (PAZ,PEL)
NBL	number of az-el data points (used for NB(L))
NB(L)	number of az-el data points from site L
NC(L,N)	az-el data point number of site L used to obtain solution point N
ND	index used to increment solution point number N by ±1
NO	number of az-el data points used in least-squares fit
NP(L)	data point number nearest solution point from site L
NS	number of observation sites
NV	particular data point within the set NO
N1	smallest solution point number
N2	largest solution point number
P,Q,R	coefficients in cubic equation

PAZ	azimuth of point P
PEL	elevation of point P
PI	π
PLAT	geocentric latitude of point P
PLATG	geographic latitude of point P
PLATI	geographic latitude of first estimated trial point P
PLON	longitude of point P
PLONI	longitude of first estimated trial point P
PR	geocentric radius of point P
R	radius of revolution of spheroid
RA	range from observation site S to point P
RAD	radians per degree
RP	projection of geocentric radius on equatorial plane
SH(L)	altitude of observation site L
SLAT(L)	geographic latitude of observation site L
SLON(L)	longitude of observation site L
SR	radius from Z-axis of observation site
TALT(N)	altitude of solution point N
TLAT(N)	geographic latitude of solution point N
TLON(L)	longitude of solution point N

WT(L) weighting factor for observation site L

X,Y,Z geocentric coordinate axes; in equatorial plane in direction of Greenwich, in equatorial plane directed to make a right-handed system, and in direction of north pole perpendicular to equatorial plane, respectively

XA,YA,ZA geocentric components of line-of-sight vector from observation site S to trial point P

XD angle between point P and data point on az-el curve

XP,YP,ZP geocentric components of trial point P

XS,YS,ZS geocentric components of observation site S

X1,Y1 independent and dependent variables, respectively, in least-squares curve fit to az-el data

ϕ,θ,ψ Eulerian rotation angles

GENERAL DISCUSSION OF MULTISTATION TRIANGULATION

Case of a Point Object

Consider first the simple case of a point object in space observed from several sites. (Fig. 1 illustrates the case of three sites.) In general, the measured lines of sight will not intersect because of errors in measuring the pointing direction from each site. One would expect the probable error in pointing to be the same at each site, where the error in pointing is defined as the angle between the measured direction and the actual direction of the object. In order to find the most probable solution, consider a trial solution point P in space and define the residual from a site as the angle between the measured direction from that site and the direction of the trial solution (D(1), D(2), and D(3) in fig. 1). The most probable solution, if it is assumed that pointing errors at each site are random, then, would be the one which minimizes the sum of the squares of the residuals $D(1)^2 + D(2)^2 + D(3)^2$. (See ref. 10, pp. 107-109.)

Case of a Curved Object

Next, consider the problem of locating a curved line object in space which is illustrated in figure 2 for three observation sites. The curve for the azimuth as a function of elevation from a single site defines a conical surface in space. With data from only two sites, intersection of the surfaces is unique, but with three or more sites, the intersections are no longer unique.

Generally, there is no distinguishable feature on any part of the cloud. The end points cannot be determined since they fade out gradually, and hence the location of the end depends on the exposure of the photograph, and brightness of the sky background. The center of the cloud length may sometimes be brighter, but it cannot be located accurately. Hence, if the curvature of the azimuth as a function of elevation is small, only errors perpendicular to the curve are important, since any error along the curve merely slides the curve on itself. It is reasonable to expect, then, that the probable error perpendicular to the surface defined by the az-el curve is the same for each site. Thus the residual of a trial point will be defined as the angle between the line of sight of that point and its projection on the conical surface defined by the curve for the variation of azimuth with elevation from the specified site. This angle, of course, represents the minimum angle between the line of sight to the trial point and any line of sight on the conical surface (D(1) in fig. 2). If the trial point is to lie on the most probable solution curve, then the sum of the squares of the residuals from all sites must be minimized. (This procedure is equivalent to minimization of the root-mean-square (rms) value of the residuals.)

It has been assumed throughout that the object is a line in space. If the object has lateral dimensions, then it must possess a center line which is identifiable as such from every observation site for the method to be applicable.

Minimization of Root-Mean-Square Residuals

Even though the desired solution is a curved line in space, the method described in this paper successively solves for specific points on that line. For convenience, the independent variable is altitude (which will be converted to geocentric radius) so that each point will be at a selected altitude; if the cloud had extended beyond the magnetic equator, then the independent variable should be latitude in order to be single-valued.

Thus, consider a trial solution point P which will remain at a fixed geocentric radius (which is essentially equivalent to a fixed altitude); the problem is to find the latitude and longitude which minimize the root-mean-square residuals. First, assume a trial latitude A and vary the longitude along A until a minimum root-mean-square residual EM_A is found, as indicated in figure 3. Next, increment the latitude to some new value B, and again vary longitude until a minimum as indicated by EM_B is obtained. Thus EM, the minimum root-mean-square residual with variable longitude can be obtained as a function of latitude, and one can readily find the latitude that minimizes EM; this latitude and its corresponding minimizing longitude is the point at the selected altitude that lies on the most probable solution curve. A description of the computer programs which use this method to find the most probable solution curve is given in the following sections.

COMPUTER PROGRAMS

The main program, called LARC (listing and flow chart are given in appendix A), in addition to reading cards, processing data, and printing results, has the main function of iterating the altitude at desired preselected intervals. Program LARC calls subroutine MINISOL which first calls subroutine LONMIN in order to minimize the root-mean-square residuals with respect to longitude, and then iterates the latitude in order to find the minimizing latitude.

The subroutines will be discussed first and are listed in appendix B. FORTRAN variables in the computer programs will have the same name in the text, but when used in algebraic equations, they will be underlined.

Appendix C describes the geographic, geocentric, and topocentric coordinate systems. The conversion from geographic to geocentric (subroutine GGRGCN), as well as the formulas for the inverse conversion (subroutine GCNGGR), are also given in appendix C.

Conversion From Geocentric to Topocentric Coordinates

Subroutine PAZEL converts a trial solution at a point P (given in geocentric coordinates, PLAT, PLON, and PR) to local azimuth and elevation from an observation site S with geographic coordinates SLAT, SLON, and SH. Figure 4 applies.

The first part of the program converts the observation site to geocentric coordinates XS, YS, and ZS by using the same method as in subroutine GGRGCN.

The geocentric coordinates of the trial point are converted to Cartesian coordinates XP, YP, and ZP. The geocentric coordinates of the line-of-sight vector from observation site S to trial point P are

$$\begin{bmatrix} \underline{XA} \\ \underline{YA} \\ \underline{ZA} \end{bmatrix} = \begin{bmatrix} \underline{XP} - \underline{XS} \\ \underline{YP} - \underline{YS} \\ \underline{ZP} - \underline{ZS} \end{bmatrix}$$

The geocentric axes X, Y, and Z can be brought into coincidence with axes DX, DY, and DZ by rotation through the Eulerian angles

$$\phi = \underline{SLON} + 90^{o}$$

$$\theta = 90^{o} - \underline{SLAT}$$

$$\psi = 0$$

Thus, from use of formulas derived in reference 11,

$$\begin{bmatrix} \underline{DX} \\ \underline{DY} \\ \underline{DZ} \end{bmatrix} = \begin{bmatrix} -\sin \underline{SLON} & \cos \underline{SLON} & 0 \\ -\sin \underline{SLAT} \cos \underline{SLON} & -\sin \underline{SLAT} \sin \underline{SLON} & \cos \underline{SLAT} \\ \cos \underline{SLAT} \cos \underline{SLON} & \cos \underline{SLAT} \sin \underline{SLON} & \sin \underline{SLAT} \end{bmatrix} \begin{bmatrix} \underline{XA} \\ \underline{YA} \\ \underline{ZA} \end{bmatrix}$$

Then the azimuth angle is

$$\underline{AZ} = \tan^{-1} \frac{\underline{DX}}{\underline{DY}}$$

and the elevation angle is

$$\underline{EL} = \tan^{-1} \frac{\underline{DZ}}{\left(\underline{DX}^2 + \underline{DY}^2\right)^{1/2}}$$

Sorting of az-el Data

Since certain subroutines require the az-el data to be ordered from one end of the curve to the other, it is necessary to insure that they are. Subroutine SORT(L) sorts the az-el data from each site L, for convenience, in such a way that the first point from each site corresponds to the high altitude end of the cloud; this is accomplished by setting the variable

$$\underline{XS} = -(-1)^{L}$$

This particular equation, of course, was made to hold for a particular orientation of the cloud and a particular set of observation sites.

Since elevation was the independent variable and single-valued, the ordering was done in terms of that variable only.

Computation of Second-Order Least-Squares Fit to

Successive az-el Curve Segments

The accurate computation of the residual is an important part of the triangulation method. The original az-el data, in a striving for accuracy, usually consists of many arbitrarily located closely spaced points along the curve, so that a least-squares curve fit can be used to reduce the random errors involved in measuring the cloud center line. A second-order fit is sufficient for defining a segment of the curve since only a short interval is needed in the vicinity of the trial point P. The number of points used in the curve fit is NO and depends on the number and quality of data points.

Subroutine BCOEF(L,N) computes the three coefficients BC(3,L,N) for a second-order least-squares curve fit from a given observation site numbered L, with the data point N as an origin. The NO data points used in the curve fit are centered about the data point N (except near the ends of the curve) and hence NO is an odd number. It is to be noted that the coefficients are calculated for every data point, of which there are NB(L).

An exact coordinate conversion to the point N as origin would involve the Eulerian angles AZ and EL, but since the angular deviations from this origin will always be small, the two orthogonal angular components are

$$\underline{X1} = \Delta\underline{EL}$$

and

$$\underline{Y1} = \Delta\underline{AZ} \cos (\underline{EL})$$

as illustrated in figure 5. The independent variable in the quadratic formula for the particular data point NV within the set NO is

$$\underline{X1} = \underline{EL}(\underline{L},\underline{NV}) - \underline{EL}(\underline{L},\underline{N})$$

The dependent variable is

$$\underline{Y1} = (\underline{AZ}(\underline{L},\underline{NV}) - \underline{AZ}(\underline{L},\underline{N}))\cos \frac{\underline{EL}(\underline{L},\underline{N}) + \underline{EL}(\underline{L},\underline{N})}{2}$$

Computation of Residuals

Subroutine RESDUE(PAZ,PEL,L,D,XD) finds the residual D of a trial solution point when the azimuth PAZ and the elevation PEL of the line of sight from site L to the trial point P are given.

The first part of the subroutine finds the data point N which is nearest the trial point PAZ,PEL, starting with the point from the previous calculation which is stored in NP(L) as a first try.

The next part of the subroutine computes the residual. Point P has the coordinates (with data point N as the origin)

$$\underline{X1} = \underline{PEL} - \underline{EL}(\underline{L},\underline{N})$$

$$\underline{Y1} = (\underline{PAZ} - \underline{AZ}(\underline{L},\underline{N}))\cos\frac{\underline{PEL} + \underline{EL}(\underline{L},\underline{N})}{2}$$

The second-order curve fit to the data points is

$$\underline{Y} = \underline{C1} + \underline{C2}\ \underline{X} + \underline{C3}\ \underline{X}^2$$

where

$$\underline{C1} = \underline{BC}(1,\underline{L},\underline{N}),\ \ldots$$

The distance between point P and the curve is given by

$$\underline{D}^2 = (\underline{X1} - \underline{X})^2 + (\underline{Y1} - \underline{Y})^2$$

To find the minimum distance, differentiate D^2 and set the result equal to zero:

$$(\underline{X1} - \underline{X}) + (\underline{Y1} - \underline{Y})\frac{d\underline{Y}}{d\underline{X}} = 0$$

Substitution for $\underline{Y}$ and $d\underline{Y}/d\underline{X}$ from the preceding equations gives the standard form for a cubic

$$\underline{X}^3 + \underline{P}\,\underline{X}^2 + \underline{Q}\,\underline{X} + \underline{R} = 0$$

where

$$\underline{P} = \frac{1.5\underline{C2}}{\underline{C3}}$$

$$\underline{Q} = \frac{1 + \underline{C2}^2 + 2\underline{C3}(\underline{C1} - \underline{Y1})}{2\underline{C3}^2}$$

$$\underline{R} = \frac{\underline{C2}(\underline{C1} - \underline{Y1}) - \underline{X1}}{2\underline{C3}^2}$$

The solution of this cubic equation is standard, but the type of solution depends on the value of the discriminant DC (as shown in the listing of subroutine RESDUE). If DC is greater than zero, there is one real root which is computed. If DC is less than zero, there are three real roots. The residual is thus

$$\underline{D} = \left((\underline{Y} - \underline{Y1})^2 + (\underline{X} - \underline{X1})^2\right)^{1/2}$$

where X is a real solution of the cubic, and Y is the corresponding value from the equation for Y given previously. The subroutine finds the real root which gives the smallest D.

For the case of a point solution, the subroutine is specialized with LP = 2, which causes the subroutine RESDUE to go to statement 10, and computes

$$\underline{Y} = (\underline{PAZ} - \underline{AZ}(\underline{L},\underline{K}))\cos\left(\frac{\underline{PEL} + \underline{EL}(\underline{L},\underline{K})}{2}\right)$$

$$\underline{X} = \underline{PEL} - \underline{EL}(\underline{L},\underline{K})$$

$$\underline{D} = \left(\underline{X}^2 + \underline{Y}^2\right)^{1/2}$$

where AZ(L,K), EL(L,K) is the data point from site L.

Calculation of Root Mean Square of Residuals

Subroutine SUMRES calls subroutine PAZEL to calculate azimuth and elevation for a trial point P, and then calls subroutine RESDUE to calculate the residual from each observation site. Then it computes the root-mean-square value of the residuals.

It usually happens that the cloud viewed at one site does not extend as far as from another site. Thus, it is desirable to extrapolate data from such sites, but with reduced weighting. Subroutine SUMRES reduces the weighting by the factor

$$\frac{\underline{A}}{\underline{A} + \underline{DIST(L)}}$$

where DIST(L) is either zero or equal to DX, the distance between the trial az-el and the nearest data point NP(L) on the az-el curve, and $\underline{A} = 0.5/57.3$ radians, a somewhat arbitrary fixed angle of 0.5°. Finally, DIST(L) is set equal to zero for the two sites having the smallest value of DIST(L).

Variation of Longitude To Obtain Minimum Residuals

Subroutine LONMIN(PLAT,PLON,PR,DR,EM) finds the minimum value of the root-mean-square residuals EM as the longitude PLON is varied while keeping the geocentric latitude PLAT and radius PR constant.

The procedure is to increment PLON by DA (which is initially equal to DR), changing directions when necessary to go through a minimum, calling SUMRES to calculate the root-mean-square residual. Consecutive root-mean-square values are labeled (and relabeled as PLON is incremented) E1, E2, and E3 so that when E2 is the smallest, the PLON which gives minimum can be approximated by using the following analysis.

Assume that the root-mean-square residual E is a second-order function of the longitude DA:

$$\underline{E} = a + b\underline{DA} + c\underline{DA}^2$$

For minimum E, the longitude is

$$\underline{DDA} = -\frac{b}{2c}$$

From the three values E1, E2, and E3, and the corresponding longitudes -DA, 0, and DA (using the longitude of the middle point as origin) one can obtain by substitution into E

$$b = -\frac{\underline{E1} - \underline{E3}}{2\underline{DA}}$$

$$c = \frac{\underline{E3} + \underline{E1} - 2\underline{E2}}{2\underline{DA}^2}$$

from which the longitude (relative to point 2) for minimum root-mean-square residuals is

$$\underline{DDA} = \frac{(\underline{E1} - \underline{E3})\underline{DA}}{2(\underline{E1} - 2\underline{E2} + \underline{E3})}$$

The increment DA is then decreased by a factor of 10, and the procedure is repeated one time.

Minimization With Latitude

Subroutine MINISOL(PLAT,PLON,PR,DR,IN,E) starts with the trial point PLAT, PLON, PR and while keeping PR fixed, varies PLAT. For each PLAT, LONMIN is called to find the minimum root-mean-square residuals with respect to PLON.

When three consecutive minimum root-mean-square residuals E1, E2, and E3 are found so that E2 is the least, then an approximation is made (same method as in LONMIN, except independent variable is now latitude DR) to determine the PLAT for minimum residual. The increment DR is then decreased and the procedure is repeated until IN iterations are made (typically three, with consecutive DR values of 0.1°, 0.01°, and 0.001°). Upon returning to LARC, the new values of PLAT and PLON give the solution at the particular PR.

Program LARC

The main program is program LARC, which first reads in the observation site data cards in geographic coordinates. For each pass through the program, a triangulation is made at one instant of time by using az-el data from simultaneous photographs at each site; thus a solution curve of latitude and longitude as functions of altitude is obtained.

After the time is read in, azimuth and elevation data cards from each site are read and stored in AZ(L,N) and EL(L,N); L refers to the site number and N to the data point number. Then subroutine SORT(L) sorts the data into either increasing or decreasing elevation angles.

The BC(I,L,N) coefficients are then calculated for each site L and for every data point N by calling the BCOEF subroutine.

The first estimated solution is chosen near the center of the cloud; since the first estimate may be far from the solution, it is better to start in a region where the data have the best quality. An integral 100-km value of altitude is used and is read from a data card. The first solution point is labeled N = 50 with altitude increments DNA = 100 km corresponding to increments of 1 in N, where N now refers to a solution point and not a data point. Subsequent trial solutions use the previous solution point.

One of the main difficulties in the solution occurs near the ends of the cloud. Because of differences in exposure, range of cloud, orientation, and visual conditions, the cloud visibility may extend farther at one site than at another. Subroutine SUMRES will automatically extrapolate curves when necessary, but will give less weight to the extrapolated parts. Hence a procedure is needed to stop the calculation when the solution curve is going beyond the data from every observation site. This procedure is accomplished with the index IS which is equal to the number of sites using a data point on the end of the curve in subroutine RESDUE. When all sites except one are extrapolated beyond the end of the az-el curve, then the solution is stopped, and started again at the middle (N = 50) with the altitude now incremented downward.

Solutions are stored in TLAT(N), TLON(N), and TALT(N). For each solution point, the point on the az-el curve for each site is stored in NC(L,N) and the root-mean-square residuals are stored in E(N) in units of degrees. Finally, the solution is printed out.

ILLUSTRATIVE CASE

As an example, actual data from the barium ion release of September 21, 1971, will be given. Three observation sites were used: (1) Mt. Hopkins, Arizona; (2) Cerro Morado, Chile; and (3) Wallops Island, Virginia. (Coordinates are given at beginning of table II.) Table II shows a printout of the az-el input data for the time 3 hrs 18 min 10 sec UT (13.307 min after release). There were 36 data points from site 1, 75 from site 2, and 28 from site 3. The number of points NO used in the least-squares fit was 25, which represents about 1.7° of arc when viewed from site 2, and 2.7° when viewed from the other two sites.

Table III shows the final solution. Total central processor time for the job on the Control Data Corporation model 6600 computer was 19.5 seconds. A test was made to see how many iterations were made in the solution. For this case, SUMRES was called a total of 2922 times, or on the average, 104 times for each solution point at each altitude. The root-mean-square residuals were of the order of 0.0045°.

CONCLUDING REMARKS

A computer program for the triangulation of azimuth-elevation (az-el) data from several observation sites has been presented. Because of the relatively short baseline used and the high accuracy required, it was necessary to make a special effort to reduce the effects of random errors. This reduction was accomplished by using several observation sites and many data points from each site.

An optimal solution was achieved by requiring the minimization of the root-mean-square residual of all the sites and by using a least-squares curve fit to the az-el data. An illustrative example using three observation sites was given for an actual barium cloud. The root-mean-square residuals were of the order of 0.005°.

Langley Research Center,
National Aeronautics and Space Administration,
Hampton, Va., February 6, 1974.

APPENDIX A

PROGRAM LARC

Program Listing

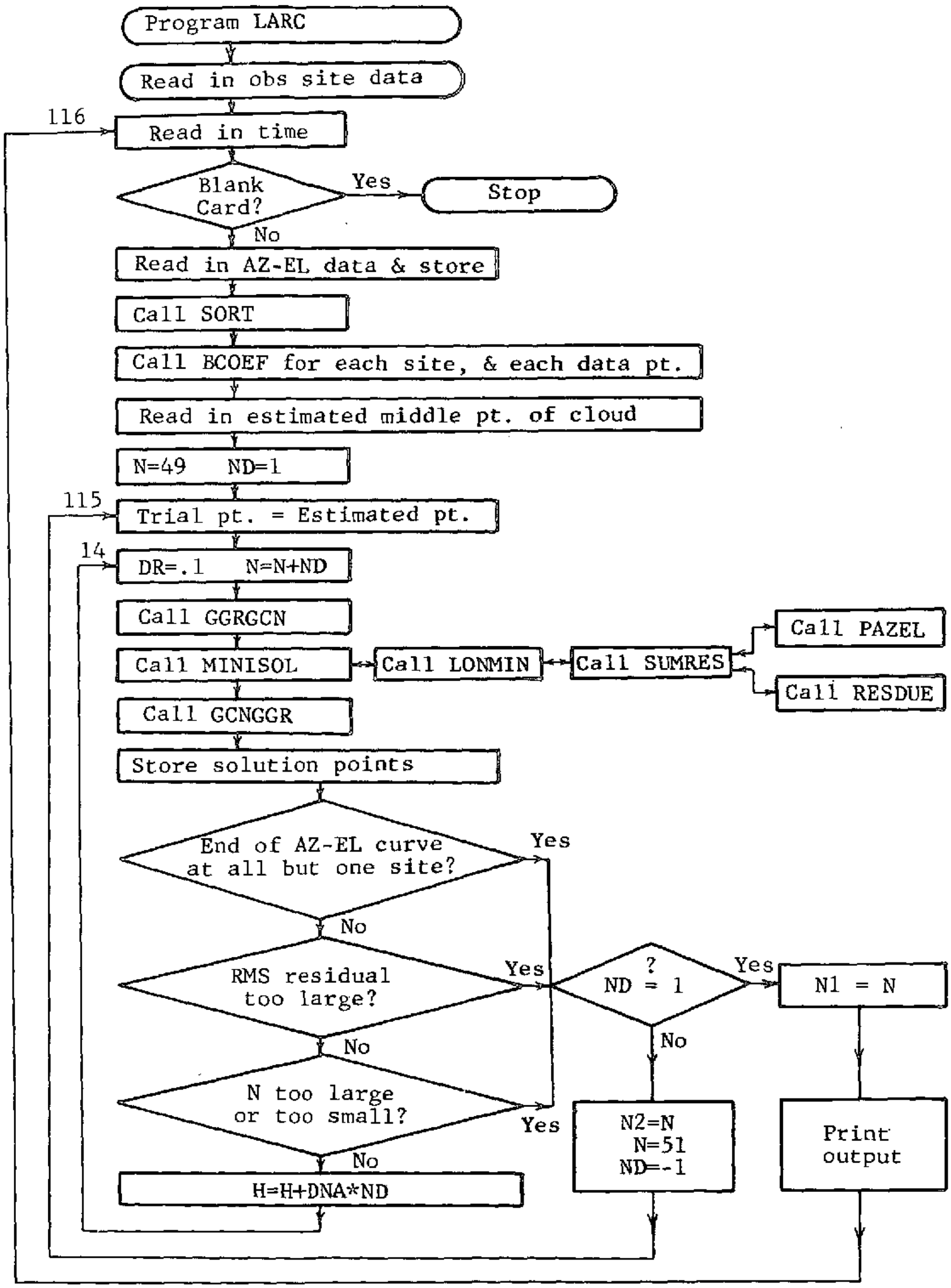
The listing and flow chart for program LARC are presented in this appendix.

```
      PROGRAM LARC(INPUT,OUTPUT)
C
****  THIS PROGRAM SOLVES THE PROBLEM OF LOCATING A CURVED LINE IN SPACE GIVEN
*     SIMULTANEOUS AZ-EL DATA FROM SEVERAL OBSERVATION SITES.  THE METHOD FINDS
*     SUCCESSIVE LAT AND LON POINTS AT SELECTED ALTITUDE INCREMENTS.  EACH POINT
*     IS VARIED IN BOTH LONGITUDE AND LATITUDE IN ORDER TO MINIMIZE THE RMS
*     RESIDUALS FROM ALL THE OBS SITES.
*     THE RESIDUALS ARE COMPUTED USING A 2ND ORDER LEAST SQUARES CURVE FIT TO
*     THE AZ-EL DATA IN THE VICINITY OF EACH TRIAL POINT.
*     THE PROGRAM FINDS A COMPLETE CURVED LINE SOLUTION FOR SEVERAL EPOCHS
*     OR TIMES
C
      DIMENSION    TLAT(99),TLON(99),TALT(99)
      DIMENSION E(99),             NC(5, 99)
      COMMON/SITES/NS,SLAT(5),SLON(5),SH(5),WT(5),DEG
      COMMON/LINE/AZ(5,190),EL(5,190),BC(3,5,190),NP( 5 ),NB(5),LP
      R=6371.2      $  PI=3.1415926535898     $  RAD=PI/180   $  DEG=1/RAD
*     SET IN ALTITUDE INCREMENT
      DNA=100.
      WT(1)=WT(2)=WT(3)=WT(4)=WT(5)=1.
*     READ IN NUMBER OF OBS SITES
      READ 10,NS
10    FORMAT(I5)
*     PRINT OUT NO OF OBS SITES
      PRINT 101,NS
101   FORMAT(*1*I2* STATION TRIANGULATION*/)
***   READ IN GEOGRAPHIC COORDINATES (IN DEGREES AND KM) FOR EACH SITE AND
*     CONVERT TO RADIANS
      DO 11 L=1,NS
      READ 12,SLAT(L),SLON(L),SH(L)
12    FORMAT(2F20.10,F10.2)
      PRINT 112,SLAT(L),SLON(L),SH(L)
112   FORMAT(*   SLAT=*F9.4*     SLON=*F9.4*     SALT=*F8.4)
      SLAT(L)=SLAT(L)*RAD     $   SLON(L)=SLON(L)*RAD
11    CONTINUE
*****THIS IS BEGINNING OF SOLUTION FOR EACH EPOCH OR TIME.
*     READ IN TIME
116   READ 111,IHR,MIN,SEC
111   FORMAT(I15,I3,F7.1)
*     IF TIME CARD IS BLANK, STOP--THE LAST EPOCH HAS BEEN PROCESSED.
      IF(IHR.EQ.0) STOP
*     PRINT  TIME
      PRINT 110,IHR,MIN,SEC
110   FORMAT(*1TIME*I5* HR*I5* MIN*F7.1* SEC*)
*     INITIALIZE STARTING POINT FOR LATER SEARCH ON AZ-EL CURVE
      NP(1)=NP(2)=NP(3)=NP(4)=NP(5)=3
***   READ IN INPUT AZ EL DATA FOR EACH SITE
      DO 21 L=1,NS
```

```
*       READ IDENTIFICATION INFORMATION ON DATA CARD, WHICH HAS COLUMN 10 BLANK.
*       ON SAME CARD ALSO READ AZ-EL FOR ONE DATA POINT.
        N=0
113     READ 23,LST,IS1,IS2,TIME,AZ1     ,EL1
23      FORMAT (9X,I1,A10,A6,A10,22X,2F11.0)
*       IF LST (COLUMN 10) IS NOT BLANK, END OF AZ-EL DATA FOR THIS SITE IS
*       INDICATED.
        IF(LST.NE.0) GO TO 22
        N=N+1          $   NB(L)=N
*       STORE AZ-EL DATA AND PRINT AZ,   EL,   AND IDENTIFICATION INFORMATION
        AZ(L,N)=AZ1   $   EL(L,N)=EL1
        PRINT 123,L,N,AZ(L,N),EL(L,N),IS1,IS2,TIME
123     FORMAT(* STA *I2*   PT *I3*   AZ=*F9.4*    EL=*F9.4,A10,A6,A10)
*       CONVERT EL TO RADIANS
        EL(L,N)=EL(L,N)*RAD
*       PUT AZ IN PROPER QUADRANT AND COVERT TO RADIANS
        IF(AZ(L,N).GT.180.) AZ(L,N)=AZ(L,N)-360.
        AZ(L,N)=AZ(L,N)*RAD
        GO TO 113
22      CONTINUE
*       SORT AZ-EL DATA IN ORDER OF EL
        CALL SORT(L)
21      CONTINUE
        DO 13 L=1,NS              $   NBL=NB(L)
        DO 13 N=1,NBL
*       COMPUTE COEFFICIENTS FOR  SECOND ORDER LEAST SQUARES LOCAL FIT AT EVERY
*       DATA POINT FOR EACH OBS SITE.
13      CALL BCOEF(L,N)
*       NOTE LP=1 IS FOR A LINE SOLUTION. IN IS NO. OF ITERATIONS IN MINISOL.
        LP=1             $   IN=3
*       READ FIRST ESTIMATED PLAT AND PLON IN GEOGRAPHIC DEGREES AT A SELECTED
*       ALTITUDE HI AND PRINT OUT.   HI SHOULD BE INTEGRAL MULTIPLES OF ALTITUDE
*       INCREMENT.
        READ 131,PLATG,PLON,HI
131     FORMAT(3F10.0)
        PRINT 114,PLATG,PLON,HI
114     FORMAT(*0ESTIMATED TRIAL POINT    LAT=*F6.3*    LON=*F7.3*    ALT=*F6
       $.0)
*       INITIALIZE N AND ND SO THAT FIRST SOLUTION POINT IS N=50 AND ALTITUDE WILL
*       INCREASE.
        N=49          $   ND=1
        PLATI=PLATG*RAD             $  PLONI=PLON*RAD
**      STARTING PLACE FOR ALTITUDE INCREASING OR DECREASING
115     PLATG=PLATI   $   PLON=PLONI        $    H=HI
*****STARTING PLACE FOR EACH SOLUTION POINT
14      ER=.1*RAD
*       INCREMENT N
        N=N+ND
*       CONVERT TO GEOCENTRIC
```

```
      CALL GGRGCN(PLATG,H,PLAT,PR)
*     FIND SOLUTION POINT, I.E. PLAT AND PLON WHICH MINIMIZES RMS RESIDUAL
*     AT RADIUS PR
      CALL MINISOL(PLAT,PLON,PR,DR,IN,E(N))
*     CONVERT BACK TO GEOGRAPHIC AND DEGREES AND STORE ANSWERS
      CALL GCNGGR(PLAT,PR,PLATG,TALT(N))   $  TLAT(N)=PLATG*DEG
      TLON(N)=PLON*DEG
*     FOR EACH SITE STORE DATA POINT NUMBER WHICH IS NEAREST THE SOLUTION
      DO 141 L=1,NS
*     DETERMINE NO. OF SITES WHICH ARE USING END POINT DATA
141   NC(L,N)=NP(L)
      IS=0
      DO 16 K=1,NS
      IF(NP(K).LE.1) IS=IS+1
      IF(NP(K).GE.NB(K)) IS=IS+1
16    CONTINUE
*     IF ALL BUT ONE SITE USED END-POINT DATA, JUMP OUT OF LOOP.
      IF(IS.GE.NS-1) GO TO 15
*     IF RMS RESIDUAL IS TOO GREAT JUMP OUT OF LOOP
      IF(E(N) .GT.0.13) GO TO 15
*     IF N IS TOO LARGE JUMP OUT OF LOOP
      IF(N.EQ.99.OR.N.EQ.1) GO TO 15
*     INCREMENT H, GO TO 14 AND START NEXT SOLUTION POINT
      H=H+DNA*ND              $  GO TO 14
*     IF ALTITUDE IS DECREASING JUMP OUT OF LOOP.  ENTIRE LINE SOLUTION IS
*      NOW FINISHED
15    IF(ND.EQ.-1) GO TO 331
*     RESET N AND ND TO START AT CENTER AND DECREASE ALTITUDE
      N2=N            $  N=51   $  ND=-1
      GO TO 115
331   N1=N
*     PRINT TIME AS HEADING FOR COMPLETE SOLUTION PRINT-OUT
      PRINT 382,IHR,MIN,SEC
382   FORMAT(*1TIME   *I2* HRS   *I2* MIN   *F5.1* SEC*)
      PRINT 38
38    FORMAT(* LINE SOLUTION*)
      PRINT 39
39    FORMAT(*0ALTITUDE     LATITUDE     LONGITUDE     RMS RES     PTS ON AZ
     3$-EL CURVE*)
*     PRINT OUT TOTAL NO. OF DATA PTS. FOR EACH SITE
      PRINT 342,(NB(L),L=1,NS)
342   FORMAT(52X,5I5)
*     PRINT OUT SOLUTION POINTS FOR EACH N
      DO 34 N=N1,N2
      PRINT 36,TALT(N),TLAT(N),TLON(N),E(N),(NC(L,N),L=1,NS)
36    FORMAT(F9.0,F12.3,F13.3,F11.4,I12,4I5)
34    CONTINUE
35    GO TO 116
      END
```

Program LARC Flow Chart



APPENDIX B

SUBPROGRAMS

```
      SUBROUTINE GGRGCN(GLAT,H,CLAT,CR)

***   SUBROUTINE TO CONVERT GEOGRAPHIC LATITUDE GLAT AND ALTITUDE H
***   TO GEOCENTRIC LATITUDE CLAT, AND RADIUS CR

      A=6378.166    $  B=6356.784     $  E=B*B/A/A
      R=A/SQRT(1.+E*TAN(GLAT)**2)     $  Z=R*E*TAN(GLAT)
      RP=R+H*COS(GLAT)         $  ZP=Z+H*SIN(GLAT)
      CR=SQRT(RP*RP+ZP*ZP)     $  CLAT=ATAN2(ZP,RP)
      RETURN        $  END


      SUBROUTINE GCNGGR(CLAT,CR,GLAT,H)

***   SUBROUTINE TO CONVERT GEOCENTRIC LATITUDE CLAT AND GEOCENTRIC RADIUS CR TO
*     GEOGRAPHIC LATITUDE GLAT AND ALTITUDE H.

      A=6378.166    $  F=1./298.3
      H=CR-A+A*F/2.*(1.-COS(2.*CLAT)+.5*(F/4.-A*F/CR)*(COS(4.*CLAT)-1.))
      GLAT=CLAT+A*F/CR*SIN(2*CLAT)+(A*F/CR)**2*(1.-CR/4./A)*SIN(4.*CLAT)
      RETURN        $  END


      SUBROUTINE PAZEL(SLAT,SLON,SH,PLAT,PLON,PR,AZ,EL)
C
C*****GIVEN AN OBS SITE S (IN GEOGRAPHIC COORDINATES) AND
C     A POINT P (IN GEOCENTRIC COORDINATES) IN SPACE, FIND THE
C     LINE-OF-SIGHT (AZ,EL) FROM S TO P
C
      A=6378.166    $  B=6356.784      $  E=B*B/A/A
      R=A/SQRT(1.+E*TAN(SLAT)**2)      $  Z=R*E*TAN(SLAT)
*     COMPUTE GEOCENTRIC CARTESIAN COMPONENTS OF OBS SITE S
      SR=R+SH*COS(SLAT)         $  ZS=Z+SH*SIN(SLAT)
      XS=SR*COS(SLON)           $  YS=SR*SIN(SLON)
*     COMPUTE GEOCENTRIC CARTESIAN COMPONENTS OF POINT P
      XP=PR*COS(PLAT)*COS(PLON)
      YP=PR*COS(PLAT)*SIN(PLON)         $  ZP=PR*SIN(PLAT)
*     COMPUTE COMPONENTS OF VECTOR DIRECTION FROM SITE S TO POINT P
      XA=XP-XS      $  YA=YP-YS         $  ZA=ZP-ZS
*     CONVERT TO TOPOCENTRIC CARTESIAN COMPONENTS
      DX=-XA*SIN(SLON)+YA*COS(SLON)
      DY=-XA*SIN(SLAT)*COS(SLON)-YA*SIN(SLAT)*SIN(SLON)+ZA*COS(SLAT)
      DZ=+XA*COS(SLAT)*COS(SLON)+YA*COS(SLAT)*SIN(SLON)+ZA*SIN(SLAT)
*     COMPUTE AZ-EL DIRECTION
      AZ=ATAN2(DX,DY)
      EL=ATAN2(DZ,SQRT(DX*DX+DY*DY))
      RETURN        $  END
```

```
      SUBROUTINE SORT(L)
      COMMON/LINE/AZ(5,190),EL(5,190),BC(3,5,190),NP( 5 ),NB(5),LP

******THIS SUBROUTINE SORTS AZ-EL DATA IN EITHER INCREASING OR DECREASING VALUES
*     OF EL, DEPENDING ON THE VALUE OF L, SO THAT FIRST VALUE ALWAYS CORRESPONDS
*     TO THE HIGH ALTITUDE END OF THE CLOUD.
***   XS MUST BE MODIFIED FOR A DIFFERENT SITE CONFIGURATION OR CLOUD
*     ORIENTATION.

1     XS=-(-1.)**L              $  N=NB(L)
      DO 100 I=1,N              $  K=N-I
**    IF ELEVATION ANGLE OF ONE POINT IS EQUAL TO THAT OF ANOTHER,
*     EXCHANGE THE ONE POINT WITH THE VERY LAST POINT ON THE LIST,
*     AND DECREASE THE TOTAL NUMBER OF DATA POINTS BY 1.
      DO 100 J=1,K              $  IF((EL(L,J)-EL(L,J+1))*XS) 100, 20,10
20    NB(L)=NB(L)-1
      TEL=EL(L,J)               $  TAZ=AZ(L,J)
      EL(L,J)=EL(L,N)           $  AZ(L,J)=AZ(L,N)
      EL(L,N)=TEL               $  AZ(L,N)=TAZ
      GO TO 1
10    TEL=EL(L,J)               $  TAZ=AZ(L,J)
      EL(L,J)=EL(L,J+1)         $  AZ(L,J)=AZ(L,J+1)
      EL(L,J+1)=TEL             $  AZ(L,J+1)=TAZ
100   CONTINUE
      RETURN          $   END



      SUBROUTINE BCOEF(L,N)
      COMMON/LINE/AZ(5,190),EL(5,190),BC(3,5,190),NP( 5 ),NB(5),LP
      DIMENSION              A(3,3),B(3),C(50,3),IP(3)

******THIS SUBROUTINE COMPUTES THE THREE COEFFICIENTS FOR A SECOND ORDER
*     LEAST SQUARES CURVE FIT TO THE AZ-EL DATA AT EVERY DATA POINT N.
*     THE COMPUTATION USES NO DATA POINTS CENTERED ABOUT N, EXCEPT AT THE ENDS
*     WHERE NO END POINTS ARE USED.  THEINDEPENDENT VARIABLE IS
*     EL(L,NV)-EL(L,N) WHERE NV IS WITHIN THE SET NO.

      NO=25
      NO=NO+2
10    NO=NO-2
**    IF THERE ARE FEWER THAN NO DATA POINTS, THEN DECREASE NO
      IF(NB(L).LT.NO) GO TO 10
**    THE MIDDLE POINT OF THE SET NO, IS CALLED NM AND IS USUALLY THE SAME AS N.
                       M=3      $  NM=N                    $   NC=(NO+1)/2
*     AT ENDS OF CURVE, NM IS SPACED NC POINTS FROM THE END.
      IF(N.LT.NC) NM=NC         $  IF(N.GT.NB(L)+1-NC) NM=NB(L)+1-NC
**    THE FOLLOWING COMPUTES THE MATRICES A AND B WHICH ARE USED TO FIND THE
*     LEAST SQUARES FIT.
      DO 20 I=1,NO
20    C(I,1)=1.
      DO 50 J=2,M               $  DO 50 I=1,NO        $  NV=NM-NC+I
50    C(I,J)=C(I,J-1)*(EL(L,NV)-EL(L,N))
      DO 100 I=1,M              $  DO 100 J=1,M        $  A(I,J)=0.
      DO 100 K=1,NO
100   A(I,J)=A(I,J)+C(K,I)*C(K,J)
      DO 150 I=1,M              $  B(I)=0.
      DO 150 K=1,NO             $  NV=NM-NC+K
150   B(I)=B(I)+C(K,I)*(AZ(L,NV)-AZ(L,N))*COS((EL(L,N)+EL(L,NV))/2)
**    SIMEQ SOLVES THE EQUATION AX=B.  THE SOLUTION X IS RETURNED IN B, WHICH
*     CONTAINS THE 3 COEFFICIENTS DESIRED.
      CALL SIMEQ(A,M,B,1,D,IP,M,IS)
```

```
      DO 200 I=1,M
*     THE COEFFICIENTS B ARE STORED IN BC.
200   BC(I,L,N)=B(I)
      IF(N.EQ.NB(L)) PRINT 301,L,N,NO
301   FORMAT(*0L=*I2*    N=*I4*    NO=*I3)
      RETURN       $ END
```

```
      SUBROUTINE RESDUE(PAZ,PEL,L,D,XD)

***** GIVEN AN EL AND AZ OF A POINT FROM STATION L, FIND THE
****  RESIDUAL (ANGULAR DISTANCE BETWEEN THE POINT AND THE AZ-EL CURVE)

      COMMON/LINE/AZ(5,190),EL(5,190),BC(3,5,190),NP( 5 ),NB(5),LP
      COMMON/SITES/NS,SLAT(5),SLON(5),SH(5),WT(5),DEG
      IF(LP.EQ.2) GO TO 10
*****FIND POINT NUMBER ON CURVE CLOSEST TO GIVEN PAZ PEL
      N=N1=NP(L)      $   ND=1
      DSN=((PAZ-AZ(L,N))*COS(PEL))**2+(PEL-EL(L,N))**2
      IF(N.EQ.NB(L)) ND=-1
1     DSO=DSN
      N=N+ND          $   IF(N.GT.NB(L).OR.N.LT.1) GO TO 2
      DSN=((PAZ-AZ(L,N))*COS(PEL))**2+(PEL-EL(L,N))**2
      IF(DSN.LT.DSO) GO TO 1 $  IF(N.GT.N1+1) GO TO 2
      IF(N.LT. N1) GO TO 2
      ND=-1           $   N=N+ND               $   DSN=DSO
      GO TO 1
2     N=N-ND          $   NP(L)=N
*****COMPUTE RESIDUAL FROM BC COEFFICIENTS CORRESPONDING TO N
      X1=PEL-EL(L,N)               $  Y1=(PAZ-AZ(L,N))*COS((PEL+EL(L,N))/2.)
      XD=SQRT(X1*X1+Y1*Y1)
      C1=BC(1,L,N)                 $   C2=BC(2,L,N)        $   C3=BC(3,L,N)
*     CALCULATE THE COEFFICIENTS OF THE CUBIC
      P=1.5*C2/C3   $   Q=(1.+C2*C2+2.*C3*(C1-Y1))/2./C3/C3
      R=(C2*(C1-Y1)-X1)/2./C3/C3
      A=Q-P*P/3.                   $   B=(2.*P*P-9.*Q)*P/27.+R
      DC=B*B/4.+A*A*A/27.
*     IF DISCRIMINANT IS LESS THAN ZERO, THERE ARE THREE REAL ROOTS
      IF(DC.LE.0.) GO TO 5
*     DISCRIMINANT IS GREATER THAN ZERO, COMPUTE THE ONE REAL ROOT.
      RDC=SQRT(DC) $   CA=-B/2.+RDC       $   CB=-B/2.-RDC $   E=1./3.
      X=SIGN((ABS(CA))**E,CA)+SIGN((ABS(CB))**E,CB)-P/3.
      Y=C1+(C2+C3*X)*X             $   DS=(Y-Y1)*(Y-Y1)+(X-X1)*(X-X1)
      D=SQRT(DS)                   $   RETURN
5     PHI=ACOS(-B/2./SQRT(-A*A*A/27.))
      DSO=100.
*     COMPUTE THE THREE REAL ROOTS AND FIND THE SMALLEST.
      DO 6  I=1,3
      X=2.*SQRT(-A/3.)*COS(PHI/3.+120.*(I-1)/DEG)-P/3.
      Y=C1+(C2+C3*X)*X             $   DS=(Y-Y1)*(Y-Y1)+(X-X1)*(X-X1)
      IF(DSO.GT.DS) DSO=DS
6     CONTINUE
      D=SQRT(DSO)                     $   RETURN
****** FOR A POINT SOLUTION LP=2, AND THE FOLLOWING IS USED
10    K=1
      Y=(PAZ-AZ(L,K))*COS(EL(L,K))
      X=PEL-EL(L,K)                $   D=SQRT(X*X+Y*Y)
      RETURN       $   END
```

```
      SUBROUTINE SUMRES(PLAT,PLON,PR,E)

*****GIVEN A POINT, CALCULATE THE ROOT-MEAN-SQUARE OF THE RESIDUALS FROM
*     ALL OBSERVATION SITES.
*****IF THE POINT IS OFF THE END OF THE AZ-EL CURVE OF A SITE,THEN THE WT FACTOR
***** F WILL DIMINISH THE RESIDUAL FROM THAT SITE AS DETERMINED BY DIST, THE
*****ANGULAR DISTANCE FROM THE END OF THE CURVE.  HOWEVER, FOR THE TWO SITES
*****WHICH HAVE THE LEAST DISTANCE FROM THE END, MAKE DIST=0, SO THAT F=1, WHICH
*****GIVES FULL WT.

      COMMON/SITES/NS,SLAT(5),SLON(5),SH(5),WT(5),DEG
      COMMON/LINE/AZ(5,190),EL(5,190),BC(3,5,190),NP( 5 ),NB(5),LP
      DIMENSION D(5),DIST(5)
      A=.5/DEG
      DO 1 J=1,NS                   $  IND=0
*     FIND PAZ AND PEL OF POINT
      CALL PAZEL(SLAT(J),SLON(J),SH(J),PLAT,PLON,PR,PAZ,PEL)
*     CALCULATE RESIDUAL
      CALL RESDUE(PAZ,PEL,J,D(J),XD)
      IF(NP(J).EQ.1.OR.NP(J).EQ.NB(J)) IND=1
1     DIST(J)=IND*XD
*****SORT DIST INTO INCREASING ORDER
      DO 100 I=1,NS                 $  K=NS-I
      DO 100 J=1,K                  $  IF(DIST(J)-DIST(J+1)) 100,100,10
10    TEMP=DIST(J)                  $  DTEM=D(J)
      DIST(J)=DIST(J+1)             $  D(J)=D(J+1)
      DIST(J+1)=TEMP                $  D(J+1)=DTEM
100   CONTINUE                      $  E=0
      DO 2 L=1,NS                   $  IF(L.LE.2) DIST(L)=0.
      F=A/(A+DIST(L))
*     CALCULATE RMS RESIDUAL
2     E=E+(D(L)*WT(L)*F)**2
      E=SQRT(E/NS)
      RETURN          $  END
```

```
      SUBROUTINE  LONMIN(PLAT,PLON,PR,DR,EM)

******FOR A GIVEN PLAT AND PR, THIS SUBROUTINE FINDS PLON WHICH GIVES THE
*     MINIMUM RMS RESIDUAL EM.

******INCREMENT PLON BY DA UNTIL E2 IS THE LEAST OF THE THREE CONSECUTIVE
*     RMS RESIDUALS E1,E2,E3.  USING THESE RESIDUALS COMPUTE APPROXIMATE PLON
*     WHICH GIVES THE MINIMUM (WITH RESPECT TO PLON) RMS RESIDUAL EM.
*     THEN DECREASE DA AND REPEAT THE PROCEDURE.

      PI=3.141592653589B      $  DEG=180./PI
      NT=0             $  DA=DR
10    NT=NT+1
      NI=0
*     FIND RMS RESIDUAL FOR FIRST POINT.
      CALL SUMRES(PLAT,PLON,PR,E1)
      NI=NI+1
      PLON=PLON+DA            $  CALL SUMRES(PLAT,PLON,PR,E2)
      NI=NI+1
      IF(E2.LT.E1) GO TO 3    $  ES=E2   $  E2=E1   $  E1=ES   $  DA=-DA
      PLON=PLON+DA
3     PLON=PLON+DA            $  CALL SUMRES(PLAT,PLON,PR,E3)
      NI=NI+1
      IF(NI.GT.20) GO TO 5
      IF(E3.GT.E2) GO TO 5    $  E1=E2   $  E2=E3   $  GO TO 3
```

```
*      FROM THREE RESIDUALS FIND NEW PLON FOR MINIMUM RESIDUAL
5      DDA=-(E3-E1)*DA/(E1-2*E2+E3)/2.   $  PLON=PLON-DA+DDA
       CALL SUMRES(PLAT,PLON,PR,EM)
       IF(NI.GE.10) PRINT 6,NI
6      FORMAT(*         ITERATIONS IN LONMIN=*I4)
*      TEST FOR 2 ITERATIONS
       IF(NT.GE.2) RETURN
*      DECREASE STEP SIZE
       DA=DA/10.
       GO TO 10
       END


       SUBROUTINE MINISOL(PLAT,PLON,PR,DR,IN,E)

***** SUBROUTINE TO FIND A SOLUTION BY MINIMIZING THE RMS OF THE RESIDUALS
*      OF ALL SITES, WHILE KEEPING PR FIXED.
******INCREMENT PLAT BY DR, CALLING LONMIN TO FIND THE MINIMUM RMS OF RESIDUALS
*      (AND THE CORRESPONDING PLON)
*      FOR EACH PLAT, UNTIL E2 IS THE LEAST OF THE CONSECUTIVE MINIMUM RMS
*      RESIDUALS E1,E2,E3.
*      USING THESE RESIDUALS COMPUTE APPROXIMATE PLAT WHICH GIVES THE
*       MINIMUM (WITH RESPECT TO BOTH PLAT AND PLON) RESIDUAL E.
*      THEN DECREASE DR AND REITERATE.

       PI=3.1415926535898     $  DEG=180./PI
       IT=0         $  DF=.1
1      DA=DR                  $  LI=0
*      FIND FIRST PLON WHICH MINIMIZES THE RMS RESIDUAL
       CALL LONMIN(PLAT,PLON,PR,DA,E1)
       PLAT=PLAT+DR           $  CALL LONMIN(PLAT,PLON,PR,DA,E2)
       IF(E2.LT.E1) GO TO 3   $  ES=E2  $  E2=E1  $  E1=ES  $  DR=-DR
       PLAT=PLAT+DR
3      PLAT=PLAT+DR           $  CALL LONMIN(PLAT,PLON,PR,DA,E3)
       LI=LI+1
       IF(LI.GE.20) GO TO 5
       IF(E3.GT.E2) GO TO 5   $  E1=E2  $  E2=E3  $  GO TO 3
*       FROM THREE RESIDUALS FIND NEW PLAT FOR MINIMUM RMS RESIDUAL.
5      DDR=-(E3-E1)*DR/(E1-2*E2+E3)/2.  $  PLAT=PLAT-DR+DDR
       CALL LONMIN(PLAT,PLON,PR,DA,E)   $  IT=IT+1        $  DE=E*DEG
*      TEST FOR NO. OF ITERATIONS, AND DECREASE STEP.
       IF(IT.EQ.IN) GO TO 6            $  DR=DR*DF            $  GO TO 1
6      CONTINUE
       E=DE
       RETURN        $  END
```

APPENDIX C

COORDINATE SYSTEMS AND CONVERSIONS

This appendix will describe the coordinate systems used and the subroutines involved.

Earth Model

For the purpose of triangulating from widely dispersed stations over the Earth's surface, the Fischer spheroid was adopted since it is believed to provide the best available global fit to the actual geoid. The relevant parameters are

Equatorial radius:	$\underline{A} = 6378.166$ km
Polar radius:	$\underline{B} = 6356.784$ km
Flattening factor:	$\underline{F} = \frac{\underline{A} - \underline{B}}{\underline{A}} = \frac{1}{298.3}$

For this model, the deflection of the vertical, that is, the angle between the normal to the geoid and the normal to the Fischer spheroid nowhere exceeds 30 arc seconds which is sufficiently accurate for the present purposes, since pointing directions used in triangulation are referenced to the stars rather than to a local horizon.

Geographical Coordinate System

The geographical coordinate system is the conventional system of latitude, longitude, and altitude. Figure 6 shows an exaggerated spheroidal surface corresponding to the Earth's sea-level surface. The geographic latitude GLAT of a point P is the angle between the equatorial plane and a line drawn from P perpendicular to the spheroidal surface. The altitude H is measured from the surface at point G to the point P. The longitude GLON is measured eastward from Greenwich.

Geocentric Coordinate System

In this system, O is the Earth's center in figure 6. The X-axis is directed toward the intersection of Greenwich meridian with the equator. The Y-axis is directed toward 90^{o} east longitude in the equatorial plane, and Z is directed toward the north geographic pole. The point P is also located by the geocentric longitude GLON, geocentric latitude CLAT, and radius CR from the Earth's center.

Topocentric Coordinates

This is a local system with center at some observation site S (see fig. 4) with the DX,DY plane coincident with the horizontal plane, with DX directed toward east, DY directed toward north, and DZ directed vertically (that is, perpendicular to the surface of the spheroid). In the polar version, a point P is located by azimuth angle AZ measured clockwise from DY (north), and elevation angle EL measured up from the horizontal plane, and range RA measured from S to P.

Conversion From Geographic to Geocentric

This conversion is accomplished by using subroutine GGRGCN. Reference to figure 4 shows that the point G on the spheroid follows the equation

$$\frac{\underline{R}^2}{\underline{A}^2} + \frac{\underline{Z}^2}{\underline{B}^2} = 1$$

where

$$\underline{R}^2 = \underline{X}^2 + \underline{Y}^2$$

The slope on the ellipse is

$$\frac{d\underline{Z}}{d\underline{R}} = -\frac{\underline{B}^2 \underline{R}}{\underline{A}^2 \underline{Z}}$$

hence

$$\tan \underline{GLAT} = \frac{\underline{A}^2 \underline{Z}}{\underline{B}^2 \underline{R}}$$

or

$$\underline{Z} = \underline{R}\ \underline{E} \tan \underline{GLAT}$$

where

$$\underline{E} = \frac{\underline{B}^2}{\underline{A}^2}$$

Substitution in the original equation gives

$$\underline{R} = \frac{\underline{A}}{\sqrt{1 + \underline{E}\tan^2\underline{GLAT}}}$$

The geocentric coordinates of point P are then

$$\underline{RP} = \underline{R} + \underline{H}\cos\underline{GLAT}$$

$$\underline{ZP} = \underline{Z} + \underline{H}\sin\underline{GLAT}$$

$$\underline{CR} = \sqrt{\underline{RP}^2 + \underline{ZP}^2}$$

$$\underline{CLAT} = \tan^{-1}\frac{\underline{ZP}}{\underline{CR}}$$

Geocentric to Geographic Conversion

This inverse conversion cannot be obtained explicitly. Subroutine GCNGGR uses the following approximate formulas derived in reference 12.

The altitude is given by

$$\underline{H} = \underline{CR} - \underline{A} + \frac{1}{2}\underline{A}\,\underline{F}\left[1 - \cos(2\underline{CLAT}) + \frac{1}{2}\left(\frac{\underline{F}}{4} - \frac{\underline{A}\,\underline{F}}{\underline{CR}}\right)\left(\cos(4\underline{CLAT}) - 1\right)\right]$$

where CLAT is the geocentric latitude and CR is the geocentric radius.

The geographic latitude is given by

$$\underline{GLAT} = \underline{CLAT} + \frac{\underline{A}\,\underline{F}}{\underline{CR}}\sin(2\underline{CLAT}) + \left(\frac{\underline{A}\,\underline{F}}{\underline{CR}}\right)^2\left(1 - \frac{\underline{CR}}{4\underline{A}}\right)\sin(4\underline{CLAT})$$

It should be noted that all angles must be expressed in radians.

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TABLE I.- COMPARISON OF VARIOUS TRIANGULATION METHODS

Reference	Author's name	Type of object	Number of observation sites	Method of solution
2	Brown	Point	Many	Least-squares method
3	Hogge	Point	Several	Least-squares method
		Straight line	2	Intersection of ray from one site with plane from other site
		Curved line	2	Intersection of ray from one site with surface defined by 3d order least-squares fit to data from other site
4	Lloyd	Points	2	Midpoint of minimum skew distance between rays from each site
		Straight line	2	Intersection of two planes
		Curved line	2	Intersection of ray from one site with surface from other site
5	Whipple	Straight line	2	Intersection of two planes
6	Justus	Point	2	Equal residuals
		Curved line	2	Intersection of ray from one site with surface from other site

TABLE II.- INPUT DATA FOR ILLUSTRATIVE EXAMPLE

```
3 STATION TRIANGULATION

SLAT=  31.6953   SLON=-110.8774   SALT=  2.3640
SLAT= -30.1666   SLON= -70.7673   SALT=  2.1346
SLAT=  37.9324   SLON= -75.4717   SALT=   .0106

TIME     3 HR    18 MIN    10.0 SEC
 STA  1   PT   1  AZ= 119.0452   EL=  43.2738 MT HOPKINS CAM C-2 3,18,1
 STA  1   PT   2  AZ= 119.1804   EL=  43.2085 MT HOPKINS CAM C-2 3,18,1
 STA  1   PT   3  AZ= 119.2918   EL=  43.1431 MT HOPKINS CAM C-2 3,18,1
 STA  1   PT   4  AZ= 119.4330   EL=  43.0773 MT HOPKINS CAM C-2 3,18,1
 STA  1   PT   5  AZ= 119.5394   EL=  43.0121 MT HOPKINS CAM C-2 3,18,1
 STA  1   PT   6  AZ= 119.6648   EL=  42.9467 MT HOPKINS CAM C-2 3,18,1
 STA  1   PT   7  AZ= 119.8046   EL=  42.8824 MT HOPKINS CAM C-2 3,18,1
 STA  1   PT   8  AZ= 119.9364   EL=  42.8175 MT HOPKINS CAM C-2 3,18,1
 STA  1   PT   9  AZ= 120.0707   EL=  42.7521 MT HOPKINS CAM C-2 3,18,1
 STA  1   PT  10  AZ= 120.1841   EL=  42.6868 MT HOPKINS CAM C-2 3,18,1
 STA  1   PT  11  AZ= 120.3110   EL=  42.6210 MT HOPKINS CAM C-2 3,18,1
 STA  1   PT  12  AZ= 120.4449   EL=  42.5552 MT HOPKINS CAM C-2 3,18,1
 STA  1   PT  13  AZ= 120.5667   EL=  42.4893 MT HOPKINS CAM C-2 3,18,1
 STA  1   PT  14  AZ= 120.6786   EL=  42.4229 MT HOPKINS CAM C-2 3,18,1
 STA  1   PT  15  AZ= 120.7976   EL=  42.3568 MT HOPKINS CAM C-2 3,18,1
 STA  1   PT  16  AZ= 120.9223   EL=  42.2899 MT HOPKINS CAM C-2 3,18,1
 STA  1   PT  17  AZ= 121.0436   EL=  42.2229 MT HOPKINS CAM C-2 3,18,1
 STA  1   PT  18  AZ= 121.1714   EL=  42.1565 MT HOPKINS CAM C-2 3,18,1
 STA  1   PT  19  AZ= 121.2812   EL=  42.0898 MT HOPKINS CAM C-2 3,18,1
 STA  1   PT  20  AZ= 121.4022   EL=  42.0228 MT HOPKINS CAM C-2 3,18,1
 STA  1   PT  21  AZ= 121.5237   EL=  41.9556 MT HOPKINS CAM C-2 3,18,1
 STA  1   PT  22  AZ= 121.6460   EL=  41.8887 MT HOPKINS CAM C-2 3,18,1
 STA  1   PT  23  AZ= 121.7643   EL=  41.8212 MT HOPKINS CAM C-2 3,18,1
 STA  1   PT  24  AZ= 121.8680   EL=  41.7536 MT HOPKINS CAM C-2 3,18,1
 STA  1   PT  25  AZ= 121.9944   EL=  41.6863 MT HOPKINS CAM C-2 3,18,1
 STA  1   PT  26  AZ= 122.1165   EL=  41.6182 MT HOPKINS CAM C-2 3,18,1
 STA  1   PT  27  AZ= 122.2271   EL=  41.5496 MT HOPKINS CAM C-2 3,18,1
 STA  1   PT  28  AZ= 122.3316   EL=  41.4809 MT HOPKINS CAM C-2 3,18,1
 STA  1   PT  29  AZ= 122.4629   EL=  41.4121 MT HOPKINS CAM C-2 3,18,1
 STA  1   PT  30  AZ= 122.5736   EL=  41.3435 MT HOPKINS CAM C-2 3,18,1
 STA  1   PT  31  AZ= 122.6864   EL=  41.2748 MT HOPKINS CAM C-2 3,18,1
 STA  1   PT  32  AZ= 122.8074   EL=  41.2057 MT HOPKINS CAM C-2 3,18,1
 STA  1   PT  33  AZ= 122.9165   EL=  41.1368 MT HOPKINS CAM C-2 3,18,1
 STA  1   PT  34  AZ= 123.0379   EL=  41.0674 MT HOPKINS CAM C-2 3,18,1
 STA  1   PT  35  AZ= 123.1260   EL=  40.9988 MT HOPKINS CAM C-2 3,18,1
 STA  1   PT  36  AZ= 123.2700   EL=  40.9294 MT HOPKINS CAM C-2 3,18,1
 STA  2   PT   1  AZ= 349.5335   EL=  49.1204 CHILE CAM D-3  3,18,10
 STA  2   PT   2  AZ= 349.5519   EL=  49.0524 CHILE CAM D-3  3,18,10
 STA  2   PT   3  AZ= 349.5625   EL=  48.9852 CHILE CAM D-3  3,18,10
 STA  2   PT   4  AZ= 349.5813   EL=  48.9175 CHILE CAM D-3  3,18,10
 STA  2   PT   5  AZ= 349.5991   EL=  48.8499 CHILE CAM D-3  3,18,10
 STA  2   PT   6  AZ= 349.6055   EL=  48.7822 CHILE CAM D-3  3,18,10
 STA  2   PT   7  AZ= 349.6160   EL=  48.7143 CHILE CAM D-3  3,18,10
 STA  2   PT   8  AZ= 349.6280   EL=  48.6468 CHILE CAM D-3  3,18,10
 STA  2   PT   9  AZ= 349.6405   EL=  48.5789 CHILE CAM D-3  3,18,10
 STA  2   PT  10  AZ= 349.6550   EL=  48.5114 CHILE CAM D-3  3,18,10
 STA  2   PT  11  AZ= 349.6689   EL=  48.4436 CHILE CAM D-3  3,18,10
 STA  2   PT  12  AZ= 349.6859   EL=  48.3762 CHILE CAM D-3  3,18,10
 STA  2   PT  13  AZ= 349.6937   EL=  48.3075 CHILE CAM D-3  3,18,10
 STA  2   PT  14  AZ= 349.7075   EL=  48.2403 CHILE CAM D-3  3,18,10
 STA  2   PT  15  AZ= 349.7223   EL=  48.1732 CHILE CAM D-3  3,18,10
 STA  2   PT  16  AZ= 349.7386   EL=  48.1047 CHILE CAM D-3  3,18,10
 STA  2   PT  17  AZ= 349.7549   EL=  48.0372 CHILE CAM D-3  3,18,10
 STA  2   PT  18  AZ= 349.7651   EL=  47.9686 CHILE CAM D-3  3,18,10
```

TABLE II.- INPUT DATA FOR ILLUSTRATIVE EXAMPLE – Continued

STA	2	PT	19	AZ=	349.7797	EL=	47.9017	CHILE	CAM	D-3	3,18,10
STA	2	PT	20	AZ=	349.7918	EL=	47.8341	CHILE	CAM	D-3	3,18,10
STA	2	PT	21	AZ=	349.8059	EL=	47.7669	CHILE	CAM	D-3	3,18,10
STA	2	PT	22	AZ=	349.8234	EL=	47.6998	CHILE	CAM	D-3	3,18,10
STA	2	PT	23	AZ=	349.8379	EL=	47.6326	CHILE	CAM	D-3	3,18,10
STA	2	PT	24	AZ=	349.8514	EL=	47.5647	CHILE	CAM	D-3	3,18,10
STA	2	PT	25	AZ=	349.8628	EL=	47.4974	CHILE	CAM	D-3	3,18,10
STA	2	PT	26	AZ=	349.8782	EL=	47.4299	CHILE	CAM	D-3	3,18,10
STA	2	PT	27	AZ=	349.8911	EL=	47.3620	CHILE	CAM	D-3	3,18,10
STA	2	PT	28	AZ=	349.9025	EL=	47.2944	CHILE	CAM	D-3	3,18,10
STA	2	PT	29	AZ=	349.9158	EL=	47.2261	CHILE	CAM	D-3	3,18,10
STA	2	PT	30	AZ=	349.9286	EL=	47.1589	CHILE	CAM	D-3	3,18,10
STA	2	PT	31	AZ=	349.9418	EL=	47.0913	CHILE	CAM	D-3	3,18,10
STA	2	PT	32	AZ=	349.9535	EL=	47.0237	CHILE	CAM	D-3	3,18,10
STA	2	PT	33	AZ=	349.9677	EL=	46.9561	CHILE	CAM	D-3	3,18,10
STA	2	PT	34	AZ=	349.9808	EL=	46.8892	CHILE	CAM	D-3	3,18,10
STA	2	PT	35	AZ=	349.9954	EL=	46.8210	CHILE	CAM	D-3	3,18,10
STA	2	PT	36	AZ=	350.0139	EL=	46.7535	CHILE	CAM	D-3	3,18,10
STA	2	PT	37	AZ=	350.0245	EL=	46.6855	CHILE	CAM	D-3	3,18,10
STA	2	PT	38	AZ=	350.0330	EL=	46.6182	CHILE	CAM	D-3	3,18,10
STA	2	PT	39	AZ=	350.0445	EL=	46.5506	CHILE	CAM	D-3	3,18,10
STA	2	PT	40	AZ=	350.0526	EL=	46.4829	CHILE	CAM	D-3	3,18,10
STA	2	PT	41	AZ=	350.0655	EL=	46.4146	CHILE	CAM	D-3	3,18,10
STA	2	PT	42	AZ=	350.0789	EL=	46.3474	CHILE	CAM	D-3	3,18,10
STA	2	PT	43	AZ=	350.0903	EL=	46.2798	CHILE	CAM	D-3	3,18,10
STA	2	PT	44	AZ=	350.1089	EL=	46.2123	CHILE	CAM	D-3	3,18,10
STA	2	PT	45	AZ=	350.1193	EL=	46.1446	CHILE	CAM	D-3	3,18,10
STA	2	PT	46	AZ=	350.1340	EL=	46.0768	CHILE	CAM	D-3	3,18,10
STA	2	PT	47	AZ=	350.1419	EL=	46.0091	CHILE	CAM	D-3	3,18,10
STA	2	PT	48	AZ=	350.1506	EL=	45.9418	CHILE	CAM	D-3	3,18,10
STA	2	PT	49	AZ=	350.1643	EL=	45.8742	CHILE	CAM	D-3	3,18,10
STA	2	PT	50	AZ=	350.1794	EL=	45.8063	CHILE	CAM	D-3	3,18,10
STA	2	PT	51	AZ=	350.1905	EL=	45.7384	CHILE	CAM	D-3	3,18,10
STA	2	PT	52	AZ=	350.2036	EL=	45.6705	CHILE	CAM	D-3	3,18,10
STA	2	PT	53	AZ=	350.2156	EL=	45.6025	CHILE	CAM	D-3	3,18,10
STA	2	PT	54	AZ=	350.2272	EL=	45.5343	CHILE	CAM	D-3	3,18,10
STA	2	PT	55	AZ=	350.2383	EL=	45.4663	CHILE	CAM	D-3	3,18,10
STA	2	PT	56	AZ=	350.2502	EL=	45.3984	CHILE	CAM	D-3	3,18,10
STA	2	PT	57	AZ=	350.2637	EL=	45.3302	CHILE	CAM	D-3	3,18,10
STA	2	PT	58	AZ=	350.2761	EL=	45.2620	CHILE	CAM	D-3	3,18,10
STA	2	PT	59	AZ=	350.2870	EL=	45.1934	CHILE	CAM	D-3	3,18,10
STA	2	PT	60	AZ=	350.2993	EL=	45.1255	CHILE	CAM	D-3	3,18,10
STA	2	PT	61	AZ=	350.3098	EL=	45.0572	CHILE	CAM	D-3	3,18,10
STA	2	PT	62	AZ=	350.3196	EL=	44.9896	CHILE	CAM	D-3	3,18,10
STA	2	PT	63	AZ=	350.3309	EL=	44.9218	CHILE	CAM	D-3	3,18,10
STA	2	PT	64	AZ=	350.3456	EL=	44.8539	CHILE	CAM	D-3	3,18,10
STA	2	PT	65	AZ=	350.3578	EL=	44.7854	CHILE	CAM	D-3	3,18,10
STA	2	PT	66	AZ=	350.3699	EL=	44.7175	CHILE	CAM	D-3	3,18,10
STA	2	PT	67	AZ=	350.3859	EL=	44.6494	CHILE	CAM	D-3	3,18,10
STA	2	PT	68	AZ=	350.3951	EL=	44.5811	CHILE	CAM	D-3	3,18,10
STA	2	PT	69	AZ=	350.4068	EL=	44.5129	CHILE	CAM	D-3	3,18,10
STA	2	PT	70	AZ=	350.4207	EL=	44.4445	CHILE	CAM	D-3	3,18,10
STA	2	PT	71	AZ=	350.4313	EL=	44.3759	CHILE	CAM	D-3	3,18,10
STA	2	PT	72	AZ=	350.4443	EL=	44.3071	CHILE	CAM	D-3	3,18,10
STA	2	PT	73	AZ=	350.4586	EL=	44.2390	CHILE	CAM	D-3	3,18,10
STA	2	PT	74	AZ=	350.4677	EL=	44.1704	CHILE	CAM	D-3	3,18,10
STA	2	PT	75	AZ=	350.4797	EL=	44.1016	CHILE	CAM	D-3	3,18,10
STA	3	PT	1	AZ=	182.3040	EL=	50.5829	AC	WALLOPS	AC-3	3,18,10
STA	3	PT	2	AZ=	182.3085	EL=	50.6889	AC	WALLOPS	AC-3	3,18,10
STA	3	PT	3	AZ=	182.3122	EL=	50.7940	AC	WALLOPS	AC-3	3,18,10
STA	3	PT	4	AZ=	182.3192	EL=	50.9018	AC	WALLOPS	AC-3	3,18,10

TABLE II.- INPUT FOR ILLUSTRATIVE EXAMPLE – Concluded

```
STA  3  PT   5  AZ= 182.3341  EL=  51.0146 AC WALLOPS AC-3 3,18,10
STA  3  PT   6  AZ= 182.3490  EL=  51.1274 AC WALLOPS AC-3 3,18,10
STA  3  PT   7  AZ= 182.3511  EL=  51.2318 AC WALLOPS AC-3 3,18,10
STA  3  PT   8  AZ= 182.3554  EL=  51.3366 AC WALLOPS AC-3 3,18,10
STA  3  PT   9  AZ= 182.3601  EL=  51.4417 AC WALLOPS AC-3 3,18,10
STA  3  PT  10  AZ= 182.3723  EL=  51.5526 AC WALLOPS AC-3 3,18,10
STA  3  PT  11  AZ= 182.3821  EL=  51.6622 AC WALLOPS AC-3 3,18,10
STA  3  PT  12  AZ= 182.3891  EL=  51.7683 AC WALLOPS AC-3 3,18,10
STA  3  PT  13  AZ= 182.3953  EL=  51.8749 AC WALLOPS AC-3 3,18,10
STA  3  PT  14  AZ= 182.4013  EL=  51.9808 AC WALLOPS AC-3 3,18,10
STA  3  PT  15  AZ= 182.4039  EL=  52.0844 AC WALLOPS AC-3 3,18,10
STA  3  PT  16  AZ= 182.4213  EL=  52.1985 AC WALLOPS AC-3 3,18,10
STA  3  PT  17  AZ= 182.4277  EL=  52.3046 AC WALLOPS AC-3 3,18,10
STA  3  PT  18  AZ= 182.4363  EL=  52.4123 AC WALLOPS AC-3 3,18,10
STA  3  PT  19  AZ= 182.4439  EL=  52.5196 AC WALLOPS AC-3 3,18,10
STA  3  PT  20  AZ= 182.4526  EL=  52.6272 AC WALLOPS AC-3 3,18,10
STA  3  PT  21  AZ= 182.4655  EL=  52.7376 AC WALLOPS AC-3 3,18,10
STA  3  PT  22  AZ= 182.4709  EL=  52.8434 AC WALLOPS AC-3 3,18,10
STA  3  PT  23  AZ= 182.4764  EL=  52.9492 AC WALLOPS AC-3 3,18,10
STA  3  PT  24  AZ= 182.4872  EL=  53.0580 AC WALLOPS AC-3 3,18,10
STA  3  PT  25  AZ= 182.4923  EL=  53.1636 AC WALLOPS AC-3 3,18,10
STA  3  PT  26  AZ= 182.5001  EL=  53.2708 AC WALLOPS AC-3 3,18,10
STA  3  PT  27  AZ= 182.5079  EL=  53.3786 AC WALLOPS AC-3 3,18,10
STA  3  PT  28  AZ= 182.5226  EL=  53.4904 AC WALLOPS AC-3 3,18,10

L= 1   N=  36   NO= 25

L= 2   N=  75   NO= 25

L= 3   N=  28   NO= 25

ESTIMATED TRIAL POINT   LAT= 6.150   LON=-76.770   ALT= 31500
```

TABLE III.- SOLUTION FOR ILLUSTRATIVE EXAMPLE

TIME 3 HRS 18 MIN 10.0 SEC

LINE SOLUTION

ALTITUDE	LATITUDE	LONGITUDE	RMS RES	PTS ON AZ-EL CURVE		
				36	75	29
30200	8.121	-76.763	.0024	35	75	28
30300	7.978	-76.763	.0024	34	72	29
30400	7.836	-76.763	.0025	32	69	28
30500	7.696	-76.762	.0027	31	67	28
30600	7.556	-76.762	.0029	30	64	28
30700	7.416	-76.762	.0033	29	61	28
30800	7.279	-76.762	.0039	27	59	28
30900	7.137	-76.761	.0040	26	56	27
31000	7.003	-76.760	.0044	25	54	25
31100	6.869	-76.759	.0046	24	51	24
31200	6.729	-76.758	.0044	22	48	23
31300	6.601	-76.756	.0047	21	46	22
31400	6.467	-76.755	.0051	20	44	20
31500	6.329	-76.754	.0054	19	41	19
31600	6.132	-76.752	.0057	17	38	18
31700	6.039	-76.751	.0058	16	35	16
31800	5.896	-76.750	.0061	15	33	15
31900	5.751	-76.749	.0062	13	30	13
32000	5.603	-76.748	.0058	12	27	12
32100	5.449	-76.747	.0055	10	24	11
32200	5.310	-76.746	.0053	9	22	9
32300	5.171	-76.745	.0048	8	19	8
32400	5.011	-76.745	.0042	6	16	6
32500	4.867	-76.744	.0042	5	14	5
32600	4.725	-76.742	.0044	4	11	4
32700	4.584	-76.741	.0048	2	8	2
32800	4.477	-76.739	.0052	1	6	1

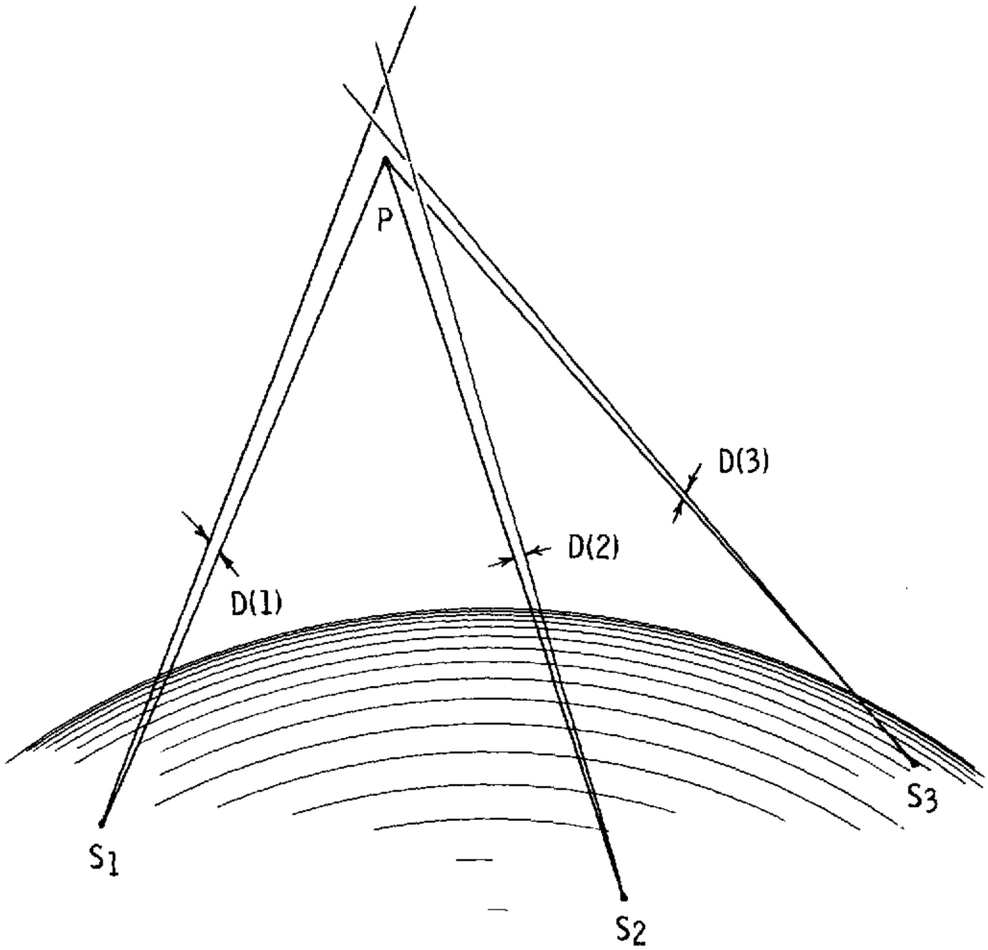

Figure 1.- Trial solution point P in space as observed from three sites illustrating residuals D(1), D(2), and D(3) due to errors in measuring lines of sight.

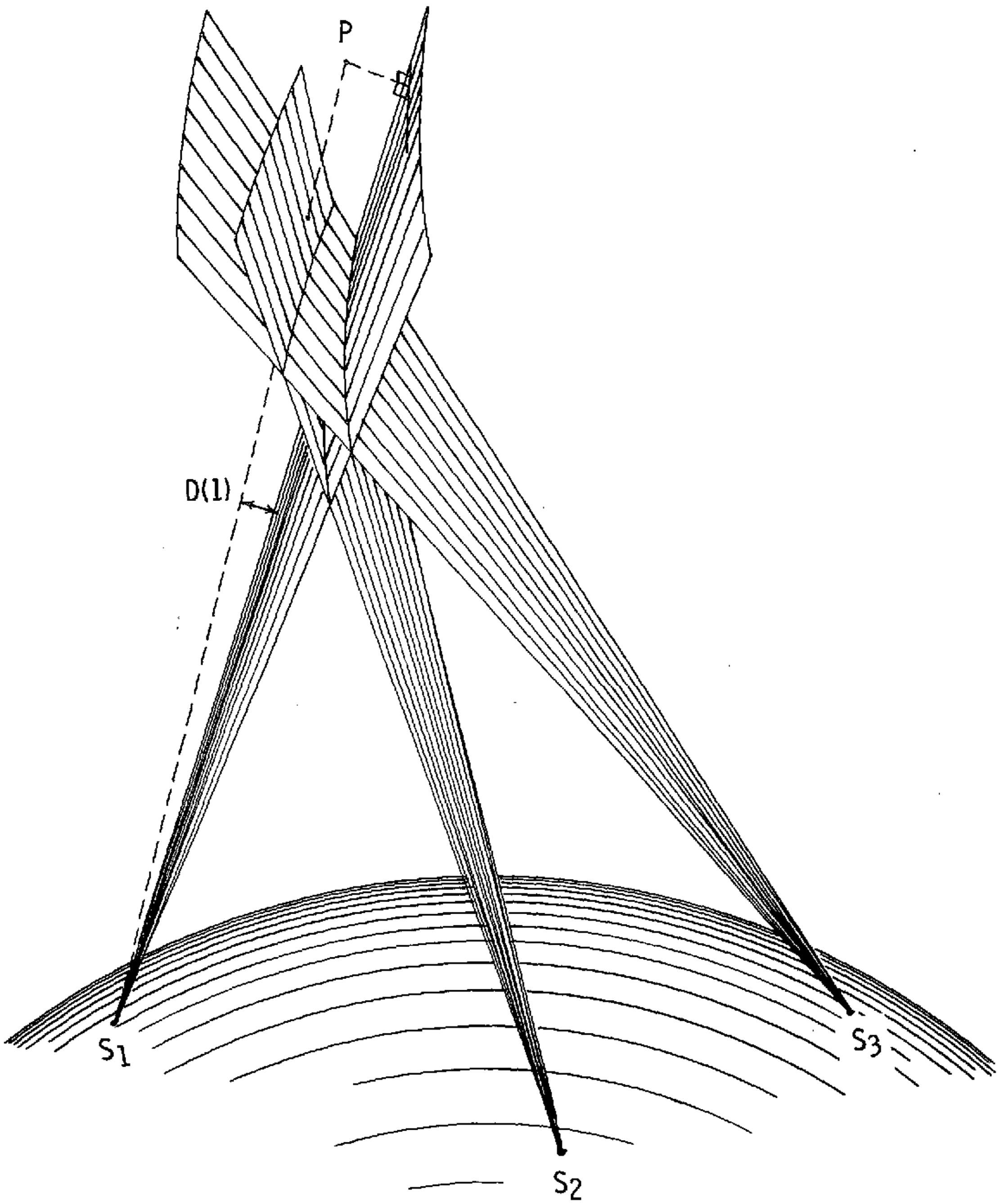

Figure 2.- Illustration of three surfaces defined by az-el data from three observation sites. For the trial point P, the residual for site 1 is D(1).

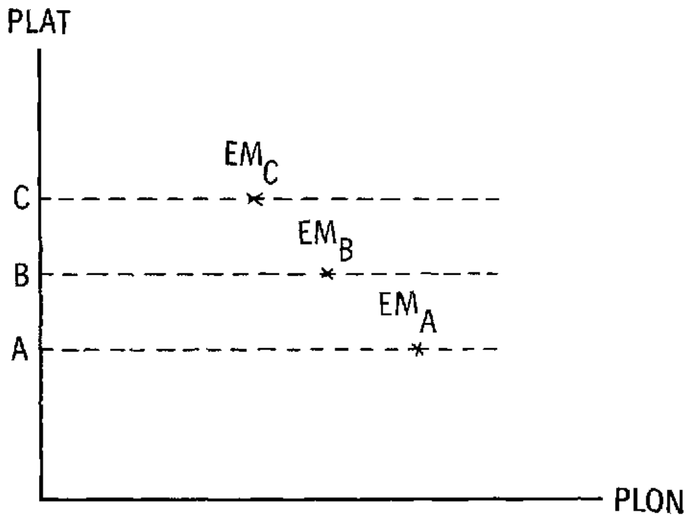

Figure 3.- Illustration of minimization of root-mean-square residuals with varying PLON and PLAT. EM is the minimum root-mean-square residual for variable PLON at a fixed PLAT.

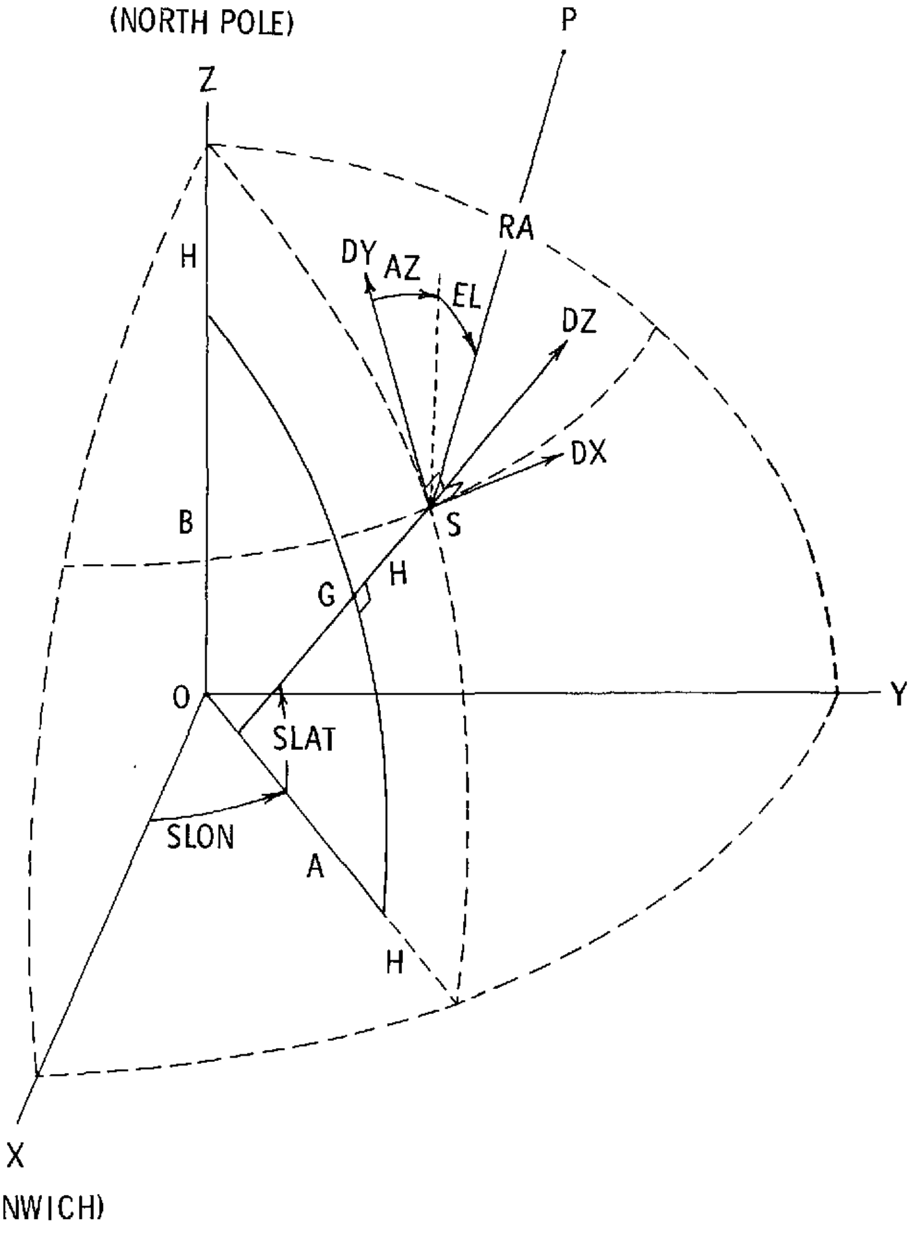

Figure 4.- Illustration of the relations between topocentric and geographic coordinates.

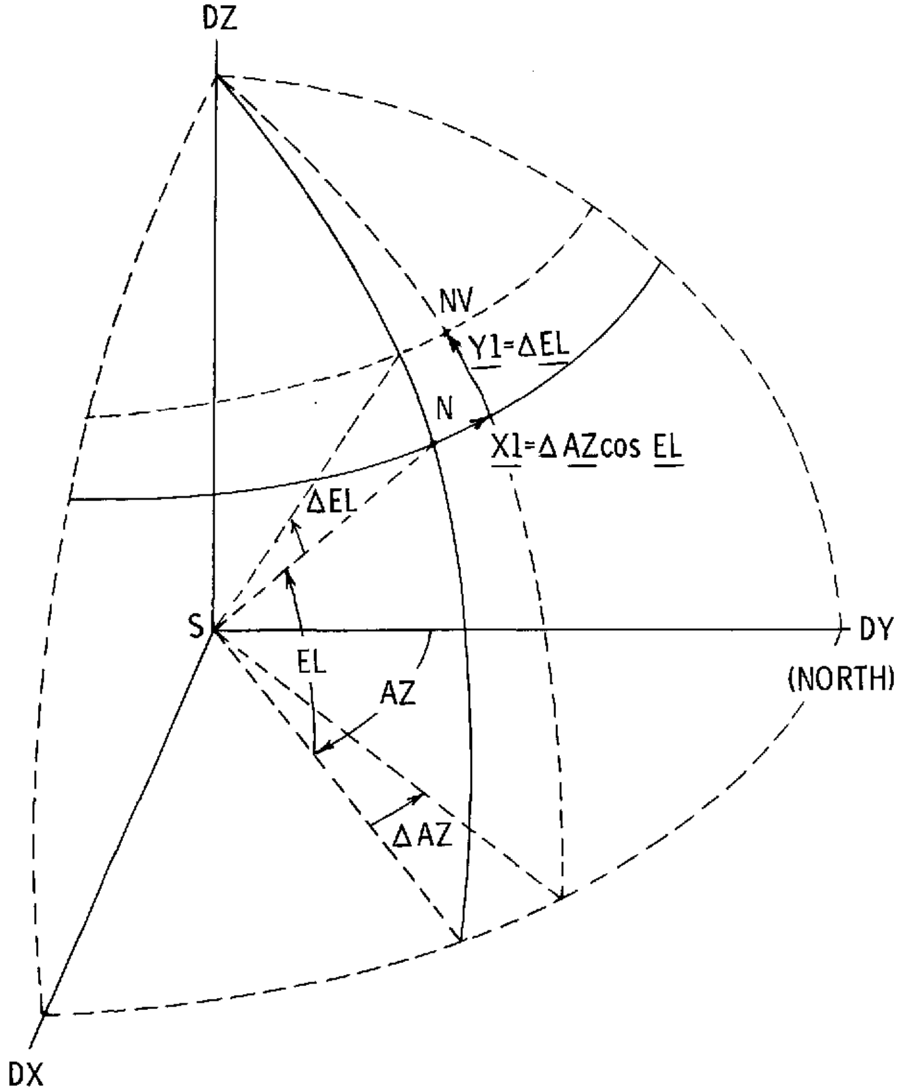

Figure 5.- Illustration of an approximate angular two-dimensional coordinate system with reference direction S-N, which is used in least-squares fit of az-el data and computation of residuals.

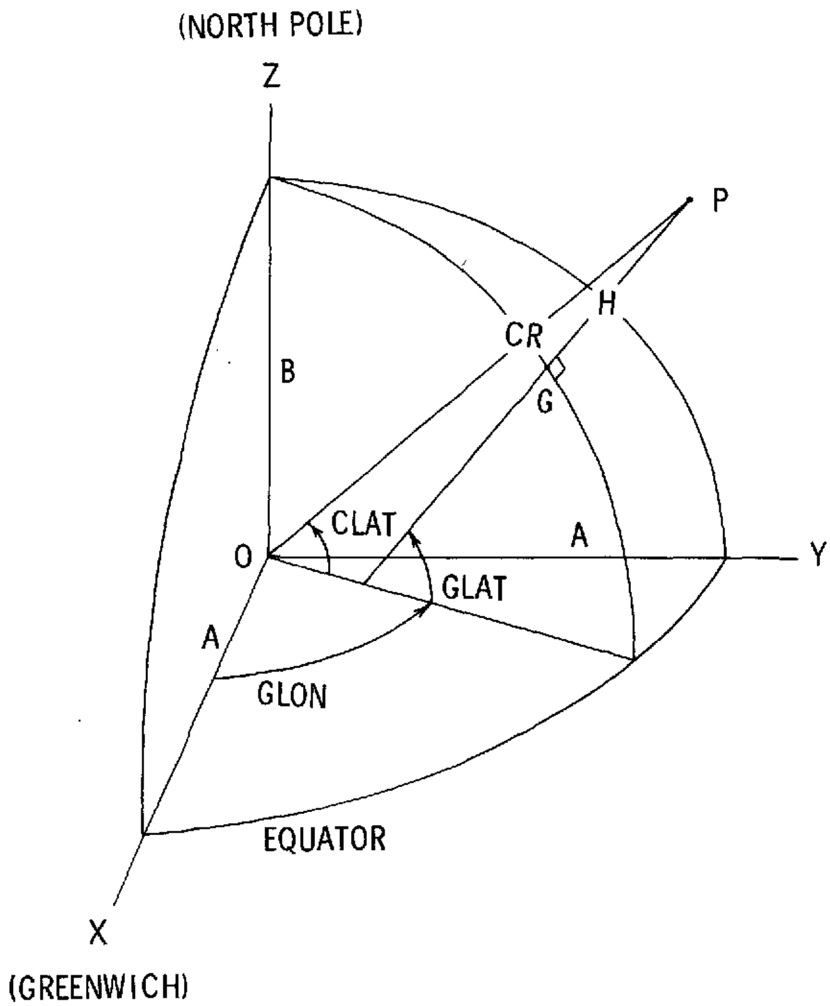

Figure 6.- Illustration of the Earth spheroid and the relations between geographic and geocentric coordinates.

NASA-Langley, 1974 L-9268