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13TH ORDER RESONANCE FROM NAVY TRACKING ON A DIADEME 2 FRAGMENT

C. A. WAGNER

MARCH 1974

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GODDARD SPACE FLIGHT CENTER
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ABSTRACT

A strong constraint on 13th order (odd degree) terms in the geopotential has been derived from Navy tracking on a DIADEME 2 fragment (1967-14F). This object (perigee height: 580 km, orbit inclination: 38.9°) is presently decaying slowly through perfect commensurability with these terms. The resonance forces will increase its inclination by 0.02° when the passage is complete by late 1974. The constraint (lumped harmonics), derived by adjustment of a pair of harmonic coefficients to the Navy inclination data (principally) is:

$$\begin{gathered}10^{9}\,(14.8 \pm 0.8,\ 48.3 \pm 0.7) = 0.023(C,S)_{13,13} - 0.172(C,S)_{15,13} + 0.505(C,S)_{17,13} \\ -0.884(C,S)_{19,13} + (C,S)_{21,13} - 0.673(C,S)_{23,13} + 0.099(C,S)_{25,13} + 0.295(C,S)_{27,13} \\ -0.279(C,S)_{29,13} + 0.018(C,S)_{31,13} + \ldots\end{gathered}$$

There should be a significant contribution to this result from terms as high as 29th degree. But current geopotential solutions (for 13th order terms) to this degree are about 20% in error when judged by this independent data.

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13TH ORDER RESONANCE FROM NAVY TRACKING ON A DIADEME 2 FRAGMENT

INTRODUCTION

Among the thousands of Earth satellites launched in the past 15 years, hundreds must have suffered or will suffer measurable effects from poorly known resonances with the Earth's geopotential (King-Hele, 1973a; Gabbard and Wackernagel, 1971). But less than 50 of these objects have actually been used geodetically. The major difficulty of course is the lack of adequate tracking for the majority of the objects which are (or were) rocket bodies or fragments (debris) of larger satellites. However, in the case of deep resonance or close commensurability to the Earth's rotation, the effects are so large that even tracking with crude instruments can reveal them (King-Hele, 1973b). Many of the observations made in support of the tracking on decaying 15th order resonant orbits were accomplished with amateur observers using binoculars and stopwatches.

In these deep resonances the change of the inclination of the orbit is of the order of 0.01°. To achieve overall inclination accuracies of a tenth of this figure, a topocentric angle good to only about a minute of arc is necessary. King-Hele's results were often much better than this, but his amateurs were backed up by more precise camera observations. (See also Winterbottom and King-Hele, 1974.) In addition, King-Hele's group has utilized minute-of-arc data from the U.S. Navy skin track Naval Space Surveillance (NAVSPASUR) system, of which there is a great abundance on low altitude satellites. In fact since 1972, the NAVSPASUR data and mean elements derived from them (by the Navy) dominate the orbits used in the 15th order resonance analysis (i.e., King-Hele and Walker, 1972). This data consists of (direction cosine) pointing angles from a fence of receiving stations across the United States. The signals are reflections from the satellite of a continuous high power radar fan illuminating space across the U.S. at about 28° latitude. The system has tracked objects as high as 18,000 km altitude. Providing the inclination of the orbit is greater than about 28° and the object is large enough to be tracked, data on each revolution will be received.

The existence of this good data on a large number of objects opens up the possibility of examining in detail all the deep geopotential resonances besides the one's of 15th order. In this study, the Navy's own mean elements were used directly. They proved sufficiently accurate to reveal the strong effect on the inclination of the decaying (13 revolutions/day) orbit of a DIADEME 2 fragment (1967-14F).

ANALYSIS

In Figure 1, the Navy Mean Inclinations are shown from February 1972 to January 1974. Also in Figure 1 is a plot of the mean (primary) resonant longitude rate for this 13 revs/day orbit:

$$\dot{\psi}_{13,0} = \dot{\omega} + \dot{M} + 13\ (\dot{\Omega} - \dot{\theta}),$$

where ω, M and Ω are the orbit's argument of perigee, mean anomaly and right ascension of the ascending node, and $\dot{\theta}$ is the Earth's rotation rate.

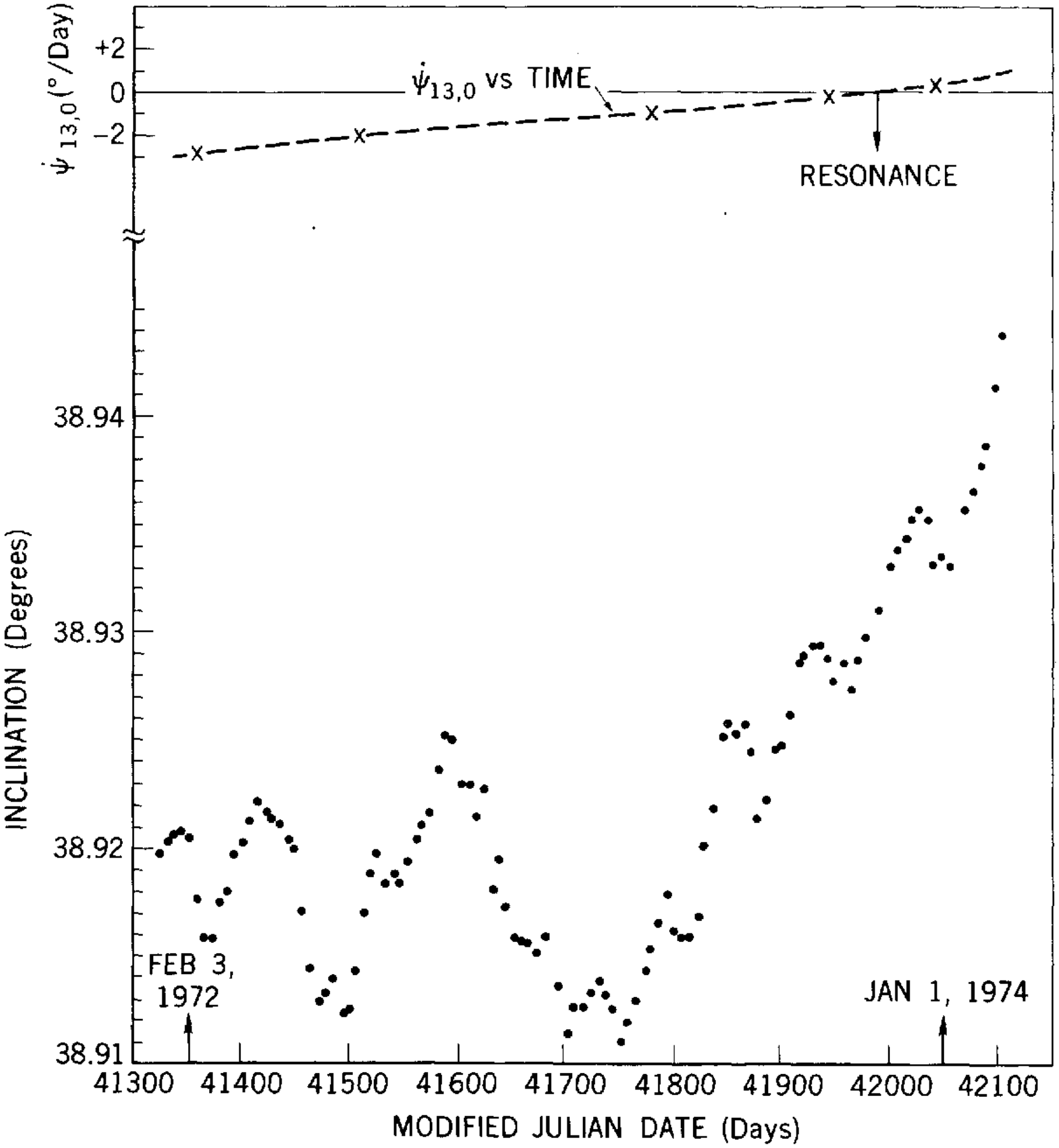

Figure 1. U.S. Navy Orbit Inclinations for 1967-14F

It is recalled that the longitude rates which determine the frequency of gravitational perturbations on an orbit in Kaula's development of the geopotential (Kaula, 1966; especially p. 40, 49 and 55) are given as:

$$\dot{\psi} = (\ell - 2p)\,\dot{\omega} + (\ell - 2p + q)\,\dot{M} + m\,(\dot{\Omega} - \dot{\theta}),$$

where ℓ and m are the degree and order of a geopotential harmonic, and p and q are additional indices related to the inclination and eccentricity of the orbit. The resonances ($\dot{\psi} = 0$, $m \neq 0$) will occur for $(\ell - 2p + q)\,\dot{M}$ close to $m\dot{\theta}$, since $\dot{\omega}$ and $\dot{\Omega}$ are small. The primary resonances (strongest) occur when $\ell - 2p + q = 1$, since then m will be minimum, the closest integer to $\dot{M}$ in revolutions/day. Secondary resonances (sub harmonics) for a given $\dot{M}$ (near a rational number of revs/day) will occur for $\ell - 2p + q = 2, 3, 4 \ldots$, but in any case all the resonant longitude rates can be characterized by the order m and the q index, by writing

$$\dot{\psi}_{m,q} = -q\,\dot{\omega} + (\ell - 2p + q)\,(\dot{\omega} + \dot{M}) + m\,(\dot{\Omega} - \dot{\theta}).$$

For a given resonant order m, there will be a series of "side band" resonances characterized by q around the (generally) dominant one for q = 0. The q = 0 resonance is also the mean of the series since q can take on all positive or negative integers.

It is seen in Figure 1 that the longitude rate $\dot{\psi}_{13,0}$ for 1967-14F goes to zero over the period of record as the inclination of the orbit suffers a progressive oscillation of increasing period and amplitude. The increasing period closely matches the period of $\psi_{13,0}$. There is clearly a strong resonant perturbation of this orbit entirely analogous to the decaying 15th order orbits first analyzed by Gooding (1971) and the 11th order orbit examined by Wagner (1973).

But there is more significant detail in Figure 1 than resonance. The "raw" mean inclinations given by the Navy (see also Table 1) are actually Brouwer elements (determined over independent 7 day arcs) with both short and long period zonal effects removed (Brouwer, 1959). Only J_2 to J_5 are used in the Navy-Brouwer model and the coefficients are not up to date. The period of the principal odd zonal inclination effect ($2\pi/\dot{\omega}$) is 65 days. The amplitude, with the correct model, is about 0.005°. The error in the Navy model is certainly seen in the raw inclination data. But, in addition, there is an 84 day lunar perturbation with amplitude 0.0014° which is also observable.

To clarify the quality of this "signal" I have added back the long period zonal effects used in the Navy-Brouwer model to produce mean elements free of this bias (also Table 1). Then I compared these (less smooth) mean elements to values calculated from a trajectory which includes all significant (but nonresonant) long period effects on the orbit, in particular the zonal perturbations from the Smithsonian (SAO) Standard Earth 2 (Gaposchkin and Lambeck, 1971), radiation pressure, atmospheric drag and direct lunar-solar gravity. The comparison (observed minus calculated values) in Figure 2, shows the resonant signal much more strongly and also reveals what appears to be a much reduced residual effect due to odd zonal error in the Smithsonian field.

EPOCH (MJD)	SEMIMAJOR AXIS (EARTH RADII) A A′	ECCENTRICITY E E′	INCLINATION (DEGREES) I I′	ARGUMENT OF PERIGEE (DEGREES) ω ω′	RIGHT ASCENSION OF THE ASCENDING NODE (DEGREES) Ω Ω′	MEAN ANOMALY (DEGREES) M M′
41302.97540	1.188114410	0.32216800000D-01	38.91900000	244.7337000	163.3735000	106.5790000
	1.188114410	0.31661532780D-01	38.92226150	244.5616746	163.3703660	106.7513946
41309.95744	1.188103680	0.82327300000D-01	38.91960000	283.7053000	133.3052000	67.26750000
	1.188103680	0.81732150014D-01	38.92310039	283.5604436	133.3069459	67.17197619
41316.93724	1.188097220	0.82314900000D-01	38.92130000	322.6472000	103.2376000	31.78590000
	1.188097220	0.31939782310D-01	38.92390563	323.1742240	103.2434829	31.45764655
41323.99672	1.188094410	0.32136500000D-01	38.91970000	2.328100000	72.83840000	358.1007000
	1.188094410	0.32205690230D-01	38.91958909	2.756983192	72.84564445	357.6703315
41330.97545	1.188091060	0.32165700000D-01	38.92020000	41.46780000	42.76650000	324.5669000
	1.188091060	0.32574383510D-01	38.91780111	41.60235163	42.77214165	324.2307752
41337.95508	1.188084250	0.32092400000D-01	38.92060000	30.64450000	12.69510000	268.6231000
	1.188084250	0.32708562600D-01	38.91693657	30.71376941	12.70032973	268.5486026
41344.93478	1.188075270	0.32114100000D-01	38.92070000	119.7039000	342.6415000	248.8082000
	1.188075270	0.32654438760D-01	38.91753040	119.4792322	342.6377582	249.0335480
41351.93539	1.188066450	0.32005900000D-01	38.92040000	159.2596000	312.2627000	204.3533000
	1.188066450	0.32273172670D-01	38.91914390	159.0491707	312.2457055	204.7650918
41358.96472	1.188064930	0.32094200000D-01	38.91760000	198.4363000	282.1952000	158.4723000
	1.188064930	0.31693708850D-01	38.91977551	198.0382001	282.1681821	158.3717783
41365.94034	1.188057230	0.32213300000D-01	38.91580000	237.3451000	252.1255000	114.2966000
	1.188057230	0.31694183680D-01	38.91884435	237.3282520	252.1215593	114.5142575
41372.99454	1.188039730	0.32234600000D-01	38.91580000	277.0921000	221.7267000	73.69040000
	1.188039730	0.31626959780D-01	38.91937032	277.1414352	221.7276047	73.64085000
41379.97059	1.188022490	0.32244600000D-01	38.91740000	316.1030000	191.6549000	37.70270000
	1.188022490	0.31815639430-01	38.91992061	316.3970495	191.6602070	37.40748697
41386.95503	1.188005210	0.82092300000D-01	38.91790000	355.1822000	161.5816000	4.140500000
	1.188005210	0.32034231440D-01	38.91824057	355.0068850	161.5850006	3.714313887
41393.93236	1.187988570	0.82068100000D-01	38.91960000	34.33120000	121.5077000	330.7904000
	1.187988570	0.32414553840D-01	38.91755379	34.69850042	121.5139000	330.4218668
41400.95099	1.187969680	0.32022600000D-01	38.92020000	73.97410000	101.1106000	294.9860000
	1.187969680	0.32622364840D-01	38.91663572	74.10013176	101.1126860	294.8595726
41407.95482	1.187958080	0.32076900000D-01	38.92110000	113.1135000	71.04050000	255.8317000
	1.187958080	0.32250084620D-01	38.91773932	112.9331133	71.03753400	256.0108095
41414.94160	1.187950490	0.32110900000D-01	38.92210000	152.1800000	40.97620000	212.5451000
	1.187950490	0.32395965790-01	38.92042757	151.7891105	40.96956747	212.9372008
41421.99380	1.187943100	0.82170600000D-01	38.92150000	191.7595000	10.58300000	166.2681000
	1.187943100	0.32045151670D-01	38.52237335	191.3459904	10.57573590	166.6831252
41428.97024	1.187934450	0.32153200000D-01	38.92130000	230.8827000	340.5181000	121.5812000
	1.187934450	0.31674632080D-01	38.92410855	230.6263210	340.5134649	121.8385605
41435.94741	1.187931660	0.82253900000D-01	38.92100000	269.9245000	310.4466000	80.71240000
	1.187931660	0.31641573550D-01	38.92459804	269.9239735	310.4465903	80.71292858
41442.92043	1.187928570	0.32241900000D-01	38.92020000	308.9307000	280.3705000	44.02060000
	1.187928570	0.31762511550D-01	38.92301556	309.2460351	280.3751276	43.76426445
41449.97392	1.187927510	0.32190900000D-01	38.91990000	348.5193000	249.9634000	9.778700000

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	1.187927510	0.32062086837D-01	38.92065534	346.9333068	249.3706736	9.363213635
41456.93365	1.187923130	0.31952300000D-01	38.91700000	27.66050000	219.8859000	336.5491000
	1.187923130	0.32275765270D-01	38.91533951	28.05233065	219.8925329	336.1559808
41463.93703	1.187921650	0.32032000000D-01	38.91440000	66.76880000	189.8105000	301.7228000
	1.187921650	0.32604662100D-01	38.91104344	66.96821038	189.8134765	301.5428306
41470.99046	1.187920190	0.32015700000D-01	38.91290000	106.3853000	159.4060000	262.8700000
	1.187920190	0.32614235540D-01	38.90939226	106.2570328	159.4038687	262.9991641
41477.96716	1.187916070	0.31957500000D-01	38.91320000	145.4500000	129.3375000	220.2418000
	1.187916070	0.32305908620D-01	38.91115965	145.0630052	129.3313233	220.6099008
41484.94319	1.187911570	0.32030100000D-01	38.91390000	164.6453000	99.27010000	174.6160000
	1.187911570	0.31973283671D-01	38.91422946	164.2201018	99.26270132	175.0426900
41491.99303	1.187903670	0.32049400000D-01	38.91230000	224.2125000	68.87610000	128.9921000
	1.187903670	0.31615078810D-01	38.91482885	223.9193219	68.87083298	129.2863883
41498.97181	1.187900670	0.32150700000D-01	38.91340000	263.3200000	38.79760000	87.32600000
	1.187900670	0.31542511720D-01	38.91697015	263.2734436	38.79674836	87.37273489
41505.94942	1.187895740	0.32154700000D-01	38.91430000	302.3631000	8.713100000	56.00150000
	1.187895740	0.31635020630D-01	38.91735064	302.5793424	8.717028796	49.78422017
41512.92765	1.187889590	0.32122000000D-01	38.91690000	341.4675000	338.6333000	15.77130000
	1.187889590	0.31920509280D-01	38.91803221	341.6651523	338.6408177	15.37217857
41519.95191	1.187883900	0.32059600000D-01	38.91880000	21.01910000	308.2244000	342.2425000
	1.187883900	0.32276746230D-01	38.91752700	21.42894083	308.2313837	341.8312964
41526.95090	1.187877590	0.32002900000D-01	38.91960000	60.12990000	278.1457000	307.8533000
	1.187877590	0.32548364260D-01	38.91643939	60.35597724	278.1454567	307.6265192
41533.93735	1.187871020	0.32004600000D-01	38.91830000	99.27690000	248.0751000	270.1696000
	1.187871020	0.32620969530D-01	38.91463346	99.20316040	248.0738815	270.2435617
41540.93952	1.187858540	0.31964200000D-01	38.91870000	136.9024000	217.6789000	227.6553000
	1.187858540	0.32369863690D-01	38.91632404	136.5647706	217.6732379	227.9940074
41547.95512	1.187854710	0.31962300000D-01	38.91830000	176.0702000	187.6086000	182.3411000
	1.187854710	0.31977407760D-01	38.91821446	177.6401497	187.6011729	182.7726337
41554.94063	1.187849440	0.32105000000D-01	38.91930000	217.1740000	157.5374000	136.9124000
	1.187849440	0.31725631640D-01	38.92150154	216.8461048	157.5316320	137.2414832
41561.99255	1.187840570	0.32126800000D-01	38.92040000	256.6974000	127.1362000	94.07570000
	1.187840570	0.31530360180D-01	38.92338916	256.6051086	127.1345154	94.16832329
41568.96743	1.187833440	0.32141300000D-01	38.92100000	295.7378000	97.05500000	56.06700000
	1.187833440	0.31587963160D-01	38.92424579	295.9127777	97.05818365	55.89134198
41575.94705	1.187828600	0.32064300000D-01	38.92160000	334.6476000	66.97540000	21.43020000
	1.187828600	0.31767477550D-01	38.92315416	335.2248000	66.98208281	21.05166022
41582.92495	1.187824180	0.31925100000D-01	38.92360000	13.97250000	36.89460000	348.2376000
	1.187824180	0.32068918450D-01	38.92273342	14.39606520	36.80183330	347.8125843
41589.97345	1.187819030	0.31940300000D-01	38.92520000	53.56800000	6.488400000	313.7965000
	1.187819030	0.32433842060D-01	38.92227546	53.53676023	6.492870691	313.5268765
41596.95343	1.187813120	0.31910900000D-01	38.92500000	92.71840000	336.4160000	276.7636000
	1.187813120	0.32535106610D-01	38.92134618	92.59666139	336.4156419	276.7854049
41603.93146	1.187811370	0.31937600000D-01	38.92290000	131.6970000	306.3426000	235.4734000
	1.187811370	0.32333684150D-01	38.92020140	131.5959050	306.3375780	235.7754239

EPOCH (MJD)	SEMIMAJOR AXIS (EARTH RADII) A A′	ECCENTRICITY E E′	INCLINATION (DEGREES) I I′	ARGUMENT OF PERIGEE (DEGREES) ω ω′	RIGHT ASCENSION OF THE ASCENDING NODE (DEGREES) Ω Ω′	MEAN ANOMALY (DEGREES) M M′
41610.93257	1.187808260	0.8197040000D-01	38.92290000	171.4370000	275.9481000	190.1284000
	1.187808260	0.8205652360D-01	38.92239574	171.0082202	275.9407365	190.5586363
41617.95762	1.187801960	0.8207230000D-01	38.92140000	210.5561000	245.8750000	144.4607000
	1.187801960	0.8175464420D-01	38.92325094	210.1992404	245.8686604	144.8188587
41624.93313	1.187790210	0.8203770000D-01	38.92270000	249.6072000	215.8033000	101.4517000
	1.187790210	0.8146255028D-01	38.92607038	249.4869493	215.8007501	101.5924968
41631.95329	1.187785260	0.8200100000D-01	38.91810000	269.0721000	185.3944000	62.28500000
	1.187785260	0.8142123375D-01	38.92149515	269.2035239	185.3967899	62.15299354
41638.96240	1.187785240	0.8198730000D-01	38.91940000	328.2342000	155.3127000	27.42870000
	1.187785240	0.8165395806D-01	38.92132311	328.5864400	155.3189609	26.77515038
41645.93983	1.187784130	0.8191460000D-01	38.91720000	7.338700000	125.2260000	353.8610000
	1.187784130	0.8198755846D-01	38.91677303	7.764394292	125.2333786	353.4298410
41652.99309	1.187783170	0.8175040000D-01	38.91570000	46.95200000	94.81660000	319.6943000
	1.187783170	0.8220275995D-01	38.91305292	47.26011107	94.82172090	319.3852185
41659.95992	1.187783760	0.8175850000D-01	38.91560000	36.07010000	64.74150000	283.3159000
	1.187783760	0.8233175633D-01	38.91195939	56.10153406	64.74201660	283.2843451
41666.94387	1.187783830	0.8176050000D-01	38.91550000	125.3057000	34.65960000	242.7033000
	1.187783830	0.8226727727D-01	38.91253959	125.0434491	34.65525735	242.9663684
41673.99580	1.187781080	0.8182520000D-01	38.91510000	164.8857000	4.258400000	197.7912000
	1.187781080	0.8193104465D-01	38.91413890	164.4633427	4.251209204	198.2149702
41680.97160	1.187779390	0.8186930000D-01	38.91590000	203.9955000	334.1881000	152.0295000
	1.187779390	0.8161368431D-01	38.91739516	203.6133351	334.1813683	152.4130246
41687.94394	1.187779650	0.8181810000D-01	38.91570000	243.1059000	304.1093000	108.3569000
	1.187779650	0.8127016223D-01	38.91690301	242.9227317	304.1059959	108.5407445
41694.99377	1.187778350	0.8186150000D-01	38.91360000	282.6374000	273.7014000	68.39350000
	1.187778350	0.8126359360D-01	38.91709724	282.7254050	273.7029968	68.30515150
41701.97363	1.187775570	0.8184430000D-01	38.91140000	321.7357000	243.6170000	32.78580000
	1.187775570	0.8145993475D-01	38.91364791	322.0594363	243.6227636	32.46082427
41708.93303	1.187772450	0.8175740000D-01	38.91260000	0.3454000000	213.5301000	359.3531000
	1.187772450	0.8176031380D-01	38.91253295	1.276161165	213.5375105	358.9208547
41715.93334	1.187769240	0.8169380000D-01	38.91260000	39.99870000	183.4399000	325.8239000
	1.187769240	0.8209017856D-01	38.91023625	40.34274349	183.4456345	325.4787615
41722.93297	1.187766490	0.8165020000D-01	38.91330000	79.57130000	153.0346000	289.6068000
	1.187766490	0.8226429886D-01	38.90971740	79.65490054	153.0359622	289.5234372
41729.95391	1.187762530	0.8165510000D-01	38.91310000	118.7110000	122.9550000	249.8258000
	1.187762530	0.8220086434D-01	38.90991534	118.4913104	122.9513911	250.0455662
41736.93395	1.187758230	0.8166360000D-01	38.91310000	157.8985000	92.87990000	205.9152000
	1.187758230	0.8191178120D-01	38.91175625	157.4894117	92.87295704	206.3256336
41743.93435	1.187754240	0.8162670000D-01	38.91240000	197.4521000	62.48060000	159.6370000
	1.187754240	0.8143619272D-01	38.91351111	197.0489303	62.47357861	160.0415589
41750.95425	1.187752090	0.8166070000D-01	38.91100000	236.5782000	32.40410000	115.4020000
	1.187752090	0.8114699006D-01	38.91399756	236.4538752	32.40008022	115.6271813
41757.93454	1.187748740	0.8170260000D-01	38.91190000	275.6701000	2.323400000	75.13230000
	1.187748740	0.8109340079D-01	38.91545763	275.7098719	2.324119912	75.09237304

EPOCH (MJD)	SEMIMAJOR AXIS (EARTH RADII) A A′	ECCENTRICITY E E′	INCLINATION (DEGREES) I I′	ARGUMENT OF PERIGEE (DEGREES) ω ω'	RIGHT ASCENSION OF THE ASCENDING NODE (DEGREES) Ω Ω'	MEAN ANOMALY (DEGREES) M M′
41764.96729	1.187743020	0.81662500000D-01	38.91300000	315.1421000	331.9134000	38.59420000
	1.187743020	0.81225284660D-01	38.91554526	315.4333307	331.9185844	38.30181529
41771.95429	1.187738840	0.81636900000D-01	38.91430000	354.3028000	301.8265000	4.888600000
	1.187738840	0.81589439010D-01	38.91469358	354.7266198	301.8356513	4.461295959
41778.94128	1.187726950	0.81504600000D-01	38.91530000	33.44550000	271.7439000	331.5264000
	1.187726950	0.81642989860D-01	38.91332957	33.81893499	271.7501214	331.1517693
41785.99362	1.187717010	0.81489800000D-01	38.91650000	73.03780000	241.3346000	295.8148000
	1.187717010	0.82086512680D-01	38.91302558	73.17179792	241.3367901	295.6803943
41792.96924	1.187706850	0.81450400000D-01	38.91790000	112.2349000	211.2582000	256.6901000
	1.187706850	0.82027416000D-01	38.91454264	112.0613449	211.2553623	256.8641846
41799.94591	1.187699180	0.81354800000D-01	38.91610000	151.3576000	181.1870000	213.4450000
	1.187699180	0.81647881800D-01	38.91439863	150.9659942	181.1864767	213.8379562
41806.99379	1.187689300	0.81392700000D-01	38.91560000	190.9551000	150.7873000	167.2296000
	1.187689300	0.81269453130D-01	38.91651665	190.5358715	150.7800815	167.6502950
41813.95500	1.187683410	0.81435700000D-01	38.91580000	230.1096000	120.7058000	122.5016000
	1.187683410	0.80962240660D-01	38.91855452	229.8463370	120.7051274	122.7658226
41820.94295	1.187680490	0.81451900000D-01	38.91680000	269.2015000	90.63060000	81.52100000
	1.187680490	0.80839514030D-01	38.92035337	269.1353721	90.63045883	81.52664864
41827.99458	1.187678070	0.81456500000D-01	38.92000000	308.7087000	60.22390000	44.33780000
	1.187678070	0.80975149140D-01	38.92280074	308.9651132	60.22845623	44.08042180
41834.97096	1.187674760	0.81409600000D-01	38.92150000	347.6241000	30.13920000	10.38490000
	1.187674760	0.81273522660D-01	38.92259125	348.2409126	30.14636544	9.966627311
41841.94747	1.187670600	0.81301600000D-01	38.92390000	26.99220000	0.54900000000D-01	337.0916000
	1.187670600	0.81578701610D-01	38.92229102	27.38959946	0.61515206970D-01	336.6929218
41848.99942	1.187665840	0.81251700000D-01	38.92570000	66.56360000	329.6541000	301.8537000
	1.187665840	0.81823626510D-01	38.92238140	66.74640036	329.6570744	301.6703454
41855.97478	1.187661410	0.81180000000D-01	38.92520000	105.6575000	299.5826000	263.5296000
	1.187661410	0.81781078090D-01	38.92171531	105.5330714	299.5605815	263.6544734
41862.94919	1.187657120	0.81234400000D-01	38.92570000	144.8873000	269.5123000	220.8296000
	1.187657120	0.81588127670D-01	38.92364794	144.5193098	269.5062157	221.1987537
41869.99577	1.187653120	0.81211400000D-01	38.92440000	184.4720000	239.1155000	174.8277000
	1.187653120	0.81157076210D-01	38.92471507	184.0420132	239.1081735	175.2591541
41876.97260	1.187649930	0.81270300000D-01	38.92130000	223.6216000	209.0453000	129.7205000
	1.187649930	0.80643249480D-01	38.92377592	223.3221380	209.0400300	130.0210205
41883.94710	1.187646780	0.81325800000D-01	38.92210000	262.7601000	178.9655000	87.58660000
	1.187646780	0.80717780990D-01	38.92563057	262.7090143	178.9665865	88.03781197
41890.99529	1.187643430	0.81353200000D-01	38.92450000	302.2414000	148.5643000	50.18460000
	1.187643430	0.80832558600D-01	38.92752497	302.4594598	148.5681776	49.86568605
41897.97429	1.187638990	0.81251900000D-01	38.92460000	341.2779000	118.4834000	15.96140000
	1.187638990	0.81048458940D-01	38.92576051	341.6796345	118.4903353	15.55824592
41904.95547	1.187633170	0.81166600000D-01	38.92610000	20.42710000	88.40450000	342.7159000
	1.187633170	0.81377824070D-01	38.92487569	20.84294051	88.41143282	342.2987038
41911.92536	1.187627920	0.81080800000D-01	38.92850000	59.59330000	58.32790000	308.2571000
	1.187627920	0.81617484660D-01	38.92539284	59.82561535	58.83167329	308.0240789

EPOCH (MJD)	SEMIMAJOR AXIS (EARTH RADII) A A′	ECCENTRICITY E, E′	INCLINATION (DEGREES) I, I′	ARGUMENT OF PERIGEE (DEGREES) ω, ω′	RIGHT ASCENSION OF THE ASCENDING NODE (DEGREES) Ω, Ω′	MEAN ANOMALY (DEGREES) M, M′
41918.97734	1.187623820	0.81034600000D-01	38.92890000	99.16290000	27.92700000	270.1737000
	1.187623820	0.8165141539D-01	38.92533103	99.08919367	27.92561097	270.2476265
41925.90156	1.187616900	0.81006900000D-01	38.92930000	138.34030000	357.8555000	228.2052000
	1.187616900	0.8141744954D-01	38.92692537	138.00131188	357.8459544	228.5452343
41932.92002	1.187600510	0.81104700000D-01	38.92930000	177.52440000	327.7904000	182.9772000
	1.187600510	0.8112527167D-01	38.92913067	177.08246060	327.7630531	183.4136565
41939.97416	1.187600430	0.81133200000D-01	38.92870000	217.08610000	297.3960000	137.0863000
	1.187600430	0.8075643136D-01	38.93087115	216.75355553	297.3901945	137.4200862
41946.94009	1.187595150	0.81178400000D-01	38.92760000	256.20330000	267.3218000	94.69310000
	1.187595150	0.8058287808D-01	38.93105212	256.10644136	267.3200740	94.79035625
41953.99063	1.187585510	0.81225500000D-01	38.92850000	295.47620000	236.9111000	56.39830000
	1.187585510	0.8067073163D-01	38.93171756	295.65167448	236.9142165	56.22216035
41960.97096	1.187568410	0.81203000000D-01	38.92730000	334.62370000	206.8280000	21.48850000
	1.187568410	0.8093586610D-01	38.92834590	335.20517668	206.8346118	21.10564031
41967.94948	1.187553200	0.81035600000D-01	38.92870000	13.92000000	176.7600000	346.2603000
	1.187553200	0.8117897584D-01	38.92757039	14.34845948	176.7671560	347.8304367
41974.92475	1.187543310	0.80972600000D-01	38.92970000	53.10190000	146.6732000	314.1335000
	1.187543310	0.8146919391D-01	38.92682596	53.37677020	146.6776653	313.8577847
41981.97014	1.187540540	0.80906800000D-01	38.92900000	92.72150000	116.2693000	276.6451000
	1.187540540	0.8153094103D-01	38.92539328	92.69947127	116.2689459	276.6671933
41988.94077	1.187532040	0.80929900000D-01	38.93100000	131.88760000	86.19680000	235.3905000
	1.187532040	0.8139125929D-01	38.92633421	131.58281133	86.19384174	235.6962222
42003.95026	1.187523340	0.81076200000D-01	38.93380000	216.17530000	21.48030000	138.1221000
	1.187523340	0.8070905659D-01	38.93582505	215.33660035	21.47442645	138.4604594
42010.92049	1.187519350	0.81087400000D-01	38.93430000	255.27570000	351.4071000	95.66000000
	1.187519350	0.8049409957D-01	38.93773451	255.17223096	351.4062624	95.76383347
42017.97036	1.187512350	0.81109500000D-01	38.93520000	294.76790000	321.0083000	57.04650000
	1.187512350	0.8055154319D-01	38.93843070	294.95911333	321.0033349	56.87457038
42024.95310	1.187506330	0.81101000000D-01	38.93570000	333.88500000	290.9179000	22.30000000
	1.187506330	0.8082483112D-01	38.93727666	334.26373373	290.9244496	21.61991382
42031.92011	1.187501080	0.81019200000D-01	38.93510000	13.01850000	260.8371000	345.0268000
	1.187501080	0.8115305143D-01	38.93432584	13.44833368	260.8442754	348.5955653
42038.97071	1.187493610	0.80862300000D-01	38.93310000	52.69350000	230.4327000	314.4870000
	1.187493610	0.8135674622D-01	38.93024652	52.97627061	230.4372065	314.2083780
42045.95261	1.187484740	0.80966400000D-01	38.93340000	91.66520000	200.3531000	277.5054000
	1.187484740	0.8159123708D-01	38.92973524	91.85010743	200.3526571	277.6206375
42052.92096	1.187478650	0.80957200000D-01	38.93300000	130.96030000	170.2741000	236.4210000
	1.187478650	0.8145534304D-01	38.93024274	130.66102239	170.2692272	236.7211943
42059.97410	1.187472980	0.81010200000D-01	38.93460000	170.51430000	139.6712000	191.1899000
	1.187472980	0.8110634730D-01	38.93404396	170.00060685	139.6639399	191.6248220

NOTE: Unprimed values are original Navy Mean Elements, the equivalent of Brouwer double primed elements. The primed values (used in the orbit determinations) are elements with Navy-used long period terms added back in, the equivalent of Brouwer single primed values. The Navy-used zonal coefficients are: $10^6 J_2 = 1082.63$, $10^6 J_3 = -2.55$, $10^6 J_4 = -1.61$, $10^6 J_5 = -0.19$.

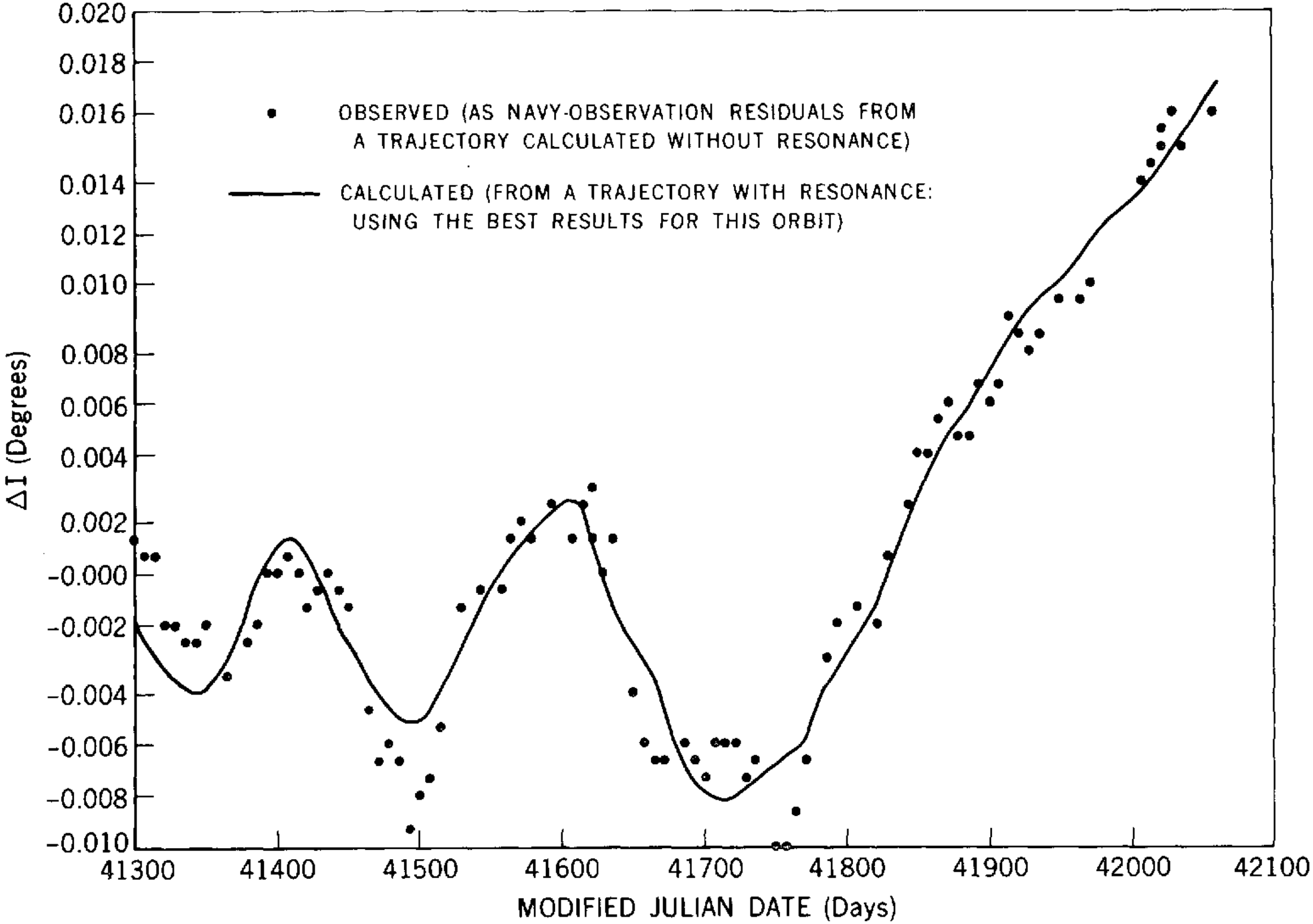

Figure 2. Resonant Variation of the Inclination for the Orbit of 1967-14F

The residuals in inclination (rms) from this trajectory (and others including those with resonant effects) are shown in Table 2. The calculated trajectories were "fit" to the mean element "observations" by differential correction of initial elements and other model parameters (see i.e., Morrison, 1970 or Wagner and Douglas, 1970) under the conditions stated in Table 2. The principal difficulty in these orbit determinations was in following the mean anomaly of the satellite which (with a perigee of 580 km) underwent fluctuations of tens of degrees from drag error. The level of this error was less than 10% after correction for a single drag coefficient. Yet it was necessary to include additional accelerations in the semimajor axis and mean anomaly to permit the critical resonant longitude $\psi_{13,0}$ to be calculated to better than 5° (rms). (This error is consistent with the formal accuracy of the resonance determination.)

A more readily available data source for satellite objects, the North American Air Defense Command's SPADATS elements were also evaluated for 1967-14F by the same mean element program. (See run 2 of Table 2.) Figure 3 shows the inclination residuals from a nonresonant trajectory with this data over a somewhat shorter arc in 1972-1973. For most of the period no detail at all can be seen. At the end of the arc some definition of the resonance appears. The NORAD-SPADATS data quality is clearly not uniform in this arc. The small "acceptable" portion could not be used for adequate resonance recovery.

Table 2

Results of Orbit and Coefficient Determinations for 1967-14F Using Navy and NORAD Data

Run	Field Used	Residuals* in Inclination (rms) (10^{-3} deg's)	Data Used	Comments
1	SAO SE 2 (non resonant)	5.41	NAVSPASUR - All elements MJD 41303 - 41989	Drag, radiation, $\dot{a}$ and $\dot{M}$ coefficients solved from data. Commensurability at 41985. Resonance in "I" clear, residuals about 0.001°.
2	SAO SE 2 (non resonant)	6.25	SPADATS - All elements MJD 41302 - 41936	Same as above. Resonance in I not seen except possibly after MJD 41700. Residuals about 0.01°.
3	SAO SE 2 (non resonant)	6.83	Same as run #1, but to MJD 42060	Same as run #1.
4	SAO SE 2 + 3,0 and (23, 13) solved from data $10^9(C,S)_{23,13} = (-20.7 \pm 1.4, -69.8 \pm 1.2)$	1.13	Same as run #1.	Same as run #1. Weight on I: 0.0002°. Maximum along track error: 6°.

*Observed – calculated values from converged mean element trajectory

Table 2 (continued)

Run	Field Used	Residuals* in Inclination (rms) (10^{-3} deg's)	Data Used	Comments
5	SAO SE 2 + 3,0 and (23, 13) solved from data $10^9(C,S)_{23,13}$ = $(-22.0 \pm 1.2, -71.8 \pm 1.1)$	1.29	NAVSPASUR - All elements to MJD 42060	Drag, radiation, $\dot{a}$, $\ddot{a}$, $M^{[3]}$, $M^{[4]}$, $M^{[5]}$ coefficients solved from data. Short period lunar terms used. Maximum along track error: 10°. Correlation coefficient: $(C,S)_{23,13} = -0.38$. Weight on I: 0.0002°.
6	SAO SE 2 + 3,0 and (23, 13) solved from data $10^9(C,S)_{23,13}$ = $(-21.2 \pm 3.4, -71.3 \pm 2.4)$	1.29	NAVSPASUR - Inclination data only - to MJD 42060	All state elements except "I" and radiation coefficient fixed from solution in run #5. Correlation coefficient $(C,S)_{23,13} = 0.34$.

*Observed – calculated values from converged mean element trajectory

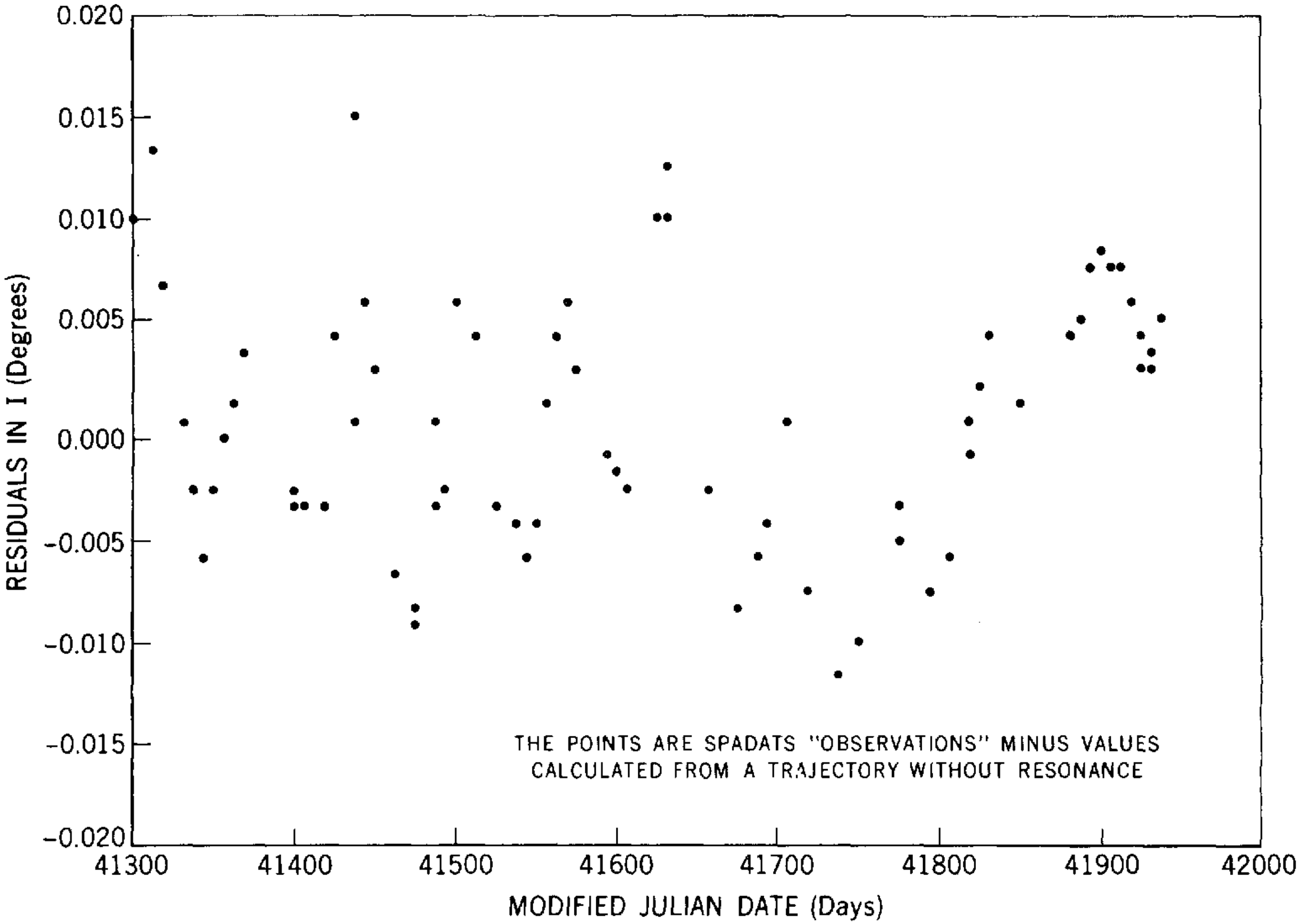

Figure 3. Inclination Residuals from a Nonresonant Trajectory for 1967-14F Using NORAD-SPADATS Observations

RESONANCE RECOVERY

The geopotential indices for this (mean) primary resonance ($\ell - 2p + q = 1$) are $m = 13$, $q = 0$ and all $\ell \geq 13$ for which

$$\ell = 2p + 1, \quad p \leq \ell.$$

Thus the ℓ are all odd and any one of the (ℓ, m) terms (13,13), (15,13), (17,13) . . . can be recovered from the resonance in inclination on this orbit.

It is noted that the first of the side band primary resonances ($q = \pm 1$) which are affected by the even degree geopotential term (and $m = 13$) have only slight effect in this data span since $\dot{\omega} = 5.6°$/day (relatively large compared to $\dot{\psi}_{13,0}$ in this period). The $q = -1$ resonance has a period of 180 days at 4300 MJD declining to 60 days at 42060 MJD. The $q = +1$ resonance has a period of only 40 days at the beginning of the data span, increasing to 70 days at the end. In addition, the amplitudes of these terms are of order $e^{|q|}$, where e is the orbit's eccentricity, [see Kaula, 1966, p. 37] or reduced by a factor of 0.08 with respect to the $q = 0$ resonance for 1967-14F.

The fact that (essentially) only a single geopotential term can be recovered from each resonance is a consequence of the fact that only a single harmonic perturbation is responsible for it (see King-Hele, 1973a or Wagner, 1973). The amplitude of this term changes only as a consequence of the change in frequency through the resonance. The fundamental amplitude (a weighted sum of the resonant geopotential coefficients) remains constant. The scale of this sum is determined by the actual resonance perturbation. However, even for the same fundamental frequency (ψ) this weighted sum is different for 4 of the Kepler element variations (a, e and "I" have the same sum, but ω, Ω and M are distinct).

Why (physically) only 4 of the 6 sums (or amplitudes) yield independent information is unclear. It may be due to the choice of the classical Kepler elements to express the perturbations. In particular, the choice of the mean anomaly instead of the true anomaly introduces an infinite number of frequencies to express the effects of a single geopotential harmonic on an orbit. In some sense this must dilute the information content of any single frequency. But in any case, for the dragged orbits, only the inclination variation (essentially free of drag error) has provided unambiguous recovery of resonance information. However, recently, King-Hele (1973c) and Winterbottom and King-Hele (1974) have shown that near circular orbits can suffer significant resonances of Ω, e and ω apparently quite distinct from drag (and other) effects. A strong resonance in e was also seen by Wagner (1973) in the very slow decay of the Vanguard 3 orbit. On the other hand no significant resonance of other elements on the DIADEME 2 fragment has yet been seen, though the full passage will not be over till late 1974.

I chose the geopotential harmonic (23,13) to absorb the inclination resonance on 1967-14F and at the same time made a 5% adjustment of (3,0) to correct odd zonal model error (see Table 2, runs 4-6). The same differential correction program was used in these adjustments as previously to reveal the resonance. But added weight was given to the inclination data in these adjustments and additional secular accelerations were used to reduce the along track or phase error of the resonance. Full data correction runs were made for 686 and 757 day arcs (runs 4 and 5). The inclination residuals were significantly larger in the longer arc, possibly showing the influence of the secondary resonance with m = 26. But this conclusion must wait till the passage is complete and the smaller (higher frequency) effect can be well separated. In any case, the (23,13) recovery is not substantially altered over the longer span. A final run was made using the inclination data only, to confirm that the other elements, influenced by drag, were not distorting this result. The (23,13) recovery in this correction (run 6) was between the values in the two full data analyses.

The solid curve in Figure 2 shows the computed resonance in inclination from the best (most representative) results to date (run 5). The values for the harmonic coefficients themselves are somewhat larger than Kaula's rule [Kaula, 1966, p. 98]. But this is not significant since it is only a linear sum of resonant terms which is well determined.

RESONANT CONSTRAINT (LUMPED COEFFICIENTS)

Following the method developed by Gooding (1971) and elaborated by Wagner (1973) the resonant inclination variation for 1967-14F ($a = 1.88$ e.r., $e = 0.082$, $I = 38.92°$) is determined by the lumped sine and cosine terms:

$$
\begin{aligned}
(C,S)_{13,0} = {} & 0.023(C,S)_{13,13} - 0.172(C,S)_{15,13} + 0.505(C,S)_{17,13} \\
& - 0.884(C,S)_{19,13} + 1.000(C,S)_{21,13} - 0.673(C,S)_{23,13} + 0.099(C,S)_{25,13} \\
& + 0.295(C,S)_{27,13} - 0.279(C,S)_{29,13} + 0.018(C,S)_{31,13} + 0.156(C,S)_{33,13} \\
& - 0.105(C,S)_{35,13} - 0.036(C,S)_{37,13} \\
& + 0.085(C,S)_{39,13} - 0.021(C,S)_{41,13} + \ldots
\end{aligned} \tag{1}
$$

The terms in this series are for fully normalized geopotential harmonics [Kaula, 1966, p. 7]. The weights are just the fundamental amplitudes (without the rate denominator) of the linear perturbation of the inclination due to a fully normalized harmonic [Kaula, 1966, p. 40]. The sum of these fundamental amplitudes (with C and S coefficients) are merely the coefficients of the cos $(\psi_{13,0})$ and sin $(\psi_{13,0})$ terms determining the rate of the resonance variation of the inclination. It is these two (lumped) coefficients which are actually "observed" in this resonance. The reason (23,13) was chosen to absorb the effect was because it had a high weight in (1). It also was the lowest degree 13th order term not present in the SAO SE 2 field. Using the (23,13) weight of -0.673, the "observed" lumped coefficients are (best results):

$$10^9 (C,S)_{13,0} = (14.8 \pm 0.8,\ 48.3 \pm 0.7), \tag{2}$$

with a correlation coefficient of -0.38.

With regard to "lumped coefficients", if their perturbations can be properly identified, the weights of their constituents can serve to extend the information in them to any degree. Such "lumped coefficients" have been reported many times from "shallow resonant" satellite orbit analyses [i.e., Yionoulis, 1965;

Murphy and Cole 1968; Gaposchkin and Veis 1967; Douglas and Marsh, 1970]. One difficulty with "shallow resonant coefficients" is that except for near circular orbits, the side band resonances (i.e., $q = \pm 1$) will also be observable. They will not be separable unless long arcs of data are analyzed spanning at least a rotation of perigee. But if the "lumped coefficients" are well determined (and identified), as here and in the previous 15th and 11th order analyses, they can serve as absolute benchmarks for geopotential determinations. An example of this use has been given by Wagner (1973) for a previously poorly observed 11th order resonance. Here, I calculate the "lumped coefficient" for the 1967-14F resonance from Equation (1) with a number of recent fields which are well represented by 13th order terms. The results are presented in Figure 4. The merit of each of these fields is now immediately apparent when compared to the "observed" coefficients.

DISCUSSION OF RESULTS

The "point" in Figure 4 representing the "observed" lumped coefficients has two uncertainty circles about it. The inner one represents the formal 1σ uncertainty of the "best" solution. The small solution shift from the abbreviated data analysis is also seen. The larger circle represents the uncertainty in the lumped coefficient if terms of degree higher than 30 are ignored (as they generally are in current global solutions). This expected "truncation error" is calculated as the root sum of squares for the neglected terms in Equation (1) using Kaula's rule ($10^{-5} / \ell^2$) for the harmonic coefficients (Table 3). It is also

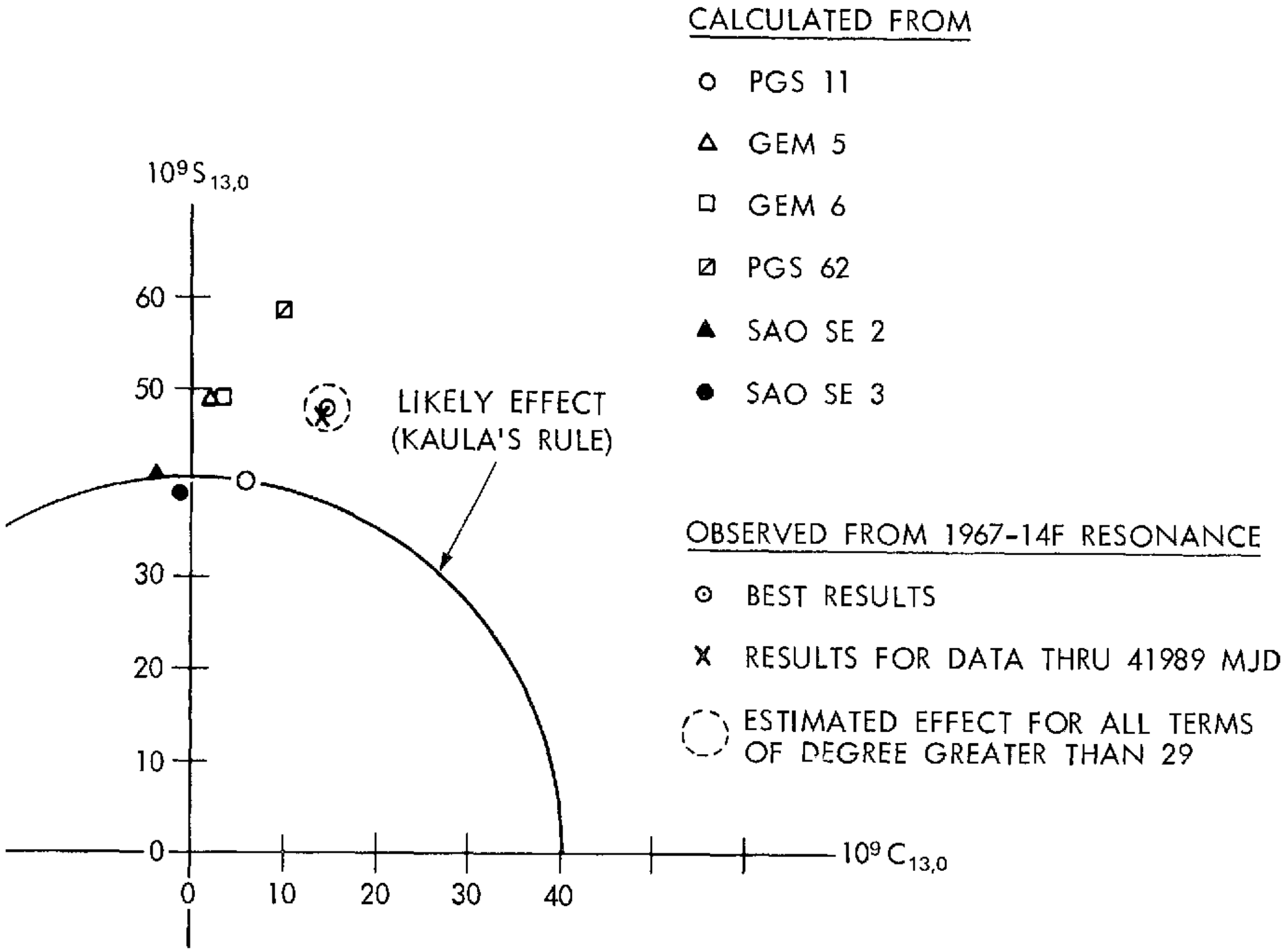

Figure 4. Lumped Harmonic for 1967-14F Resonance

Table 3

Estimated Cumulative Effect on Lumped Harmonic for 1967-14F, From Geopotential Terms

	ℓ	13	15	17	19	21	23	25	27	29	31	33	35	37	39	41
10^9 RSS* (all terms $\geq \ell$)		40.9	40.9	40.2	36.2	26.6	13.9	5.8	5.5	3.8	1.9	1.9	1.3	0.9	0.8	0.6

*Using Kaula's rule and Equation (1).

of interest to calculate the a priori variance of the total lumped coefficient, as its square root can be thought of as a reasonable (1σ) value for it. Using Kaula's rule again [over all the coefficients in Equation (1)] this value is indicated by the circular arc centered on the origin in Figure 4.

The convergence of the series in Equation (1) is actually stronger than it appears. The weights can be shown to decrease (on average) at least as fast as $1/\ell$ while the coefficients behave as $10^{-5}/\ell^2$. Comparison of the observed lumped harmonic with its a priori standard deviation shows its value is slightly greater than Kaula's rule would have predicted. In terms of Kaula's rule, the formal statistics say the lumped harmonic is determined to better than 1 part in 40.

The fields chosen for comparison with the "observed" harmonic all used basically the same shallow 13th order resonant orbits but with different amounts and kinds of tracking data. They all extend to at least the 22nd degree in 13th order terms. The purely satellite data fields are GEM 5 and PGS62 (F. Lerch, Personnal Communication, 1973 and 1974). The others are combination solutions with surface gravimetry data. The Smithsonian fields (Gaposchkin and Lambeck, 1971; Gaposchkin, 1973) used analytic techniques for both orbit and geopotential determination. The satellite data in these consisted only of camera and laser observations. The fields originating at Goddard Space Flight Center employed numerical integration to rationalize the orbital data. GEM 5, 6 and PGS-62 contain significant amounts of electronic data (radar range and range rate, and

Doppler observations) on the 13th order shallow resonant orbits. PGS 11 (F. Lerch, Private Communication, 1974) contains only camera data for the satellite observations.

As for the truncation of 13th order terms, GEM 5, 6 and SAO SE 2 extend to (22,13), SAO SE 3 goes to (23,13), and PGS 11 and 62 include all 13th order terms to (29,13).

Yet in spite of all these differences, the fields are clustered fairly closely in Figure 4, showing the dominance of the similar 13th order satellite information. The Goddard fields (with substantially more data) are somewhat closer to the observed harmonic, but the truncation is significantly different for them. In fact the (probable) truncation error alone would account for all of the distance of the GEM 5 and 6 models from the observation. On the other hand the SAO SE 2 (at the same truncation) is significantly farther from the observed value. (It is somewhat beyond the probable truncation error for all the terms of degree greater than 22.) The SAO SE 3 (with 23,13 terms) is marginally closer than SAO SE 2 to the observation but the added terms should have made a much greater improvement still. The same situation holds (and even more strongly) for the more recent Goddard fields (PGS 11 and 62) which extend to (29,13). Here, no improvement is seen over the earlier GEM solutions but the truncation error (given by the dotted circle about the observation) should be much reduced.

The simplest way to interpret these comparisons is to say that the higher degree terms (i.e., above about 21) for this resonance are not well determined.

The conclusion then follows that these terms will be significantly improved with the use of the lumped harmonic (i.e., Equation (2)) for the DIADEME 2 fragment. The simplest example of such use would be to add a single high degree term to an existing field. Of course this would not be a realistic solution on two counts. First, it would upset the 13th order ties (correlation) in the existing field. Secondly, it would ignore the contributions of still higher degree terms. But it does give a representative value (exact for 1967-14F) and a first approximation of the realistic numbers. Using GEM 6, I find the added (23,13) term (C,S) to be 10^{-9} (-16.9, 0.3) which has an rms of 12.0×10^{-9} compared to 18.9×10^{-9} for Kaula's rule.

SUMMARY AND CONCLUSIONS

A strong 13th order resonance has been observed and analyzed from U.S. Navy Tracking Data on the slowly decaying orbit of a DIADEME 2 fragment (1967-14F). The exact commensurability for the orbit occurred in late 1973 and the major changes due to the resonance will be over by late 1974. Nevertheless, apparently stable and well determined values of a lumped harmonic for this resonance have been found which should significantly improve 13th order geopotential terms to about as high as degree 33. There is fairly close agreement of calculated values from recent fields with the "harmonic" observation (within 20%). The major part of the discrepancy is probably due to poorly known coefficients above degree 21 in these fields.

Substantial improvement of 13th order and high degree terms will be seen with use of the lumped values (and the linear constraint) in combination solutions with other data.

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