

VISCOELASTIC MATERIALS  
FINAL REPORT - JANUARY 1974

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VISCOELASTIC MATERIALS Final Report  
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FAILURE CRITERIA FOR VISCOELASTIC MATERIALS

FINAL REPORT

TO THE

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

Research Grant No. NsG-172-60 and NGL-05-002-005

GALCIT 120

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Firestone Flight Sciences Laboratory

Graduate Aeronautical Laboratories

California Institute of Technology

Pasadena, California

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The following report summarizes work performed under the Grant by the National Aeronautics and Space Administration to the California Institute of Technology. Initiated under Prof. M. L. Williams as principal investigator, together with Prof. P. J. Blatz and carried on later by this writer, the program covered a rather wide range of problems associated with the structural integrity of solid propellant rocket motors. The writer as well as all the researchers associated with this work are indebted to the technical cognizance of NASA personnel, in particular to Messrs. R. Ziem, P. Wetzel, R. Wassel, and G. Lewis (JPL) with the assistance of Mr. E. Duran (JPL) in fostering a research atmosphere that has, we believe, produced a significant advance in the understanding of failure behavior in visco-elastic materials.

## REVIEW

The work under this grant covered a wide range of topics and produced a large number of reports. It would seem unnecessarily repetitive to review the contents in any detail here since these reports are available. There appeared a total of about 75 reports which have appeared as GALCIT (Graduate Aeronautical Laboratories, California Institute of Technology) reports, in the bulletin of the Interagency Chemical Rocket Propulsion Group, as well as in archive journals and in books. But instead of presenting here a comprehensive review it appears more appropriate to trace the major developments and subjects covered under this grant, referring to reports listed in the bibliography at the end of this report. In addition, it appears desirable to highlight these remarks with the appending of several publications of those papers for ready reference which may be considered milestones in the progress of work under this grant. As is the case with such scientific work, such milestones soon become superseded as new ideas are derived from them and are developed further, thus leading to a progressively better understanding of the problems under consideration.

### Properties of Composite Propellant, Binder and Granular Aggregate

The initial effort was expended on understanding the mechanical properties of composite propellants excluding fracture. In this endeavor it seemed obvious at the outset that one would have to understand first the mechanical behavior of the granular constituents and that of the polymeric binder separately. Accordingly several

studies dealt with viscoelastic material characterization (1-8)\* and with the time-dependent deformation characteristics of a granular, unbonded medium (9,10). It did not become clear until we understood the time-dependent fracture of the binder material better at a later time, that some relaxation or creep phenomena should occur only for the composite material, that is relaxation and creep phenomena resulting from growing voids or cracks which are induced by the granules in the binder matrix.

Large Deformation Analysis. Since the deformations usually encountered in solid rocket motors tend to be larger than those considered admissible under a linearized theory of (visco)elastic deformations, it seemed appropriate to tool up for handling large deformation problems relating to motor deformations. Such problems are recorded in references 11 through 16, including one relating to stress concentration in a sheet containing a hole under biaxial tension as a model for a flaw in a solid undergoing large deformations (17).

Fracture Theory. The major aim of the performed work was to obtain understanding of the fracture behavior of viscoelastic solids. Accordingly studies were performed on the distribution of stresses in cracked structures. These analyses drew on linear (18-20) as well as nonlinear elasto-statics (17,21), and emphasized a thorough understanding of the stress distributions in geometries used in experimental work.

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\*Numbers in parentheses denote references at the end of this summary.

Starting from molecular concepts of fracture the time-dependent failure in elastomers was the subject of detailed study. At the time, characterization of failure was usually discussed in terms of uniaxial tensile data, and therefore most of the early work in failure analysis dealt with that special case of stress state. A theory based on a marriage of chemical reaction rate and fracture mechanics produced good correlation with experiments under monotonic load histories (22-25) and illustrated the problem of cyclic fatigue (29-32) and the role of dissipation played in that process (29). The same theory pointed out the shortcomings of the failure-envelope concept as a criterion for failure (25).

It soon seemed evident, however, that this rate theory was inadequate to describe fracture in more complicated stress states; at any rate, it appeared that there should exist a less cumbersome way of computing fracture behavior. A more mechanics oriented approach with the behavior of cracks appeared more promising than one tied to the physical chemistry. A start in that direction was the development of a model for the transient growth of cracks in a viscoelastic sheet by M. L. Williams (35), with a further development devoted to a generalization of the Orowan-Irwin fracture energy of plastic energy dissipation to include energy dissipation due to visco-elastic processes (36). A series of reports on fracture behavior followed which became increasingly more refined both with respect to the physical modelling and with respect to the analytical treatment (37-54). They range from quasi-elastic analysis (40, 41, 49, 50), via elastic-viscoelastic analogy (52) to full-fledged viscoelastic

treatments (43-46, 48, 53, 54) and resulted in a review of fracture in viscoelastic solids (55). The improvements in understanding the fracture process in viscoelastic solids came primarily from a more detailed treatment of the material in the immediate vicinity of the crack tip than had been attempted before (44, 47, 48, 53, 54). The newest developments of this theory of failure by time-dependent crack propagation show excellent corroboration with experiments. They contain the fracture of brittle solids as a special case and bring the fracture of viscoelastic solids in line with fracture treatments of rate-insensitive solids (53, 54).

PRESENT STATE OF KNOWLEDGE

The work under this grant has produced a theory of fracture which is well corroborated by experiments. Furthermore we also understand the limitations of that theory although that understanding has yet to be fortified by further experiments. Within these (possible) limitations we are now able to understand what controls the fracture process. (See, e. g. ref. 54, the second to the last paper appended.)

We are able to identify how the geometry of the structure and the loads acting on it influence the failure process. Next we know how the viscoelastic deformation characteristics of the bulk elastomer influences the failure behavior. In addition we have established what additional material parameters control the fracture process. These parameters have been found to be material constants for the polyurethane elastomer used in our experimental investigations. Moreover, these parameters should depend on the molecular parameters of the material which are commonly employed in characterizing polymers from the viewpoint of physical chemistry.

We believe that we have thus provided a framework by which the macroscopic failure behavior can be linked in a rational way to the molecular structure of polymers. Such an association is very important in providing the chemical physicist and the chemical syntheticist with information for designing materials with improved strength characteristics. Although this potential bridge between macroscopic and microscopic properties, the detailed nature of this bridge, i. e. , the functional dependence of the two fracture parameters on molecular parameters must yet be delineated through experiments.



We believe that such experiments carry great hope for further understanding the failure behavior of viscoelastic solids.

### APPENDED PAPERS

In order to present some detail of past work for ready reference, several papers are included in this report. These papers represent a chronological development of the fracture work performed under this grant and represent the essence of the work in scope if not in full detail. They are identified in the list of references by numbers in square brackets which indicate their order of appearance in the appendix.

### EDUCATIONAL RESULTS OF THE GRANT

It seems of interest to add to the technical accomplishments those derived from the grant measured in less tangible educational benefits. There were associated with the program some 21 students, each for three months or longer. Of these, 12 have gone on to receive, or are in the process of obtaining a Ph. D. Of the latter, 4 have obtained their Ph. D. degree with almost continuous support under this grant.

In addition, 9 postdoctoral fellows and 3 professional researchers have contributed to the work under this grant.

REFERENCES

1. Knauss, W. G.: "On the Mechanical Properties of an H-C Rubber." 1963. Bulletin of the 2nd Meeting of ICRPG Working Group on Mechanical Behavior, CPIA Publ. No. 27, October 1963, pp. 179-198. (GALCIT SM 63-1).
- [1]\* 2. Williams, M. L.: "The Structural Analysis of Viscoelastic Materials." 1963. AIAA Journal, Vol. 2, *A64-17717* No. 5, May 1964, pp. 785-808. (GALCIT SM 63-15).
3. Knauss, W. G.: "Modulus Determination of Polymers at Low Temperature." Presented at 3rd Annual Meeting of ICRPG Working Group on Solid Propellant Mechanical Behavior, November 16-19, 1964, Huntsville, Alabama. (GALCIT SM 64-28).
- [8] 4. Knauss, W. G.: "Uniaxial Wave Propagation in a Viscoelastic Material Using Measured Material Properties." J. A. M., Vol. 35, Sept. 1968, pp. 449-453. *A68-42344*
- [9] 5. Clauser, J. F., Knauss, W. G.: "On the Numerical Determination of Relaxation and Retardation Spectra for Linearly Viscoelastic Materials." Trans. Soc. Rheol. vol. 12, No. 1, pp. 143-153, 1968. *A69-12605*
6. Knauss, W. G., Mueller, H. K.: "The Mechanical Characterization on the Swollen and Unswollen State." 1967. (GALCIT SM 67-8).
7. Pucik, T. A., Knauss, W. G.: "On a Problem of Thermo-Viscoelasticity." 1968. (GALCIT SM 68-17).
8. Mueller, H. K.: "On the Relaxation Modulus of the Equivoluminal Composition of Solithane 113." 1969. (GALCIT SM 69-7).
9. Blatz, P. J.: "Studies in Granular Mechanics - I." 1963. (GALCIT SM 63-2).
10. Blatz, P. J.: "Studies of the Mechanical Behavior of Ammonium Perchlorate Particles, A Glass-Bead Filled Polyurethane Binder, and a Typical Continuum Binder." 1963. Presented ICRPG-Nov. 1963 Meeting. (GALCIT SM 63-28).
11. Blatz, P. J., Levinson, M.: "Stress Induced Anisotropy in Pressurized Thick Walled Cylinders. 1962. (GALCIT SM 62-6).

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\*Numbers in brackets refer to the order in which the appended papers appear.

12. Levinson, Mark: "The Finite Strain Behavior of Incompressible Hollow Cylinders and Spheres under Internal Pressure." 1962. (GALCIT SM 62-48).
13. Blatz, P. J.: "Finite Elastic Slump of an Infinitely Long Hollow Cylindrical Incompressible Neo-Hookean Body Bonded to a Rigid Outer Tube." 1962. (GALCIT SM 62-54).
14. Blatz, P. J.: "The Finite Elastic Plane Strain Deformation of a Neo-Hookean Body." 1963. (GALCIT SM 63-14).
15. Blatz, P. J.: "Application of Finite Elastic Theory to the Behavior of Rubberlike Materials." 1963. Rubber Chemistry and Tech., Vol. 36, No. 5, December 1963, pp. 1459-1496 (GALCIT SM 63-21).
16. Blatz, P. J.: "The Effective Compressibility of a Hollow Sphere." 1963. Bulletin of 2nd Meeting ICRPG Working Group on Mech. Behavior, CPIA Pub. 27, October 1963, p. 1. (GALCIT SM 63-27).
- [6] 17. Yang, W. H.: "Stress Concentration in a Rubber Sheet under Axially Symmetric Stretching." J. A. M., Vol. 34, December 1967, pp. 942-946.
18. Knauss, W. G.: "On the Effect of a Boundary on the Stresses in the Vicinity of a Crack." 1962. (GALCIT SM 62-46).
19. Knauss, W. G.: "Stresses in an Infinite Strip Containing a Semi-Infinite Crack." J. A. M., Vol. 33, No. 2, June 1966, pp. 356-362.
20. Chao, C. C., Holte, J. M., Westmann, R. A.: "Stresses Around a Penny Shaped Crack." 1965. (GALCIT SM 65-27).
- [7] 21. Knauss, W. G.: "Stresses Near a Crack in a Rubber Sheet." Exptl. Mech., Vol. 8, April 1968, pp. 177-181.
22. Knauss, W. G.: "Rupture Phenomena in Viscoelastic Materials." Ph. D. Thesis, June 1963. (GALCIT SM 63-10).
23. Knauss, W. G.: "Energy Considerations Relating to Viscoelastic Materials." 1964. (GALCIT SM 64-11).

24. Knauss, W. G.: "On the Propagation of Failure in a Biaxial Stress Field." 1964. Presented at 3rd Annual Meeting of ICRPG Solid Propellant Working Group on Mechanical Behavior, Nov. 16-19, 1964, Huntsville, Ala. (GALCIT SM 64-26), V.1, pp. 437-454
- [4] 25. Knauss, W. G.: "The Time Dependent Fracture of Viscoelastic Materials." 1964. Presented at Proc. of the 1st Int'l Conf. on Fracture, Sendai, Japan, Sept. 1965. (GALCIT SM 64-47).
26. Knauss, W. G., Hufferd, W.: "Failure Analysis of a High Mass Fraction Solid Propellant Rocket Motor." 1967. (GALCIT SM 67-1).
27. Knauss, W. G.: "Failure of Polymeric Continua." 1962. (GALCIT SM 62-28).
28. Betz, E.: "Some Mechanical Properties of Solithane 113." 1965. (GALCIT SM 65-12).
29. Knauss, W. G., Betz, E.: "Some Characteristic Functions Pertinent to Failure of Viscoelastic Materials." 1965. For presentation at 4th CPIA ICRPG Meeting, San Diego, Cal., Nov. 17, 1965. (GALCIT SM 65-16).
30. Betz, E.: "Failure of Viscoelastic Materials Under Cyclic Strains - Part I." 1965. (GALCIT SM 65-22).
31. Knauss, W. G.: "Time-Dependent Failure of Viscoelastic Materials Under Cyclic Loads." 1966. Presented at the AIAA/ICRPG Solid Propulsion Conference, Washington, D. C. (GALCIT SM 66-6).
32. Williams, M. L., Knauss, W. G., Wagner, F. R.: "Fatigue Failure in Linearly Viscoelastic Materials." 1966. (GALCIT SM 66-13). *A68-18937*
- [5] 33. Knauss, W. G.: "An Upper Bound on Failure in Viscoelastic Materials Subjected to Multiaxial Stress States." Int. J. Fract. Mech, Vol. 3, No. 4, Dec. 1967, pp. 267-277.
34. Knauss, W. G.: "A Review of Failure Analysis Related to Solid Propellant Rocket Motors." 1967. (GALCIT SM 67-11).
- [2] 35. Williams, M. L.: "The Fracture of Viscoelastic *A63-24859* Material." Fracture of Solids, Drucker, D. C. & Gilman, J. J. eds.; Intersci. pp. 157-188, Publ. 1963.

- [3] 36. Williams, M. L.: "Initiation and Growth of Viscoelastic Fracture." *Int'l J. Fract. Mech.*, Vol. 1, No. 4, Dec. 1965, pp. 292-310. *A66-32304*
37. Mueller, H. K.: "Research Summary on Fracture of Swollen Rubber." 1966. (GALCIT SM 66-15).
38. Wnuk, M. P., Knauss, W. G.: "Delayed Fracture in Viscoelastic-Plastic Solids." *Int'l J. Solid Struct.*, Vol. 6, 1970, pp. 995-1009.
39. Mueller, H. K.: "Stable Crack Propagation in a Viscoelastic Strip." Ph.D. Thesis, 1968, Caltech. (GALCIT SM 68-11).
- [10] 40. Knauss, W. G.: "Quasi-Elastic Failure Analysis of a Solid Propellant Rocket Motor under Thermal Cooling." 1968. *AIAA Jour.*, Vol. 6, No. 11, Nov. 1968, pp. 2101-2106. *A68-44715*
- [11] 41. Knauss, W. G., Hufferd, W.: "Stresses in a Long High Mass Fraction Solid Propellant Rocket Motor." 1968. *AIAA Jour.*, Vol. 6, No. 11, Nov. 1968, pp. 2200-2202. (GALCIT SM 68-12). *A68-44739*
- [12] 42. Knauss, W. G.: "Stable and Unstable Crack Growth in Viscoelastic Media." *Transact. Soc. Rheol.*, Vol. 13, No. 3, 1969, pp. 291-313.
- [13] 43. Mueller, H. K., Knauss, W. G.: "The Fracture Energy and Some Mechanical Properties of a Polyurethane Elastomer." *Trans. Soc. Rheol.*, Vol. 15, No. 2, 1971, pp. 217-233.
- [14] 44. Mueller, H. K.: "Stress Intensity Factor and Crack Opening for a Linearly Viscoelastic Strip with Slowly Propagating Central Crack." *Int'l J. Fract. Mech.*, Vol. 7, No. 2, 1971, pp. 129-141.
- [15] 45. Mueller, H. K., Knauss, W. G.: "Crack Propagation in a Linearly Viscoelastic Strip." *J. A. M.*, Vol. 38, Series E, 1971, pp. 483-488. *A70-42937*
- [16] 46. Knauss, W. G.: "Delayed Failure--The Griffith Problem for Linearly Viscoelastic Materials." *Int'l Jour. Fract. Mech.*, Vol. 6, 1970, pp. 7-20. *A70-28671*
47. Mueller, H. K.: "Response to Uniaxial Tension in the Swollen State and Some Network Characteristics of Various Compositions of Solithane 113." 1969. (GALCIT SM 69-6).

- [17] 48. Knauss, W. G., Dietmann, H.: "Crack Propagation under Variable Load Histories in Linearly Viscoelastic Solids." Int'l J. Engrg. Sci., Vol. 8, 1970, pp. 643-656.
49. Palaniswamy, K.: "Crack Propagation under General In-Plane Loading." Ph.D. Thesis, CIT 1971. (GALCIT SM 71-5).
- [18] 50. Palaniswamy, K., Knauss, W. G.: "Propagation of a Crack under General In-Plane Tension." Int'l J. Fract. Mech., Vol. 8, 1972, pp. 114-117.
- [19] 51. Knauss, W. G.: "An Observation of Crack Propagation in Anti-Plane Shear." Int. J. Fract. Mech., Vol. 6, No. 2, June 1970, pp. 183-187.
- [20] 52. Knauss, W. G.: "Fracture Mechanics and the Time-Dependent Strength of Adhesive Joints." J. Comp. Mat'ls, Vol. 5, April 1971, pp. 176-192.
- [21] 53. Knauss, W. G.: "Mechanics of Fracture in Polymeric Solids." Proc. 3rd Int. Conf. on Fracture. Munich, Germany, April 1973.
- [22] 54. Knauss, W. G.: "On the Steady Propagation of a Crack in a Viscoelastic Sheet: Experiments and Analysis. Deformation and Fracture of High Polymers," Kausch, Hassel, Jaffee, eds., Plenum Press, 1972.
- [23] 55. Knauss, W. G.: "The Mechanics of Polymer Fracture." Lead article in Appl. Mech. Reviews, January 1973.

There appeared about 20 progress reports (GALCIT SM Reports) which are not listed here.

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