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ABSTRACT

In this paper the optimal Bayes detector for a general periodic waveform having uniform delay and additive white Gaussian noise is examined. It is shown that the detector is much more complex than that for the well known cases of pure sine waves (i.e. classical noncoherent detection) and narrowband signals. An interpretation of the optimal processing is presented, and several implementations are discussed. The results have application to the noncoherent detection of optical square waves.

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Introduction

Various modulation techniques are presently under study for communicating digital information over an optical channel. The most common method is by the use of pulse position modulation (PPM) in which digital words are transmitted as narrow optical pulses properly located within a data frame. Such systems however are hampered by the requirement to maintain a close tolerance on timing and synchronization in order to perform detection over the narrow pulses. An alternative encoding scheme that avoids the short pulse timing problem is by the use of coded frequency division modulation (FDM). In this case information is sent as frequencies, rather than pulse positions, and the synchronization problem is relaxed. One possible implementation scheme is to transmit the digital words as bursts of square waves of different frequencies, where the length of the square wave is selected to generate sufficient energy levels for detection. The encoded square wave is used to intensity modulate the optical beam. (A square wave is used rather than a sin wave because it has maximum baseband energy in a finite time for a fixed power contraint on the optical transmitter.) Following direct (non-coherent) optical detection in the photo detector the subcarrier square wave is detected (a decision is made as to which square wave frequency is being received) in order to decode the digital word. The timing need be maintained only to within the length of the square wave signal, which is many times the length of an optical pulse in a PPM system.

It is desired to implement the optimal detector for the set of square waves. Although the bit timing problem has been considerably reduced, there still exists a time referencing problem, since the square waves will be received with random delays. Hence, coherent correlation techniques cannot be used, and the optimal noncoherent FDM square wave detector is required. Unfortunately, noncoherent detectors for waveforms that are not narrowband are not known, even for the classical additive Gaussian noise channel. In this report we present the results of an initial study to derive the optical noncoherent detector for an arbitrary periodic waveform not necessarily of the narrowband type; e.g., square waves. Attention is confined to only an additive Gaussian noise channel. The latter model is valid in an optical system when strong optical fields are detected. Future work will extend the results to the low power optical (poisson) channel.

Analysis

Classical non-coherent detection is generally understood to be the detection of a sin wave with random phase or time delay in additive gaussian noise. The problem is well documented in communication texts, and the Bayes optimal detector has been derived as both a matched envelope detector and a quadrature correlator-squaring device. These results have been expanded to include narrowband bandpass signals as well [1]. However, the extension to a general non-coherent problem involving the detection of an arbitrary periodic signal with random time delay has received little attention. Closest documentation appears in the radar literature where the problem is formulated as non-coherent detection of periodic RF pulses [2], but in all cases the narrowband assumption is imposed in order to derive an interpretable solution. Admittedly, the general non-coherent problem may not be of great practical interest because of the bandwidths required to transmit all harmonics. Also, perhaps, the complexity of the general solution may have discouraged academic pursuit. Nonetheless, in this paper the general non-coherent problem is re-examined with the objective of interpreting the processing required by the optimal detector.

Let p(t) be a general periodic, deterministic signal having period t_0 and bounded energy. The signal is observed for T seconds with a random delay τ in the presence of additive white gaussian noise r(t). The observation time T will be taken as an integer multiple of t_0 for convenience, although our results become an accurate approximation if $T >> t_0$. The observable can therefore be written

$$v(t) = p(t-\tau) + n(t) \qquad t\varepsilon(0, T) \tag{1}$$

For the non-coherent problem we assume τ is uniformly distributed over $(0, t_0)$. The optimal (Bayes) detector for the signal is desired. Mathematically, the Bayes detector is that which computes the generalized likelihood ratio Λ obtained by averaging over τ. For the observable of (1) this becomes

$$\Lambda = C\int_0^{t_0} \exp\left[\frac{2}{N_0}\int_0^T v(t)p(t-\tau)dt\right]d\tau \tag{2}$$

where N_0 is the one-sided noise level and C depends upon v(t) but not on τ. Since C can be computed without use of p(t) it is brought along simply as a constant in subsequent equations. This property of C also requires our assumption concerning the relation of observation time and signal period. Since p(t) is periodic, it admits a Fourier expansion which allows its delayed version to be written as

$$p(t-\tau) = \sum_{k=0}^{\infty} a_k \sin\left[k\left(\frac{2\pi}{t_0}\right)t + \psi_k - k\theta\right] \tag{3}$$

where (a_k, ψ_k) are the harmonic amplitudes and phases of p(t), and $\theta \overset{\Delta}{=} 2\pi\tau/t_0$ is the uniformly distributed phase variable over $(0, 2\pi)$. The delay τ therefore introduces a random phase to each harmonic of p(t), but note that these phases are related as rational multiples of each other. Using (3) in (2), and manipulating trigonometrically, yields

$$\Lambda = C\int_0^{2\pi} \exp\left[\sum_{k=0}^{\infty} X_k \cos k\theta + Y_k \sin k\theta\right] d\theta$$

$$= C\int_0^{2\pi} \exp\left[\sum_{k=0}^{\infty} E_k \cos(k\theta + \varphi_k)\right] d\theta \qquad (4)$$

where

$$X_k = \frac{2a_k}{N_0}\int_0^T v(t)\cos\left[k\left(\frac{2\pi}{t_0}\right)t + \psi_k\right] dt \qquad (5a)$$

$$Y_k = \frac{2a_k}{N_0}\int_0^T v(t)\sin\left[k\left(\frac{2\pi}{t_0}\right)t + \psi_k\right] dt \qquad (5b)$$

$$E_k = [X_k^2 + Y_k^2]^{\frac{1}{2}} \qquad (5c)$$

$$\varphi_k = \tan^{-1}[Y_k/X_k] \qquad (5d)$$

Here (X_k, Y_k) are the in phase and quadrature harmonic correlations, and (E_k, φ_k) are the corresponding harmonic envelope and phase variables. Unfortunately, (4) does not appear to integrate to an immediately obvious system implementation. In particular, it does not collapse down to a simple in phase and quadrature correlation with p(t) and $p[t-(t_0/2)]$, as might be conjectured from the well known bandpass case. The latter correlator would develop only if sin θ or cos θ terms factored out of every term in the exponent of (4). That this factorization does not occur in general is simply a reiteration of the fact that a single sin wave is the only periodic function satisfying the condition that shifted versions of itself are always uniquely decomposable into in phase and quadrature components.

Nevertheless, several analytical procedures are possible to reduce (4). One is to define the random variable

$$z(\theta) \triangleq \sum_{k=0}^{\infty} E_k \cos(k\theta + \varphi_k) \tag{6}$$

and to note that Λ/C is the characteristic function of z evaluated at $j\omega = 1$. Unfortunately, z is a sum of dependent random sin variables, and its probability density is not easily computed. A more fruitable procedure is to derive an infinite series solution by using the expansion

$$e^{\alpha \cos \beta} = \sum_{m=0}^{\infty} \varepsilon_m I_m(\alpha) \cos m\beta \tag{7}$$

where ε_m is the Nueman parameter and $I_m(\alpha)$ is the m^{th} order imaginary Bessel function. When used in (4), the latter expands to

$$\Lambda = C \sum_{\underline{m}} \prod_i \varepsilon_{m_i} I_{m_i}(E_i) \int_0^{2\pi} \cos\left[\sum_{\underline{m}} m_i i\theta + m_i \varphi_i\right] d\theta \tag{8}$$

where $\underline{m} \triangleq \{m_1, m_2, \ldots\}$ is the vector of integer coefficients m_i, $m_i \varepsilon (-\infty, \infty)$. Each vector $\underline{m}$ produces a different harmonic in the integrand. However, each such harmonic will integrate to zero in (8), except for those in which

$$\sum_{i=0}^{\infty} i m_i = 0 \tag{9}$$

This reduces (8) to

$$\Lambda = C \sum_{\underline{m}(0)} \prod_{i} \epsilon_{m_i} I_{|m_i|}(E_i) \cos \left[\sum_{\underline{m}(0)} m_i \varphi_i \right] \tag{10}$$

where $\underline{m}(0)$ is the set of integer vectors whose components satisfy (9). The optimal detector therefore involves a search and summation over an infinite number of integer vectors. Note that the detector makes use of the envelope of each harmonic of p(t), but processes it in a rather complicated way. At this point, all that can be concluded is that the general detector involves a bank of matched envelope detectors producing $\{E_i\}$ and $\{\varphi_i\}$, followed by a complicated computer processor that instantaneously computes (10). Furthermore, the Bessel functions must be evaluated, unless one appeals to high and low signal-to-noise ratio arguments to substitute limiting forms.

Let us examine the implications of (10). Theoretically, one wonders why the optimal detector utilizes such complex processing for detection. If the harmonic random phase angles in (3) had been statistically independent of each other (i.e., $\{k\theta\}$ replaced by $\{\theta_k\}$, where the latter is an independent, uniform sequence) then the Λ obtained by averaging over the sequence of phase angles would be

$$\Lambda = C \prod_{i=1}^{\infty} I_0(E_i) \tag{11}$$

as previously reported [3]. We see that this is one term of the sum in (10). Thus the remaining terms of the sum must be takirg advantage of the integer phase relation between the random phase angles. From a practical point of view, one may also inquire if any type of physically realizable system can produce (10), precluding the use of infinitely fast computers.

A partial answer to those inquiries can be obtained by noting that (10) is reminescent of the intermodulation terms arising when a sum of carriers is passed through a nonlinearity [4]. In fact, (10) is proportional to the average, or "d.c.", value of the output of the nonlinearity e^x when impressed with the input

$$x(t) \triangleq \sum_{n=0}^{\infty} E_n \cos(nt + \varphi_n) \tag{12}$$

That is, if $y(t) \triangleq C \exp[x(t)]$, then since x(t) in (12) is periodic with periodic 2π,

$$\begin{bmatrix}\text{Time average}\\ \text{of } y(t)\end{bmatrix} = \lim_{T\to\infty} \frac{C}{2T}\int_{-T}^{T} \exp[x(t)]dt = C\int_{0}^{2\pi} \exp[x(t)]dt \tag{13}$$

which is identical to the desired Λ in (4). The terms in (10) involve precisely those output harmonic terms that contribute (beat down) to this average value. The optimal processing implied is therefore used to take advantage of the phase relation among the harmonics, making use of all beat frequencies that contain useful information for detection. In the independent phase case of (11), the harmonics are not phase related and the available beat frequencies do not aid detection, on the average. Hence, only the zero order component is used. Note that the processing is not simply angle shifting each harmonic of p(t) so as to overlap in time, but instead using the nonlinearity to intentionally generate all possible beat frequencies that cause harmonic overlap.

Equation (13) also suggests a method of implementation. The receiver must generate (10), then pass it through the nonlinearity e^x, followed by averaging (low pass filtering), as shown in Figure 1. The processor generating x(t) involves determination of $\{X_n, Y_n\}$ from v(t), according to (5), then adjusting the amplitude and phase of harmonically locked oscillators, as shown in Figure 2. The computation of X_n and Y_n involve in phase and quadrature harmonic correlation over the T sec observation inverval. The overall processor would then be a bank of such harmonic subsystems, one for each signal harmoni c. Since the averaging implied in (13) must be done after these correlations, Figure 1 may be interpreted as a non-real time implementation. The processor in Figure 1 can also be interpreted by comparing (12) to (6), and noting that

$$x(t) = z(\theta)\Big|_{\theta=t} \tag{14}$$

However, $z(\theta)$ is also the exponent in (2), with $\tau = t_0\theta/2\pi$. Thus

$$x(t) = \frac{2}{N_0}\int_0^T v(\rho)p\left[\rho - \left(t/\frac{2\pi}{t_0}\right)\right]d\rho \tag{15}$$

When written as above, the processor output x(t) is the output of a filter at the normalized time $t(t_0/2\pi)$, when the input is v(t) and the filter impulse response is p(-t), $(t\epsilon 0, T)$. This is simply a matched filter for the periodic signal p(t), but the filter is non-causal since p(t) is not zero for negative t. [The non-causality is indicative of the fact that all the observable over (0, T) is used to generate x(t) at any t within (0, T).] The non-causality implies

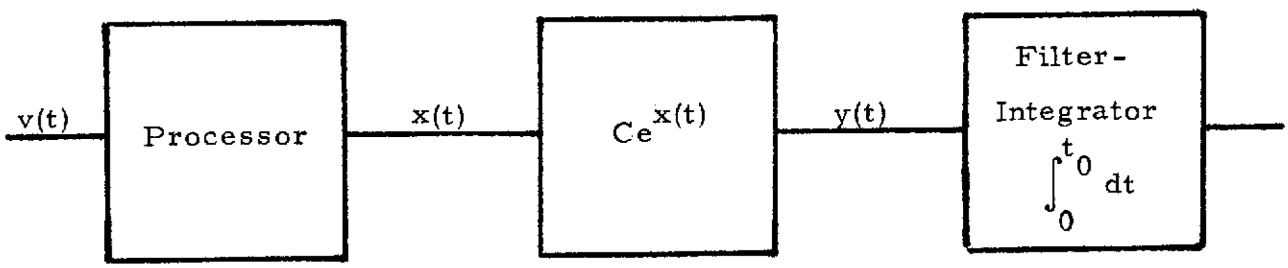

Figure 1.

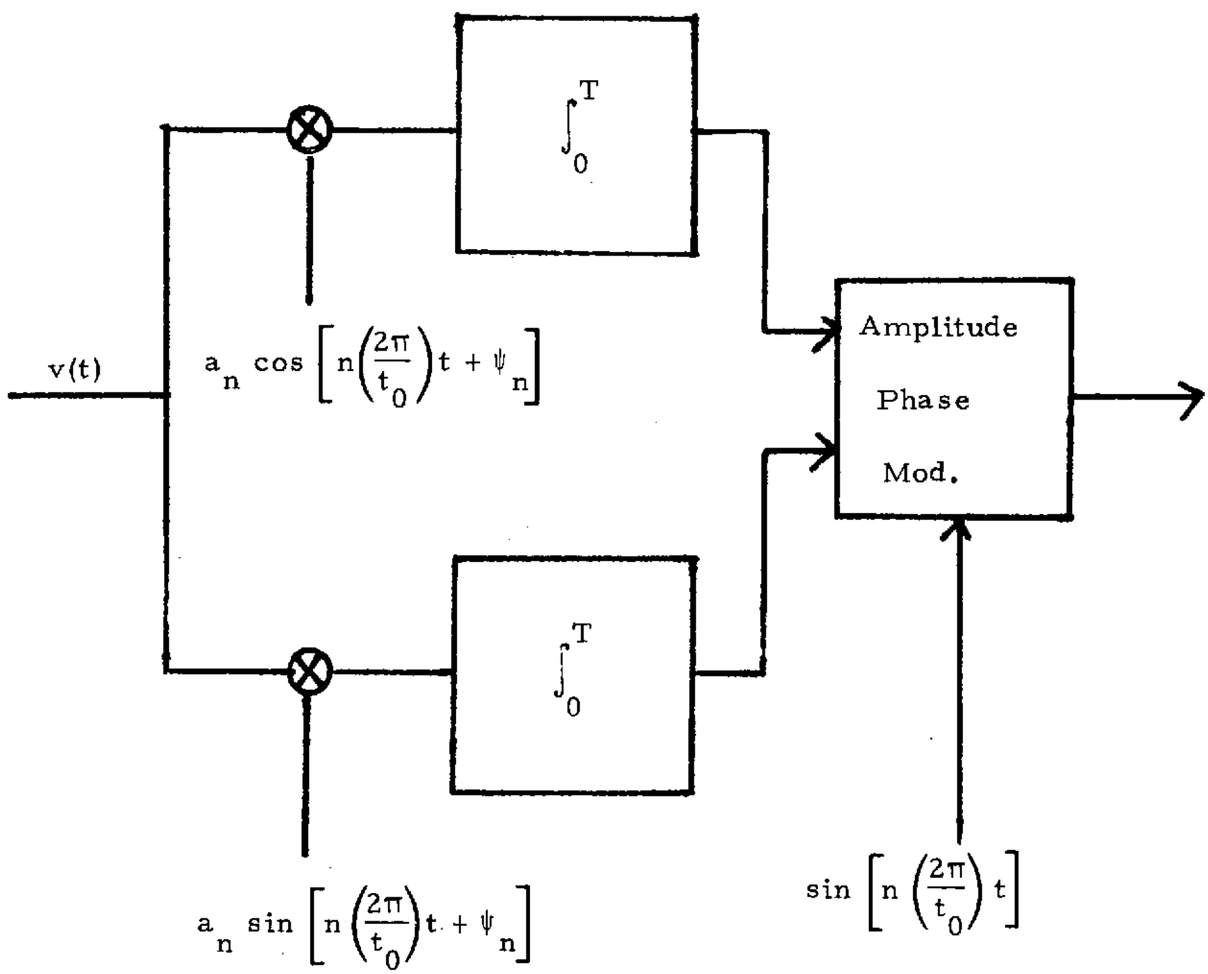
v(t)
$\int_0^T$
$\int_0^T$
$a_n \cos\left[n\left(\frac{2\pi}{t_0}\right)t + \psi_n\right]$
Amplitude
Phase
Mod.
$a_n \sin\left[n\left(\frac{2\pi}{t_0}\right)t + \psi_n\right]$
$\sin\left[n\left(\frac{2\pi}{t_0}\right)t\right]$

Figure 2.

again the non-real time implementation required for Figure 1. It is interesting that a particular non-linearity (exponential) is specified by the Bayes detector.

The extension to non-uniform densities on the delay τ can be easily accounted for in Figure 1. A non-uniform density, $\sigma(\theta)$, in the integrand of (4) would convert to a correlation rather than an integration in (13). The detector in this case would simply replace the low pass filter following the non-linearity by a correlator of y(t) and $\sigma(t)$ over the 2π sec interval. The receiver would therefore be required to locally generate this probability density as a function of t.

It may be of interest to further examine why in phase-quadrature (I-Q) correlation is not the optimal processor. The I-Q detector for an arbitrary periodic p(t) is shown in Figure 3. The input v(t) is simultaneously correlated for T sec with p(t) and $p(t-t_0/2)$, and the outputs are squared and summed. Consider the behavior of the system when only the signal portion of v(t) [i.e. $p(t-\tau)$] is impressed at the input. The output of the in phase correlator is

$$X = \int_0^T p(t-\tau)p(t)dt$$

$$= TR_{pp}(\tau) \tag{16}$$

where $R_{pp}(\tau)$ is the correlation function of p(t) evaluated at the point τ. Similarly, the quadrature correlator produces

$$Y = \int_0^T p(t-\tau)\hat{p}(t)dt$$

$$= TR_{p\hat{p}}(\tau) \tag{17}$$

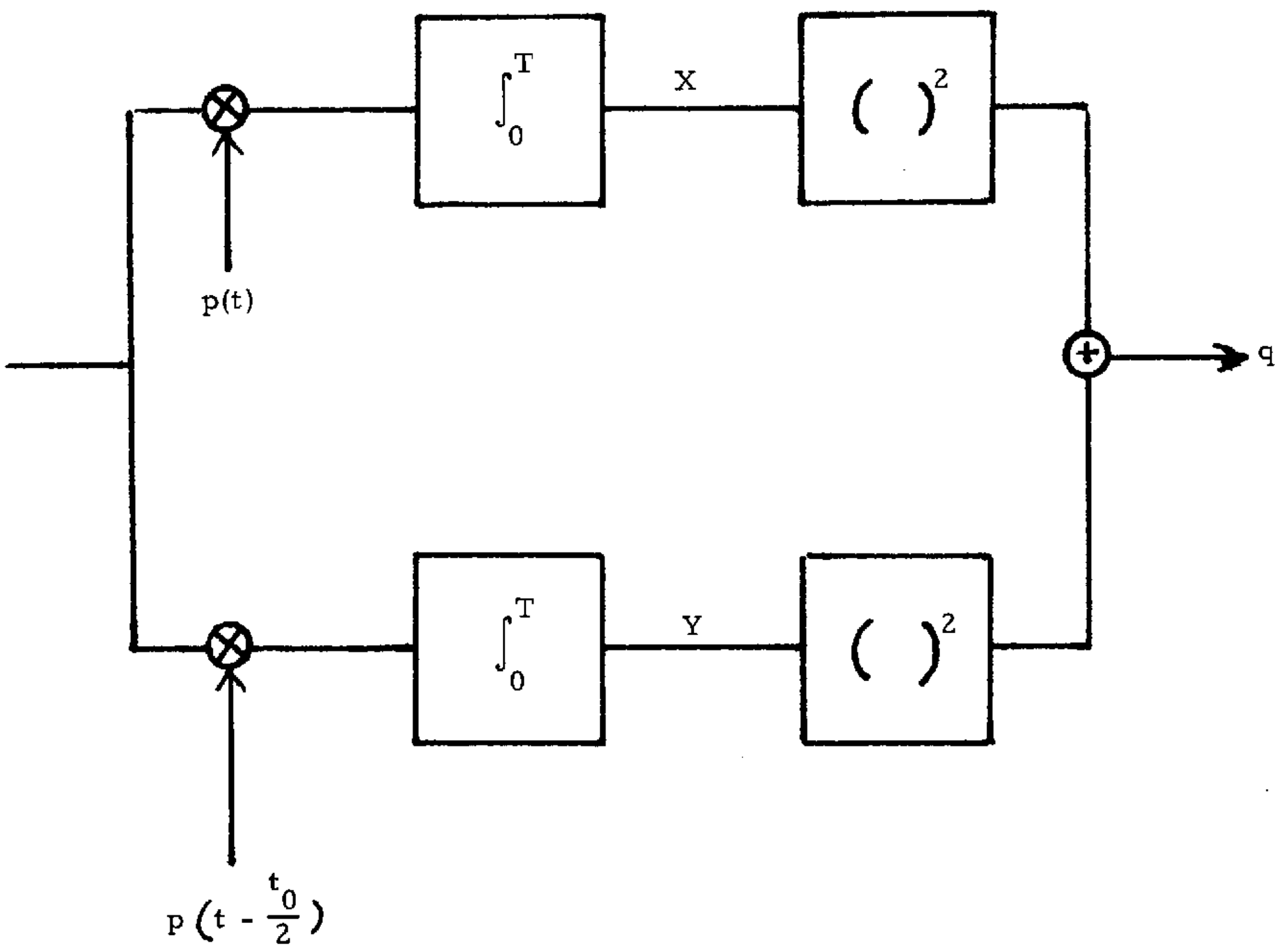

Figure 3.

where $\hat{p}(t)$ is the shifted version of p(t). Since p(t) is periodic, $\hat{p}(t)$ is also the Hilbert transform of p(t). From a well known property of such transforms [5]

$$R_{p\hat{p}}(\tau) = \hat{R}_{pp}(\tau) \tag{18}$$

Defining the complex correlation pre-envelope process $\Omega(\tau) = R_{pp}(\tau) + j\hat{R}_{pp}(\tau)$ allows us to express the I-Q correlator output as

$$\begin{aligned} q &\triangleq R^2_{pp}(\tau) + R^2_{p\hat{p}}(\tau) \\ &= |\Omega(\tau)|^2 \end{aligned} \tag{19}$$

Since $\Omega(\tau)$ is a pre-envelope process, its magnitude equals $\sqrt{2}$ times the magnitude of its real part [1, p.80]. Hence, we write q in (19) as

$$q = 2|R_{pp}(\tau)|^2 \tag{20}$$

Thus, in the noiseless case the I-Q detector always produces an output equivalent to sampling the squared correlation envelope at the delay τ. Since this τ is random it would be expected that a useful detection system should not depend on τ. The output of the I-Q detector will not depend on τ only if the envelope of the correlation function of p(t) does not depend on τ. For a pure sin wave the correlation function is a cosine wave and its envelope is indeed constant. For a narrowband bandpass p(t) the envelope is approximately constant over the range of τ [i.e., $\tau \in (0, t_0)$ and $t_0 \ll$ envelope variations]. For both of these examples the I-Q detector is in fact optimal.

However, for the general periodic function, q in (19) will depend on τ, and I-Q correlation is not a plausible detector.

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