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M. L. GOLDSTEIN
A. J. KLIMAS
G. SANDRI

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GODDARD SPACE FLIGHT CENTER
GREENBELT, MARYLAND

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(Telephone 301-982-4488)

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MIRRORING WITHIN THE FOKKER-PLANCK FORMULATION OF COSMIC RAY PITCH ANGLE SCATTERING IN HOMOGENEOUS MAGNETIC TURBULENCE

by

M. L. Goldstein* and A. J. Klimas**
Laboratory for Extraterrestrial Physics
NASA-Goddard Space Flight Center
Greenbelt, Maryland 20771
G. Sandri
Aeronautical Research Associates of Princeton, Inc.
Princeton, New Jersey 08540

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*NAS-NRC Postdoctoral Resident Research Associate
**NAS-NRC Senior Postdoctoral Resident Research Associate

Abstract

The Fokker-Planck coefficient for pitch angle scattering, appropriate for cosmic rays in homogeneous, stationary magnetic turbulence, is computed from first principles. No assumptions are made concerning any special statistical symmetries the random field may have. This result can be used to compute the parallel diffusion coefficient for high energy cosmic rays moving in strong turbulence, or low energy cosmic rays moving in weak turbulence. Because of the generality of the magnetic turbulence which is allowed in this calculation, special interplanetary magnetic field features such as discontinuities, or particular wave modes, can be included rigorously. The reduction of this result to previously available expressions for the pitch angle scattering coefficient in random field models with special symmetries is discussed.

The general existance of a Dirac delta function in the pitch angle scattering coefficient is demonstrated. It is proved in this paper that this delta function is the Fokker-Planck prediction for pitch angle scattering due to mirroring in the magnetic field. The conditions under which this delta function contributes to pitch angle scattering are determined, and shown to be identical to the conditions under which first order mirroring occurs in the random field. These conditions are generally fulfilled in interplanetary and probably interstellar space. The implications of the delta function for the validity of the Fokker-Planck approximation are discussed.

I. Introduction

The parallel diffusion coefficient for low energy cosmic rays in a random magnetic field has often been calculated from the small gyro-radius (or guiding center) approximation to the Fokker-Planck pitch angle scattering coefficient (Jokipii, 1966, 1967, 1968, 1971; Hall and Sturrock, 1967; Hasselmann and Wibberenz, 1968). But Klimas and Sandri (1973b), in a numerical calculation using the exact pitch angle scattering coefficient, have computed a parallel diffusion coefficient which differed markedly from that predicted using the small gyro-radius approximation. This calculation was limited to the special case of statistically isotropic magnetic turbulence with a Gaussian correlation function (and thus, a Gaussian power spectrum). Because of this limitation, the significance of this discrepancy was not fully appreciated because 1) in any case the small gyro-radius approximation was not expected to be accurate for Gaussian power spectra, (Jokipii, 1971) but was still proposed as an accurate approximation for the more familiar power-law, power spectra, and 2) the source of the discrepancy, and especially its physical interpretation, was not discernable in the numerical computation.

Recently, Fisk et al. (1974), and Klimas and Sandri (1973c), still within the framework of isotropic turbulence, but for power-law power spectra, calculated the pitch angle scattering coefficient without making the small gyro-radius approximation. They found that the exact result differed significantly from the approximate one, especially for pitch angles, θ (measured relative to the mean field) near 90° where the Fokker-Planck coefficient contains a Dirac delta-function in $\mu = \cos \theta$. The discrepancy found by Klimas and Sandri was then completely explained. For low energy cosmic rays in the Gaussian power spectrum, the delta function dominated all other contributions to the parallel diffusion coefficient. Furthermore, Klimas and Sandri (1973c) in calculating the parallel diffusion coefficient, showed that for power-law spectra, with any reasonable spectral index, the contribution of the delta function still dominated the contribution of the small gyro-radius approximation. For these spectra, Fisk et al. (1974) through a numerical calculation found that for $\mu \neq 0$, the pitch angle scattering coefficient is overestimated by the small gyro-radius approximation. Thus, the delta function is essentially the sole contributor to the Fokker-Planck estimate of the parallel diffusion coefficient in realistic models of the interplanetary magnetic field. A physical interpretation of the delta function, as well as a re-examination of the validity of the entire Fokker-Planck procedure in light of its existence, becomes necessary.

Concurrent with the above developments, a number of papers (Jones, Birmingham, and Kaiser, 1973; Jones, Kaiser, and Birmingham, 1973; Kaiser, 1973; Jones and Birmingham, 1974; Volk, 1973; Volk, Morfill, Alpers and Lee, 1974) were presented in which it was argued that the Fokker-Planck formalism fails to correctly describe pitch angle scattering near $\theta = 90^{\circ}$ ($\mu = 0$). For mathematical simplicity, these papers have invariably discussed an idealized model of the random magnetic field in which the $\delta(\mu)$ does not appear. We will demonstrate in this paper that the delta function is the rule, rather than the exception, and, therefore, must be taken into account in studying the apparent failure of the Fokker-Planck formalism near $\mu = 0$.

In this paper we derive the Fokker-Planck pitch angle scattering coefficient with no restrictions on the statistical behavior of the random magnetic field, other than the assumption that it is homogeneous and stationary over several correlation lengths. Our calculation is based on the Liouville equation for the cosmic ray distribution function. We apply the quasi-linear and adiabatic approximations which, together, give us a Fokker-Planck equation for the <u>mean</u> cosmic ray distribution function. The mean distribution function is assumed gyrotropic in order to focus our attention on pitch angle scattering. This derivation is done in a way which clearly demonstrates the existence of the $\delta(\mu)$ contribution to pitch angle scattering. But, the generality of our derivation enables us to study the conditions under which the strength of the delta function is zero or non-zero. In this way, we are able to determine that the delta function part of the pitch angle scattering coefficient is the Fokker-Planck prediction for the contribution of mirroring (Alfven and Falthammar, 1963; Northrop, 1963) to pitch angle scattering. It has been thought that the effects of mirroring were not contained, a priori, in the Fokker-Planck formalism, but had to be developed separately. Work along these lines has been done by Noerdlinger (1968), Quenby, et al. (1970), Cesarsky and Kulsrud (1972), and Jokipii (1973). In this last reference, Jokipii concludes that mirroring is not a significant pitch-angle scattering mechanism for an isotropic particle distribution. We show below, at least for isotropic magnetic turbulence, that the effects of mirroring dominate the spatial diffusion of low energy cosmic rays within the Fokker-Planck formalism.

In general, the interplanetary or interstellar magnetic field will have fluctuations in field magnitude which will produce particle mirroring to first order in the weak random field strength. For the isotropic models of the random magnetic field discussed above, the Fokker-Planck formalism predicts that mirroring is the most important pitch angle scattering mechanism for the parallel diffusion of cosmic rays. On the other hand, the existence of the delta function makes the Fokker-Planck formalism suspect. We feel that the isotropic field model calculation indicates that mirroring cannot be neglected, nor even included in a casual manner. In particular, further efforts at constructing a correct description of the behavior of those cosmic ray particles with pitch angles near 90° should probably not be based on idealized models of the random magnetic field which do not allow first order mirroring.

II. Pitch-Angle Scattering Coefficient: Arbitrary Homogeneous Turbulence

We have been able to construct the Fokker-Planck pitch angle scattering coefficient using two independent methods. The first method, which we will present here, follows directly from first principles; i.e., the Liouville equation for the cosmic ray particle distribution function. The second method, outlined in Appendix A, is based on the standard Fokker-Planck approach (Chandrasekhar, 1943; Jokipii, 1971; Hasselman and Wibberenz, 1968). Although these methods are similar in some respects, they also contain surprisingly disimilar reasoning. The results of both methods are identical.

Klimas and Sandri (1973a) have shown that, starting from the Liouville equation for the cosmic ray distribution function, the following _truncated master equation_ for the _mean_ cosmic ray distribution function can be constructed:

$$\frac{\partial f}{\partial \tau} + \alpha \mathcal{H} f + \epsilon \mathcal{L} f = (\epsilon')^2 \langle \mathcal{L}' G_o \mathcal{L}' \rangle f \tag{1}$$

The right hand side of this equation represents the leading significant term in an infinite series expansion in the small parameter, ϵ'. By, truncating this expansion at this point, we, in effect, make the familiar quasi-linear approximation. As a consequence of this truncation, we approximate the actual trajectory of a particle in the magnetic field, by the helical trajectory it would have in the mean magnetic field. We assume this helical trajectory for how ever long it takes the particle to travel approximately one correlation length along the mean field. Particles with pitch angles near 90^o may take a long time to move this distance, however, and in this time the assumed helical approximation to the actual particle motion becomes suspect. It is this point which has lead to recent modifications of the quasi-linear approximation in the region of particle phase-space near, $\mu = 0$. Our point of view, here, is to determine the actual predictions of the quasi-linear theory, which we may then use, in confidence, to further investigate the apparent failure of this theory and its possible modifications.

Equation (1) has been written in a dimensionless form. The time variable is given by, $\tau = t\ (v/\lambda_c)$, where t is the dimensional time, v is the particle speed, and λ_c is the correlation length in the random field. The term $\alpha \mathcal{H} f$, is non-zero only if the distribution function depends on spatial position. Our purpose, here, is to fully investigate the pitch angle scattering coefficient that this theory predicts. Consequently, we can drop the term, $\alpha \mathcal{H} f$, by assuming f independent of position.

The parameters, ϵ and ϵ' are defined by,

$$\epsilon = \frac{\lambda_c \langle B \rangle}{P} \qquad \epsilon' = \epsilon\eta = \epsilon\left(\frac{B'_{rms}}{\langle B \rangle}\right) \tag{2}$$

where $\langle B \rangle$ and B'_{rms} are, the mean and the rms magnetic field strengths respectively, and where P is the particle rigidity. We note that Klimas and Sandri (1973a) have given a rescaling of equation (1) which is necessary when $\epsilon \geq 1$. The version given here is appropriate when $\epsilon \leq 1$. However, the results of the calculation we will present here can be applied to any range in ϵ, so long as we remember that we must have $\epsilon' \ll 1$ when $\epsilon \lesssim 1$, but when $\epsilon \geq 1$, we must have $\eta \ll 1$ instead.

The differential operators, $\mathcal{L}$ and $\mathcal{L}'$, generate the effects on the distribution function of the Lorentz force on a charged particle in the mean and random components of the magnetic field, respectively. Convenient representations of $\mathcal{L}$ and $\mathcal{L}'$, can be given in terms of the spherical coordinate system variables, θ, the polar, or pitch angle between the particle momentum and the direction of the mean magnetic field, and, ϕ, the azimuth, or phase angle of the momentum vector. With the definition of $\mu \equiv \cos\theta$, we are able to express these operators as,

$$\mathcal{L} = -\frac{\partial}{\partial\phi} \tag{3}$$

and,

$$\mathcal{L}' = \frac{\partial}{\partial\mu}\left(\hat{\underset{\sim}{p}}\cdot\underset{\sim}{\Omega}\cdot\underset{\sim}{\beta}'\right) - \frac{\partial}{\partial\phi}\left(\underset{\sim}{\beta}'\cdot\underset{\sim}{n}\cdot\hat{\underset{\sim}{\beta}}\right)(1-\mu^2)^{-1} \tag{4a}$$

or,

$$\mathcal{L}' = -\left(\underset{\sim}{\beta}'\cdot\underset{\sim}{\Omega}\cdot\hat{\underset{\sim}{p}}\right)\frac{\partial}{\partial\mu} + (1-\mu^2)^{-1}\left(\underset{\sim}{\beta}'\cdot\underset{\sim}{n}\cdot\hat{\underset{\sim}{\beta}}\right)\frac{\partial}{\partial\phi},$$

where, $\hat{\underset{\sim}{p}}$ is a unit vector in the momentum direction, $\hat{\beta}$ is a unit vector in the direction of the mean field, $\underset{\sim}{\beta}' = B'/B'_{rms}$, and the tensors,

$\underset{\sim}{\Omega}$ and $\underset{\sim}{n}$ are defined by,

$$\Omega_{ij} = \epsilon_{ijk}\,\beta_k, \tag{5}$$

and,

$$n_{ij} = \delta_{ij} - \frac{p_i p_j}{p^2} \tag{6}$$

A full description of the integral operatior, G_o, has been given by Klimas and Sandri (1973a). This operator, when operating on an arbitrary function of position, momentum, and time, operates as follows:

$$GA(\underset{\sim}{x},\underset{\sim}{p},\tau) = \int_o^\tau d\lambda \; \mathcal{J}_\epsilon(\epsilon,\lambda) \; A\;(\underset{\sim}{x},\underset{\sim}{p},\tau-\lambda) \tag{7a}$$

$$= \int_o^\tau d\lambda \;\; A\;(\underset{\sim}{x}(\lambda),\; \underset{\sim}{p}(\lambda),\; \tau-\lambda) \tag{7b}$$

The streaming operator, $\mathcal{J}_\epsilon(\epsilon,\lambda)$, in equation (7a), shifts the phase space position $(\underset{\sim}{x},\underset{\sim}{p})$ to $(\underset{\sim}{x}(\lambda),\ \underset{\sim}{p}(\lambda))$ in equation (7b) which is along the helical particle trajectory in the mean magnetic field with $(\underset{\sim}{x},\ \underset{\sim}{p})$ for the starting point; i.e.,

$$\underset{\sim}{x}(\lambda) = \underset{\sim}{x} - \underset{\sim}{r}(\lambda) \tag{8}$$

where

$$r(\lambda) = \underset{\sim}{\mathfrak{D}}(\epsilon,\lambda) \cdot \hat{\underset{\sim}{p}} \tag{9}$$

and

$$\hat{\underset{\sim}{p}}(\lambda) = \underset{\sim}{\mathfrak{C}}(\epsilon,\lambda) \cdot \hat{\underset{\sim}{p}} \tag{10}$$

where

$$\underset{\sim}{\mathfrak{C}}\;(\epsilon,\lambda) = \underset{\sim}{P} + \underset{\sim}{N}\cos\epsilon\lambda - \underset{\sim}{\Omega}\sin\epsilon\lambda \tag{11}$$

and,

$$\underset{\sim}{\mathfrak{D}}\;(\epsilon,\lambda) = \underset{\sim}{P}\lambda + \frac{1}{\epsilon}\left[\underset{\sim}{N}\sin\epsilon\lambda + \underset{\sim}{\Omega}(\cos\epsilon\lambda - 1)\right] \tag{12}$$

The skew-symmetric tensor, $\underset{\sim}{\Omega}$, is defined above in (5) and the parallel and normal projection operators, $\underset{\sim}{P}$ and $\underset{\sim}{N}$, are defined through,

$$P_{ij} = \beta_i \, \beta_j \tag{13}$$

and

$$N_{ij} = \delta_{ij} - P_{ij} \tag{14}$$

Now, we can rewrite equation (1) as,

$$\frac{\partial f}{\partial \tau} - \tilde{\varepsilon}\,\frac{\partial f}{\partial \phi} = (\varepsilon')^2 < \Big[\, \frac{\partial}{\partial \mu}\,(\hat{\boldsymbol{p}}\cdot\boldsymbol{\Omega}\cdot\boldsymbol{\beta}') - \frac{\partial}{\partial \phi}\,(\boldsymbol{\beta}'\cdot\boldsymbol{n}\cdot\hat{\boldsymbol{\beta}})\;(1-\mu^2)^{-1} \Big] \cdot$$

$$\cdot \int_0^{\tau} d\lambda\, \mathcal{G}_o(\varepsilon,\lambda)\;\Big[-(\boldsymbol{\beta}'\cdot\boldsymbol{\Omega}\cdot\hat{\boldsymbol{p}})\,\frac{\partial}{\partial \mu} + (1-\mu^2)^{-1}(\boldsymbol{\beta}'\cdot\boldsymbol{n}\cdot\hat{\boldsymbol{\beta}})\,\frac{\partial}{\partial \phi}\Big] > f(\mu,\phi,\tau-\lambda) \tag{15}$$

The integro-differential operator on the right side of equation (15) is quite complicated, as it acts on an arbitrary $f(\mu,\phi,\tau-\lambda)$. For the purpose of computing pitch angle scattering only, we assume a gyrotropic distribution function; i.e., we assume f independent of ϕ. Then, after averaging equation (15) over ϕ, we obtain,

$$\frac{\partial f}{\partial \tau} = (\varepsilon')^2\, \frac{1}{2}\,\frac{\partial}{\partial \mu}\, D_\mu\,(\tau) * \frac{\partial f}{\partial \mu}\,, \tag{16}$$

where,

$$D_\mu(\tau) = -\frac{1}{\pi}\int_0^{\tau} d\lambda \int_0^{2\pi} d\phi < (\hat{\boldsymbol{p}}\cdot\boldsymbol{\Omega}\cdot\boldsymbol{\beta}'(\boldsymbol{x})\,\mathcal{G}_o(\varepsilon,\lambda)\;(\boldsymbol{\beta}'(\boldsymbol{x})\cdot\boldsymbol{\Omega}\cdot\hat{\boldsymbol{p}}) > \tag{17}$$

and where the star (*) in equation (16) indicates the convolution integral operation on the function of time to the right. Notice that this term, because it is assumed to be a function of μ and τ only, is not affected by the streaming operator in $D_\mu(\tau)$.

We proceed by introducing the Fourier integral transform representation of $\boldsymbol{\beta}'(\boldsymbol{x})$, and then allowing the streaming operator to operate on the resulting explicit dependence on the phase space variables. We obtain,

$$D_{\mu}(\tau) = -\frac{1}{\pi}\int_0^{\tau} d\lambda \int_0^{2\pi} d\phi \; \left(\frac{1}{2\pi}\right)^3 \int d^3k \int d^3k' \; e^{i\,\mathbf{k}\cdot\mathbf{x}} \;\cdot$$

$$\cdot\; e^{i\,\mathbf{k}'\cdot\,(\mathbf{x}-\mathbf{r}(\lambda))} [\hat{\mathbf{p}}\cdot\boldsymbol{\Omega}\cdot < \boldsymbol{\beta}'(\mathbf{k})\; \boldsymbol{\beta}'(\mathbf{k}') > \cdot\boldsymbol{\Omega}\cdot \;\; (\lambda)\cdot\hat{\mathbf{p}} \tag{18}$$

With the assumption of homogeneous magnetic turbulence we can introduce,

$$< \boldsymbol{\beta}'(\mathbf{k})\; \boldsymbol{\beta}'(\mathbf{k}') > \; = (2\pi)^{3/2}\; \mathbf{R}(\mathbf{k})\;\; \delta(\mathbf{k}+\mathbf{k}') \tag{19}$$

where $\mathbf{R}$ (k) is the Fourier integral transform of the correlation tensor,

$$\mathbf{R}(\mathbf{r}) = < \boldsymbol{\beta}'(\mathbf{x})\; \boldsymbol{\beta}'(\mathbf{x}+\mathbf{r}) > \tag{20}$$

No other assumptions concerning $\mathbf{R}$ ($\mathbf{k}$), or $\mathbf{R}$ ($\mathbf{r}$), will be made.

Through the assumption of homogeneity we are able to rewrite the exponential terms appearing in equation (18) as,

$$e^{i\,\mathbf{k}\cdot\mathbf{x}}\; e^{i\,\mathbf{k}'\cdot(\mathbf{x}-\mathbf{r}(\lambda))} = e^{-i\,\mathbf{k}\cdot\mathbf{r}(\lambda)} \tag{21}$$

where $\mathbf{r}$ (λ) is given by equations (9) and (12). This result can be further rewritten to bring out the explicit dependence on the phase angle, ϕ. Then, the averaging over phase can be done. We will proceed in a particular coordinate system. Since $D_{\mu}(\tau)$ is a scalar quantity, our choice of coordinate system is irrelevant. The end result of this calculation will, however, be written in an invariant form which then will apply in any coordinate system.

In a cartesian coordinate system denoted by (1,2,3), we let the mean field be in the 3-direction. Then, both $\hat{\mathbf{p}}$ and $\mathbf{k}$ have parallel and perpendicular components denoted, for example, by $(\hat{\mathbf{p}})_{\parallel} \equiv \mathbf{P}\cdot\hat{\mathbf{p}}$ and $(\hat{\mathbf{p}})_{\perp} \equiv \mathbf{N}\cdot\hat{\mathbf{p}}$ with similar definitions for $\mathbf{k}_{\parallel}$ and $\mathbf{k}_{\perp}$. The magnitudes of these vector components will be denoted by $|(\hat{\mathbf{p}})_{\parallel}| = \mu$ and $|(\hat{\mathbf{p}})_{\perp}| = \mu' = (1-\mu^2)^{1/2}$, and by $k_{\parallel}$ and $k_{\perp}$. The cartesian components of these vectors are defined through,

$$p_1 = \mu' \cos\phi \qquad\qquad k_1 = k_\perp \cos\psi$$
$$p_2 = \mu' \sin\phi \qquad \text{and} \qquad k_2 = k_\perp \sin\psi \tag{22}$$
$$p_3 = \mu \qquad\qquad k_3 = k_\parallel$$

Now, we can rewrite equation (21) as follows:

$$\begin{aligned} e^{-i\,\underset{\sim}{k}\cdot\underset{\sim}{r}(\lambda)} &= e^{-i\,\underset{\sim}{k}_\parallel\cdot\underset{\sim}{r}_\parallel(\lambda)}\; e^{-i\,\underset{\sim}{k}_\perp\cdot\underset{\sim}{r}_\perp(\lambda)} \\ &= e^{-i\,k_\parallel \mu\lambda} \sum_{m,n=-\infty}^{\infty} e^{i(m-n)(\psi-\phi)}\; e^{-i\epsilon m\lambda}\; J_m\left(\frac{k_\perp\mu'}{\epsilon}\right) J_n\left(\frac{k_\perp\mu'}{\epsilon}\right) \end{aligned} \tag{23}$$

where the J's are the Bessel functions of the first kind. On substituting equations (19) and (23) into equation (18), we find,

$$\begin{aligned} D_\mu(\tau) = -\frac{(2\pi)^{-3/2}}{\pi} \int_0^\tau d\lambda \int_0^{2\pi} d\phi \int d^3k\; e^{-i\,k_\parallel\mu\lambda} \sum_{m,n=-\infty}^{\infty} e^{i(m-n)(\psi-\phi)} \\ \cdot\, e^{-i\,\epsilon m\lambda}\, J_m\left(\frac{k_\perp\mu'}{\epsilon}\right) J_n\left(\frac{k_\perp\mu'}{\epsilon}\right) [\hat{\underset{\sim}{p}}\cdot\underset{\sim}{\Omega}\cdot\underset{\sim}{R}(\underset{\sim}{k})\cdot\underset{\sim}{\Omega}\cdot\underset{\sim}{\mathbf{C}}(\lambda)\cdot\hat{\underset{\sim}{p}}] \end{aligned} \tag{24}$$

Then, by introducing the tensor elements of $\underset{\sim}{R}(\underset{\sim}{k})$, denoted by $R_{11}(\underset{\sim}{k})$, $R_{22}(\underset{\sim}{k})$, etc., we are able to exhibit explicitly the phase angle dependence of the square bracket in equation 24. We find

$$\begin{aligned} [\;] = \frac{(1-\mu^2)}{2}[-(R_{11}+R_{22})\cos\epsilon\lambda + (R_{11}-R_{22})\cos(\epsilon\lambda+2\phi) + \\ +(R_{12}+R_{21})\sin(\epsilon\lambda+2\phi) - (R_{12}-R_{21})\sin\epsilon\lambda] \end{aligned} \tag{25}$$

From our earlier assumption of homogenity, we find

$$R_{ij}(\underset{\sim}{k}) = R_{ji}(-\underset{\sim}{k}) \tag{26}$$

and, since the magnetic field is real

$$R_{ij}(-\underset{\sim}{k}) = R^*_{ij}(\underset{\sim}{k}) \tag{27}$$

where the star (*) denotes complex conjugation. We will use these symmetry properties of the tensor elements in the following development.

Upon introducing equation (25) into equation (24), we explicitly exhibit all ϕ - dependence in the integrand of equation (24). The phase angle averaging can now be done. Because of the orthogonality of the trigonometric functions in the interval, 0-2π, only certain values of n, relative to m, in the infinite summation will contribute, thereby reducing the double infinite sum to a single one. The result is,

$$D_\mu(\tau) = (2\pi)^{-3/2} \frac{(1-\mu^2)}{2} \int_0^\tau d\lambda \int d^3k \sum_{m=-\infty}^{\infty} J_m\left(\frac{k_\perp \mu'}{\epsilon}\right) e^{-i(k_\parallel \mu + m\epsilon)\lambda} \cdot$$

$$\cdot J_m\left(\frac{k_\perp \mu'}{\epsilon}\right) \left[(R_{11}+R_{22})(e^{i\epsilon\lambda}+e^{-i\epsilon\lambda}) - i(R_{12}-R_{21})(e^{i\epsilon\lambda}-e^{-i\epsilon\lambda})\right] +$$

$$- (R_{11}-R_{22})\left[J_{m-2}\left(\frac{k_\perp \mu'}{\epsilon}\right) e^{i(2\psi+\epsilon\lambda)} + J_{m+2}\left(\frac{k_\perp \mu'}{\epsilon}\right) e^{-i(2\psi+\epsilon\lambda)}\right] +$$

$$+ i(R_{12}+R_{21})\left[J_{m-2}\left(\frac{k_\perp \mu'}{\epsilon}\right) e^{i(2\psi+\epsilon\lambda)} - J_{m+2}\left(\frac{k_\perp \mu'}{\epsilon}\right) e^{-i(2\psi+\epsilon\lambda)}\right]$$

At this point, we must remember that $D_\mu(\tau)$ is actually an integral operator (see equations (16) and (17). Further simplifications of this expression for $D_\mu(\tau)$ depend on the function on which $D_\mu(\tau)$ operates. We must make the adiabatic approximation at this point to obtain the Fokker-Planck pitch angle scattering coefficient, even though there are good reasons for suspecting that this approximation may not be valid (Klimas and Sandri, 1971, 1973a). Klimas and Sandri (1973b) have shown that in the special case of isotropic, homogeneous magnetic turbulence, with a cosmic ray distribution finction which is essentially isotropic, the adiabatic approximation to the parallel transport of the cosmic ray particles can be formally constructed, but its accuracy remains in doubt. In the general case being considered here, no formal justification of the existence of the adiabatic approximation is available. However, because it is our intention to compute the Fokker-Planck coefficient as a basis for further arguments on its validity, we will proceed anyway.

The adiabatic approximation to equation (16) is the Fokker-Planck equation for pitch-angle diffusion, which can be written as,

$$\frac{\partial f}{\partial \tau} = \frac{1}{2} \frac{\partial}{\partial \mu} \left(\frac{<(\Delta\mu)^2>}{\Delta\tau}\right) \frac{\partial f}{\partial \mu} \tag{29}$$

where,

$$\frac{<(\Delta\mu)^2>}{\Delta\tau} = (\epsilon')^2 \, D_{\mu}(\infty) \tag{30}$$

In Appendix B we show that, by using the symmetry properties of equations (26) through (28), we find,

$$\begin{aligned}\frac{<(\Delta\mu)^2>}{\Delta\tau} = \frac{(\epsilon')^2(1-\mu^2)}{2\sqrt{2\pi}} \int d^3k \sum_{m=-\infty}^{\infty} \delta(\epsilon m + k_{\parallel}\mu) \, J_{m-1}\left(\frac{k_{\perp}\mu'}{\epsilon}\right) \cdot \\ \cdot\{(R_{11}+R_{22}) \, J_{m-1}\left(\frac{k_{\perp}\mu'}{\epsilon}\right) - (R_{11}-R_{22}) \, J_{m+1}\left(\frac{k_{\perp}\mu'}{\epsilon}\right) \cos 2\psi + \\ - (R_{12}+R_{21}) \, J_{m+1}\left(\frac{k_{\perp}\mu'}{\epsilon}\right) \sin 2\psi + i \, (R_{12}-R_{21}) \, J_{m-1}\left(\frac{k_{\perp}\mu'}{\epsilon}\right)\}\end{aligned} \tag{31}$$

This result is applicable in a coordinate system with the 3-direction in the direction of the mean field. However, by using the relationships between the angle, ψ, and the components of $\underset{\sim}{k}$ exhibited in equation (22), as well as the recursion relationships, $J_{m-1}(z) + J_{m+1}(z) = \left(\frac{2m}{z}\right) J_m(z)$ and $J_{m-1}(z) - J_{m+1}(z) = 2J'_m(z)$, we are able to rewrite equation (31) as,

$$\begin{aligned}\frac{<(\Delta\mu)^2>}{\Delta\tau} = \frac{(\epsilon')^2(1-\mu^2)}{\sqrt{2\pi}} \int d^3k \sum_{m=-\infty}^{\infty} \delta(\epsilon m + k_{\parallel}\mu) \, \cdot \\ \cdot\{-J_{m+1}\left(\frac{k_{\perp}\mu'}{\epsilon}\right) J_{m-1}\left(\frac{k_{\perp}\mu'}{\epsilon}\right) \left[\frac{\underset{\sim}{k}_{\parallel}\cdot \underset{\sim}{R}(\underset{\sim}{k})\cdot \underset{\sim}{k}_{\parallel}}{k_{\perp}^2}\right] + \\ + \left(\frac{\epsilon m}{k_{\perp}\mu'}\right)^2 J_m^2 \left(\frac{k_{\perp}\mu'}{\epsilon}\right) \mathrm{Tr}\,[\underset{\sim}{N}\cdot\underset{\sim}{R}\,(\underset{\sim}{k})] + \\ -i\left(\frac{\epsilon m}{k_{\perp}\mu'}\right) J_m \left(\frac{k_{\perp}\mu'}{\epsilon}\right) J'_m \left(\frac{k_{\perp}\mu'}{\epsilon}\right) \mathrm{Tr}\,[\underset{\sim}{\Omega}\cdot\underset{\sim}{R}\,(\underset{\sim}{k})]\}\end{aligned}$$

which now may be applied in any coordinate system. (The symbols, "Tr [ ]", indicate that the trace of the tensor inside the square bracket should be taken). Equation (32) can be viewed as the pitch angle scattering coefficient which results from resonant interactions between the particles and waves in the magnetic field with wave numbers $k_{\parallel} = \frac{\epsilon m}{\mu}$. Similar expressions of this scattering coefficient have appeared previously for special forms of $\underset{\sim}{R}\ (\underset{\sim}{k})$ (Hasselman and Wibberenz, 1968; Volk, 1973; Fisk, et al., (1974). In the next section we will consider the reduction of this general expression of the pitch angle scattering coefficient for <u>any</u> homogeneous magnetic turbulence to the previously available special cases.

III. <u>Isotropic, and Slab, Random Field Models</u>

i) <u>Isotropic model for magnetic turbulence</u>

The magnetic turbulence is statistically isotropic if the tensor, $\underset{\sim}{R}\ (\underset{\sim}{k})$, has the form,

$$\underset{\sim}{R}(\underset{\sim}{k}) = R(k)\ (\underset{\sim}{I} - \underset{\sim}{\hat{k}\hat{k}}) \tag{33}$$

where $\underset{\sim}{I}$ is the unit matrix, and R (k) is a arbitrary scalar function of $k = |\underset{\sim}{k}|$ which can, however, be related to the 1-dimensional power spectrum for the field components parallel to the displacement vector (Klimas and Sandri, 1973a) through the relation,

$$R(k) = \frac{1}{2}\, k \frac{d}{dk} \left(\frac{1}{k} \frac{d\, P_{\parallel}(k)}{dk}\right) \tag{34}$$

By substituting equation (34) into equation (32), we find, for isotropic turbulence,

$$\frac{<(\Delta\mu)^2>}{\Delta\tau} = \frac{(\epsilon')^2(1-\mu^2)}{\sqrt{2\pi}} \int d^3k \sum_{m=-\infty}^{\infty} \delta(k_{\parallel}\mu+\epsilon m)\ R(k)\ \cdot \tag{35}$$

$$\cdot \left\{ -J_{m+1}\left(\frac{k_{\perp}\mu'}{\epsilon}\right) J_{m-1}\left(\frac{k_{\perp}\mu'}{\epsilon}\right) \left(\frac{k_{\parallel}}{k}\right)^2 + \left(\frac{\epsilon m}{k_{\perp}\mu'}\right)^2 J_m^2\left(\frac{k_{\perp}\mu'}{\epsilon}\right)\left(1+\left(\frac{k_{\parallel}}{k}\right)^2\right)\right\}$$

This result, written in a dimensional form, has been given by Fisk, et al. (1974).

All terms in equation (35) which correspond to $m \neq 0$ do not contribute to $<(\Delta\mu)^2>/\Delta\tau$ at $\mu = 0$. Notice that no part of the integrand depends on ψ. Thus, the integrations over ψ and $k_\parallel$ can be carried out. Then,

$$\frac{<(\Delta\mu)^2>}{\Delta\tau}\Big|_{m\neq 0} = (\epsilon')^2\sqrt{2\pi}\,\frac{(1-\mu^2)}{|\mu|}\int_0^\infty dk_\perp\, k_\perp\, R\Big(\sqrt{k_\perp^2+\big(\tfrac{\epsilon m}{\mu}\big)^2}\Big)\cdot$$

$$\cdot\Big\{-J_{m+1}\Big(\frac{k_\perp\mu'}{\epsilon}\Big)\,J_{m-1}\Big(\frac{k_\perp\mu'}{\epsilon}\Big)\Big(\frac{k_\parallel}{k}\Big)^2 + \Big(\frac{\epsilon m}{k_\perp\mu'}\Big)^2 J_m^2\Big(\frac{k_\perp\mu'}{\epsilon}\Big)\Big(1+\Big(\frac{k_\parallel}{k}\Big)^2\Big)\Big\}_{k_\parallel=\frac{\epsilon m}{\mu}} \tag{36}$$

For a continuously differentiable magnetic field which is statistically homogeneous, R (k) must approach zero, as k approaches infinity, at least as fast as k^{-4} (Erdelyi, 1956; Batchelov, 1960). Therefore, from equation (36), $<(\Delta\mu)^2>/\Delta\tau = 0$ at $\mu = 0$.

The $m = 0$ term in equation (35) contributes to $<(\Delta\mu)^2>/\Delta\tau$ at $\mu = 0$ only. In fact,

$$\frac{<(\Delta\mu)^2>}{\Delta\tau}\Big|_{m=0} = \delta(\mu)\,\frac{(\epsilon')^2}{\sqrt{2\pi}}\int d^3k\,\frac{R(k)}{|k_\parallel|}\Big(\frac{k_\parallel}{k}\Big)^2 J_1^2\Big(\frac{k_\perp}{\epsilon}\Big) \tag{37}$$

Thus, it is the $m = 0$ term which contributes the $\delta(\mu)$ part of the pitch angle scattering coefficient. Looking back at equation (32), we can see that this conclusion is completely general.

The infinite series of resonant terms ($m \neq 0$) has been investigated by Fisk, et al. (1974), for power-law, power-spectra. They found that, for $1 \leq \epsilon \leq 30$, the entire series can be very well approximated by μ times the small gyro-radius approximation given by Jokipii (1971). This result applies over a wide range of values of the power law spectral index.

Klimas and Sandri (1973b, 1973c) have shown that the m = 0 term dominates the parallel spatial diffusion coefficient for both power law and gaussian power spectra for low energy particles ($\epsilon \gg 1$). The parallel diffusion coefficient is given by Klimas and Sandri (1973b) as,

$$\mathcal{K}_{\parallel} = \frac{1}{3}\,\frac{\nu\lambda p}{\eta^2}\,(\epsilon^2 \Delta_{\parallel}(\epsilon))^{-1} \tag{38}$$

for $\eta \ll 1$ and $\epsilon \gg 1$, where λp is the parallel integral length (essentially the correlation length in the random field). The quantity, $\Delta_{\parallel}(\epsilon)$, is related to the pitch angle scattering coefficient through (Fisk, et al. 1974),

$$\frac{1}{3}\,(\epsilon')^2\,\Delta_{\parallel} = \frac{1}{4}\int_{-1}^{1} d\mu\,\frac{\langle(\Delta\mu)^2\rangle}{\Delta\tau} \tag{39}$$

For $\epsilon \gg 1$, equation (37) can be simplified considerably, Using $J_1\left(\frac{k_\perp}{\epsilon}\right) \underset{\epsilon\uparrow}{\simeq} \frac{1}{2}\left(\frac{k_\perp}{\epsilon}\right)$, and equation (34), we find

$$\frac{\langle(\Delta\mu)^2\rangle}{\Delta\tau}\Big|_{m=0} \underset{\epsilon\uparrow}{\simeq} \delta(\mu)\,\eta^2\sqrt{\frac{\pi}{2}}\int_0^{\infty} dk\; kP_{\parallel}(k) \tag{40}$$

On substituting equation (40) into equations (38) and (39), we reproduce the earlier results of Klimas and Sandri (1973b, 1973c) as well as obtain an excellent approximation to the parallel diffusion coefficient, as predicted by Fokker-Planck theory, for low energy cosmic rays. A general feature of this result (in fact, for the $\delta(\mu)$ contribution to $\langle(\Delta\mu)^2\rangle/\Delta\tau$ in any kind of homogeneous turbulence) is that the term $(\epsilon^2\Delta_{\parallel}(\epsilon))^{-1}$, in equation (38), is independent of ϵ,, for $\epsilon \gg 1$. Thus $\mathcal{K}_{\parallel}$ can be written,

$$\mathcal{K}_{\parallel} = \frac{1}{3}\,\nu\,\lambda_{MFP} \tag{41}$$

where the mean free path, which is given by

$$\lambda_{MFP} = \frac{\lambda p}{\eta^2}\,(\epsilon^2\Delta_{\parallel}(\epsilon))^{-1} \tag{42}$$

is independent of regidity.

ii) Slab model for magnetic turbulence

In the slab model we let

$$\underset{\sim}{R}(\underset{\sim}{k}) = \underset{\sim}{N}\; R(k_{\parallel})\; \delta(\underset{\sim}{k}_{\perp}) \tag{43}$$

where $R(k_{\parallel})$ is an arbitrary even function of $k_{\parallel}$. Thus, the random field components are normal to the mean field, and they are functions only of distance along the mean field. The correlations in the random field are assumed cylindrically symmetric about the mean field.

This model is closely related to others which have been studied previously. In Jokipii's (1972) plane polarized model, the random field is assumed to have only one orthogonal component. We will see, shortly, that the pitch angle scattering coefficient computed here is simply twice Jokipii's. Jones, Birmingham and Kaiser (1973) have used the plane polarized field model to investigate modifications of quasi-linear theory near $\mu = 0$. Volk (1974) has also studied such modifications in a random field made up of transverse Alfven waves. A common feature of all of these models is that the $\delta(\mu)$ does not appear in the Fokker-Planck pitch angle scattering coefficient.

With the choice for R(k) given in equation (43), we find, from equation (32), that,

$$\frac{<(\Delta\mu)^2>}{\Delta\tau} = 2\frac{(\epsilon')^2(1-\mu^2)}{\sqrt{2\pi}} \int d^3k\; \delta(k_{\perp})\; R(k_{\parallel}) \sum_{m=-\infty}^{\infty} \delta(k_{\parallel}\mu+\epsilon m)\; \cdot$$
$$\cdot\; \left(\frac{\epsilon m}{k_{\perp}\mu'}\right)^2 J_m^2\left(\frac{k_{\perp}\mu'}{\epsilon}\right) \tag{44}$$

We see immediately that there can be no $\delta(\mu)$ contribution to $<(\Delta\mu)^2>/\Delta\tau$ since the m = 0 term in equation 44 is zero. In fact, because of the $\delta(\underset{\sim}{k}_{\perp})$, only the $m = \pm 1$ terms are non-zero. All other terms in the expansion are zero because $\left(\frac{\epsilon m}{k_{\perp}\mu'}\right) J_m\left(\frac{k_{\perp}\mu'}{\epsilon}\right) \to 0$ as $k_{\perp} \to 0$, and the integrand can only contribute at $k_\perp = 0$. Thus

$$\frac{\langle(\Delta\mu)^2\rangle}{\Delta\tau} = (\epsilon')^2 \sqrt{2\pi} \left(\frac{1-\mu^2}{|\mu|}\right) R\left(k_\parallel = \frac{\epsilon}{\mu}\right) \tag{45}$$

(Note: $\delta(\underset{\sim}{k}_\perp)$ is normalized to 2π instead of 1). This result is exactly twice that of Jokipii's (1972) plane polarized field model calculation in which there is only one orthogonal field component instead of two. On substituting equation (45) into equation (39), we find,

$$\begin{aligned} \Delta_\parallel &= 3\sqrt{\frac{\pi}{2}} \int_0^1 \frac{d\mu}{\mu} (1-\mu^2)\ R\left(\frac{\epsilon}{\mu}\right) \\ &= 3\sqrt{\frac{\pi}{2}} \int_\epsilon^\infty \frac{dz}{z} \left(1-\left(\frac{\epsilon}{z}\right)^2\right) R(z) \end{aligned} \tag{46}$$

which is identical to the small gyro-radius approximation to $\Delta_\parallel$ that Klimas and Sandri (1973a) have computed in their isotropic model. This correspondence between the small gyro-radius approximation in isotropic turbulence and the exact result in the slab model was first pointed out by Jokipii (1971).

IV. Mirroring and the Delta Function

The fact that the $\delta(\mu)$ contribution to $\langle(\Delta\mu)^2\rangle/\Delta\tau$ vanished in the slab model was one of the first strong indications that this contribution represents mirroring in the random magnetic field. The guiding center approximation (Alfven and Falthammar, 1963; Northrop, 1963) to the motion of charged particles in the random magnetic field applies to particles whose motion along the field is so slow that they can resonate only with very short wavelengths where the power density is typically negligible. This could be the situation for a very low energy particle, or a higher energy particle moving in a weakly turbulent field with pitch angle very near 90°. The latter situation is the one being studied here.

Within the guiding center approximation, the pitch angle changes with time according to,

$$\frac{d\mu}{dt} = - M \hat{e} \cdot [(\hat{\underset{\sim}{e}}\cdot\underset{\sim}{\nabla}) \underset{\sim}{B}] \tag{47}$$

where, $M = \frac{1}{2} p_\perp^2/Bm$, is the magnetic moment, and $\hat{e}$ is a unit vector in the direction of the local magnetic field. Of course, the pitch angle that enters into this expression is the angle between the direction of the particle momentum, and the local exact field (actually the angle averaged over a gyro-period). In Appendix C we demonstrate that, to first order in η in a weakly turbulent magnetic field, this pitch angle is identical to the pitch angle relative to the mean field averaged over a gyro-period. By introducing $\underset{\sim}{B} = \langle\underset{\sim}{B}\rangle + \underset{\sim}{B}'$, we can rewrite equation(47). To first order in η, we find,

$$\frac{d\mu}{d\tau} = - \frac{1}{2} (1-\mu^2) \eta \hat{\underset{\sim}{\beta}} \cdot [(\hat{\underset{\sim}{\beta}} \cdot \underset{\sim}{\nabla}) \underset{\sim}{\beta}'(\underset{\sim}{x})] \tag{48}$$

where $1/2(1-\mu^2)\eta$ is the magnetic moment, which we consider a constant of the motion in this guiding center approximation. Since the pitch angle scattering coefficient is quadratic in $\Delta\mu$, we see that, in order to obtain an $O(\mu^2)$ contribution to that coefficient, we must find an $O(\eta)$ contribution to $d\mu/d\tau$ through equation (48). Equation (48) is non-zero only if the random field contains a component in the direction of the mean field which also has spatial gradients in that direction. In the slab model, $\hat{\beta}\cdot\beta'(x) = 0$, and there is also no $\delta(\mu)$ contribution to $\langle(\Delta\mu)^2\rangle/\Delta\tau$. In this section we will show that <u>if, and only if</u>, equation (48) is non-zero, do we find a $\delta(\mu)$ in the pitch angle scattering coefficient. Thus, only when there are $O(\eta)$ changes in μ due to guiding center motion, is there a $\delta(\mu)$ in $\langle(\Delta\mu)^2\rangle/\Delta\tau$. In view of the fact that the $\delta\ (\mu)$ is the <u>only</u> part of $\langle(\Delta\mu)^2\rangle/\Delta\tau$ which is non-zero at $\mu = 0$, and therefore, is the only part which can possibly reverse the parallel motion of the particles, we ascribe the $\delta\ (\mu)$ to mirroring. The fact that mirroring exhibits itself as a $\delta\ (\mu)$ in $\langle(\Delta\mu)^2\rangle/\Delta\tau$ can be understood from the asymptotic nature of the Fokker-Planck equation. This equation is asymptotic in small η, and, in an infinitesimal random field, only particles very near $\mu = 0$ could possibly be mirrored. In fact, the delta function is probably a crude approximation to a sharply peaked function of μ.

It is interesting to note the results of a calculation which we present in Appendix D. There, we compute $\langle(\Delta\mu)^2\rangle/\Delta\tau$ from equation (48) using the Fokker-Planck formalism. For the position of the particle, $\vec{x}$, along its trajectory in the field, as it appears in $\underset{\sim}{\beta}'(\underset{\sim}{x})$, we substitute the position of its guiding center. This approximation to the position of the particle is exact in the limit of zero particle energy ($\epsilon = \infty$). The result of this calculation is identical to the result which we will present in this section when we study the m = 0 term (the $\delta\ (\mu)$ contribution) of equation (32) in the limit, $\epsilon = \infty$. Thus, the Fokker-Planck coefficient for pitch angle scattering, computed in the guiding center approximation to the particle trajectory, also contains a $\delta\ (\mu)$ term which is in exact agreement with the $\delta\ (\mu)$ term obtained from the quasi-linear approximation to the particle trajectory, in the limit, $\epsilon = \infty$.

From equation (32), with m = 0, we find,

$$\frac{\langle(\Delta\mu)^2\rangle}{\Delta\tau}\bigg|_{m=o} = \delta(\mu)\ \frac{(\dot{\epsilon}')^2(1-\mu^2)}{\sqrt{2\pi}} \int d^3k \left(\frac{\underset{\sim}{k}_{\parallel}\cdot\underset{\sim}{R}(\underset{\sim}{k})\cdot\underset{\sim}{k}_{\parallel}}{|k_{\parallel}|}\right)\left(\frac{J_1^2(\frac{k_{\perp}\mu'}{\epsilon})}{k_{\perp}^2}\right) \quad (49)$$

A necessary property of $\underset{\sim}{R}(\underset{\sim}{k})$ is that it be non-negative i.e., the quadratic form,

$$\underset{\sim}{V} \cdot \underset{\sim}{R} \cdot \underset{\sim}{V}^* \geq 0 \tag{50}$$

for any complex vector, $\underset{\sim}{V}$ (Batchelor, 1960). In particular, $\underset{\sim}{k}_\parallel \cdot \underset{\sim}{R} \cdot \underset{\sim}{k}_\parallel$ must be either positive or zero for any $\underset{\sim}{k}$. Notice also, that the rest of the quantities in the integrand of equation (49) are non-negative. Therefore, barring special situations which we will discuss in a moment, $R_{\parallel,\parallel}(\underset{\sim}{k})$, must be zero everywhere in $\underset{\sim}{k}$ in order for the $\delta(\mu)$ to not contribute to pitch angle scattering. But then, $R_{\parallel,\parallel}(\underset{\sim}{r})$ is zero for all $\underset{\sim}{r}$, from which we can conclude that there is no random field component parallel to the mean field (slab or plane polarized field model). With no random field parallel to $\hat{\beta}$, we see from equation (48) that mirroring is impossible. This argument can be carried in the reverse order. It is clear that if equation (48) indicates no mirroring because $\underset{\sim}{\beta} \cdot \underset{\sim}{\beta}'(\underset{\sim}{x})$ is zero for all $\underset{\sim}{x}$, then $R_{\parallel,\parallel}(\underset{\sim}{r}) = 0$, and finally $R_{\parallel,\parallel}(\underset{\sim}{k}) = 0$ for all $\underset{\sim}{k}$. In this case there is no $\delta(\mu)$ contribution to $\langle(\Delta\mu)^2\rangle/\Delta\tau$.

There is a special situation in which $R_{\parallel,\parallel}(\underset{\sim}{k}) \neq 0$ everywhere in $\underset{\sim}{k}$, and yet the integral in equation (49) is zero, and $\langle(\Delta\mu)^2\rangle/\Delta\tau$ does not contain a $\delta(\mu)$. It is possible that $R_{\parallel,\parallel}(\underset{\sim}{k})$ is non-zero only at point in $\underset{\sim}{k}$ where other terms in the integrand are zero. The Bessel function cotains isolated zero's which, nevertheless we will not consider because the positions of these zeros are rigidity dependent through ϵ, and $\underset{\sim}{R}(\underset{\sim}{k})$ does not depend on rigidity. On the other hand, we could have $\underset{\sim}{R}(\underset{\sim}{k})$ non-zero only when $k_\parallel = 0$ without contributing to the integral. But, we still have,

$$k_\parallel \; R_{\parallel,\parallel}(\underset{\sim}{k}) = 0. \tag{51}$$

This statement is the Fourier transform of

$$\frac{\partial}{\partial r_{\parallel}} R_{\parallel,\parallel}(\underset{\sim}{r}) = 0 \tag{52}$$

Barring non-differentiable functions of the type discussed by Wiener (1933), we can conclude that the random field component in the direction of the mean field is independent of position along the mean field; i.e., $(\hat{\underset{\sim}{\beta}}\cdot\underset{\sim}{\nabla})(\hat{\underset{\sim}{\beta}}\cdot\underset{\sim}{\beta}'(\underset{\sim}{x})) = \hat{\underset{\sim}{\beta}}[(\hat{\underset{\sim}{\beta}}\cdot\underset{\sim}{\nabla})\underset{\sim}{\beta}'(\underset{\sim}{x})] = 0$. But this is exactly the condition that equation (48) be zero. This argument can also be reversed. Starting from a zero for equation (48), we can conclude equation (52), then equation (51), and then we can conclude no $\delta\ (\mu)$ in $<(\Delta\mu)^2>/\Delta\tau$. In general, a random field component along the mean field is allowed, but this component must be a constant along the field; in the slab model the constant is zero.

<u>From the above arguments, we conclude that the $\delta\ (\mu)$ in $<(\Delta\mu)^2>/\Delta\tau$ represents the physical phenomenon of mirroring in the random magnetic field.</u>

In isotropic turbulence, we have seen, when $\varepsilon \gg 1$ (low energy particles) the $\delta\ (\mu)$ contribution to $<(\Delta\mu)^2>/\Delta\tau$ becomes independent of rigidity. From equation (49), we find in the same approximation,

$$\left.\frac{<(\Delta\mu)^2>}{\Delta\tau}\right|_{m=o} \underset{\varepsilon\uparrow}{\approx} \left(\frac{(1-\mu^2)\eta}{2}\right)^2 \frac{\delta(\mu)}{\sqrt{2\pi}} \int d^3k \left(\frac{\underset{\sim}{k}_{\parallel}\cdot\underset{\sim}{R}(\underset{\sim}{k})\cdot\underset{\sim}{k}_{\parallel}}{|k_{\parallel}|}\right) \tag{53}$$

Thus, the $\delta\ (\mu)$ part of $<(\Delta\mu)^2>/\Delta\tau$ becomes rigidity independent, for low rigidities, in any homogeneous magnetic turbulence. For isotropic turbulence, Klimas and Sandri (1973b,c) have shown that this $\delta\ (\mu)$ contribution dominates over all other scattering mechanisms. Thus the rigidity dependence of λ_{MFP} is determined by the effects of mirroring: i.e., λ_{MFP} is rigidity independent. For arbitrary turbulence, however, we do not know the relative strengths of the contributions of the $\delta(\mu)$ terms, and the non-δ (μ) terms, to the parallel diffusion coefficient. Thus, in contrast to the isotropic turbulence, we cannot determine the rigidity dependence of λ_{MFP}.

As we mentioned above, equation (53) is identical to the result (given in Appendix D) of computing $<(\Delta\mu)^2>/\Delta\tau$ within the guiding center approximation to the particle motion.

V. Conclusion

We have constructed the Fokker-Planck coefficient for pitch angle scattering of cosmic rays in otherwise arbitrary, but statistically homogeneous, magnetic turbulence. Our result was obtained both from first principles, and through the Fokker-Planck formalism. The reduction of our expression to previously available scattering coefficients, calculated in special models of the random field, has been discussed.

We have shown that the pitch angle scattering coefficient contains a Dirac delta function, δ (μ), in $\mu = \cos\theta$, where θ is the pitch angle. We have, further, proved that this delta function is the Fokker-Planck prediction for the contribution of mirroring to pitch angle scattering in a weakly turbulent magnetic field.

The δ (μ) does not contribute to pitch angle scattering when, within the guiding center approximation to the particle motion, the pitch angle is a constant of the motion of the particle O (η) in the random field strength. This condition is met when the vector component of the random field, which lies in the direction of the mean field, is independent of distance along the mean field. The slab, or plane polarized, or linearized Alfven wave models of the random field, all of which have no random field component along the mean direction, fall within this class. Magnetosonic waves propagating across the mean field have a random parallel field component which is independent of distance along the mean field, and fall within this class. Typically, however, the observed interplanetary field does not fall within this class. We expect mirroring in the interplanetary field, and we expect a delta function in the appropriate Fokker-Planck pitch angle scattering coefficient.

The fact that mirroring exhibits itslef as a δ (μ) in the Fokker-Planck pitch angle scattering coefficient indicates that this particular pitch angle scattering mechanism is misordered within the Fokker-Planck formalism. In this formalism it is assumed that the effects of the random field on the motion of the particles is $O(\eta^2)$ in the random field strength. When the effects, on the cosmic ray distribution function, or mirroring are computed correctly we will most certainly find them entering in a lower order than $O(\eta^2)$, in a non-Fokker-Planck type equation.

Other investigators have concerned themselves with the modifications of the Fokker-Planck formalism which are necessary even in the special random field models in which mirroring does not occur. Even though these modifications are extremely difficult to compute, they nevertheless, in principle, are minor modifications to the quasi-linear, or weak coupling approach which leads to the Fokker-Planck equation. The phenomenon of mirroring has not been properly ordered within the approach, and the presence of the delta function in the pitch angle scattering coefficient presents a serious obstacle to these modification schemes. In this paper we have demonstrated the existence of the delta function, as well as its connection to mirroring, in order that further discussions of the validity and/or modifications of the Fokker-Planck approximation can be based on the actual Fokker-Planck predictions in realistic field models.

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Appendix A

With our notation, the momentum equation of motion for a charged particle in a magnetic field, can be written as,

$$\frac{d\hat{p}}{d\tau} = \epsilon\,[\Omega + \eta\,\Omega'] \cdot \hat{p} \tag{A.1}$$

where $\Omega_{ij} = \epsilon_{ijk}\beta_k$ and $\Omega'_{ij} = \epsilon_{ijk}\beta'_k$. Thus, for $\mu = \beta \cdot p$ we find

$$\frac{d\mu}{d\tau} = \epsilon'(\beta \cdot \Omega' \cdot p) \tag{A.2}$$

or,

$$\frac{d\mu}{d\tau} = \epsilon'(p \cdot \Omega \cdot \beta') \tag{A.3}$$

where, in obtaining equation (A.2), we have used $\Omega\cdot\hat{\beta} = 0$. By formally integrating equation (A.3) we can obtain,

$$<(\Delta\mu)^2> = -\,(\epsilon')^2 \int_0^{\Delta\tau} d\lambda \int_0^{\Delta\tau} d\lambda\; \hat{p}(\lambda)\cdot\Omega\cdot <\beta'(\lambda)\beta'(\lambda')> \cdot\Omega\cdot\hat{p}(\lambda') \tag{A.4}$$

where $\beta'(\lambda) \equiv \beta'[x(\lambda)]$, and $x\,(\lambda)$ and $\hat{p}\,(\lambda)$ are the time dependent coordinates given by equations (8) thru (12) which describe the zeroth order helical approximation to the particle motion. Now, by introducing the Fourier integral transform representation of $\beta'\,(x)$, as well as the assumption of homogeneity, as expressed through equation (19), we find,

$$<(\Delta\mu)^2> = -\frac{(\epsilon')^2}{(2\pi)^{3/2}} \int_0^{\Delta\tau} d\lambda \int_0^{\Delta\tau} d\lambda \int d^3k\; e^{ik_\parallel \mu(\lambda-\lambda')} e^{ik_\perp\cdot(x_\perp(\lambda)-x_\perp(\lambda'))}\cdot \left[\hat{p}(\lambda)\cdot\Omega\cdot R(k)\cdot\Omega\cdot\hat{p}(\lambda')\right] \tag{A.5}$$

We also choose a special coordinate system in which to proceed here. In a cartesian coordinate system, we let the mean field lie in the 3-direction, as in the main text, but we further rotate our coordinate system so that the perpendicular component of the momentum, $(p_\perp)$, lies in the 1-direction.

Then, on introducing the expansion, given by equation (23), of the exponentials in equation (A.5), we obtain,

$$<(\Delta\mu)^2> = -\frac{(\epsilon')^2}{(2\pi)^{3/2}}\int_0^{\Delta\tau}d\lambda\int_0^{\Delta\tau}d\lambda'\int_{\infty}^{\infty}dk_{\parallel}\; e^{ik_{\parallel}\mu(\lambda-\lambda')}\int_0^{2\pi}d\psi\sum_{m,n=-\infty}^{\infty}e^{i(m-n)\psi}\cdot$$

$$\cdot\; e^{in\epsilon\lambda}e^{-im\epsilon\lambda'}\int_0^{\infty}dk_{\perp}k_{\perp}\; J_n\left(\frac{k_{\perp}\mu'}{\epsilon}\right)J_m\left(\frac{k_{\perp}\mu'}{\epsilon}\right)\;\cdot$$

$$\cdot\;[\hat{\underset{\sim}{p}}(\lambda)\cdot\underset{\sim}{\Omega}\cdot\underset{\sim}{R}(\underset{\sim}{k})\cdot\underset{\sim}{\Omega}\cdot\hat{\underset{\sim}{p}}(\lambda')] \tag{A.6}$$

By introducing the tensor components of $\underset{\sim}{R}$ $(\underset{\sim}{k})$, we find,

$$<(\Delta\mu)^2> = \frac{(\epsilon')^2(1-\mu^2)}{2(2\pi)^{3/2}}\int_0^{\Delta\tau}d\lambda\int_0^{\Delta\tau}d\lambda'\int_{-\infty}^{\infty}dk_{\parallel}\; e^{ik_{\parallel}\mu(\lambda-\lambda')}\int_0^{2\pi}d\psi\; e^{i(m-n)\psi}$$

$$\cdot\; e^{in\epsilon\lambda}\; e^{-im\epsilon\lambda'}\int_0^{\infty}dk_{\perp}\; k_{\perp}\; Jn\left(\frac{k_{\perp}\mu'}{\epsilon}\right)J_m\left(\frac{k_{\perp}\mu'}{\epsilon}\right)\;\cdot \tag{A.7}$$

$$\cdot\;[(R_{11}+R_{22})\cos\epsilon(\lambda-\lambda') - (R_{11}-R_{22})\cos\epsilon(\lambda+\lambda') +$$

$$-\;(R_{12}+R_{21})\sin\epsilon(\lambda+\lambda') - (R_{12}-R_{21})\sin\epsilon(\lambda-\lambda')]$$

Now, following the usual Fokker-Planck arguments, we must find the parts of this expression which grow as $\Delta\tau$. Consider, for example, the integral,

$$I_1 \equiv \int_0^{\Delta\tau}d\lambda\int_0^{\Delta\tau}d\lambda'\; e^{ik_{\parallel}\mu(\lambda-\lambda')}\; e^{in\epsilon\lambda}\; e^{-im\epsilon\lambda'}\cos\epsilon(\lambda-\lambda') \tag{A.8}$$

which is associated with the term $(R_{11}+R_{22})$ in equation (A.7). By introducing a new variable of integration $S = \lambda'-\lambda$, for λ', and inverting the order of integration, we find,

$$I_1 = [\int_0^{\Delta\tau}dS\int_0^{\Delta\tau-S}d\lambda + \int_{-\Delta\tau}^{0}dS\int_{-S}^{\Delta\tau}d\lambda]\; d^{-i(k_{\parallel}\mu+\epsilon m)S}\cos\epsilon S\; e^{i(n-m)\epsilon\lambda} \tag{A.9}$$

When $n = m$, we do obtain a term which grows linearly with $\Delta\tau$, for large $\Delta\tau$, given by,

$$\Delta\tau\;\pi\;[\delta(k_{\parallel}\mu + (m+1)\epsilon) + \delta(k_{\parallel}\mu + (m-1)\epsilon)] \tag{A.10}$$

A further examination of equation (A.9) reveals no other terms with the linear growth property. In order to obtain the Fokker-Planck coefficient from equation (A.7), we replace I_1, in equation (A.7) with

$$\Delta\tau \ \delta_{m,n}\pi \left[\delta(k_{\parallel}\mu + (m+1)\epsilon) + \delta(k_{\parallel}\mu + (m-1)\epsilon)\right] \qquad (A.11)$$

This procedure can be continued for the rest of the time integrals, and the steps which follow are identical to the steps taken in the alternate derivation of $<(\Delta\mu)^2>/\Delta\tau$ given in the main text. As we mentioned there, the results of the two calculations are identical.

The point of this appendix is to note the difference between the arguments given here, and those given in the main text. From Equation (A.9), we actually find a variety of terms, some of which do not grow unbounded with $\Delta\tau$, some which grow linearly with $\Delta\tau$, and some which grow as $(\Delta\tau)^2$. From the other time integrals in equation (A.7) we find similar results with, in addition, terms which grow as $\Delta\tau$ times trigonometric functions of $\Delta\tau$; these terms oscillate with amplitudes which grow unbounded in time. In the face of these unbounded oscillations, the usual Fokker-Planck argument for choosing the terms which grow as $\Delta\tau$ in $<(\Delta\mu)^2>$, and dropping all others, becomes difficult to support. Even ignoring this problem, we still have the problem of choosing the $\Delta\tau$ terms by arguing that $\Delta\tau$ is large compared to the interaction time, and small compared to the relaxation time, when in fact these two time scales are not clearly separated for particles in a magnetic field.

In comparison, from equation (24), notice that we never face these problems in the derivation of $<(\Delta\mu)^2>/\Delta\tau$ given there. The inner time integration in equation (A.7) is replaced by the phase averaging integration over $\emptyset$ in equation (24). The effects of carrying out the Fokker-Planck argument here, and the phase averaging there are identical;

both procedures pick the same terms out of the infinite expansions for retention. Thus, the results of both approaches are identical, but, the reasoning contained in the Fokker-Planck approach is much more difficult to support.

Appendix B

The adiabatic approximation to equation (28) permits the time integration to be done. The result is

$$D_\mu(\infty) = (2\pi)^{-3/2} \left(\frac{1-\mu^2}{2}\right) \int d^3k \sum_{m=-\infty}^{\infty} J_m \left(\frac{k_\perp \mu'}{\epsilon}\right) \cdot$$

$$\cdot \Big[J_m \left(\frac{k_\perp \mu'}{\epsilon}\right) \{ i(R_{11}+R_{22}) [\zeta(\epsilon(1-m)-k_\parallel\mu) + \zeta(-\epsilon(1+m) - k_\parallel\mu)] +$$

$$+ (R_{12}-R_{21}) [\zeta(\epsilon(1-m)-k_\parallel\mu) - \zeta(-\epsilon(1+m) - k_\parallel\mu)] \} + \tag{B.1}$$

$$- i(R_{11}-R_{22}) [J_{m-2} \left(\frac{k_\perp \mu'}{\epsilon}\right) e^{i2\psi} \zeta(\epsilon(1-m)-k_\parallel\mu) + J_{m+2}\left(\frac{k_\perp \mu'}{\epsilon}\right) e^{-i2\psi} \zeta(-\epsilon(1+m)-k_\parallel\mu)$$

$$- (R_{12}+R_{21}) [J_{m-2} \left(\frac{k_\perp \mu'}{\epsilon}\right) e^{i2\psi} \zeta(\epsilon(1-m)-k_\parallel\mu) - J_{m+2}\left(\frac{k_\perp \mu'}{\epsilon}\right) e^{-i2\psi} \zeta(-\epsilon(1+m)-k_\parallel\mu)\Big]$$

The function $\zeta(x)$ is the zeta function defined by (Heitler, 1954)

$$\zeta(x) = - i \lim_{T\to\infty} \int_0^T d\lambda \; e^{i\lambda x} = \frac{P}{x} - i\pi\delta(x) = - \zeta^*(-x) \tag{B.2}$$

where $\frac{P}{x}$ is the principle value of $1/x$.

In the terms that multiply $\zeta(\epsilon(1-m)-k_\parallel\mu)$, let $m' = - m$ and $\underline{k}' = - \underline{k}$, and use the relation $J_{-m}(z) = (-1)^m J_m(z)$ along with the symmetry relations (26) and (27) and (A.2) to rewrite (B.1) in the form

$$D_\mu(\infty) = (2\pi)^{-3/2} \left(\frac{1-\mu^2}{2}\right) \int d^3k \sum_{m=-\infty}^{\infty} J_m \{J_m(\zeta-\zeta^*)[i(R_{11}+R_{22}) +$$

$$- (R_{12}-R_{21})] - J_{m+2}(R_{11}-R_{22})[e^{i2\psi}\zeta - e^{-i2\psi}\zeta^*]$$

$$- J_{m+2}(R_{12}+R_{21})[e^{i2\psi} \zeta + e^{-i2\psi} \zeta^*]\} \tag{B.3}$$

where $J_m = J_m(\frac{k_\perp\mu'}{\epsilon})$, $\zeta = \zeta(\epsilon(1+m)+k_\parallel\mu)$, and $\zeta^* = \zeta^*(\epsilon(1+m)+k_\parallel\mu)$.

The terms containing zeta functions can be rewritten as

$$\zeta(x) - \zeta^*(x) = -2\pi i\ \delta(x)$$
$$e^{i2\psi}\zeta(x) - e^{-i2\psi}\zeta^*(x) = 2i[\sin 2\psi\ (\frac{P}{x}) - \pi \cos 2\psi\ (\delta(x))]$$
$$e^{i2\psi}\zeta(x) + e^{-i2\psi}\zeta^*(x) = 2[\cos 2\psi\ (\frac{P}{x}) - \pi \sin 2\psi\ (\delta(x))] \tag{B.4}$$

Now consider terms of the form

$$\int d^3k \sum_{m-\infty}^{\infty} J_m\ J_{m+2} \begin{Bmatrix} (R_{11}-R_{22}) \sin 2\psi \\ (R_{12}+R_{21}) \cos 2\psi \end{Bmatrix} \frac{P}{\varepsilon(1+m)+k_{\parallel}\mu} \tag{B.5}$$

and let n = m + 1, so that equation (B.5) becomes

$$\int d^3k \sum_{n=-\infty}^{\infty} J_{n-1} J_{n+1} \begin{Bmatrix} (R_{11}-R_{22}) \sin 2\psi \\ (R_{12}+R_{21}) \cos 2\psi \end{Bmatrix} \frac{P}{\epsilon n+k_{\parallel}\mu} \tag{B.6}$$

If we let $n \to -n$ and $\underset{\sim}{k} \to -\underset{\sim}{k}$ in equation (B.6), then equation (B.6) is equal to minus itself and all terms in (B.3) which contain principle value contributions are identically equal to zero. Equation (31) follows immediately.

Appendix C

Just for the purpose of developing this argument, in this Appendix, we will introduce a special notation which is different from that contained in the rest of this paper. Let,

$$\mu(\tau) = (\frac{\underset{\sim}{B}}{B}) \cdot \hat{\underset{\sim}{p}}\,(\tau) \qquad (C.1)$$

be the cosine of the pitch angle relative to the local (exact) field. Within the guiding center approximation to the particle motion, we assume $(\underset{\sim}{B}/B)$ constant over a gyroperiod, and assume that the particle moves in a helical trajectory in the field. Thus,

$$\mu(\tau) = (\frac{\underset{\sim}{B}}{B}) \cdot [\overline{\underset{\sim}{P}} + \overline{\underset{\sim}{N}} \cos \epsilon\tau - \overline{\underset{\sim}{\Omega}} \sin \epsilon\tau] \cdot \hat{\underset{\sim}{p}}(o) \qquad (C.2)$$

where the projection matrices, $\overline{\underset{\sim}{P}}$, $\overline{\underset{\sim}{N}}$, and $\overline{\underset{\sim}{\Omega}}$, are identical to those defined by equations (5), (13), and (14) except that they are based on the local, rather than the mean field. We now introduce the notation $\langle\mu\rangle \equiv \hat{\underset{\sim}{\beta}} \cdot \hat{\underset{\sim}{p}}$, for the cosine of the pitch angle relative to the mean field. The quantity which enters equation (47) is $\overline{\mu\ (\tau)}$, which is the average over one gyroperiod, of $\mu\ (\tau)$.

$$\overline{\mu\ (\tau)} = (\frac{\underset{\sim}{B}}{B}) \cdot \overline{\underset{\sim}{P}} \cdot \hat{\underset{\sim}{p}}(o) \equiv \mu \qquad (C.3)$$

The time dependence of $\langle\mu\rangle$ is given by,

$$\langle\mu\rangle = \hat{\underset{\sim}{\beta}} \cdot [\overline{\underset{\sim}{P}} + \overline{\underset{\sim}{N}} \cos \epsilon\tau - \overline{\underset{\sim}{\Omega}} \sin \epsilon\tau] \cdot \hat{\underset{\sim}{p}}(o) \qquad (C.4)$$

and its average over one gyroperiod is,

$$\langle\mu\rangle = \hat{\underset{\sim}{\beta}} \cdot \left[\frac{\hat{\underset{\sim}{\beta}} + \eta\, \underset{\sim}{\beta}'}{[1+2\eta(\hat{\underset{\sim}{\beta}}\cdot\underset{\sim}{\beta}') + \eta^2\ (\underset{\sim}{\beta}'\cdot\underset{\sim}{\beta}')]^{1/2}}\right]\ \mu \qquad (C.5)$$

Thus,

$$\overline{\langle\mu\rangle} = \mu + O(\eta^2) \qquad (C.6)$$

To O (η) in the radom field strength, $<\mu>$ and μ are identical when averaged over a gyroperiod. This averaging is implied in equation (47) since it follows from the guiding center approximation. In the rest of this paper we therefore, simply use the symbol μ to stand for the cosine of any of the relevant pitch angles.

Appendix D

By introducing the Fourier transform representation of the random field into equation (48), we find,

$$\frac{d\mu}{d\tau} = -\frac{1}{2}(1-\mu^2)\eta\left(\frac{1}{2\pi}\right)^{3/2}\int d^3k\, \hat{\underset{\sim}{\beta}}\cdot[(\hat{\underset{\sim}{\beta}}\cdot\underset{\sim}{\nabla})\, e^{i\underset{\sim}{k}\cdot\underset{\sim}{x}(\tau)}\underset{\sim}{\beta}'(\underset{\sim}{k})] \tag{D.1}$$

or,

$$\frac{d\mu}{d\tau} = -\frac{}{2}(1-\mu^2)\eta\left(\frac{1}{2\pi}\right)^{3/2}\int d^3k(\hat{\underset{\sim}{\beta}}\cdot\underset{\sim}{\beta}'(\underset{\sim}{k}))(\hat{\underset{\sim}{\beta}}\cdot\underset{\sim}{k})e^{i\underset{\sim}{k}\cdot\underset{\sim}{x}(\tau)} \tag{D.2}$$

Now, on remembering that, in this notation, $(1-\mu^2)\eta/2$ is the magnetic moment of the particle which is an adiabatic invariant of the motion of the particle in the guiding center approximation, we can formally integrate equation (D.2) and form its square to find,

$$<(\Delta\mu)^2> = -\left(\frac{(1-\mu^2)\eta}{2}\right)^2\left(\frac{1}{2\pi}\right)^3\int_0^{\Delta\tau}d\lambda\int_0^{\Delta\tau}d\lambda'\int d^3k\int d^3k'\; e^{i\underset{\sim}{k}\cdot\underset{\sim}{x}(\lambda)}e^{i\underset{\sim}{k}'\cdot\underset{\sim}{x}(\lambda')} \tag{D.3}$$

$$\cdot\;[(\underset{\sim}{k}\cdot\hat{\underset{\sim}{\beta}})\hat{\underset{\sim}{\beta}}\cdot<\underset{\sim}{\beta}'(\underset{\sim}{k})\underset{\sim}{\beta}'(\underset{\sim}{k}')>\cdot\;\hat{\underset{\sim}{\beta}}\;(\hat{\underset{\sim}{\beta}}\cdot\underset{\sim}{k}')]$$

If the magnetic turbulence is statistically homogeneous, we have equation (19), which can be inserted into equation (D.3), to obtain,

$$<(\Delta\mu)^2> = \left(\frac{(1-\mu^2)\eta}{2}\right)^2\left(\frac{1}{2\pi}\right)^{3/2}\int_0^{\Delta\tau}d\lambda\int_0^{\Delta\tau}d\lambda'\int d^3k\; e^{i\,\underset{\sim}{k}\cdot(\underset{\sim}{x}(\lambda)\;-\;\underset{\sim}{x}(\lambda'))}$$

$$\cdot\;[\underset{\sim}{k}_{\parallel}\cdot\underset{\sim}{R}(\underset{\sim}{k})\cdot\underset{\sim}{k}_{\parallel}]$$

As e xplained in the main text, we approximate the position of the particle, by the position of its guiding center. Thus, $\underset{\sim}{k}\cdot(\underset{\sim}{x}(\lambda)-\underset{\sim}{x}(\lambda'))= =k_{\parallel}\mu(\lambda-\lambda')$. Following the usual Fokker-Planck argument (see Appendix B) for obtaining the part of equation (D.4) which grows linearly with $\Delta\tau$, when $\Delta\tau$ is large, we obtain,

$$\frac{<(\Delta\mu)^2>}{\Delta\tau} = \left(\frac{(1-\mu^2)\eta}{2}\right)^2 \frac{\delta(\mu)}{\sqrt{2\pi}} \int d^3k \left(\frac{\underset{\sim}{k}_\parallel \cdot \underset{\sim}{R}(\underset{\sim}{k}) \cdot \underset{\sim}{k}_\parallel}{|k_\parallel|}\right) \tag{D.5}$$

This result is identical to that given by equation (53) which was obtained, in the low energy limit, from the usual quasi-linear approximation to the particle motion.

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