(Final Report)

SATELLITE TO AIRCRAFT MULTIPATH SIGNALS OVER THE OCEAN

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National Aeronautics and Space Administration
Langley Research Center
Hampton, Virginia 23365

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ABSTRACT

A satellite-to-aircraft communication link over the ocean may include a specularly reflected signal and a diffusely scattered signal, as well as the direct path transmission. This report provides a computer program for estimating two statistical properties of the diffuse signal, namely the Doppler spectrum and the delay spectrum. A convenient representation of the total signal is also provided. Examples of the Doppler and delay spectra for a number of representative link geometries are given to illustrate the features of the diffuse signal.

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I. INTRODUCTION

One of the design limitations[1] in the development of communication links between satellites and aircraft is the presence of a secondary signal reflected from the surface of the earth. It is the purpose of this report to provide a convenient method for estimating the statistical properties of this multipath signal[2,3], particularly the power spectrum and delay spectrum, for ocean surfaces. Because the scattering properties of the sea are rather complex, it is not possible to obtain closed form expressions for these statistics. Instead a computer program, and its output for a number of representative link geometries, have been provided. These results should provide an insight into the characteristics of the multipath signal. In addition, they may be used to construct a representation of the total signal which should be adequate for most detector design studies.

The heart of the report is the computer program in Appendix I. Although conventional designations for range, angle, frequency etc. are used in the body of the report, a number of the more important identifications between report variables and the corresponding program names are provided. The latter are given in capital letters enclosed in parentheses; for example the distance R_{dir} from aircraft to satellite is called (REF) in the program.

II. THE MULTIPATH MODEL

Before giving the details for estimating the statistical properties of the multipath signal, it will be useful to describe the overall geometry of the link. This will permit an understanding of the mechanisms which produce the total receiver signal, without having to consider such extraneous factors as antenna patterns and polarizations. The basic problem in modelling the link is to estimate the relative amplitudes, statistics, and signal distortions in the three components of the total signal transmitted from the satellite to a moving receiver.

The first component (see Fig. 1) is the direct transmission from transmitter to receiver. If a voltage reference level V_o is chosen such that the far field radiated by the transmitter has an electric field strength V_o/R at distance R from the transmitter (i.e., the transmitter power, gain etc. are subsumed in V_o) then the receiver voltage due to the direct component will be, at time t

(1) $$V_{dir}(t) = V_o(t-R_{dir}/c)\, h_d/R_{dir}(t)$$

where c = velocity of electromagnetic waves

R_{dir} = TP = distance from transmitter to receiver

and where h_d (the effective height of the receiving antenna) accounts for the receiver properties, including pattern and polarization, in the direction of the transmitter. The distortion of the signal (the Doppler shift for a cw signal) is accounted for by the change of R_{dir} with time

(2) $$R_{dir}(t) = R_{dir}(0) + (\vec{v}\cdot\vec{n}_d)t$$

where $\vec{v}$ is the constant aircraft velocity and $\vec{n}_d$ is a unit vector along TP.

The second component is that reflected from the surface (as if the sea were a smooth sphere) at the specular point O. This component is given by

(3) $$V_{spec}(t) = V_o(t-R_{spec}/c)\rho\, e^{-(kh\cos\theta_i)^2}\, h_{sp}/R_{spec}(t)$$

where

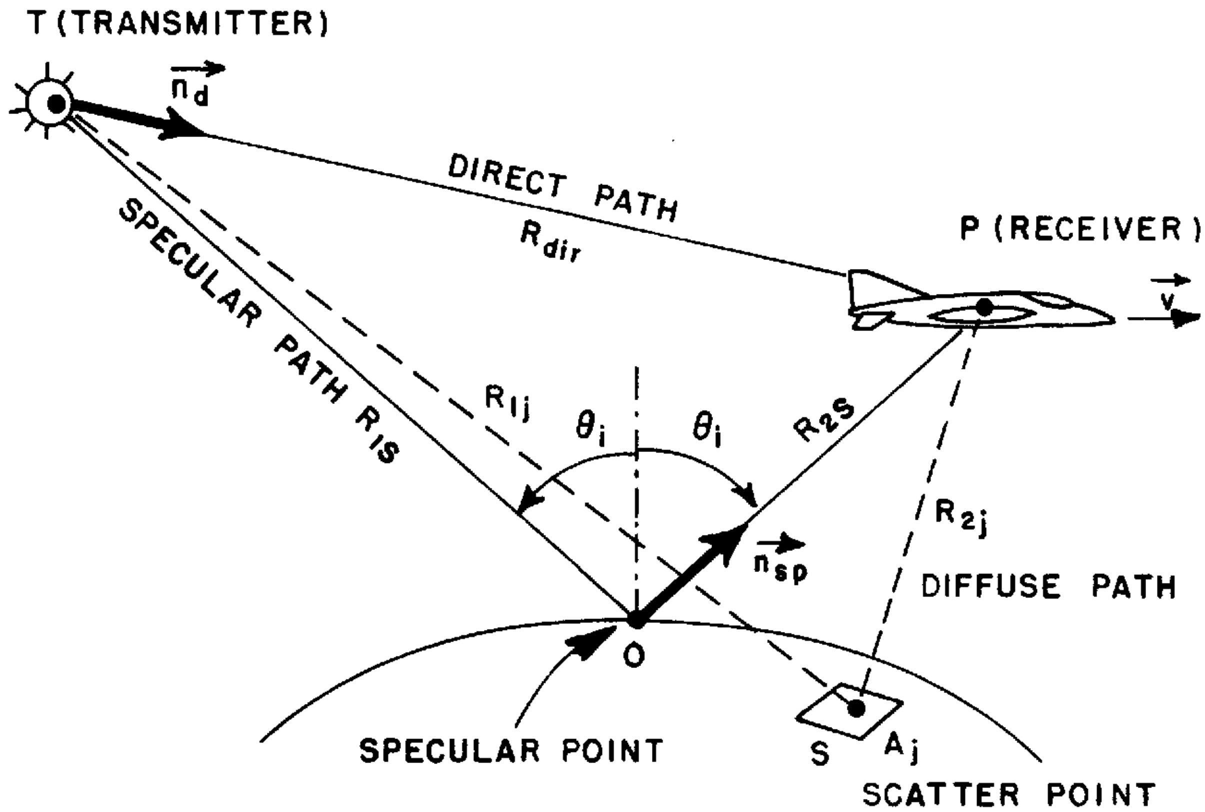


Fig. 1. Multipath links over the ocean.

R_{spec} = TO + OP
= $R_{1s} + R_{2s}$
= distance along specular path
ρ = voltage reflection coefficient at surface
k = $2\pi/\lambda$ where λ is electromagnetic wavelength
h = root mean square height of ocean surface
θ_i = angle of incidence at the specular point
h_{sp} = antenna effective height.

The exponential factor (the "Rayleigh roughness factor") accounts for the fact that since the sea surface is not smooth, some of the energy is scattered out of the specular signal. The distortion of the specular signal is also produced by the change in R_{spec} with time. If the satellite is much further from the surface than the aircraft,

(4) $$R_{spec}(t) \approx R_{spec}(0) + (\vec{v}\cdot\vec{n}_{sp})t$$

$\vec{n}_{sp}$ = unit vector from 0 to P.

Clearly the direct and specular terms are coherent, albeit with differing delays or doppler shifts, so their sum $V_{dir}(t) + V_{spec}(t)$ will fluctuate (exhibit "interference", or "height gain effect") with time[3].

Finally, the diffuse or "multipath" component is a superposition of reflected fields from suitably oriented "glint points" on facets on the surface. If the location (which determines the delay) and the curvature (which determines the amplitude) of each properly oriented facet were known, one could construct the diffuse signal by superposition of the reflections from each facet. The actual sea surface is random, however, so that one must in practice be content with a representation of the diffuse signal based on certain statistical properties of the ocean surface. The simplest such surface is found by dividing up the mean surface into convenient areas A_j (see Fig. 1) and associating with each area a reflected signal whose time delay is determined by the center point, and whose amplitude is determined by the average scattering cross-section of the area. Thus the diffuse component can be represented by

$$V_{diff}(t) = \sum_j \frac{V_o(t-(R_{1j}+R_{2j})/c)}{R_{1j}\,R_{2j}} \sqrt{\frac{\sigma_{oj}A_j}{4\pi}}\; h_j \tag{5}$$

where
R_{1j} = TS distance from T to scatterer
R_{2j} = SP distance from P to scatterer
h_j = receiver effective height for j-th direction
σ_{oj} = average cross-section per unit area[4] of j-th sub-area.

For any given choice of sub-areas, each individual term in Eq. (5) will be in a coherent relationship with the direct and specular contributions. Because the range terms R_{1j} and R_{2j} are continuously changing, the total diffuse signal will have a random character with well defined statistical properties (power density vs frequency, correlation coefficient, power density vs delay etc.).

The sum of the three terms given by Eqs. (1), (3) and (5) now provide a direct representation of the total receiver signal. Insofar as the cross-sections σ_{oj} are known, and the areas A_j are appropriately chosen, the representation will possess the same statistical properties as the actual multipath signal. It is difficult to estimate the minimum number of terms in Eq. (5) needed for a statistically valid representation, and this problem is not considered here. In practice, the sizes for A_j are probably most conveniently determined empirically, by reducing their size and increasing their number until the statistics of interest show no significant change. In this report, we will not make use of this direct representation of the total signal, since we calculate only two simple statistics of the multipath signal.

III. THE SYSTEM GEOMETRY

In order to introduce the program for estimating multipath signals, it is necessary first to establish a precise system geometry. This geometry is fixed by the positions of the transmitter T, the moving aircraft P, and the spherical earth, but is a good deal easier to visualize than to specify algebraically. The basic coordinate systems are most conveniently defined (see Fig. 2) by the specular point O, the center of the earth C, and the positions P and T. The specular point is that point on the mean surface of the earth in the plane TPC for which the angle θ_i(THI) between the local vertical CO and the line of sight OP is equal to the angle between CO and OT. An xyz coordinate system with origin O, with z-axis along the local vertical, and with x-axis in the POT plane may now be constructed. The tangent plane at the specular point is then the xy plane. The aircraft is at altitude H above the tangent plane with coordinates

$$x_p = H\tan\theta_i$$
$$y_p = 0$$
$$z_p = H.$$

The satellite has range R_{1s} (RSAT) from O, with coordinates

$$x_s = - R_{1s}\sin\theta_i$$
$$y_s = 0$$
$$z_s = R_{1s}\cos\theta_i.$$

The scatter point S on the surface of the earth has coordinates x,y,z, or may be defined by the polar coordinates Re,θ,ϕ (where Re (RE) is the radius of the earth) in the coordinate system x'y'z' with origin at C. In the computer program, variables X, Y define the scatter area center point at S, and the polar coordinates (STHR) $\equiv \sin\theta = (x^2+y^2)^{1/2}/Re$ and (SPH) $= \phi = \tan^{-1}y/x$ are computed. The distance (ZP) = $(-z) = Re(1-\cos\theta)$ of the J-th scattering point below the tangent plane, and the ranges

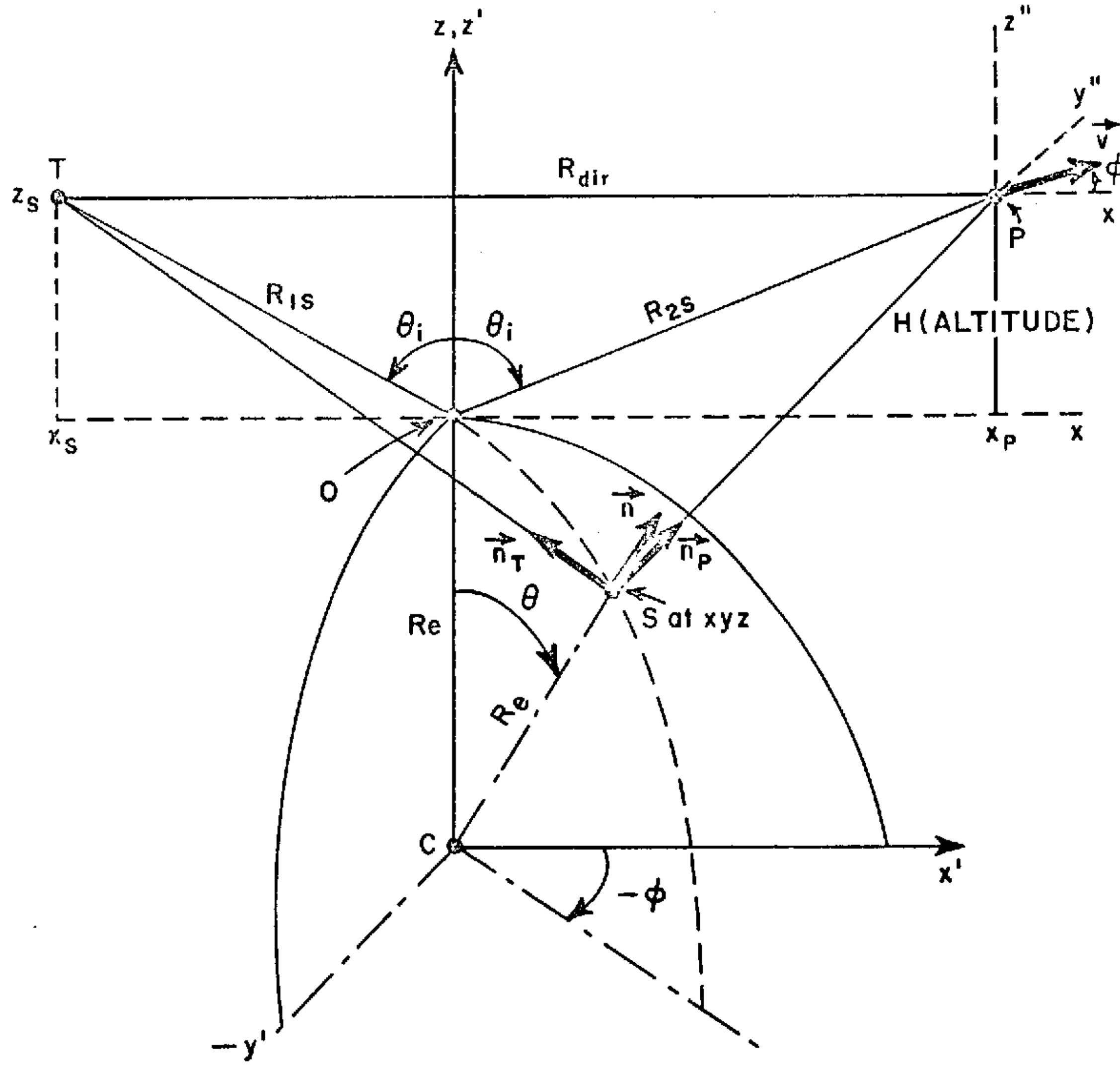


Fig. 2. Multipath geometry.

$$
(6) \qquad
\begin{aligned}
R_{1j} &= (RS(J)) = ST \\
R_{2j} &= (R(J)) = SP \\
R_{dir} &= (REF) = TP \\
R_{spec} &= (RELSP) = TO+OP
\end{aligned}
$$

are now easily found. Unit vectors $\vec{n}_P$ (along S to P), $\vec{n}_T$ (along S to T) and $\vec{n}$ (the local vertical at S) have components (CNI(I,J)), (CNS(I,J)) and (CN(I,J)) along the x(I=1), y(I=2) and z(I=3) axes respectively.

The aircraft, with position P is at the origin of a second coordinate system x", y", z" parallel to xyz, and is assumed to be moving with velocity $\vec{v}$ in the x" y" plane, with the velocity vector making an angle ϕ_v (PHV) with the x" axis.

The antenna patterns are specified as functions of polar angles θ_a, ϕ_a (see Fig. 3) in a polar coordinate system attached to the metal of the aircraft, with polar angle θ_a measured from the z" axis and azimuth angle ϕ_a measured from the velocity vector.

IV. ANTENNA PATTERNS AND POLARIZATIONS

One must next specify the antenna patterns and polarization states of the transmitter and receiver. For this purpose, it will be assumed that T is sufficiently far from O and P so that the satellite antenna gain and polarization state is uniform over the entire scattering area, including the specular point and the direct path. (Otherwise it would be necessary to introduce still another set of coordinates centered on the transmitter.) Two unit vectors $\vec{i}_{T1} = -\vec{i}_x\cos\theta_1 - \vec{i}_z\sin\theta_1$ and $\vec{i}_{p1} = -\vec{i}_y$ transverse to the line of sight may be used to specify the transmitter polarization via the two complex numbers A = (ATH) and B = (BPH). More precisely, the transmitted electric field in the vicinity of the earth may be written, at range R_1,

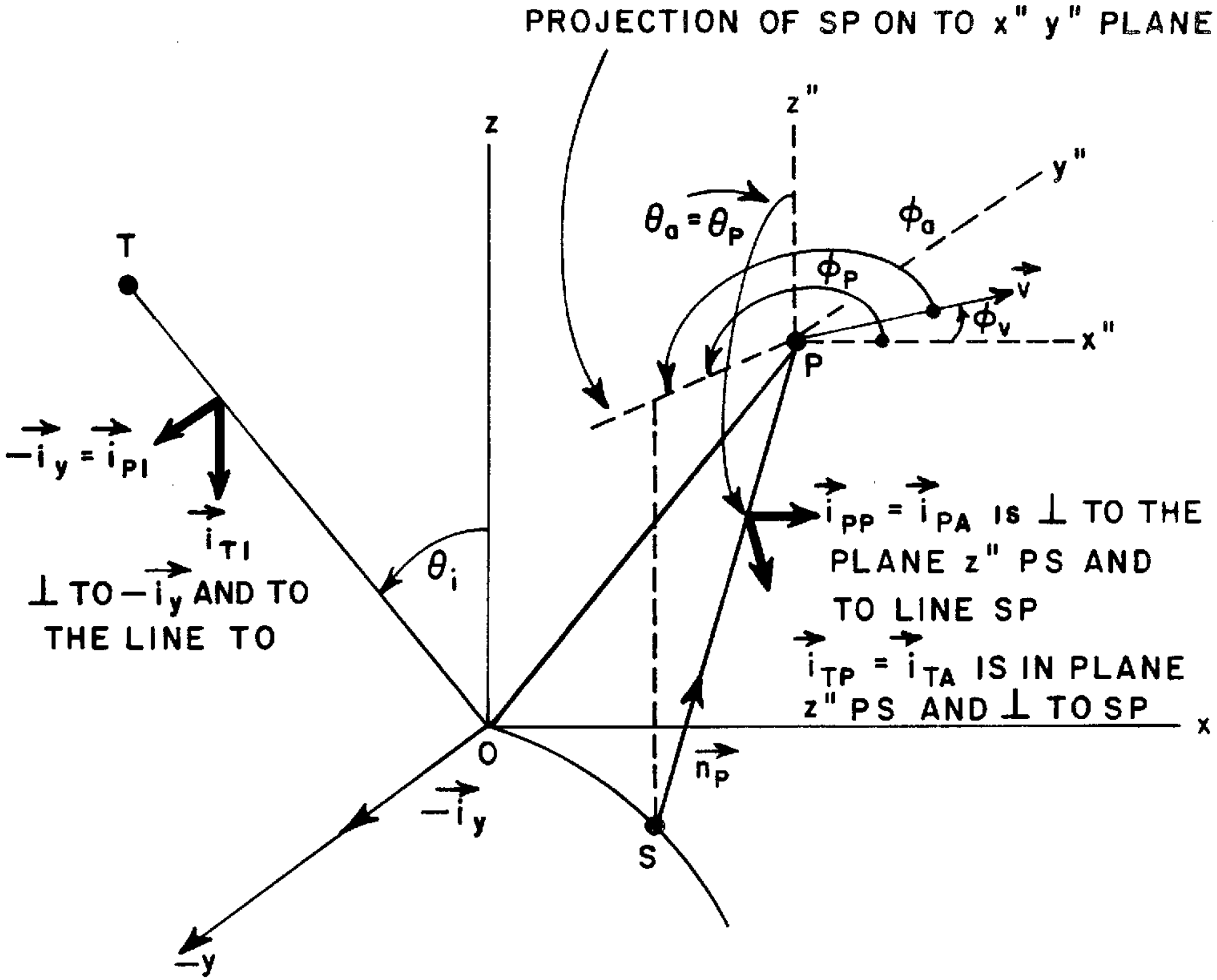


Fig. 3. Polarization unit vectors and antenna pattern coordinates.

(7) $$\vec{E}(x,y,z,t) = \frac{V_o(t-R_1/c)}{R_1}\,[A\vec{i}_{T1}+B\vec{i}_{P1}]$$

where the complex unit vector $\vec{P}_T$

(8) $$\vec{P}_T = (A\vec{i}_{T1} + B\vec{i}_{P1})$$

is the transmitter polarization. Since $\vec{P}_T\cdot\vec{P}_T{}^* = 1$, one must have AA* + BB* = 1. For example

A = (0,0) B = (1,0) "horizontal" polarization
A = (1,0) B = (0,0) "vertical" polarization
A = $(1/\sqrt{2},0)$ B = $(0,+j/\sqrt{2})$ "right circular".

If the transmitter is radiating a cw signal, of angular frequency ω_o, or frequency f_o (F), then the reference level voltage is

(9) $$V_o(t) = \left(\frac{P_T G_T Z_o/4\pi}{R_1}\right)^{1/2} \exp\left[j\omega_o\left(t - \frac{R_1}{c}\right)\right]$$

where P_T = transmitted power
G_T = transmitter antenna gain
Z_o = 120π ohms.

The receiver antenna is specified by its maximum receiving aperture A_m, its pattern function $f(\theta_a,\phi_a)$ and its polarization state

(10) $$\vec{P}_R(\theta_a,\phi_a) = C(\theta_a,\phi_a)\vec{i}_{TA} + D(\theta_a,\phi_a)\vec{i}_{PA}$$

where again CC* + DD* = 1, and the unit vectors $\vec{i}_{TA}$ (unit θ_a vector) and $\vec{i}_{PA}$ (unit ϕ_a vector) are shown in Fig. 3. In the x"y"z" system, the polar coordinates defining the line of sight, P to S, are θ_p,ϕ_p with unit vectors $\vec{i}_{TP}$, $\vec{i}_{PP}$. These two sets are related by

$$\theta_a = \theta_p \qquad \vec{i}_{TA} = \vec{i}_{TP}$$
$$\phi_a = \phi_p - \phi_V \qquad \vec{i}_{PA} = \vec{i}_{PP}.$$

The receiver complex polarization states C = (CTH) and D = (DPH) are specified in terms of the polarization that the antenna would produce if it were operated as a transmitter. That is, the receiver antenna, if excited by a cw signal, would produce an electric field $E_{ref}[\vec{P}_R]\exp[j\omega_0(t-R/c)]$ in the vicinity of S where E_{ref} is a reference level.

Thus, for example, in direct transmission from satellite to receiver, the receiver voltage would be proportional to

$$V_{rec}(t) = \frac{V_0(t-R_{dir}/c)}{R_{dir}} (\vec{P}_T \cdot \vec{P}_R) \sqrt{f(\theta_a', \phi_a')} \; K$$

where $\theta_a', \phi_a' = \pi - \phi_v$ are the values of θ_a, ϕ_a in the direction P to T of the transmitter, and K is a proportionality constant related to the receiver gain.

The factor $\vec{P}_T \cdot \vec{P}_R$ = AC + BD is the polarization mismatch factor. For example, for the receiver,

$C = (0,0)$	$D = (1,0)$	"horizontal"
$C = (1,0)$	$D = (0,0)$	"vertical"
$C = (0,j/\sqrt{2})$	$D = (1/\sqrt{2},0)$	"right circular"

Thus the polarization mismatch factor between a right circular transmitter ($A = 1/\sqrt{2}$, $B = j/\sqrt{2}$) and a right circular receiver ($C = j/\sqrt{2}$, $D = 1/\sqrt{2}$) is AC + BD = j; that is, the "right circular" receiver completely absorbs "right circular" incident radiation.

V. THE SCATTERING PROBLEM

Since the intensity of the multipath signal depends on σ_0, the scattering cross-section per unit area, we next consider scattering model which will provide this parameter. The predominant contribution to the diffuse energy comes from a series of specular (forward) scatter points; the cross-section for scattering of this type at microwave frequencies is derived from the "optical" or "physical optics" model[4]. In this model, the total cross-section is the product of two factors, the first a polarization transforming reflection coefficient, and the second a probability density function for surface slope. If we consider an element of the mean surface of area $dS = dxdy \sec\theta$, at a point where the local vertical is $\vec{n}$, then the geometry for specular reflection will be as illustrated in Fig. 4. Those facets in dS which are suitably oriented for specular reflection will have a local normal $\vec{n}_0$ which is the coplanar bisector of $\vec{n}_T$ and $\vec{n}_P$. The angle θ_n = (TH) = $\cos^{-1}(\vec{n}\cdot\vec{n}_0)$ between the local vertical and the normal to the tilted specular facet then determines, from the known slope distribution of the surface, the probability that the facet normals are properly oriented. The angle between the incident polarization vector $\vec{P}_T$ and the vector perpendicular to the plane of incidence $\vec{q} = \vec{n}_P \times \vec{n}_T / |\vec{n}_P \times \vec{n}_T|$ determines the reflection coefficients and polarization transformation.

Thus the computational problem is to determine for each point dS, the voltage response of a polarized receiver due to a plane wave of known polarization and direction which has been reflected by a plane surface of complex dielectric constant ε. The subroutine (COEF) determines this in the following manner. Consider (see Fig. 4) a coordinate system ξ,η,ζ with ζ the local vertical and ξ, η, the plane of the mean surface dS. A plane wave is incident from a direction θ_{inc} in the ξ,ζ plane. A scattering direction is defined by polar angles θ_{scat} and ϕ_{scat}. The problem is to find the polar angles θ_n ϕ_n which define $\vec{n}_0$ the tilted surface normal required to produce specular reflection from the given incident wave into the chosen scattering direction. The normal clearly lies in the plane of, and bisects the angle TSP. The scattering angle (the angle of incidence in the "plane of incidence" system) is α with $2\alpha = \angle TSP$. The subroutine finds θ_n, ϕ_n and α, given θ_{inc}, θ_{scat}, ϕ_{scat} by standard trigonometric manipulation. If the incident plane wave has polarization $\vec{P}_T = A\vec{i}_{T1} + B\vec{i}_{P1}$ and the receiver has polarization state

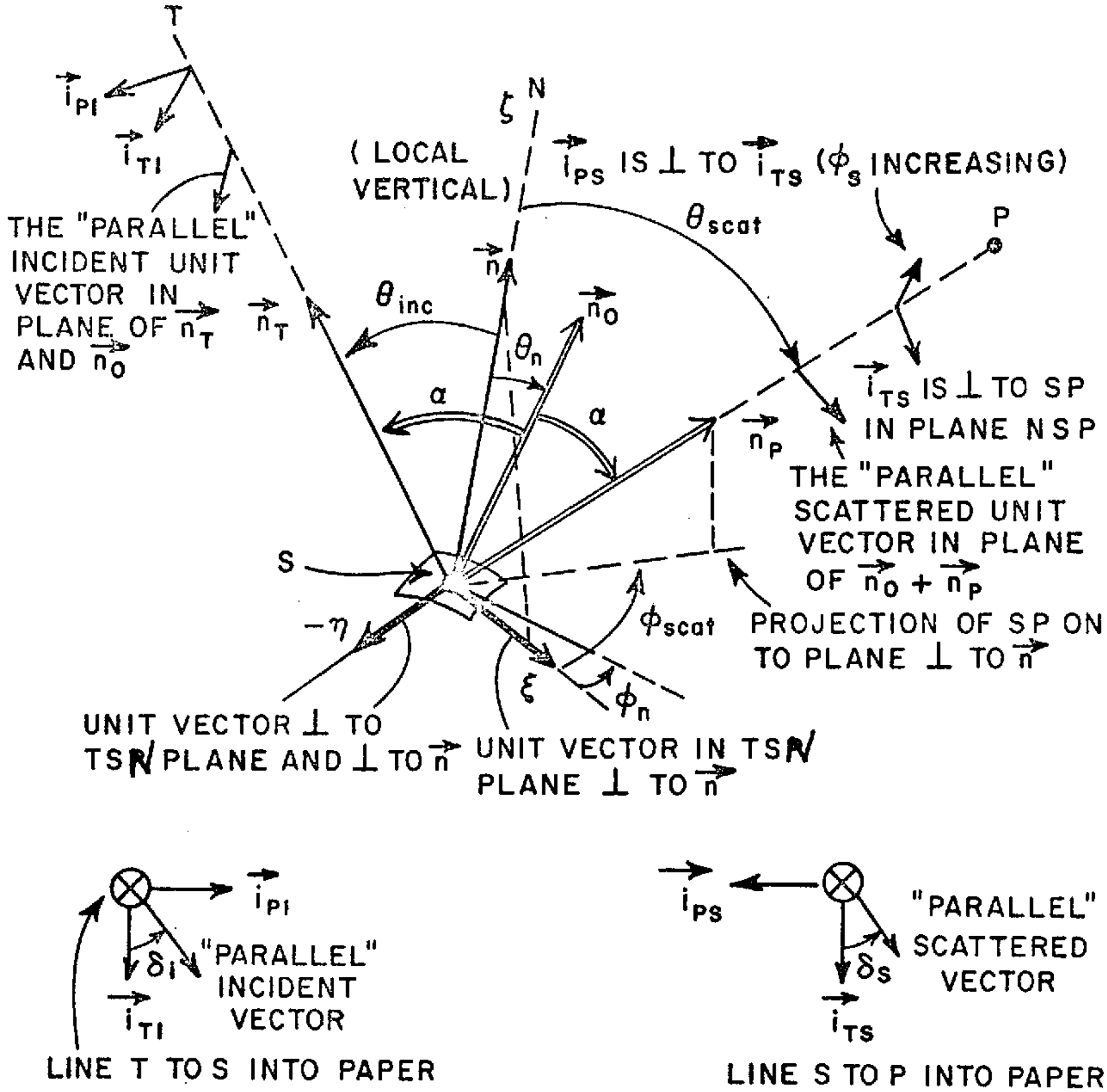


Fig. 4. The scattering coordinate system.

(11) $$\vec{P}_R = C_1 \vec{i}_{TS} + D_1 \vec{i}_{TS}$$

where the unit vectors are as indicated in Fig. 4, then it is simple to show, by resolving $\vec{P}_R$ into components parallel and perpendicular to the plane of incidence, that the effective reflection coefficient and polarization mismatch factor R_{cf} (RCOEF) is

(12) $$R_{cf} = R_{\parallel}(\alpha)\,[A\cos\delta_i + B\sin\delta_i][C_1\cos\delta_s - D_1\sin\delta_s] - R_{\perp}[B\cos\delta_i - A\sin\delta_i][D_1\cos\delta_s + C_1\sin\delta_s]$$

where

(13)
$$R_{\parallel}(\alpha) = (\varepsilon\cos\alpha - E)/(\varepsilon\cos\alpha + E) \qquad \text{(RPAR)}$$
$$R_{\perp}(\alpha) = (\cos\alpha - E)/(\cos\alpha + E) \qquad \text{(RPER)}$$
$$E = (\varepsilon - \sin^2\alpha)^{1/2}$$

where δ_i (see Fig. 4) is the angle between i_{T1} and the incident "parallel" unit vector, and δ_s is the angle between $\vec{i}_{TS}$ and the reflected "parallel" unit vector. (A complete discussion of this simple but tedious problem is given in Ref. 4). Thus the complete subroutine

$$\text{COEF}(\theta_{inc}, \theta_{scat}, \phi_{scat}, \varepsilon, A, B, C_1, D_1)$$

accepts calculated values of the calling parameters, and returns α, δ_i, δ_s, θ_n, ϕ_n and R_{cf}. The only problem remaining in the use of the subroutine is the connection between the scatter geometry of Fig. 4, and the system geometry of Fig. 3. By comparing these figures it is clear that $\vec{i}_{TS} = \vec{i}_{TP}$ and $\vec{i}_{PS} = -\vec{i}_{PP}$. In the program, this is accounted for by changing the sign of D, i.e., $C_1 = C$ and $D_1 = -D$ (DPHI = - DPH). Notice also that a slight error (entirely negligible for satellites in stationary orbits) is incurred in the program by assuming that the incoming wave polarization vectors do not depend on the scatter point position.

VI. THE RECEIVED POWERS

With the coordinates and scattering parameters defined, we are now in a position to compute the relative power level, the doppler shift, and the time delay in each of the three transmission paths.

A. The Direct Power

The direct power received by the aircraft from a distant satellite is simply

(14) $$P_{dir} = \frac{P_T G_T A_m}{4\pi\ R_{dir}^2} f(\theta_i, \pi-\phi_v)\ |AC + BD|^2$$

$$C = C(\theta_i, \pi-\phi_v)$$

$$D = D(\theta_i, \pi-\phi_v)$$

except for a small error in the mismatch factor due to the finite range of the satellite. The direct signal is received at a frequency $f_o + f_{dir}$, where f_{dir} is the doppler shift,

(15) $$f_{dir} = - \frac{\vec{v}\cdot\vec{n}_d}{c} f_o \qquad \text{(DDD)}$$

and f_o is the carrier frequency. The direct path doppler is printed out in H_z, a positive doppler indicating the received frequency is higher than the transmitter carrier frequency. In the program, the direct power is computed relative to a reference power level, namely that received by a polarization matched receiver at the specular point. That is, the direct power is calculated and printed as

(16) $$\text{PDIR} = (R_{1s}/R_{dir})^2\ |AC + BD|^2\ f$$

so that the actual direct power in watts is

(17) $$P_{dir} = (P_T G_T A_m / 4\pi R_{1s}^2)\cdot(\text{PDIR})\ .$$

The time delay on the direct path is taken to be zero, so that all other delays are relative to the direct path delay.

B. The Coherent (Specular) Reflected Power

The magnitude of the specular path power is less than the direct path mainly because of the surface roughness (Rayleigh Factor) but also because of the reflection coefficient and the slightly longer path length. In the program the specular power is computed as

(18) $$PSPEC = (R_{1s}/R_{spec})^2 f(\pi-\theta_i, \pi-\phi_v) |R_{cf}|^2 \exp(-2(kh\cos\theta_i)^2)$$

where
$$R_{cf} = RCOEF(\theta_i,\ \theta_i,\ 0,\ \varepsilon,\ A,\ B,\ C,\ D_1)$$
$$C = C(\pi-\theta_i,\ \pi-\phi_v)$$
$$D_1 = -D(\pi-\theta_i,\ \pi-\phi_v)$$

so that again the actual specular power in watts is

(19) $$P_{spec} = (P_T G_T A_m/4\pi R_{1s}^2)\cdot(PSPEC).$$

However, the value of PSPEC printed out is the ratio 10 log (P_{spec}/P_{dir}) i.e., it is the ratio of specular to direct (not reference) power in decibels. This is a more significant ratio than the actual specular power for system studies. At L-band, P_{spec} is negligible except for very smooth seas, or near grazing incidence.

The specular doppler shift is given by

(20) $$f_{spec} = -\frac{\vec{v}\cdot\vec{n}_{sp}}{c} f_o \qquad \text{(DDO)}$$

and is printed out in Hz. It is almost identical to the direct doppler. The time delay of the specular path power is calculated from

(21) $$\tau_{sp} = DELSP = (R_{1s} + R_{2s} - R_{dir})/c$$

and is printed out in microseconds.

C. The Diffuse Power

The program computes the diffuse power by dividing the xy plane into squares of side dx(DX) and dy(DY) centered at $x = (J + 1/2)dx$, $y = (K + 1/2)dx$ (J,K integers). Each square is then projected onto the spherical earth surface, for which the actual area becomes $dxdy/\cos\theta$. The width of the square is taken as

$$dx = \left(\frac{\eta}{JJJ}\right) \frac{2H}{\cos^2\theta_i}$$

where JJJ is an integer of order 20 to 35. This choice of square size ensures a smaller integration area for surfaces of small root mean square slope η. The incremental power scattered by this area and absorbed by the receiver is then given by

$$(22) \qquad dP(J,K) = \frac{P_T G_T A_m}{4\pi\ R_{1j}^2} \cdot f(\theta_a,\phi_a)\ \sigma_{oo}\ \frac{|R_{cf}|^2\ dxdy}{4\pi\ R_{2j}^2\ \cos\theta}$$

where σ_{oo} is the scattering cross-section per unit area of a perfectly conducting rough surface, and R_{cf} accounts for the reflection coefficient of the actual surface, and receiver polarization mismatch. The scattering cross-section for the sea surface, based on the physical optics model, is given by[4]

$$(23) \qquad \sigma_{oo} = \pi \sec\theta_n\ P(\theta_n \phi_n)$$

where θ_n, ϕ_n are polar angles of the normal $\vec{n}_o$ required for reflection (see Fig. 4). Here $p_n(\theta_n\phi_n)$ is the probability density function for the surface normal $\vec{n}_o$, i.e., $p(\theta_n,\phi_n)d\Omega_n$ is the probability that the normal lies in the cone of solid angle $d\Omega_n = \sin\theta_n\ d\theta_n\ d\phi_n$. For the sea surface the actual slope distribution is a rather complicated function of the wind velocity and its history[5]. For the purposes of this report it is sufficient to assume the rather simple isotropic form

$$p(\theta_n,\phi_n) = \frac{\cos\theta_n}{\pi\eta^2}\; e^{-\frac{\tan^2\theta}{\eta^2(1+2\eta^2)}} \qquad \text{(PDEN)} \tag{24}$$

where η^2 = mean square surface slope
= $\langle\tan^2\theta_n\rangle$ (average value of $\tan^2\theta_n$ over surface).

This form is quite similar to a cross-wind section of the standard (Cox-Munk) distribution[5] and, for surfaces of moderate slope, very nearly satisfies the normalization conditions

$$\begin{aligned} &\int P(\theta_n\phi_n)d\Omega_n = 1 \\ &\int P(\theta_n\phi_n)\tan^2\theta_n d\Omega_n = \eta^2 \quad . \end{aligned} \tag{25}$$

Consistent with the power normalization used for the direct power, the normalized power increment (DP) is computed as

$$DP = f(\theta_a,\phi_a)|R_{cf}|^2\, \frac{e^{-\tan^2\theta_n/\eta^2(1+2\eta^2)}}{\pi\eta^2}\, \frac{dxdy}{R_{2j}^2\cos\theta}\, (\tfrac{1}{4}) \tag{26}$$

so that the actual incremental power in watts is

$$dP = DP[P_tG_tA_m/4\pi\, R_{1s}^2]. \tag{27}$$

(It will be noted that a small error is incurred in Eq. (26) because the transmitter to surface distance is assumed to be R_{1s} at all times, instead of the variable R_{1j}. For satellites in stationary orbit the difference between Eq. (26) and the correct power level is insignificant.) The doppler and delay for this element, as before, are given by

$$\begin{cases} f_{diff} = -\dfrac{\vec{n}_p \cdot v}{c} f_o \\ \tau_{diff} = (R_{1j} + R_{2j} - R_{dir})/c \end{cases} \tag{28}$$

In the actual program, the incremental powers are calculated for four cells (at ± x, ± y) at a time, beginning with the four cells nearest the specular point. These four cells determine a reference power level (DPI). The subsequent sets are obtained by increasing $|x|$ with $|y|$ constant until the power from a set is less than some predetermined fraction of the power found in the first set. (1/1000 in the program of Appendix I.) At this point y is incremented, x returned to zero and the process repeated until further increase in y fails to produce sufficient power to satisfy the above condition. At this point the iteration stops, and the maximum values of x(XMAX) and y(YMAX) are printed. The values XMAX and YMAX then define an ellipse within which all significant scattering occurs.

As the power from each set is computed, the doppler shift and the time delay for the path is also found, and the incremental power assigned to a doppler or delay bin. In this program 128 doppler bins cover the range of $2v/c\ f_o$, the maximum possible range of doppler frequencies. The delay bins are 2 nanoseconds wide. Thus one can determine the doppler spectrum, (the power per Hz vs frequency) and the delay spectrum. In addition, the total diffuse power (PT), the sum of the power in all bins, is calculated in dB below the direct power.

(29) $$PT = 10 \log(\text{Total Diffuse Power}/P_{dir}).$$

VII. THE PROGRAM PARAMETERS

To utilize the program, listed in Appendix I; a number of parameters must be specified. These are:

A. Ocean Temperature and Salinity

TEM The temperature of the ocean in °C.

SAL The salinity of the ocean in parts per thousand. (Typically SAL = 35 ppt.)

F The carrier frequency in GHz.

These three parameters determine the complex dielectric constant of the ocean via the complex function (EPS). This subroutine calculates ε according to the formula given by Stogryn in Reference [6]. The frequency F should probably be limited to the range 0.4 GHz $<f<5$ GHz in order for the ocean scatter mechanism to be valid.

B. Ocean Slope and Roughness

ANG is the root mean square slope of the ocean in degrees.
RUF is the root mean square height of the ocean in meters.

In terms of the report variables, ANG = $(180/\pi)\tan^{-1}\eta$ and RUF = h. These two oceanographic parameters may be chosen in a number of obvious ways. For many types of calculation however, it is convenient to relate them to a single variable, the wind speed W over the ocean. Thus, for example one may modify a standard[5] expression for the mean square slope of the ocean surface to obtain

(30)
$$\eta^2 \approx 0.006\ \ln(K_m/K_{oc}).$$
$$K_{oc} = g/W^2$$
$$g = \text{Acceleration of Gravity} = 9.81\ \text{m/sec}^2$$
$$K_m = \gamma k = \gamma 2\pi/\lambda$$
$$\lambda = \text{electromagnetic wavelength}$$
$$\gamma = \text{empirical constant} \approx 4$$

where K_{oc} is the wave number for the largest ocean waves developed at a wind velocity W and K_m is the wave number for the smallest ocean waves that are effective in the electromagnetic scattering process. That is, expressions for the mean square slope in the oceanographic literature are based on the actual ocean surface, including many small waves (ripples) superimposed on the large scale wave structure. These small scale waves do not participate in the forward scattering process and thus the effective mean square slope is smaller than the oceanographic value. The estimate

$K_m = 4\ k$ is an empirical result which should be valid for frequencies up to 5 GHz and moderate to large wind velocities. Similarly, the roughness may be estimated from

$$h^2 = .003/K_{oc}^2 = .003\ W^4/g^2$$

$$h = .055\ W^2/g \text{ meters}$$

for wind velocities greater than a few meters/second.

C. Aircraft Velocity

VEE = aircraft speed in meters/sec

PHV = angle of velocity vector from x-axis in degrees.

Thus when PHV = 0 the aircraft is flying directly away from the satellite.

D. Geometry of Path

THI = angle of incidence at the specular point in degrees

H = aircraft altitude above x,y plane in meters.

E. Antenna Polarization and Patterns

ATH, BPH	Two complex numbers defining the polarization state of the transmitting antenna. (A, B in report)
$CTH(\theta_a,\phi_a)$ $DPH(\theta_a,\phi_a)$	Two complex functions defining the polarization state of the receiving antenna (C, D in report)
$ANTP(\theta_a,\phi_a)$	Antenna power pattern function $f(\theta_a,\phi_a)$ in report.

In the program in Appendix I, the antenna power pattern $f(\theta_a,\phi_a)$ is taken as isotropic, i.e., f = 1. The two functions C and D are also taken as constants, i.e., the polarization of the receiver is independent of look direction.

F. <u>Integration Parameter</u>

JJJ = an integer defining increment in x.

If JJJ is taken too small, (say less than 10), the density and delay spectra appear erratic because the elementary areas are too large. If JJJ is too large, (say greater than 40) the running time becomes prohibitive. Reasonable values appear to lie in the range 20 < JJJ < 30.

G. <u>Program Output</u>

The output of the program, an example of which is given in Appendix II, are the direct power, the specular power in dB and the total diffuse power in dB; the specular and direct doppler shifts; the specular delay; and the bin widths of the doppler and delay spectra. Note that the unshifted carrier frequency is always in bin 65, and that the bin which would contain the direct and specular returns is also indicated. The complex dielectric constant is printed out as EPS.

The graphical output consists of two graphs, one of power per doppler bin vs doppler bin number, and the other of power per delay bin vs delay bin number. The widths of the doppler and the delay bins are indicated. The total power in the doppler spectrum should sum to the output diffuse power level when converted to dB.

Note that the diffuse and specular power are stated in dB "below" the direct power. However a negative number indicates that the specular or diffuse power is smaller than the direct power. Note also that when the receiver is blind to the direct power (right circular transmitter to left circular receiver, for example) the direct power is zero, so that the program must be modified if the specular or diffuse power is desired for this case.

VIII. RESULTS AND CONCLUSIONS

Although the purpose of this report is to document the program of Appendix I, and to provide, through Eqs. (1), (3) and (5) convenient realizations of the multipath signal, a number of cases have been run to illustrate the principal features of the diffuse multipath spectra.

In Fig. 5, the doppler spectrum is plotted vs doppler bin for several values of ϕ_v, the aircraft flight direction. In each graph the location of the specular power is shown. Evidently, the spectra tend to be centered on the specular signal, indicating that the major part of the diffuse power is emanating from a scattering region near the specular point. Figure 6 indicates the effect of increasing the surface roughness (mean square slope) on the shape and intensity of the scattered signal. Notice that for aircraft velocities nearly along the line of sight, the spectra are skewed towards the extreme doppler bins, especially for the rough surfaces. Note also a tendency for the diffuse power to saturate as the mean square slope increases. Figure 7 illustrates the variation of doppler spectrum as the angle of incidence is varied, and Fig. 8 shows the effect of polarization on the spectrum, and on the direct and diffuse total power. Clearly an appropriate choice of transmitter and receiver polarization can significantly influence the character of the total signal. Figure 9 shows several typical delay spectra and illustrates the fairly rapid initial decay and the extended tail echo for certain geometries. Note that in the program, the power in delay bins beyond the hundredth has been accumulated in the last bin.

It may be concluded that the program of Appendix I provides a convenient means for estimating a number of features of the multipath signals which occur when a satellite to aircraft link is maintained over the ocean. It should be possible to undertake the study of a variety of detectors using either the statistical signal properties given by the program, or the signal realization given in Section II. In situations where standard detection schemes must be used, the program may be easily modified to investigate the feasibility of multipath rejection by antenna pattern or polarization control.

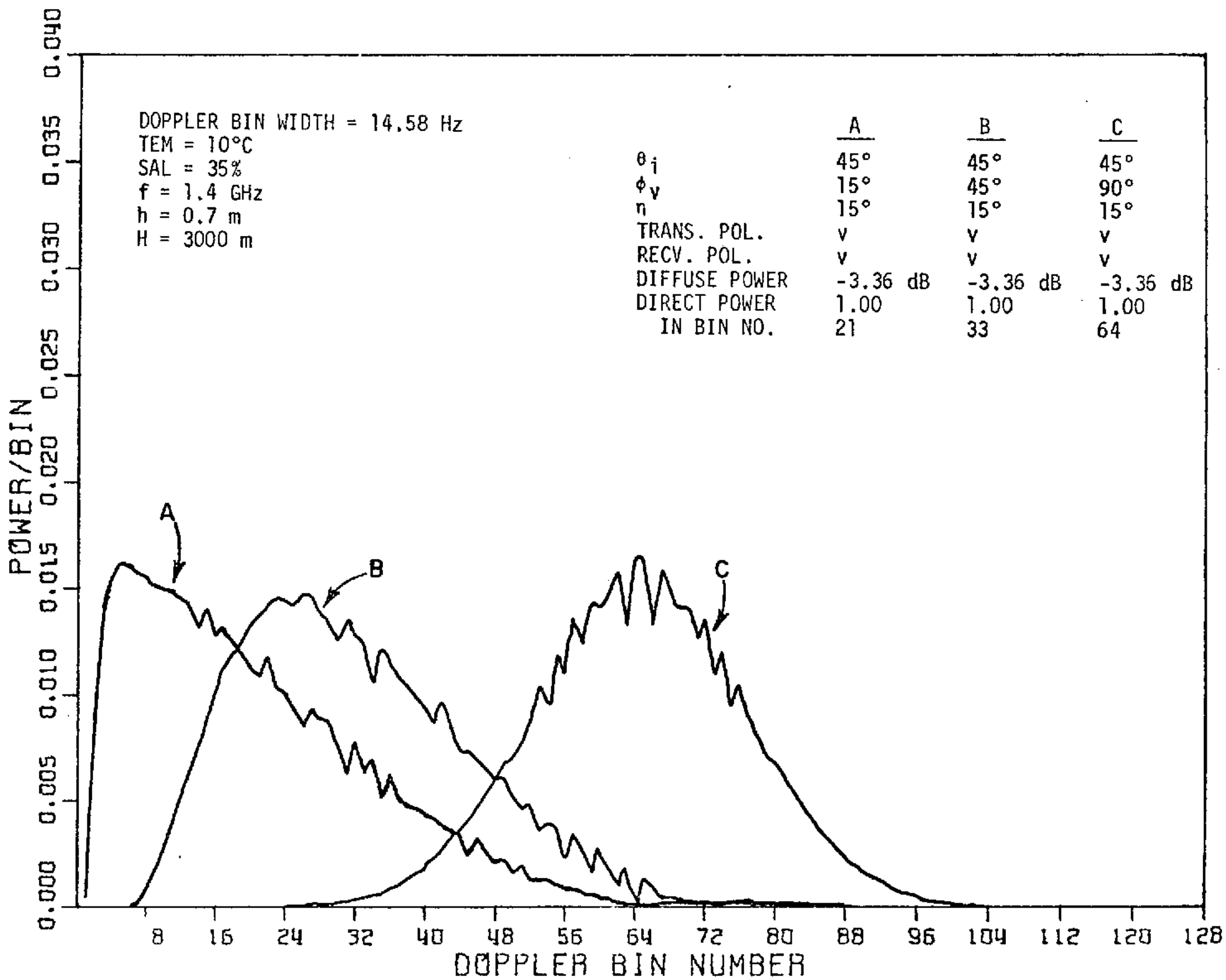


Fig. 5. Doppler spectrum with aircraft direction as parameter.

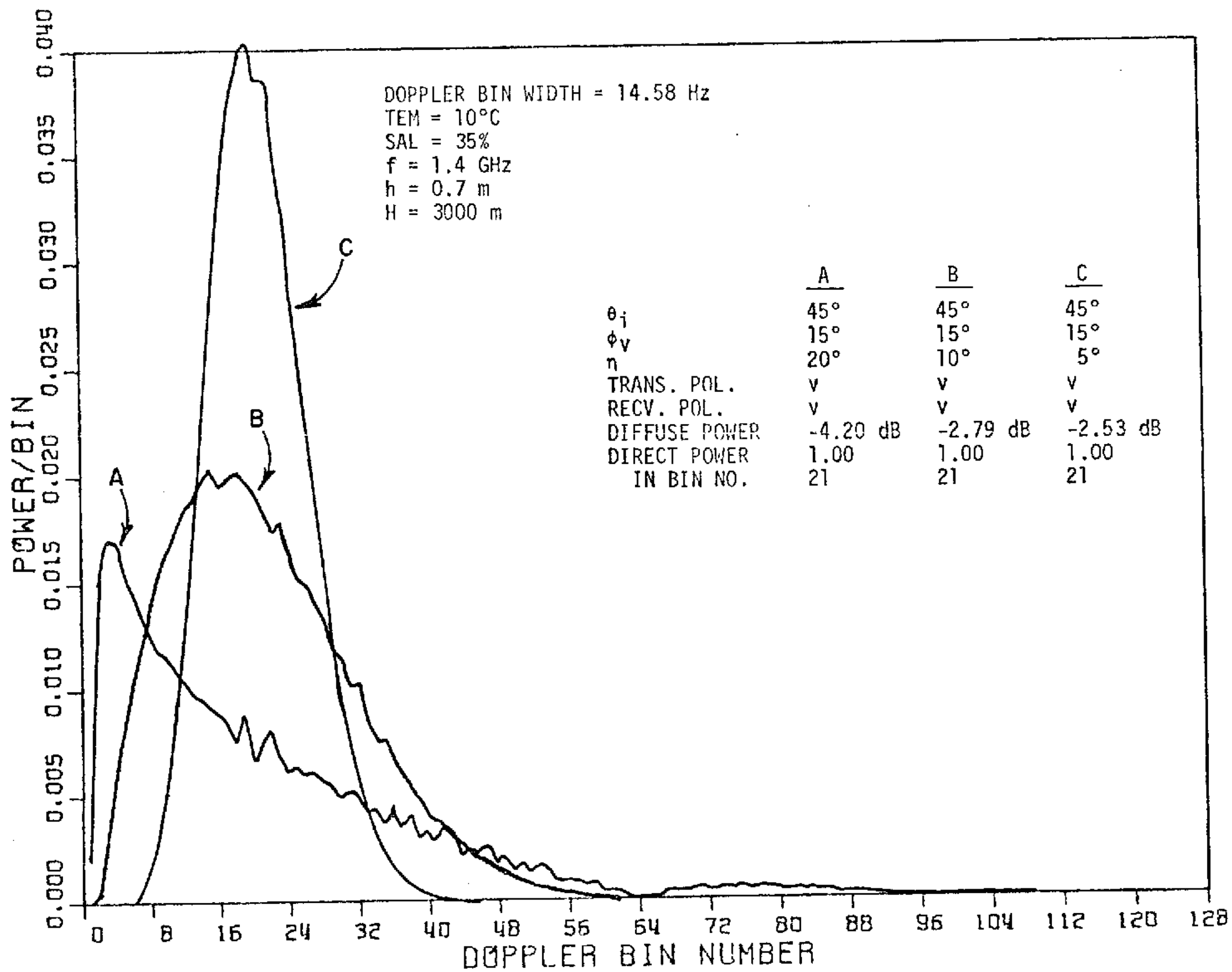


Fig. 6. Doppler spectrum with surface slope as parameter.

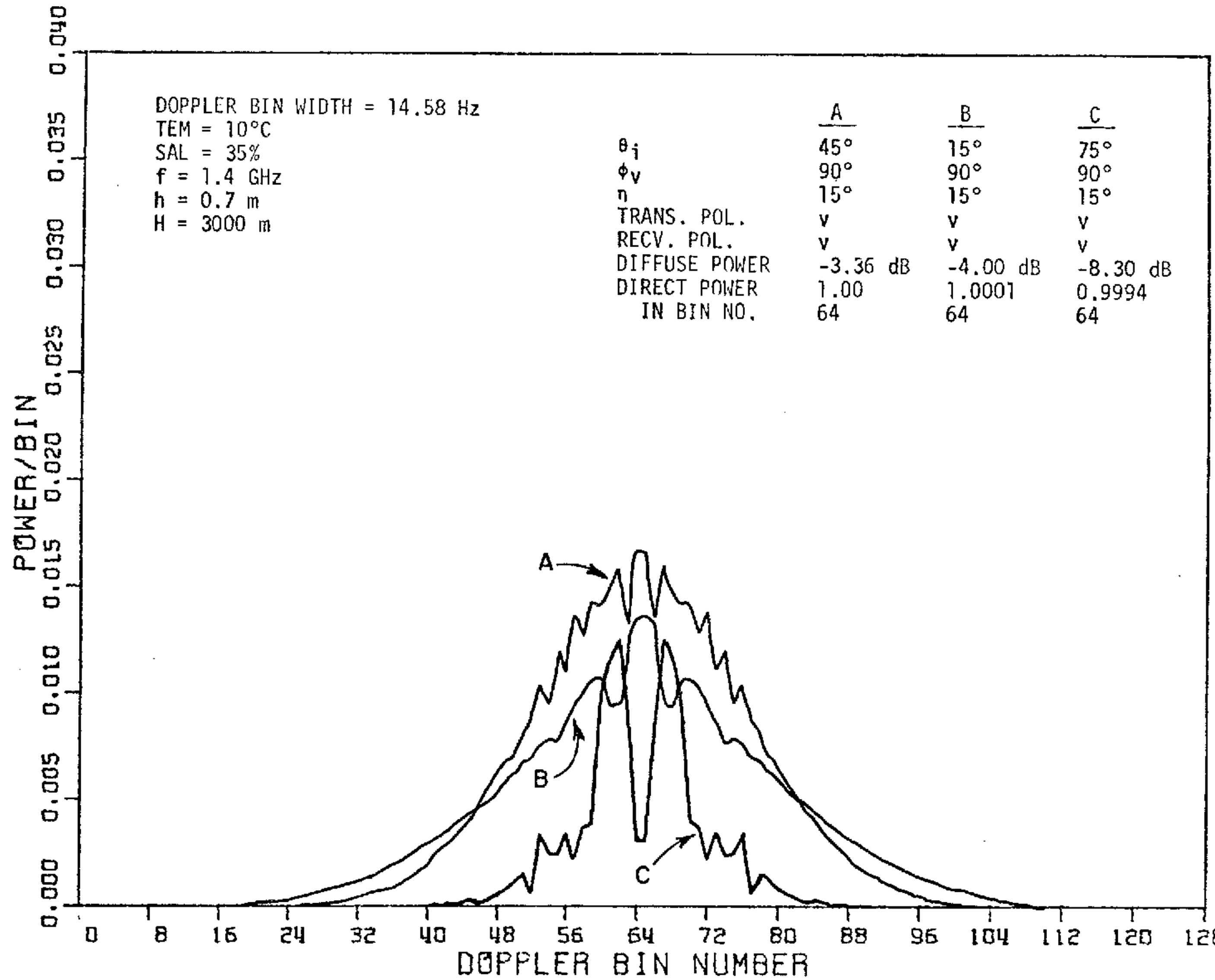


Fig. 7. Doppler spectrum with angle of incidence as parameter. The oscillation in curve C between bins 58 and 70 is an artifact of the program, and illustrates the effect of choosing JJJ too small.

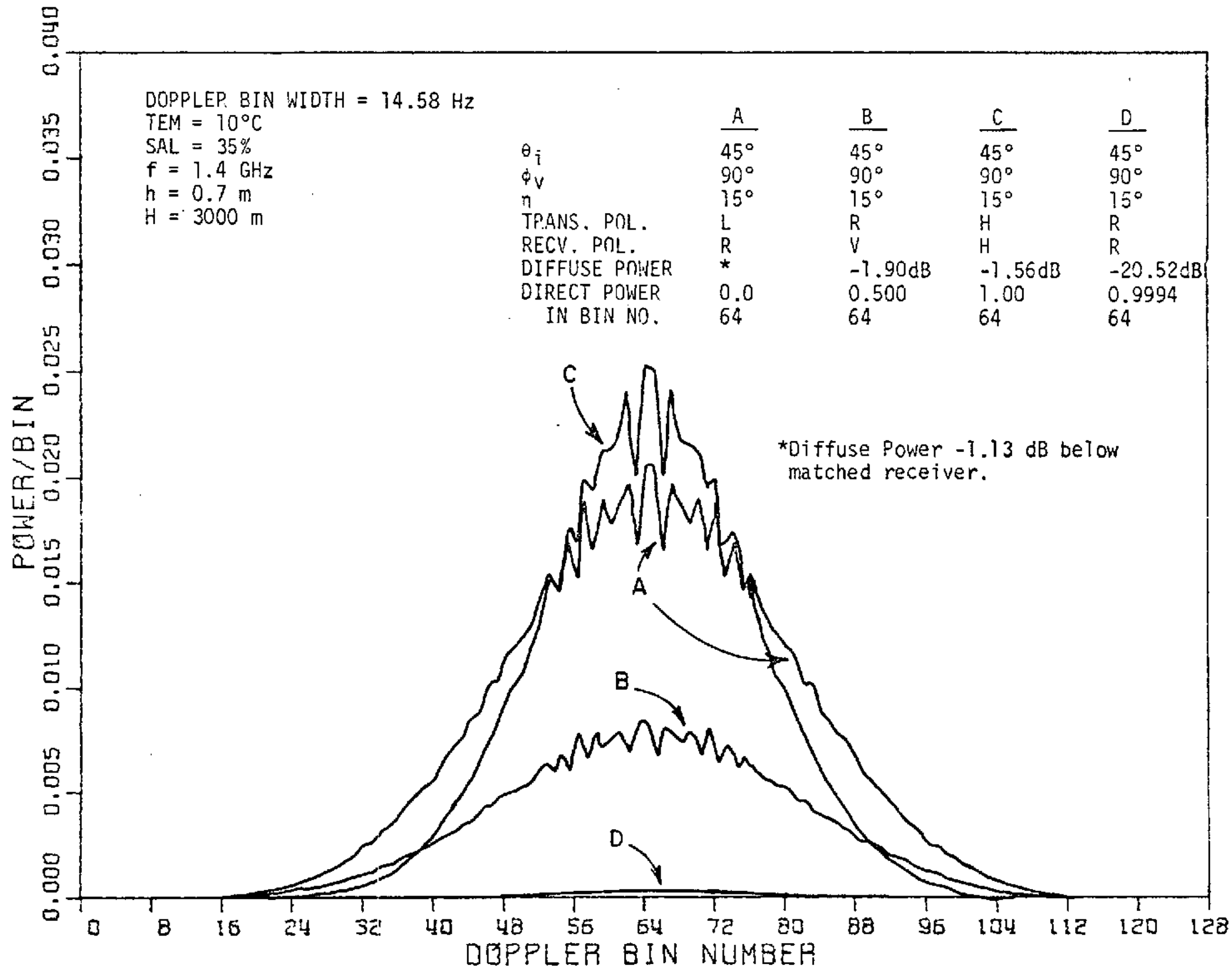


Fig. 8. Doppler spectrum with polarization as parameter. Note that the diffuse power in curve B is -1.90 dB below the direct power of 0.5, or -4.90 dB below a polarization matched receiver.

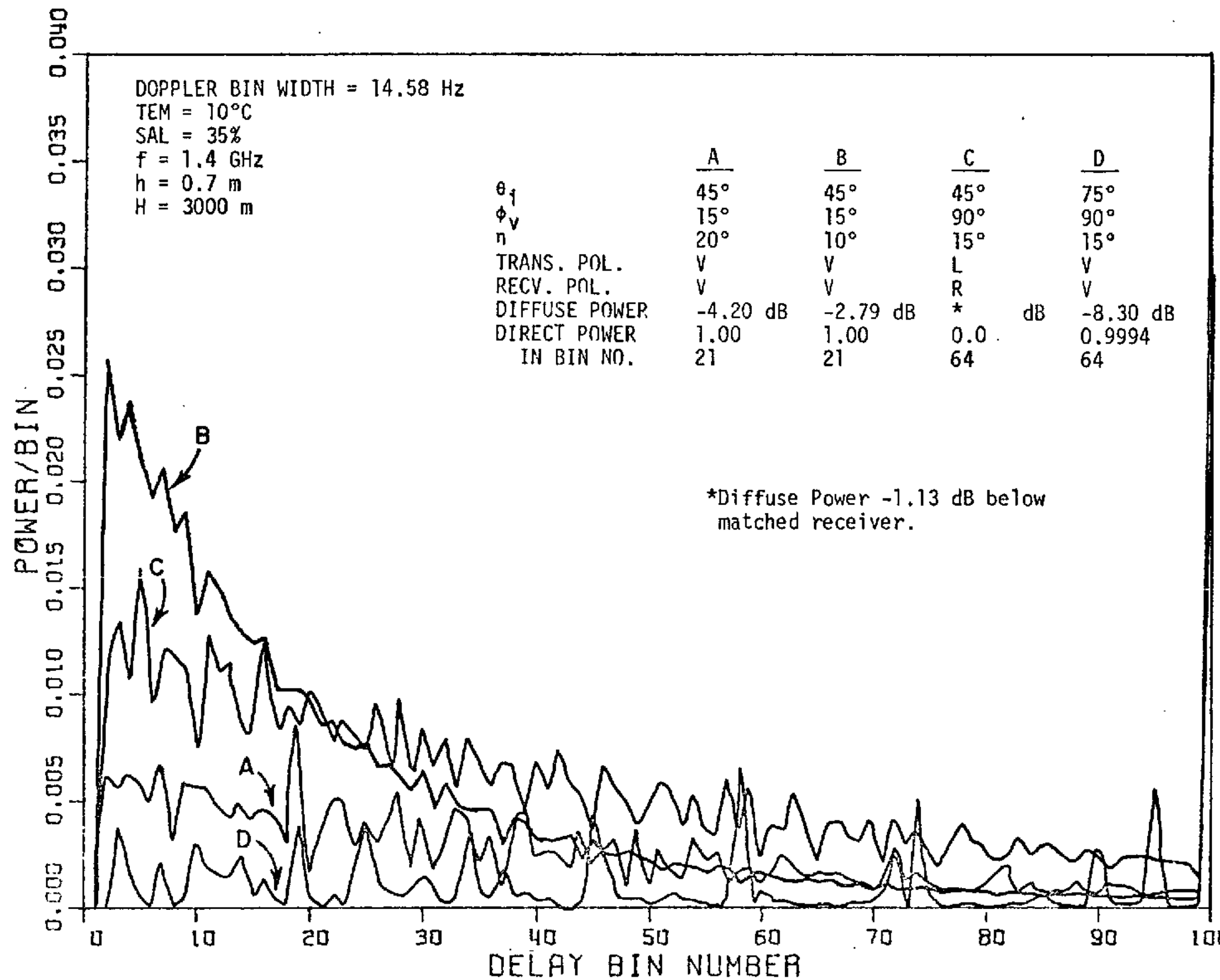


Fig. 9. Delay spectrum for various geometries. The delay bin width is 2 nanoseconds. A time delay of 0 (bin 1) is coincident with the specular signal, i.e., 14.1413 microseconds later than the direct signal for this geometry.

APPENDIX I

```
C      THIS PROGRAM GIVES THE POWER SPECTRAL DENSITY AND TOTAL
C     *POWER IN THE DIFFUSE MULTIPATH SIGNAL FROM SATELLITE
C     *TO AIRCRAFT OVER THE OCEAN, AND DISTRIBUTION OF POWER
C     *VERSUS TIME DELAY
       DIMENSION SPH(4),RS(4),CN(3,4),CNI(3,4),CNS(3,4),STHI(4)
       DIMENSION CTHI(4),CTHS(4),CTHR(4),STHS(4),CPHS(4),SPHS(4)
       DIMENSION PDEL(250),DEL(4)
       DIMENSION PD(250),R(4),PHS(4),DP(4),FD(4),YY(2)
       COMPLEX EP,EPS,RCOEF,ATH,BPH,CTH,DPH,DPH1
    50 FORMAT(10X,'TEM(C)=',F6.3,5X,'SAL(PPT)=',F6.3,5X,'F(GHZ)=',F6.3,
      *5X,'SEA SLOPE(DEG)=',F5.2,5X,'SEA HEIGHT=',F5.2,'M')
    51 FORMAT(10X,'THETA INC=',F8.4,5X,'PHI(PLANE VEL)=',F8.4,
      *5X,'PLANE HEIGHT(M)=',F12.6,5X,'VELOCITY=',F8.4,'M/S')
    52 FORMAT(10X,'EPS=',2F10.2,'  J')
    53 FORMAT(//,30X,'GRAPH OF POWER SPECTRUM    BIN WIDTH IS',F8.4,'HZ')
    54 FORMAT(22X,I3,11X,E12.6)
    55 FORMAT(//,10X,'DIFFUSE POWER IS',F8.4,'DB;COHERENT POWER IS'
      *,F10.4,'  DB BELOW DIRECT POWER OF ',F9.5,)
    56 FORMAT(/,10X,'MAX VALUES OF X AND Y IN INTEGRATION',5X,'XMAX=',
      *E12.6,5X,'YMAX=',E12.6,'      DELTX =  ',F10.4)
    57 FORMAT(10X,'ATH=',2F8.4,5X,'BPH=',2F8.4,5X,'CTH=',2F8.4,5X,
      *'DPH=',2F8.4)
    58 FORMAT(//,10X,'FINITE RANGE CASE',5X,'JJJ IS  ',I3)
    61 FORMAT(//,30X,'POWER VS DELAY;SPECULAR DELAY IS',F8.4,'MICROSEC')
    62 FORMAT(/,30X,'SPEC POINT IN BIN 2;BIN WIDTH IS ',I3,'  NANOSEC')
    64 FORMAT(/,10X,'DIRECT DOPPLER IS ',F10.3,' HZ;
      *SPECULAR DOPPLER IS ',F10.3,' HZ IN  BIN  ',I3)
C
C
C      INPUT DATA; TEM IS SEA TEMPERATURE IN DEG CENTIGRADE
C      SAL IS SALINITY IN P.P.THOU.    35 IS TYPICAL
C      ANG IS RMS SLOPE OF SURFACE IN DEGREES   15 IS TYPICAL
C      RUF IS RMS ROUGHNESS HEIGHT OF OCEAN IN METRES
C
C      F IS CARRIER FREQUENCY IN GIGAHERZ
C      H IS HEIGHT OF AIRCRAFT ABOVE SURFACE IN METRES
C      VEE IS AIRCRAFT VELOCITY IN METRES/SEC
C      PHV IS ANGLE OF VELOCITY FROM X AXIS;PLANE RECEDING IF PHV=0
C      THI IS ANGLE OF INCIDENCE IN DEGREES ; 90 DEG IS GRAZING
C      JJJ GIVES FINENESS OF INTEGRATION   JJJ=30 IS TYPICAL
C      RE IS RADIUS OF EARTH IN METRES
C      RSAT IS DISTANCE FROM SATELLITE TO SPECULAR POINT IN METRES
C      ATH,BPH,CTH,DPH ARE COMPLEX POLARIZATION STATES;SEE LATER COMMENT
C
C
       READ(8,-)TEM,SAL,F,ANG,VEE,RUF
       READ(8,-)THI,PHV,H,JJJ
       READ(8,-) ATH,BPH,CTH,DPH
C
C
       PI=3.14159265
       RE = (20./PI)*1.E6
       RSAT = 3.5E7
       WRITE(6,50) TEM,SAL,F,ANG,RUF
       WRITE(6,51) THI,PHV,H,VEE
```

```
      WRITE(6,57) ATH,BPH,CTH,DPH
      WRITE(6,58) JJJ
C
C
      ETA2=TAN(ANG*PI/180.)**2
C     INITIALIZE STORAGE BINS
      DO 42 IK=1,250
      PD(IK)=0.0
      PDEL(IK) = 0.0
   42 CONTINUE
      PT=0.0
      XP=H*TAN(THI*PI/180.)
      ZS = COS(THI*PI/180.)*RSAT
      XS = -SIN(THI*PI/180.)*RSAT
      REF = SQRT((XP-XS)**2 + (ZS-H)**2)
      IDX = 2
      RELSP = RSAT + SQRT(XP**2 + H**2)
      DELSP = (RELSP - REF)/300.
      H2=H*H
C
C     INCIDENT POLARIZATION DATA
C     ATH IS THETA COMPONENT OF SATELLITE POLARIZATION  THE THETA UNIT
C     VECTOR IS -IX*COS(THI) -IZ*SIN(THI);   BPH IS PHI COMPONENT WITH
C     PHI UNIT VECTOR ALONG -IY; ATH*ATHCONJ +BPH*BPHCONJ = 1.0
C     VERT POL ATH=(1,0) BPH=(0,0);HORIZ POL ATH=(0,0) BPH =(1,0)
C     RIGHT CIRC POL HAS ATH = (.707,0) BPH = (0, .707) .
C
C     CALCULATE COMPLEX DIELECTRIC CONSTANT OF SEA WATER
      EP=EPS(F,TEM,SAL)
      WRITE(6,52) EP
C
C     CALCULATE INCREMENTAL AREA
      DTH=SQRT(ETA2)/JJJ
      DX=2.*H*DTH/COS(THI*PI/180.)**2
      DY=DX
      XXMAX=0.0
      YYMAX=0.0
C     INTEGRATION STARTS
      Y=-DY/2.
      IA=1
   30 CONTINUE
      Y=Y+DY
      IF(Y.GT.YYMAX) YYMAX=Y
      Y2=Y*Y
      X=DX/2.
   31 CONTINUE
      IF(X.GT.XXMAX) XXMAX=X
C     TRUE POLAR COORDINATES OF SCATT POINT ON SPHERICAL EARTH
      STHR = SQRT(X*X+Y*Y)/RE
      CTHR = SQRT(1. - STHR**2)
      THR = ATAN(STHR/CTHR)
C     ZP IS DISTANCE OF SCATTER POINT BELOW TANGENT PLANE
      ZP = RE*2.*SIN(THR/2.)**2
      DO 99 J = 1,4
      XJ = X
```

```
      IF( J .GE. 3)   XJ = -X
      YJ = Y
      IF( J .EQ. 1 .OR. J .EQ. 3)   YJ = -Y
      SPH(J) = ATAN2(YJ,XJ)
      R(J) = SQRT((H+ZP)**2 + YJ**2 + (XJ-XP)**2)
      RS(J) = SQRT((XJ-XS)**2 + YJ**2 + (ZS+ZP)**2)
      CN(1,J) = STHR*COS(SPH(J))
      CN(2,J) = STHR*SIN(SPH(J))
      CN(3,J) = CTHR
      CNI(1,J) = (XP-XJ)/R(J)
      CNI(2,J) = -YJ/R(J)
      CNI(3,J) = (H+ZP)/R(J)
      CNS(1,J) = (XS-XJ)/RS(J)
      CNS(2,J) = -YJ/RS(J)
      CNS(3,J) = (ZS+ZP)/RS(J)
      CTHI(J) = 0.
      CTHS(J) = 0.
      CTHB(J) = 0.
      STHS(J) = 0.
      DO 77 K = 1,3
      CTHI(J) = CN(K,J)*CNI(K,J) + CTHI(J)
      CTHS(J) = CN(K,J)*CNS(K,J) + CTHS(J)
      CTHB(J) = CNI(K,J)*CNS(K,J) + CTHB(J)
77    CONTINUE
      STHS(J) = SQRT(1. - CTHS(J)**2)
      STHI(J) = SQRT(1. - CTHI(J)**2)
      IF (ABS(STHS(J)) .LE. .001 .OR. ABS(STHI(J)) .LE. .001) GO TO 78
      CPHS(J) = (CTHI(J)*CTHS(J) -CTHB(J))/(STHS(J)*STHI(J))
      SPHS(J) = -(YJ/ABS(YJ))*SQRT(1. - CPHS(J)**2)
      PHS(J) = ATAN2(SPHS(J),CPHS(J))
      GO TO 99
78    PHS(J) = 0.
99    CONTINUE
C     CALCULATION OF INCREMENTAL POWERS STARTS
      DO 40 II=1,4
      PHSCAT =PHS(II)*180./PI
      THINC =  ATAN(SQRT(1.-CTHS(II)**2)/CTHS(II))*180./PI
      THSCAT =  ATAN(SQRT(1.-CTHI(II)**2)/CTHI(II))*180./PI
C
C     THE ANTENNA PATTERN ANTP(THA,PHA)IS A FUNCTION OF THE POLAR COORDINATES
C     THA AND PHA FIXED IN THE AIRCRAFT. THE POLAR AXIS IS Z(VERTICAL)
C     AND THE X** AXIS IS ALONG THE VELOCITY VECTOR,FROM WHICH
C     THE ANGLE PHA IS MEASURED. THE ANTENNA POLARIZATION IS GIVEN AS TWO
C     COMPLEX FUNCTIONS OF THA,PHA.  FUNCTION CTH(THA,PHA) IS THE THETA
C     COMPONENT AND DPH IS THE PHI COMPONENT. CTH*CTHCONJ+DPH*DPHCONJ=1
C     VERT POL RECVR HAS C=(1,0) D=(0,0); HORIZ POL RECVR HAS C =(0,0)
C     D=(1,0); RIGHT CIRC POL RECVR HAS C=(0,.707)  D=(+.707,0)
C     THA = 180. - THINC
C     PHA = PHSCAT - 180. -PHV
      ANTP = 1.0
C     THE NEXT LINE CONVERTS UNIT VECTORS FROM AIRCRAFT TO SCATTER COORDS
      DPH1=-DPH
C
      CALL COEF(THINC,THSCAT,PHSCAT,EP,ATH,BPH,CTH,DPH1,
     *AL,TH,PH,CELI,DELS,RCOEF)
```

```
      DP(II)=ANTP*CABS(RCOEF)**2*PDEN(TH,PH,ETA2)*DX*DY/(4.*R(II)*CTHR*
     *R(II)*COS(TH*PI/180.))
C     DOPPLER SHIFT IN HZ FROM CARRIER IS FD*(F*1.E9)*(V/C)
      FD(II)=-SIN(THSCAT*PI/180.)*COS((PHSCAT-PHV)*PI/180.)
C     DELAY IN MICRO SEC REFERRED TO DIRECT PATH
      DEL(II) = (R(II)+RS(II)-RELSP)/(300.*DELSP)
   40 CONTINUE
      IF(IA.EQ.1) DP1=DP(1)
      IF(IA.EQ.1)IA=IA+1
C
C     CALCULATE BIN STORAGE OF POWER; 2**7 BINS NUMBERED 1 TO 129
      PAVG=0.0
      DO 41 IJ=1,4
      IBS=IFIX((1.+FD(IJ))*64.)+1
      PD(IBS)=PD(IBS)+DP(IJ)
      PT=PT+DP(IJ)
      PAVG=PAVG+ABS(DP(IJ))/4.
C     CALCULATE BINS FOR DELAY
      IBT = IFIX(1. + 1000.*DEL(IJ)/IDX) + 1
      IF (IBT .LE. 1)   IBT = 1
      IF (IBT .GE. 100)   IBT = 100
      PDEL(IBT) = PDEL(IBT) + DP(IJ)
   41 CONTINUE
C
      X=X+DX
      IF(PAVG.LT.1.E-3*DP1.AND.X.LT.3.*DX) GO TO 32
C     ABOVE TERMINATES INTEGRATION
      IF(PAVG.LT.1.E-3*DP1) GO TO 30
C     ABOVE TERMINATES X INTEGRATION AND STEPS TO NEXT Y
      GO TO 31
   32 CONTINUE
      FDO=-SIN(THI*PI/180.)*COS(PHV*PI/180.)
      FDD = COS(PHV*PI/180.)*(XS-XP)/REF
C     DOPPLER SHIFT IN HZ
      DDO = FDO*F*VEF*10./3.
      DDD = FDD*F*VEF*10./3.
      IBS=IFIX((1.+FDO)*64.)+1
C
C     AIRCRAFT ANTENNA PATTERN IN SATELLITE DIRECTION IS
C     ANTP = ANTP(THI,PP)
C     PP = 180. - PHV
C     RECEIVER POLARIZATION IN SATELLITE DIRECTION IS
C     CTH = CTH(THI,PP)
C     DPH = DPH(THI,PP)
C
      ANTP=1.0
C     DIRECT POWER FROM SATELLITE
      PDIR=ANTP*CABS(ATH*CTH+BPH*DPH)**2*(RSAT/REF)**2
C     THE RECEIVED POWER IN WATTS IS PDIR*POWER*GAIN*APERTURE/DIST
C     WHERE PDIR HAS BEEN CALCULATED, DIST=4*PI*RSAT**2 AND THE
C     OTHER VARIABLES ARE TRANSMITTERR POWER,TRANSMITTER ANTENNA GAIN,
C     RECEIVER COLLECTING APERTURE AND RANGE FROM SATELLITE TO SPECULAR POINT
C
C     CALCULATE THE COHERENT POWER
      CALL COEF(THI,THI,0.,EP,ATH,BPH,CTH,DPH)
```

```
     *AL,TH,PH,DELI,DELS,RCOEF)
      EX1 = 2.*((2.*PI*F*RUF*10./3.)*COS(THI*PI/180.))**2
      IF (EX1 .GE. 60.)  EX1 = 60.
      EX2 = EXP(-EX1)
      PSPEC = EX2*ANTP*(RSAT*CABS(RCOEF)/RELSP)**2
      PS = 10.*ALOG10(PSPEC/PDIR)
      PT=10.*ALOG10(PT/PDIR)
      WDOP = VEE*F/10.2
      WRITE(6,55) PT,PS,PDIR
      WRITE(6,64)DDD,DDD,IHS
      WRITE(6,56) XXMAX,YYMAX,DX
      WRITE(6,58)WDOP
      NN=1
      YMAX=.05
      YMIN=0.0
      DO 45 IL=1,128
      XX=FLOAT(IL)
      YY(1)=PD(IL)
      IND=IL-1
      CALL PLOT(XX,YY,NN,IND,YMAX,YMIN)
   45 CONTINUE
      WRITE(6,61) DELSP
      WRITE(6,62)IDX
      NN = 1
      YMAX = .05
      YMIN = 0.
      DO 46 IM = 1,100
      XX = FLOAT(IM)
      YY(1) = PDEL(IM)
      IND = IM-1
      CALL PLOT(XX,YY,NN,IND,YMAX,YMIN)
   46 CONTINUE
      CALL EXIT
      END
C
C     ***** SUBROUTINES AND FUNCTION SUBPROGRAMS *****
C
      SUBROUTINE COEF(THI,THS,PHS,EPS,ATI,BPI,CTS,DPS,AL,TH,PH,DELI,DELS
     *,RCOEF)
      COMPLEX ATI,BPI,CTS,DPS,RCOEF,RPAR,RPER,EPS
C     GIVEN THE INCIDENCE AND SCATTERING DIRECTIONS,THE POLARIZATION
C     STATE OF THE TRANSMITTER AND RECEIVER,AND THE COMPLEX DIELECTRIC
C     CONSTANT;THIS SUBROUTINE DETERMINES THE DIRECTION OF THE NORMAL
C     TO THE SURFACE REQUIRED FOR SUCH REFLECTION,THE ANGLE ALPHA
C     BETWEEN THE NORMAL AND EITHER THE INCIDENT OR REFLECTED
C     DIRECTIONS,AND THE COMPLEX REFLECTION COEFFICIENT.
C     ALL ANGLES REQUIRED IN DEGREES.
      PI=3.14159265
C     THI-THETA ANGLE OF INCIDENCE
C     THS-THETA ANGLE OF SCATTER
C     PHS-  PHI ANGLE OF SCATTER
C     ATI-THETA COMPONENT OF THE INCIDENT FIELD
C     BPI-PHI   COMPONENT OF THE INCIDENT FIELD
C     CTS-THETA COMPONENT OF THE SCATTERED FIELD
C     DPS- PHI  COMPONENT OF THE SCATTERED FIELD
```

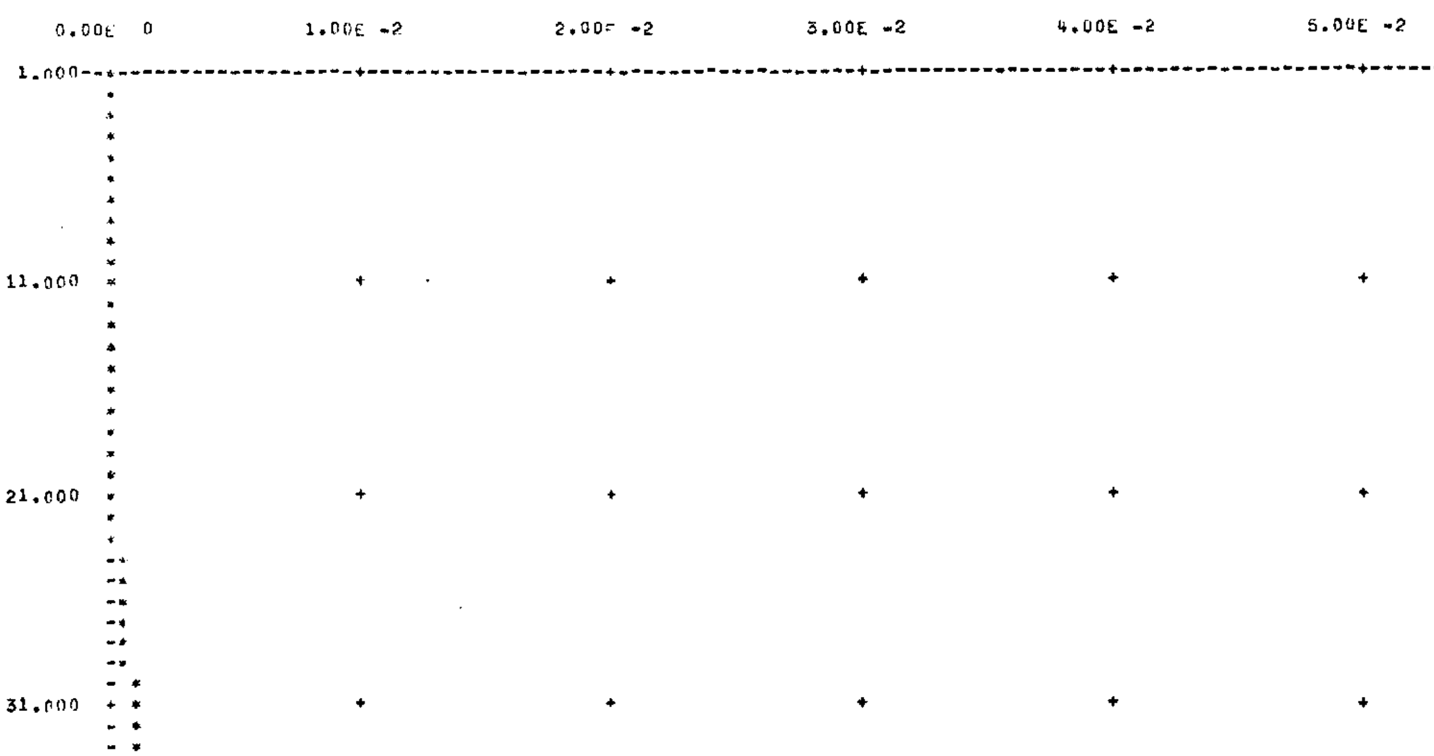
```
C      EPS-COMPLEX DIELECTRIC CONSTANT
       THI=THI*PI/180.
       THS=THS*PI/180.
       PHS=PHS*PI/180.
C      AL-ANGLE ALPHA BETWEEN INCIDENT DIRECTION AND NORMAL
C      PH-PHI ANGLE OF THE NORMAL
C      TH-THETA ANGLE OF THE NORMAL
       C2AL=-(SIN(THI)*SIN(THS)*COS(PHS)-COS(THI)*COS(THS))
       CAL=SQRT((1.+C2AL)/2.)
       AL=ATAN2(SQRT(1.-(CAL)**2),CAL)
       IF(AL.LE.1.E-3) GO TO 100
       PH=ATAN2(SIN(THS)*SIN(PHS),SIN(THS)*COS(PHS)-SIN(THI))
       CTH=(COS(THS)+COS(THI))/(2.*CAL)
       TH=ATAN2(SQRT(1.-CTH**2),CTH)
C      DELI-ANGLE DELTA INCIDENCE
C      DELS-ANGLE DELTA SCATTERED
       S2AL=SIN(2.*AL)
       CDELI=(SIN(THS)*COS(PHS)*COS(THI)+SIN(THI)*COS(THS))/S2AL
       SDELI=SIN(THS)*SIN(PHS)/S2AL
       DELI=ATAN2(SDELI,CDELI)
       CDELS=(SIN(THI)*COS(THS)*COS(PHS)+COS(THI)*SIN(THS))/S2AL
       SDELS=SIN(THI)*SIN(PHS)/S2AL
       DELS=ATAN2(SDELS,CDELS)
       GO TO 90
   100 CONTINUE
       AL=0.0
       TH=THI
       PH=PI
       DELI=0.0
       DELS=0.0
       CDELI=1.
       SDELI=0.0
       CDELS=1.0
       SDELS=0.0
    90 CONTINUE
C      RPAR-FRESNEL REFLECTION COEFFICIENT FOR PARALLEL POLARIZATION
C      RPER-FRESNEL REFLECTION COEFFICIENT FOR PERPENDICULAR POLARIZATION
C      RCOEF-COMPLEX REFLECTION COEFFICIENT
       RPAR=(EPS*COS(AL)-CSQRT(EPS-SIN(AL)**2))/(EPS*COS(AL)+CSQRT(EPS-SI
      *N(AL)**2))
       RPER=(COS(AL)-CSQRT(EPS-SIN(AL)**2))/(COS(AL)+CSQRT(EPS-SIN(AL)**2
      *))
       RCOEF=-RPER*((RPI*CDELI-ATI*SDELI)*(DPS*CDELS+CTS*SDELS))+RPAR*((A
      *TI*CDELI+RPI*SDELI)*(CTS*CDELS-DPS*SDELS))
       AL=AL*180./PI
       TH=TH*180./PI
       PH=PH*180./PI
       THI=THI*180./PI
       THS=THS*180./PI
       PHS=PHS*180./PI
       DELI=DELI*180./PI
       DELS=DELS*180./PI
       RETURN
       END
       FUNCTION FDEN(TH,PH,S2)
```

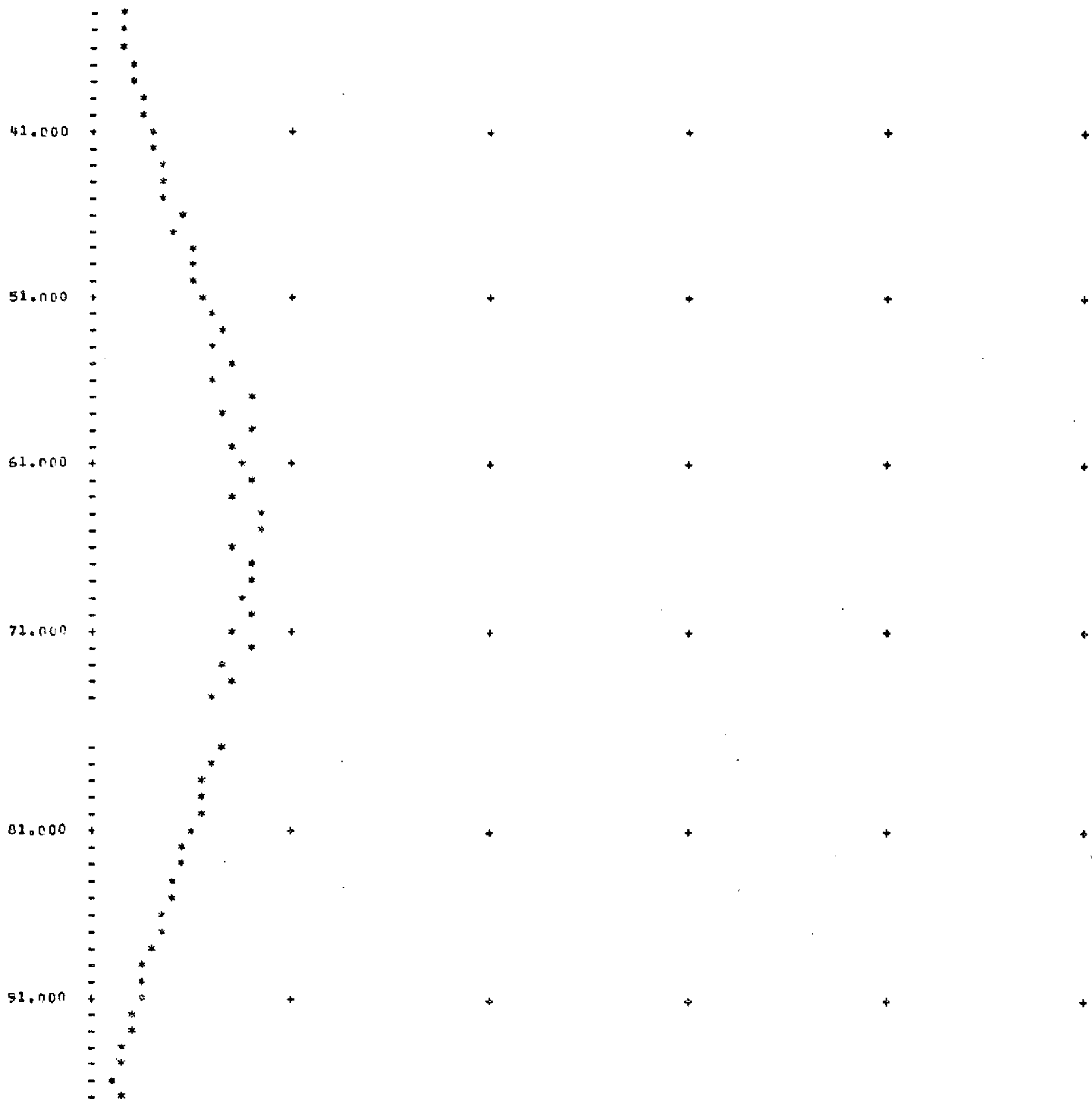
```
C     THIS FUNCTION CALCULATES THE VALUE OF THE PROBABILITY DENSITY
C     FUNCTION (PDEN) FOR A SURFACE NORMAL WHERE TH IS THE THETA-ANGLE
C     IN DEGREES AND PH IS THE PHI-ANGLE IN DEGREES OF THE SURFACE NORMAL
C     AND S2 IS THE MEAN SQUARE SLOPE. THE PDF IS GAUSSIAN.
      PI=3.14159265
      TH=TH*PI/180.
      A=-TAN(TH)*TAN(TH)/(S2*(1.+2.*S2))
      IF(ABS(A).GT.60.) A=-60.
      PDEN=COS(TH)*EXP(A)/(PI*S2)
      TH=TH*180./PI
      RETURN
      END
      COMPLEX FUNCTION EPS(F,T,S)
C      COMPLEX DIELECTRIC OF SEA WATER EPS (STOGRYN MTT)
C      T IN CENTIGRADE, F IN GIGAHERTZ, S IS SALINITY IN P.P.THOU.
       REAL N
C      NORMAL SEA WATER RANGES FROM 20. TO 35. PPT
       N = S*(.01707 + .00001205*S + S*S*4.058E-9)
       C = 25. - T
       AA = 1.649E-5 - C*(1. - .1*C)*2.551E-7
       D = EXP(C*(.02033+.0001266*C+C*C*2.464E-6-S*AA))
       SIG = S*(.162521-.0014619*S+.00002093*S*S -S*S*S*1.282E-7)/D
       DD =.11109 - .003824*T + .0000694*T*T
       W = (1. + .001463*N*T - .049*N - .02967*N*N)*DD
       E0 = (1.-.2551*N+.0515*N*N)*(87.74-.4008*T+.00094*T*T)
       EPS = 4.9 +(E0-4.9)/CMPLX(1. , W*F) - CMPLX(0.,17.95*SIG/F)
       RETURN
      END
      SUBROUTINEPLOT( X,Y,N,IND,YMAX,YMIN)
      DIMENSIONM(119),YLABEL(6),Y(10),MARK(10)
      DATA MARK(1),MARK(2),MARK(3),MARK(5),MARK(6),MARK(7),MARK(8),
     2MARK(9),MARK(10),MARK(4)/1H*,1H.,1HI,1HO,1HN,1HH,1H1,1HZ,1H-,1HX/
      DATA IBLANK,NOPT,IPLUS/1H ,1H$,1H+/
      IF(IND)1,1,11
    1 WRITE(6,3)
    3 FORMAT(//25X,'ORDER IN WHICH PLOT SYMBOLS ARE USED *.IXONH1Z-',
     *//30X,'THE SYMBOL ($) INDICATES OFF-SCALE DATA',//)
      DO7J=9,119
    7 M(J)=MARK(10)
      NCOUNT=10
      SCALE=100.0/(YMAX-YMIN)
      LLL=(-YMIN*SCALE)+11.5
      DO8J=1,6
      R=J-1
    8 YLABEL(J)=R*20.0/SCALE+YMIN
      WRITE(6,9) (YLABEL(I),I=1,6)
    9 FORMAT(6X,1PE9.2,5(1PE20.2) /  )
      GOTO132
   11 NCOUNT=NCOUNT+1
      DO99J=1,119
   99 M(J)=IBLANK
      IF(LLL.GE.11.AND.LLL.LE.110)M(LLL)=MARK(10)
      IF(NCOUNT-10)133,132,133
  132  DO89J=11,111,20
   89  M(J)=IPLUS
```

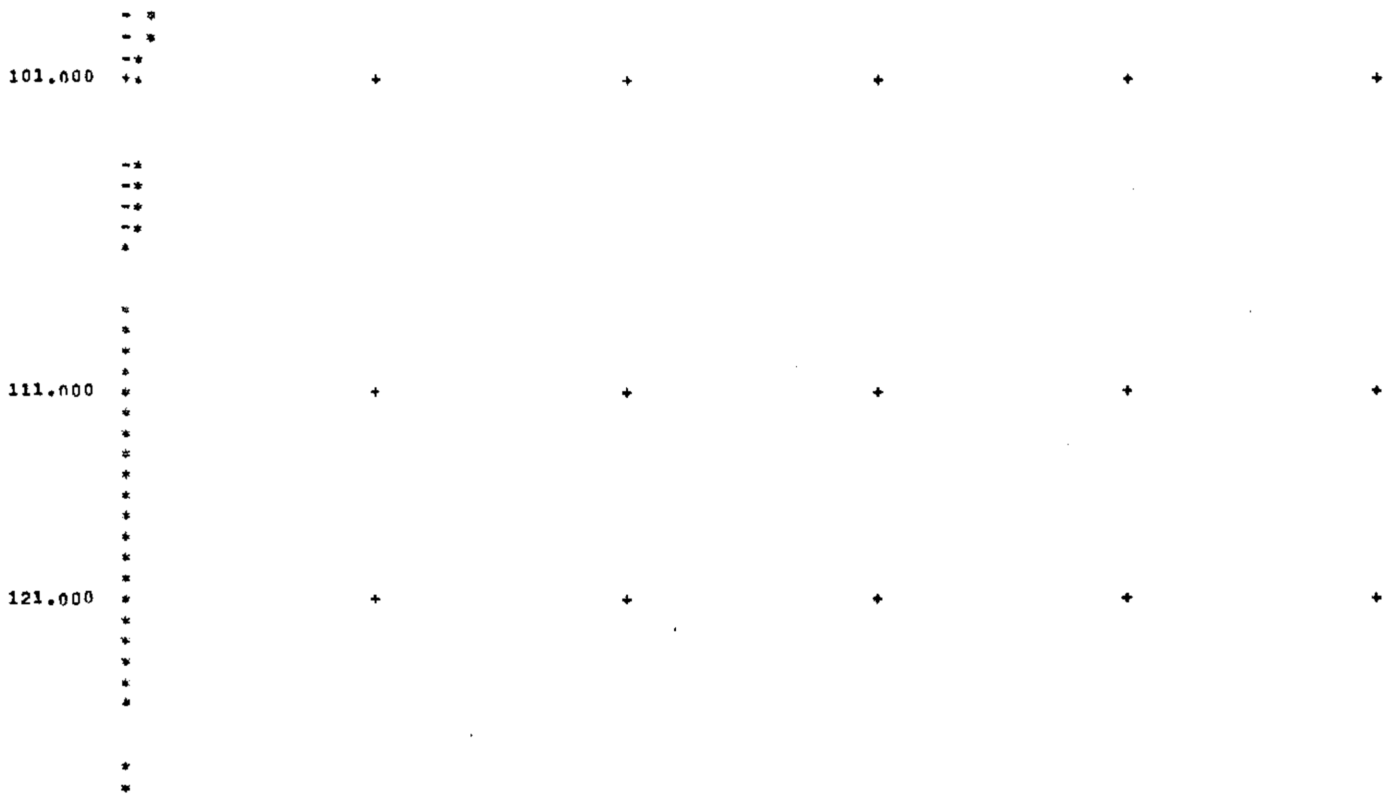
```
   13 DO20J=1,N
      L=(Y(J)-YMIN)*SCALF+0.5
      IF(L)14,17,17
   14 IF(L+10)15,16,16
   15 M(1)=NOPT
      GOTO20
   16   LL=L+11
       M(LL)=MARK(J)
      GOTO20
   17 IF(L-108)18,19,19
   18  LL=L+11
       M(LL)=MARK(J)
      GOTO20
   19 M(119)=NOPT
   20 CONTINUE
      IF(NCOUNT-10)21,25,21
   21 WRITE(6,24) (M(J ), J=1,119)
   24 FORMAT(1X,119A1)
      GOTO27
   25 WRITE(6,26) (X,(M(J),J=9,119))
  26  FORMAT(1X,F8.3  ,111A1)
      NCOUNT=0
   27  CONTINUE
      RETURN
      END$
```

APPENDIX II
SAMPLE OUTPUT (Curve B Fig. 8)

```
TEM(C)=10.000      SAL(PPT)=35.000      F(GHZ)= 1.400      SEA SLOPE(DEG)=15.00      SEA HEIGHT=  .30M
THETA INC= 45.0000      PHI(PLANE VEL)= 90.0000      PLANE HEIGHT(M)= .300000E  4      VELOCITY=200.0000M/S
ATH=   .7070  0.0000      BPH=  0.0000   .7070      CTH=  1.0000  0.0000      DPH=  0.0000  0.0000


FINITE RANGE CASE      JJJ IS   30
EPS=     71.57    -56.03  J


DIFFUSE POWER IS -1.8943DB;COHERENT POWER IS -262.3041  DB BELOW DIRECT POWER OF    .49985

DIRECT DOPPLER IS      -.000 HZ;                    SPECULAR DOPPLER IS      -.000 HZ IN  BIN   64

MAX VALUES OF X AND Y IN INTEGRATION     XMAX= .310285E  5     YMAX= .519821E  4     DELTX =   .1072E  3

                    GRAPH OF POWER SPECTRUM    BIN WIDTH IS 14.5833HZ

               ORDER IN WHICH PLOT SYMBOLS ARE USED *.IXONHIZ-

                    THE SYMBOL ($) INDICATES OFF-SCALE DATA
```




41.000
51.000
61.000
71.000
81.000
91.000

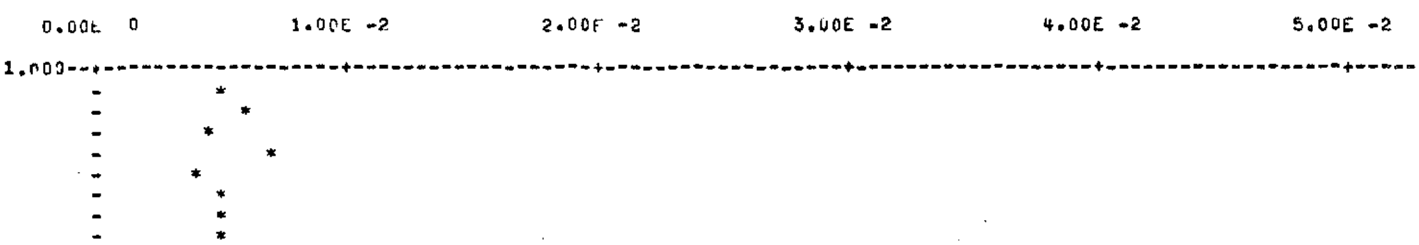


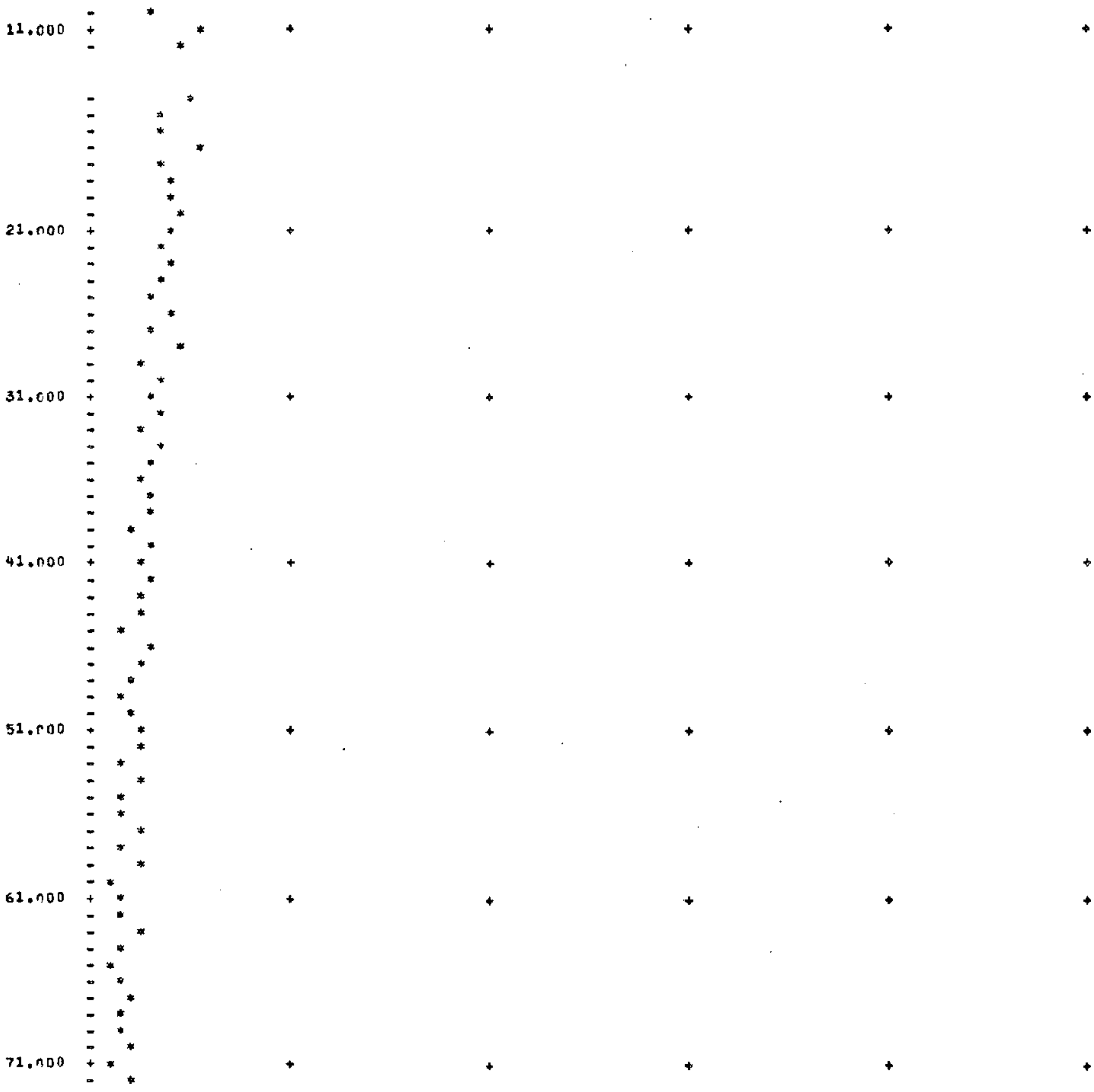
POWER VS DELAY;SPECULAR DELAY IS 14.1413MICROSEC

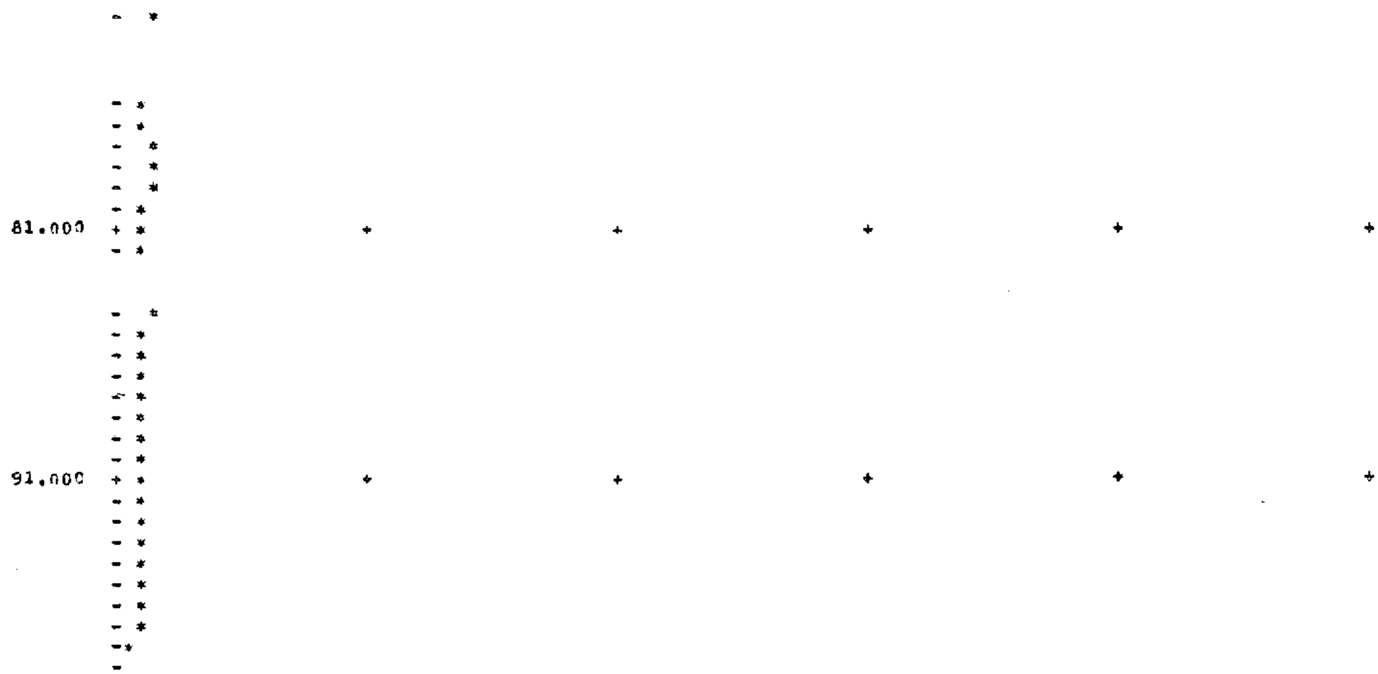
SPEC POINT IN BIN 21BIN WIDTH IS 2 NANOSEC

ORDER IN WHICH PLOT SYMBOLS ARE USED *.IXONH1Z-

THE SYMBOL ($) INDICATES OFF-SCALE DATA




11,000
21,000
31,000
41,000
51,000
61,000
71,000


81.000
91.000

REFERENCES

1. Painter, J. H., Gupta, S. C., and Wilson, L. R., "On the Technology of Aerospace Communication in Multipath, IEEE Nat. Telecomm. Cont., Houston, Texas, December 1972.

2. Durrani, S. H. and Staras, H., "Multipath Problems in Communications Between Low Altitude Space Craft and Stationary Satellites," RCA Review 29, 1968, p. 77.

3. Gager, D. R., "Signal Fading Characteristics due to Reflection From the Sea, Marconi Rev., First Quarter, 1971, pp. 27-48.

4. Ruck, G. T., Barrick, D. E., Stuart, W. D., and Krichbaum, C. K., Radar Cross-Section Handbook, Plenum Press, New York, 1970.

5. Phillips, O. M., The Dynamics of the Upper Ocean, Cambridge University Press, 1966.

6. Stogryn, A., "Equations for Calculating the Dielectric Constant of Saline Water," IEEE Trans. MTT, 1971, pp. 733-735.